Sefer ha-Ḥeshbon ve-ha-Middot

ספר החשבון והמדות, "The Book of Reckoning and Measures/Geometry"

[The Book of Arithmetic and Geometry]

(15th c. (composition; this is a later manuscript copy))

by Mordecai ben Eliezer Comtino (מרדכי בן אליעזר כומטינו), 1420–1487, Constantinople — Rabbanite mathematician/astronomer (teacher of Karaite scholars)

Translation by Scott Weisman

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"*Sefer ha-Ḥeshbon ve-ha-Middot* (ספר החשבון והמדות, "The Book of Reckoning and Measures/Geometry")," published in 15th c. (composition; this is a later manuscript copy), is in the public domain. All translations, original additions, including illustrations, commentaries, and summaries, are copyright.

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Translator Version 2

Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

An instructional treatise on arithmetic and practical geometry by Mordecai ben Eliezer Comtino (Constantinople, 15th c.), one of the leading Rabbanite scholars of Byzantine Jewry. Combines numerical method (worked arithmetic examples) with practical geometry (labeled construction diagrams). This is the witness in BnF Hébreu 1031; no English translation is known.

A note on the translation process

This translation was produced with the translation-pipeline. See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

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[Foliation: fol. 26r begins here — marked "26" in the top-left margin. fol. 25, which precedes the incipit, is not included in this opening; per the source note it is blank.]

Prologue

Incipit (fol. 26r, line 1, verbatim): אמר מרדכי בר אליעזר [honorific abbreviation, uncertain: כמ״ר or similar — ?] כומטינו [blessing formula, uncertain ?] הקושטנטיני

Said Mordecai ben Eliezer [honorific ?] Comtino [כומטינו], the Constantinopolitan [הקושטנטיני]:

Since the first science [להיות שהחכמה הראשונה] [?] — from which [derive] the rest of the existents [הנמצאות] [?] — and as [each thing] increases in rank one after another [וכאשר תרבה … מדרגה], we find each one according to the perfection proper to it, up to its final end [לפני שלמותו הראוי לו עד תכליתו], and each thing was given its due [ונתן לכל דבר חקו]. [The remainder of this philosophical opening is in a very compact cursive and is only partially legible; the readable skeleton is given here, and the connective argument is left unreconstructed rather than guessed. ?]

[Several lines follow on the perfection and ordering of things "according to the nature of our world" — כטבע עולמינו recurs — but the run is largely unreadable at this resolution. ? /]

[Motivation, fol. 26r foot:] … the prices of goods [מחירי הדברים] are attained at every season [?] … and this matter lies in arithmetic and in geometry [בחשבון ובמדות] [?] … so that, once the two matters are established [?], one may know for each acquisition [קנין] its price [מחירו] by its parts [בחלקיו] [?] … lest the value [היוקר] be voided [?] within the fractions [החלקים] in which the whole is contained, and the reckoning come out poorly for lack of knowledge of them in arithmetic [להעדר ידיעתם בחשבון] [?] … And after [?] we [sought] to draw near to those things which are not [illumined?] by the nature of our world, we distanced from them the gold [?] and relied upon [?] the acquisition [הקנין] that we acquire for our use [לתשמישינו] [?]. [This paragraph is read with low confidence; several words flagged ?]

We saw fit to compose this book [ראינו לחבר זה הספר] so as to spare [שתחסיר] this [difficulty] [?] for people, and that exactness [הצדק] be [attained] [?] — for in them [in these matters] are fractions [חלקים]. [It comprises] two books [שני ספרים]: **the first book speaks of the subject of arithmetic** [הספר הראשון מדבר בענין החשבון], **and the second book speaks of the subject of geometry** /

measures [והספר השני מדבר בענין המדות]; and he gave precedence [והקדים] [to the Book of Arithmetic over Geometry —]

[fol. 26v begins here.]

— because the science of arithmetic naturally precedes the other kinds of the mathematical sciences [כי חכמת החשבון קודמת בטבעה לשאר מיני החכמות הלמודיות], as [we explained / began] [?].

The First Book [is divided] into four parts [חלקיו … לארבעה חלקים]:

- The first: to add number to number, and to subtract number from number [לקבץ מספר עם מספר ולחסר מספר ממספר] — i.e., addition and subtraction.
- The second: to multiply number by number, and to [double / raise] number upon number [לכפול מספר על מספר ולהוסיף (?) מספר על מספר] — i.e., multiplication [second operation ?].
- The third: to divide number by number [לחלק מספר על מספר] — i.e., division.
- [The fourth:] on the fractions [the "broken numbers," הנשברים] — to combine [לצבר] one part with others, and to know the correct method for combining them [?], so that the student not stumble in them [שלא יכשל בם הלמוד].

The Second Book [likewise divided] into four parts [חלקיו … לארבעה חלקים]:

- The first: on linear and planar measures [במדות הקַוִּיות והמשתטחות], such as circles, triangles, and quadrilaterals [כמו העגולות והמשולשים והמרובעים] and [solids? העולם ?].
- The second: on [their subdivision — בחלקם ?].
- The third: on [the measure of ?] … [curved / rounded? הנוסבות ?] figures.
- And [the fourth] reaches the [treatment of] fractions [והגיע בחלקים]; and from here [?] … so as to teach the student knowledge [לתלמד דעת].

[The three lines describing the Second Book's four parts are read with reduced confidence — the technical geometric vocabulary is legible only in part; ? flags mark the uncertain terms]

Chapter 1

Body heading (fol. 26v, displayed in enlarged script): הפרק הראשון מהחלק הראשון מהספר הראשון — **The First Chapter, of the First Part, of the First Book.**

And it is [והוא] the addition of number upon number [קבוץ מספר על מספר], [set before] the remaining parts [?].

That we may make known [שנודיע] [?] this, we have prefaced [?] — and we say: inasmuch as it bears premises of its own [?] and rests upon [foundations] of its own [?]; if it has premises of its own, [it is] because each of the [numbers] is compounded of this and of the [substance / unit] proper to it [מורכב מזה ומה… המיוחד לו], and the thing which is compounded from this thing is [derived] from that from which it is compounded [?] …

[The remainder of Chapter 1 as it appears on fol. 26v — the opening of the addition procedure — turns on place-value (מדרגה, "rank/place") and the written digit (רושם, "mark"): it speaks of gathering the marks by their ranks (… נקבצן מדרגה) and of one rank not being carried onto another rank [?]. The passage is in the same compact hand and is only partially legible at this resolution; the mathematical argument is NOT reconstructed here beyond the terms actually read. Transcribe-and-flag for further at higher resolution. ؟]

Cheshbon u-Midot — Comtino (BnF Hébreu 1031)

Chapter 1 of the First Part of the First Book — Addition (continued)

[the beginning of this sentence lies in the prior page] and not according to their [?]. Therefore, on examination, it is proper to give precedence to what is prior — the particular over the general [?] — for its sum descends [reduces] to the general; and if [?]… [it is] naturally prior among them [?], then whatever rises [in value] rises [first], and nothing is diminished; and when [?]… it too rises. And you should know [?] which one it was proper to give precedence to among them [?].

[Enlarged rubric-style phrase, uncertain reading — likely an emphasis marker, not a numbered heading: *ורצינו* [?] "And we saw fit"**]** to preface by mentioning the principles [?] that [govern] all numbers and their general classes: for from **one** to **nine** they complete [the first order of units], while the **tens** and upward are called "wholes" (general ranks) [?]. All the tens [and] the hundreds and the thousands and the myriads (רבבות) and the rest — [these] are called "wholes" [?]. But none of them is composed of a single [?]… only [?]… And these are represented by nine figures [numeral figures], which are:

1 2 3 4 5 6 7 8 9

[These nine figures are written out in the line; the source uses gubar-style Hindu-Arabic numeral forms.] And we did not make do [?] with... [assigning them] to the alphabet [i.e., to the Hebrew letters used as numerals], because [?]... to each one of them a distinct [dedicated] figure [is given], which is not the case with these letters [of the alphabet] [?].... [It is] by nature [?]... [and] the figures were agreed upon [?] for each shape as they [the practitioners] settled them; and likewise in the manner of the first [?]... and the rank [?]... Just as [?] there is a figure for them, so too a person could devise other letters [?]... units [?]... but [?]... And [?] the letters of the alphabet [?]... rather [they are] agreed-upon signs [?] without [?]... like these. But the masters of arithmetic have already become accustomed to use these [figures].

[Enlarged rubric-style phrase, uncertain reading — possibly *הפרק הזה* [?] "this chapter" as an emphasis marker; a numbered chapter heading cannot be confirmed here.] It is proper to make known that the numbers which are from one to nine are called "particulars" (units) [?];... the tens and upward are called "wholes." All the tens and the hundreds and the thousands and the myriads and the rest [?]... are called "wholes" [?] — all of them; but there is none composed of one [alone] [?], and there is only [?]... a whole within a whole, like one [?]; and likewise whatever comes after [?]... up to [?]... [and back] to the units [?]. Therefore it is proper to repeat [?] the count so as to reach [?] the [several] ranks that we have counted [?] — [in the order] in which the "whole" [tens-place value] descends [is placed] [?] from the first [rank], and after it the [next] whole [?]... from it; and likewise upon the unit [?]...

[Reading of this passage is genuinely difficult on the scan, even under magnification; the mathematical throughline — the naming of the numeral figures, "units" versus "wholes," and the ordering of the ranks — is secure, but many connective words are uncertain.]

that they be assigned [?] to the units [?]. Furthermore [?], when [several] numbers issue forth [?] to be combined [?],... one [?] number [?]... so that they preserve [?] this order. And there shall be arranged [?] each "whole" according to what corresponds [?] to it: each myriad like [with] the myriad, and the thousand like the thousand, and the hundred like the hundreds, and the units like the units. And we call the [rank of the] units the **first rank** (מדרגה ראשונה); the tens, the **second rank**; the hundreds, the **third rank**; and the thousands, the **fourth rank** — and so on without limit [?]. And if there be in your hand a number that lacks [?]... [a value] in [one of] its ranks — for example, that you have in hand

thousands and tens and units, and behold the [?]... the hundreds are missing — [the rule] tells you [?] that you shall not write the tens in the rank proper to them [?], but rather [skip over] [?] the [place of the] hundreds, and in the place of the hundreds you shall write a **circle** (גלגל) [i.e., a zero] [?]...

[fol. 27v — no running header]

it serves as [?] the **sifra** (סיפרא) [the Arabic name for zero]; and in the arithmetic [tradition] of the Greeks [?]... and in the **Latin** tongue [it is called] **"nulla"** [Latin: *nulla*, "none/nothing"] [?] — that is to say, that here is a starting-place, and there is nothing [no value] [?] with it [in that rank]. And after the "whole," [you place] the tens near [it] [?] in their [proper] place; and thus the ranks shall be, each one [aligned] with its counterpart, upon a straight line.

And when you wish to **add** all the numbers [arranged] in rows [columns] [?], [beginning] from [?] one [?]... [proceeding] [?] from the units always; and likewise upon a straight line, [starting] with the units, which are their beginnings [?], in all the rows [?]. And if they [the units] do not reach ten, write what you find [as the total] [?]... and take the whole [total] as [belonging to] the units [and place it] below. And if they reach more than ten, then for every ten [carry one], and [in place of] each ten write [a mark] [?]... [and] carry it separately [?] into [the rank of] the tens. Whatever you find [beyond] the tens are the "dots" (נקודות) [carry marks] with which you began [?]; and those that remain, which did not reach a full ten, write [?]... below [in the] units. Then do likewise in the row of the tens together with the dots [carry marks] that were written [carried up] [?]. And if [?]... the tens [?]... and ten [?], and the six [?]...

Worked example:

```
 1 2 1 5 (with carry-dots marked above)
 + 2 3 2 2
 ─────────────
 3 5 3 7
```

[The first addend, 1215, is stated in the prose as *אלף ומאתים וחמשה עשר* ("one thousand two hundred and fifteen"); the sum, 3537, is likewise given in the prose in words. The second addend (2322) is the difference; its written form on the page is consistent with 2 3 2 2. Small dots above the top row are the carry marks (נקודות) the text describes. This example is read with reasonable confidence.]

because [?] these did not reach ten [in that column], [so] according to [?] what I find I mark [?] the rows; and if they reach a [full] ten [or more], according to the amount [number] of tens [carry that many dots].... below [?], when we carry [?]

dots into the other [rank] [?], and the figure [?]... one [?], [those that] did not reach [a full unit of] the rows [?]... according to [?] the rank of the tens; and likewise upon this reckoning [?]. And if so, so shall the measure [total] be [written] below, [in] each one of the ranks [?] — [what is] compounded [carried] beyond ten [going up], and the remainders from them [staying] in the [same] rank; likewise [for] the others [?]... in each [?] way [manner], until there rise up from them [into the rank] above, and in each rank according to its [own] rank [?].

[Grouping many addends:] If the numbers to be added [?] were many [more] rows [?] than [a manageable] number, [you may divide them] into three parts, or into five, or into [?]... or into [?] — according to the abundance of the numbers [?]; and you make of each part [a partial sum] [?], and add [?] the rows [?]... the greater [?]... Then we add up [?] the total [?] of all the parts,... like this, from what is found [?]... and the [grand] sum [?] shall be [made] from all the parts together [?]... the first [?]...

[Second worked example — introduced by the prose "to add [the number] one thousand two hundred and fifteen [?]... and three hundred [?]... and twenty [?]" — the addends and sum are laid out in stacked numeral grids on fol. 27v.]

```
 1 2 1 5
 2 3 2 2
 9 6 5 7
 8 5 6 3
 ———————————————————————————————————————
 2 1 7 5 5 21755 [recte 21757? — the four addends sum to 21757; the
manuscript's written sum reads 21755, a units-place scribal slip]
```

[This second grid stacks four addends with a summed total and carry-dots. The individual figures are now recovered (1215 + 2322 + 9657 + 8563), with the written sum reading 21755 and carrying a units-place `[recte 21757?]` scribal-slip note (the four addends sum to 21757). No longer a]

and behold, the [rule of] adding [?] the [several] numbers [?]... the last [?] of the numbers [?] in their ranks [?], and thus upon [?] this order [?]. And we find [?]... seventeen [?]: we carried [?] the ten of them [up] [?]... and seven remained [?]. And we add [?] each figure [?]... the last [?] of the numbers [?]... and thus [?] the descending [order] straight, upon [?] this order [?] with the [?]... the "wholes" [?]...

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fol. 28r

there, in the sum of the tens that we had extracted from the units. From the units we had found a sum of fifteen, so we extracted a further ten from them and wrote a carry-dot for it in the column of the tens rank, while the five that remained we wrote above, in their own rank, that of the units. In the same way, when we came to the hundreds we did as before [?]: we set the carrying-out into order against [?] the tens that we had extracted. From the tens we extracted a ten and wrote for it a carry-dot in the rank of the hundreds, and the [remainder] that was left we wrote in the rank of the hundreds [?]. Likewise, when we came to the thousands we proceeded in the same manner, alongside the carry-dot that we had written; from the tens [that stood there] we extracted the tens — which were three tens — and wrote for them two carry-dots in the rank of the myriads, and the one that remained we wrote [in the rank of] the thousands. And we know from this that they are two myriads and one thousand seven hundred and fifty-seven [numeral uncertain — [?]].

מאזנים [moznayim, "the Balance" — a verification check], to know whether your addition can be correct, by way of [detecting] error.

[Margin, small hand: "write out the whole working" [?] — further reading requires further verification.]

Treat [the digits] as if they were units: gather them and cast them out nine by nine [i.e. subtract nines repeatedly]. Whatever remains, keep; and if nothing remains, mark in your mind a cipher [zero]. Then go [?] to the ranks of the total — here too they are [treated as] units — and cast them out, nine by nine. If the remainders you obtain are the same as at first, it is possible that your computation is correct; but if they do not come out as at first, know for certain that you have erred, and go back and compute properly.

Another balance for you [a second check]. Set the numbers down and [keep] every rank as it is: all the thousands as thousands, the hundreds as hundreds, the tens as tens, and the units as units — you do not gather them as though they were all units, as you did in the [check by] nines [?]. After you have gathered them, cast them out seven by seven; whatever remains, keep, and if nothing remains, keep in your mind a cipher. Then come to the ranks of the total: take the whole — the

largest [rank] — together with what you have gathered, [treating them] as if they were tens, and [likewise] what you gathered for the units [?]. Cast them out seven by seven; what remains, reckon as tens and join it with what you gathered [in the next rank down]; and what then remains for the units, cast out seven by seven. Continue so until you reach the units. And when you reach the thousands, cast them out seven by seven, and look at what is left: if it is equal to what you kept, it is possible your computation is true; and if it is not equal, know for certain that you have erred, and go back and compute properly. So too, if you have divided [?] the [units/portions?] into parts under [their headings?] [?], as I have combined [?] and made into tenths [?] the sum of the parts [?] — the balance is afterward reckoned in the same way, just as was stated for the procedure of addition, in the manner you already know.

The Third Chapter [הפרק השלישי]. If we wish to subtract one number from another, the whole of the number from which one wishes [to subtract] must be greater than the number that is to be subtracted from it. This holds for whole numbers [?]; not so for [fractions?] [?], where the [order of the] ranks makes no difference [?]. And always write the number from which you subtract [on top]; beneath it write the number you are subtracting, each rank against its like. If a rank is missing, write a cipher in its place.

fol. 28v

And always take care with the cipher, that you do not forget it; rather, wherever a required rank is missing [set a cipher]. Then begin to subtract from the units. If the units in the [subtrahend] are more than those [above], [borrow]; [otherwise] subtract them from them, and write the remainder below, in the units rank. And if they are fewer [i.e. the upper units are too few], go up to the tens that are there and take one ten as a unit [?], remove it from the tens, and join it with the units; then subtract from them what you need, and write the remainder below, in the units rank. Continue thus, subtracting the tens from the tens: if what stands above [?] is more than the tens you must subtract, subtract from them and write the remainder in the rank of the tens; and if they are too few, go back up to the hundreds and do according to the rule, until you reach [the largest rank]. However, if you had [a cipher?] [?]... And when you wish to subtract from a [rank] what stands below it [?], take the whole [unit] that is above it.

And this is the worked example [?]: we wish to subtract [subtrahend, numeral uncertain] from [minuend, numeral uncertain]. The legible number-words are: "thousands... two hundred and twenty... and five," "and one hundred and fourteen," and "we extracted the five... five from the parts [portions]." The actual

computation is set out in the left margin in ghubar (dust-numeral) figures as a stacked subtraction:

- Upper row (minuend): [numeral: ?] (four ghubar figures; degraded)
- Lower row (subtrahend): [numeral: ?] (four ghubar figures; degraded)
- Below the rule (difference): [numeral: ?] (leading cipher followed by three figures; degraded)

[The marginal ghubar figures and the compressed number-words of this worked subtraction could not be resolved with confidence on the scan, even under magnification. Do not reconstruct the digits from the method; they must be read from the source.]

we extracted the five, and there remained [?]... — here these are the units [of the difference]; this is [called] the first [remainder]. And so [proceed with] the tens [?] and the [higher ranks], subtracting rank against its like, writing each remainder below its rank, until the whole subtraction is complete.

מאזנים [The Balance] by way of nine [the nines-check for subtraction]. Take all the ranks [and treat the digits] as units; cast them out nine by nine, and whatever remains, keep — it is called "the first kept." Do likewise with the number we wish to subtract, which we wrote beneath: cast it out nine by nine, and this is called "the second kept" [?]. Then take the remainder [the difference] that is left after the subtraction, [treat its digits] as units, and cast them out nine by nine; if the result equals the first kept [less the second kept] [?], your computation is true; if not, know for certain that you have erred. And if the first kept is less than the [second] kept, add nine to it, then subtract the smaller kept from it, and proceed by the rule.

מאזנים אחרים על דרך שבעה [Another Balance, by way of seven — the sevens-check for subtraction]. [Take] the whole of the numbers we [are working with]... as tens... [The passage breaks off at the foot of the folio.]

fol. 29r

Completing the Balance by seven (subtraction check)

[The sevens-check for subtraction, begun at the foot of the previous folio side, continues:] ...and those beside them are units; cast them out seven by seven, and what remains is their count as tens. And the number [that is left over] again falls short as units, so cast them out seven by seven — that which bears upon the units — and cast them out seven by seven; and what then remains is called the **first**

kept (השמור הראשון). Then proceed in the same way with the numbers that you subtracted [i.e., the subtrahend], and what remains from them, keep it: this is called the **second kept** (השמור השני). Subtract the second kept from the first kept, and what results from this is called the **third kept** (השמור השלישי).

Do this in the manner of the numbers that remained when you cast the numbers out of the numbers, and you will find it comes out just as we said in the earlier cases [the nines-check] — that is: when you write the wholes as tens and those beside them as units, and cast them out seven by seven, and [take] the remainder as tens and those beside it as units, and cast them out seven by seven — that which bears upon the units. And after you have cast them out seven by seven, look at the two remainders: if they are equal to the third kept, then your computation may well be sound; and if not, know for certain that you have erred.

Another Balance, by way of addition

Another Balance, by way of addition (מאזנים אחרים על דרך הקבוץ) — and these are honest scales (מאזני צדק). Add the numbers [of the subtrahend] together with the numbers that remained after the subtraction [the remainder], and observe: if [their sum] is equal to the numbers [you began with — the minuend], then certainly your computation is complete (תמת); and if not, then you have certainly [erred].

[Marginal/scribal mark before the following heading: "עשג" (?) — function unclear.]

The Second Part of the First Book — Chapter One: On Multiplication

החלק השני מהספר הראשון · הפרק הראשון · לכפול — "The Second Part of the First Book. Chapter One. On Multiplication (לכפול)."

A number upon a number. One who wishes to multiply a number by a number [uses] the table (הלוח), which is this — a "gate of doubling/multiplying" (שער כפל), for [it is] like this:

The multiplication table (לוח הכפל) on this folio side is a ruled 10 × 10 grid. Both the top row and the rightmost column run 1–10 (א–י) as the factors, and each interior cell holds the product of its row-factor and column-factor. Reading the rightmost column downward gives the multiplier 1, 2, 3, … 10; the top row read right-to-left gives 1, 2, 3, … 10. The transcription below preserves the logical order (factor ×1 at the right in the source). The upper rows register cleanly on the scan; the largest-product rows (7–10) were read at the limit of legibility.

row \ ×(1→10)	×1	×2	×3	×4	×5	×6	×7	×8	×9	×10
1 (א)	א 1	ב 2	ג 3	ד 4	ה 5	ו 6	ז 7	ח 8	ט 9	י 10
2 (ב)	ב 2	ד 4	ו 6	ח 8	י 10	יב 12	יד 14	יו 16	יח 18	כ 20
3 (ג)	ג 3	ו 6	ט 9	יב 12	טו 15	יח 18	כא 21	כד 24	כז 27	ל 30
4 (ד)	ד 4	ח 8	יב 12	יו 16	כ 20	כד 24	כח 28	לב 32	לו 36	מ 40
5 (ה)	ה 5	י 10	טו 15	כ 20	כה 25	ל 30	לה 35	מ 40	מה 45	נ 50
6 (ו)	ו 6	יב 12	יח 18	כד 24	ל 30	לו 36	מב 42	מח 48	נד 54	ס 60
7 (ז)	ז 7	יד 14	כא 21	כח 28	לה 35	מב 42	מט 49	נו 56	סג 63	ע 70
8 (ח)	ח 8	יו 16	כד 24	לב 32	מ 40	מח 48	נו 56	סד 64	עב 72	פ 80
9 (ט)	ט 9	יח 18	כז 27	לו 36	מה 45	נד 54	סג 63	עב 72	פא 81	צ 90
10 (י)	י 10	כ 20	ל 30	מ 40	נ 50	ס 60	ע 70	פ 80	צ 90	ק 100

[The interior products above are transcribed from the ruled grid; in so dense a grid the individual cells are not independently confirmed.]

You will need to multiply a certain number by a certain number — say [some number] by [some number] [numeral: ? — the two exemplar values beside the table are illegible on the scan]. If the number you wish to multiply lies together with another number [such that] each is [in] a single rank, or [spans] a portion of a rank, then take a likeness for each [factor down to] the rank of units, and multiply the one by the other as though they were units; and the result you obtain, you then reckon: assign to each [digit] a number according to its rank. For if the one were in the tens and the other [numeral/rank uncertain] in the hundreds, we take for the tens [a value] and for the hundreds [a value]; and we reckon them.

Worked example. For instance, suppose we wished to multiply **80 by 500**. We took, as a likeness reduced to units, **8** for the eighty and **5** for the five hundred, and we multiplied 5 by 8, and the sum came out **40**. Then we took for the hundreds **3** [numeral: ?] and for the eighties **8** [numeral: ?] — for these are of the [higher] ranks — and [summing the ranks] they became **5**; we cast out **1**, and **4**

remained; and these four are the rank of thousands. Thus we know that the four which came out when we multiplied the eight by the five hundred are **forty thousand** (40,000).

Now when we wish to multiply a two-place number by a two-place number, we must write each one under [the sign of] its rank, and we multiply four times: each of the [digits] of the upper [number] with each of the [digits] of the lower [number]. We then gather the resulting number. [We begin] with the first column [rank], and the number set opposite it [we] multiply upon [its] column; and we begin to multiply the units, taking them upon the units…

fol. 29v

[if the product] does not reach ten, we write it below in the units rank. But if it does reach ten, we take out the tens from it and keep them [as a carry]; and the remainders that did not reach a full ten we write in the units rank. Next we multiply the units below with the tens rank above, and whatever there is we combine also with the tens number that we kept, [adding it] to the units of this product — for although these are [reckoned as] tens, what comes out of them is treated as though they were units and tens, since it is upon the likeness of units that we perform all the multiplying. And the units that gather from the product we write in the tens rank; and the tens, if there be any, we write in the third rank. Afterward we view the total; and this is the reckoning of the multiplication of the [first] unit, and it is the likeness [pattern] by which…

[Worked multiplication, digits set out in ghubar (Arabic-form) numerals:]

The example is **348 × 235**, giving the product **81780** (the "348" and the product "81780" read clearly; the multiplier is read as **235** — the units-and-tens "35" are clear and the leading hundreds digit **2** is pinned by the inked product, since 348 × 235 = 81,780). A short intermediate figure "**4 3**" [numeral: ?] and companion digits (partly given also in Hebrew letters — "ד ס ו" [?]) appear above a rule line as partial-product workings [numeral: ? — the partial products are read at the limit of legibility on the scan].

[The scribe then walks the product through digit by digit:] we multiplied the **3** with each of them [the digits of the other factor], and we multiplied the **5** with the **3**, and it became [a partial product] — for the numbers we multiplied are of the first rank, the three… Again we multiplied the **4** with the **3**… Again we multiplied the **4** with the **2**… [the successive partial products are stated with their kept carries; the specific partial-product totals are only partially legible on the scan].

Folios covered: fol. 30r–30v **Script:** compact Sephardic semicursive, single primary hand (continuous with prior batch).

fol. 30r

[Continuation of the digit-by-digit computation of 348 × 235.]

with the second. Since the tens [that arose] we retained for ourselves, there came to be 4 to be gathered in the hundreds rank, because the numbers we had multiplied lie in the second rank. Again we multiplied the four by the two, and there were 8; we combined this with the ten we had retained, and there came to be 9 to be gathered in the third rank, which are the [figures?] [?] we had multiplied — they were of the graded [rank] [?]. So now we have completed the multiplication.

Again we return to multiply the 2 by each and every [digit], to work through the numbers we set down in the upper [row]. Again we return to multiply the 2 by each and every one of them [?]. [The 2] belongs to the third [rank], and its product with the units[?] came out as 5 [?], and there was a 2 [?]; and reckon that there are none [remaining] [?] [with] the others. Again we multiplied [it] by [the digit] in the second rank [?], and it was 8 [?]; and its product went into the [second?] rank[?] — thus for the rest of the ranks.

Diagram. A stepped (staircase-bounded) numeral grid to the right of the text — the partial-products layout for the 348 × 235 multiplication just described. Two short rows sit above a rule, then a block of three rows (one of which is a row of three zeros/circles), then a final summary row beneath a lower rule giving the assembled product. The interior cells are two-letter gematria values in a very small hand; they do not resolve reliably on the scan, even under magnification, and are transcribed as `[numeral: ?]` rather than pattern-filled.

(rank →)				
[numeral: ?]	[numeral: ?]	[numeral: ?]		
[numeral: ?]	[numeral: ?]	[numeral: ?]		
[numeral: ?]	[numeral: ?]	[numeral: ?]		
0	0	0		
[numeral: ?]	[numeral: ?]	[numeral: ?]		
[numeral: ?]	[numeral: ?]	[numeral: ?]	[numeral: ?]	[numeral: ?]

Again we multiplied the one [digit] [?] by [each], and there were [figures] to be gathered in the first rank after the [?]. [The digit] of the second rank [?]… again we multiplied [?]… and the sum was [?] in the [?] rank, and the [carry] was from the third rank. [When?] we gathered them, they are 3 thousand and 5 hundred [?] — and the like [?].

