

Astronomia Danica

(1640)

by Christen Sørensen Longomontanus (1562–1647)

Translation by Scott Weisman



ARS ASTRONOMICA

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Translator Version 2

Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

A comprehensive textbook of astronomy and the definitive technical account of the Tychonic system — Earth at rest, the Sun circling it, the planets circling the Sun. It works systematically through the structure of the cosmos, the motions of the Sun and Moon, eclipses, the planets, the fixed stars, and precession, with working tables and geometric diagrams throughout; an appendix treats the new stars of 1572 and 1604 and the comets of 1607 and 1618. It is the fullest pre-Newtonian exposition of the Tychonic system.

A note on the translation process

This translation was produced with the [translation-pipeline](#). See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

Translated with the assistance of Claude. The translator thanks the [Anthropic](#) team for making this work possible.

[Pasted-down endpaper / inside front cover. Blank apart from an oval library ownership stamp near the top.]

[Approval stamp: oval purple library stamp, partly faded. Reads approximately "FR. C. C[?]IAN[?] / PROFESSOR D[r]. MED. / [K][Ø]BENHAVN." — a Copenhagen medical professor's ownership stamp. Several letters illegible. [?]]

[Flyleaf, recto. Blank. Faint show-through of text from the title page on the following leaf is visible but not independently legible. A red-crayon shelf/inventory mark reads approximately "3-14" or "13-14". [?] No printed text.]

[Flyleaf / free endpaper, recto. Blank apart from modern pencil cataloguing annotations in the upper-left corner.]

[Margin: pencil shelfmark annotations, top-left — "Bm[?] 5034 / 48", then "Q / 155 /.S395 / 1640 / OVERSIZE". Modern library call number (a Library of Congress–style shelfmark, QB155-range astronomy), not part of the original work. [?]]

Astronomia Danica

Title page.

[Margin/manuscript: an ownership inscription in brown ink across the top of the title page, partly trimmed and faded — reads approximately "Collegij [?] Societ[atis] Jesu [?] Grandani [?] 1640"; not fully legible.]

[Manuscript: "M 11" in ink at upper left.]

ASTRONOMIA DANICA,

Produced through the night-labors and effort of **Christian S. Longomontanus** [Christen Sørensen Longomontanus], Professor of Mathematics in the Royal Academy of Copenhagen [Regia Academia Hauniensis], and divided into two parts. Of these:

THE FIRST sets out, in two books, the doctrine of the apparent daily revolution of the stars about the armillary sphere of the ancients, here restored.

THE SECOND will likewise comprise, in two books, the theories of the motions of the planets — reconstructed in a threefold form, drawn from the observations of D[ominus] Tycho Brahe [Tycho Brahe] and from the author's own.

With an Appendix on the adventitious phenomena of the heavens [Asciticia Caeli Phaenomena], namely, NEW STARS AND COMETS.

Now once again corrected and enlarged by the author in a number of passages.

AMSTERDAM, at the press of Joh[annes] & Cornelius Blaeu [the Blaeu firm]. In the year 1640.

- Banner (below the sphere): INDEFESSVS AGENDO ("Untiring in action")

[The verso of the title leaf is blank; only show-through of the title-page text and foxing are visible.]

Epistola Dedicatoria — Dedicatory Epistle

TO THE RENOWNED KING OF DENMARK, NORWAY, THE GOTHES, THE VANDALS, ETC.

D[ominus] CHRISTIAN IV [Christian IV], HIS OWN MOST GRACIOUS LORD AND KING.

Suppose, most powerful King, most gracious Lord, that we wish to observe and weigh attentively the diverse cast of men's talents, the inclinations of their minds, and finally their pursuits and their actions in life. These differ, it is true, with an almost infinite variety; yet they are so constituted, and arranged under such an opposition, that they may be sorted, not unreasonably, into two kinds. And since the characters acquired in representing these same things — as is usually done in comedies and tragedies — call for just as many roles, those two kinds are not unlike two captives. By their own fault these men were thrown into prison, bound there in chains, and so held under the power of a great king that, until death, they cherished no hope of escape. Yet matters stood thus: both of them were granted, by a singular clemency, one and the same favor — that, if they wished, they might (even while in chains) be carried about and set their eyes upon the varied and astonishing works of that Prince, most beautifully built and laid out in every quarter of his realm, and so relieve their misery for the whole of their lives.

Meanwhile, the one made ready with a most joyful spirit to kiss and to follow the kindly will of this Lord; the other, spurning that same will with a temper more than impious, dull, and stubborn, chose rather to withdraw into the cell of his prison — as a tortoise does — and there, like an owl, to be forever dulled, to pour out mournful lamentations, and at last to be wasted away by decay, by foul squalor and filth, than to share with his companion in the constant contemplation of those most beautiful sights. By what right, then, I ask, could this reprobate —

most wretched throughout his whole life — either envy the happiness of the other, or cast the blame for his own unhappiness onto anyone but himself?

We too are captives in this mortal body, since we carry about with us that very condition which the sorrowful fall of our first parents brought upon us all — so much so that, like the captives of Plautus [Plautine captives], doomed to death, we groan in these words: *Shall we flee? Where shall we flee?* Never in so great a prison of guilt could we be treated more kindly, invited more honorably and graciously, or [be granted leave] to contemplate

still more astonishing riches and works. For these riches and works of the supreme Monarch and Creator of us all — which most beautifully fill this whole world, and are shut off neither from our eyes nor even from our use, except at the outermost circuit of heaven — are carried about [before us] for our contemplation. Since, then, we enjoy so great a clemency of Almighty God that He not only founded from the beginning this most beautiful Theater of the world for our use, but also, after the deplorable transgression and captivity that deservedly followed upon it, has continually sustained the same — and daily sustains it by His almighty word — so that even in this our exile and mortal state, while we journey far from our true homeland, there might be something by which we may console our sick mind, admire the riches of the Creator in heaven and earth, and over them rightly dissolve into His praises:

Above all this, however, [we owe Him gratitude] that this same most gracious Father took such pity on our wretched condition and captivity that He was willing to send His only-begotten Son down to the lowest parts of the earth — indeed, into our very mortal prison — so that this Savior and Redeemer of ours, by taking flesh from our flesh and by the fellowship of His Spirit, might sanctify the captivity of all believers, lead it back with Himself into heaven, and at last bestow upon it an unheard-of freedom, the victory over sin and death having been won by His own precious death.

We therefore hold it to be the mark of a degenerate and utterly stupid spirit so to retreat into one's prison and flee the light that one is touched by no happiness in coming to know the best of things, and stirred by no spur to the exercise of virtue — but rather is infected by shadows, by the pleasures of the belly, by idleness, sloth, listlessness, and that sluggishness, though the mind itself is a particle of the divine breath. And just so, surely, we must judge it to belong to a noble and truly heroic spirit so to conduct oneself in this most splendid theater of the world as to express and represent fittingly the role of the condition which that actor and dispenser of all things, GOD, has laid upon each one; and, while playing that part,

not to neglect to thrust forth his own power through the two windows of this prison — that is, through the body's pair of openings (as Plato calls the eyes).

For this is the practice of the virtues and the beginning of Philosophy: that, with the aid of the eyes, we should first behold the divine works — the miraculous works, I say, in heaven and earth — then admire them; thirdly, scrutinize them more intently with the keen edge of mind and intellect; and finally turn them to the praise of the Creator and, so far as may be, to our own use. The beginning, then, is from the eyes; but the middle, which carries us through to the end, must be taken from the arts and disciplines — and above all (unless we are gravely mistaken) from the Mathematical ones. For by these, nature herself is bound fast in an indissoluble bond — as, after the sacred pages, which affirm that God disposed all things by measure, number, and weight, Plato testifies in the *Epinomis* [Plato, *Epinomis*] in these words: *There is one bond of nature — the doctrine of figures, of numbers, of harmonies, and of the celestial motions; and if anyone*

seeks to find another way to knowledge, and (as the saying goes) invokes fortune. No state can be happy without these arts; one must travel this road, whether the easy way or the hard, and God must not be neglected, who has spread abroad the most welcome reputation of himself in these arts." So says Plato.

From these objects laid out in advance by Plato, just as many names of the arts arise in order: Geometry, Arithmetic, Music, and Astronomy. Now since the preceding three are shown by Plato himself to have been discovered chiefly for the sake of mastering this last art properly, the earlier two — namely Arithmetic and Geometry — are called the "wings" and the "ladders" by which we climb the heavens and measure the perpetual motions of the stars; while Music is the proportioning art, by which at last we rightly grasp the harmony of the higher things with the lower. No one, therefore, should be surprised that these arts have always stood in the highest honor and favor among great emperors, kings, and princes, and among men of public affairs. Indeed, so far as the just governance and defense of their subjects allowed them leisure, they applied themselves to these studies — drawn by the innborn greatness of their minds and the leading of a loftier spirit — or at the very least favored them. And not without reason, since they saw that no small part of happy government, after the greatest virtue (namely religion and true piety toward God), lay in these arts. For they judged nothing more absurd than "to rule the world, yet not know what the world is."

Among the number of these heroes who cultivated the mathematical disciplines — and above all Astronomy, for its antiquity and the wonderful contemplation of

the celestial motions, most worthy of a lofty and royal genius — one may count not only the Magi of the Persians and the philosophers of the Egyptians (from whose ranks first kings, then priests, were drawn), but also princes of royal lineage who came after. Take the first and supreme emperor, Julius Caesar [Julius Caesar]: did he not, in the midst of arms and trumpet-calls, measure the heavens with his rod — chiefly for this reason, that he might thereby restore the calendar of that most corrupted age, with its true annual cycles, to its proper integrity? So that at last he might justly boast of himself in these words:

My year shall not be surpassed by that of Eudoxus. [Eudoxus]

And what of Ptolemy of Alexandria [Ptolemy]? He was certainly so industrious in advancing Astronomy that, aided by the observations of his predecessors and his own, he was the first to transmit to posterity in complete form this fairest of all arts, founded and preserved upon his own traces and monuments. Among Christian princes, the first to come forward is the great-souled Alphonso [Alfonso X], that king of Aragon and Sicily, who is said to have been so lavishly devoted to this art that he laid out four tons of gold for the making of the Tables that to this day bear his name. And why should I mention the emperors Maximilian [Maximilian I] and Charles the Fifth [Charles V]? The former kept with him the celebrated astronomer Vögelin [Vögelin], the latter Peter Apian [Peter Apian] — not only maintaining them honorably, but also present in person while the art was being practiced, indeed drawing equatorial instruments [aequatoria] with his own hand.

And what of Wilhelm, the most illustrious Landgrave of Hesse [Wilhelm IV, Landgrave of Hesse-Kassel], at once a most skilled astronomer? What too of Maurice [Maurice of Nassau], the most excellent Duke of the Netherlands in our own time, no less a most practiced mathematician? But why foreign examples, when our own domestic ones suffice — those that attest that the most serene and powerful kings of Denmark and Norway have likewise held these arts in the highest honor?

That Your Royal Majesty's paternal grandfather, the lord Christian III [Christian III], that most serene and most Christian king, was lavishly devoted to this sublime study, foreign historians proclaim (even were we to keep silent); but so too, even today, do the most artful [instruments — Greek: αὐτόματα (automata, "self-moving devices")] that he left here, and his other monuments, worthy of the heaven and of the lofty spirit of a king. And what shall I say of the lord Frederick II [Frederick II], the father and predecessor of Your Royal Majesty, that most clement king? When he had obtained the lord Tycho Brahe [Tycho Brahe] — a

man most noble by birth, and by genius born to restore Astronomy — did he not, through this exceptional man, the greatest astronomer of all ages and as it were a second Atlas, bring Urania down from heaven, gloriously settle her among us, and adorn her with the greatest royal expense? And Your Royal Majesty likewise continued this, with truly royal generosity, up to your twenty-first year — the year which, with the sevenfold number tripled, is wont to play its fatal trick in human affairs. And not in vain, since nowhere in the whole world have there existed observations of the stars of greater accuracy or moment than on our island of Hven [Hven].

After occasion — or rather, as I see, greater divine providence — had added me too to the apprenticeship of toiling through these labors in that place, and after the length of time, joined to an inborn inclination of mind toward these studies, had brought it about that at last I needed no teacher other than nightly and daily practice: during that eight-year residence with Tycho in that place (in which I not only took part in the constant Uranian labors, but for the most part directed them — as all who know me know) it came about, after the canonical mapping of the fixed stars had been deduced from the sky by our own incredible labor, that I attempted, at my master's command, the restoration of the Moon's course (begun here but completed elsewhere) and likewise a similar reconstruction of the remaining planets. And from this it came about that a certain synopsis of selected observations gathered for this use remained in my possession — not, indeed, kept then to this end, that I should ever hope for a comparable reward from my labors (though even this is owed, above all, by right to one who labors), but much more that I might more correctly and truly judge the industry of others concerning the restoration of Astronomy, whenever the chance to complete it by anyone's hand might arise. Meanwhile, once released from these Uranian labors, when I was applying my mind to other studies and to another way of life that I supposed more profitable, He did not permit it — He who turns in his hands the fortunes of us all — but called me back, beyond my hope, to my old study.

And for my advancement He appointed the illustrious Royal Chancellor, the lord Christian Friis of Borreby [Christian Friis of Borreby] — my excellent patron and outstanding supporter, indeed my prompter as well — who, having summoned me, maintained me for two years in his own household and held me in an honorable position; nor, when on several occasions I was summoned elsewhere honorably enough, did he permit me to depart, but rather took pains that I should place my service with my homeland, and indeed with this Royal Academy [the University of Copenhagen] — which also happened fifteen years ago now. And so, when the Professorship of Mathematics devolved upon me two years later, since I

perceived that I had gained an opportunity for further adorning Urania in my homeland, I gladly accepted it, and as the first thing that had to be done, I had instruments newly built — few of them, indeed, but necessary, and no less capacious or fit for purpose than those I had once handled with my own hands at Hven. This I did so that I might add to them the complements of the observations of the planets that were wanting (as was afterward done here).

For now that I had been kindly refreshed by the serene King's countenance and graciously enriched with royal benefits — through the care of that same aforementioned Patron, the Magnificent Chancellor — it behooved me to ponder with no greater effort by what means Astronomy might be cared for, and the traces of Urania preserved here to the perpetual praise of the Fatherland: traces that, before she had passed over into Denmark, had been so consumed and disfigured by oblivion and neglect that she could scarcely be recognized by her face as having been an offspring of heaven. But while I was pressing down — and all but suppressing — so great a venture in my mind (since, as I had been taught before, it far exceeded the powers of my talent and the means of a private man), God, who had led me to this kind of study and led me back to it, raised up for me another patron and supporter of Astronomy in the homeland: namely, the illustrious man most worthy of succeeding to the Royal Chancellorship, the lord Christian Friis of Kragerup [Christian Friis of Kragerup], Magnificent Chancellor, golden knight, and foremost Senator of the Realm. By his encouragement and singular favor — with which he has lately graciously attended me on account of the professorship of these arts, which he wonderfully cherishes and knows how to handle most expertly — I set down these pages, in which the *Astronomia Danica* (so called, since it was at once born and elaborated in Denmark) may be read, that they might come forth into the public light, the sooner; and two years ago I sent them into Belgium [the Low Countries], so that they might at once be submitted to the discerning judgment of the Netherlanders and to the hands of a most skilled printer there. This is the very work that I now humbly present to Your Royal Majesty — the work that in former years I completed publicly, among other matters, in this your Royal Academy, hoping that this endeavor of mine may be clemently received with royal favor. For what shall we not hope from so great a King — one whom no one ever surpassed in ruling the North, whether in true piety, virtue, and clemency, or in authority and power, or in a loftier and wholly heroic spirit, or in a subtler genius for rightly judging these and other liberal arts and directing them to use; nor, finally, in greater elevation of mind, care, and diligence

in governing his subjects with clement justice, in increasing the wealth of these kingdoms, and in fortifying and justly defending their borders? Indeed, the fame of his renown, crossing the Ganges as once did that of Alexander the Great [Alexander the Great], has long since struck the Indians, and has enticed them into the friendship of a happy confederation — as we hope and pray. These praises of Your Royal Majesty we can by no right pass over in silence, since the Academy and schools of the Church of Christ proclaim them through these realms — schools that persist in the confession of an unblemished religion and besides apply themselves unceasingly to the practice of all the more humane arts. The defended inhabitants attest them; the citadels and cities ringed about with the strongest fortifications display them — some of them, too, entirely new, rising up under the auspices of so great a King, on whom God and nature so smile that he may deservedly be a living and true exemplar of kings throughout Europe.