Said Mordecai: I saw fit to make known to you this multiplication by another route, easier than this one — namely, the way the sages of the Ishmaelites [i.e., the Arab mathematicians] do it. It is carried out in the manner of the doublings, in [aligned] rows, one [beneath the other] [?] as in a table [?]: whatever descends [in a column] [?]… And as we do it: the number you wish to multiply, [set it in] the first row as usual; then set the number by which you wish to multiply it in the [second] row, in a rank matching it [i.e., aligned by place-value]. After that, begin to multiply the units by the units, and as it proceeds [?]…

fol. 30v

[The tabular ("Ishmaelite") method, continued step by step.]

below. Again, multiply the units of the lower [number] by the tens of the upper, as though they were units upon units; and if tens are left over to you from the first reckoning, count [them as] [?] units and add them to these. Likewise multiply the units that are in the upper [row] by the tens that are in the second row, and combine the whole, and write the sum below; and if tens came up, place [them in] the higher rank, apart from the simple [digit], in your mind. Likewise the units in the lower [row] with the hundreds in the upper… with the hundreds in the lower; also the tens in the lower with the tens in the upper.

And the rule: the ranks in the computation, when you sum them, come out equal [i.e., the rank of a product is fixed by the ranks of its factors]. For the first with the first make [the first]; likewise the third with the first [?], and the second with the [second?] and the first; also the fourth with [the second make] five [?]; and likewise the seventh with the third [?], and so on without end. Whatever results, add the number that was in the rank [holding] the tens we had retained in the mind together with them, and write [it] below — and that is the total sought.

Diagram. The 348 × 235 example re-worked by the tabular method — the multiplicand (348) in the first row, the multiplier (235) beneath it in the second row aligned by place-value, and a boxed grid of partial products with the running total. Small-hand two-letter gematria cells; interior values do not resolve reliably

on the scan, even under magnification. Transcribed as `[numeral: ?]`, not pattern-filled.

[numeral: ?]	[numeral: ?]	[numeral: ?]	
[numeral: ?]	[numeral: ?]	[numeral: ?]	
[numeral: ?]	[numeral: ?]	[numeral: ?]	[numeral: ?]

Illustration: we wished to multiply three hundred forty-eight [348] by two hundred thirty-five [235], thus [as figured above]. We multiplied the 8 by the 5, and there were 40; we set [the 0] [in the first rank] and retained the 4, since there are no units [beyond the zero]. [We continued:] we multiplied [the units] [?] and gathered [the result] in the first rank; and the whole of the simple [digits] you reckon in your mind and write down. Again we multiplied [the 8] by the 3 [?], and there were [24?] [numeral: ?]; likewise we multiplied [the 8] by [the 2] [?]… and multiplied [the 4] by [?]… [and so through the remaining digits, assembling the product]. [The interior single products beyond 8 × 5 = 40 do not read cleanly; flagged `[numeral: ?]` rather than reconstructed.]

[Opening of a new topic — squaring.] …[This is] the case of a number [?]: [take] the [root/third?] [?] of the number and multiply it by itself, and it becomes a square [מרובע], and we take its like [?] in the next rank [?]. **Illustration:** we wished to multiply seven [?]… as a square: we took [its root?] [numeral: ?] and multiplied it by itself, and it came to be [?]; and we took its counterpart [in] the rank next above [?], and it was 3 [?]; we brought it out beneath — know that it is the square of the [root/third] [?] — and there remained [88?] [numeral: ?], and that is what was sought. And if you wished to multiply a number that has no exact [third/root] [?], take a [third/root] of the nearest number to it [?], square it, and proceed by the same rule [?]; […] add to it the deficiency [?] of the numbers and add them to the total, and that is what was sought. **Illustration:** we wished to reckon the square of ten [?]…

Cheshbon u-Midot — Book One, Part Two, Chapter One (On Multiplication): Squaring (continued)

Compact Sephardic semicursive; a single hand throughout. Folio 31 (recto and verso). This opening continues the topic of the square (מרובע) computed by ranks (מדרגות).

This number has no third-rank part [שלישית]; the number nearest to it is one whose part does have a third-rank part, and it divides it evenly [טהור]. We took the square of the third-[rank digit], which is 3, and from the next rank [המדרגה הבאה] we took the like of it, which is 3; from it there issued [numeral: ?], and there remained [numeral: ?].

Understand that the number from which we took the third [rank] is less than the sought number [המבוקש]. We summed the two numbers and added the excess, and they became 100 — and that is the square of ten [מרובע העשרה].

The rule [הכלל]: if the number from which we took the third [rank] had been greater than the sought number, we would subtract the two numbers from the total, and the remainder is the sought result. [And if it were less, we add, as above.]

Now, we took one that has no third-rank part, and its part that has a third-rank [part] divides evenly. Illustration [דמיון]: we wished to know the square of a number whose part has a third-rank [part], which is 12 [numeral: alt reading uncertain]; and it was [?].

I found [מצאתי] a way to raise a number to its square by way of the ranks [בדרך המדרגות]: you multiply it and double it [כפל], set it out by way of the ranks, and it comes out to the sought result. Its square, which is the square of the third-[rank digit] — and it is 12 — is separated [יוכך] from the sought number; and we subtracted [וגרענו] from 144 [numeral: alt reading 104 = ק״ד], and there remained [numeral: ?].

And we doubled [וכפלנו] by way of the ranks. Set it out [i.e. the number] upon itself, take its fifth-[rank part], and double it upon [its rank]; [numeral: ?] upon [numeral: ?], and it is the sought result [המבוקש]. So too, by placing it upon itself they became 144, the [?]; and four behind [?] the fourths [רביעית], placed [?]. We wished to double [לכפול] the fifths [חמישית] of the fifth-[rank] [?].

and we doubled it once [פעם אחת], since the difference [ההפרש] falls between [numeral: ?]; and they became 100(?), and this is the sought result [המבוקש]. Illustration [דמיון]: another — since the illustrations are similar. This number has no fifth-rank part [חמישית], and [?].

We doubled it by way of the fifths [בדרך החמישיות], and it rose accordingly. Likewise, the [number] we doubled is 12(?) times, since it falls [beneath?] and the difference [ההפרש] between them; the number nearest to it has no fifth-rank part. This is such-and-such [?], since the difference falls between us and between what has a fifth-rank part; and the number nearest [to it] [?].

It has no fifth-rank part, and it was [?] once more, [and] afterward we doubled it and it became 100(?) upon [?]; and behold, this number has no fifth-rank part, and the number nearest to it whose part has a fifth-rank [part] is [numeral: ?]. Illustration: another — for the number [?] has no fifth-rank part, and the number nearest [to it] [?].

Afterward we doubled it and it became [numeral: ?], and there remained [numeral: ?]; and behold, the difference [ההפרש] fell between our number and between it — it is one. Illustration: another — after we doubled our number and were left with 104(?); and behold, this number has no fifth-rank part [?].

The number [?] has no fifth-rank part, since the numbers are far [רחוקים] from the whole [הכלל]; so, to double the whole [לכפול הכלל] upon itself... and behold, the ranks [numeral: ?].

Said Mordecai [אמר מרדכי]: You must, when you wish to double a number so that its number should be [numeral: ?] — that is, [when] you double a number by way of the units [פרטים], and it was [numeral: ?] and became [numeral: ?] — take the whole [הכלל] and its part, add [it] as a whole and set it out. And behold, a whole of [numeral: ?] — they were 70(?) and 6(?) times [ע׳ ו׳ פעמים, alt reading = ע״ו 76]; hence [?] [numeral: ?].

The Balance [מאזנים] — [verification] of the doubling [על הכפל]: reduce [?] the circles [העגולין] all of them as though they were units [כמו אחדים], and cast [them] out, and the remainder — keep it; it is called the first kept [השמור הראשון]. Then do likewise with the lower [number] [השפל], and the remainder is called the second kept [השמור השני]. Multiply the first kept upon the second kept; what results is called the third kept [השמור השלישי]. Then do likewise with the third [kept] and the [?]; gather it as though they were units [כמו אחדים], and cast them out.

Book One, Part Two, Chapter One — On Multiplication: conclusion of the מאזנים (Balance / casting-out) verification, followed by a set of propositions on the properties of a number divided into parts.

fol. 32r

[Margin, top-left: **32** (folio number)] …and the remainder [that comes out] will equal the third kept; if it does, your computation is correct [?], and if not, know your error in it.

מאזנים [enlarged rubric — *the Balance*]. In this way: take the digits of the first number — the tens [?] and, beside them, the units — and cast them out [nine by nine? — numeral: ?]; whatever still remains, keep as prescribed. And whatever remains beside them [as] units, cast out likewise [numeral: ?]; and do the same for the higher [ranks]. What remains, keep — this is called the **first kept**. Next, take the digits of the second number and do just as [with the first]; whatever remains is called the **second kept**. Then multiply the second kept by the first kept; whatever [results], complete it likewise [numeral: ?], and the remainder is called the **third kept**. After that, cast out the digits of the total number [of the product], which is called **the sum** (הסך), treating it by the rule of the upper numbers; and look at the remainder. If it equals the third kept, your computation is correct [?]; and if not, know your error. [End of casting-out procedure.]

Said Mordecai: I have decided to write out for you some matters concerning the nature of number, to give you practice in them, for it is good [?]…

ואומר [rubric — *And I say*]: Every number that you divide into parts, however the division may fall out — the [sum of the] product of each one of the parts by the whole number equals the square of the whole number.

Illustration. The number 12, which we divided into 3, 4, and 5. We multiplied the 3 by the whole [12]: it was 36. Again we multiplied the 4 by the whole: it was 48. [And the 5 by the whole: it was 60 — numeral: ?] We joined them all, and the whole is 144, which equals the square of 12 [= 144].

Again: every number that you divide into parts, however it may fall out — the square of the whole number equals [the sum of] the squares of the two parts together with twice the product of the one part by the other.

Illustration. The number 10, which we divided into 3 and 7. The square of the 3 [is] 9 [ט']; the square of the 7 [is] 49 [מ"ט]; and the product of the 3 by the 7 [is] 21

[numeral: ?], taken twice — that is 42 [numeral: ?]. This with that [together] is 100 [ק'], equal to the square of 10 [= 100].

Again: every number that you divide into parts [however it falls out — the square of the whole equals the sum of the products of each part with the whole? — reading uncertain]…

fol. 32v

the parts, whether the part be equal or [unequal] [?] — [it is] equal to the square of the parts of the number.

Illustration. The number 10: we divided it into 5 and 5, which are equal parts, and we also divided it into 3 and 7 [numerals read: 5,5 and 3,7], which are unequal parts. We multiplied the 5 by itself [5 × 5 = 25], and it was 25 [כ"ה]. Then we took the square of the 7 [7^2 = 49 — numeral: ?]… [comparison of the equal against the unequal division; several worked values on this line not cleanly legible — numeral: ?].

Again: every number that you divide into two parts [equally], [the product of the whole with a part, together with the square arising from half the number, equals the square of half the number and its … together] [statement partly uncertain].

Illustration. The number 10, which we divided into two [equal] parts, namely 5 and 5. We multiplied the 5 by the 10 [numeral: ?]… [and the several derived squares — worked values not isolable on the tile — numeral: ?]… which is the square of half the number, and it was [numeral: ?]; and this equals the product [numeral: ?]…

Again: every number that you divide into parts, however the division falls out — [the difference (הקוסמן) between the parts, its square, together with …] [reading uncertain]…

Illustration. The number 10, which we divided into two parts [unequally], into 3 and 7 [?]… the square of the 5 [half]… which is the square of the difference (הקוסמן)… [worked values on these lines not cleanly legible — numeral: ?].

Again: every number that you divide into parts — [the square of one part and the square of the difference by itself are the two squares which are twice the two parts …] [statement uncertain].

Illustration. We have a number, the count 10, and we divided it into two parts, [5 and 5?], and [also] divided it into 3 and 7… [and we returned to …] [worked values not isolable — numeral: ?].

Again: every number that you divide into parts, from 10 [?]… and we divided it into 3 [?]… [text continues onto the following folio].

Cheshbon u-Midot (Arithmetic and Geometry) — Mordecai Comtino

[f. 33r]

(Continuing the tail of the chapter on multiplication — a worked example of squaring, small-hand and abraded across the top lines.)

another number, 3, and it becomes 12. Behold, the square of 12, which is the product taken with itself, is [numeral: ?]; and the square of 3, which is the product [numeral: ?], and its value is [numeral: ?]; and it is double the square, which is [numeral: ?]; and the square of 4, which is [numeral: ?], and its value is [numeral: ?], which is the square of the product taken twice; and the product… [the top three lines here are numeral-dense and abraded — the individual squares and totals do not resolve cleanly on the scan, even under magnification].

Behold, this is so. Now we shall transcribe some matters that are needed, [drawn] from the treatise… for the ways of reckoning are of several differing kinds, brought under a single computation, one from another, to show [that] one way is shorter than another. And with this the discussion of multiplication is complete.

The Third Part — To divide a number by a number, and to add[?] a number to a number

[Verbatim rubric: החלק השלישי · לחלק מספר על מספר ולהוסיף[?] מספר על מספר] *(The second verb of the rubric reads ולהוסיף, "and to add," which is unexpected in a division section; the reading is flagged.)*

Chapter One

Said Mordecai: We have already prefaced this — the reason [being] that [the divisor] is prior by nature, for we cannot… the whole and the part except… so that [the divisor] is composed of this and of what is set opposite it, [some part] of the number.

And behold, the division is performed by the practitioners of computation in [two ways]: the one way is as the sage Rabbi Avraham [ibn Ezra?], of blessed memory, explained it[?]; and the second way is the one that the Romance-speakers call *galion* [Romance: *galea* / *galion* — the "galley" (scratch) method of division] or *pi dosho dono* [?]. And we choose this one. So here too there are two methods.

The first method is when one wishes to divide a number by a number — whether it be one [digit] upon many, or many upon one, or many upon many. Write the number that you wish to divide first (and it must always be greater than the number divided into it), and in another line below it write the number by which you wish to divide, so that [the ranks align]…

Set the number to be divided… in the arrangement of the ranks of the upper row, each [digit] opposite its counterpart. Take a portion to divide: the greater [is placed] in the… and the smaller… and divide it by the greatest whole in the upper row. If the greatest whole in the upper row, when you reckon it by the count of the [digits], is greater, do [thus]; and from the whole in the last row… And if it is greater than the whole in the lower row, then divide [what is] in the upper row… And [as to] the number by which you divide: if it is of the first rank, write [the quotient] at the first rank as you divide; if it is of the second rank, do so at the [numeral: ?] rank; if of the third, at the third rank; and so on accordingly.

And if there remains any number in the first whole that cannot be divided at its[?] rank, place it [aside]… and return it [as] a remainder — each of the [digits] that cannot be divided — and join it with the nearest rank toward the lower [end], and divide it. And likewise, if there remains also of the [carried] one at that rank, do the same, until you reach the units rank. And likewise it is proper, whenever any [remainder] occurs, [to follow] the rule of the parts by which you are dividing… And you divide the whole in the first row by the whole in the [lower] row… something that you carry back… and divide it also by the rest of the number in the lower row; and so at every single rank.

Illustration. We wished to divide [numeral: ?] … [the dividend and the "…and fifteen by fourteen" continue onto the next folio side].

[f. 33v]

and fifteen by fourteen, as in the figure.

Diagram. A boxed galley-division ("galleon") layout worked upon 14 as the divisor. The divisor appears in the lower row as **1 4** (= 14). The dividend row and

the intermediate/quotient rows are scratch digits that do not resolve confidently on the scan.

- Upper row (dividend): [numeral: ?] (reads approximately 5–1–2–4, uncertain)
- Middle row (working): [numeral: ?] (reads approximately 1–0–3, uncertain)
- Lower row (divisor): **1 4** (= 14)

We divided the whole in the upper row — which is in the **tens** rank — by the one in the lower row, and gave it **three**, so that we might also divide by the four, as you will see. And if [the quotient] at the second rank [were] one… the division [would come] from the wholes, since the one by which we divided was at the second rank of the [digits].

As for the one that remained, we set it back as a remainder in its place[?] and joined it with the second [digit], which was **nine**[?] in the rank of the units[?], and they became **12**[numeral: ?]; we subtracted from them the [product with] three, three times, and nothing remained, since they were nine… And so that nothing should go outside… behold, they are in the tens rank. Again we returned to divide what is in the lower row by the four[?] in the lower row, and gave it one…

(There follows a second, larger worked division. The prose narrates a dividend reaching into the ten-thousands rank, divided by 114, with successive quotient digits and remainders; the accompanying figure:)

Diagram. A second boxed galley-division layout. The divisor in the lower row reads **1 1 4** (= 114) and the quotient row reads **9 1 2** (= 912); the dividend row reaches into the ten-thousands rank.

- Upper row (dividend): 104034 [units digit 4/2 residual] (into the ten-thousands rank)
- Middle row (quotient): **9 1 2** (= 912)
- Lower row (divisor): **1 1 4** (= 114)

the whole in the upper row, which is in the tens rank, [reaching] to the ten-thousands, up to the ten [thousands?]; and we were able to divide **five** as a portion upon the… And behold, the fifth[?] rank… from the [digits], because the whole [could] be divided so that [it stood] in the tens rank of the wholes; and the one that remained at that rank, [being] the rest of the wholes, we set it back as a remainder [of] ten[?] in the rank of the [digits], and joined it with the four[?] [with] its remainder — and it was 16[?] — and there remained still [something] of

this, again a remainder. Three times… what remained in the tens rank, and the rest in the rank of the [digits]…

Again we returned to divide the whole [numeral: ?] in the [numeral: ?] rank of the [digits], and behold, the whole that is in the tens rank stood upon [the divisor]; and the whole that is in the… rank of the [digits], we divided… and gave it one, since the one by which we divided [stood] at the tens rank, and nothing remained, and there remained no [further] matter.

Again we set [it] as [digits] … [what] remained in the [rank of the] [digits], and there remained still… of the number in the rank of the tens[?]; and so upon every whole in the tens rank, we divided[?] by 114[?], and gave it two[?], and there remained [numeral: ?] of them.

Again we set [it] to divide the [numeral: ?] that remained in the tens rank of the wholes; and behold, the wholes that are in the tens rank, we set back as a remainder in the tens rank upon [numeral: ?]; and we gave [it] two[?] to divide, and returned them as a remainder… so that we might carry them back as remainders. And their portion, together with the rest of the number in the tens rank, [we] divided[?] with all the number in the upper row; and behold, there were 14[?], and there remained of them the [digits], and there remained… the [digits] in the tens rank. And so, know and understand — however much you divide — that the number of the wholes and of the [digits is such].

**** ("and there remained…")

and there also remained [numbers] that had not been divided.

You should know that when you divide the numbers of the upper row by all the numbers of the lower row, it may happen that — before you complete [a digit of] the quotient — you find there is no remainder to be carried and joined to the [digits] that come after. In that case you must set down a zero (*sipra*) in place of [that digit of] the quotient; and afterwards, when you go back to divide the following [digits] as before, you proceed with the zero you have placed, just as in the first illustration above.

Another illustration. We wished to divide five thousand two hundred and fourteen [5214] by one hundred and eight [108].

We divided the [leading] 5[?] [numeral: ?], and it sufficed; we gathered [the remainder] into the third rank, returning to the 4, for the one that we had taken upon it stood in the third rank [numeral: ? — the intermediate scratch values here are not legibly recoverable]. Returning to the units, we [subtracted] four

times one from it, and there remained [numeral: ?] in the units rank. They became twelve, for we joined them with the [numeral: ?] in the last rank; and since it still fell short, we carried the whole back to the tens and joined it with the [one hundred–odd, numeral: ?] standing in the tens rank; and there remained [numeral: ?] in the returned rank. We continued to subtract in the tens rank until we came back to divide the eight upon it [numeral: ?]. We saw how many times [it went], subtracting from it, until we joined [the remainder] with all that is in the upper row; they became [one hundred–odd, numeral: ?], and after subtracting the 108 [numeral: ?] there remained 3 in the tens rank.

Thus we knew that the quotient is forty-eight [48], and there remained also thirty [30] that were not divided.

This is the first of the two ways of the [art of] division.

Diagram. A boxed galley/scratch-division layout for the worked example 5214 ÷ 108, set beside the word *כזה* ("thus"). The divisor **108** stands on the lowest line; the quotient **48** is written as the pair of digits **4 8**; the dividend and the successively cancelled scratch remainders occupy the rows above. Only the divisor and quotient digits are securely legible; the interior scratch digits are abraded/compressed and read `[numeral: ?]`.

- Bottom row (divisor): 1 0 8
- Quotient: 4 8
- Upper scratch rows: 4 [numeral: ?] 8 [numeral: ?] · [numeral: ?]

The Second Method. This is the one they call in their language *galea* [Romance: *galea*, "galley"]; it is the method the merchants of the Marches [מארקישיס — Marchese, the March of Ancona region] use. It consists in this: you set down the numbers you wish to divide in one row — dividing after the manner of the division, as the upper row — and the numbers by which you divide as the lower row; but not ranged rank-for-rank directly beneath the upper ones. Rather, they stand in whatever rank accords with their highest place, aligned with the highest place of the first row; and the number that follows accordingly, matching the [highest] number, so that you may divide the [digit of the] upper row of the highest place by the highest place of the lower row.

You need do nothing of all that preceded [in the first method]. If the numbers [of the dividend at hand] are not sufficient to be divided by the numbers you divide by, you carry down one, or descend [a place], as you require; and you consider how many times it goes, and how much [remains] of the whole against a single

[digit of the] whole, [namely] the remainder left over by us. And if what remains [is to be joined to] the digits that will come after it, [set it] in its proper place;

and likewise, [operate] with the number in its proper place upon the whole of these numbers you divide by; and subtract from the whole of the number being divided, and [what is left is] the remainder of the dividend. And if a remainder is left, carry it up above it, and [take] the next [digit], until it reaches — together with the number in its proper place — the number that comes after the leading [digit] of the divisor; and subtract it from the number that comes after the leading [digit] of the numbers being divided; and do as was kept [i.e. as recorded]. And so you continue until you have divided all the numbers.

[the numbers] you divide by. And if the number being divided does not suffice for the [amount] fixed by it, you take [what is left of] the one digit that remains — or more — if they had been carried back to the tens, and you divide them by the number you divide by; and so you continue until you reach all the numbers to be divided by, until [you come] near to cancelling the numbers you divide by. Then carry [to] another rank and begin afresh, and bring [the divisor] near again, and see how many times it goes; and subtract the whole [of the product] from the numbers you divide by, as [many times as] the number [of the quotient digit]. Multiply the leading [digit] of the divisor by the numbers being divided; and that number, according to the amount of the multiplication, multiply it as [with] the number you multiplied in its proper place, and [carry] with it upon all the numbers you divide by; and do as was kept. And [take] what is joined from the numbers you divide by, from the first of the lower numbers, as the remainder; and cancel the neighbouring number. And if a remainder is left, write it above it; so you do until it is done.

And if you cancel the number before you reach — within [the row] — all the numbers you were dividing by, and no remainder is left, carry a zero (*sipra*) within, there where you write the quotient, as [in] the divisor you divided by, until you return to divide the [remaining] numbers likewise, until you complete [it] within, and [set down] the quotient digits together with the zero.

Illustration. We wished to divide [three?] thousand [numeral: ?] by [numeral: ?], and we wrote it in this form [see fig-p18-1]. The [divisor] was as the lower row; we set it down and divided; [numeral: ?]... we saw how many times [it went], and [numeral: ?] we subtracted, and there remained [numeral: ?]... until it sufficed to divide the rest as well, within, as it should be.

Diagram. A boxed galley-division layout with a divisor cell attached at the right. The right-hand cell reads **301**; the main scratch block shows a row **1 4 1 4** with further partly-cancelled digits above (a leading **4** and others). The interior scratch digits are largely uncertain.

- Right cell (divisor?): 3 0 1
- Scratch row: 1 4 1 4
- Upper row: 4 [numeral: ?] [numeral: ?]

Then, [taking] the [numeral: ?] units, there remained [numeral: ?]; we subtracted [numeral: ?] and carried it, and set it in the rank above [see fig-p18-2]. [numeral: ?] units — [numeral: ?] remaining and [numeral: ?] carried — and we joined them in the last rank; and its neighbour we brought near and subtracted them with the [carried]. And what remained, [numeral: ?]... twelve, for the leading [digit], one, was worth ten; and we carried [numeral: ?] to the tens, and there remained [numeral: ?]. Twelve [12], and no remainder was left; and we set down the whole as [it should be]. And still we returned to divide the [numeral: ?] that stood above it [as] the ten... going down within, as [with] the leading [digit]...

We subtracted from the tens the remainder [numeral: ?], as [in] the rank of the tens and the units. And thus you do to divide all the numbers, and it is done as [was kept]; and so you do upon all the numbers [numeral: ?].

Diagram. A tall boxed galley-division layout, larger than the two above, with a divisor/quotient cell attached at its right — the small clean box reading **[9?]12** (leading digit 9/6 ambiguous, [numeral: ?]), with a *ʊ* mark — and the numeral **26** written above the box. The interior holds several stacked scratch rows of successively cancelled digits. Only a few interior digits are securely legible (e.g. a **1 7 1** row and a lone **1** at the foot); the remainder read `[numeral: ?]`. Marked as a (>2 labels/digits flagged).

- Above box: 2 6
- Right cell (small clean box): [numeral: ?] 1 2 [with *ʊ*]
- Scratch rows (top → bottom):
- 1 [numeral: ?] 4 [numeral: ?]
- [numeral: ?] 4 9 6
- [numeral: ?] 0 3 4
- 1 2 4 [numeral: ?] 4
- 1 7 1
- 1

- Left external note: [numeral: ?] / ט

the leading [digit].

Book One, Part Three — continuing Chapter One (On division), the closing verification steps and the "Balance" (מאזנים, casting-out-nines proof); then opening Chapter Two.

in the [digit] that stands in the multiplication frame, and they came to 15. We carried the single unit that stands in the ten-thousands rank back to the [tens], joined it with the [tens] of the thousands, marked them, and set them back in their place; the thousands then stood at 4.

Again we multiplied the 4 that stands in the multiplication frame by the 2[0] [numeral: ?], and they came to [numeral: ?]. We carried the 4 from the [numeral: ?] that stands in the thousands rank, marked it, and set it back in its place.

The remaining difference we placed among the [units], and did with them likewise; we marked them and set them back in their place. And the [digit] of the ten-thousands rank between the [numeral: ?] — we did with them as [before], and there remained of them [numeral: ?]. And still the reckoning [does] not [come out], since there is no remainder in the [column]; so we carried them and gathered [them], and set them back in their place, all of them, [numeral: ?] of them.

Before you [reach] the highest rank, mark whatever there is [in it] according to the first stage — for what is joined [from] the first stage is what we set to be divided; and we placed the [digit] in the multiplication frame [numeral: ?], and multiplied the [factors] with the [numerals], and [they were] equal upon them, and behold, they were nine, and [they] enter into them a number of times [numeral: ?]; and we carried them.

[This step] enters into every [number] whose measure is on the [descending line?], as [with] the [descending] mark; and multiplied it, [and it came] to [numeral: ?]; and we carried them. Again we multiplied the [factor] and it came to [numeral: ?]; and we marked [them] and set them back in their place. And the [digit] in the multiplication frame [came] to [numeral: ?], and it came to [numeral: ?]; and we did with them as we did.

By this reasoning we [proceed] from the [numeral: ?] that is in the tens rank; and they did not suffice to subtract from the tens; and we joined it, and marked the [numeral: ?] and set it back in its place; and the tens stood at 2. And they came to

[numeral: ?], and the difference of them was [numeral: ?]; and we did with them, [reaching] the number in the tens rank, and marked it, and set it back in its place, [numeral: ?] of them.

We reasoned that from the [numeral: ?] which is in the [column] the tens did not suffice, and the [remainder] did not [complete]; and the leftover from the tens rank stood at one [column]. The number of the tens [was] the difference [with] the tens, and we marked it and set it back in its place, [numeral: ?]; and the tens rank stood at [numeral: ?], and they were 1[?]; and they came to what remained of the difference [numeral: ?]; and still the reckoning did not [come out].

By the reasoning by which we marked and set them back in their place [numeral: ?] — [reaching] the highest stage before the [descending?] stage, according to the [ranks] that are [carried] — and behold, the [operation] entered the reckoning as the remainder, which was not divided; and we found still, and we found that they came out to [numeral: ?]; and set them back in their place. By the reasoning in which we marked, [reaching] the descending [column] from the [numeral: ?] that is in the multiplication frame — [multiply] it by the [factor], and there was no remainder, and we marked [numeral: ?] and set them back in its place, [numeral: ?] of them, and it came to [numeral: ?].

The highest stage [being] the nearest one in the ranks, and the tens, and the [remainder], and we marked them and set them back in their place. And likewise we marked the [numeral: ?] in the multiplication frame; and again we multiplied [numeral: ?] with the [numeral: ?] in the multiplication frame, and it came to [numeral: ?]; and behold, [numeral: ?] — [reaching] the tens rank and the [remainder], the [factor?] [numeral: ?]; and this is the [digit] which is called [the residue] of the difference of the parts whose measure was upon it. Again this is done in the ranks, so that the [remainder] set them back in their place, [numeral: ?]; and the [remaining] joined its neighbors with the [numeral: ?], and they were 1[?]; and they came to [numeral: ?] of them, and there remained [numeral: ?].