Therefore, in this flourishing commonwealth (under divine grace), it is fitting that the mathematical disciplines — which have surely been the companions of all happy empires — should flourish here as well; and no less among these, Astronomy, that most excellent and fairest of all the sciences, the science of the stars. For it consists in the knowledge of the shining stars and of their revolution, joined with a most tempered harmony — whence there emerges, in a well-ordered commonwealth, an almost infinite usefulness by land and by sea: whether to investigate from this the nature and distance of remote places; or to traverse the earth itself without error; or to plow the seas on the longest voyage; or to search out the truth of histories; or to recount and understand the poets; or to know, from the daily revolution, the annual changes of season, the lengths of days and nights, and any moments of time whatever; or to construct machines that rival the revolutions of the stars; or to predict the imminent times and magnitudes of the eclipses of the luminaries that occur from time to time. And among the other planets, to demonstrate from their own fixed hypotheses their motions, stations, and retrogradations [repedationes]; and finally — for anyone who so wishes — to have the causes of all these things laid open to him. Besides countless other things that flow from this science of the stars, joined with the highest delight and pleasure — so much so that no art can be more excellent than this, nor more worthy or more fruitful, since it also opens the fountain of all Philosophy. For what else do the poets mean when they fashion Pegasus adorned with the oarage of wings, by which he poised himself in the air, and at last, reaching the summit of Helicon, opened with his hoofs a fountain for the Muses — what else, but that Astronomy gave Philosophy its beginning? As the divine Plato also discusses in plain words in his *Timaeus*. "Nor indeed," says Cicero [Cicero] (in his fifth *Tusculan*), "would Atlas be said to hold up the sky, nor

Prometheus to be fixed to the Caucasus, nor the starry Cepheus, with his wife, his son-in-law, and his daughter, be handed down [in story], had not the divine knowledge of the heavens carried their names over into the error of fable." This art, therefore — the most ancient, fairest, most pleasant, and most useful of all — among the earliest men (as Josephus [Josephus]

as Josephus attests in his *Jewish Antiquities*. From there it was carried on among the Chaldeans, Egyptians, Greeks, Italians, Spaniards, French, Germans, Britons, and Poles — and nowhere has it been reborn with greater care or finer industry than here in our Denmark (and this I can truly say). Now, since I judged that it pertained in no small degree to the honor of both King and kingdom that this knowledge be shared with every liberal mind throughout the world — so that the revolutions and configurations of the stars, coeval with the world, might (so far as is at all possible) be derived from it readily, by means of the [Greek: ψηφοφορία (psēphophoria, "computation, reckoning")], for any time whatever, and turned to uses far greater than those just mentioned, with the heavens left to all as their inheritance — I have for that reason chosen to devote the slender powers of my own talent, as best I could, to bringing this very work to light one day, rather than to pursue other, more profitable arts. I am content so long as I obey God and my magistrate humbly, and so long as I expend my efforts in these labors, first for the praise of my dearest fatherland and then for all good and honest minds. And if these labors should ever be dragged into accusation by perverse minds, following the appetite innate in such people, I rejoice all the same: for when they are weighed and judged — both under so great a King's patronage and against the very face of the heavens, which we enjoy daily — they will the sooner vanish into the smoke of slander.

May the eternal God grant that the great and most powerful Monarch of Denmark and of the North, Lord Christian IV, together with his illustrious Royal House and with these kingdoms and dominions, may live and flourish; and that with him true piety and the other royal virtues may endure unbroken, and be happily propagated to all posterity. Amen. At Copenhagen, in the year from the birth of the world's Savior 1620, and the twenty-fourth from the coronation of the present most serene King.

To Your Sacred Royal Majesty.

Most humble servant,

Christianus Severini F. Longomontanus.

On the Astronomical Work of Christianus Longomontanus, Professor in the Royal Danish Academy

Επίγραμμα [Greek: epigramma, "epigram"] by W. Westhovius.

As far as the stars surpass the remaining parts of the world, so far has that care withdrawn from all other pursuits of things. As the stars hold dominion over all the bodies of the world, and so, by your skill, deep expertise in the stars belongs to all. To surpass on this great and wondrous path those who profess it — this, Christian, Urania [Urania], more excellent than the rest, here grants to you by your teaching, which she rarely grants to the Muses. Envy herself, with unwilling lips, confesses it. [?]

To the Distinguished and Most Ingenious Mathematician Christian Longomontanus, the Dane from Cimbria

an incomparable astronomer, most excellent Professor of the higher Mathematics in the Royal Danish Academy at Copenhagen, Canon of Lund, etc.;

a colleague, kinsman, and most closely joined brother, on his publishing the *Astronomia Restaurata* [Astronomy Restored].

Epigram

by Caspar Bartholinus [Caspar Bartholin], Doctor and likewise Royal Professor in the same place.

Urania had seen light given from LONG MOUNTAINS [a play on "Longomontanus"] to the stars, and unaccustomed glimmerings bestowed. [?] She added: until now the stars have shone from the heart of the world — the stars of the Danes glow red, but now with a new Sun.

Another.

The first Danish Tycho [Tycho Brahe] shone upon the stars, that Sun of his fatherland and of the heavens, the summit of nobility. A new Sun has shone upon the stars, a second Atlas: thus the Danish land has given two suns.

To the Distinguished and Most Skillful Mathematician Christian Longomontanus, Author of the *Astronomia Danica*.

Once the worshipper and the love of Venusian Urania, [?] not without reason do you bear your name from a long mountain. To scale the heavens, you need not pile Pelion upon Ossa: you approach by a more convenient way. You lay lofty and wondrous labors upon the mountain — a mountain that is high, not merely long, departs. [?] It rises, lifts you sublime to the walls of the world, and grants you to enjoy the aether no less than the air. Like a neighbor, you almost touch the stars with your finger, and you see how they are composed and to what place each is assigned. Hence what the ancients taught and what the moderns have taught, you, O astronomers, test with a more skillful balance. [?] And in what place each thing is, and by what motion they are borne through the heaven, you arrange and set in motion in your books. Well done, with your new art (for thus the road is short to the stars) — press on duly to lay open the abstruse paths of the pole. Let the King, the Sun, stand in the midst: let the five royal stars [planets] move around him, now far off, now close at hand. Let Cynthia [the Moon] change her horns around the earth, and circle whatever comes to be, perishes, or varies. Let the fixed throng run constantly across the pole with the same steps, by night and day, around the airy globe. [?] You arrange the rest as well, just as you see them to be arranged by the supreme arranger, God.

Nicolaus Georgius Ærylæus.

To the Distinguished Christian Longomontanus, Author of the work.

What does the long-lived day of an age not change? By this law the very goods of minds hold firm: arts perish and rise again anew; this region is cultivated, that one squalid with filth; and things that had grown obsolete shine bright again afterward. Time and the age turn such changes about. Why should I prove each one singly by testimony? You, Christian, whose skilled hand

and the circles of whose learned mind we behold, by which you render the heavens and the very poles passable, and bring the North nearer to God in his chorus — here you could be a rich witness and authority. For Hipparchus [Hipparchus] indeed, and that sublime Conon [Conon] — each of these shone as a great glory of Urania: but the one [Hipparchus] was born of Rhodes, while Conon set out from Samos and turned his lofty head to the heavens. Ptolemy [Ptolemy] settled within the shallows of the Nile and held a bright torch before his

descendants, the sure path by which they might strive toward the stars. And many others, learned men whom of old either the Greek land or Arab culture brought forth. But these came earlier; now they lie in decay, and there all things wither in the gloom of night. And so the chorus of the arts turned its yoked team and sought Helicon, where the cold of the North is stiff. Here, born of lofty research, Copernicus [Copernicus], among the farthest Prussians of Germany, led the way to heaven by a path not trodden before; here Regiomontanus [Regiomontanus], here Peurbach [Peurbach]; whom, following soon after, learned in the starry regions beyond the rest, came that Reinhold, the Erasmus. [Erasmus Reinhold] [?] But all these were surpassed by the illustrious Tycho Brahe [Tycho Brahe], that glory of his Denmark, a man supreme in mind and in the gifts of genius. To him, that great Atlas, you — though lesser — lending a hand, now lay open the more hidden sacred things of the heavens, and if anything in the art remains gaping, you plane it smooth with shrewd hand and learned mind. And you do this too: that the Danish youth, roused by your leadership, may learn from here the changes of the world, the orb of Phoebe [the Moon], and the fixed courses of the Sun, and may claim this praise as their own. O Christian, and you too, happy Denmark, whom the Greek, the Arab, the Nile, and Germany courted in vain, to be granted to their own brows — this palm is yours, this laurel is prepared for you.

Iohannes Isacius Pontanus [Johannes Isacius Pontanus] Royal Historiographer.

Astronomia Danica — Part One (Pars Prior)

[Internal title]

THE FIRST PART OF THE *ASTRONOMIA DANICA*,

containing the doctrine of the sphere in two books. Of these:

THE FIRST

presents the structure of the sphere, restored from the armillary circles of the ancients; with brief preliminaries on the Foreknowledge [Prognorismata] of Astronomy through the analysis of triangles, on the substance of the heavens, and so forth.

THE SECOND

explains the use of the same sphere succinctly and demonstratively, through various and most useful problems.

To the Reader [Ad Lectorem] — on the *Astronomia*

Although from the outset I had intended to bring our Astronomy into the light without an attached analysis of triangles, I came to a different decision. I judged it easy to see that the mathematicians, for whom in particular I had long ago designed this labor, would each prefer to follow the triangular solution most familiar to them in unraveling the work itself. I also thought that, in this same field, the part I had publicly lectured on needed to be worked out more exactly, and that a portion of geometry — the part that governs its chief application — ought to be held in reserve, so that the work might come out with a fuller apparatus of its own: namely, together with the Canon and with the construction and correction of this Canon derived from the true Cyclometry I had discovered.

Nevertheless, once I had found among the Belgians a thoroughly suitable craftsman for printing this work — Willem Blaeu [Guilhelmus Blaeu], a man of the greatest skill, excellently versed in mathematics, and once my closest friend at Hven [Huana] — who spares no effort to make even the products of other workshops in this field as polished as possible, I judged it advisable to compress the trigonometry of plane as well as spherical triangles into the briefest and easiest compendium, and to set it before this same work of ours. In this way our astronomical study might be made more accessible and more welcome to lovers of astronomy and to students in the universities. For the foundation of astronomy — and of nearly all practical mathematics — rests on the analysis of any given triangle, as the first of all figures, the one into which the rest are resolved. It was on this account that our master, the lord Tycho [Brahe], once bestowed on it this praise:

The triangle holds everything that learned Mathematics pursues: It opens whatever locked thing Olympus contains.

Indeed, the most excellent F. Viète [Vieta] also had this in view, when he prefaced the hypothesis of his Mathematical Canon with these words:

To obtain the sides from the angles, or the angles from the sides, and mixedly in triangles both plane and spherical, is the highest glory of the mathematician; for in this way he measures heaven, lands, and seas by a fortunate and admirable calculation.

The same had also to be done after the example of the ancient astronomers Ptolemy and Copernicus — lest I should seem to want to fly into the heavens without these wings or ladders, acquired through Arithmetic and Geometry, and

to point out an unknown road to the upper air. This is also the reason why I have added the second part of the astronomical *Prognorismata* [Προγνωρισμάτων] — namely the διδάσκειν [Greek: didaskein, "the teaching" — i.e. the doctrinal portion], concerning the substance of the heavens and the great bodies of the world — to the earlier part on the measurement of triangles by numbers, according to the judgment of my own ability, in keeping with how I thought it agreed with Sacred Scripture, with nature, and with reason. For without a moderate prior knowledge of these matters, Astronomy, the queen of all the disciplines, is neither successfully taught, nor grasped, nor resolved into its causes, nor at last rightly applied to its uses. But on such matters we have discussed better views throughout, without prejudice to anyone who brings forward something better. I thought only that the kindly Reader should be warned, in the first place, about the *prognorismata*; about the rest, more frequently, wherever there is need. For to commend with elaborate effort, before discerning readers and at the very threshold, an Astronomy newly restored, I judge to be no less superfluous than to hang up ivy beside wine that already sells itself. Meanwhile, if anyone gains some profit here from our watching and labor, I shall reckon that I myself have gained it; since from these efforts I have looked for nothing else in life than that they might be rightly invested in the benefit and use of a grateful posterity.

ΠΡΟΓΝΟΡΙΣΜΑΤΩΝ [Prognorismatōn] of Astronomy — PART ONE

On the compendious analysis of plane and spherical triangles by means of numbers

[Margin: Definition of the analysis of triangles.] The analysis of triangles is the most ingenious method of producing, from three given parts [διδόμενοις — Greek: didomenois, "(things) given"] of a proposed triangle, any fourth part you please — or all the remaining parts latent within it — in numbers, according to the same proportion.

The doctrine of the analysis of triangles, insofar as it is here treated by us in compendium, has two members.

First come certain general matters concerning the definition and partition of the triangle, together with certain geometric theorems. With the help of these, the chief reasoning about the analysis of any triangle is set up. At the same time it is

shown from where the threefold Canon — which consists of the sines, tangents, and secants, and is destined for solving triangles — depends; and likewise the construction and continuation of this Canon by numbers.

Then follow the matters that accompany the analysis of the triangle itself, and these in turn are twofold.

For either the analysis is directed generally to any question whatever, and falls under the single golden rule of proportion — where the quantity of the unknown, fourth (and so on), latent term is found either by the customary procedure handed down in Arithmetic, or by the more compendious route through prostaphaereses [prostaphaereses].

Or else a solution is fitted to the particular kinds of the proposed triangle, both plane and spherical, according to what is sought, with the reasoning about each kind suitably adapted from the foregoing theorems.

This, then, is the order, this the sum of our analytic doctrine concerning triangles. If it is asked why this is placed first of all, in the very vestibule of an astronomical work, the answer is ready at hand: because it embraces the measurement of the whole universe. But if it is asked further why most of the theorems to be produced below as aids to the analysis are presented with their proofs by suitable demonstrations [ἀποδείξεις — Greek: apodeixeis, "demonstrations, proofs"], this too can be answered: that those proofs are taught in Geometry, and are here only cited, lest, while we aim at brevity, we be overwhelmed by the multitude and mass of such things — which, more rightly cultivated and demonstrated in their own ground, are channeled by little streams to this present use.

ON THE FIRST MEMBER.

[Margin: Definition of the triangle.] A triangle is a figure consisting of three angles and as many sides. Therefore, in every triangle six things are set out for consideration: three angles and three sides. But so that what follows may be more conveniently dispatched for beginners, the division of the circle — from which the solution of the triangle chiefly springs — must first be set down here.

[Margin: First division of the circle.] It must therefore be held that every circle is commonly divided into 360 degrees, or parts; each degree into 60 first minutes; and again each first minute into 60 seconds, and so on, as far as the requirement of precision demands, to infinity. From this arises the sexagesimal Arithmetic, which is otherwise called Logistic. In certain places the sign ° is to be written

above the degree, ' above the first minute, " above the second, and so on, when Logistic numbers are worked through in practice.

Further, this Arithmetic of two species contains analytic trigonometry: namely, by Addition and Subtraction. On each of these, very briefly here.

[Margin: Addition.] Addition is the gathering of the proposed numerical species into one sum. For example, the numbers $28^{\circ} 35' 23''$ are to be added to $36^{\circ} 25' 46''$. With the order preserved, so that homogeneous species are placed beneath one another, the operation proceeds — as in other cases — from the right toward the left. Then, in each species, whatever falls below sixty is set down in its place; and when sixty is completed, a unit for it is carried over into the preceding species. The reason for this rests on the division of the degree above it into the smaller species by 60.

	mi				mi		
	gr.	.	fec.		gr.	.	fec.
ut	36.	25.	46.	Item	27.	39.	56.
	28.	35.	23.		56.	28.	27.
Summa fit (Sum is)	65.	1.	9.	Summa (Sum)	84.	8.	23.