We marked the [numeral: ?] and set them back in their place, [numeral: ?]; and we marked its neighbor the [numeral: ?] and set them back in their place, [numeral: ?]; and we marked ten parts, [reaching] the number ten parts, and what remained after them, [that] which was not divided.

The Balance is by nine. [I.e. the proof by casting out nines.] Apply the doubling of our balance-figure — the number whose residue you took — together with our balance of the parts [i.e. the residue of the quotient]… [Margin/degradation:

several tokens here read as "the ring/seal of winter (?)" — genuinely uncertain; see QA]. Reckon it according to the first method and according to the second method: what was [written]… [that] which the descending mark had already tested.

And after you have gathered the wholes — [numeral: ?] — and whatever remains as the measure of our [residue]… behold, these — there did not remain, from the number that was divided, [any remainder]… whatever [it was] that was not divided… we gathered them, [and] there remained [a residue] far off; also our balance-figure, and join the [residue] with the number…

whatever remains, and if [the residues] are equal one to another, then it is [correct]; and the remaining balance-figure is the [residue of the] result. Afterward, if the numbers came out equal, so that they are equal, then the balance-figure [numeral: ?]. But if they were equal to one another and there did not remain [any discrepancy], know for a certainty that they are equal, and so [the numbers] are entered, [a number of] times [numeral: ?] of them, and [thus] the balance-figures are equal.

And what remained in the reckoning, since it did not divide, we found still, and we found what we sought, from the parts, and set them back [in their place] — as we set the highest stage before the descending stage — and behold [it was] equal with the balance-figure, [and] it came to [numeral: ?], and they were [numeral: ?] of them; that whose measure did not divide [means] we found it, since our finding [was as] the [numeral: ?] of them equal, and set them back [in their place].

By the reasoning in which we [proceed], the balance-figure is what we called "the parts"; and whatever remains — that whose measure did not divide — set them back in their place [numeral: ?], upon the [column] the parts, and whatever remains will be the [numeral: ?]; and whatever remains, its neighbor, is called "the residue." And behold, these did not remain in the numbers, since the parts were divided; a number whose measure did not divide, take the number that is [it], its residue is a certain [number] [numeral: ?] according to this method; and its neighbors [numeral: ?], and there remained the residue [numeral: ?], and the residue [was]… and whatever remains — a number of times — that whose measure did not divide — behold, one matter, [namely] that the number is the [balance] itself, and its neighbors [numeral: ?].

As it was [written], after them, and the neighbors [numeral: ?], and so its measure is equal, and the residue [numeral: ?]. And whatever remains a certain [number],

that whose parts did not divide [numeral: ?], is one [matter], since our reasoning is that there is no remainder; know for a certainty that you have erred, and if not [i.e. if it does come out even], know that your reckoning was true — for you found [what was sought]; and if there is [a discrepancy], know that you have erred.

The Balance is by nine. [Second statement of the casting-out-nines proof.] There are other Balances, which are the very [same in] substance; [each is applied] to the operation whose measure the parts [give]. If they were equal, [that is] the correct one; and if there remained of them any discrepancy, know that you have erred.

The Balance is by nine. [Third statement.] Its reasoning is that the number whose parts we found — set them back [in their place]; and whatever remained will be equal. And with the numbers whose measure did not divide their parts — if the whole is equal, then one of them is equal, so that its measure is [drawn] from the numbers whose measure did not divide their parts, [meaning] you have not [erred]; for it is my reasoning that when you find, there is no remainder; and if not, know that you have erred.

Chapter Two. [הפרק השני] To add a number to a number according to their arrangement or in another arrangement — the intention being to add [numbers], one number upon another — the [descending] mark; and the result is the sought [check-number, המבוקש].

Illustration: we wished to add the numbers which are 41 [numeral: ?] and 74 and… [numeral: ?]; behold, they came to [numeral: ?] [numeral: ?], and they were [numeral: ?], and we added them.

Another illustration, [for practice]: we reckoned by how much the whole number was gathered — this being the sought [check] — and behold, [numeral: ?]; and there remained [numeral: ?], and they were 1[?] and 1[?]; and we added upon this yet one part, and there was [numeral: ?] behind it; and behold it was [numeral: ?] — and this is the sought [check]. And upon [numeral: ?] there was [numeral: ?] of them.

And if you say [take] the nature of the whole number, and find the residue upon it, and take a certain [figure] — behold, the balance-figure is what we said upon its name; and behold it is [correct]. We added upon it its residue, so that it was [numeral: ?], and behold, these are [numeral: ?].

Said Mordecai: Another method, a very choice one. It is that you gather the total of the sought number [the running check-residue] together with the number that

comes last — and [half] of it will be the sought [check]; and this is like the whole [reckoning] of the sought number [numeral: ?] with the balance-figure. And [the residue of] the sought number that comes after it — behold it is the number that comes after them.

And if this [is] the sought number — an illustration: we wished to know by how much the numbers 460 [numeral: ?] and 41 [numeral: ?] and 7[4] [numeral: ?] gather; behold, they gather to [numeral: ?] [with] the number that comes after them, and they came to [numeral: ?]; and its residue [numeral: ?], and this is the sought [check].

f. 36r

Question. If someone asks you, "I gathered numbers [in sequence] and they came to 55 [נה], how many numbers are there?" **Answer:** [Method partly illegible — the recipe involves doubling the total and taking a square root:] take the [square] root of the [quantity] that results, and see [whether it is a whole number]. [numeral: ?] And if it equals the number remaining from the [square] that resulted, then just so are the numbers, according to the count [produced by] the operation. [numeral: ?] [Worked steps of the recipe are only partly legible.] And so the numbers you gathered, which came to 55, are [numeral: ? — the sought count]. [The classic answer for 1 + 2 + … = 55 is ten.]

Question. "I gathered numbers … how many are they?" **Answer:** Proceed as in the first rule, and you will find them [numeral: ?]. Then [take/subtract] [numeral: ?] from 55, and what remains of the 55 [numeral: ?] is the sought answer (המבוקש).

And if someone asks you, "How many are the squares of the numbers, up to a certain number, taken in order?" — in the way [that] you find their sum [as simple numbers]. Afterward [you add to it a third of its double / proceed by the following rule]: [recipe partly illegible] with a count of one third, and the result is the sought answer. **Example:** How many are the squares written from 1 to 4 [i.e. $1^2 + 2^2 + 3^2 + 4^2$]? We [computed by the rule] … [intermediate steps partly illegible] … and they came to **thirty (30)**, and that is the sought answer.

Part Four — to set in ratio[?] a number to a number. This [Part] will deal with three [kinds of ratio], for the [Arabs / others — לעברים/לערבים [?]] have another method, whose path we shall explain in the next chapter. And we say: since [these ratios] are of two kinds, the ancients chose only three species of ratios (ערכים) — namely the ratios of arithmetic (ערכי החשבון) and the ratios of geometry

/ measure (ערכי המדות) … [and] we shall mention here [the] two ratios of [money / magnitude] [?], for they are needed by most of those occupied with computation.

And it is already established in computation [the theorem of proportion]: for any [four] numbers such that the first is to the second [numeral: ?] as [the third] is to the fourth — if [we take] the square of the means, it equals the squares of the first and the fourth taken together, together with the square of the difference (הפרט) that lies between [the first / the smaller] and the third. Likewise, when the squares of the second and the third together, with the square of the difference that lies between the first and the fourth, will be equal in this computation. And likewise [for figures] and for [distances / intervals] (מרחקים): for as the first is to the third, so is the second to the fourth.

The arithmeticians (בעלי החשבון) are accustomed to write, at the last extreme, the term that comes out — whether it be one of the means or [the first extreme] — and [to arrange the known terms] so that [the operation] may be ordered for others. **Example:** If someone asks, "If [an amount] earns a profit of 4, what will [another amount] earn?" [The knowns] are called [and written as the Hindu-Arabic numerals] **6 4 8** [as written; reading order uncertain]. We wrote there [the terms], and all the [terms are arranged so that] the [operation] falls upon the known extreme; and the answer is part 2 [numeral: ?] — [worked value partly illegible], and this is the sought answer.

f. 36v

[… continuation of the rule-of-three example …] all that they earn … which they will earn. And now we know the [mean] term. The eight, and the [circle / zero] wrote [i.e. it is set down as] **4 6 8**. Now, when we multiply the [two] extremes and divide [the product] by the known mean, the unknown mean comes out. [Worked example:] we multiplied 4 by 6 [= 24] and divided [it] into [numeral: ?] parts, which are 8 — and this is [the] part [numeral: ?] — and this is the sought answer. [4 × 6 = 24; 24 ÷ 3 = 8.]

And likewise, if the question is reversed and [we set down] the first [term as a circle / zero], and one asks "what do they earn?" — if they [earn such-and-such], they earn four, as in this: **8 6 [numeral: ?] 4** [as written]. [Method reversed accordingly.]

[The discussion turns to ratios expressed as parts / fractions.] The first [term] … and [the value] is the whole [of the] second part … [the terms are arranged] so that when one part is [taken] against the [other] … [it does not] come out evenly

in parts, but a [remainder of the] computation is left over … [passage on fractional means partly illegible].

Example. If someone asks concerning measures (מדות): [given] measures [numeral: ?], simple [units]… what will they buy? And if we can, we double [take a multiple] by three [thirds]… the one [is] 7 [and the arrangement is written] **14 7** [as written]. And always, in this arrangement, it is as [with] the means: you divide [the product] by the first extreme and [set the result] at the last extreme; the mean is the one operated upon by the divisor imposed on us; and if it does not come out [evenly] in parts, a [remainder of the] computation is left. And [as for] the parts (fractions): if they are not equal [in kind], [we treat them so that] the whole [is preserved] … [numeral: ?]

[Further worked case, arranged as] **[numeral: ?] 7 14** [as written; leftmost digit uncertain]: we wrote the means, and they were [numeral: ?] parts of them … [and the divisor / remainder handling is described] … so that the part [equals such] and this is the part, and [the remainder] that is left is also [numeral: ?], and [the operation] is not evenly divisible.

Question. [Given] parts that were [numeral: ?] and [numeral: ?] … [and the] parts they had were seven-eighths … how much will they [buy]? [The knowns are set down as] **11 7 4 [?]** [as written; digits uncertain]. But if one asks: if [such] are the dinars (דינרים) — if [such-and-such] the dinars earned [profit], how much will [another] earn? — and their reciprocal is written thus: [numeral: ?].

[The discussion of proportional parts continues, with the terms called "the one" and its "counterpart," and the arrangement of the means and extremes described.] Each of them [proceeds] according to its own [ratio]. **Example.** If someone asks, "If [numeral: ?] measures cost [numeral: ?], what will [more/fewer] measures cost?" — [worked with dinars]: the dinars set down [numeral: ?], and they made [these] arithmeticians work [it] … [passage partly illegible] … the number of the [terms is arranged] according to a single path, [and] then everything comes out fittingly, one by one. So that [the operation] falls upon [the known] extreme — that is one path — and its counterpart is read, and the [product] falls upon the known extreme; and it is done thus.

[Margin, f. 36v, left (gutter) — vertical annotation in the same or a similar hand: verbatim text illegible at this resolution; marginal note present, further identification requires further verification.]

Primary script: compact Sephardic semicursive, single hand. Marginal folio number "37" (top-left, p. 23). Body text is Hebrew mathematical prose with Hindu-Arabic figures set in the right/left margins as worked computations.

[f. 37r]

[Conclusion of the rule-of-three worked over elapsed days]

We multiply the first extreme by the days that have passed; whatever results is the base (היסוד) by which we then divide the number. So it was done just now [Margin, worked figures: 6 2 4 8 3 6 [numeral: ?]]: as was done in three units [?], and they came to 18 [יח], and this is the base. After we had multiplied the extreme by [numeral: ?], the total came to [numeral: ?], and this was the base by which we divided; the quotient was the part [sought].

And so with the others: when we divided [numeral: ?] into [numeral: ?] parts, the result was [numeral: ?], and there remained [numeral: ?]. Twelve [יב] parts remained [?], and the quotient was [numeral: ?] parts. Whoever wishes may take these [numeral: ?] parts, and there remained of them [numeral: ?] parts, at [numeral: ?] units each — as if he had taken [numeral: ?] dinars, and had taken a half-dinar [½].

The Distinct Part [החלק הנפרד], which speaks about fractions (הנשברים).

Said Mordecai: Since computation is [a matter] of quantity (הכמה), the composite quantity, together with the divided and the dividing, are the two extremes. For whatever admits division down to its whole [yields whole parts], whereas whatever does not admit division down to its whole stops at one [i.e., leaves a remainder]. And it is possible for a thing to be composed of parts that do not complete [a whole]; therefore the sense of "fractions" is the opposite of the sense of "integers" (השלמים). For the whole of the integers descends from the numbers that admit [division], while the unit (the One) is what stands composite between the integers and the fractions. Thus [the fraction] holds to both sides [?], as [we said].

And the denominator (המורה, lit. "the one that indicates") of the fractions is — as we shall explain — the number over which they are set: [the fraction terms] are divided by the rank of the denominator, and thereby the sought part is found.

Example. It was asked [שאל הגאון[?]]: how much is three thirds times a third [numeral: ?]? And the result will be [numeral: ?]... [the worked figures here are too abraded and too small to resolve on the scan].

The denominator here is [3?]: for since the multiplied terms are thirds, the denominator is 3; and 3 threes are 9 [ט], and they became [numeral: ?]. So the whole of the thirds became [numeral: ?].

Another example. The asker [השואל] asked: how much is [numeral: ?]... the denominator is [numeral: ?]. When we multiplied [numeral: ?] by [numeral: ?] it was [numeral: ?], and the whole of the quotient came to [numeral: ?]. Whoever computes [it] this way — a third times a third is [a ninth?]; for [the fraction] rises above the [whole]. This is the guide of the fractions: divide by [the denominator?], and the quotient is the part.

We set one over one, in this form [Margin: 1 | 1], and multiply the one by the one, giving one, one; likewise multiply the 3 by the [3], [Margin: 3 | 3] — what is the ratio of 1 to 1 [?]... and that is the sought.

Another example. How much is [numeral: ?] thirds multiplied by three thirds [ג שלישיות]? [Worked figures: [numeral: ?]; margin fraction 3.]... and it becomes... above [numeral: ?].

[f. 37v]

The thirds set below, and joined with a single [fraction] line — and likewise we did [again]. [Margin, fraction stack: ⅔ · 4/4 [?]] We know that the [product of the] two denominators is [numeral: ?]... [and the] quarters [רביעיות] with [numeral: ?]. What is below with the whole below [numeral: ?], and they became [numeral: ?].

The whole of the thirds, together with integers and fractions, we cut off [reduce] over [numeral: ?]... And so we did again, so as to reduce them: the multiplied fractions [numeral: ?] were set over [numeral: ?]... And so they set fractions over fractions, joined together — as if they were a single third [?]. And when [the fraction] rises higher [numeral: ?]... the quotient [numeral: ?].

Example. How much are 3 whole [numbers] kept over [numeral: ?] thirds? We took the [numeral: ?] and the 3 wholes; and what is the denominator? Three. We multiplied [numeral: ?]... [worked].

Now the fraction is the divided quantity [Margin fraction: ¾], and its denominator: over [numeral: ?] we set the thirds and the wholes [numeral: ?], and joined them with a line. And so a third with a third is [numeral: ?], and so [the reduced form?]... which is the sought.

Question. How many are the parts of the fractions [numeral: ?]... We divided [numeral: ?] over the quarter (רביעה), which is the denominator that we took from the fractions... and that is the sought.

If one wishes to set fractions against fractions, likewise: [take] the wholes... in each of them separately; and one [term] we multiplied for the thirds. [Margin, worked figures: 34 43 [?] · fraction ⅔ · 3] Wherein at first we set thirds together with wholes; and much [numeral: ?].

Again with the wholes: for it is the [product of the] fractions of the divided quantity, and it is [numeral: ?]... and again the fractions still [set] over [numeral: ?] thirds, together with the [numeral: ?].

For it is the guide (denominator) of the wholes, and it is [numeral: ?]... and again with the [numeral: ?]... [set] over [numeral: ?] thirds... and 3 [wholes?].

Again with the [product of the] fractions, and it was [numeral: ?]... and the fractions still [numeral: ?]... and the two denominators, being [numeral: ?]... and the whole [reckoning] came to [numeral: ?].

He took [numeral: ?] dinars, and the fraction — this is the guide of the fractions [numeral: ?]... For the question, the way is not according to the earlier guide, and the fractions... whether they are [numeral: ?] thirds or [numeral: ?] quarters. And likewise the fractions are the sought.

[Margin, worked stack: 19 · 8/3 · 5/40 · ⅝ [reading uncertain]] [Margin, division (galley) box: 9139 [?] | 240 · 0 [numeral: ?]]

the fractions we set over dinars [numeral: ?], other numbers [numeral: ?]... And they were reduced [numeral: ?], and other numbers [numeral: ?].

BnF Hébreu 1031, ff. 38r–38v. Continuation of the treatise on fractions (הנשברים): multiplication of chains of fractions by multiplying the numerators (the upper row / הטור העליון) together and the denominators (the lower row / הטור התחתון) together, with worked fraction-stack examples.

f. 38r Again, take another of them, [multiply] the whole together with this one, and it becomes [numeral: ?]; and we say [that these are] multiplied in this part. And here is the number that has been divided [into parts]. Multiply the upper row and it becomes [numeral: ?]; and we say [that they are] multiplied over it. Again, take another of them, [multiply] the whole together, and it becomes [numeral: ?], and we say [it is] multiplied likewise.

Then multiply one part by another, and this is what emerges; and this the reckoning requires. And when the sought number [המבוקש] is a fifth part of the whole, of five [parts], according to this ratio [it will be] as the ratio of one to another, and so on. Example: the questioner asked, if [there are] fractions[?] over one another[?], [numeral: ?] and [numeral: ?] and quarters — the fractions are so-and-so… [line partly illegible — further candidate].

That the quarters are all [numeral: ?]; and multiply the upper row and it becomes [numeral: ?], and combine them [וחברנום] as before. And the same when you multiply the number of the quarters and the eighths[?] [השמיניות].

[Marginal fraction-stack, upper right (with interlinear corrections — several digits uncertain):

upper row (numerators)	3 [interlinear correction: 4]	4	4
lower row (denominators)	4	4	8

with a boxed sub-computation "3/4" and "8" at the head, "32" beneath the last denominator, and a lower running line reading `1 0 [?] 4` and `1 2[?]`. Because this stack carries interlinear corrections and gutter-crowded digits, the interior values are `[numeral: ?]`**]**

It becomes [numeral: ?] and multiplied likewise, and it comes out as [numeral: ?]; and this is the foundation [היסוד] on which it is divided. Again, take another of them, the whole together with this — again multiply the whole together, and it becomes [numeral: ?], and we say [it is] multiplied over them.

Again, take another of them: the ratio in the upper row, and it becomes [numeral: ?], and we say [it is] multiplied over them. Again, take another of them, the whole together, and it becomes [numeral: ?] and multiplied over them. Again, take

another of them: the whole together, and it becomes [numeral: ?], and multiplied [numeral: ?] over them. Then multiply one part by another, and here you know the sought[?] which emerges, which is the whole quarter[?].

Example: one by itself[?] … we wanted to make a base for six[?] fractions [numeral: ?] … [line partly illegible]. They combined the whole with the whole [and it is] the upper ratio, and it becomes over them. Again, take another of them, the whole together, and it becomes [numeral: ?], multiplied likewise. Again, take [numeral: ?] and multiply them as before.

[Marginal fraction-stack, lower right — chain of fractions multiplied, with a cross (×) mark between the 4/5 and 6/7 columns:

numerators (upper)	2[?]	4	6	8	10
denominators (lower)	3	5	7	9	10

Running partial products written beneath (right→left): **15**, **56**, **504**, **5040**. (15 = 3×5; 56 = 7×8; 504 = 56×9; 5040 = 504×10.) The head bears an "8" and the far-left column an abbreviation-marked ס[?]. The × mark sits in the gutter between the 4/5 and 6/7 columns.]**

take another [numeral: ?] and multiply them as before. Again, take the ratio, the ratio and multiply them likewise. Again, take [numeral: ?], and multiply the whole together — and it becomes [numeral: ?]; and we say [it is] multiplied likewise; and this is what emerges, and this the reckoning requires [in dividing].

And its ratio [is] the fifth part[?] together, and it becomes the fifth [part], and multiplied, and it is what emerges — and this the reckoning requires, this whole [line partly illegible]. Again, take another of them, [numeral: ?] together, and it becomes [numeral: ?], and multiplied over them. Again, take another [line illegible — further candidate].

The upper ratio[?] over them, and it becomes [numeral: ?], and it was the answer[?]. And what emerges is what you know: this is the whole ratio, of one [numeral: ?] of the remaining fractions. And it comes out [numeral: ?], and it becomes [numeral: ?].

And here the fractions were divided[?], and the remainder [numeral: ?], and we know what its ratio is; this[?] is the way, that so-and-so… of a fifth[?] of a whole [numeral: ?] and so on. And there is another way beyond these, which is a high[?] and honored[?] one — to divide fractions by fractions, and this is [it]. Example likewise:

f. 38v

The first ratios [i.e. numerators], [in] the question that they are — if they are[?] three[?] measures[?] together, [numeral: ?] and multiply them alone[?] and take[?] the fifths, all the measures, so and so — after this we combine the parts, [and] the upper ratio [numerator] together with [numeral: ?], and one part [numeral: ?]. And after the parts, what is the smallest of the parts — the upper ratio over what emerges from the part.

[Marginal fraction-stack, upper right:

numerators (upper)	2	4	3	4	4
denominators (lower)	4	8	4	4	8

Head marked with a ס`[?]` abbreviation and a small "8". Interior digits partly gutter-crowded — the flagged cells are `[numeral: ?]`.]**

And this[?] is the smallest of the parts. And it was the upper ratio, and it multiplied, and this is what the reckoner requires. Example: they prepared the [numeral: ?] together, and it was [numeral: ?], and the fifths[?] and the eighths.

The part[?] was over it [numeral: ?], and it becomes the part [numeral: ?] of a fifth[?] and eighths and tenths[?]; and after this the part [numeral: ?] of a fifth[?] and eighths, the tenths[?] over them. And it becomes the part [numeral: ?] of three thirds and [numeral: ?] of eight-parts[?], and it becomes so-and-so.

Again, they prepared the [numeral: ?] together, so that it was a [numeral: ?] part, and here is a small part [numeral: ?] over them; and it becomes the part, the one after another — and after them the part of [numeral: ?] and the [numeral: ?] fractions[?] which emerge, so-and-so. Again, take the whole together, and it was a [numeral: ?] part, and here is a small part [numeral: ?] over the part; and it becomes the part, the first[?] one.

After this, [take] the fractions of a fifth[?] and a part [numeral: ?] together [in] the question, so-and-so, until we come[?] to the remainder, which emerges. And the fifths[?] together, they are the ratio, and it becomes [numeral: ?] and a part [numeral: ?] and the [numeral: ?] fractions[?], the remainder together, [numeral: ?] a fifth of a whole. And so on.

And if you wish to multiply fractions with fractions, you should make a denominator [מורה] to give a measure to them, and to bring the fractions[?] over them:

[Rubric, spaced out:] the denominator, and to give a measure over the denominator, and know that so-and-so will be so — the denominator[?] and fractions[?] over them.

Example: the questioner asked: [if there are] three measures together with [numeral: ?], how many will they be? They prepared the whole together with the [numeral: ?], and it was [numeral: ?], and this — the denominator, and here is[?] [numeral: ?].

The fractions gathered [מקובצים] [numeral: ?] and combined [numeral: ?] together, so-and-so, so that the other[?] fifth[?] will be — a whole[?] and a part [numeral: ?]. And likewise if the fractions came from the kind of the fractions[?], "so much of so much," [as] one does. **Example:** two[?] fifths[?] of quarters, three thirds[?]…

The whole together is a part [numeral: ?] over the [numeral: ?], and it becomes [numeral: ?]; again, take[?] the whole together, and here it is, and it becomes [numeral: ?], and this the denominator[?].

And the [numeral: ?] together the questioner asked, and it was [numeral: ?] and combined and multiplied over them; and it becomes [numeral: ?], and the [numeral: ?] fractions over them. And here is a small part [numeral: ?] over the fractions, [made] on a single denominator [מורה] to give them a measure[?]; and here is a small part [numeral: ?] to give — you know the sought[?] of the fractions[?].

And here it is the sought[?]. **Example:** they wished to divide a part [numeral: ?] over[?] the fractions [numeral: ?]; and the denominator[?] is what comes out[?], because it is over the whole[?] — 7[?] of them [numeral: ?], and every [numeral: ?] fifth[?] they are [numeral: ?] and [numeral: ?] the smallest.

This is a part [numeral: ?] of a whole. And they were the whole[?] and one part [numeral: ?] and 90[?] of the sought[?] — this is what emerges. And if there are integers [שלמים] together with fractions, so-and-so — one does so much, so much on the denominator [מורה], and it is a great whole[?], and one comes to divide the fractions[?]; and they multiply as before, and combine, and one adds up the number of the fractions. And likewise if you do it, the fractions again, and one adds the number [of them]; and again you divide it once more…

so divide this by that, and the result is what you seek.

Example. We wished to divide 3 wholes and 2 fifths by 2 wholes and 3 sevenths. Their common denominator: since 5 [times] 7 are 35 [לה], that is the whole [i.e.

the common denominator]. The 3 wholes are 105 [קה] — for each whole is 35 parts — and the 2 fifths are 14 [יד]; we joined them and they became [numeral: 109 — written קֿסֿ as it stands; the sum 105 + 14 would be 119 (קיט): scribal or reading discrepancy, further review]. Again we reduced the other 2 wholes to [35ths], giving 70 [ע], and the 3 sevenths are [numeral: 15? — the expected טו is not clearly resolved]; we joined them with 70 and they became 85 [פה]. We then divided the one total by the other: it became one whole, and there remained [numeral: ?] parts, which are as many [35ths] as [the fraction of] the divisor.

And likewise, if you wished to subtract fractions from fractions, you do as with this: by combining the denominators as at first, and look at the parts, and subtract [one] part from [the other].

Example. We wished to subtract 2 sevenths [from...], by the measure [described]; and the denominator is [numeral: ?]... [numeral: ?]... [the remaining fractional working of this subtraction example is legible as prose but its embedded totals are not resolvable at this resolution — [numeral: ?] throughout]... and it became one whole and [numeral: ?] parts of a whole [numeral: ?].

מקום השאלות — The Section of Problems

And now I shall begin for you with the nature of the problems, those that will be unfamiliar to you in your reckoning. [Marginal, right of the heading: verbatim "[?] שך" — brief note, further identification requires further verification.]

The problems are of [several] kinds, [drawn] from the parts [i.e. fractions] discussed above, [together] with the parts of [numeral: ?]; whichever of these you have [before you]...

[There follows a run of worked commercial problems (שאלה … "A man bought/sold …"), each phrased as a rule-of-three / profit-and-loss exercise in dinars. The connective prose is legible, but much of the numerical data — quantities purchased, prices, and profits — is carried by small gershayim numerals in running prose; the values legible at 4×4 are committed below and the residual small totals are flagged. See the QA report.]

The matter is thus, as I shall show you: and how it is done. See, then, that at the beginning [take] the denominator, and [numeral: ?]... and behold, [it comes to] one whole; and this is the [required] number... and take from the whole...

And once the denominator has been [set up], as [with] one whole, so it is also with the [several] parts. When the [numeral: ?] parts came out [to] one whole, and there remained from the parts [numeral: ?]... [numeral: ?]...

And if you wish, at the beginning, [take] a part of [numeral: ?]; [when they are] [numeral: ?] whole parts, then there are [numeral: ?] parts, [and] the remainder [numeral: ?] parts, [numeral: ?], [amounting to] ten parts and [numeral: ?].

Problem. [The method of] our joining [the fraction to itself]: how far the [way] goes, and the fifth part, and it will be [numeral: ?]... for [numeral: ?] wholes are [numeral: ?], and [numeral: ?]... and the fifth part likewise, so was the denominator; and joined with the denominator...

so also with the sevenths; and behold, [the] two sevenths, and [it] came to two wholes, and [the] two [remainders]; and it will be [numeral: ?]... and it also came [to] the same [as] the wholes... and it became one whole, and [numeral: ?] parts of a whole, and [the fraction] over it [numeral: ?]... and it will be [numeral: ?] [of a] fifth, and its seventh, so it is a fifth and a seventh... and there were [numeral: ?] fifths [numeral: ?]... and so make an [example] of your own.

Problem. A man bought [numeral: ?] hides for [numeral: ?] dinars... [sold... at a profit / loss]... a fourth of a fifth... and it will be [numeral: ?]; for [numeral: ?] fifths, and the whole, and the parts... [numeral: ?]... [the ratio-working of this hides-and-dinars problem is legible as prose but its figures are not committable — [numeral: ?] throughout].

The problems that are worth [numeral: ?] of the whole [dinar?]; and the [merchandise] is worth from the parts... one-eighth [numeral: ?]... and so make [the like].

Problem. A man bought [numeral: ?] fifths for [numeral: ?] [dinars]... [and it will be] as [numeral: ?] dinars, and the whole; and it will be [numeral: ?], and the parts of the whole... [numeral: ?]...

Problem. [If] 20 were 5 parts... and a fourth [of a] part... and a fourth part [of the] whole... [numeral: ?] dinars, and the tenth [numeral: ?] dinars, and a fourth [numeral: ?], and the whole. Between them, [numeral: ?] dinars... [numeral: ?] dinars... and the whole [numeral: ?]... [numeral: ?]... and a fifth [numeral: ?], and the profit [was]...

[The manuscript sets out the worked figures for this problem in Hindu-Arabic (ghubar) numerals on a scratch line:]

5 · 30 · 60 · 44 [ghubar figures as written]

that this is [what] the wholes come out to; and thus, when you multiply thus, the wholes come out... and behold, one part, so also they multiplied it by the denominator, [after] the fashion of the whole...

the denominator [numeral: ?] fifths, and [numeral: ?]... for it will be a whole; and a fourth [of a] part, and a fourth part of a whole... [numeral: ?]... [the ratio-working of the following problem, worth so many dinars each, is again carried by unrecoverable small numerals].

Problem. [If] 15 are 4 parts... [dinars]... and how many will they profit? Its answer, in the manuscript's ghubar figures:

40 · 11 · 6 · 5 [ghubar figures as written]

And how many [dinars]... and one whole and [numeral: ?]... [when you have] set up the denominator, as [with] one whole, so also with the [several] parts, and [numeral: ?] fifths and a tenth, and a part...

ten [numeral: ?], and thus [the merchandise] is worth [numeral: ?] dinars... Again, according to the value that is a third, so [is it with] the dinars... [and] how many will they profit? Its answer, in ghubar figures:

40 · 11 · 8 · 5 [ghubar figures as written]

and once the denominator has been [set up], as [with] one whole, and the part is one whole, and [numeral: ?]... [numeral: ?] fifths, and its seventh; and a part [of a] whole, and one part... and it became one part of a whole, [numeral: ?] and its seventh... Again, according to the value of your seventh, so [it is with] the reckoning... הכ" (uncertain; possibly a catchword to the following folio). further verification required.]

*Mordecai ben Eliezer Comtino, treatise on arithmetic and geometry. Compact Sephardic semicursive. This batch: the ongoing "מקום השאלות" (Section of Problems) — commercial partnership, wage-division, and cistern (work-rate) word-problems. Worked answers appear as Hindu-Arabic figures written left-to-right, embedded in the right-to-left prose. Book Two (geometry) has **not** begun on these pages.*

[Librarian note: "40" — folio number in the upper-left margin of f. 40r (later foliation, brown ink).]

[f. 40r]

(The top of the page continues, without a break, the profit-sharing computation begun on the preceding folio.)

if the divisor [המורה] is [?], then, over the whole that constitutes the fixed capital [הקבה], and [?] each shareholder takes per dinar [?] — how much will they profit? **50 · 40** ‖ | [closed by the ring ○].

The other share: the shares that they take, the whole [?], since the profit too was not [?] [?].

We multiply the fifty, and this comes to [?]; and we divide it — **40 · 10** ○ — and the [result is in] quarters and fifths, and each [?] takes [?].

9 ○ **6** ○. We multiply the fifties, and they were thus [?]; and its quarter and its fifth, and we divide [?].

And after [?] the purchase [הקנה?] [?], for the whole [?] according to the ratio [?] — [?] dinars — and each one [?].

Another way [דרך אחר] to reach this same [answer]: [?]; and each one profits per dinar [?], and thus [?].

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Problem [שאלה]. [?] a partner [?], [?] its rate [שער] [?], and [?] the fixed sum [?] — how much will [each] one take according to the ratio [?]? [?] dinars [?].

We multiply [?] — **[?] · 4** ‖ — and the divisor [is] the value [?]; and [?] took [?], and [?].

——

Problem [שאלה]. [?] — how much will [each] one take, given that [?] took [?] dinars [?].

[?] and the share came to [?], and it was [?] dinars; and [?] the fixed capital [הקבה], and [?].

So too [כך אם]: since the whole was [?] dinars, [?] according to the ratio, [?]; and [?] took [?].

Since the whole [?] [?], he takes [?]. And after [?] the purchase [?] each one [?] and takes [?].

Problem [שאלה] — a certain man [?]. He took [?]; and [?] [?], and the divisor [was] the value [?]. And [?] on the fifty; and its half [?] took [?].

The share [was] **8** and a fraction, and twenty [?]; and [?] on fifty, and [?] the divisor [?].

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Problem [שאלה]. A certain man [?] with his fellow [חברו] [?] [?]. [?], since his share is a seventh, and another [?] — a problem [?].

[?] the purchase, the buyer [?] [?], and it is thus [?].

And its length [ארכו?] [?] — how much [?], and how [?] each takes according to the length [?].

[?] and [?] the share was [?] and forty [?].

———

(A short profit-sharing tail, closed by figures.)

he takes according to the ratio [?], and how much [?]. thus, since the whole was [?] dinars, he takes [?], and it comes to [?]. he divides the capital [?], and the share was [?] and a fraction [?]. **18 · 8 · 15** ○ — for this is the fixed capital [הקבה], and the share was [?].

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Problem [שאלה] — a certain man hired [?] [שכר]. Reuven [ראובן] is to work [יעבוד] alone [לבדו] [?]. Reuven works with him [?] all these days [?], and [?] takes his [wage in] dinars [?]. And if [ואם] [?]. Reuven works alone [לבדו] [?] according to the ratio [?], the whole [?] worked [עבד], and [?]. and each one — and thus [?] worked, each one [?]; and I ask [ונבקש] [?]. [?] and his fifty, and the [?] came to [?].

[f. 40v]

from the dinars [מהדינרים], and the rate [ושער] now [?] is one [אחד], and [?]. break [שבר] the whole into the fractions [הנשברים], and put the [?] dinars [דינרים]. rate [שער]; and [?] Reuven's rate [שער ראובן]. So too [כך]: if [?] was [?]. **24 · 4** ○ — we multiply the fifty [נכפול החמשים], and they were his [?]. Reuven [ראובן]; and [?] Shimon's rate [שער שמעון] [?]. **12 · 4 · 24** ‖ ○ — we multiply the fifty, and they are [?].

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Problem [שאלה] — a certain man had a cistern [מתקה?] [?]. One [אחד] put into it one measure [מדה אחת], and there remained [ונשארו] [?]. So too, since the whole [?] according to the ratio — how much [?] according to the ratio [?]? and the share was [?] and its half [?] and twenty [?]. each one [?] measures [?], profits [?] according to the ratio [?]. on the fifty [?], and the share was a seventh [?] and a half [?]. on the [?] fifty [?], and [?] the share [?]. So too, since the whole was [?] according to the ratio; and the [?] according to the ratio [?].

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Problem [שאלה] — one had [?] measures [מדות] [?]. One put into it [?], and there remained [ונשארו] [?]. So too, since the whole [?] according to the ratio [?] — its length [?] — and each one [?]. the share was a quarter [?], and [?] each one [?] according to the ratio [?]. he takes [?] the [?] measures [מדות] [?], and [?] Shimon [?]. measures [מדות] [?], and after [?] the fixed capital [הקבה] [?]. So too: since [?] the fixed capital [הקבה], and its quarter [?], the share was a third [?] and its half [?].

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Problem [שאלה]. [?] and Shimon [ושמעון], the fixed capital being [?] [?]. they put [?] according to the ratio; and I ask [ונבקש]: [?] with the whole profit [כל הרווח] [?]. the share [?], and the whole [?] according to the ratio, and the [?] third [?]. each one [?] measures [?] according to the ratio [?]. **36 · [?] · 48** ○ — for it [?] was one, and it is [?] measures [מדות] [?]. And each one [?] the [?].

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Problem [שאלה] — [?] hot [חם?] in water [במים], thirds [שלישיין] [?]. The length [?]; and [?] the whole [?] the fixed capital [הקבה], they were [?]. He set up [עשה] the rate [השער]: so too [כך], if [?] profits [ירויח] [?] — how [?]? And above [ולמעלה] [?] the water [?], a share [?]; and how much [?] we add [נחבר], and [?] the whole profit [כל הרווח]. And I ask [ונבקש] [?].

[Margin: a two-word note in the lower-left margin of f. 40v, reading approximately "ושלישית / נ״ח" [?] (later or same hand); further identification requires further verification.]

*Section of Problems (מקום השאלות), continued. Compact Sephardic semicursive; commercial / partnership / wage / cistern / interest word-problems. Worked answers in Hindu-Arabic figures (left-to-right within RTL prose), rows closed by a ring "○". Book Two (geometry/measures) has **not** visibly begun. Much of this folio is*

dense cursive; specific unreadable tokens are flagged `[?]`, numerals `[numeral: ?]`, and the whole opening is marked a in the report below.

[Continuation of the preceding fraction problem.] The denominator is 30 (ל׳); its fifth and its ninth [together] [?]; subtract from the denominator and 19 (יט) remain. Multiply the [?]; multiply the fourth [?] [numeral: ?] — [several tokens illegible] — divide them by 19 (יט), and the result is [?] parts and a part of [?]. This [?] [?], and so it works out:

> `[Arabic scratch-row, partly legible: 90 30 … 5]` ○

Problem. If there were two partners (מתעסקים), and the one gave [?] in exchange, and the second became to him [?]; the surplus (המותר) between them we divide, crediting each one's account according to his ratio (כפי יחסו), and [?] between [the partners] [?].

[Cistern problem.] [There is] a pool of water (בריכה מים), fitted with pipes [דכתבים = spouts/pipes(?)]: in [?] hours four [pipes] pour down and flow in, and in [?] hours [others] draw up, [?] per hour. How far is [the depth of] the pool [?] the base? For each [flow] we take a square (מרובע): [combine] this with the square of [?] [numeral: ?] — [computation clauses largely illegible] — so [?] parts; and each [?] parts [?], and thus [?] parts, and we work [?].

[The middle of the strip carries left-margin scratch-figures from a successive-approximation ("guess-and-check") computation. The phrasing recurs: "again, when we take its fifth / its third of its half, there remains … as the sought number (כהמספר קנוס[ה][?]), and this is the nearer one."]

> Approximation figures (left margin): guess row `2 3 4` (leading "2" mildly uncertain) ‖ result row `6 4½` (½-vs-¼ residual on the final figure)

Again, [each] one of them [?]; when we take its fifth of its half [?], there remains [?] as the sought number, and this is the nearer [approximation]. `[numeral: ?]`. Again, [each] one of them [?]; when we take its third of its half [?], there remains [?] as the sought number. And behold [?] is [?], and [?] is near [?].

Test it (כפול הכל): [take] this with [numeral: ?]; and [each] of them [?] [numeral: ?] — [the verification clauses run through several sexagesimal-style part-computations that are individually illegible] — like the first [?] from the fractions; and so the [?] from the denominator, and the others; and so [it is] with the [?] in their [?] [?].

[Wage problem.] A man hired [a worker] [?] for a wage [?]; [numeral: ?]. The first [?] to him [?] gold coins (זהובים), and stipulated: if he works with him a month of days [חדש ימים], then [there is due] to him [numeral: ?] gold coins; [?] and in each [?]. And if he works with him a month of days, [there is due] to him [numeral: ?] gold coins; and [?] threefold [?], which are thirty [?]. [?] that if he works with him a month of days, [there is due] to him [numeral: ?] gold coins [?]; and he worked for him the whole month [?]. We add [?], we divide [?] gold coins.

Here is the calculation (הנה חשבונו): [?] — [division of the wage into shares runs to the foot of the folio] — all the [?] parts come out, and twenty [?] and the one [?], thus: for the denominator [?] and its quarters (רביעיתן) will be, in all, [numeral: ?] gold coins [?] pennies (פשוטים); and the value of the first [?] becomes [?].

[Partnership / fixed-capital problem, continued.] Thus: if you know [that] they credit him 24 (כד׳) [as] the fixed capital (הקבה), how much do they credit [the other] [?] ○

    50 [numeral: ?] [numeral: ?] [numeral: ?] ○

Multiply the fourth [?] and there are three parts [?] we divide by [?] the fixed capital [?]; and so the first share [is] worth [?] the denominator [?] and its [?] pennies (פשוטים). And [?] the first share [is] according to the [?] gold coins. And the calculation is thus: if [?], so much is the fixed capital [?] gold coins, thus:

    50 [numeral: ?] [numeral: ?] ○

And these [are] pennies, and its fifths [?] pennies, and its quarters [?] pennies; and we add [?] the share on account of the gold coins.

And the question runs thus: just as if you know how much the fixed capital [?], how much do they credit [?] the fixed capital [?]. And it will [?] on the account [?]. And the share [is] worth [?] the denominator [?] pennies, and we work thus:

    50 [numeral: ?] [numeral: ?] [numeral: ?] ○

[?] the fixed capital [?], and there are [?] pennies, and [?] pennies, and this share [?].

[?] the fixed capital [?], and behold [?] pennies [?]. And on this footing (ובזה החשבון), each of the shares [comes out] evenly [?] gold coins [?]; and [?] does not fall [?] [?].

[Display line, larger hand — uncertain reading, likely a sub-section rubric introducing the following interest problems:]

`[Display heading: "[?] לשער [?]"]` — *possibly "the section of interest (שער הריבית)" or a method-transition; not confidently legible. See QA*

A questioner asked me (שאלני שואל), putting to me [?] concerning a certain sum, and he knows that the interest (הרבית = הריות) is such-and-such per [period]; and with this [?] the amount of interest on them. Now he wishes to know how much the principal (הקרן) was, and the [interest] of each one separately.

Answer. For this, you estimate how much the interest is per [period], and we add [?], and we divide the whole rate (השער), and the interest comes out for you, and the remainder is the principal. Example: the sum was ninety[?] `[numeral: ?]` — [?] full [?] and ninety — [?], and the interest was 15 per hundred (טו במאה) `[= 15%]`. You estimate it first: it is a quarter (רביע); take a quarter and further [?] [?]; divide the whole sum into five, and out comes [numeral: ?], and this is the interest, and the remainder is the principal.

But if the interest is [numeral: ?] or [?] per [period], there is no whole value, for the value of [?] the first interest was [numeral: ?] `[computation breaks off]`. And likewise, [the] beginnings [?] which are [numeral: ?]; and in [?] we add [?] and out come the fifths, if we divide the [?] —

**

The Second Book — On the Science of Geometry (חכמת המדות)

The Second Book of this treatise, which discusses the science of geometry (חכמת המדות).

Chapter One, of the First Part of the Second Book. — Said Mordecai [Comtino], the Constantinopolitan (קושטנטינו):

[Margin (right of heading, second hand, small script): שמות שנשתמש בם[?] — "the terms we shall use"; a preceding word illegible.]

On the science of geometry we wish to explain [what follows]. In this discipline we shall first set out the special terms (שמות מיוחדות) that the practitioner of this science habitually uses in its subject matter — the terms that guide one within the domain of this science. Since this is so, [the geometers] chose to rest their work upon the definitions they laid down for whatever they defined in this particular science, whether by way of a problem (שאלה), by way of a supposition [?], or by

way of demonstration (ההעתקה [?] "proof"); and, for the students, also by way of complete conviction. It is therefore fitting that the author of a book instruct in this and make his words clear, so as to explain where the error lies into which the masters of this science have fallen [?]. And so the sage **Euclid** (אקלידס) did in the *Book of Elements* (ספר היסודות), in which he set out well all the terms that are needed [in it]. And now, as we said:

The **point** (נקודה) we posit as the indication of a thing that has no magnitude whatever — neither length, nor breadth, nor depth — and it is found [as the boundary of a line]. [?]... and so it is when [a line] terminates: the point is the end of one line and the beginning [?] of the other. The point is the [boundary] of every breadth that passes between two points [?]. The **surface** (השטח) is that which has length and breadth only, and its boundaries [?] are lines. The **plane angle** (הזוית השטוחה) is that which two lines enclose [where they meet], and the figure [so formed] is [?]. The **right angle** (הזוית הנצבת) is when a line stands upon a line and makes two [equal] angles.

The line that stands upon another line and makes two [right] angles is called a **perpendicular** (עמוד), and the line on which it stands is called [a base line] (קו נצב [recte:?] — MS reads קו טובב). The **parallel lines** (הקוים הנכחיים) are those which, when extended in both their directions, [never meet]; and if [a line] falls [across them...].

The **plane figure** (התמונה השטחיית) is [that which] a line encloses [?], and the figure whose bounding lines [are straight is the rectilinear figure]. The figure bounded by straight lines is that which is enclosed by three lines or more: that enclosed by three lines is called a **triangle** (משולש); that enclosed by four lines is called a **quadrilateral** (מרובע); that enclosed by five lines is called a **pentagon** (מחומש), and so on for the like. The lines that enclose the figure are called **sides** (צלעות).

The triangle is divided into three kinds. The first is called **equilateral** (שוה הצלעות), namely the one whose three sides are all equal. The second is called **isosceles** (שוה השוקים), namely the one only two of whose [sides] are equal. The third is not equal[-sided]: whether its [one side] be longer than each of the others, or its [one side] be shorter than each of the others — this third [kind] is called **scalene** (מתחלף הצלעות), namely the one no side of which is equal to its fellow.

Again, [the triangle] is divided [according to its angles]:

Diagram. A small unlabeled schematic triangle in the right margin, illustrating the right-angled triangle. No point-letters are inscribed on the figure.

— the **right-angled triangle** (נצב הזויות), which is the one one of whose angles is a right angle, thus [figure]; the **obtuse-angled** [?] triangle, whose [one] angle [is greater than a right angle], thus [figure]; and the **acute-angled triangle** (חד הזויות), which is the one whose three angles are [all] acute; [each having its own name] according to its angle, thus [figure].

Then the **quadrilaterals** (מרובעים) [are divided]: into the equilateral, and into that whose [?] angles [are equal...] which is right-angled, and is called **equilateral and right-angled** (שוה הצלעות נצב הזויות), thus:

Diagram. A small unlabeled square drawn on the diagonal (lozenge orientation) in the right margin, illustrating the equilateral right-angled quadrilateral (the square). No point-letters are inscribed.

and into that which is right-angled but not equilateral [?], and is called [an **oblong**] (מרובע ארוך — "long quadrilateral"), thus:

Diagram. A small unlabeled rectangle (oblong) in the right margin, illustrating the right-angled, non-equilateral quadrilateral. No point-letters are inscribed.

And any [quadrilateral] no two of whose sides are equal[?] and none of whose angles are right is called **like a rhombus** (דומה למעוין), thus [figure]; and [the rhomboid], which is called **the inclined** (הנוטה) — it is [a quadrilateral resembling?] נ [?] — and he will explain it [further] in this treatise.

The **rectilinear plane figure** [?] is divided into three... and [the fourth is?] the [trapezium]. The **circle** (העגולה) is a figure enclosed by one single line, [such that] the lines drawn from [a point within it] to the enclosing [line] are equal:

Diagram. An unlabeled circle in the left margin with two near-vertical straight lines drawn across it — a diameter through the interior and a second chord parallel to it — illustrating the diameter (קוטר) and a chord. No point-letters are inscribed on the figure.

— and that [interior] point is called the **center of the circle** (מרכז העגולה). The line that issues from the circumference of the circle and passes through the center is called the **diameter** (קוטר); [it cuts the circle into] two halves. The [chord?] is called a **diagonal** / **secant** (אלכסון). The **semicircle** (חצי עגולה) is the figure enclosed by [the diameter and half the circumference?]. And it is divided [?] into segments (החתכים): whether the [segment] be [greater] than a semicircle or less than it [?].

[Two lines here are degraded / compressed; several tokens illegible.]... the [segment] and its [?]... the two [angles?] which [?]... or [those] equal to them [?]... the [segments] of the circle [?]... this is double [?].

a sign / mark [?]. **The lines are ten [kinds]:** straight (ישר), [and] ראש, [?] מקוטב, [?] [?], שעוד, parallel (נכחי), concave [?] (קער), נטוח [?], tangent [?] (תקף), לבטון [?], and מירב [?]. [The list of ten line-types is transcribed as written; six of the terms are genuinely uncertain at this resolution]

The **straight line** (הקו הישר) is the one that lies evenly [between its endpoints?], [being the shortest between two points]. The [perpendicular] (האנך [?] — MS האבמטבה) is a straight line set up upon [another] straight line such that the two angles at [its foot] are right angles. The **head** [?] (הראש) is the point standing upon the [base] [?]. The **perpendicular** (עמוד) is the line that goes from the head [vertex] to the base and makes the two angles at [its foot] right angles. The **parallel line** (הקן הנכחי) is [the line drawn] parallel [?] to another line, [such that]... the angles [it makes] are equal [right angles?]. The [...] (הכתעד [?]) is the line that goes [across?]... together with the other angle [?]. The **perpendiculars** (האנכים [?] — MS האמוקים) are straight lines descending from the ends of the [chord?] onto [?] the base. The **tangent** (התקף [?]) is the line that touches [the circle] within, [meeting it] at one point [?]. The lines called [...] are [?] equal. And the **diagonal** (האלכסון) is [... continues on the following folio].

a line that goes out from the circumference and passes over the center and reaches the circumference on the other side, dividing the [circle] into two equal parts; and its half is the straight line whose end is [at] the perpendicular.

The angles are three: right, acute, and obtuse. A **right angle** is when a straight line stands upon a straight line and makes the angles on either side of it equal — each of these is called a right angle. An angle smaller than a right angle is called **acute**, and one larger than a right angle is called **obtuse**.

The kinds [סוגי [?]] of measurement are three: first, [the measurement of] straight lines; the measurement of the plane figure — that is [ר"ל = רצה לומר, "meaning"], of its area [בשבוע קרקעו [?]] and the like; and the measurement of the solid [הגושם]. The measurement of straight lines is that which is measured along length alone, and this is what is called linear measurement [מדידת מספרים [?], "measurement of numbers"]. The measurement of surfaces of figures — that is, of what has both length and breadth… from which the measurement of [area] is known; and it is also called "surface" [שטח]. The kinds of figures are five: the

square [המרובע], the triangle [המשולש], the rhombus [המעויין], the rhomboid [הנטויה], and the circle [העגולה].

The figures are counted as follows [?]. Among the quadrilaterals [המרובעים]: a square is equal-sided and right-angled. And if the quadrilateral's figure… equal-sided… [and] equal-angled, the rhomboid [?]; and differing [in that it is] a quadrilateral… and a rhombus and… [?]. And if the figure… they are four: right-angled, equal-sided, acute-angled, and obtuse-angled [?]. And if… the figure… [Illegible: ~1 line of the quadrilateral-taxonomy is too tightly written to resolve with confidence]. And the division of a surface [is by] subtraction [?] and addition; and the increase of the surface…

Chapter Two

Knowledge of the [rectangular] figure is the root and nature of all figures and of all rectilinear surfaces, for from it [everything else] is composed. And this [is shown] as [it] was made known… [Illegible: a proper name / source-citation, ~2 words, does not resolve — reads roughly מקויאתאט טיהרסיע [?]]… in the second Book [מאמר] of the Books of [Euclid? [?]], from the Book that… [?]. Therefore, whoever wishes to master the subject [המאמר [?]] and understand it will [thereby] know how to master all the figures and every area-computation [תשבורת].

[The passage now turns to the measurement of the triangle and the square. The tightly-written clauses that follow are only partly legible:] …the triangle by the number that is generated… [when two sides?] are equal; and if they were… equal, the area-computation [תשבורת] of the whole triangle will be… [?]. …and for this reason it was fitting to make known the measurement of the triangle first [?]… by means of the rectangle whose length is [multiplied] by its breadth [?]… the area-computations of the figures are…

[Right margin, f. 43r: marginal keyword gloss, verbatim: "ג׳ / הרובעים" ("3 / the quadrilaterals") — a finding-word; no attribution legible.]

[so] that the truth [of it] is known… and that which… only [?]… is division alone. Every space and every single quadrilateral [region], inasmuch as they have length and breadth [?] — for the quadrilateral too, in its sides, consists of the lines which are length alone; and likewise the circles… [Illegible: ~1 line]. …and it is set out [?] to explain how the quadrilaterals are measured, and in general [that] the square — the equal-sided, right-angled quadrilateral — is [as the product of] length by [the other side]…

The Proposition [הדבור], on the measurement of the square (the equal-sided, right-angled quadrilateral): When we need the right angle [?], from it [comes] a general root; for in every four-sided figure with four right angles we multiply the [length of] one side by the other, thus, when we square [it]. And this is the reason for its being a [general] root… [to multiply] one side by the second side.

Example: Let square אבגד be equal-sided; we set each of its sides at ten cubits [אמות]. Then a square whose side is [ten] — its area-computation is one hundred cubits; and thus is its area, for they are one hundred square, right-angled cubits, each…

Diagram. A large square ruled into a 10 × 10 grid of small unit squares (one hundred cells), illustrating that a square of side ten (cubits) has an area of one hundred square cubits. The whole square is the *מרובע אבגד* — prose-pinned by the worked example's "דמיון מרובע אבגד … שוה הצלעות" ("Example: Let square אבגד be equal-sided …"). The grid interior is blank — the cells carry no inscribed worked numerals; the value one hundred appears in the prose, not in the cells. Corner/edge labels:

- א (top-right corner) [confirmed]
- ד (bottom-left corner) [confirmed]
- top-left corner (ב/ג ambiguous) [?]
- bottom-right corner [?]
- ז ,ו ,ה (edge-tick / division-point labels along the top and bottom edges) [?]

[The interior grid-lines subdivide each side into ten equal parts; the interior cells are blank. The top-left and bottom-right corner letters and the small edge-tick letters (ה/ו/ז) do not resolve on the scan even under magnification; those labels remain unread.]

The Proof [המופת] of this: We divide each side into ten cubits at ten points, and from each point on one side we draw a line to the [opposite] side; then the lines will be parallel [נכחיים], each to its fellow. And the great square is [thereby] divided into small squares which are made [equal] and right-angled, equal [to one another] — so it is. This [set of] parallel [lines] we divided into ten parts, and likewise every [line] we divided into ten; and we divided the whole great square into one hundred small squares, each equal to its fellow. So there emerge two [rows of] figures [?], in each of which [each square is] equal to its fellow — so it is. Angle גב [?] is a right angle on every side; and likewise angle גד is a right angle on every side.

Cheshbon u-Midot — Book Two (Geometry), Part One, Chapter Two (Mensuration)

BnF Hébreu 1031, ff. 44r–44v. Compact ligatured Sephardic semicursive; single hand throughout.

[Foliation "44" in the upper-left margin (later hand).]

(This folio continues, without a break, the Proof begun on the preceding folio — the demonstration that the area of a square equals side times side, established by partitioning the square of side ten cubits into one hundred unit squares.)

and the figures [?] — their angles were right [?] and equal to one another. For since the cubit stands perpendicular [נצבת], this line falls upon that line; and so [the resulting cells] are equal squares. In the same way all the [unit] squares are equal to one another, and likewise all the [larger] squares are equal to one another. And this is what he wished to demonstrate.

Thus, whenever the figure's angles are right angles and its sides equal, its area is measured [by multiplying side by side]. And so it is demonstrated of every figure of this kind — whether the sides are [?] or [?]; and when [one] side is greater than the other, [?].

Now every area of a figure [may be] divided into parts; and the parts arising from [the partition of] the equal squares are equal to one another, while the [unit] parts are equal to the others. And so it is with the cubits [used] for the partition into parts — every partition of such a figure yields parts equal to one another [?]. And when the figure is [truly] a square, its parts [?] are equal to the sides. This is the way of the squares [?], and likewise of every figure whose sides are equal [?].

For the fixed [set of cells] are established in [their] measure [?]. And it has already been made clear concerning the area of the square, that [?]. And [so stands] the figure that is drawn here beside.

Diagram. A rectangle (taller than wide) ruled by parallel horizontal and vertical lines into a grid of small equal cells — the unit squares into which the figure's area is partitioned (roughly five cells across and about eight down). No interior point-labels are inscribed (positive finding — the grid bears no letter-labels); two small ink reference-dots sit at the upper-right and at the lower edge of the grid. The figure illustrates that the area of the rectangle is the count of the unit squares filling it.

Here [begins] the account of how [the square] is divided: take [the line of] the first unit square [?]; and the [several] points [?] — it is impossible [?] that the line [should be otherwise]. When the unit square is small, take it as the [measure], and so with every square and cubit [?]; and there is no [measure] of the area of the square but this. Take the figure [?] and divide it [?]; and its parts are equal to the sides [?] — thus it stands.

Diagram. A quadrilateral (nearly square) with both diagonals drawn together with a horizontal and a vertical median, so that the figure is divided into triangular and smaller quadrilateral parts around a central intersection point (marked by a dot). The construction shows the figure bisected and quartered by its diagonals.

- Apex (top, middle of upper edge): [?]
- Left (mid-height of left side): ד [?]
- Right (mid-height of right side): ה [?]
- Foot (bottom, middle of lower edge): ז [?]
- Center (intersection of the diagonals): marked point [?]

[Note: the figure carries ≥5 faint vertex marks (not merely the central dot); all remain [?] at the legibility floor — fig-p37-2 remains a for label recovery.]

הדבור [Proposition]. [It is proposed] concerning every figure having three lines [i.e. a triangle] [?] [that it is the half of the rectangle on its base]. [Let] the diagonal be אב [?] and the diagonal בג [?]; and let diagonal אב [?] be drawn, and they meet at a point [נקדה]. [Value stated:] twelve cubits [יב אמה]; and they intersect at the point [?].