[Margin: Subtraction.] Subtraction is the taking of a smaller proposed number away from a larger, so that the difference may appear in the remainder. Accordingly, with the order observed that was kept in Addition, the smaller number will conveniently be placed below the larger — and this with regard not to the lower species, which bear the name of minutes, but rather to the whole quantity of each number being compared. Then, the procedure of Subtraction having begun here too from the right, if a higher species is somewhere insufficient for the one to be subtracted from it, a unit will be summoned from the next preceding species, diffusing itself by 60 into the species immediately following. It is no different than if you wished to take more coins of a smaller denomination away from a larger one — which cannot be done otherwise than by resolving the value of the larger into the smaller denomination. Let the preceding examples be repeated.

	mi				mi		
	gr.	.	fec.		gr.	.	fec.
	36.	25.	46.		56.	28.	27.
	28.	35.	23.		27.	39.	56.
Differentia (Difference)	7.	50.	23.	Diff.	28.	48.	31.

These few remarks on Logistic Addition and Subtraction had to be inserted, if only for the benefit of learners; they will become easier through the several examples below.

Furthermore, there is another division of every circle — namely, into its four quadrants — which we follow in practice, and which is most apt for the distribution of the triangle that follows.

Diagram. A circle with center A, lettered B C D E around its circumference. Two diameters — B D (horizontal) and C E (vertical) — meet at the center A at right angles, dividing the circle into four equal quadrants. The labels run: C at the top, B at the right, D at the left, E at the bottom, with A at the center.

- A (center): center of the circle
- B (right): endpoint of the horizontal diameter B D
- C (top): endpoint of the vertical diameter C E
- D (left): endpoint of the horizontal diameter B D
- E (bottom): endpoint of the vertical diameter C E

For when, in the adjacent circle B C D E whose center is A, the diameters B D and C E meet at right angles in A, it is plain that the said circle is cut into four equal parts, all four angles around the center A being right angles. And since the measure of an angle proceeding from the center of a circle is determined by the circumference of that same circle, and is double that of the angle which, standing on the same circumference, begins from the opposite side (by Euclid, *Elements* III.20), therefore the individual quadrants — that is, the 90 degrees — enclose a right angle in every triangle, and conversely. An angle that exceeds this measure is called obtuse; one that falls short of it, acute.

[Margin: Division of the triangle by the disposition of its angles.] From this consideration of the angles in a single triangle, the proposed triangle is correspondingly either right-angled, when it possesses some right angle, or oblique-angled, when there are no right angles in it but either all are acute, or one is obtuse and the rest acute.

[Margin: Another division of the triangle by the disposition of its sides.] Further, another division of the triangle arises from the character of its sides, so that it is called either Equilateral [Isopleurum], or Isosceles, or Scalene. An equilateral triangle is one with equal sides and angles. An isosceles triangle is one with equal legs. A scalene triangle, finally, is one with all its sides unequal. But this division of the triangle was the more general one; the special division follows.

A triangle is either plane (that is, rectilinear) or spherical (that is, curvilinear).
[Margin: Division of the triangle, with respect to its special class, into plane and spherical.]

A rectilinear triangle is made up of three straight lines. A spherical triangle is made up of three circular lines lying on one and the same spherical surface.

GENERAL THEOREMS

Serving for the Solution of Triangles

Some geometrical theorems bearing on the solution of triangles are considered, as it were, from the outside; these are called the prior and more general theorems. Others enter into the construction of the Canons, which carry out the analysis of triangles by means of numbers; these are more special, since they are restricted to the sines, tangents, and secants alone. The latter, to distinguish them from the former, we call *Enunciations*.

THEOREMS

[Margin: General theorems.]

1. A line — whether straight or circular — falling upon a line of the same kind makes two angles equal to two right angles.

Therefore, if one of these angles is given, the other is the complement of the given angle to a semicircle, that is, to 180 degrees.

2. The angles at the vertex are equal. Hence, if one of the four angles that arise at the point of intersection of two lines of the same kind is given, the three remaining angles are given: first in a general way, since they are the complements of the given angle to the measure of the whole circle; then in a specific way, by the theorem just stated.
3. Every triangle proposed can be resolved into two right triangles by dropping from one of its angles a perpendicular line of the same kind onto the opposite side — and this is done conveniently, namely so that the perpendicular falls opposite the given angle.
4. In every plane triangle the three angles are equal to two right angles, that is, to 180 degrees. Therefore, when two angles of a rectilinear triangle are

given, the third is the complement of the two given angles to two right angles, that is, to 180 degrees.

5. In a spherical triangle the three angles are greater than two right angles. Here it must be noted that, although this holds geometrically and ἀκριβῶς [Greek: akribōs, "exactly"] for any spherical triangle whatever, nevertheless in an extremely small spherical triangle — where none of the sides exceeds one degree — the excess of such three angles over two right angles can scarcely, if at all, be discerned. For this reason the solution of a spherical triangle of this kind does not differ from the analysis of a plane one.
6. A straight line cutting across parallels makes the alternate angles equal on either side.
7. If within a rectilinear triangle a line is drawn parallel to one of the sides, it cuts the remaining sides proportionally. For thus equiangular triangles are formed on both sides, and they therefore have proportional sides. [Margin: Props. 2 and 4 of *Elements* Book 6.]
8. Since all reasoning is carried out by means of straight lines, it is clear that in the analysis of a plane triangle there are substituted, for the angles, their sines — that is, the halves of the chords in the circle — or else their tangents or secants; while in a spherical triangle this is done for the angles together with the arcs (that is, the sides). But what these lines are, and in what ways they must be adapted to the analysis, must now be briefly set out.

Definitions of the Sines, Tangents, and Secants

Let a circle B C be described about the center A (in the following figure), and in one of its quadrants — sufficient for the whole undertaking — let some arbitrary point D be conceived. Draw the radius A D, and to the opposite diameters of the circle D E and D F. Now since the *right sine* of an angle or of some arc is defined as the straight line which descends perpendicularly into the radius from the end of the proposed arc, under that same arc, it follows that

Diagram. A circle centered at A, with a horizontal diameter through A and a vertical diameter through A defining a quadrant at the upper left. A point D lies on the circumference within that quadrant; the radius A D is drawn and extended outward to H. From D, the perpendiculars D E (onto the horizontal radius, meeting it at E) and D F (onto the vertical radius, meeting it at F) are

dropped. The tangent line C G is erected at C (end of the horizontal diameter) and B H is erected at B (top of the vertical diameter), both meeting the extended radius A H at G and H respectively.

- A (center): center of the circle
- B (top of vertical diameter): endpoint of the vertical diameter
- C (end of horizontal diameter): endpoint of the horizontal diameter
- D (on circumference, upper-left quadrant): end of the proposed arc
- E (on horizontal radius): foot of the perpendicular D E
- F (on vertical radius): foot of the perpendicular D F
- G (on extended radius, near C): intersection of tangent C G with the extended radius
- H (on extended radius, beyond B): intersection of tangent B H with the extended radius

it follows that D E is the right sine of the conceived arc D C, that is, of the angle D A C. Likewise D F is the right sine of the complement of the arc D C — that is, of the arc D B, or of the angle D A B.

Furthermore, since the *versed sine* of an angle or of some arc is defined as the portion of the semidiameter, continued from the end of the right-sine line to the circumference, the versed sine of the arc D C will be the line E C. Likewise the versed sine of the complement of the arc D C — that is, of the arc D B — is the line F B. [Margin: What the right sine is.] [Margin: What the right sine of the complement is.] [Margin: What the versed sine is.] [Margin: Note. The versed sine is in no use among us.] But it must be noted that, although the versed sines we have just described are in use among [Giovanni Antonio] Magini [Magini] and others, we nevertheless set them aside and consistently employ the right sines. So much for the sines, or lines, within the quadrant of the circle. Next come the tangents — lines outside the circle — and likewise the secants, drawn through the circle.

Further, let the line A D be drawn out to infinity from A, the center of the circle, through D, the end of the proposed arc; and let the perpendicular lines C G and B H be erected from the ends of the diameters, meeting the infinitely extended line A H. [Margin: What the tangent is.] Now since the tangent of some part of the circumference is the straight line, perpendicular at the end of the diameter (the point, that is, which the said part of the circumference reaches), continued into the radius through the end of the given arc, it follows that C G is the tangent of

the arc C D, that is, of the angle D A C; and B H is the tangent of the arc of its complement, namely B D, that is, of the angle B A D.

[Margin: What the secant is.] Finally, the secant of a part of the circumference, being the straight line drawn from the center through the given end of the part of the circumference into the tangent of that same part, will be A G, the secant of the proposed arc C D, that is, of the angle D A C; while A H is the secant of the complement of that same arc — that is, of the arc B D, or of the angle B A D.

Thus far we have had the definitions of the sines, tangents, and secants. There follow

CERTAIN ENUNCIATIONS

Premised to the Analysis of Triangles

1. Given the sine, tangent, or even secant of some arc, there is given in the Canon the sine, tangent, and secant of the complement of that same arc to 90 degrees. For example, given D E, the right sine of the arc C D, the arc C D itself is given, and therefore its complement D B, and consequently the right sine of this latter, D F. The same thing holds for tangents and secants to be obtained from the Canon, so that here nothing but the use of the table is required — and to this, a correction for the second minutes may also be commended, since the numbers extend at least to the first minutes, or to fractions of them, as was indicated above.
2. The right sine of any angle is proportional to the side subtended by that same angle; and conversely.

Hence also, by conversion of the terms in the rule of proportion: as the sine of one given angle is to the sine of another angle, so is one given side to the other side sought — namely, the sides that subtend those same angles of the triangle.

Now this Enunciation, whose use in analysis is most frequent, is demonstrated as follows by the 7th theorem above. Let A B C be any triangle, and let the side A B be extended to E, so that A E becomes equal to B C. Then, A E and B C having been made equal in this way and set up as radii, let the arc E H be described, which is the measure of the angle at A, whose right sine is E F; and the arc B G, which is the measure of the angle at C, whose right sine is B D. Therefore, within the right triangle A E F, since B D is parallel to the base E F, it will be that, as A B — the side

opposite the angle at C — is to B D, the right sine of that same angle, so is the side A E, that is (by hypothesis)...

Diagram. An oblique triangle with apex B and base running along the bottom. From the base, points are marked A, G, D, F, H, C from left to right; a curved arc rises from the base up toward apex B and beyond toward E at the top. Lines are drawn from E and B down to the base, forming the right triangle A E F with the perpendiculars described in the text.

- A (base, left): vertex of the original triangle
- B (apex): vertex of the original triangle
- C (base, right): vertex of the original triangle
- E (top): point such that A E equals B C, set up as a radius
- F (on base): foot related to the right sine E F
- G (on base): point on the arc B G measuring the angle at C
- D (on base): foot related to the right sine B D
- H (base, far right): end of the arc E H

(by hypothesis) B C, which is opposite the angle at A, to the right sine of that same angle, namely E F. Which was to be shown.

3. If the given angle is obtuse, one must work with its complement to a semicircle, that is, to 180 degrees.
4. In every right triangle, when one of the sides about the right angle is set as the radius (that is, the whole sine), the other side about the right angle will be the tangent of the angle opposite to it — as in the next-to-last figure, recalled here. For when the side A E of the right triangle A E D is set as the radius, D E will be the tangent of the angle D A E. Or, when D E is set as the radius, A E will be the tangent of the angle A D E — by the 7th theorem of this section.
5. By the same reasoning, the radius A C is the mean proportional between G C, the tangent of the arc D C, and the tangent of its complement, B H. For as C G is to A C, so is A B to B H.

Diagram. A circle with a horizontal and a vertical diameter crossing at the center A, forming four quadrants. A point D lies on the circumference of the upper-right quadrant; the radius is extended outward beyond the circle to H. Tangent constructions at the top (B) and right (C) produce the points G (near the upper-right, where the extended radius meets the tangent) and the line to H. The figure

repeats the tangent–secant construction with the quadrant placed in the upper right.

- A (center): center of the circle
 - B (top): endpoint of the vertical diameter
 - C (right): endpoint of the horizontal diameter
 - D (on circumference, upper-right quadrant): end of the proposed arc
 - E (on horizontal radius): foot of the perpendicular
 - G (on extended radius, near C): intersection of tangent with the extended radius
 - H (on extended radius, beyond B): intersection of tangent with the extended radius
6. The radius is the mean proportional between the secant of an angle and the right sine of the complement of that same angle — as in the figure before the last. For as A G, the secant of the angle D A C, is to A C, the radius, so again is A D, the radius, to A E, the right sine of the complement of that same angle at A.

Therefore, in a right spherical triangle, as the radius A D is related to the right sine of the angle D A E, namely D E, so does A H, the secant of the remaining angle B A D, relate to the secant of the side A E — that is, of the side opposite the remaining angle, at D.

Since by these few Enunciations — serving, as it were, as a kind of directory of the reasoning — the analysis of a proposed triangle is for the most part carried through, and since, accordingly, in our striving for brevity we have had to pass over the other proportions (which could still be drawn from the sines, tangents, and secants among themselves in manifold ways), this much will suffice concerning the first member.

ON THE SECOND MEMBER

Up to this point we have had the things that are προπαρασκευαστικά [Greek: *proparaskeuastika*, "preparatory"] to the analysis of triangles, that is, preliminaries. The things that accompany that analysis are, first, in a general way, the δεδόμενα [Greek: *dedomena*, "things given"] — namely the data that are required in the solving of any triangle — and the ζητούμενα [Greek: *zētoumena*, "things sought"], that is, the things sought, which result from them.

Now since the ἀνάλυσις [Greek: analysis, "analysis"] of triangles falls universally under the Golden Rule of proportion — in which, three terms being given, a fourth is sought — it is easily understood from this that in every triangle, in order to solve it numerically, three of the things proposed must be given, so that from these the fourth, or all the remaining three in the said triangle, may be brought to light according to the analogy of the data.

That rule, therefore, which on account of its most excellent use is called by us the rule of the Golden Proportion — by which, when a fourth is sought, the answer is rendered easier each time the individual quantities are tracked down — will undoubtedly be the more acceptable and commendable when, especially in cases where that prolixity of numbers must be worked through in practice, the Canon of triangles offers, in order to avoid fractions, a more compendious way than the one that requires the multiplication and division of those numbers in the usual manner. [Margin: The foundation of the Golden Rule is handed down by us in Prop. in *Arithm.*, and is demonstrated in Props. 16 of Book 6 and 19 of Book 17 of the *Elements*.] On this account the diligence of those is especially to be praised who have profitably bestowed their labor on this — among whom certainly may be reckoned the most ingenious invention of Lord John Napier [John Napier], Baron of Merchiston, a Scot, set forth in his admirable *Logarithm*, published in the year 1614. Nevertheless, because — in my judgment — such an undertaking removes too far the continual inspection of the demonstrations, which is more necessary for beginners, I therefore conclude that for these readers a compendium is rather to be chosen which retains that golden rule in the very transaction of the δεδόμενα [Greek: dedomena, "things given"], and no less by mere...

by mere addition and subtraction of numbers (which they call by a single word) it carries out: especially since such a procedure is most suited to the solution of certain spherical triangles — those, namely, whose use in the sphere is most frequent. But if anyone should ask about the inventor of this compendium, the analemmatic writings of the Arabs, or of John Regiomontanus [Regiomontanus], make plain that it was not they; and I certainly have no one older than our Tycho [Tycho Brahe] and Vitichius of Breslau [Paul Wittich], by whose mutual labor, in the year 1582, certain spherical triangles were first subjected to such a procedure at Hven, for the benefit of the students of Urania.

Afterward [Christopher] Clavius [Clavius] rendered this prosthaphaeretic doctrine somewhat broader. But last of all, the most excellent geometer and mathematician of Wittenberg, Master Melchior Jöstel [Melchior Jöstel], made it so

universal that it could readily be adapted to the resolution into numbers of all triangles, both plane and spherical, whenever this was required. Of this matter that most courteous man first gave me a demonstration in the year 1598, showing it to me in person as to an intimate friend. That doctrine — altered only in a few points — is the one I now set forth, with all the brevity that is possible, for carrying out the said Golden Rule; and I heartily commend it to lovers of learning, in case they should prefer to make use of it. Should they desire anything in this compendium to be demonstrated in every respect with regard to prosthaphaeresis, they will find it in due time either in our *Geometry*, or with that most distinguished man, Master Ambrosius Rhodius [Ambrosius Rhodius], mathematician of Wittenberg, formerly the genuine [Greek: γνήσιον (gnēsion, "genuine, true-born")] disciple of the late, blessed Master Jöstel.