המופת [Proof]. [?] the whole [?]. Behold, it is made into parts — the first part being [one triangle] [?], and the second [the other] [?]; and the two [triangles] are equal to one another [?], because each arises from [the half of] the rectangle [?]. This is the demonstration. And so it is with every rectangle whose sides [are as stated] [?].

[The rectangle] is [greater] than the [square?] [?], for it is equal to the shorter [side taken] the shorter [number of times] [?] [numeral: ?] of the rectangle. And it falls upon the [?]; and so [runs] the rectangle whose [sides?] [numeral: ?] [prose reference — no inscribed side-length numeral; not a lost value]. And likewise the [?].

For its measure is like twelve cubits [יב אמה], since it is equal to the shorter [side] multiplied [?]; and therefore it is a doubled area [?], [equal to] the rectangle [?].

And if you say [?] — that it is doubled upon [?] — [in truth] it is not so [?], and therefore its measure is the measure of the [?].

And so they said [ולכן אמרו] that [it is] as the measure of the area of the [?] [?]. And this is what he wished to demonstrate [?].

המופת [Proof]. That the [longer] measurement is the [?].

For every cubit [כי כל קטר] [?] is [?]; and when the shorter [side] goes forth [יצא הקצר], taken [the other] time [?], [there arise] figures [?] and the rectangle [?]. And upon it the diagonal is drawn, dividing it into two equal parts [?], each of them [being half of] the rectangle on [the same base] [?]. [Value:] twelve [יב] [?]; and they meet at [?].

For every cubit of area is equal to the others [?]; and when the shorter [side] goes forth it produces equal figures [?]; and they are [numeral: ?] figures [?].

Now [as to] their [parts] equal to the sides [?]: the [second] measure arises from the rectangle [?], and it lies between the two lines [?] — between the one [?] and [?].

For to recall the area of the square from the rectangle [?]: when its area is equal to the [square's] area [?], and when it is not [equal] [?], he [?] to the other, and [?].

The rectangle is [equal to] the [square] whose area is [likewise] [?]; and it is divided [וניחלק] into parts [חלקים], each of them being [half] [?]. This is the area [שטח] of the rectangle [?]; and it is the [portion] [numeral: ?] [?].

For when the parts [חלקים] [?] shall be divided, the first part [?] [numeral: ?]; and so it stands with the area of the rectangle [?]. When its area [?] the rectangle is [?] — [its] shorter measure [?], and it is not [?].

For to recall the [area of the] square, take the rectangle [?] and multiply it [?]; then it will be a rectangle [?]. And when the shorter [side] goes forth [?] and the rectangle [?].

And thus, [when] you know the shorter [side] and the [longer] [?], you will know the [area] and the [portion] [?]. And it is not known until you know its length; take it [?].

Each one [of them] is [equal to] the area of the rectangle [?], and the remaining [portion] is [?]. And when the rectangle is [?] — [its] shorter [side] known — it is the area of the rectangle. Take twelve [יב] cubits [?] [and] multiply [לקחת] the rectangle [?].

And [take] four [?] from it, [so that] it will be a rectangle [?]; and multiply them [ותכפלם] by [?] cubits [?], and its [portion] [numeral: ?]. And [its] measure [is] the length [?]; divide it into two [שני] parts [?].

The rectangle is [?] the [square] whose remainder is a rectangle [?]. Thus [runs] the reckoning [דמיון] of the long rectangle [מרובע ארוך].

The rectangle [?] the shorter [side], and it is a [?]; and the rectangle [?] when you know [?] and [?] figures [?].

Diagram. An oblong rectangle (wider than tall) with a single diagonal drawn across it (from the lower-left toward the upper-right corner), dividing the rectangle into two triangles. The figure illustrates that a diagonal bisects the rectangle into two equal triangles.

- Lower-left corner: ז [?]
- Lower-right corner: ג/ו [?]
- Upper corners: blank (no inscribed label)

and so it is: take the [long side] [numeral: ?] and multiply the [?]; and there arises from this a length [?] and its [portion] [numeral: ?]. And it is [?] the shorter [side] of the [?], and its [portion] a third [שלישי] [?].

[?] the rectangle [?]. **המופת [Proof].**

[Marginal note, right margin of p. 38 (44v), c. 13 short lines, same hand — a constructive/explanatory gloss; no named authority is cited.]

> וזה יתבאר / לפי הנחות / זה החכם [?] / מהמרובע ה[?] / ואם תאמר [?] / ולרוחב אחר [?] / שבארכו וזולת [?] / זה המקום והוא [?] / שכאשר [?] וכן / כן חלוקת [?] ולא / שוה הצורות [?] / ויחלק לשני / חלקים [?] הנצבת [?] / עוד הנצבת [?] כב [?]

"And this will be made clear according to the premises [הנחות] of this scholar [?] … from the rectangle [?] … And if you say [?] … and for another breadth … in its length, and apart from this place; and it is that, when [?] … so [too] the division [?], and the figures are not equal [?] … and it is divided into two parts … the perpendicular [הנצבת] … again the perpendicular …" [Further identification requires further verification.]

By these two operations it is known that [the product] resolves into two equal squares [?] … the sides themselves of the line, since they are [?]. When we carry out the operation and obtain the result, take one [part] within the whole square and adjoin it [?]; and the product is the area of the surface itself [?].

For if we already know the number of the parts of the one surface [its area] together with the parts of the one side, then the number that is divided by the other number necessarily yields the quotient [?]: the one [factor] measured by the quantity of the other number, and likewise the other by this one; and they are one number [i.e. the same product read either way]. Consider now in this way the number of the one [side]. Accordingly, we may return: if we divide the number [the area] by the quantity of the known side, then—the known number [area] being set before us—the quantity of the [unknown side] becomes known to us. And therefore we divided the [area] by the known side, and what came out is the unknown side.

And if the rectangle has equal sides—that is, its length equals its breadth [?]—the line [side] is such that it is called an "equilateral square" [מרובע משוה הצלעות, i.e. a square proper]. Suppose its diagonal is [numeral: ?] cubits, and we take [numeral: ?] cubits: how large will its side be? Divide the diagonal into two [halves], and take from each a square [square each half], and join the two squares—they become one number; take its root, and that is the side of the square [?].

Proof [המופת]. As in proposition [numeral: 9 = ט]: you will find that the total number of the parts of the [area] is contained in the parts of the [known] side and the [resulting] number; for the number divided by one number is necessarily found [as a quotient] measured by the quantity of the other, and likewise the other by this—so they are equal. Hence, when we divide the [area] by the known side, the quotient is the unknown side, and this is what was to be shown [?].

Example [דמיון]. We divided the diagonal … it came to [numeral: 7 = ז?], and we took [numeral: ?] … the square of [numeral: ?] … [?]. … its one diagonal is [numeral: ?] cubits and [numeral: ?] cubit[s] … how much is the one diagonal? A quarter [?] and the half of the diagonal … the known diagonal, which is … [numeral: ?], and the square of [numeral: 24 = כד?] … and it is [numeral: 7 = ז?], the beginning of … the diagonal as its completion … [numeral: 12 = יב], and that is the one diagonal.

Proof of what we said: every diagonal stands to the diagonal [?] at a right angle … the square … equal-sided. And the equilateral square, which is a square [proper], its side … For when any line is divided into two parts, the square [on the whole], as in the figure, equals the square … possessing the two lines [i.e. the squares on the segments, together with the rectangles] [?]. And by this it is made clear … the measurement of the [figures] for the eyes [i.e. as they appear] [?].

The Third Chapter [הפרק השלישי]

We already know what … the triangles are divided into two [kinds] … for the sides: three legs, and their rank equal [i.e. three equal legs — the equilateral triangle] [?]. And we begin to explain … the triangle … the equilateral, when … [?].

[The triangle is treated first] because these—triangles—are the foundation of the [rectilinear figures] [?]; for every [figure] can be broken up … into triangles that partition it, or [is reduced] to a [right?] angle. And accordingly let us explain the measurement of the triangles for their own sake.

Draw the perpendicular [העמוד, the altitude] in thought upon each side, extending it to the side … upon the other triangles [?]. And afterward … [proposition] [numeral: 9 = ט] you will find doubled for all the triangles [?]; and according to the method of … the perpendicular … in accordance with the sought area. Let us state, then, how at the outset the perpendicular [altitude] is obtained in every triangle, and it is completed until it becomes … known [?] … the quantity of the triangle that it divides [?]. And we say:

In the [equilateral / acute-angled] triangle the perpendicular always falls **inside**, at a point … since [its] angles [?] are always acute. And we say [its] side[s] are equal. Now I speak of the perpendicular [altitude]: the [line] falling from the apex point of the triangle is called the "perpendicular" [נכח / ניצב] dropped upon [the base].

Diagram. A triangle with its apex at the top and a horizontal base along the bottom. A straight altitude is drawn from the apex, perpendicular to the base, meeting it at an interior point; this altitude divides the triangle into two smaller triangles (left and right). The base vertices and the foot of the altitude are lettered.

- Apex (top): [?] (single small letter, not securely legible)
- Base, left vertex: ד
- Base, foot of the altitude (center): ז
- Base, right vertex: ג

[Text to the right of the figure:] … the altitude divides the triangle into two triangles … the [perpendicular] came out from … the one [?] … And we say that the point … [?].

Proof [המופת] of this: the altitude divides the triangle into two triangles … whose … are equal [in the measures], each one … [The concluding steps of this

proof carry values and point-letters that are not securely legible at this resolution.]

Cheshbon u-Midot — Book Two (Geometry), Part One, Chapter Three (mensuration of triangles)

[f. 46r]

equal to line ז״ב, and the altitude ח״ב is equal to [line] כ״לב [?], because [the two triangles] stand on equal bases — line ט״לב [?] being equal to base ד״ב and to base [?] — and they lie between the same parallels; therefore all their angles are equal, and so too each one of them [equals its counterpart].

Since, then, the two triangles into which the figure was divided are [each] equal, it follows that the whole rectangle is double the triangle — that is, the triangle is half of it. And [the ground of] this is that line [?] falls upon [?] at point [?], parallel to line ק״ן ב״ב [?], and you have shown that when you take the half [of the base] the [resulting figure] is [?].

The proof of this is that whenever [a triangle and a rectangle stand] on one base and between two parallels, the triangle is half the rectangle. For angle ז״ב[?]ד is equal to [angle] [?], and the two triangles [?] are equal; and when [the rectangle] is bisected through its middle, [each half] is equal to the whole [triangle]. So we bisected it — the first [half] like the second — and we equated the two halves [?].

Therefore the area of the rectangle is the product [כפילת] of the altitude by the base [?]; and the area of the triangle is half of that — namely, the product of the altitude by half the base. And this is what we wished to demonstrate. ●

[And so the triangle is] half of this rectangle [?], and thus the area of the triangle is the product of the altitude by half the base [?]. ●

Diagram. A triangle inscribed in a rectangle: the rectangle is drawn with its top and bottom sides, and a diagonal is carried across it so that the triangle occupies exactly half of the rectangle — the "triangle = half the rectangle" construction underlying the proof above. Two vertices are marked at the top; the base points are lettered along the foot.

- Top corner (apex, dotted): א/ה [?]
- Top corner: ד/ה [?]
- Bottom-left: ד

- Bottom-middle: ז
- Bottom-right: ב

**

therefore the area [?]... is the product [כפילת] of the altitude [?]... and this is what we wished [to show]. ●

[f. 46v]

Since you now know [that the triangle is half its rectangle], take any triangle you please: its area is the product of the altitude by half the base — and this holds whether the triangle be right-angled, acute-angled, or obtuse-angled, for the [proofs] all proceed alike.

[?] the area of the rectangle is the product of one of the two sides containing the right angle by the other; likewise you take the altitude and multiply it by half the base [?], and this yields the area of the triangle.

Example [דמיון]. Let this be an acute-angled triangle, [triangle] א-ד-ג [?]. I drop from its apex א the altitude [העמוד] to its foot upon the base [?], and I draw an interior line parallel [נכחי] to the base [?]. Then the completed rectangle stands upon the [same] base, and the triangle is half of it, for [the altitude] cuts it and its [halves are] equal [?].

Diagram. An acute-angled triangle enclosed within a rectangle. The apex א stands at the top; an altitude descends from א to a foot on the base; the base runs along the foot of the figure, and the top side of the enclosing rectangle carries several marked points. An interior line is drawn parallel to the base.

- Apex (top center): א
- Top edge (right to left): ט/ש — ה — ב — ד [?]
- Base-left: ד
- Base-middle (foot of the altitude): ז
- Base-right: ג [?]

**

The Proof [המופת]. [?] the triangle is equal [?]... and I say [that we take] the altitude and [obtain] the area — [namely] that whenever [a triangle and a rectangle stand] on one base and between two parallels, the altitude which they share [?] and its half [?].

And [as for] the triangle, [it is] equal [?]: its area is half the area of the rectangle [?]; and this holds whether [the perpendicular] falls within [the triangle] or outside it. All this is demonstrated in the science of geometry [בחכמת המדות].

By these two other ways [?], along the paths of geometry [?]... take [?] and multiply [?] — for indeed [?] it is the altitude [?].

Behold, you already know [הנה כבר ידעת] [?]... that the area of the rectangle is the product [כפילת] of [?]; and this is what [we said] concerning its altitude being [?]... and I have set it before you in another way, and with other [figures], as [I shall] explain [?].

[Marginal note, left/gutter margin, second hand, running alongside the last lines: "[?] כאלכסון [?] / העברתי חצו [?] / מדבר כאלב [?] / כאחד תמונ[ת] [?] / [?]דבר וקבל" — a constructive geometric helper tying the diagonal/altitude of the figure to the prose; no named authority is legible. Further identification requires further verification.]

Comtino, *Cheshbon u-Midot* (BnF Hébreu 1031) — Book Two, Part One, Chapter Three (continued)

(f. 47r–47v; triangle mensuration — the altitude/area method and its two proofs)

f. 47r …and from this the base-segment is found. If we wish to know how to find the segment of the base upon which the altitude stands: the altitude that is erected upon the base — in the acute-angled triangle it falls **within** the base, so that the base is divided by it into a **near** segment and a **far** segment.

You must, however, distinguish where the altitude falls according to the angle of the triangle. When the angle standing on the base is a **right angle**, the altitude falls upon the very end of the base [and coincides with the side]; when the angle is **obtuse** [זוית נוחת], the foot of the altitude falls **outside** the base, beyond its end; and when the angle is **acute** [זוית חדה], it falls **within** the base [between its two ends].

To find the length of the altitude: take the square [מרובע] of the [known] side, and subtract from it the square of the base-segment adjacent to that side; the remainder [הנשאר] is the square of the altitude, and its root is the altitude itself.

And to find the two segments [the near and the far]: since the difference between the squares of the two [unequal] sides is equal to the difference between the squares of the two segments — take the square of the longer side and subtract [ונחסר] the square of the shorter [?] — then divide [ונחלק] the remainder by the whole base; the base and this quotient together yield the two segments, the shorter [הקצר] and the longer [?]. We have already established the length [of each]. Then take the square of this [segment] and add [ונחבר] it [?]…and the square of the altitude [is recovered from the base].

Then multiply [כפל] the altitude by half the base — or half the altitude by the whole base — and the product is the mensuration [area] of the triangle. Do likewise [in every case].

Diagram. A single composite ("M"/"W"-shaped) drawing in the bottom-left of the page: a left triangle with an interior altitude adjoined to a right triangle with crossing lines, illustrating together the **acute** case (altitude falls **within** the base) and the **obtuse**/external-altitude case (altitude falls **outside** the base and is completed by an external perpendicular beyond the base-end). What earlier appeared as three separately-placed diagrams is one composite construction.

- ג (bottom-left): base vertex
- ז (bottom-center / shared foot): the foot at the shared base position
- ד (bottom-right): base vertex
- apex(es) and interior points: [?] (at 600-dpi legibility floor)

f. 47v

Its proof [המופת שלה]. We have already established that the altitude, when it is erected upon the base, makes with it a **right angle** [נצב]. And since, in each of the right-angled [sub-]triangles so formed, the square of the side is equal to the square of the altitude together with the square of its adjacent base-segment, it follows that when the square of the [longer] side is enlarged [/taken] and the square of the [shorter] side is subtracted from it, what remains [הנשאר] is the difference between the square of the far segment and the square of the near segment. Hence the segment [is found], and it is the [shorter] one [?]; and likewise the altitude, whose square is the side's square less the segment's square. Divide it [as shown] upon the base, and this is the position of the altitude. Likewise, if we

wished to know the length of the altitude, we would do exactly so, and thus the altitude is found. [numeral: ?]

And its proof [of the area-rule]. That the rectangle [מרובע] of every side… belongs to the **science of geometry** [i.e., it is demonstrated by Euclid], where it is shown that the triangle is half of the [enclosing] rectangle [built on the same base]; this has already been established there, in the language of the practitioners [מתלמדים(ה) בלשון] [?]. And when [the two sides are] equal, the rectangle [is equal-sided]…and so likewise for **all** the triangles set before the eyes: whatever [triangle] does not have its sides equal [?] — since they have equal measures [מדות שוות]… [the same reduction applies]. [Refer this to the science of geometry.]

Cheshbon u-Midot — Book Two (Geometry), Part One

[Conclusion of Chapter Three — the triangle and its altitude]

[The passage continues the treatment of the triangle's altitude from the preceding folio.] When the angle [in question] is a right angle, there is no need [?] to derive an altitude [separately], since [the side itself already stands as the altitude] [?]. And we say that when the [triangle is such] [?] that the angle has no [proper] measure [for] deriving the [altitude] [?] … [several clauses here are densely written and only partly legible: they run from the erecting of the altitude on the base, through its foot, to the segment cut off — but the exact wording cannot be recovered with confidence] [?].

The erected altitude [is dropped] from the apex to the base [?], and it divides [the base] at the place where the altitude falls [?]. Up to the point where the altitude meets [the base] [?] … and the segment that is cut off [הנגזרת, "the derived segment"] … [?].

Diagram. Triangle אבג drawn with its apex at the top and its base along the bottom. From the apex a straight line (the altitude, אד) descends to the base, meeting it at its foot ד and dividing the base into two segments. The base is lettered at its vertices and at the foot of the altitude.

- א (apex, top): apex of the triangle
- ב (base, lower-left vertex): base vertex
- ד (base, at the altitude foot): foot of the altitude אד
- ג (base, lower-right vertex): base vertex

Chapter Four, on the measurement of the quadrilaterals

Some quadrilaterals are equal in their sides and equal in their angles [i.e. the square], and its form is like this:

Diagram. A small square (equal sides, right angles), drawn inline as the type-illustration for the square. No interior point-labels.

And some are equal in their angles but longer [in one pair of sides than the other, i.e. the oblong/rectangle], and its form is like this:

Diagram. A small rectangle (oblong), drawn inline as the type-illustration for the oblong. No interior point-labels.

And some are equal in their sides but not [right] [?] in their angles [i.e. the rhombus, מעוין], and its form is like this:

Diagram. A small rhombus (a diamond — equal sides, oblique angles), drawn inline as the type-illustration for the rhombus. No interior point-labels.

And some resemble the rhombus but [with unequal sides — the rhomboid, הנוטה] [?], and its form is like this:

Diagram. A small rhomboid (a slanted parallelogram), drawn inline as the type-illustration for the rhomboid. No interior point-labels.

And some are other, differing quadrilaterals [i.e. the trapezoids and irregular quadrilaterals] [?], and their form is like this:

Diagram. A small trapezoid (a four-sided figure with one pair of unequal parallel sides), drawn inline as the type-illustration for the irregular quadrilaterals. No interior point-labels.

[The lower lines of the folio state that the measurement (מדידה) of each of these will now be taken up in turn, beginning with the first type; the precise wording of the transition is only partly legible.] [?]

The measurement of the quadrilaterals [continued]

[This folio side gives the rule for the area, which is obtained by the product of the altitude — כפילת העמוד — with the base, and works it through on a figure.]

The [area] is [found by] the product of the altitude [with the base] [?] … and this is [demonstrated] [?]. When the two [opposite] sides [are multiplied] one by the other [?], the area is derived; and the remainder [?] … [the derivation continues

through several densely written clauses that cannot be recovered with confidence, involving the altitude and the segments of the base] [?].

(interlinear: with the altitude (עם העמוד))

For the altitude [drawn] from the one [end] [?] up to the [other] [?] … and it is known [ידוע] that [the figure] is [equal to] [?] … [numeral: ?] … and they are twelve [י"ב] [?] … [The worked numbers on this line — apparently a small example using the value 12 — are too compressed to transcribe cell-by-cell with confidence; the specific quantities are left flagged rather than reconstructed.] [numeral: ?]

Therefore the product of the altitude [with the base] [?] is the [area], as was explained [כמו שנתבאר] [?]; and so [the rule holds] equally [for the rhombus and the rhomboid] [?], since each of them [can be reduced to] a rectangle [מרובע] of the same base and the same altitude [?].

[Margin: a short computation note in a lighter/faded ink beside these lines, reading approximately "כ[?]ד י"ב חמות · וי"ב ו חמות" — the exact figures are uncertain; further identification requires further verification.]

Diagram. A larger construction demonstrating that an oblique parallelogram (or the triangle upon a base) has the same area as a rectangle on the same base and altitude. A rectangle is drawn with its top edge running from an upper-left corner to an upper-right corner. From the top edge a vertical altitude descends to the base; one or more oblique (diagonal) lines run from the top region down to the lower-right corner, forming triangles inside the rectangle. The base is divided by the foot of the altitude into segments, and interior and base points are lettered.

- א (top-right corner): confirmed
- ב (top-left corner): confirmed
- ג (bottom-right, base): confirmed
- ד (base, center-right): confirmed
- ז / ו / ה (top-centre convergence cluster, at the head of the altitude): confirmed
- ח, ט (interior points on the diagonals): confirmed
- remaining base and interior marks: not securely legible [?]
- יב (interior label, near the altitude — possibly the value 12 rather than a point) [numeral: ?]

[The text below the figure continues the same demonstration — that the triangle/parallelogram equals a rectangle of equal base and altitude, dividing the

base and reasoning from the equal-area rule — but the lines run to the foot of the folio and the closing clauses are only partly legible.] [?]

Book Two — Geometry (חכמת המדות), Part One: The measurement of quadrilaterals (continued)

[The demonstration of quadrilateral area — the product of the altitude with the base, extended to the oblique/trapezoidal cases — continues from the preceding folio. The hand here is a compact, heavily ligatured Sephardic semicursive; the paper is well preserved, but the density of the script leaves a substantial minority of words unresolved. Genuinely unreadable words and spans are flagged `[?]`; recovered diagram labels are given per figure.]

the figure and all its parts [?], and it will be [?] known [?]. Therefore [?] the figure [as a whole] and [each of] its parts [?] are known.

For when the altitude and the base are both known, the figure is likewise known [?], and one may learn the area (התמונה כמו שאמרנו) of the figure from [the product of] the altitude [?]. Thus [?] both the figure and its parts, and the product [?] of the two [dimensions] together [?].

[Illegible span — approximately two lines of dense semicursive: the argument that a figure and its equal are of equal area, reducing the oblique figure to a rectangle of the same base and altitude; specific wording not resolvable `[?]`.]

and the two [parallel] sides [?], and the altitude [?] between them [?], and its measure [is such-and-such — the value is not legibly recoverable] `[numeral: ?]`. And this [?] is the altitude, and it stands [perpendicular] `[?]` at [point] ה [?].

and its parts [?] upon the base [?], the altitude being the beginning `[?]`, and [it is] known, and the [product?] `[?]` …

Length by breadth `[רוחב … אורך, ?]`: [take] the altitude [?] and multiply it (וחתו כפול) with the [base/altitude] `[?]`.

and the figure will be [as] a small [rectangle?] `[קטן?]`, and they are the area [?] of the figure `[כשבורת/תשבורת הנוטה, ?]`. (And [the same holds?] `[?]`.)

and behold [?] the figure of ב-ד [?] and figure ז [?], the figure being thus [?].

The figure `[?]` is one figure [?]; it is the area [?] as we have shown [?].

and it stands [?] the altitude in [figure] ל [?] `[?]` …

[Illegible span — the transition into the worked example; dense, not resolvable [?].]

Diagram (below) — the worked example, altitude יב (= 12).

Diagram. An upright rectangle enclosing an inscribed triangle whose apex sits at the top edge; the triangle's base coincides with the rectangle's base, and its two sides run down to (near) the base corners — the figure by which a triangle is shown equal to a rectangle of the same base and half its altitude. One or more interior vertical rules divide the rectangle.

- א (apex, top edge): apex of the inscribed triangle
- ב (bottom-left corner): base vertex
- [additional base labels near the foot, e.g. י / ט: faded, [?]]

Diagram. An oblique quadrilateral (trapezoid / rhomboid) with a slanted upper edge descending left-to-right and a horizontal base; an altitude is dropped from the upper edge to the base, and an internal line (cevian) runs from an upper vertex across to the base — the construction reducing the oblique figure to an equal rectangle.

- [top vertex (upper edge, left): not legibly recoverable [?]]
- כ (mid-right, on the slanted side): vertex on the descending edge
- ד (bottom-right corner): base vertex
- ב (bottom-left corner): base vertex
- [bottom-edge intermediate point (along the base): [?]]
- מ (interior, near the cevian): foot of the altitude / cevian

Diagram. A rectangle (near-square) with a diagonal drawn from the top-left corner to the base, plus a second internal line to a point on the base — the worked example. The left (vertical) side bears the value **יב (= 12)**, the altitude taken for the computation.

- [top-left corner: not legibly recoverable [?]]
- יב (left side): the altitude, value 12
- [bottom-left corner: [?]]
- ה (base, center): division point on the base
- [bottom-right corner: [?]]

Caption/lead-in above the figure (partly legible): "*… a known [rectangle] to measure*" (…ידוע למדוד), "*the figure and [its] side*" (תמונה וצלע).

the figure and its side [?]; a known [rectangle] to measure [?]. And when [?] the altitude [?] and [the base?] [?], [the area follows] [?].

and [they are] as in this figure [והם … בזה התמונה, ?].

Diagram (below).

Diagram. A triangle with an internal line (cevian) descending from the upper vertex to the base, dividing it. The apex is at the upper right; the base runs along the bottom; a point is marked on the right-hand (descending) side.

- א (apex, upper-right): apex vertex [label a small mark, read א, ?]
- יו / י (right side): point on the descending side [?]
- ד (bottom-right corner): base vertex [ד or ר, ?]
- ג / ב (bottom-left corner): base vertex [?]

and similarly [?] the product [יכפול, ?] [of] the altitude upon the base [על הבסיס, ?].

Upon [the sides / the base?] [על צלעות/שאם, ?], and they will be [?] thus (ק) [?], and both of them upon [?] the sides [?].

[The value?] with its part [חצות עם הלק, ?], and it will be [?] their product [כפלם, ?]. And [?] the product upon [?] the sides, and they will be [?] …

and behold [?] the two of them [?]; they are added [יאספו, ?] [to give?] [?], and it counts [?], and the halving [of them?] [וחצים, ?] [?], and the remainder [הנשאר, ?] is [equal to] [?] the product [?].

Diagram. A square (מרובע) with both diagonals drawn, crossing at the center.

- ב (top-left corner)
- ג (bottom-right corner)
- ד (bottom-left corner)
- [top-right corner label: not legibly recoverable [?]]

upon the base [על הבסיס, ?], and they will be ג [?]; and they are added [ונחברם, ?] and the sides [?] and the product [?] …

upon the base and they will be ק [?]; and its part [?] with the part [עם הלק, ?], and it will be [ויהיה, ?] … their product [?] …

upon the base, and they are [מרובעים, ?], and they revert [ויהיו, ?] [to] [?] … and its part [?] upon [it?] [?], and it will be thus [ויהיה כן, ?], and it is [?] …

and their product [כפלם, ?] upon the base [?], and they will be [?] from [?] … and the base [?], and behold [?] the squares [?], and they will be [?], and near [?] …

as [in] the figure [כתמונה, ?], from [?] … the two sides [שני צלעים ?] known [ידועים, ?]; and its [?] product [?] …

[The demonstration of the trapezoid / oblique-quadrilateral area — half the sum of the two parallel sides multiplied by the altitude, established here through the lettered figures and the worked example on 12 — continues onto the following folio. The bulk of the connective prose on this opening is written in a very compressed hand and resists confident word-level reading at the resolution available; the recoverable structure (altitude × base as the product; the two parallel sides; halving of their sum; the equal-figure reduction) is rendered above with the specific wording flagged [?].]

and one wishes to find the area. Take, then, the altitude of the one part [?], which is [numeral: ?]... and if it is [numeral: ?] the product will be [?]; divide it into portions [?], and if it is 4 [ד]... [?]

And if it is as in the figure [כתמונה], and it is [numeral: ?]... but if ב״ג was ten cubits [עשר אמות]... [?] so too here, for the part of this line is [?], and it is ten [עשר] and a fifth [וחמישית]. [?] Take, then, [its] root [?] and multiply it by itself [וכפלהו בעצמו]; the result is the product [כפל]. [?]...and a fifth, and [the remainder?] [?].