THE PROSTHAPHAERETIC DOCTRINE

Adapted to the Golden Rule, to be Worked Out Everywhere

The foundation of prosthaphaeresis is this: that the radius, or whole sine — namely the number 10000000 (or extended to more figures) — be placed in the first position among the data of the Golden Rule of proportion.

Now the radius is either granted as an intermediate term — as in right triangles — or it is not granted, as in the other cases that are content with a single proportion set up. Again, it is apparent that the radius is either given in the first position of the Rule, as required, or at least in some one of the following positions, and therefore here it needs to be transferred. In accordance with this consideration, then, this doctrine is set forth in order, and is restricted to three general Rules.

THE THREE PROSTHAPHAERETIC RULES

[Margin: Theor. 8.]

When the radius occupies the first position of the rule, and the two following terms are right sines, the prosthaphaeresis is carried out for finding the fourth term in this way:

1. Let the smaller of the two remaining given arcs, and the complement of the larger, be added to and subtracted from each other; and let the right sines be sought both for the sums and for the differences of the arcs.

[Margin: Prior case.]

2. If the smaller arc is greater than, or equal to, the complement of the larger, then half the sum of the sines will be the fourth term sought. Therefore here the sines found are added to each other, and so on.

[Margin: Posterior case.]

3. But if the smaller arc is less than the complement of the larger, then half the difference will be the fourth term sought. Therefore here the sines found are subtracted from each other, and so on.

PARASCEVE.

Demonstration and example of the prior case.

The meaning of the foregoing rule is sufficiently clear from the present diagram and the example attached to it. Let the larger of the given arcs, DE, be 65 degrees, and the smaller, FG, be 54 degrees. The former is the angle BAC, the latter the side BA of the right triangle BCA, whose measure is the arc FG — or rather the right sine of that same arc — with the right angle at C. The problem is to find from these the side BC together with the arc corresponding to it. This is easily accomplished by Propositions 2 and 1.

By the Golden Rule of proportion [S.T.], we have:

	BA (or FG)	BAC (or ED)	BC
As 90	to 54 degrees	so 65 degrees	to 47° 9½'
10000000	S.R. 8090170	S.R. 9063078	S.R. 7332184

The same by prosthaphaeresis:

G F 54 degrees smaller arc
 G K 25 degrees complement of the larger arc
 F K 79 degrees sum } | 9816272 F M
 } Resp. S.R. {
 K H 29 degrees difference } | 4848096 H Z (or M N)
 sum of the sines 14664368 F N
 half 7332184 F R (or BC sought)

Here is the proof. The chord FH, subtended by the arc FH (which is formed from the sum and the difference, and whose total is double the given smaller arc FG), stands to FN (the sum formed from the sines of the sum and the difference) as half of FH — that is, FL or BA — stands to half of FN, that is, to FR or BC sought, by Theorem 7.

Demonstration and example of the latter case.

Let there first be given the maximum obliquity of the Sun, 23° 32', which in the attached diagram is represented by the arc DG, or the angle BAC. Let there also be given as the larger arc EF, 50 degrees, whose right sine is TF or AB. Then in the right triangle ACB the sine BC is sought from these, and from it afterward its arc is established — namely, the declination of the Sun corresponding to 20° of ♉ [Taurus]. By Theorem 7, BC is easily found in this way:

Diagram. A circle inscribed within a square frame, used to demonstrate the prior case (larger arc DE = 65°, smaller arc FG = 54°). Radii and chords are drawn from the center A to lettered points on the circumference; a dotted arc traces the prosthaphaeretic construction. Labels read clockwise/around the figure.

- F, I (top of circle): upper points where the construction chords meet the circumference
- E (upper left): endpoint of an arc
- B (interior, upper): vertex of the right triangle
- O (right): point on the circumference
- G (left): point on the circumference
- L, R (interior): points on the construction
- K, Z (left): points on the circumference / horizontal
- M, A, C (interior, center): A is the center; M and C lie on the horizontal diameter
- D (right): point on the horizontal diameter extended
- H (lower left): point on the circumference
- N (lower interior): foot of a perpendicular
- S (lower right): point on the circumference
- P (lower left): point on the circumference
- Q (bottom): lowest point of the circle

GV (or GAD)	
As S.T.	to 23° 32'
10000000	S.R. 3992826
EF	BC
So 50 degrees	to 17° 48' 36"
S.R. 7660445	S.R. 3058682

But by prosthaphaeresis in this manner:

F G complement of the larger arc 40 degrees

G D smaller arc $23^{\circ} 32'$

F D sum of the arcs $63^{\circ} 32'$ S.R. 8951939 F M

D H difference of the arcs $16^{\circ} 28'$ S.R. 2834574 H Z (or M N)

Difference 6117365 L M

Half 3058682 R M (or BC)

The proof here too is plain from the diagram itself, and from the marks attached to the working of the numbers. For since RM must be equal to BC: first, as FH is to half of FB, so FN is to half of it, FR. But once MN has been subtracted from FM, LM remains, whose half is RM, equal to BC.

Diagram. A circle inscribed within a square frame, illustrating the latter case (maximum obliquity $23^{\circ} 32'$; larger arc EF = 50°). Radii and chords run from the center A to lettered points; a dotted arc marks the prosthaphaeretic construction.

- I, F (top): upper points on the circumference
- E (upper left): endpoint of an arc
- T (upper interior): point on a chord
- L (interior): point on the construction
- G (right): point on the circumference
- O (left): point on the circumference
- B (interior, upper): vertex of the right triangle
- R (interior): point on the construction
- Z (right): point on the horizontal
- K (lower left): point on the circumference / horizontal
- A, M, C, V (along the horizontal diameter): A is the center
- N (lower interior): foot of a perpendicular
- H (right): point on the circumference
- S (lower left): point on the circumference
- P (bottom): lowest point of the circle
- Q (bottom): lowest point of the circle

[Margin: When the remaining terms are not sines.]

Furthermore, when the whole sine first occupies the place of the rule, and one of the following terms — or indeed both — are not right sines, but tangents, or any other numbers whatever, provided they are not greater in number of digits: then to those that are not right sines, enough zeros are added at the end so that they

match the sines in length. In this way the arcs corresponding to them are sought from the Table of Sines, as if they were right sines, and they are handled according to the first rule of prosthaphaeresis prescribed above. Then, from the fourth term sought that results, as many zeros are removed as were previously added.

Example 1.

Let there be given, in a plane right triangle ABC — besides the right angle at C — also the base BA, 351 parts; likewise the angle at A, 24 degrees. From these the side BC is sought.

The rule, according to Proposition 2, will stand thus:

C	BA	CAB	BC
As 90	to	so 24 degrees	to about 143.
S.T. 10000000	351 p.	S.R. 4067366	

But prosthaphaeresis proceeds thus, according to the second case:

Complement of the given angle $66^{\circ} 0'$
 BA 3510000 as S.R. Resp. arc $20^{\circ} 33'$ —
 sum $86^{\circ} 33'$ S.R. 9981877
 difference $45^{\circ} 27'$ 7126385
 difference of the sines 2855492
 half 1427746

Therefore, once the last four zeros have been removed, the side BC sought is about 143 parts.

Example 2, where neither of the following terms is a right sine.

Let there be given, in the adjacent right triangle ABC, the side CA, 8 parts, and the angle at A, $36^{\circ} 53'$. Then by Theorem 4 the angle at B will be $53^{\circ} 7'$, provided the proposed triangle is plane. But if, without this angle, from the given data

the side BC is to be sought, it is found by Proposition 4. For if CA is taken as radius, BC will be the tangent of the given angle at A. For:

C	BAC	CA	BC
As 90	to $36^{\circ} 53'$	so 8 p.	to 6 p.
S.T. 10000000	T. 7503663		

The prosthaphaeresis in this example, from the foregoing advice, is as follows:

C A 8000000 as S.R., to which corresponds the arc $53^{\circ} 7' +$
 C A B Tang. 7503663 as if S.R., corresponds the arc $48^{\circ} 38' -$

These are worked according to the rule of the first case, in this manner:

48° 38' smaller arc
 36° 53' complement of the larger.
 85° 31' sum S.R. 9969401
 11° 45' difference S.R. 2036418
 sum of the sines 12005819
 half 6002909 sought.

Therefore, with six zeros removed from the result, the side BC comes out to 6 parts.

Diagram. A small plane right triangle with the right angle at C. A short dotted arc near the upper vertex marks an angle.

- B (top): apex vertex
- A (lower right): vertex with the given angle (24° , Example 1)
- C (lower left): right-angle vertex

Lastly, when the following given terms are found to be greater than the radius — whether only one is greater, or both. If only one is greater than the radius, let it be divided by the radius (keeping in it, of course, as many figures as there are zeros equal in number to the degrees of the radius); and with the quotient preserved, one proceeds with the retained figures as the right sine to be set down.

Then, once the prosthaphaeresis has been carried out, the given smaller number is multiplied by the quotient preserved earlier; the product is added to the half found, and the fourth term sought results.

Example.

In the adjoining spherical right triangle ABC, there are given — besides the right angle at C — also the angle at A, 30 degrees, together with the opposite side BC, $23^{\circ} 30'$. From these the side AC is sought, by Proposition 4. For:

Diagram. A small spherical right triangle with the right angle at C, the side CA along the base and a curved hypotenuse BA.

- B (top): apex vertex
- C (lower left): right-angle vertex
- A (lower right): vertex with the given angle (30°)

C (right angle)	T. compl. BAC	T. BC	S.R. CA
As 90 degrees	to 60 degrees	so 23° 30'	to 48° 51½' nearly.
S.T. 10000000	T. 17320508	T. 4348124	S.R. 7531081.

By prosthaphaeresis the procedure is carried out thus:

T. 17320508 S.R., corresponding arc 47° 4'
T. 4348124 S.R., corresponding arc 25° 46'
Complement of the larger arc 42° 56'
minor arc 25° 46'
sum 68° 42' S.R. 9316913
difference 17° 10' S.R. 2951523 subtract.
difference 6365390
half 3182695
Smaller multiplied by the quotient, namely 1. 4348124
Sought S.R. 7530819 nearly, as before.

But when both of the following given terms are greater than the radius, let each be divided separately by the radius, the quotients being preserved; and from the figures cut off let the arcs be sought from the table of sines, by which the prosthaphaeresis is carried through according to what was prescribed. To the half found are added the products, which are formed: one from the quotient of the third number into the

second residue, the other from the quotient of the second into the whole third — or vice versa, if one prefers. The sum of these three will give the fourth term sought.

Example.

Let there be given, in the right triangle ABC with the right angle at C, the base AB, 60 degrees, and the angle at A, 30 degrees.

From these the remaining angle at B is sought. Then by Propositions 5 and 6 it is:

C	BA	Compl. A	B
As 90	to 60 degrees	so 60	to 73° 54'
S.T. 10000000	Sec. 20000000	T. 17320508	34641016

The same by prosthaphaeresis, from the prescriptions:

20000000 S.R., corresponding arc 0° 0'
7320508 S.R., corresponding arc 47° 4'
Complement of this 42° 56'
Minor arc 0° 0'

Sum of the arcs $42^{\circ} 56'$ S.R. 6811470
Difference of the arcs $42^{\circ} 56'$ S.R. 6811470 subtract.
Difference of sines 0000000
Half 0000000

There is added the product of the quotient of the third — namely unity — into the second residue, 0000000; which is therefore 0000000, that is, nothing. And from the whole third — namely 17320508 — by 2, that is, the quotient of the second. So in this example the product is the same, namely 34641016, the fourth term sought, as before.

The calculation also succeeds in these cases if numbers greater than the radius are removed from the result, until they are reduced to right sines.

Likewise, if a tangent or a secant, falling somewhere into the proportion, extends two zeros beyond the radius of the circle, let the first on the left be taken as the quotient, and the last on the right be rejected, while the rest are taken as the right sine. But in these cases the working will not give the value sought so precisely, unless the arcs for the seconds have been most exactly corrected and drawn from the *Opus Palatinum*. And even if this happens to the working above, the prosthaphaeresis in it is nevertheless more reliable, and will not differ perceptibly from the truth.

With these examples that we have left, all the cases of prosthaphaeresis are included, wherever the radius has occupied the first place of the rule. And certainly, although prosthaphaeresis sometimes brings little or no abbreviation — as in the last cases of this rule — yet it is pleasant, and useful for δοκιμασίαν [Greek: δοκιμασία (dokimasia, "testing" or "proof")], to make it thus universal: so that the product of any two numbers proposed for multiplication is obtained by addition and subtraction alone. This procedure is altogether expeditious, since right sines alone, as in the first examples, have sufficed.

SECOND RULE OF PROSTHAPHAERESIS.

When the radius, or whole sine, is placed in the second or third position of the rule, a transposition must be applied so that it occupies the first place — since every prosthaphaeresis demands that the radius be put in the first place, as was said above. This transposition rests upon two precepts, but a single demonstration.

First precept of transposition.

If any number greater than the whole sine occupies the first place of the rule, to this

Second precept.

If the number found in the first place of the rule is smaller than the radius (the whole sine), add one or more zeros to it, until a matching arc can be drawn from the table of secants. Its complement likewise takes the place of the radius and moves into the first place of the rule, and here too an example for the first rule of prosthaphaeresis will stand ready. Once this is done, append to the fourth term as many zeros as were earlier added to the number smaller than the whole sine.

The demonstration of this rule depends on the sixth Proposition. For when the diagram of that passage is repeated here, it was shown there that the radius AD is the mean proportional between the secant AG — namely of the arc DC — and DF, the right sine of the complement of that same arc DG. That is: as AG is to AC, so in turn is AC (or AD) to DF. Therefore, once the radius (the whole sine) AD is put in place of the secant AG, FD will be the right sine of the complement of the given arc DC, found by means of the given secant AG, which we imagine to have been placed in the first place of the rule in place of the radius AD.

Diagram. A circle with two perpendicular diameters meeting at center A (the horizontal running left toward C, the vertical running up toward B). Within it a geometric construction illustrates the secant–sine relationship for arc DC.

- A (center): the center of the circle, the vertex of the right angle
- B (top): upper end of the vertical diameter
- C (left): left end of the horizontal diameter
- H (upper left): point on the upper-left arc
- D (interior, near center): point where the radius meets the construction
- G (interior, left of D): foot of the secant AG
- F (interior, right of D): foot of the perpendicular, the right sine of the complement
- E (lower, below D): point on the lower portion of the construction

Example of the earlier case, where the first number in the rule is the larger.

Let there be a right-angled spherical triangle ABC with the given angle at A of 30° , and the side opposite to it, BC, of $25^\circ 39' 32''$. From these the base AB is to be found, by Proposition 6. The proportion is therefore as follows:

	sec. compl. A	S.T.	sec. compl. BC	sec. compl. Base
a	60° 0'	to 90°	so 64° 20' sec. 28	to 30° 0'
s,				
	20000000	1000000	23094008	11547004
		0		

The prosthaphaeresis is carried out thus:

Sec. 20000000 corresponds to 60° 0'; its complement, 30°, is set in place of the radius, promoted into the first place, and the proportion stands: as S.T. is to 30° (S.R.), so is Sec. 23094008, and so on.

Because the third number, too, exceeds the whole sine, divide it by the radius according to the precept set out above, in this manner: 23094008 (S.R.) — the arc cut off is 18° 2'; the other was 30°.

Greater complement	60° 0'	
Lesser arc	18° 2'	
Sum	78° 2'	S.R. 9782684
Difference	41° 58'	S.R. 6686981 (subtract)
Difference of sines	3095703	
Half	1547851	

The sine (S.R.) of 30° — as of the smaller number posited above in the proportion — is 50000000, which, multiplied by the second quotient, creates a product 10000000 to be added.

(product)	10000000
Half	1547851
and they make	11547851

— almost as before.

Example of the later case, when the number placed in the first position is smaller than the radius.

Diagram. A right-angled spherical triangle drawn as a narrow curvilinear figure. The right angle is at C (lower right), with B at the top and A at the lower left, the hypotenuse arcing between them.

- B (top): vertex
- A (lower left): vertex of the angle sought
- C (lower right): vertex of the right angle

Let there be a right-angled spherical triangle ABC, with the two sides about the right angle at C given — namely AC of 60° and BC of 30° — from which the angle at A is sought, by Proposition 4, in this manner:

	AC	rad.	BC	BA C
as 60	to 90	so 30	to $33^\circ 41'+$	
S.R. 8660254	10000000	T. 5773502	T. 6666665	

The prosthaphaeresis runs thus:

8660254 (sec.) corresponds to the arc $83^\circ 22' 10''$; its complement is $6^\circ 37' 50''$. T. 5773502, as a quasi-S.R., corresponds to the arc $35^\circ 15' 50''$; its complement is $54^\circ 44' 10''$.

$54^\circ 44' 10''$	
$6^\circ 37' 50''$	
$61^\circ 22' 0''$	S.R. 8777044
$48^\circ 6' 20''$	S.R. 7443763
	1333281
	6666455 = T. BAC, almost as before.

THIRD RULE OF PROSTHAPHAERESIS.

When the radius (the whole sine) is found nowhere in the rule of proportions, then a double prosthaphaeresis must be applied.

For first proceed by the preceding second rule: as the first of the given terms is to the second, so is the whole sine to another.

Then, by the first rule: as the whole sine is to the result found, so is the third to the fourth term sought.

Or, if it seems more convenient, proceed thus: as the first is to the third, so is the whole sine to another.

Then: as the whole sine is to the result found, so is the second to the fourth.

The reason is that the first, together with the second or the third, is reduced to two other terms, of which the first is the whole sine; and so at last the example is fitted to the first rule.

The demonstration is offered clearly enough in the proportion of the numbers, that is, of the four terms of the rule of proportion. Let the terms of the proportion be, apart from unity, with figures of this kind: 8. 12. 9. ($13\frac{1}{2}$ — which quotient stands in the same ratio to 10 as 8 to 12 (for 9 and $13\frac{1}{2}$ are in the same ratio with those, namely $1\frac{1}{2}$, or sub-sesquialter). 10, or the radius, is inserted, and the rule is formed thus: As 8 is to 12, so is 10 to (15.

Second: As 10 is to 15, so is 9 to ($13\frac{1}{2}$, as before.

And so, the prosthaphaeresis being repeated, the fourth number found is to be reckoned, according to the first rule, together with the one that fell to the radius in the first operation; and thus every variation that can occur will be judged according to that same first rule.

But this rule too must be illustrated by an example:

Example.

Diagram. A plane scalene triangle. A is at the lower left, B at the lower right, and C at the upper right, with the longest side AB along the base.

- A (lower left): vertex of the angle of 20°
- B (lower right): vertex of the angle of 60°
- C (upper right): vertex

Let there be a plane scalene triangle with two given angles: at A, 20° ; at B, 60° ; together with the side BC of 125 parts. The side CA is sought. Therefore, by Proposition 2, it will be:

	CAB	CB	CBA	CA
as 20°	to	so 60°	to	
S.R. 3420201	125 p.	S.R. 8660254	317	

By Prosthaphaeresis I:

Sec. 34202010 corresponds to the arc $73^{\circ} 0'$, and the complement of this is $17^{\circ} 0'$ — the smaller arc. The greater arc is $60^{\circ} 0'$. Therefore:

its complement	$30^{\circ} 0'$	
	$17^{\circ} 0'$	
sum of arcs	$47^{\circ} 0'$	S.R. 7313537
difference of arcs	$13^{\circ} 0'$	S.R. 2249511
		5064026
		2532013 = the half, which is the prior number sought.

II. Then:

	90	grad. $14^{\circ} 41'$	grad. $7^{\circ} 11'$
as	to	so	
S.T. 10000000	S.R. 2532013	S.R. 12500000	

Greater complement	$75^{\circ} 19'$	
Lesser arc	$7^{\circ} 11'$	
Sum	$82^{\circ} 30'$	S.R. 9914449
Difference	$68^{\circ} 8'$	S.R. 9280532
Difference of sines		633917
Half: the fourth term sought		316958 — almost as above.

Now our customary prosthaphaeresis rests on these three rules, since it furnishes a great aid for finding the fourth term everywhere required in the rule of proportion — especially in large numbers, where otherwise multiplication and division would be burdensome. In comparing the numbers through the worked examples, however, one small discrepancy arose: that I have for the most part declined to correct the sines, tangents, and secants — and the arcs derived from them — for the seconds that sometimes cling to them, content to leave the worked examples to prosthaphaeresis alone. Accordingly, whoever insists on ἀκρίβεια [Greek: akribeia, "exactness"] should turn to the canon of the *Opus Palatinum*, or

correct the sines accurately for the seconds from other sources, and so will test the truth of the comparison more precisely.

So much for the analysis of triangles in general by prostaphaeresis alone — which, as is understood, can most conveniently be applied to the foregoing cases whenever the radius (the whole sine) can be had in the first place of the rule, and the remaining given terms are also right sines. There now follows the solution of triangles of the specific kinds proposed.

This concerns specifically each individual triangle. And since it is understood from the corollary of the definition of a triangle that six things are to be considered in it; and likewise, from the beginning of this second part, that three data are required to solve any triangle, so that the remaining three hidden ones may be brought to light: the analysis proceeds correctly through the variety in the kinds of plane and spherical triangles, by means of the διδόμενα [Greek: didomena, "the things given"].

Triangles, as διδόμενα [Greek: didomena, "given quantities"], or ζητούμενα [Greek: zētoumena, "sought quantities"], are either:

- **Pure**, as:
 - three angles — I.
 - three sides — II.
- **Mixed**, as:
 - two sides with an angle, either:
 - enclosed — III.
 - opposite one of them — IV.
 - two angles with a side, either:
 - adjacent — V.
 - opposite one of the angles — IV.

Next, since right-angled triangles have great use in themselves, and sometimes serve conveniently for solving the others, the analyses of these — in plane figures as well as in spherical ones — will rightly come first. With these matters considered, we shall give the following doctrines, most suited to those learning the practical solution of triangles in either kind.

DOCTRINES FITTED TO THE PRACTICAL SOLUTION OF PLANE TRIANGLES, IN NUMBERS.

DOCTRINE ONE.

To make manifest, for a right-angled plane triangle whose three angles are given, the proportions of the sides to one another.

Since an angle in a semicircle is a right angle (by Proposition 31, Book 3, of the *Elements*), let the side opposite [the right angle] be made the diameter of the circle. The remaining two sides will then be the chords subtending the remaining opposite angles; therefore their right sines are sought and doubled, and the proportions of these sides emerge — of the kind in which the diameter of the circle is taken as 20000000.

Or, more compendiously, by Proposition 2: if the side opposite the right angle is set as the radius of the circle, that is 10000000, then the right sines of the remaining angles will be the sides opposite to those angles individually. So, for a first example, let there be a triangle ABC having a right angle at B, and an angle at A of 30° , and therefore an angle at C of 60° (by Theorem 4). Further, about this same triangle let a circle be described, by Proposition 5, Book 4, of the *Elements*.

Diagram. A circle with an inscribed right triangle ABC, the hypotenuse AC drawn as a diameter through the center. A is at the left, C at the right, B at the top; D marks the center of the circle on the diameter AC. The two acute angles (at A and at C) are marked with small circular symbols.

- A (left): vertex of the 30° angle, left end of the diameter
- C (right): vertex of the 60° angle, right end of the diameter
- B (top): vertex of the right angle
- D (center): center of the circle, on the diameter AC

The diameter of this circle is therefore AC. If it is set at 20000000, then BC will be 10000000 and AB 17320508. Further, if CA is supposed to be 10000000, then BC will be 5000000 and AB 8660254 — namely, the halves of the former values, that is, the right sines of the given angles.

DOCTRINE II.

For a right-angled plane triangle whose three sides are given, the angles are found.

Since, by Proposition 2, the side opposite the right angle stands to the radius as the side opposite any one of the remaining angles stands to the right sine of the angle to which it is opposite — therefore:

Multiply the side opposite the angle sought by S.T., and divide the product by the side opposite the right angle; there results the S.R. of the angle sought, and hence

also the angle sought itself, together with the remaining one, which is its complement to 90° .

Example.

Let A B C be a plane right triangle whose sides are given: A C, opposite the right angle at B, is 15 p.; likewise A B is 12 p., and B C is 9 p. Then the angle B A C is $36^\circ 52'$, and therefore B C A is $53^\circ 8'$, by Theorem 4.

DOCTRINE III.

Given two sides of a plane right triangle together with their included angle, the remaining angles and the remaining side are found.

First, when the included angle is the right angle: since, by Enunciation 4, if one of the sides about the right angle is taken as the radius, then the other given side about the right angle is the tangent of the opposite angle. So multiply the side opposite the angle sought by the radius, divide the product by the remaining side, and the result is the tangent of the angle sought.

Example.

Keep, in the right triangle above, the given sides about the right angle, namely A B 12 p. and B C 9 p. Then the tangent of the angle sought, B A C, is 7500000, and the angle B A C itself is $36^\circ 52\frac{1}{2}'$. Consequently its complement also, namely the angle B C A, is $53^\circ 8'$.

Next, for the side (or base), proceed by Enunciation 2. Multiply the side of one of the angles other than the right angle by the radius; divide the product by the right sine [sinus rectus] of the remaining angle. The quotient gives the side opposite the right angle. For:

B A C	B C	A B C (right)	A C
a gr. $36^\circ 52\frac{1}{2}'$ — S. R. s 60000000	to 9 p.	so 90 — S. T. 10000000	to 15 p.

[The right sine S.R. of B A C = 60000000; the tangent S.T. of the right angle = 10000000 (the radius).]

Second, when the given angle included by the given sides is acute, its complement to 90° is also given, by Theorem 4, and so the third side too is found, by Enunciation 2.

No example is needed here.

DOCTRINE IV.

Given two sides of a plane right triangle together with the angle opposite one of them, the rest are found.

Since, besides the given right angle, another angle in this triangle is also granted, all three angles are therefore given, by Theorem 4.

If, then, the side given is the one opposite the right angle, lying between the two given angles, by the second Enunciation, for the third of the sides:

As S. T. is to the opposite side, so the right sine of the given angle is to the opposite side sought.

But if the base — the side opposite the right angle — is sought, then: as the right sine of the one angle is to the given opposite side, so S. T. is to the base.

DOCTRINE V.

Given two angles of a plane right triangle together with the adjacent side, the rest are found.

First, in this way all three angles are given, by Theorem 4.

Then the two sides that are wanted are found by Enunciation 2, as in the preceding Doctrine.

DOCTRINE VI.

Last: given two angles of a plane right triangle together with the side opposite one of the angles, the rest are also found.

This Doctrine is solved in the same way, by Enunciation 2, exactly as the fourth and fifth of the preceding ones.

So much for the solution of plane triangles, which follow most plainly from what has been laid down. There follows the treatment of oblique-angled plane triangles.

DOCTRINE VII.

Given the three angles of an oblique-angled plane triangle, the proportions of the three sides to one another are found.

Here the given oblique-angled triangle is circumscribed by a circle, by Proposition 5 of Book 4 of the *Elements* [Euclid]. Then the right sines of the given angles are found and each is doubled, whereupon they become the chords [subtensae] of the angles, and so the proportions of the sides sought to one another — and indeed in those numbers in which the diameter is 20000000. Or else: if the diameter is taken as 10000000, the right sines of the given angles themselves serve as the sides, preserving the same ratio among themselves as before.

Example.

Diagram. A triangle A B C inscribed in a circle. Vertex A is at upper left, B at upper right, C at the bottom; the circle's circumference passes through the three vertices. A point D is marked on the circle at the top and a point E at the lower left; a dotted line (a diameter or chord, D–E) runs across the figure.

- A (upper left): vertex of the triangle, angle 80°
- B (upper right): vertex, angle 60°
- C (bottom): vertex, angle 40°
- D (top of circle): endpoint of the diameter D E
- E (lower left of circle): other endpoint of the diameter D E

Let A B C be an oblique-angled triangle, all of whose angles (or at least two of them) are given. Let the angle at A be 80° , the angle at B 60° , and the angle at C 40° . Then the corresponding sines are these:

Angle	Right sine (S. R.)	Side	Note
gr. 80	9848078	B C	in which the diameter of the circle D E is 10000000
gr. 60	8660254	C A	
gr. 40	6427876	A B	

Or: in the numbers in which the diameter D E is commonly taken as 20000000 — since these right sines are doubled — B C will be 19696156, C A 17320508, B A 12855752.

DOCTRINE VIII.

Given the three sides of an oblique-angled plane triangle, the three angles are found.

The given sides of an oblique-angled triangle are either equal or unequal.

1. If the given triangle has its given sides equal — that is, if it is equilateral [isopleurum] — then, since equal angles are subtended by equal sides, and since the three angles (by Theorem 4) equal two right angles, or 180° , the number divided in three assigns 60° to each of the angles.
2. Further, if the proposed triangle has unequal sides, it will either have two equal and be isosceles, or it will have all three unequal and be scalene.

An isosceles triangle is resolved into its constituents sought [ζητούμενα (zētoumena, "the things sought")] by Theorem 3, by dropping a perpendicular from the angle contained by the equal sides (or legs) onto the base. For in this way the isosceles triangle is divided into two equal right triangles, and in each of them, along with the right angle, two sides are given — namely half the base and one of the legs, which is opposite the right angle. Hence the solution is carried out by the first case of the fourth Doctrine of this book.

Example.

Diagram. An isosceles triangle A B C with apex B at the top, base A C at the bottom. A perpendicular B D drops from the apex B to the midpoint D of the base.

- B (apex, top): vertex of the equal legs A B and C B
- A (lower left): left base vertex
- C (lower right): right base vertex
- D (base midpoint): foot of the perpendicular B D

Let A B C be an isosceles triangle whose given sides — namely A B and C B — are each 12 p. (these are therefore called the equal legs [equicrura]); and let the base A C be 16 p. The perpendicular B D dropped from the angle B divides the base into A D equally and on both sides at right angles about D. Thus, in the one right triangle A D B, when the angle A B D is sought, by Enunciation 2:

A B	D	A D	A B D
a 12 p. — S. T. 10000000 s	to 90	so 8 p. — S. R. 6666666	to gr. 41 mi. 48 sec. 37

The angle A B D being thus found, all the remaining angles of the proposed triangle A B C are obtained. For, by Theorem 4, the angle B A D is given as $48^\circ 11' 23''$ — namely the complement of the one found to 90° — to which the angle A C B is equal, because the equal sides B C and A B are subtended. Therefore, with these

equal angles added together, and their sum subtracted from 180° , the angle A B C finally stands in the remainder, thus:

	gr.	mi.	sec.	
	48	11	23	
	48	11	23	
sum	96	22	46	sum of the angles at A and C, subtracted from the semicircle
	180	0	0	the semicircle
	83	37	14	the angle A B C

The angle A B C also comes out in the same numbers through the doubling of the one found at the outset, A B C, thus:

	gr.	mi.	sec.
	41	48	37
	41	48	37
	83	37	14

3. Or, finally, the proposed triangle of given sides will be scalene — that is, having all three sides unequal.

For this case, although several methods present themselves through geometry by which such a solution of a proposed triangle of given sides into its angles may be carried out, none is more convenient than the one indicated in Proposition 36 of Book 3 of the *Elements* [Euclid]. This may therefore be kept, especially since the present Doctrine chiefly serves a good part of geodesy [Geodesia], where, in order to obtain the area of a triangle of given sides, the perpendicular — conveniently dropped from the largest of the triangle's angles onto the base — must be known. But, to return more quickly to practice: let the smallest of the given sides be made the radius of a circle, and with this as center, fixed at the largest of the angles of the given triangle, let a circle be described. By its remaining circumference it cuts off the other sides in such a way that the parts remaining outside, continued together with the remaining parts through the circle, observe the proportions for forming similar rectangles, by the cited Proposition 36 of Book 3 of the *Elements*. Now once the portion of the largest side outside the circle is known, the remaining base is found, and the solution devolves upon the preceding Doctrines concerning right triangles.