[The connective computation across the top lines of this page — a worked area reduction carrying the values ten, five, one-fifth and the boxed value 25 (כה) beside the figure — is heavily ligatured and abbreviated; individual quantities and their operators are at the limit of legibility. The readable frame is the standard mensuration procedure — square the side, or multiply length by width for an oblong — but the specific intermediate numerals cannot all be committed. Flagged as a]

Diagram. A lettered construction at the left of the text column: a quadrilateral (an oblong/rectangle) drawn with a diagonal, surmounted by a triangle, with a right-angle mark at the lower-right corner. A boxed numeral sits to the right of the figure at mid-height. The construction illustrates the reduction of the figure to an equal rectangle for the area computation discussed in the surrounding prose.

- (apex, top): ג [?] (candidate)
- ה (far left, mid-height): vertex label
- ב (upper-left of the quadrilateral): vertex label

- ד (bottom-left corner): vertex label
- ח (bottom-middle): vertex label
- ז (bottom-right corner, marked with a right angle): vertex label
- [25] כה (boxed, to the right of the figure): value label

[More than two labels are uncertain — figure marked a]

And if [ואם] we come to demonstrate this [?]... [a second conditional demonstration follows, extending the area rule to a further case of the oblong/rectangle — its connective prose is at the reading ceiling]. [?]

For the area of the square [מרובע] is length × width [?]; and this is known [ידוע], and the rectangle [מרובע] is likewise known. [?]...for the area of the oblong [מרובע ארוך] is length by width [אורך ברוחב]; and if the side ב״ד is known, the area is known, and it will be [?]... And if the oblong is known, so too its area is known. [?] Therefore the area of the square upon the diagonal [?] is [?]... And this [?] is a marvel [?], for the area upon the [?] is known. [?]

[The lower half of page 1 continues the same worked demonstration on the areas of squares and oblongs and their diagonals; the argument is legible in outline (recurring: מרובע "square/oblong", אורך/רוחב "length/width", אלכסון "diagonal", ידוע "known"), but the chained numerals and operators are not all recoverable at this resolution.]

to its side ב״ג [?] it will be known.

The Fifth Chapter [הפרק החמישי]

[Subtitle in smaller script, partly legible: "to know the [measure/figure of?] the circle" — לדעת ‹?›מנה ‹ה›עגולה [?].]

Know that the circle [העגולה] is a single line [קו אחד], closed upon itself, surrounding one point at its center [?], such that all the lines going out [from the center] to the circumference [ההיקף] are equal [שוים]. And the parts of the circle [וחלקות העגולה]... [?]

for the circumference [ההיקף] is the enclosing line [?]. Now, the ratio of the circumference to its diameter [ההיקף אל קוטרה] is [as] 3 and a seventh [ג׳ ושביעית — i.e. 3⅐]; and what remains... know [?]... The ancient scholars [החכמים הקדמונים] labored over this in [their] science; they set down all their computations in the book of [Ptolemy?] [בספר ה‹תלמסי?› — reading uncertain]. [?]

the circle is [named/measured] by its lines [?], and the diameter [?] upon all [?]... [?]

If the circumference is [numeral: ?], the diameter is [numeral: ?] [?]; and the ratio of the circumference to the whole diameter [כל האלכסון/הקוטר] is 3 and a seventh [ג׳ ושביעית]. [?]

And if [ואם] we come to demonstrate the area of the circle [תשבורת העגולה], we take the diameter... [?] The example [המשל]: in this figure, the circumference has been added upon [?]... [?]

Diagram. The right-hand of a pair of circles, each drawn with a single horizontal diameter. Above the diameter line is written "[½35 =] "לה וחצי; below the diameter line, near the center, is written "[22 =] "כב. The two figures illustrate the circumference-to-diameter relation of the worked example.

- [½35] לה וחצי (above the diameter): value label
- [22] כב (below the diameter, center): value label

Diagram. The left-hand circle of the pair, likewise drawn with a single horizontal diameter, labeled "[½35 =] "לה וחצי above the diameter line. (Its lower label is carried off the tile at the gutter.)

- [½35] לה וחצי (above the diameter): value label

for the ratio of the circumference to the whole diameter, and it is 3 [and a seventh?] [‹והוא ג‹ושביעית] [?]; thus the ancient scholars [?], in truth, as a settled matter [?], measured [?] the circle [?]... [?]

The area of the circle is [computed] from the circumference and the diameter [?]... take the circumference... [?] and multiply [?]. This is the area of the circle. [?] And Ptolemy [ובטלמיוס — reading uncertain] in another place [?]... [?] there is no difference between this computation and the computation of the sages of [?]... [?]

take from it a seventh part [שביעית] [?], and this is the area of the circle [וזה תשבורת העגולה] [?]...

And now let us begin to set out the method of measuring [the parts of the circle] [?]. We say that [the circumference of] the circle [?] is that which encompasses it, [running] from one end [of the enclosing line] to its other end.

A straight line is called the *koter* (diameter): it is the line that passes [across the circle] from one side of the enclosing line to the other side. A line [drawn] to the circumference at a right angle is called a *yeter* (chord) [?], and the measure of

each segment [?] [is reckoned] [?]. And if the [half] is equal to half [the... of] the circle. And if [the...] is [longer] [?] — [several words illegible] —

for the circumference [?] is longer than the diameter, [as] the ancient scholars [have said] [?]; and if the [chord] is [equal] [?], then this is [the segment] [?], and this is the diameter, and [the...] of the circle. And if [the...] is [the...] of the segment, it is larger, and [the...] of the segment is [wider] [?] than the [...] of the circle. So they said [?]: the way to know [the area of the segment] [?] is [as follows] [?] — [reading uncertain across the line] —

for the multiplication of half the circumference [?] into the diameter [?] — [word illegible] — is [equal to] the area [?]; and if the [chord/quantity] be [as the...] of the circle, [the...] is [wider] [?] than [the...] of the circle. And they said further [?]: the way to know the area of the circle [?] is that you multiply the whole circumference [?] into the diameter [?], and [the result] is the area [?] of the whole circle [?]. And they said further: [the way is] that you take a quarter [?] of [the product of the circumference and the diameter] [?], for every reckoning [computed] in this manner [yields] half [?] —

[reading uncertain] the diameter [?]. This is [the model/rule] [?]. Illustration [?]:

Diagram. A full circle with a horizontal diameter drawn across it. The labels are numerical measurements (continuing the manuscript's convention of labelling circle figures by their measured circumference and diameter in cubits).

- (near top, inside): (22) כב [כ/ל residual — 32 not excluded] אמות ["22 cubits" — further re-read leans (22) כב on paleography: the glyph lacks the tall lamed ascender this scribe uses elsewhere (lean is NOT arithmetic-driven). כ/ל still not certain at the resolution ceiling, so a [?]-class residual is retained]
- (on/along the diameter): אמות [7] ז ["7 cubits"]

Diagram. A semicircle (dome), its diameter as the base and a line rising within it. The base bears the caption *חצי העגולה* ("half the circle" / semicircle).

- (apex, small label): [numeral: ?] אמות
- (base, verbatim): חצי העגולה ["half the circle"]

They knew [?], and how much [is] the area, [namely] the [half] of the circle [?]; and it was in [Ptolemy] [?] [numeral: ?] — [reading uncertain] — take the one part from it [?], set it upon [the...] cubits [?], and multiply it by [the...] according to the square of the [line] [?], which is [numeral: ?] cubits, and [the...] of the [chord] [?] is [numeral: ?] cubits. And [multiply] — [reading uncertain] — and it is a quarter

[?] of the square of the diameter, and it is [numeral: ?] cubits and a quarter [?]; and these are the area of half the circle [?].

And if [the...] is [the...] the circle [?] and [you wish] to know the area [?] — [reading uncertain] — and so it is done [?]. Take a quarter [?] of [the product] and multiply it [?] upon [the...], and it is [numeral: ?]; and a quarter [of] [numeral: ?] cubits and [the...] is [numeral: ?]. And so it is done [?]: take a quarter [of] its square [?] and multiply it by half [?], and it is [numeral: ?]; and half [the square of] the diameter [?], and the center of the circle [?] —

Diagram. A quarter-circle (quadrant) at the lower left, with a vertical and a horizontal radius. The labels are numerical cubit-measurements.

- (upper side): ג ורחב [numeral: 3?, "and width" ?] אמות
- (base): אמות [?7] ז
- (side, small label): [numeral: ?] אמות

And [reading uncertain] cubits, and [the...] area [?] — [reading uncertain] — and [it] is [numeral: ?], and these are the area [?].

[Reading uncertain: this closing line reads approximately] "...and a quarter [?], and what is the area [?]."

We wished to know the area of a [segment] [?] — [reading uncertain] — and this is the matter [?]: to find the diameter of the whole circle [?] from it. And this [Euclid's *Elements*] (*Sefer HaYesodot*, Euclid) has already shown at length.

[Reading uncertain] the part of [this line] the other; and we joined [added] twice the part of [the...] — [reading uncertain] — [the...] the diameter upon [the...] of the [ancients / Elements] [?]. And we multiplied half the diameter by [the...], and it [came out] [?] as [it should be] [?] — [several words illegible] —

upon the [enclosing line] [?], and [the...] [?]; and as we knew that the length of the diameter [?] — [reading uncertain] — and the number that is [the...] of [the...] [?] upon [the...]; and multiplied [?] [the...] [?] — [reading uncertain] — the length [?] upon [the...]; and it is a segment [?] [of] the circle, [with the chord?] [numeral: ז / 7?] and it will be [a...] [?] — [reading uncertain] —

Diagram. A semicircle (dome) with its diameter as the base and a vertical radius rising from the midpoint of the base to the apex.

- ד (apex, top): vertex
- ג (base, left end): vertex

- ז (base, midpoint — foot of the vertical radius): point

and it is also [?] a chord [?] upon [the...] of the area [?]. And in [this] I [?] recognized [?] that the length [of...] to [the...] the diameter [?]; and it was recognized [?] that it [runs] within [the...] the diameter [?]. And in [this] it is recognized [?] that [the...] within [the...] — [reading uncertain] —

the [radii] [?]. And so we knew that the arc [?] of the circle [?], as the [half] of the circle [?] passes upon [the...]; and behold, the part of the diameter [?] upon [numeral: 4 / ד?] — [reading uncertain] — so that when you multiply half [the diameter] [?] upon [its] length [?] — [reading uncertain] — and there is no [number] [?] within [the...] over it [?], and we knew that they [are] — [reading uncertain] — the length [is] all the [enclosing line] [?], and the center of the circle [?].

The chord of the [radii] [?] is [numeral: 5 / ה?] — [reading uncertain] — with the [half] [?] together, and it is [a single line] [?]. And this is the whole [enclosing line] [?], and it is [done] [?]. And behold [the...] is [a point] [?] which [meets] [?] [the...] the [radii] [?], for it is the center [?]. And behold it is [numeral: ?] — [reading uncertain] — and behold [the...] within [the...] [?] the circle [?], and it is [the...] [?]; and this is the area [?] —

[reading uncertain: the enclosing line] [?] which he [?] [numeral: ?], and the seven [radii] [?] — [reading uncertain] — the [radii] which [meet] [?] [the...] a point [?], for [it is] [?] the center [?]; and the [seven] [?] which [meet] the [radii] [?] — this is the area [?] of the [right] segment [?].

And it is fitting [?]:

Diagram. A full circle with its center marked and radii drawn to the circumference, together with an inscribed line making a triangle in the upper part (the sector/triangle-summation figure for the area of the circle).

- ד (top, on circumference): point
- ב (upper left, on circumference): point
- ג (center of the circle): the center, from which the radii issue
- [an additional radius/point, label illegible] [?]

That we knew [the area of...] the circle [?] to the diameter [?] upon its circuit [?] separately [?] — [reading uncertain] — in order to bring out the whole area [?] of the circle: reckon the diameter [?]; take half [?] and take [half] again [?] — [reading uncertain]. And if [the...] be equal, take [half] of the diameter upon its

length [?], and multiply [?] the [half] of the diameter [?] by [numeral: 30 / ל?] of [numeral: 4 / ד?] fourteen [?] — [reading uncertain] — the remainder [?], and add [it] [?] upon the [half] of the diameter [?], and the [half] [?]. So for example [?] the diameter [numeral: 12 / י״ב?] with a line [of] [numeral: 7 / ז?] — [reading uncertain] —

[numeral: ?] cubits [?] and fifty [?] — [reading uncertain] — its [half] [numeral: 32 / ל״ב?] and a half [?]; and half the diameter [?] upon [the...].

Cheshbon uMidot — Book Two, Chapter Five (the circle)

(Sephardic semicursive, single column, clean paper; compact and heavily ligatured. Continuing the treatment of the circle and its parts.)

on the [chord] it will be a little [smaller], and there will result the area (תשבורת) of the segment [?]. And it returns to the circle as [?] we said. The figure is this: the segment is a segment [smaller than a semicircle] whose diameter is like the line [?]; and so we take the square of half the chord [and keep it].

Diagram. A circular segment (a low arc standing on a horizontal chord), the segment enclosed by the arc above and the chord below. A short interior mark labels the segment; the chord is captioned beneath.

- ד (within the segment): reference letter of the segment
- Caption beneath the chord: 12" — **יב אמות** cubits" (the length of the chord)

For example: this segment stands on a chord of **12 cubits** (יב אמות) [and its height/arrow is [numeral: ?] cubits]. [?] its diameter [?], and its area is [?]. He doubles half the diameter, and the perpendicular (עמוד) [is the arrow/height of the segment]; we divide the square of half the chord by it [and add to it the perpendicular], and there results the diameter [?].

We take the square of half the chord and divide it by [the perpendicular]; then [numeral: (3) ג] cubits remain [?], and the area of the segment [is] [numeral:) לב (32 or (22) כב?] [?]. And this is [?] the area of the segment; and so [?] we know that it is [?] the area of the segment.

The Proof [display rubric: הַמּוֹפֵת(?)]. [?] we take the square of half the chord, and it will be [?]. And its diameter [?]; and we found that its area [is] [?]. And this is what we said [?].

Diagram. A circle with a vertical diameter drawn from the top of the circumference through the center to the bottom; a triangle is inscribed with its apex at the top of the diameter and a slanted side running down to a point on the lower-left of the circumference.

- ג (top of the circle, apex of the vertical diameter): reference point
- ה (center of the circle): center
- ד (bottom of the vertical diameter): reference point
- ב (lower-left of the circumference): reference point

[Proof, referred to the points ב, ד, ה, ג:] the line ג–ד is the diameter and ה is its center; and the segment [?] its area [?]. [?] the square of the half [?], and we divide it [?], and the perpendicular ה[?] [?].

and this is explained (מבואר) in the tables of the arrows [sagittae / versines] (לוחות החיצים) and the arcs (והקשתות), [of] Ptolemy [בטלמיוס(?)]. [?]

[?] and it will be [?]; and afterward [?]. So we take the square of half the chord [?] and add to them [?], and there results [?].

the chord [?] and the arc [?]. He returns [?] to find the area [?]. So we take half [the chord] [?] and the perpendicular (עמוד) [?], and there results [?] which are **quarters** (רביעים) [?]. [?] add to them, and it will be [?].

We divide it [?]; the segment [?] its area [?] straight [?]. And he doubles [?] until [numeral: (3) ג] [?], and the circle [?] the perpendicular.

Diagram. A semicircle: a horizontal diameter is drawn, with a triangle standing on it (apex above the diameter, its two sides descending to the diameter's endpoints) and the semicircular arc bulging below the diameter; a vertical line runs from the apex down through the center.

- א (apex, above the diameter): reference point
- ד (left endpoint of the diameter): reference point [?]
- ב (right endpoint of the diameter): reference point

[Construction referred to ב, ד, א:] [?] a straight line from [?] to ג [?]; and the circle [?] the perpendicular [?].

Diagram. A tall narrow figure at center-right: an apex at the top with a long side descending to a point at the lower-left, an arc closing the figure on the right. [Two labels visible; further detail is uncertain]

- א (apex, top): reference point [?]

- ב (lower point): reference point [?]

Diagram. A full circle with a horizontal diameter; a point marked at the top of the circumference.

- א (top of the circumference): reference point
- ד (left endpoint of the diameter): reference point
- ב (right endpoint of the diameter): reference point

[?] and they will be like the first area (תשבורת) [?].

Chapter Six [הַפֶּרֶק הַשִּׁשִּׁי — large display heading at the foot of the page]

[?] the lines [?] is divided into [?] [?], which are circles (עגולות); it is the [?] and the foundation (יסוד) for all the [?] [?] — [rectangles / rectilinear solids (מלבנים) [?], rectilinear (ישרות הקוים)] [?]. [The chapter's opening; its subject beyond this point is not resolved on this folio and continues past the foot of the page.]

[Book Two — Geometry (חכמת המדות); Chapter Six (הפרק הששי), continued]

[?] the figures [?] and their [?] [numeral: ?]. Once the practitioner already knows [וכבר ידע[?]] the [rule of the] triangles [המשולשים[?]] as against [?] the outer [figures], and [knows] the diameter [ולקוטר[?]] … [several words illegible] … he can measure all the triangles [כל המשולשים[?]] and the squares [ורבוע[?]], and [reckon] their areas by means of the diameters [תשבורתיהם עם הקוטרים]. And once every [?] rises … he reckons every [?] and measures the [?] [Illegible: roughly one and a half lines of dense connective prose in the upper block of p. 55 that I cannot resolve word-for-word at this resolution].

and this is the general rule [וזהו הכלל[?]] … [?] the aforementioned figure [התמונה הנזכרת[?]], and this is the [?] within it [בתוכה[?]] … [?] and these are the segments [חתיכות[?]] … [Illegible: continuation of the rule statement, ~1 line].

For each of its sides [וכל צלע מצלעותיה[?]] … [?] as [it was done] for the first figure [כתמונה הראשונה[?]], so [one does] here [כן …]. Take half [וקח חצי] of [?] and the [?] [numeral: ?], and it becomes [?] [numeral: ?], and this is the area [וזהו התשבורת[?]] … [Illegible: ~1 line with one or two embedded worked values I cannot read]. And this is like the area of the polygon [כתשבורת הפלגון] [פלגון — Greek: πολύγωνον, "polygon"; the inscribed rectilinear figure].

the first figure [התמונה הראשונה] … as it did, so shall it multiply/square [כמו שעשה כן יכפול[?]] … [numeral: ?] parts [חלקים] … and this is the area of the figure [וזהו

תשבורת התמונה]. For example [המשל]: we have [יש לנו[?]] a figure [?] whose [?] … and each of its sides [וכל צלע מצלעותיה] … take half [וקח חצי] … it becomes [numeral: ?], and a square [ומרובע[?]] like the area of the polygon [כתשבורת הפלגון].

Diagram. A circle with an inscribed regular polygon (a pentagon, apparently: an apex vertex at the top with chords descending to vertices on either side and a base chord below). Point-labels partly legible around the circumference; the interior bears a short line of text that is illegible at this resolution.

- ה (apex, top of circle): vertex label
- ב (upper right on circumference): vertex label
- ז (lower right): vertex label
- ד (lower left / base): vertex label
- כפלה וחציה ("its product and its half") [interior caption]

Diagram. A second circle with an inscribed regular polygon (more vertices than the first — apparently a hexagon), its vertices marked with small letters and numeral-like strokes, and a short interior caption.

- ה [?] (upper): vertex label
- ב [?] (upper right): vertex label
- ד / [?] ג [?] (further vertices): labels
- ז [?] (lower): vertex label
- כפלה וחציה ("its product and its half") [interior caption]

after [?] [numeral: ?], and it is [?] [numeral: ?], and these are the areas of the [?] [ישבורות ה[?]]. And to each figure [?] as [with] the figure [כתמונה[?]] … [Illegible: ~1 line].

so shall it multiply/square [כן יכפול[?]] … [numeral: ?] … and this is the area of the polygon [והוא תשבורת הפלגון[?]] … For example [המשל], we have [יש לנו] a figure [?] … [Illegible: embedded worked value, unread] … and each of its sides [וכל צלע מצלעותיה[?]] … take half [וקח חצי] … it becomes [numeral: ?].

as it did [כמו שעשה[?]], so [one does here]. Take [וקח] half [חצי] … it becomes [numeral: ?] and it is [?] [numeral: ?] … and this is the area of the aforementioned figure [וזהו תשבורת התמונה הנזכרת[?]] … [And this is like] the area of the polygon [כתשבורת הפלגון[?]].

and know [ודע[?]] that whatever the reckoning increases [מה שהחשבון יוסיף[?]] … so shall its area increase [כן יוסיף תשבורתו[?]] … [Illegible: ~1 line]. For example

[המשל], for a [seven-sided?] figure [numeral: ?] … take half [וקח חצי] … [numeral: ?] … and this is like the area of the polygon [כתשבורת הפלגון[?]] of [numeral: ?] sides [צלעות].

[The remainder of p. 56 is a run of further inscribed-polygon figures (heptagon, octagon, and so on), each with the same computational template and each accompanied by its own circle-and-inscribed-polygon diagram. The embedded numeric values are not securely legible.]

Diagram. A circle with an inscribed regular polygon of roughly six or seven sides; the vertices carry small letter/numeral labels and the interior bears a short caption.

- ג (vertex): vertex label
- ד (vertex): vertex label
- ז (vertex): vertex label
- [Remaining vertex labels illegible]
- כפלה וחציה ("its product and its half") [interior caption]

Diagram. A further circle with an inscribed regular polygon, vertices marked, short interior caption.

- א (vertex): vertex label
- ג (vertex): vertex label
- ז (vertex): vertex label
- ד (vertex): vertex label
- כפלה וחציה ("its product and its half") [interior caption; clearest instance]

Diagram. A circle with an inscribed regular polygon of roughly eight sides (an octagon), vertices marked with small numeral-like strokes; the interior bears a short abbreviated caption written with gershayim.

- [Vertex labels illegible]
- כפל וכ״ [?] ("its product and its half", contracted) [interior caption, written with abbreviation strokes]

Diagram. A further circle with an inscribed regular polygon at the foot of the page, partly cut by the page edge; vertices marked ב, ה, and further small labels along the upper arc.

- ב [?] (upper): vertex label
- ה [?] (apex): vertex label

- [Remaining vertex labels illegible]

[Editorial note: No worked area-numerals are inscribed in any of these figures. The circle interiors carry only the method caption (כפלה וחציה, "its product and its half" — the polygon-area rule), not computed values. The apparent "worked area numerals" seen crossing the circle interiors are **Latin printed bleed-through from the verso**, not Hebrew figures, and are disregarded. Accordingly the [numeral: ?] slots in the surrounding worked-example prose are retained as [numeral: ?] — they are not lacunae and no area value is committed.]

[Text continues on the following folio.]

Cheshbon u-Midot — Book Two, Chapter Six (continued): the area of regular polygons

Script: Sephardic semicursive, single hand, compact and heavily ligatured on clean paper. Density (not damage) is the limiting factor throughout; the folios are otherwise sound.

PDF p. 57 (upper margin numbered "57")

[This side continues the procedure of Chapter Six for the area (*תשבורת*) of a regular polygon (*פלגון*) inscribed in a circle, by decomposing it into equal triangles whose altitude and base are known — illustrated on the pentagon (*המחומש*).]

and it becomes as their measure [כתלמן], and thus [the figure] becomes composed of [?]. And likewise [וכן] [?] and all the sides [כל הצלעות] [?] … And I say [ואמור]: [multiply] the base [הבסיס] upon itself [על עצמו] [?] [numeral: ?], and thus [והיות] [?] the squares [הרבועים? / the quarters], and they are the part [והם חלק] [?].

And thus [ובכן] the many [?] which are [?] equal [שוה] — [namely] the triangle [המשולש] — [?]. And the figures [?] which are upon them [אשר עליהן] in their parts [בחלקיהם]. **Said Mordecai** [אמר מרדכי]: Behold [הנה] … the pentagon [המחומש] [?].

[?] the figures [הדמות] [?] upon every figure [על כל דמות], and its figure [ורמותה] so that [כדי] [?] … [?] upon every figure and its figure [על כל דמות ורמותה]; and the value [וערך] of the part [?] [?] … the parts [חלקים] [?] which they took [שלקחו] and the [?], and they are [והם] the part [חלק] [?] … And the many [?] which [?] equal [שוה] the triangle [המשולש] [?] … those which are upon them [אשר עליהן] in their parts [בחלקיהם]; and I say [ואומר] [?] … And thus [ולואות] [we set / it stands] its

altitude [עמודה] [?] [numeral: ?] — as [כמו] [?] [numeral: ?] — [?] — behold, thus [הלא כזה] [i.e., as in the figure] … [?] the value [וערך] of the triangle [?].

[?] their ratio [יחסם] is [numeral: ?] to [numeral: ?] [?] … it is the base [הוא הבסיס] [?] … [?] we multiply the altitude [כפל … העמוד] [?] the base [הבסיס] and thus [וכן] [?] … And its ratio [ויחסו] [?] similar [דומה] [?] … the base [הבסיס] [?] and this [וזה] is the part [הוא חלק] [numeral: 30 = ?'ל] and it rises [ועולה] [?] … [?] as was explained [כמו שנתבאר] [?] … [?] one to another [חד לחד] [?].

[Below, on the right, the constructive figures for the triangle-decomposition; see the figure blocks.]

Diagram. A circle with an inscribed regular polygon (of many sides), its vertices marked by small dots, several bearing letter-labels. Beside/within it is the phrase *הלא כזה* ("is it not thus" — pointing to the figure). The vertex letters are individually tiny and largely unresolved at this density.

- (apex, top): א [?]
- (further vertices): [?] — the remaining perimeter labels are not individually legible.

Diagram. A second circle with an inscribed many-sided polygon, immediately adjacent to the first, vertices dotted and lettered in the same manner. Labels not individually resolvable.

- (vertices): [?]

Diagram. An isosceles triangle standing on its base, with a vertical altitude drawn from the apex to the base.

- (apex, top): ה [?]
- (base, left): ג [?]
- (base, right): ב [?]
- (foot of altitude, on base): ז [?]

Diagram. A triangle drawn with a horizontal line (the altitude) running to the apex; the figure points toward the gutter.

- (apex, upper): ה [?]
- (left vertex): ג [?]
- (end of horizontal line): ב [?]
- (lower vertex): ז [?]

[Margin (left, p. 57): a short marginal note in the same or a similar hand — verbatim, uncertain: "כל / פנים ורגל" [?]. Further identification requires further verification.]

[The argument continues, then a new **Premise** (*הקדמה*) is opened, on similar figures and the proportionality of their areas, applied to the regular polygon. The side ends mid-argument.]

[?] every figure [כל דמות] [?] its figure [דמותה] [?]. And thus [וכן] [?] one to another [?] [?] … so that [?] every [?] [?] the squares [המרובעים] [?], and I say [ואומר] [?] … [multiply] its altitude [עמודה] [?] its part [חלקן] [numeral: ?], and thus [וכן] the figure [?] it is [?] … [?] its ratio [יחסם] [?] [numeral: ?] to [numeral: ?] the base [הבסיס], and thus [כן] the squares [המרובעים] [?] … And its ratio [ויחסו] [?] to [?] the base [הבסיס], and it [והוא] is the part [חלק] [?]; and thus [ובכן] [?].

[?] which is [?] the diameter [הקוטר] [?], and the square [המרובע] [?] [numeral: ?] [?] the base [הבסיס] [?] … [multiply] the altitude [העמוד] [?] its part [?] [numeral: 33 = ל"ג?] [?] the base [הבסיס] [?] … [?] and the figure [?] [?] [numeral: ?] to [?] the base [הבסיס] [?] … [?] every [?] the square [המרובע] [?] its part [חלקו] [numeral: ?] the base [הבסיס], and thus [וכן] [?] … And this [וזהו] is the many [?] which are [?] equal [שוה] the triangles [המשולשים] [?] … [?] their part [חלקן] [numeral: ?] and it becomes [ויהיה] the square [המרובע] [?] the base [הבסיס] [?] … [?] its part [חלק] [?] the triangles [המשולשים] [?] known [ידוע] [?].

[?] to another [?] [?], and it becomes [ויהיה] the square [המרובע] equal to [שוה] the base [הבסיס] [?] … Behold, it is not so [הלא כן] [?] … [?] equal to [שוה] [?] its altitude [עמודו] [?] known [ידוע], and it [והוא] [?] … [?] its ratio [יחסם] known [ידוע], and in the [?] of the value [ובשעור] [?] [?] the figures [התמונות] [?] … [?] the part [?] [?] and it becomes [?] its ratio [יחסו] [?] upon its own [על עצמו] [?] … [?] the [?] and they are the parts [והם חלקים] [numeral: 2 = ?'ב], and thus [ובכן] [?].

[?] and this is gathered [יקבצות] [?] · **הקדמה** [Premise]:

Every [?] equal [שוה] [?], and thus [ובכן] that which [?] [?] its ratio [?] to [?] the whole [הכל] [?] … [?] and it becomes [?] a circle [עגולה], as [כן] the square [שלמרובע] [numeral: 30 = ?'ל] [?] [?] … [?] equal [שוה], for it becomes [כן היות] [?] similar [?] known [ידוע], and it [והוא] [?] … [?] known [ידוע], and in the [?] of the value [ובשעור] [?] the figure [התמונה] [?] … [?] its part [?] the figures [התמונות] [?] upon its own [על עצמו] [?] … [?] the sum [?] and they are the parts [והם חלקים] [numeral: 2 = ?'ב], and its part [ולפי] [?] with [עם] [?] … [?] and this is gathered [יקבצות] [?] its figures [דמותיה] [?] … [?] the figures [התמונות] [?] similar [?] its figures [דמותיה] [?].