Example.

Diagram. A scalene triangle $A B C$ with a circle centered at B (radius = the smallest side $B C$). Vertex A is at lower left, B near the top, C at lower right. The circle, centered at B , cuts the side $A C$ at E and the side $A B$ (extended) at G ; the extension of $A B$ meets the circle again at F (top). A dotted perpendicular $B D$ drops to the base.

- A (lower left): vertex at the largest angle
- B (upper, center): apex; center of the circle $C E G$
- C (lower right): vertex
- D (on base $A C$): foot of the perpendicular from B
- E (on side $A C$): point where the circle cuts $A C$
- F (top): where the extension of $A B$ meets the circle
- G (on $A B$ near A): point where the circle cuts side $A B$

Let $A B C$ be a plane triangle of unequal given sides, where:

Side	Length
$A B$	25 p.
$A C$	28 p.
$B C$	17 p.

Then, with the smallest of the sides, $B C$, made the radius, let a circle $C E G$ be described with center B , whose circumference cuts off the remaining sides at E and G .

Diagram. The same scalene construction as on the preceding page, redrawn at upper left: triangle $A B C$ with the circle centered at B , the side $A B$ extended to F at the top, points G and E where the circle cuts the sides, and the perpendicular $B D$ dropped to the base.

- A (lower left): vertex at the largest angle
- B (upper center): apex; center of the circle
- C (lower right): vertex
- D (on base): foot of the perpendicular $B D$
- E (on side $A C$): where the circle cuts $A C$
- F (top): where the extension of $A B$ meets the circle
- G (on side $A B$): where the circle cuts $A B$

Now, with the side A B extended to F — since the point A is conceived to lie outside the circle — by Proposition 36 of Book 3 of the *Elements* the rectangle under F A and G will equal the rectangle under C A and E. Accordingly: as A C 28 p. is to A F 42 p. (namely the sum of the sides A B and B C), so A G 8 p. (the difference of those same sides) is to A E 12 p. The remainder of that same side A C is therefore also given, namely E C 16 p., and its half D C — or E D — 8 p. The perpendicular dropped to this point D divides the proposed triangle B A C into two right triangles B D A and B D C, by Theorem 3, and so divides it that, on both sides, the two sides together with the right angle at D are known. There, A B is 25 p. and A D 20 p.; here, B C is 17 p. and D C 8 p. Therefore, by the fourth Doctrine preceding and its earlier case, the angles become known: at A, $36^{\circ} 52'$; at C, $61^{\circ} 56'$; and therefore the third, at B, $81^{\circ} 12'$, by the fourth Theorem.

If the perpendicular B D is at the same time wished to be had — since from its multiplication into half the side A C the area of the proposed triangle B A C is established — this is most easily offered, also by prosthaphaeresis, through Enunciation 2, since the sides are given that contain the given acute angle, whether at C or at A.

DOCTRINE IX.

Given two sides of an oblique-angled triangle together with their included angle, to find the remaining two angles and the third side.

Here too the two given sides are either equal or unequal.

If the sides are equal, the proposed triangle is isosceles. So, since the two unknown angles at the base are likewise equal, and their measure is the complement of the given angle contained by the sides to 180° , by Theorem 4, then — that complement being bisected — the measure of each of the angles at the base becomes known.

If the sides are unequal, a twofold path to solving the proposed triangle remains. Either one of the given sides is extended outside the triangle, as may be more convenient, and a perpendicular is dropped from the opposite angle to the outside, so that, by the construction of a single right triangle, two triangles are made and the solution proceeds in the manner of a right triangle. Or else the sum of the latent angles, divided by means of the proportions of the opposite sides, brings each of the unknown angles to light, and afterwards the side opposite the given angle becomes plain, by Enunciation 2.

Example of the former method.

Diagram. A triangle A B C with A B taken as the radius. A circular arc is drawn; B C is extended to D, and a perpendicular is dropped from A to D. Vertex C is at the top, B just below it, A at the bottom, D at lower left.

- A (bottom): vertex; A B is the supposed radius (100000)
- B (upper, below C): vertex of the given included angle A B C
- C (top): vertex
- D (lower left): point where B C extended meets the foot of the perpendicular from A

We use this method especially when one of the given sides is the radius — for example: let A B C be a triangle of given sides, A B being the supposed radius 100000, but B C 3571, with the contained angle A B C of 150° . Here the prosthaphaeretic angle at the Sun, B A C — together with the side A C, the distance of the Sun at C from the earth at A — is sought. So, by extending C B to D, and dropping a perpendicular from A to D, the triangle A C D will be right-angled, having the right angle at D, and the angle A B D the given 30° . For it is the complement of the given angle A B C, by Theorem 1. Also, by Theorem 4, the angle B A D is given as 60° . And since B A is the radius, the remaining sides will be the right sines of the opposite angles, and therefore

and therefore AD is 50000, and BD is 86602; adding to this the side CB, 3571, the whole DC comes to 90173. With this side set in place of the radius, or the whole sine, AD will be the tangent of the angle ACD, by Enunciation 4; and the solution is carried out by the preceding Doctrine 3. For:

a	DC 90173	t	D: 90 [recte: S.R. 90?]; S.T. 100000	s	AD 50000
s		o		o	

Therefore, in the proposed triangle ABC, the angle ABC is given as 150° , and the angle ACB has now been obtained as $29^\circ 0' 28''$. The complement of these two to 180° — namely the angle BAC, $0^\circ 59' 32''$ — is the solar prosthaphaeresis sought at a distance of 30° from the apogee, according to its proper correction.

For the side AC, in the right triangle ADC: as the right sine of the angle at A is to the side DC, so is the radius to the sought AC, by Enunciation 2.

There is another method, which is easier and does not depend on the radius being given. It rests on this problem:

If the sum of two angles is given, together with the ratio between their right sines (that is, between the sides opposite them), then those angles are also given separately.

In the semicircle BC, whose center is A, let the sum of two angles DAE and EAF be given — namely $DAE = 80^\circ$. Let the ratio of the right sines — HD of the greater angle and EI of the lesser, or, by Enunciation 2, of the side DM opposite the greater angle and the side EM opposite the lesser — be as 86 to 34. I claim that the angles DAF and EAF are given separately.

For draw from the center A, to the point of the half-sum of the angles, the line AG; then from the same center A describe the arc KN, which is touched by the chord of the sum of the angles — that is, by the sum of the given sides DE — at L. Then LD, half the sum of the given sides, 60 parts, is the tangent of the half-sum of the angles, 40° ; and LM, the difference between the lesser side and half the sum of the sides (or between half the sum of the sides and the greater side), namely 26 parts, is the tangent of the difference of the angles. So it will be: as DL, 60 parts, is to the tangent of the angle DAL, 40° — which the canon gives as 8390996 — so is 26 parts to the tangent 3636098, whose arc is about $19^\circ 59'$, and this is the angle GAF. Accordingly, since DAG, 40° , is added here, the angle DAF comes to about $59^\circ 59'$ — the angle that was subtended by DM, 68 parts, of the given sides. Or, if it is subtracted from GAF [recte: from the half-sum 40°], it leaves the angle EAF, $20^\circ 1'$, subtended by the lesser of the sides, EM, 34 parts.

Diagram. A semicircle on diameter CB with center A at the midpoint of the base. Several radii are drawn from A up to points on the arc and to constructed points, illustrating the separation of two angles from their given sum.

- C (lower left): left endpoint of the diameter
- A (bottom center): center of the semicircle
- B (lower right): right endpoint of the diameter
- D (right, on arc): endpoint of one radius (the greater angle's side)
- E (upper left, on arc): endpoint of another radius (the lesser angle's side)
- F (top center, on arc): point at the half-sum of the angles, where radius AG meets the arc
- G (upper right, near top): labeled point on the bisecting radius / arc
- H (left of center): foot of the right sine of the greater angle (HD)
- I (top center, below F): point on the small constructed arc
- K (lower center): point on the inner described arc KN
- L (center right): point where the chord meets the inner arc KN

- M (upper center): intersection point on the constructed perpendicular
- N (center left): point on the inner described arc KN

Note that, since here the subtended angles are drawn out from the proportion of the given sides alone, the ratio of the sum of the sides to their difference is the same as the one seen in the halved values. Thus the ratio that 60 parts has to 26 parts is also the ratio of the sum of the sides, 120, to their difference, 52 — terms which can likewise be reduced to least numbers, as is taught in arithmetic, or carried over into a prosthaphaeresis of translation, as was set out above and will shortly be shown below by an example.

There follows the procedure, with an example.

So the given sides are added, and their sum is the first found value.

Then those same given sides are subtracted from each other, and the difference is the second found value.

Next, the given angle is subtracted from the semicircle, and the remainder is halved; the tangent of this half is sought, and is taken as the term, or third found value. Then the second found value is multiplied by the third (or the third by the second), and the product is divided by the first found value. Once the operation is carried out, the quotient yields the tangent of the arc of the difference of the proposed angles. This, then, having been sought in the tangent canon, is added to the halved angle (whose tangent was earlier taken as the third found value), and the sum is the greater of the sought angles; or, subtracted from it, the difference gives the lesser of the sought angles.

Example.

Let the same example be repeated that was given before in the case of the Sun.
So:

AB	100000
BC	3571
	103571 first found value (I)
	96429 second found value (II)
ABC	150° given angle
	30° difference
	15° half; T. 2679492 third found value (III)

Therefore: as 103571 is to 96429, so is 2679492 to the tangent 249472, to which corresponds an arc of $14^{\circ} 0' 28''$.

This arc, taken from half the complement of the given angle to the semicircle — namely 15° — leaves the prosthaphaeresis in the case of the Sun exactly as before, $0^{\circ} 59' 32''$.

Diagram. The same Sun-triangle figure as in the preceding example: a small circle near the top with point B on it, point C above B, and long lines descending to A at the bottom and to D at the lower left, forming the triangle ABC with the obtuse angle at B.

- C (top): apex above the small circle
- B (in the small circle): vertex of the obtuse angle
- A (bottom): vertex
- D (lower left): point completing the auxiliary construction

As for the construction of the prosthaphaeresis tables for the planets — which proceeds almost entirely from this single present Doctrine — in order to lessen the labor and make the calculation far easier (since the two prior found values remain fixed throughout, arising from the given radius and eccentricity within the orbit of each planet), the division here is most conveniently removed by translation, and the multiplication, if it must be employed, is rendered far quicker, in accordance with what is now set out in this same example.

First, for the translation by Rule 3 of Prosthaphaeresis, it will be:

a	first found value (I)	103571	t	second found value (II)	s	S.T. 10000000
s			o	96429	o	

Therefore, by translation:

as	10000000	to	9310424	so	T. 2679492 (15°)

And here only multiplication was needed, since the division has been carried out into the radius. The multiplication by the same number throughout is made very easy by the abridgment set out in our revised *Arithmetic*. For if all the numbers of the second found value proposed are multiplied by every digit from unity up to 10, and the products are kept in order, the calculation is completed by addition

alone, by adding these products for the several digits of the multiplier in turn, as follows:

Let the second found value be 9310424, multiplied by:	Product
1	9310424
2	18620848
3	28831272
4	37241696
5	46552120
6	55862544
7	65172968
8	74483392
9	83793816

These products, for the several digits below 10, will be applied in the following manner.

Multiplication with the application of the products by addition alone.	Or more compendiously from the right, in this way:

Multiplication with the application of the products by addition alone:

	9310424
	2679492
2.	1 8620848
9.	83 793816
4.	372 41696
9.	8379 3816
7.	65172 968
6.	558625 44
2.	18620848

2494720 found value IV (Inv. IV.)

Or more compendiously from the right, in this way:

	9310424
	2679492
2.	18620848
6.	558625 4
7.	65172 9
9.	8379 4
4.	372 4
9.	83 8
2.	1 8
	2494720 found value IV (Inven. IV.)

DOCTRINE X.

Given two sides of an oblique-angled plane triangle together with the angle opposite one of them, to find the rest.

The analysis is accomplished from these [Greek: δεδομένα (dedomena, "the givens" / data)], by Enunciation 2 alone — Enunciation 3 also being employed if the given angle is obtuse. There is no need of an example here.

DOCTRINE XI.

Given two angles of an oblique-angled plane triangle together with the side adjacent to them, to obtain the rest.

This Doctrine is completed by Theorem 4, and afterwards likewise by Enunciation 2. For since two angles are given, the third in the plane triangle is also known, from the complement of the given ones to 180° . Then, because the given side is opposite one of the given angles, the solution follows by Enunciation 2 and the preceding Doctrine.

DOCTRINE XII.

Finally, given two angles of an oblique-angled plane triangle together with the side opposite one of the angles, the rest are given.

Just as in the preceding case, this Doctrine is resolved into the things sought, since three things are thus given together with a side opposite one of them, by Enunciation 2 of this section.

DOCTRINES ACCOMMODATED TO THE PRACTICE OF SOLVING SPHERICAL TRIANGLES IN NUMBERS.

DOCTRINE I.

On right-angled spherical triangles in general.

Given any two parts of a right-angled spherical triangle, besides the right angle, to bring to light the rest that lie hidden.

Let there be (as in the following figure) a right-angled spherical triangle ABC, having the right angle at A. With B made a pole on the sphere, let the arcs

BA and BC be continued to quadrants as far as D and E. Then let the arc AC likewise be drawn out to a quadrant and end at the other pole F; and finally let another quadrant descend from that same pole through DE.

With the figure so constituted, the solution of right-angled spherical triangles accommodates itself for the most part to all the varieties premised — and this by Theorem 8 and Enunciation 2 of this section.

I. First, in the right-angled spherical triangle ABC, taken as primary, when all the angles are given, DE will be the measure of the angle at B; and therefore in the secondary triangle FEC, FE is given, the complement of DE — that is, of the angle at B — to 90° [recte: gr. 90]. Besides this, the angle at C is given, by Theorem 2 of this section. And because the angle at E is right, it will be: as the right sine C is to EF, so is E to CF. Once this is obtained, the complement of it, AC, is also given; and therefore in the prior right triangle BAC, the remaining two sides — namely BC and BA — are also easily obtained by Enunciation 2, since the angles opposite these, at A and C, are given. For:

as B is to CA, so is the radius A to BC; and so is C to BA.

II. Moreover, when all the sides of the right triangle BAC are given, the same follows. For since the right angle at A is always given, it will be: as BC is to A, so is BA to C; and so is AC to B.

III. Third, let two sides be given together with the included right angle, as in the same right triangle ABC, when the sides AB and AC are granted, enclosing the right angle at A — since thus in the secondary right triangle CEF too there are

exhibited, through the complements of the givens, CF the complement of the given AC; and likewise the angle CFE the complement of the given side AB.

So it will be: as the radius E is to CF, so is F to CE; from which its complement BC is also established. Hence in the primary triangle BAC, because all the sides are given, the angles at C and B are given, according to the case just preceding.

IV. Fourth, two sides being granted together with the angle opposite one of them — as in the primary right triangle ABC, given AB and BC together with the right angle at A opposite the side BC — it will first be:

as BC is to A, so is AB to C.

Then, in the secondary right-angled triangle CEF, because all its angles are thus given — namely C just found, F through the complement of the given side BA, and finally E right, together with the side CE, the complement of the given BC — first, then: as F is to CE, so is the radius E to CF, the complement of the side AC. Then: as the radius E is to CF, so is C to EF, the complement of the angle at B last sought.

V. Given two angles together with the adjacent side — as in the primary right-angled triangle ABC, when two angles are given, namely the right one at A and another at B, together with the adjacent side AB. Since therefore neither the proportion of the givens, in the present right triangle ABC, nor likewise in the secondary CEF, can be had in such a way that the solution may be made through sines alone — therefore, the arc CA being drawn out to the measure of a quadrant at G; and AB likewise continued to that same measure at I, and CB at H; and finally, from the pole C, when the fourth quadrant is drawn from G through H to I — it is manifest how, in the right-angled triangle BHI, with the right angle existing at H, there are given the angle HBI by Theorem 2, BI the complement of the given side AB, together with the right angle, as I said, at H. Therefore

Diagram. A spherical right-triangle construction: the right-angled spherical triangle ABC with arcs extended to quadrants reaching poles D, E, F, forming the secondary triangle FEC. Curved arcs connect the labeled points.

- B (upper left): pole / vertex
- A (right): vertex of the right angle of the primary triangle
- C (center left): vertex
- D (lower right): quadrant endpoint
- E (bottom center): quadrant endpoint / pole
- F (lower left): the other pole, quadrant endpoint of arc AC

Diagram. A second spherical construction for case V: the primary right triangle ABC with arcs CA, AB, CB extended to quadrants meeting at G, I, H, and a fourth quadrant from pole C through H to I, forming the right-angled triangle BHI.

- H (top left): vertex of the right angle of triangle BHI
- G (top right): quadrant endpoint of arc CA
- I (left): quadrant endpoint of arc AB
- B (center): vertex
- A (right): vertex
- C (lower center): pole / vertex
- D (lower right): point on the construction
- E (bottom center): point on the construction
- F (lower left): point on the construction

Therefore: as the radius H is to BI, so is B to HI. This gives, readily enough, the prosthaphaeresis HI, which is the complement of the angle sought at C.

From this, in the primary right-angled triangle ABC, for the remaining unknowns we have, first: as C is to BA, so is the radius A to BC.

Then: as the radius A is to BC, so is B [?] to AC.

VI. Sixth and finally, when in a spherical right-angled triangle two angles are given together with the side opposite to one of them — as in the primary right-angled triangle ABC, where, besides the right angle at A, the angle at B is also given, and finally the side BC opposite the right angle — the first step is: as the radius A is to BC, so is B to AC.

From this, in the secondary right-angled triangle CEF: since CF (the complement of AC) and EF (the complement of the angle at B) are given, we have: as CF is to the radius E, so is CE to the angle F — that is, to the complement of the side sought, AB. And likewise: as EF is to the angle sought, C.

Or here — which agrees directly with the prosthaphaeresis — in the third right-angled triangle of the last figure, BHI: since BI is given equal to DA, we have: as the radius H is to BI, so is B to HI, the complement of the angle at C that is sought.

Appendix to the Solution of Right-Angled Spherical Triangles.

Although these few preliminaries suffice to carry through the analysis of any proposed right-angled spherical triangle by means of right sines alone — and indeed so completely that all the varieties suited to practical use can be embraced

under a single doctrine — nevertheless it sometimes happens that, working from these or those givens (whose order has here been kept as it was for plane triangles), not every problem presents itself at once without the roundabout of a repeated computation. I add, moreover, that the radius does not always hold the first place in the rule of proportion (which would be most convenient), but sometimes the second or the third. For this reason, in cases of this kind, tangents and secants will be called in as a shortcut, so that through them, in accordance with Enunciations 5 and 6 of this section, the solution may be carried out in a single operation — as I shall now show below in certain instances.

In the first arrangement, namely when all the angles of the spherical right-angled triangle are given — that is, the angle at A, at B, and at C — any side whatever, BA or AC, is found at once by Enunciation 6 of this section. For:

as the radius A is to the right sine [S.R.] of ABC, so is the secant of the angle BCA to the secant of BA.

Likewise:

as the radius A is to the right sine [S.R.] of C, so is the secant of the angle ABC to the secant of the side AC.

Further, in the third supposition, when the two sides about the right angle are given — say AB, the right ascension of the Sun, and AC, its declination — then, in order that the angle ABC of the obliquity of the ecliptic may be found from these givens in a single operation, Enunciation 5 of this section must be applied. For: as the right sine [S.R.] of AB is to the radius, so is the tangent of AC to the tangent of B.

In the sixth enumeration of givens, finally, where two angles are given — namely the right angle at A and the angle of the greatest obliquity of the ecliptic at B, together with the side BC, which fixes the Sun's distance along the ecliptic from the equinoctial point B — let the side AB, the right ascension of the Sun, be sought, again in a single operation, by Enunciation 5 of this section. For in the secondary right-angled triangle CEF we have: as EF, the sine of the complement of the given B, is to the radius, so is the tangent of CE (the complement of the given side BC) to the tangent of the angle F, which is the complement of the side sought, AB.

More items, perhaps, may be noticed that should be referred to this appendix — especially in Philips van Lansbergen [Philippus Lansbergius], who, more than anyone else, had very diligently collected nearly all the proportions that can be

set up for the solution of the right-angled spherical triangle from right sines, tangents, and secants.

So much for right-angled spherical triangles; the oblique-angled ones follow.

DOCTRINE [DOGMA] THE SECOND.

Given all the sides of an oblique-angled spherical triangle, to find any desired angle.

First, the order is varied a little here, because the following doctrine — in which pure angles are given — can be demonstrated from this one more directly and more concisely.

Furthermore, the analysis of such a triangle, very common in spherical work and otherwise quite laborious, is made easy when reduced to prosthaphaeresis. Accordingly, the present doctrine is to be illustrated and, as it were, demonstrated by means of some example, in the manner that suits the diagram of the first rule of prosthaphaeresis. Then its remaining varieties, which can arise in nearly every case, are to be set out in order, since the brevity I aim at does not allow me to display each one here through its own appropriate figure.

[Margin: Christmann [Christmannus], Book 2, *Observations*, ch. 8.]

The examples that follow are deliberately drawn in their numbers from Lansbergen's data, so that both the truth of the solution by the prosthaphaeretic method — and also the far more expeditious practice — may be tested, and at the same time the authority that Christmann [Christmannus] claimed for himself in refuting Lansbergen may be rightly weighed by the kindly reader.

[Margin: See Lansbergen, *Trigonometry*, p. 200.]

So let there be a spherical triangle ABC, with

Given [Διδόμενα, "the givens"]: | Side | deg. | min. | | | ---|---|---|---| | AB | 50 | 0
| | | AC | 60 | 0 | | | BC | 26 | 22½ | |

From this let the angle BAC be sought, namely the one opposite the side BC. These data are fitted to the following prosthaphaeretic diagram in such a way that everything in it may be carried out according to the brief explanation below.

Diagram. A large circle (the primary circle) carrying the prosthaphaeretic construction for the oblique spherical triangle. Around the upper circumference the points A, B, C, D, E are marked, with arc-chords drawn between them; dotted

arcs cross the interior. A horizontal diameter runs from I (left) through the center to F (right), and a vertical diameter from D (top) down to G (lower portion). The interior is crossed by a grid of lines meeting the horizontal diameter at the lettered division points. H lies at the lower left of the circle, L at the lower right region.

- A (upper-left circumference): vertex; endpoint of minor arc AB
- B (top): vertex; endpoint of arc AB
- C (upper-right, interior of arc): vertex of the triangle
- D (top right): endpoint marking the complement arc DF
- E (right of D): division point
- T (right, on circumference): division point [?]
- F (right end of horizontal diameter): endpoint of arc, with EF the chord
- I (left end of horizontal diameter): endpoint
- O (center): center of the circle
- P, R (just above center, right): division points
- M (right, on horizontal line): endpoint of the half-sum line DM region
- N, K, Q, V, S (along the horizontal diameter, left to right): sine-foot division points
- R (left of center, on horizontal diameter): division point (a second R appears on the left arm)
- H (lower left): endpoint of diameter DH of the minor circle
- L (lower right): foot of the sine LS
- G (bottom): lower endpoint

Now the substance of the demonstration in this place is as follows. Since AB, likewise AC, and finally BC — the side opposite the angle sought at A — are granted, and since within the arc AB there remains the measure of the smaller of the given sides that enclose the angle to be found (to which EF is always equal, on account of the quadrants AE and BF of the circle); while the complement of the greater of those same sides is CT, or DE — it is therefore clear that DS is the right sine of the arc DF, that is, of the complement of the greater side and the smaller side added together.

The difference of the two sides themselves is HI, whose right sine is HR, or LS. When the sines DS and LS are added, the whole DL results; its half is DM, which in this case is the first found quantity [inventum primum].

Further, when the first found quantity DM is subtracted from DS, the sum of the sines, there remains the second found quantity [inventum secundum]: SM, NO, or PQ.

Third, the complement of the given opposite side BC is CV, whose right sine is CQ; so when PQ is subtracted from CQ, there remains CP, the third found quantity [inventum tertium].

But since DH is also the diameter of the minor circle, whose measure equals the periphery in number — as is clear from what we set down above concerning the earlier division of the circle — once these preparations are duly made, the portion CO of it is now sought, so that it may be the complement of the angle BAC that is sought.

Accordingly, by Theorem 7 of this section: as DM, the first found quantity, is to the radius DO, so is CP, the third found quantity, to CO, the fourth — which is the right sine of the complement of the angle sought at A.

A Precept Suited to Practice.

Therefore, here, according to the first rule of prosthaphaeresis, add and subtract from one another the smaller of the arcs and the complement of the greater — that is, of those that flank the angle sought. Add the sines of the two together if the smaller arc is greater than the complement of the greater; but if it is smaller, subtract; and half of the aggregate or of the difference will be the first found quantity. Then, when this first found quantity is subtracted from the sine of the aggregate, the remainder will be the second found quantity.

The third found quantity is the difference between the sine of the complement of the side opposite the angle sought and the second found quantity.

Next, multiply the third found quantity by the whole sine [radius], and divide the product by the first found quantity; there comes out in the quotient the sine of the complement of the angle sought — if the sine of the complement of the opposite side is greater than the second found quantity. But if it is smaller, there comes out the sine of the excess beyond the quadrant, by which excess, added to the quadrant, the angle sought is made obtuse.

The practice in the foregoing follows.

	deg.	min.	sec.
smaller arc AB	50	0	

	deg.	min.	sec.			
compl. of greater, DE	30	0				
aggregate DF	80	0		S.R.	9848078	D S
difference HI	20	0		S.R.	3420201	H R

aggregate of sines	13268279	DL				
half of the sine	6634139	DM, first found quantity				
difference of first found qty. & aggregate of sines	3213939	SM, second found qty.				

	deg.	min.	sec.			
opposite side BC	26	22	20			
its compl. CV	63	37	40	S.R.	8959272	C Q

second found qty. subtracted					3213939	
there remains the third found qty.					5745333	

Rule of Proportion.

As the first found quantity is to the radius, so is 5745333, the third found quantity, to the fourth found quantity — whose corresponding arc is 60°, and its complement 30°, the angle sought.

Therefore the calculation in this example of Lansbergen is sufficiently sure.

If the whole analysis of this doctrine by prosthaphaeresis is carried out without the division, it will be done by transposition in this way:

	deg.	min	sec.			
Sec[ant]	6634139	.		S.R.	5745333	

	deg.	min			
		.	sec.		
corresponding arc	81	19	49	corresponds to	35 4 1
complement	8	40	11	complement	54 55 59

	deg.	min			
		.	sec.		
	8	40	11		
aggregate	63	36	10	S.R.	8957332
difference	46	15	48	S.R.	7225248 Subtract
difference of sines					1732084
half					8660420

— to which correspond 60°, its complement being 30°, as before.

Variations of the Present Doctrine.

I. If one of the sides enclosing the angle sought is greater than a quadrant, the excess of it above the quadrant is taken for its complement; and the second found quantity is added to the sine of the complement of the side, from which the third found quantity is formed; and so the operation proceeds in the rest as above.

II. If the opposite side is greater than a quadrant, the second found quantity is likewise added to the sine of the excess of the opposite side, and the arc finally obtained is joined to its quadrant of 90° to give the angle sought.

III. Finally, if both one of the sides flanking the angle sought and the opposite side exceed the measure of a quadrant, the operation must again be carried out by the excess of the opposite side; but the second found quantity is to be subtracted from the sine of the excess of the side; the arc finally emerging is also to be added to 90°, and the angle sought will be established.

Examples of the individual varieties the following spherical doctrine will supply in abundance.

DOCTRINE [DOGMA] THE THIRD.

Given all the angles of an oblique-angled spherical triangle, to find any side whatever.

Since this doctrine is, as it were, the converse of the prior one — on account of the analogy between the right sines of the sides and of the opposite angles, by Enunciation 2 of this section — I shall accordingly take the angles here in place of the sides in the foregoing, and undertake its solution without a laborious demonstration, showing its likeness to the prior case by a numerical example, and then briefly advising learners about its varieties.

Taking here too Lansbergen's data [Διδόμενα], let there be an oblique-angled spherical triangle ABC of given angles, and let the angle at A, to which the side BC is opposite, be 30° ; the angle at B, which the side CA faces, $102^\circ 53' 31\frac{1}{2}''$; and finally the angle at C, opposite the side BA that is sought, be $59^\circ 34'$, and a little more.

Diagram. A small oblique-angled spherical triangle with vertices labeled. B is at the upper left, C at the upper right (joined by an arc bowing slightly upward), and A at the lower point; the sides BA (left) and CA (right) curve to meet at the sharp vertex A.

- B (upper left): vertex of the angle at B
- C (upper right): vertex of the angle at C
- A (bottom): vertex of the angle at A

Therefore, taking the angles A and B in place of the very sides to which they are opposite, and which enclose the angle BCA opposite the side BA that is sought — and likewise taking that same BCA for the side we seek — the solution is fitted to the practice of the preceding precept in this way:

The worked example of Doctrine III continues, completing the solution for side BA from Lansbergen's data.

gr. 30 mi. 0	minor angle	
gr. 12 mi. 53 sec. 31	excess of the greater beyond 90°	
gr. 42 mi. 53 sec. 31	sum	S.R. 6806179
gr. 17 mi. 6 sec. 29	difference	S.R. 2941747
	sum of sines	9747926
	half	4873963 — first found value
		1932216 — second found

		value
gr. 59 mi. 34 (+)	angle C	
gr. 30 mi. 26 (-)	compl.	S.R. 5065355
	second found value	1932216
	subtracted	
		3133139 — third found value

Rule of proportion [Regula proport.]

As the first found value, 4873963, is to the radius 10000000, so is the third found value, 3133139, to 6428320, the right sine of an arc of $40^{\circ} 0'$. The complement of this, $50^{\circ} 0'$, is the side BA sought — just as Lansbergen's computation gives it.

The variations to be observed here, in certain cases contrary to those above.

I. If the third angle is acute and one or the other of the adjacent angles is obtuse, then the second found value must be subtracted from the complement of the third angle — unless this second found value is the greater, in which case, putting the difference in the same way for the third found value, the right sine that results for the fourth value is one exceeding the quadrant.

II. But if the third angle is obtuse, the operation is set up using its excess beyond 90° ; and if the second found value is less than the right sine of the said excess, the resulting side will be obtuse, that is, greater than 90° .

III. Finally, if both the third angle and one or the other of the adjacent angles are obtuse, then the second found value is to be added to the right sine of the excess of the third angle.

Chapter 6 of Book 2 of the *Spherics* will supply several further examples of this Doctrine.

DOCTRINE IV [DOGMA QVARTVM]

Given two sides of an oblique-angled spherical triangle together with the angle they contain, to determine the third side, the one opposite the given angle.

This Doctrine too has a very broad use, not only in the celestial doctrine of the sphere but also on the terrestrial globe, for finding the distances of two places that differ in both longitude and latitude.

Its analysis proceeds in almost the same way as that of the second and third, being reduced to prosthaphaeresis, and is here completed with even less effort.

Diagram. A circle (the great circle of the prosthaphaeretic construction, the same figure used for Doctrine II) with an inscribed network of chords, perpendiculars, and arcs. The construction is the prosthaphaeretic diagram carried over from the second Doctrine, here re-applied to the present problem. Readable labels around and within the figure:

- B (upper region, near top): vertex
- D (upper right, beside B): point on circumference
- E (right of D): point on circumference
- A (left, on circumference): vertex
- T (interior right): point
- F (far right, on circumference): point
- I (far left, on circumference): point
- R (left interior, two occurrences — one upper-left interior, one lower-left near circumference): points
- O (interior center-left): center / pole point
- P, R, M (interior, center, in a row): points (P and R adjacent, M to the right)
- N, K, Q, V, S (interior, lower row left→right): points
- H (lower left, on circumference): point
- G (lower, on circumference): point
- L (lower interior): point
- C (interior, upper-center): vertex of the triangle

The accompanying marginal text reads:

[Margin: I shall first present the diagram for this Doctrine too, adapted to Lansbergen's data, and then add the precepts and the practice, and last append the observations on the variations.]