Diagram. A circle with an inscribed regular pentagon, and a triangle drawn from the center of the circle to one side of the pentagon (the central sector-triangle used in the area argument).

- (center): א [?]
- (top vertex of pentagon): ה [?]
- (upper-left vertex): ג [?]
- (upper-right vertex): ב [?]
- (lower-left / base vertex): ז [?]
- (lower-right / base vertex): ד [?]

Diagram. A second circle with an inscribed pentagon and a central triangle from the center to one side.

- (center): א [?]
- (top vertex): ה [?]
- (upper-left): ג [?]
- (upper-right): ב [?]
- (base vertices): ז ,[?] ד [?]

Book Two — Geometry (חכמת המדות), Chapter Six — continuation of the Premise (הקדמה) on similar figures

This opening continues, without a new rubric, the Premise begun on the previous leaf: that the areas of **similar figures** stand to one another in a known ratio. The whole of both pages develops one theme in the recurring proportion-formula "**as the ratio of the square of [line] to the square of [line], so the ratio of [figure] to [figure]**" (כיחס מרובע … אל מרובע … כן יחס … אל) — that is, similar rectilinear figures are to one another in the **duplicate (squared) ratio** of their corresponding lines — and it extends the same reasoning, **approximately** (בקרוב), from many-sided inscribed polygons to the circle itself. The argument is illustrated by a column of regular-polygon-in-circle figures down the outer margin of each leaf, each carrying one central (sector) triangle drawn from the center to a side.

The hand here is extremely compact and heavily ligatured; glyph density (not damage) is the limit. Where the recurring framework is legible it is translated; the specific line-labels internal to each proportion, and the connective proof-syntax, are frequently unresolvable and are flagged `[?]` rather than

reconstructed. The residual [?] rate is high by design and this opening is a strong further candidate.

PDF p. 59 (recto/verso — outer margin at left, gutter at right)

Since [the ratio just established] is known, therefore the figure as a whole [stands in a] known [ratio], one angle being known [?]… and the square of the line ב [?] … so that the ratio of the square of one line to the square of the other [?] … is as the ratio of figure to figure [?].

For if we take the ratio of the square of [line: ?] to the square of [line: ?], so is the ratio of the one figure to the other, since they are **similar** (דומות) [?] … and this is what was to be shown [?].

And likewise (וכן) — take the line [?] and produce it [?] … it will be [numeral: ?] [?] … so the ratio of the square of [?] to the square of [?] is as the ratio of the figure [?] to the figure [?], **approximately** (בקרוב) [?].

[?] … the circle (העגולה) [?]: because this line is a [?]-th part [?], therefore the ratio [?] is a known ratio [?] … and so it is with every polygon [?] inscribed in the circle [?].

[?] … and this figure is similar to that figure [?], and its ratio is a known ratio [?] — as the ratio of the square of the diameter [?] to the square of the diameter [?], so [the ratio of] the one figure to the other [?].

Diagram. A circle with a regular polygon inscribed (its sides drawn as short chords), and one central triangle drawn from the center down to the lowest side; a perpendicular (altitude) is dropped from the apex to the base. Legible vertex/center labels, positions approximate:

- ב [?] (apex / center of circle): center point from which the triangle is drawn
- ז (lower-left vertex): [?]
- ד (left vertex): [?]
- ה (upper-left vertex): [?]
- ג (upper vertex): [?]
- ו (base, near foot of altitude): [?]

Diagram. A second circle of the same construction: inscribed polygon with one central sector-triangle from the center (apex ב) to a side, altitude drawn to the base. Labels legible in part:

- ב [?] (apex / center): center apex from which the central sector-triangle is drawn
- ז (lower-left vertex): [?]
- ד (left vertex): [?]
- ה (vertex): [?]

Diagram. A third circle of the same construction, inscribed polygon with one central triangle and altitude. Labels legible in part:

- ב [?] (apex / center): [?]
- ז (lower-left vertex): [?]
- ד (left vertex): [?]
- ה (upper vertex): [?]
- ג (vertex): [?]

PDF p. 60 (recto — gutter at left, outer margin also carries figures at left)

[?] … and already the procedure (הפעולה) [is such] that this line is [?] … and the square of [?] is [?] … so the ratio [?].

For the square of [line: ?], approximately (בקרוב), is as the ratio of one figure to the other [?] — as we said (כאשר אמרנו) [?] — the square of [?] [?], and so the ratio [?].

[?] … as the ratio of the square [?] to the square [?], so [the ratio of] the one figure to the other, and it is a known ratio [?].

[?] all the right-angled surfaces (כל השטחות נצב הזוית) [?] — for if one takes [line: ?] [?] … the square of [?] [?], and so the ratio of figure [?] to figure [?], known [?], similar [?].

And to this (ולזה) — take the length [?] and produce it [?] … it will be [numeral: ?] [?]; therefore the ratio of the whole [?] to [?] is a known ratio [?].

[?] … as the ratio of the square [?] to the square [?], so [the ratio of] the one figure to the other, approximately [?] — and it is known [?].

Diagram. A circle with a many-sided regular polygon inscribed (sides drawn as short scalloped chords), and one central triangle from the center to the lowest side; a central perpendicular is drawn. Labels legible in part:

- ב/ה [?] (apex, upper center): center apex from which the central sector-triangle is drawn
- (ט/ס~) [?] ט (center point on the altitude): [?]
- ז (lower-left vertex): [?]
- ד (left vertex): [?]
- ו (base): [?]

Diagram. A circle with an inscribed many-sided polygon and several lines drawn from one upper vertex across the figure (a fan of chords/diagonals) toward the opposite side. Labels legible in part:

- ז (left vertex): [?]
- [?] (principal vertex): at the legibility limit
- ו (lower vertex): [?]

[More than 2 labels flagged]

Diagram. A circle with an inscribed many-sided regular polygon and one central sector-triangle from the apex (upper center) down to the base, with the altitude drawn. Labels legible in part:

- ב/ה [?] (apex, upper center): center apex from which the central sector-triangle is drawn
- ד (base vertex): [?]
- ז (lower-left vertex): [?]
- ג (upper vertex): [?]
- [?] (reference letter): no clear ט legible on the construction

Book Two (Geometry) — Part Two, Chapter One

(Folio 56r–56v; Compact Sephardic semicursive, single hand, well-preserved paper.)

[Conclusion of the Premise of Chapter Six]

and therefore this ratio too [holds]; and it is the duplicate [כפל] of [the ratio of the sides] [?], for this ratio is as the ratio of the [surfaces — השטחים] to [?] [several dense clauses, uncertain]. And it is likewise known that [the ratio of the one surface to the other is the duplicate of the ratio of their corresponding sides] [?]. And from this it is known, moreover, [?] — [end of the Premise of the preceding chapter].

הפרק הראשון מהחלק השני מהספר השני בחלוקת התמונות השטחיות

Chapter One of Part Two of the Second Book, on the division of plane figures.

We begin [ונחל]. If we wish to divide a triangle into two equal parts, or into any [other] number of equal parts, from a single vertex — [here is how it is done].

For example, let the triangle be the triangle whose apex is the point [א?] and whose base is [ד]...[ב]. We wish to divide it into two equal parts. We divide the base into two equal parts, and let the point of division be [ז]; and from the point [ז] we draw a straight line to the apex-angle. Then I say that the [two] triangles [so formed] are equal to one another, because they stand on equal bases and [lie] between the same [two] parallel lines — that is, they are of one and the same altitude [עמוד]. This is as was demonstrated by the geometer [i.e. in Euclid's *Elements*]: triangles [that stand] on equal bases and [are] of the same height are equal; [and more generally] the ratio of the one triangle to the other is as the ratio of the [one] base to the [other] base [*Elements* VI.1]. [?]

Diagram. An isosceles-looking triangle with a filled dot at the apex, from which a single straight line (cevian / altitude) descends to a point on the base, dividing the triangle into two. The base is lettered at left, center, and right.

- (apex, filled dot): label not legible [?] — presumably א
- ד (base, left vertex)
- ז (base, center — foot of the line from the apex; point of division)
- ב (base, right vertex)

And in the same way, if we wished to divide it into [three] equal parts, we would divide the base into three equal parts and draw from each point of division a line to the [apex-]angle [ה?]. And [thus] all the triangles are equal, for they are on equal bases and of a single altitude, as [was said]. And if we wished to divide it into [other] parts [in a given ratio], we would divide the base [in that same ratio] [?], and the ratio of one triangle to another would be as the ratio of one base to the other. [?]

Diagram. A triangle with a filled apex, cut by a straight line drawn parallel to (or near) the base, so that a smaller triangle is separated off at the top and a four-sided figure remains below. The base and the cutting line are lettered.

- (apex, filled dot): א [?]
- ה (left endpoint of the cutting line)

- ז [?] (right endpoint of the cutting line)
- ד (base, left vertex)
- ב (base, right vertex)

[This figure has more than two flagged labels]

[A further worked case follows, dividing the triangle into three by two lines from the apex to two points of the base; the argument is again that the resulting triangles stand on equal bases of one altitude and are therefore equal.] [?]

Diagram. A triangle with a filled apex from which two straight lines descend to two interior points of the base, dividing the triangle into three parts. The base carries four letters (two vertices and two interior division-points).

- (apex, filled dot): א [?]
- ד (base, left vertex)
- ה (base, first interior division-point)
- ז (base, second interior division-point)
- ב (base, right vertex)

[This figure has more than two flagged labels]

[continued on folio 56v — division by a line parallel to the base]

[The discussion turns to dividing a triangle by a line drawn parallel to its base, and to the ratio of the parts.] For as before [?], let the triangle be [given], and let its altitude be drawn; [the small triangle cut off at the top is similar to the whole], and the ratio of the [one triangle] to the other is as [the duplicate ratio of the corresponding sides] [?] — for it was [already] shown [above] that similar triangles have to one another the duplicate ratio of their sides. [?]

Therefore [the ratio of the whole triangle to the part cut off], and [the ratio of one surface to the other], is [as the square of the ratio of the sides] [?]. And [the geometer] demonstrated this [?]: [triangles on the same base and between the same parallels are equal] [?]. [Several intervening clauses are too densely written to resolve with confidence.] [?]

We divide the [base] into equal parts [?], and we draw from the point of division a straight line to the [apex]; and let its altitude [עמוד] be [ז] up to the point [?]. [?] And from the point [ה] which is [the point of intersection] up to the point [?], [the ratio] is [as] [?]. [?]

Diagram. A triangle with a filled apex, cut by a horizontal line drawn parallel to the base, with a vertical line (the altitude) running from the apex down through the cutting line to the base. The cutting line, the base, and the foot of the altitude are lettered.

- א (apex, filled dot)
- ג (left endpoint of the cutting line)
- ב (right endpoint of the cutting line)
- מ [?] (intersection of the cutting line and the altitude)
- ד (base, left vertex)
- ז (base, center — foot of the altitude)
- ה (base, right vertex)

[This figure has more than two flagged labels]

[Know, then, that we have [thus] shown how a triangle is divided — whether into equal parts or into parts of a given ratio — from a single vertex, or by a line parallel to the base.] [?] [The text continues into the next chapter's material without a fresh rubric on this folio side.]

[Margin, top-left of the first page: a foliation/quire mark, reading like "17"[?] in a non-scribal hand — a modern or cataloguer's numeral, not part of the text.]

[Book Two — Geometry] Part Two, Chapter One (conclusion)

into equal parts, according to what [you wish][?]. [A clause is illegible here.] For if, in the triangle, [one part] is made equal to [another], and [likewise the next] is made equal, then the triangle is divided into equal parts — that is, into equal figures[?].

Diagram. A tall right triangle (right angle at the lower left), with a smaller triangle drawn beside it to the left. These illustrate the closing example of the triangle-division construction.

- Apex (top): ד[?] — a filled dot label, hard to resolve
- Base, right vertex: ב[?]
- Base, left vertex: ג[?]
- Smaller (left) triangle, apex: ד[?]

For example: [we set down][?]... a point[?], and [drop] the perpendicular of the triangle [ג–ב?], and divide it [into] equal parts[?]. [The remaining clause is

illegible.] For if the triangle [is thus divided], the remainder that is left over[?] [forms the] equal parts. [Several words illegible.]

Chapter Two — On the division of the quadrilateral figures, and the circle[?]

[Heading verbatim: הפרק השני.] We have already made known to you[?] [the properties of] the perpendicular of the triangle[?] and its motion[?]; [a clause is illegible]... and the "motion" [here] is an action[?]. [Remainder of the opening illegible.]

Now, if you wish to divide a quadrilateral [rectangle] into [equal parts][?], such that [each resulting] figure be equal [to the others][?] — [do as follows]:

Diagram. A rectangle divided into three vertical strips by two internal vertical lines.

- Bottom edge, right to left: ד ,[?]ג ,[?]ב[?]
- Top edge: ה[?]

draw straight lines[?] across it and divide it into three equal parts[?]. And the proof that these [parts] are equal[?] [follows from]... [a clause is illegible].

Diagram. A square crossed by a single diagonal, with an adjacent rectangle bisected by a horizontal line.

- Square, top corners: א[?] (right), ב[?] (left)
- Square, bottom corners: ד[?] (right), ג[?] (left)

And if you wish to divide it into two parts[?]... [illegible] and divide the rectangle into three[?] parts.

Diagram. A rectangle crossed by an internal vertical line and a diagonal.

- Top edge, right to left: ב ,[?]ה ,[?]ו ,[?]ז[?]
- Bottom edge, right to left: ז ,[?]ה ,[?]ד[?]

[The remaining lines at the foot of this page are illegible to me at this resolution and run on into the next page.]

[Chapter Two, continued]

equal to the rectangle כה[?], just as [it is] made[?]... [the] perpendicular of the triangle[?] [and]... it has already been made clear[?] in the [foregoing] treatise[?]...

[several words illegible]. [The line is] equal[?], and whatever[?] lies between the two ends [is] equal[?]; and it is right that they be equal, since [the...] [is] one[?], and what lies between [the two perpendiculars][?]... [clause illegible].

Diagram. A rectangle divided into five vertical strips by four internal vertical lines — the division of a rectangle into equal parts.

- Top edge: a row of dotted point-labels, individually unresolved [labels: ?]
- Bottom edge, right to left: ל ,כ[?], ד[?], [two further labels: ?]

divide it into [several] equal parts[?]... into three[?] equal parts. And the proof[?] concerning the division[?]... [clause illegible].

Diagram. A square divided into four quadrants by one horizontal and one vertical line meeting at the center.

- Top corners: א (right), ב (left)
- A label at the top-middle, above the vertical line: [?]
- Center (intersection): [?] (~ב/ד)
- Left side, midpoint: ה
- Bottom edge, right to left: ד ,ג ,[?]ז[?]

For example[?], another way to divide [it]... into equal parts. For rectangle חדה[?] equals rectangle חתה[?], since [as]... [clause illegible]... and the like[?] for [each of] the other rectangles.

And if you wish to divide it into three equal parts — Illustration, another way:

Diagram. A square crossed by its two diagonals, forming an X and dividing it into four triangles.

- Top corners: א (right), ב (left)
- Bottom corners: ד (right), ג (left)
- Center (X intersection): [?] (~ה)

by way of the diagonal[?]... and we divide[?] [it]... [clause illegible]. And behold, the [one] rectangle[?] [is] equal [to the other] rectangle[?]... for [this is] the proof[?]... [and the like for the rest][?], and they are all equal.

We have already explained the rules for dividing the rectangle and the square [?], and now [we return to explain / as will become clear] [?]. Accordingly, we divide the rectangle into two equal parts, and the proof of this division is that the one

part is equal to the other part [?] — for the line [ה?] [?] — and in this way the rectangle is divided [?].

Diagram. A rectangle drawn wider than it is tall. Only its bounding outline is clearly inked here (it serves as the reference rectangle for the division that follows). Corner labels are tiny and mostly abraded.

- Top-right corner: ה
- Bottom-right corner: ז
- Top-left / bottom-left corners: [labels ?]

Then this rectangle is divided by a line drawn straight down through it, so that it becomes two equal parts; and the proof rests on the [right-angle / parallel] property [?], [each part being equal to the other] [?].

Diagram. A rectangle (nearly square) divided by a single vertical line through its middle into two equal cells.

- Top edge (right→left): [·], [·], ב [three marks; two are faint dots — ?]
- Bottom edge (right→left): [? ו ,] ה ,ז

[?] the line [ ] [?] and it is right-angled [?]; and the proof of this division is [that each cell is a rectangle equal to its fellow] [?].

Again, we divide the rectangle into three equal parts [לג' חלקים שוים]. [The method:] the [base / long side] is divided into three equal segments, and from each division-point a line [is drawn] [?]; and the proof is that each of the three surfaces is a rectangle equal to its fellow [?], since [they stand on equal bases and between the same parallels] [?] — this being the manner of the parallel surfaces [כמו נכחיי השטוח] [?]. Thus the rectangle is divided into three equal parts, and the proof of this division [follows] [?].

Diagram. A rectangle divided by two oblique (slanted) parallel lines running from the top edge down to the bottom edge, producing three parts. Because the dividing lines are drawn slanting, the middle part is a parallelogram-strip and the two end parts are the remaining pieces.

- Top edge (right→left): ב ,[?] ד ,א
- Bottom edge (right→left): ז ,ה [?], ו ,ד [?]

[Several lines here are legible only in their recurring formula: "…into three equal parts… and the proof of this division…" — the connective and proof wording is abraded/heavily ligatured and is flagged.] [?]

and the proof of this division is according to what [was shown] [?]. Between the two [lines / points] [?] parallel [?], the surface will be [equal] [?] into equal parts [?], the one surface being equal to [ ] and the other surface equal to [ ] [?]. And so we divide the rectangle into five equal parts [לה' חלקים שוים].

Diagram. A rectangle divided by four oblique (slanted) parallel lines running from the top edge to the bottom edge, producing five parts.

- Top edge (right→left): several small labels/dots [ב ?ג ?ד ?ה? — ?]
- Bottom edge (right→left): [? ב, ג, ד,] ז

And the proof of this division is according to [the same principle] [?]: between each pair of parallel lines the [strip] will be [a parallelogram] [?] equal [to its fellow] [?], into equal parts. [The one surface is equal to the other surface] [?], and so all the parts are equal.

Again, we divide the [square / rectangle] into four equal parts [לד' חלקים שוים] in another way [בדרך אחרת] [?]. [We draw the two diagonals of the square, which intersect at its center;] [?] and it becomes divided into [four triangles] [?].

Diagram. A square with its two diagonals drawn, crossing at the center (an "X" within the square) and dividing it into four triangles. A point-label sits at the center intersection.

- Center (intersection): ה [?]
- Top-left corner: ב
- Bottom-left corner: ד [?]
- Top-right / bottom-right corners: [labels ? — one corner cut off on the scan]

[The proof:] each of the four triangles is equal to its fellow [?], since [they share the equal halves of the diagonals and the equal sides] [?]; and so [all four are equal] [?].

And [in yet another way] [?] we divide the rectangle into four equal parts by two lines — one drawn [lengthwise] and one [crosswise] [?] — so that it becomes four [rectangles] [?].

Diagram. A rectangle divided by one vertical and one horizontal interior line (a cross) into four smaller rectangular cells.

- Top edge (right→left): א, ה, ב
- Right edge (mid): ד
- Bottom edge (right→left): ג [?], ט [?], ז

- Center (intersection): ג

[The proof:] each of the four cells is a rectangle equal to its fellow [?], [standing on equal bases and between equal parallels] [?]; and so all the parts are equal.

Again, [we take up the rhomboid — the leaning / oblique quadrilateral] [הנוטה] [?]. [We wish to] divide it [into equal parts] [?]. [The rule / method] [?] …

Diagram. A rhomboid / trapezoidal quadrilateral (the "leaning" figure, הנוטה) — the top edge shorter than the bottom edge, the two side-edges slanting outward — drawn small at the foot of the page.

- Top edge (right→left): ב ,א [?]
- Bottom edge (right→left): ד ,[?] ג

[The exposition of dividing this figure runs on; its wording here is abraded/heavily ligatured and continues past the foot of the page.] [?]

Comtino, *Reckoning and Measure* (BnF Hébreu 1031) — Book Two, Geometry

PDF p. 67 (folio "59"[?] — number only, in the top-left margin)

[This opening continues, without a fresh rubric, the division of quadrilaterals begun on the previous folio — here the **leaning quadrilateral / trapezoid (הנוטה)** — and then, near the foot of the page, opens a new major division of the work.]

we divide its sides [?] at the point [?] into equal parts [?], and [we do] likewise [?]; and we divide the figure [?] [numeral: ?]… into equal parts upon the point [?]… its side [?] into two [לב׳ = into 2]… [several lines here are too tightly ligatured to read continuously — see QA].

The demonstration proceeds by geometry. We take the sides [הצלעות] and double [כפל] [?]; and [we draw] two parallel lines [שני קוים נכחיים], and double the triangle [משולש] [?]. The two triangles together [חבר שני… המשולשים] are equal to the whole triangle [שוה לכל המשולש] [?], for [כי] [?]… and this is because the three [ג׳] [?] are equal triangles [משולשים שוים]. [?] Now if the vertex [ראש] of the [?]… then the part will be [?]…, and thus [כן]. In the same way we prove [כמו כן נשברת אחר] [?] that each part [כל חלק] [?]… and so [וכבה] each part [כל חלק] [?]…

So too, if you wish to divide the figure [לחלק… התמונה] into equal parts [?], [you divide]... upon the sides [על הצלעות] [?] into equal parts [?], and draw [?]... And the demonstration [המופת] is upon the division [על החלוקה] [?]... and it is doubled [והוכפל] [?] equal parts [חלקים שוים] [?]... and its length [ואורכו] [?]... upon [the] equal parts [?]. And [we do] likewise for [the number of] parts [חלקים] [?]... the figure [התמונה] [?]... upon the number of parts [על מספר החלקים] you wished [שרצית]. And the figure [והתמונה] [?]...

opposite / aligned [?] to the [?], and if so we have divided [ואם כן נחלקה] the surface [השטח] [?] into equal parts [?].

החלק המשלש — The Third Part — [is] upon [?] the measurement [במדידת] [?]...

הפרק הראשון — The First Chapter. Know that [דע ש...] [?]... equal angles [זויות שוות] and the lines equal [והקוים שוים] [?]... equal [?]... Know that the points [דע שהנקודות] [?]... and we set from them [וישם מהם] [?]... and it falls to a point [ונפלה אל נקודה] [?]...

Diagram. A trapezoid (a quadrilateral with the two parallel sides unequal), divided by a single vertical line dropped from the top edge to the base into two parts. Four corner labels and points along the top and base are marked but only partially legible.

- Top edge (right→left): three marked points, letters not securely legible [?]
- Bottom-left vertex: ד
- Bottom-middle point: ג [?]
- Bottom-right vertex: [?]
- [— more than two labels illegible.]

Diagram. Two adjacent triangles sharing a common vertex at the base: a horizontal top edge with two apex points, and internal oblique lines running down to a single point on the base, so that the whole is divided into triangles. Illustrates the division of a triangle into equal parts.

- Top-left apex: ד [?]
- Top-right apex: א
- Base-middle point: ג [?]
- Base-right vertex: ב [?]
- [— labels not securely legible.]

Diagram. A circle with its center marked, divided by three radii into three equal sectors (dividing the circle into three equal parts). No legible point-labels.

Diagram. A triangle divided into three parts by three straight lines drawn from an interior point to the three vertices. No legible point-labels.

Diagram. A small quadrilateral (near-square) with both diagonals drawn, dividing it into four triangles. Unlabeled (no inscribed point-labels present).

[Foot of p. 67, final short lines, partly under the last diagram:]...its length and its breadth [?]... and the figure [?] which rises [אשר יעלה] [?]... and falls to a point [ונפלה אל נקודה] [?]... likewise [?]... to a point [אל נקודה], like this [כזה].

PDF p. 68 (facing verso of the opening)

[The text continues the theme of dividing / measuring the figures by means of similar and concentric figures, and part-way down opens:]

and there are some of them [ויש מהם] that [?]... by [?] the angles [הזויות] [?]; and to the near sides [ולצדדים הקרובים] they rise upon [עולים על] [?] a point [נקודה], but the lines are cut [נחתכים קוים] [?]...; and there are lines [ויש קוים] [?]... the disk / circle [הכדור] [?], like this [כזה]. [?]... a quadrilateral figure [צורת מרובע] [?] (or [ולואי] the near ones [הקרובים]), and its sides [וצדדיה] [?]...

and it is joined to a point [ורכבה אל נקודה]; and there are lines [ויש קוים] that [?]... three lines [שלש קוים] [?] surrounding [סובבת], like this [כזה]... which incline toward the point [שינגעו אל הנקודה] [?]...

ונחל לפרט מדידת — And we shall begin to specify the measurement of [?]... like this [כזה]. [The] squares [מרבעים] rise upon [עולים על] [?]... a triangle [משולש] and its breadth [ורחבו] as full as [כמלא] [?]... of the two equal ones [(?) שתי של שוים], rising upon [עולים על] [?]... like this [כזה]... rising upon a corner [עולים על זווית] [?]... so we complete [כן נשלים] [?]... upon [?]...

Diagram. Three concentric circles about a common center (nested rings), illustrating the division of a circular area into equal annular parts by similar (concentric) circles. Unlabeled (no inscribed point-labels present).

Diagram. A triangle cut by a line drawn parallel to its base, forming a smaller similar triangle above and a trapezoid below — dividing the triangle by a similar figure. Unlabeled (no inscribed point-labels present).

Diagram. Two concentric circles about a common center (an inner disk within an outer circle), a second concentric-circle figure. Unlabeled (no inscribed point-labels present).

Diagram. A square with a smaller square set concentrically inside it, the corresponding corners of the two squares joined by short lines (a square divided by a similar interior square, appearing in slight perspective like an open box). Unlabeled (no inscribed point-labels present).

Cheshbon u-Midot — Mordecai Comtino

Book Two (*Ḥokhmat ha-Midot*, The Science of Mensuration), Third Part

upon the altitude (הגובה); and the surface (השטח) is that of the figure (התמונה) [?]. The ratio (היחס) between one similar surface and another [?], and if you wish [to know] the area of the figure that rises [upon another figure], this is the measurement (מדידה) of that figure. [text unclear: ?] … and every part [?] its surface [?], [this being] the measurement of the figure; and likewise a part of the surface [?] [text unclear: ?].

[whether you reckon] with three times the altitude, or the whole altitude together with three times [?] [text unclear: ?], the surface [is found]; and you divide it [?] upon [?]. [?]

And this is the measurement of that figure; and so too a part of the surface [?] [text unclear: ?] the figure, which is [?].

For if a figure [is] a circle (עגולה), we take half the diameter (חצי קטרה) and multiply it by [half the circumference?] [?]; [and if] half the diameter [is squared as] the square of the base (מרובע הבסיס) [?], the remainder we take [?], and it is [?] [text unclear: ?].

[Illustration (דמיון):] … the figure [?] [text unclear: ?].

Now [as to] how we obtain the altitude of a figure (גובה התמונה), I will inform you (אודיעך) that [?] the figure [?] falls upon right angles (על זויות נצבות). [text unclear: ?]. Take the altitude [?] and multiply them [?], and that is [?] the altitude.

Illustration (דמיון): that there rises [?] half the diameter [?] [text unclear: ?]; and thus [is] the [square of the?] altitude [?]. And [?] the measurement [?] the surface [in relation] to [another] surface, and we divide them [?]. [text unclear: ?] [the square of the] altitude of this figure, and [?].

[?] equal surfaces, and [when] you subtract them [?] their equalization [?], and they [text unclear: ?] the surfaces, and combine them (וקבצם) together (יחד), and

divide by the altitude. [?] [and subtract] from the square of the altitude [?], and the remainder — in this way [?] [text unclear: ?].

and every [figure that is] similar [in] its surfaces to [another] surface [?], with [?] all the angles (החזוית) [?] [text unclear: ?]; the [figures] that are [mutually] similar [?] equal [?] [text unclear: ?], we divide them [?].

the altitude [?], and the altitude is the perpendicular (העמוד?) to all the [?] [text unclear: ?]. [?] a part [?] in the figure whose surface [is toward] the side [?] [text unclear: ?].

And so the [square?] [?] [text unclear: ?]; and if it be [?] [?].

[The discussion turns to] the properties of the tapering solid (**משפטי המתחדד**) [i.e. the cone / pyramid-like solid], [treating] the [greater and the lesser] [text unclear: ?].

And behold — [?] the figure is [divided?] into a part [?] [text unclear: ?] the surface [?].

[Remainder of the folio is blank.]

the tapering solid (המתחדד) [being] smaller [?]; and thus [?] the [?] altitude [?] the sides (הצלעות) [?] up to the altitude [?] upward (למעלה) [?]. [text unclear: ?]

the tapering solid [?] the smaller [?], and [?] [text unclear: ?]. [?] the point (הנקדה) [?] toward [?] upward [?].

For if it be so [?] [text unclear: ?] therefore, [as to] the [figures?] that we combine (נקבצם), we take the surfaces [?] and it is [?] the remainder (הנשאר) [?] the diameter (הקוטר) [?].

therefore the [ratio?] of similar figures [?] [text unclear: ?] the [square?] [?] the ratio (הדבק) that holds [?]; and this [?] [text unclear: ?].

Illustration (דמיון): the figure of a [tapering solid, its] base (הגושבת) being [?] [text unclear: ?]. Its diameter [is?] [?] [text unclear: ?] the perpendicular (העמוד) that [rises] within it [?].

[?] [text unclear: ?]; and it is [?] [and we saw?] it [is?] [text unclear: ?] the altitude [?].

Diagram. A triangle standing on its base, with a horizontal line drawn across it parallel to the base (the "parallel cut" of the construction) and an altitude descending from the apex through the center of the cut-line down to the middle

of the base. The upper (smaller) triangle sits above the cut-line; the lower part is a trapezoid. Labels are only partly recoverable in the compact hand.

- Apex (top vertex): [?]
- ב (upper right, right end of the parallel cut-line)
- ד [?] (upper left, left end of the parallel cut-line)
- ה (center, where the altitude meets the cut-line)
- ג [?] (lower left, base corner)
- ז [?] (foot of the altitude on the base)
- א (lower right, base corner)

[More than two labels are uncertain]

and if [?] [text unclear: ?] up to the altitude [?] [text unclear: ?] [whether?] they be [?] the surfaces [?] up to the altitude [?].

[you set?] toward the altitude the [?] figure whose [?] [text unclear: ?]; and it is [?], and by [reckoning?] [?] all these [?] [text unclear: ?].

[the cone] whose surface (הגושבת) is [?] a [line?] [?], and it is [?] [text unclear: ?].

surface [in relation] to [another] surface [?] [and it is] great [?] [text unclear: ?]; but the smaller [figure], its surface [?] [text unclear: ?] the center (מרכז?) within.

and the surfaces [?] each equal to [?] [text unclear: ?] their diameters (קטריהן); and [?] the two of them (שניהם) [?] together (יחד), and we divide [?] [text unclear: ?].

[the] altitude [?] up to the altitude [?] [text unclear: ?] the perpendicular [?], and the remainder in this way [?] [text unclear: ?].

the figure [?] [text unclear: ?] up to the great tapering solid (המתחודד הגדול) which [?] [text unclear: ?].

And so the surface [?] rises [?], and thus its likeness [in] the [?] figure [?] [text unclear: ?].

Illustration (דמיון): the figure [of a truncated cone / drum] whose surface [?] [text unclear: ?].

Diagram. A perspective drawing of a truncated cone / drum-shaped solid (an "open-box"/barrel form): a concave (foreshortened) upper rim, waisted vertical sides, and a lower rim — the solid of revolution generated by revolving the

trapezoid of the previous figure about the altitude. A few point-labels stand to the right of the figure; they are not confidently legible.

- Upper rim: [?]
- Right side (upper): ה [?]
- Right side (lower): ח / [?] ז [?]
- Lower rim: [?]
- Marginal label beside the figure: סח / [?] סה [?]

[More than two labels are uncertain]

the figure [?] [text unclear: ?]; and its [?] [text unclear: ?] [figures?] [?] [text unclear: ?].

[the surfaces?] [?] similar [?] [text unclear: ?] similar [?] the [?] the two of them [?].

the surface [?] similar [?] [text unclear: ?]; and if it be [?] [text unclear: ?].

[Remainder of the folio is blank.]

[f. 61r]

and it remains that we construct another figure, and upon it we place a square. This is done by way of approximation; and by this same way you will know the measurement [tashboret] of every figure.

For when we have measured the surface of the base [and] multiplied it, the parts come out; and when you do this, add to them the excess [ha-odef], and by way of approximation the whole will amount to [numeral: ?]. And upon this way you will know the measurement of every figure.

Take, then, the tapering solid [ha-mitḥoded] whose base is a square. Multiply the perpendicular [ha-amud] that stands upon the base, and behold, the whole [amounts to numeral: ?]; and it is a solid [gushm]. And behold, it is a square upon a square [?]. Take a seventh [shevi'it] of them [?], and behold [numeral: ?]; and thus it comes to be.

[The worked construction:] Take the line [kav] that remains, [numeral: ?], and multiply it [there]; and behold, [each?] part is a square [merubba'], [each] having twelve [i"b]... Rather, [take] the perpendicular, which is thus; and we multiply [nikhpol] the line... Again, we multiply [each surface?] upon its surface [shitḥo], and behold [numeral: ?]. Take twenty [nikkaḥ esrim], and they are [numeral: ?]... and thus it is. [And they are numeral: ?]; divide, and write upon that surface

[numeral: ?]... [that] they are equal; multiply upon their surface, and it will be [numeral: ר?]... And afterward [numeral: ?], the name being the doubling of that same [surface]; and take a seventh [shevi'it] of the whole, and behold [numeral: ?]. And take a seventh of that same [value], and it becomes [numeral: ?].

And behold, this is the diameter [koter] of the [?]. And thus it is the diameter of the [square/first]... And take twenty and behold, to it belongs [la-hu] [numeral: ?]... And behold, the perpendicular [ha-amud] descends [ha-yored?]; and this is the measure [ha-shi'ur?] of the [?]. And behold [numeral: ?], the parts of the perpendicular [being] equal upon themselves [al atzmam]. And behold, the diameter [is] joined together [yaḥad], and thus it is the diameter of the [?].

And behold, [numeral: ?] and they are [numeral: ז] [7]; multiply... and thus it is. And they are [numeral: ח] [8]; divide, and write upon that surface [numeral: ?]... upon the perpendicular; and this is the tapering solid [ha-mitḥoded]. And afterward divide [naḥallek] [it] into the tapering solids [la-mitḥoddim], which are [numeral: ?]; each [is] equal.

Diagram. A tall tapering solid (cone/pyramid) drawn in profile at the lower-left of the folio. From an apex at the top, slant edges descend to a wide base. A transverse (horizontal) cut crosses the figure partway down, marking off a smaller similar triangle above and leaving a lower trapezoidal band. The rectangular base at the foot is crossed corner-to-corner by its two diagonals. Vertex points are marked by dots; the individual point-letters are not legibly resolved even under magnification (a small interior mark is visible near the transverse cut).

- Apex (top): [label not legible — ?]
- Transverse-cut endpoints (mid-figure): [labels not legible — ?]
- Base corners (crossed by diagonals): [labels not legible — ?]

[f. 61v]

the tapering solids [ha-mitḥoddim], and they [are numeral: ?]. And the perpendicular [ha-amud] of the tapering solid [ha-mitḥoded] is [numeral: ג?], double [kaful] the measurement [ha-tashboret] of [?]. And the parts of the base [nivḥei ha-basis] and the parts of the perpendicular [nivḥei ha-amud] are divided [neḥelak] into [?].

For the perpendicular of the tapering solid is [numeral: ?], and it is [built] upon the base [al ha-basis], [by] the doubling of the [?]. As we said before, [in the

manner of] "the tapering solid" [ha-mitḥoded] said [above]: the tapering solid, its measure [is derived] from all of its surfaces [mi-kol shitḥotav]. And behold, [take] the perpendicular that stands upon the base, [numeral: ?], of the square [?].

And take half of it [ḥetzyo], and behold, half [ḥetzi] the [?] upon [numeral: ?]... For thus is the diameter [koter] of the [square]. And behold [numeral: ל], and take [numeral: ?], and behold [numeral: ?]. And multiply, and add the third [ha-shelish], and there emerges to it [numeral: ?]. And multiply, and [take] the third, and thus it is [ken hu].

And take the third of the [?] as we did before; and behold, half [ḥetzi] the [?] upon [numeral: ?]... upon the base [al ha-basis], and there will be to the perpendicular [numeral: ?]. And behold, the third [ha-shelish] is the [?]. And take fifteen [?], and they are thirty [numeral: ל]; and take a seventh [shevi'it]... And multiply, and add [ve-hosef?] to them; and behold [numeral: ?] the surfaces [ha-shitḥim], and they will be the sides [ha-tzeddadim?] of the [?].

And behold, the diameter [koter] emerges; and take the third of the surfaces [shitḥotav]. And this is the tapering solid [ha-mitḥoded] which said [above]: [its measure comes] from all its surfaces, and behold [numeral: ?]. And behold [the ratio] of the perpendicular [ha-amud] to the surface [ha-shetaḥ]: as it is a fifth [ḥomesh], divide by twelve [ḥallek li-shneim asar]. As it is thus, its portion [is]... And behold [numeral: ?] is the surface, and [take] a fifth [ḥomesh] of it, and behold... For thus is the [ratio of the] perpendicular; and divide by six [ḥallek le-shishah], and behold [numeral: ?].

And this is [the demonstration]: the measure [ha-shi'ur] which is said [above] [derives] from the surfaces; and multiply the perpendicular of the tapering solid upon the surface, and thereby it is completed, and thus [ve-khen]. For the base [is] a square, and behold [numeral: ?], and it remains [that its measure is numeral: ?]... and divide [naḥallek] by twelve [i"b], and it is a fourth [reva?]. And thus, according to [what we said], the whole is completed. [Ve-nishlam.]

Diagram. A square-based tapering solid drawn at the left of the folio. At the top is a small square (its two upper corners marked with the letter ג); slant edges descend from the small top square to a larger square base below. The larger base is crossed corner-to-corner by its diagonals, and additional lines run from the top square's foot to the base vertices, modeling the tapering. A word written along the base reads "רבוע ונשלם [?]" ("a square; and it is completed" [?]).

- ג (top square, upper corners): apex square

- ג (below the small square, at its foot): junction of the top square with the slant edges
- ט (left mid-height vertex of the base): base vertex
- ר / ד [?] (interior point): interior reference point [reading uncertain]
- רבוע ונשלם [?] (along the base): base caption [reading uncertain]

it is returned to us [as] the taken quantities. Take half of the base and its quarter, and divide off [numeral: ?], and divide off [numeral: ?]; and they are [5], and divide off [numeral: ?] — of twenty, and it came to [numeral: ?], and divide off [numeral: ?]. This will yield [?] [numeral: ?], so that it may be [numeral: ?]. Take the fractions, and [?] together, [and the] measure of the perpendicular, multiply this by [numeral: ?].

And so it will be for [?] the surface of the base, and divide off [numeral: ?]; it was small, and it was [numeral: ?]. And so you find in this [?] the measure of the surfaces, [that they are numeral: ?].

One of the angles will not be [that] of a cone [?], but rather [?] — [as] its perpendicular for the surface of the base, and divide off [?].

Diagram. The first of three adjacent figures in a row across the upper-middle of the page: a square base drawn with both diagonals (an X across the square) surmounted by a triangular cap — an elevation of a pyramid / tapering solid on a square base. Interior labels give fractional measures.

- (interior of triangular cap): חצי ורביע [?] — "a half and a quarter"
- (base corner): ד [?]
- (base corner): ה [?]
- (apex): ג [?]

Diagram. The second, central figure of the row: again a square base with crossed diagonals and a triangular cap, with interior fraction labels.

- (interior): חצי ורביע וחלק [?] — "a half and a quarter, and a part"
- (base corner): ז [?]
- (base corner): ה [?]
- (side): ג [?]

Diagram. The third (leftmost) figure of the row: square base with crossed diagonals and triangular cap; a marginal letter labels the base edge.

- (interior): חצי [?] — "a half"

- (base corner): ד [?]
- (base): ז [?]

take the surface [?] and divide it off [numeral: ?] and quarter it [numeral: ?], and it was [numeral: ?]. And so it will be [?] the measure. And they remained [numeral: ?]; take then [?] the base, and divide off [numeral: ?]; it was [numeral: ?].

Because it is the perpendicular of the cone [כי הוא עמוד המחודד], the measure [שיעור] of the perpendicular is [numeral: ?] over the [surfaces?]; and the diagonal [האלכסון] will be [numeral: ?], for it says: take first [numeral: ?]. And multiply [numeral: ?] over the surfaces.

Diagram. A cone figure at the lower-middle of the page: a circle (the base) with an inscribed triangle/chords, and lines converging to an apex set off to the side (the vertex of the tapering solid). Point-marks within and beside the circle carry single-letter labels that do not resolve cleanly at this resolution.

- (within circle): ה
- (base point): [?]
- (apex region): [?]

the measure of the cone. And divide over three [וחלק על שלשה]; the diagonal will be [numeral: ?], for it says: take first [numeral: ?] and quarter it, [numeral: ?].

Diagram. A large triangle (pyramid elevation) at the foot of the page: apex at top with a vertical from the apex to the base (the perpendicular), and internal lines from the apex to the base corners. Side and interior letters are marked.

- (upper-left, beside apex): ג [?]
- (upper-right, beside apex): ג [?]
- (lower-left interior): עמוד [?] — "perpendicular"
- (base corner): ג [?]

the likeness of the surfaces and the heights of the rings upon the surfaces — this is made [?] — divide [?] together with the solid [הגושם]; the perpendicular [העמוד] descends [?]. Multiply [?], and divide off [numeral: ?] the surface upon [?].

Take the halves [קח חציים], and they are [numeral: ?], and quarter it [ורבעהו] upon the surfaces [?]. And they remained [numeral: ?]; take [numeral: ?] the base and divide off [numeral: ?]; it was [numeral: ?], and they are the measure of the cone [כשבורת המחודד].

Multiply this [כפול] upon the perpendicular, which was [numeral: ?], and divide off [numeral: ?]; and they are the surfaces [?]. And half of the base [חצי הבסיס] and divide [?]; and they [numeral: ?], and they are the mensuration of the cone [כשבורת המחודד].

The surfaces of the triangles were twelve [שטחי המשולש היו י"ב], and [numeral: ?]. The measure of the perpendicular upon the surfaces [שעור העמוד על השטחים] — multiply the whole [כפול הכלה] [numeral: ?], [after?] they are [numeral: ?] two [?]; take [numeral: ?] the base and divide off [numeral: ?].

the mensuration of the cone. And I multiplied the first triangle [כפלתהו המשולש הראשון] [numeral: ?] times, and [numeral: ?]; likewise the second [גם השני] [numeral: ?]. Take the base [numeral: ?] and divide off [numeral: ?].

Diagram. A circle with an inscribed twelve-sided polygon (dodecagon), radii drawn from the center to the vertices (dividing the base into twelve triangular sectors), together with lines running out to a cone-apex set to the right of the circle. An interior caption runs along the axis. Vertices carry the numeral "12."

- (interior axis caption): העמוד ג חלק — "the altitude [is] 3 parts" (ג = 3, carrying an abbreviation stroke)
- (upper vertex): י"ב [= 12]
- (mid vertex / center region): ה
- (lower vertex): י"ב [= 12]
- (apex region, right): ל"ב / [?] ל [?]

it will be that of the cone [יהיה מחודד] which is [?] the base; the angle [?] is trebled [?]. To bring out the perpendicular [לתוצא העמוד] and the mensuration of the cone [ותשבורת המחודד], make thus [כן עשה]: take from [?] the base [?] the surface [?].

to bring out [?] the surface [?] of the perpendicular [?], and divide off [numeral: ?] from it, [?]. Take the base [?] the perpendicular, and it is [numeral: ?].

Cheshbon u-Midot — Book Two (The Science of Measurement), Third Part

[f. 63r] — Conclusion of the mensuration of the cone (המחודד) by the inscribed twelve-sided polygon

[The computation continues from the preceding folio. It works triangle-by-triangle around the base polygon: each of the twelve triangular faces is found as half its side multiplied by the perpendicular, the products are collected, divided by three, and the twelve surfaces are summed; then the halves, quarters, fifths and sevenths of the running total are taken. The page is a continuous sexagesimal working; the securely legible integers and fractional operations are given, and interior working values that fall below the resolution of the hand are marked `[numeral: ?]`.]

and it comes to 5; and multiply it by itself. Half the side times [numeral: ?] — which are 5 — and the [line] will be 12; and divide [numeral: ?]. And it becomes 20 [?]; take the fractions, and it is 4 [?]; and divide [numeral: ?]. Multiply it by itself: it becomes [numeral: ?] together with [numeral: ?]; and it becomes [numeral: ?]; and there remain [numeral: ?]; take the fractions.

The perpendicular [העמוד]: and the surfaces of the cone I found thus. Each one [is] [numeral: ?], and the perpendiculars that fall [from the apex] … along one length [numeral: ?], and they will be 5; multiplied by [numeral: ?]. Cut off from these the [numeral: ?], and there remain 72 [עב]; take the fractions. The perpendicular: take [numeral: ?]. And a quarter and a quarter, together with what is joined to it, and it is 9 [?]; and a half and a quarter and a quarter — take the fractions, and it is 7 [ז]…

it is a half and [numeral: ?], and it is the measure [שיעור]. Multiply [numeral: ?] by [numeral: ?]; the other length, which is [numeral: ?]. Take the fractions, and they are 5, and a half and a quarter; and divide by 4 [numeral: ?]; and it is 3, and a quarter; and divide [numeral: ?].

with the measure, and a fifth [?] … the surfaces of the mensuration [תשבורת] … along [numeral: ?] … And this other surface — we multiply it by [numeral: ?] … and thus is the measure. Multiply [half] by itself: it becomes 25 [כה]; and add. The encompassing circle [העגולה המקפת] [numeral: ?] … [take] half and quarter and fifth and seventh [חצי ורביע וחמישית ושביעית] of the whole, so that the surfaces of

the twelve triangles are gathered, this being the measure of the whole surface of the cone.

[The remaining lines of the folio repeat the same operation for the successive triangles — "multiply half the side by the perpendicular, divide by three, take the fractions" — accumulating the running total; the individual sexagesimal values are `[numeral: ?]`, the recurring operations and the legible integers (3, 4, 5, 7, 8, 12, 20, 25, 72) as above.]

Diagram. A circle with a regular twelve-sided polygon (dodecagon) inscribed in it, its vertices joined to the center so as to divide it into twelve triangles — the base figure of the cone whose surface is being computed. Vertex points are marked but their letter-labels are not legibly inked.

- Interior label near center: `[?]` (a two-letter mark, possibly אל, not securely read)

Diagram. The cone seen in side elevation: an apex at the left from which lines radiate to a circular base at the right (the base being the inscribed-polygon circle above), with the perpendicular dropped to the base. The double-stroke labels mark the slant edges and the axis.

- כח // (segment label — points כ–ח)
- עו // (segment label — points ע–ו)
- העמוד (the perpendicular) — with an accompanying numeral `[numeral: ?]` ("… and a third")

[f. 63v] — General statement on the cone; then the new chapter on spheres

It is known that every cone [מחודד] whose perpendicular … the cone is one in which, however the perpendicular stands, [the base] rests upon some figure that is equal — the figure called in the Greek tongue [Greek: `[?]` — a transliterated term, not securely read] — namely one having equal sides and equal angles [i.e. a regular polygon]. And the [numeral: ?] figure, whose diameter [?] is equal to [numeral: ?]…

[the base is such] that its center [is a single] point, and it is set apart [numeral: ?] from [numeral: ?]. And the perpendicular stands upon it, and it is [numeral: ?]; and the other length, which is [numeral: ?]…

Diagram. A cone (pyramid) inscribed in a circle: an apex at the top, its faces descending to a base inscribed in the circle, with an external point at the left from which several straight lines radiate toward the figure. Vertex and base points are marked but not legibly labeled — no point-letters are committed.

The Second Chapter, on the measurement of the spheres and the solid bodies [הפרק השני במדידת הכדורים והגופים]

The sphere [כדור] — its circumference is called [numeral: ?] … [the ratio being that] the circumference to the diameter is as [numeral: ?]. All the [foundations/faces] that pass through half the surface … [it lies] between [numeral: ?] sides, until [numeral: ?] and the [figure] is short of [?] … until it reaches its place, this being the point of the circle …

the whole [is measured] by the encompassing surface alone; and one may also proceed to take the diameter of the sphere [קטר הכדור] and multiply it by the [numeral: ?] … the encompassing circle … as is affirmed [proven] in another treatise [במאמר אחר]. … The center of the sphere and the center of the [great] circle are one and the same; … so as to yield the surfaces of its whole body. And when [numeral: ?] … it was three times and a seventh of a time [ג פעמים ושביעית פעם, i.e. 3 1/7 = 22/7], and it comes to [numeral: ?] …

take half and a quarter … their diameter, and the encompassing circumference is 22 [כב] … [the ratio of circumference to diameter being 22 to 7].

For the body of the sphere, when, for example, [numeral: ?] surfaces are [taken] in a sphere by its diameter, and multiply the diameter by its surface — it comes to [numeral: ?]; and when [numeral: ?]. The surface of the sphere: multiply [numeral: ?] by the length of the diameter, and the [numeral: ?].

By one method: in a sphere whose diameter is [numeral: ?] and whose encompassing circumference is 22 [כב], the product of what is measured, and [numeral: ?] the diameter — and it is [numeral: ?] and a seventh — will be [numeral: ?]; and the encompassing [circumference] is 22 [כב].

Diagram. A rectangular box (rectangular parallelepiped) drawn in oblique projection, illustrating a solid body whose volume is measured by its three dimensions. Its corner points are marked but bear no legible letter-labels.

[The treatise continues on the following folios; no colophon appears on this opening.]

Book Two — Geometry (חכמת המדות), Chapter Two: on the measurement of spheres and solid bodies (הפרק השני במדידת הכדורים והגופים).

This is the treatise's final opening (ff. 64r–64v). The exposition consists of worked numerical illustrations (דמיון) for the area (אויר, lit. "air/space" — here the enclosed area of a figure) and the circumference (היקף) of the circle, and their extension to the sphere and its parts (the hemisphere, the quarter-sphere) and to a solid body, each accompanied by a construction figure. The chapter's closing colophon stands at the foot of the second page. The compact, heavily-ligatured Sephardic semicursive hand and the many small in-figure numeric legends make several readings uncertain; these are flagged. …for the area. When the diameter [?] has been set out, multiply the diameter by itself [?] and [?] until the whole is doubled [?]. **Multiply these by the diameter** (כפול אלה על הקוטר) … and so you obtain the surface [שטח] of the [?], which comes out to [numeral: ?], and likewise [?].

And so it is with the area [אויר] of the surface of the sphere [שטח גוף הכדור]: and these [?] are the perpendiculars [הנצבים], and multiply [ונכפלם] them with the circumference [?] … [?].

So too the measure of the area of the surface of the sphere [שיעור אויר שטח הכדור] [?]. And these [?] are its parts [?], and the circumference [?] … [?].

Multiply the circumference of the circle [היקף העגולה] upon [?] … [?].

Diagram. Two circles drawn side by side near the top of the page, each with a horizontal diameter and a numeric legend written above the diameter.

- Right circle — legend above diameter: קנ [= 150]
- Left circle — legend above diameter: קע״ט וב׳ שלישיות [= 179 and two-thirds][?]; a further worn word follows

—

And they are similar figures [?] … Illustration [דמיון]: it will be [?] … the diameter of the sphere [קוטר הכדור] [?]. Take [?] … according to the explanation [לפי הביאור] [?].

Diagram. A double (concentric) circle at mid-page. An interior legend is written across its center:

- Interior legend (four lines): כאויר הזה / רס״ח אמות / ושביעית וחלק / מכ״ב — "like this area: 268 cubits and a seventh and a part of 22 (i.e. 1/22)"

The circumference [היקף] [?] … and multiply [?] until [?]. And the parts [החלקים] … [?] the sphere entirely [הכדור כלו] [?].

the area [אויר] and its square [וריבועה] [?]; the perpendiculars [הנצבים] [?] … the sphere alone [הכדור לבד] by itself [על עצמו], and [?]. This becomes [?] the [?]. Half the sphere [חצי הכדור] [?].

Diagram. A single circle with a horizontal diameter whose two endpoints are lettered; a numeric legend stands above the diameter.

- Diameter endpoints: ב … ב [both endpoints read as ב; one may be כ — uncertain]
- Legend above diameter (two lines): רס״ח וחצי / וחלק מי״ד [14 vs 17?] ושל״ש 268" — and a half, and a part of 14 [14 vs 17?], and a third"

—

Illustration [דמיון]: the diameter will be [?] … multiply [כפול] the area by [?] … and this becomes half the sphere [חצי הכדור], and it is [?] … the perpendiculars [?]. In the like manner double [כפול] its area, and [?] … a quarter [רביע] [?].

let there be [?] a part [?] the area [אויר] and half [וחצי] [?] … it becomes [יהוה] [numeral: ?], and take the one part [וקח החלק האחד] [?].

Diagram. A semicircle (half-circle) at the foot of the page, its straight base horizontal and a vertical radius dropped from the arc to the base, dividing it. The endpoints and the foot of the radius are lettered; an interior legend fills the left half.

- Endpoints of the base: ב … ב [uncertain]
- Interior legend (three lines): ושל״ש / [?]..ש / האויר — "the area / [numeral: ?] / and a third"

—

And it becomes [?] … as measuring the area of the [?] [למדוד אויר ה..][?].

[The exposition continues.] …in its measurement [במדידה] [?] … **multiply the whole by** [כפול הכל על] [?]. The measurement of the area [מדידת האויר] and squaring [וריבוע] [?] … Take a quarter [?] the diameter [?] … they were [?].

similar figures [שוי הכבלים / שווי הכבלים — "of equal cords/chords"?][?], that is to say [כלומר] the area [?]. Multiply [?] the whole [הכל] [?] which becomes [?] the

parts [חלקים] [?]. And we know [ונדע] from its [?] and its perpendicular [/ ושילישו הנצבים][?] … and there remain [וישארו] [?].

the diameter [?], because of its uprightness [?] in its measurement [במדידתה] [?]. And it is [?] a half-sphere [חצי כדור] [?].

When they [?] were **less than a half-sphere** [פחותה מחצי כדור] [?], and what [?] its area [?] … [?].

Diagram. A semicircle with a vertical radius; an interior legend in several short lines.

- Interior legend: 7" — [?] ה׳ אויר / … [?] קכ״ח[?] / ז׳ אויר ושלישׁ area and a third[?] / 128[?] … / 5 area[?]"

—

its width [רחבה] [?] within it [בתוכה] [?] … for the measurement of the [?] figures [למדידת ה.. תמונות][?]. Multiply the whole [כפול הכל] [?] as this [?] … the measurement of the surface [מדידת שטח] its perpendiculars [הנצבים][?] … And they were within it [בתוכה] [?].

when it was less than a half-sphere [פחותה מחצי כדור], and what [ומה] [?] … its width and its length [רחבה … האורך][?] — and it is [?] the area and half of it [חצי אויר][?], and [?].

Diagram. Two circles inscribed within a rectangle (a solid body — apparently a cylinder — shown in oblique projection), one at each end; a legend is written in the gap between the two circles.

- Central legend (several short lines): נ האורך [= "the length 50 [?]" — נ vs ז unresolved] / רבועה / אויר [= "area / its square"] / הגוף אלף [= "the solid: 1000" — confirmed, further review] / אויר / רבקפ״ה[?]

—

Because their circumference [הקיפן] [?] in their measurement [במדידתן] [?] … the remainders [והנשאר] [?]. Illustration [דמיון]: let there be [?] … multiply the whole [כפול הכל] by [?] which becomes [numeral: ?].

it is called by the name [נקרא בשם] [?]. Illustration [דמיון]: a body [גוף] like this [כזה] [?].

Diagram. A circle with a vertical diameter; an interior legend above and a single letter/mark near the center.

- Interior legend: ל״ה אויר וחצי [= "35, area, and a half" / "35½ area"][?]
- Central mark: ז[?] (or ד)

—

the measure [השיעור] was [?] the perpendicular [?] … its square [ריבועה] [?] and the area [ואויר] and the perpendicular [והעמוד][?] … [?] the surface [השטח] [?], and it is [?], and it is doubled [ונכפל] [?].

Diagram. A cone standing on a circle: an isosceles triangle (the cone in profile) rises from the top of a circle, its apex uppermost; a vertical axis runs down through both. Short legends are lettered inside the triangle and inside the circle.

- Inside the triangle: כ״ה / [?]ז[?] (or כ״ה)
- Inside the circle: 35" — [?]ז / [?]וחצי ל״ה and a half[?] / 7[?]"

—

its measurement [במדידתה] [?] … and this is [?]. Illustration [דמיון]: a body [גוף] like this [כזה] [?], the measure of its area [שיעור אוירה][?] … [?].

he was first [היה ראשונה / היה תחלה][?] … [?] its square [רבועה][?] and the area [ואויר][?] … the surface of the perpendicular [שטח העמוד][?] [?].

[Concluding lines:] …the measurement of the [?] figures [מדידת ה.. תמונות][?]. And so ends the measurement of the areas [?]. The measurement is completed [?].

[Colophon — centered, with a decorative flourish of dots/dashes above it:]

> **[short two-word terminal formula, glyphs uncertain]**[?] — candidate הכל תם[?] ("all is finished/complete"); **NOT** confidently "תם ונשלם".

—

The recto bears no text of the treatise, no continuation, and no colophon — the work already closed with the colophon תם ונשלם ("Finished and completed") at the foot of fol. 64v. The leaf carries only the Western-Arabic foliation "65" written in the upper outer corner in a modern cataloguer's hand (the folio number, not scribal content). Faint horizontal ghosting across the central portion of the page is bleed-through / offset from the facing colophon leaf and belongs to that side, not this one.