[Margin: Let there be recalled here the prosthaphaeretic diagram set beneath the second Doctrine, since it was constructed on these same data, and let the demonstration attached to it there be applied throughout — except that, when the side BC opposite the given angle at A is sought, it will then be: as the radius OD is to DM, so is OC, the complement of the given angle A, to CP, the right sine of the complement of the side sought CR.]

The precept of this Doctrine.

Add and subtract from one another the lesser arc and the complement of the greater of the two arcs that flank the given angle. Add their right sines if the lesser arc was greater than the complement of the greater; otherwise subtract them. Half the sum or difference will be the first found value.

Next, subtract this first found value from the sum of the sines, and the remainder will be the second found value.

The third found value is the sine of the complement of the given angle.

When this is multiplied into the first found value, and the product divided by the radius, the quotient will be the fourth found value. Add the second found value to it, and the sine of the complement of the side sought is produced.

Example in numbers.

Let it be the same oblique-angled spherical triangle ABC as above, whose two sides about A are given — namely AB and AC — together with the included angle BAC, which in numbers, in the arrangement we customarily use, stand as follows:

	grad.	min.
B A	50	0
C A	60	0
B A C	30	0

From these the side BC is sought — that is, the one opposite the angle BAC — and it is found to be $26^{\circ} 22' 20''$.

Practice [Praxis].

grad. 50 min. 0	minor arc	
grad. 30 min. 0	compl. of the greater	
grad. 80 min. 0	sum	S.R. 9848078. D.S.
grad. 20 min. 0	difference	S.R. 3420201. H.R.
	sum of sines	13268279
		6634139 — first found value
		3213939 — second found value

compl. 60 of the angle BAC, corresponding	S.R. 8660254 — third found value

Rule [Regula]. Therefore:

As the radius is to 6634139, the first found value, so is 8660254, the second found value [recte: third found value], to 5745332, the fourth found value.

Add the second found value, 3213939.

C.P. [the accumulated sine of the complement] 8959271, corresponding to the arc $63^{\circ} 37' 40''$, whose complement — namely $26^{\circ} 22' 20''$ — is the side sought.

These last operations, carried out by this rule through the multiplication of the first found value into the third for the fourth found value, when they are then subjected to prosthaphaeresis, give about the most expeditious method there is — especially when the seconds of arc are neglected. For since the complement of the given angle is always known in its own arc, only the arc corresponding to the first found value is sought. So here, to the first found value 6634139 the corresponding arc is about $41^{\circ} 33' 40''$:

grad. 41 mi. 33 sec. 40	(arc of first found value)	
grad. 30 mi. 0 sec. 0	complement of the third found value	
grad. 71 mi. 33 sec. 40	sum	S.R. 9486615
grad. 11 mi. 33 sec. 40	difference	S.R. 2004130
	fourth found value, about as above	11490745
		5745372

By this one Doctrine I once completed the whole inquiry, treated long ago, into finding the distances of terrestrial places (their longitudes and latitudes being supplied from a geographical table) — and that with scarcely any labor, compared with the older method, which of course relied on continuous prosthaphaeresis.

The various cases of this Doctrine to be observed.

I. If one of the given sides exceeds the quadrant, use this excess in place of the complement, as in the Doctrine above; and subtract the second found value from the final product sought, and the remainder will be the sine of the complement of the side sought — or, if the second found value is the greater, the remainder will be the sine of the excess of the side sought. But if the final product is less than the second found value, the difference is the right sine of the complement of the side sought.

II. If the given angle is obtuse, one must likewise operate by its excess beyond 90° in place of the complement, and the second found value must be compared with the final product; and if the latter is the less, the difference is the right sine of the excess of the side sought.

III. Finally, if both one of the given sides exceeds the quadrant and the angle it includes is at the same time obtuse, the second found value will be added to the final product, and the sum that emerges is the sine of the excess of the side beyond the quadrant.

DOCTRINE V [DOGMA QVINTVM]

Given two angles of an oblique-angled spherical triangle together with the side adjacent to them, the remaining angle is determined.

This Doctrine too is the converse of the immediately preceding one. For by positing — or at least disregarding — the given angles that include the given side, in place of the sides opposite those same angles, and the included side in place of the angle to be found, the calculation proceeds toward the result no differently than in the preceding case, while a few variations are here also to be noted,

Diagram. A small right-angle-style figure in the upper-left corner: an arc descending from A (upper left) curving down to B (lower left), with C marked at the upper right, forming the spherical triangle ABC of the worked example.

Labels:

- A (upper left): vertex
- C (upper right): vertex
- B (lower left): vertex

as is observed in the third Doctrine, are likewise adapted [Greek: κατ' ἀναστροφὴν (kat' anastrophēn, "by inversion / conversely")] to the practice. We shall therefore begin at once with an example, drawn from Lansbergen's numbers.

Let there be an oblique-angled spherical triangle ABC, with given angles at A of $102^{\circ} 53' 31''$ and at C of 30° , together with the side AC of 50° . The angle at B is sought from these.

Operation [Operatio].

major of the given angles	grad. 102 min. 53 sec. 31	
its excess	grad. 12 min. 53 sec. 31	
grad. 30 min. 0 sec. 0	minor angle	
grad. 12 min. 53 sec. 31	excess of the greater	
grad. 42 min. 53 sec. 31	sum	S.R. 6806179
grad. 17 min. 6 sec. 29	difference	S.R. 2941747
		9747926
		4873963 — first found value
Given side grad. 50 min. 0		1932216 — second found value
Its compl. grad. 40 min. 0		S.R. 6427876 — third found value

From the multiplication of the first found value into the third, and the division of the product by the radius, the fourth found value 3132923 comes out; and when the second found value, 1932216, is added to it

(it will be done contrary to the case above):

there is accumulated the right sine of the complement of the angle sought, 5065139, which is $30^{\circ} 25' 55''$.

Therefore the angle sought is itself $59^{\circ} 34' 5''$, differing from Lansbergen by at least the 15 seconds [sec. 15] by which he, having worked the analysis through the method of right angles, had completed it.

The whole reduced to prosthaphaeresis [Ad Prosthaphaeresin totum revocatum].

4873963 first found value, corresponding arc	gr. 29 mi. 10 sec. 10	minor arc
arc of the third found	gr. 40 mi. 0 sec. 0	major arc

value		
grad. 50 min. 0	complement of the greater	
grad. 29 min. 10 sec. 10	minor arc	
grad. 79 min. 10 sec. 10	sum	S.R. 9821872
grad. 20 min. 49 sec. 50	difference	S.R. 3556054
		6265818
		3132909 — fourth found value, about as before

DOCTRINE VI AND LAST [DOGMA SEXTVM & VLTIMVM]

Given two of the [sides / angles] of an oblique-angled spherical triangle, together with one of the [angle / side], opposite to one of the given [sides / angles], the remaining parts are determined.

[The Doctrine is set out with bracketed alternatives, to be read in parallel: given two **sides** with **one angle** opposite one of the given sides — or given two **angles** with **one side** opposite one of the given angles — the remaining **side / angle** is found.]

These givens [Greek: διδόμενα (didomena, "the given quantities")] are conveniently enough gathered here into a single Doctrine, and are solved by the second Enunciation [Enunciat.] in what follows [Greek: ζητούμενα (zētoumena, "the things sought")]. Or, if one wishes all the parts to be had at once, let it be done by the 3rd theorem, the proposed triangle being divided into two right-angled

when one angle has been resolved and the perpendicular drawn, the prosthaphaeresis is very quickly carried out and the result soon obtained — especially whenever the angle adjacent to the side opposite the right angle is itself given at the same time.

If at any point one of the angles is obtuse, substitute its complement to 180 degrees by the third Enunciation. As for the other variations — where the given side exceeds a quadrant, or the angle, or the side sought happens to exceed a quadrant — these are easily recognized with the help of the Globe or of our Sphere.

So, of all the many varieties of examples by which Philippus Lansbergius [Philippus Lansbergius] illustrates problems of this kind — taking the given quantities [διδόμενα (didomena), "the things given"] and from them deriving the things sought [ζητούμενα (zētoumena), "the things sought"] concerning right-angled triangles — I shall take up just one. It is the example belonging to the second case: two given angles together with the side opposite to one of those angles, that angle being obtuse. [Margin: Lansb. p. 194.] I do this for two reasons: to show that the method by the 2nd and 3rd Enunciations holds equally true in spherical triangles, and at the same time to correct an error of Lansbergius that may have crept in through some slip in his computation of the given quantities [διδομένων ἀταξίαν (didomenōn ataxian), "a disorder among the given quantities"].

Diagram. An oblique spherical triangle ABC with the perpendicular construction. Vertex A is at the top; C is at the lower left, B at the lower right, and D lies just beyond B on the prolongation of the base. The curved base runs from C through B to D, the side CA rises from the lower left to A, and from A two lines descend to B and to D, AD being the perpendicular arc dropped onto the prolonged base CD.

- A (apex): vertex A
- C (lower left): vertex C
- B (lower right, on base): vertex B
- D (lower right, beyond B): foot of the perpendicular from A

We are given an oblique spherical triangle ABC whose angle ABC is $102^{\circ} 53' 31''$. This angle is therefore obtuse, by what was set out above concerning the division of the triangle. We are also given the side AC, 60° , and finally the angle at C, 30° .

First, then, let us seek the side AB opposite the given angle at C, by the second and third Enunciations. For:

	Compl. of obtuse at B	Side AC	C	AB sought
a	$77^{\circ} 6' 29''$	to 60°	so 30°	to $26^{\circ} 22' 24''$
s				
	S.R. 9747927	S.R. 8660259	S.R. 5000000	S.R. 4442188

If still more is to be sought, then — since the obtuse angle is at B — prolong CB, let the perpendicular arc descend from A to D, and let ACD be made a right-angled triangle. Now since the base AC is given, together with the adjacent angle C, AD is found by prosthaphaeresis in this way:

AC 60° , its complement 30° Lesser arc, i.e. the angle at C, 30°

sum	60°	S.R. 8660254
difference	0°	S.R. 0000000

Therefore 4330127 is the right sine of the perpendicular arc DA, the first quantity sought, namely $25^\circ 39' 32''$.

Next, in the right-angled triangle ADC, for the side DC, by the 5th Enunciation:

C	D	AD	DC
a $30^\circ 0'$	to radius	so $25^\circ 39' 32''$	to $56^\circ 18' 36''$
s			
T. 5773502	S.T. 10000000	T. 4803844	S.R. 8320503

From this, for the angle DAC:

CA		DC	DAC
a 60°	to radius	so $56^\circ 18' 36''$	to $73^\circ 53' 53''$
s			
S.R. 8660254		S.R. 8320503	S.R. 9607690

Third, in the spherical right-angled triangle DAB, first for DAB, again by the 5th Enunciation:

Compl. AD	Compl. AB		Compl. DAB
a $64^\circ 20' 28''$	to $63^\circ 37' 36''$	so radius	to $75^\circ 39' 47''$
s			
T. 20816708	T. 20168433		S.R. 9688578

So the angle DAB itself is given as $14^\circ 20' 13''$.

And since the angle DAC was previously found to be $73^\circ 53' 53''$, the angle BAC is therefore $59^\circ 33' 40''$.

Lastly, in the right-angled triangle DAB, for the side DB:

Diagram. The same oblique spherical triangle repeated. A is at the top; C at the lower left, B at the lower right with D just beyond it on the prolonged base. The side CA rises to A, lines descend from A to B and to D, and AD is the perpendicular arc onto the base CD.

- A (apex): vertex A
- C (lower left): vertex C
- B (lower right, on base): vertex B
- D (lower right, beyond B): foot of the perpendicular

D	BA	DAB	DB
a radius	to $26^{\circ} 22' 24''$	so $14^{\circ} 20' 13''$	to DB
s			

By the following prosthaphaeresis:

$63^{\circ} 37' 36''$, complement of the greater $14^{\circ} 20' 13''$, the lesser given

$77^{\circ} 57' 49''$	sum	S.R. 9780154
$49^{\circ} 17' 23''$	difference	S.R. 7580173
		2199981
		1099990 S.R. of the side DB sought

So the side DB itself is given as $6^{\circ} 18' 55''$. And this, when subtracted from the side DC (previously found to be $56^{\circ} 18' 36''$), leaves BC as $49^{\circ} 59' 41''$.

And so, in the closing section of this trigonometric compendium, we have restored that example of Master Lansbergius, which in fact agreed very closely with our own calculation — except that in the sides AB and BC something had been transposed in error. In the foregoing passages, moreover, Christmannus [Christmann] was finding fault with him to no purpose, while at the same time setting out to demolish the demonstration of the ancients.

THE END.

ΠΡΟΓΝΩΡΙΣΜΑΤΩΝ [Greek: Προγνώρισμάτων (Prognōrismatōn, "Preliminary Notions / Things to be known beforehand")] OF ASTRONOMY, PART TWO

On the matter that contains the heavens; on the form of the greater bodies of the world and their activity [ἐνεργεία (energeia, "activity, operation")] while in motion; and finally on the analogous mutual consideration of time and of motion.

No one should rightly be surprised, or count it a fault in us, if — before laying down hypotheses suited to the tracks and motions of the greater bodies of the world (I mean the stars and the earth) — we first set before the restoration of Astronomy a few remarks: on the nature and definition of the matter that is the place of those bodies, and of their motions in general; likewise on their nature and form, especially of the bodies of the world themselves, as will seem more fitting for their preservation and for their motions, kept up by the divine influence; and finally, a brief consideration of time and motion in relation to one another. For without at least a moderate grasp of these things, the astronomer will be unable to trace adequately the proper positions and motions of the phenomena [φαινομένων (phainomenōn, "appearances, phenomena")], nor to investigate their influences and operations upon these sublunary regions in a way that accords sufficiently with nature. And although we admit that this inquiry belongs properly to Physics, we nonetheless judge that it will not be foreign to the astronomer's profession, but rather especially appropriate to it — since the one who contemplates the supralunary, that is, the celestial, nature is the astronomer. For that reason (other things being equal) no one is raised higher or more truly into that nature than he, by sense and reason together.

As to the first part of our undertaking: first, we reject the opinion of those who hold the celestial region to be hard and impenetrable — packed, that is, with such material orbs, as the Peripatetics suppose. Their error without doubt descends from the Pythagorean doctrine of the musical harmony of the celestial orbs, wrongly understood. Next, the opinion of those who supposed the aethereal region to be fire, like certain Stoics, whom Theophrastus Paracelsus [Paracelsus]

imitated — unless perhaps this be understood of the heavenly bodies themselves, and of the primeval light scattered everywhere throughout the world, yet leaving manifest traces of itself in the heavenly bodies. This last I would rather grant than believe that any created thing has perished again — as will one day be discussed more fully in the Appendix to the Astronomy, On the New Phenomena of the Heavens. Third and finally, the opinion of those who hold that same celestial region to be either fairly thick air or water. For since in our own times, from a truer recording of the positions and motions of the more recent celestial phenomena in particular, and from the refractions of the stars, and from other signs that everywhere meet the practiced astronomer, most of those theories are refuted — without any lengthy refutation, and amid the wavering opinions of the many, we shall set forth what we judge most consistent (as it seems to us) with Holy Scripture, with nature, with reason, and finally with experience, concerning the matter here proposed, adding a short description of it together with the proof of our explanation.

The matter adequate to the place of the whole visible and finite universe, by its own nature and fullness, we call an extension most rarefied and most subtle — being indeed most like to the incorporeal and the insensible — homogeneous and of like parts throughout [ὁμογενές καὶ ὁμοιομερές (homogenes kai homoiomeres, "homogeneous and of like parts")]; and besides, in its own essence stable...

indeed, yet permeable; in quality cold, created by God the best, so that it might be a kind of substrate and, as it were, vehicle of light and of luminosity, and in the first place of the heavenly bodies; and also the cause of the separation of the waters — and for that very reason, the primary cause.

The first reason for the asserted definition can be this: that no natural light, in the space through which it emanates from its luminous body, is carried through a void or lacks any substrate — since light, or luminosity, is an accident outside its own root. But fire and all luminous bodies cast light and luminosity out from themselves, as is manifest from the effect of the heavenly bodies (and especially of the Sun), whose rays not only reach all the way to us but even penetrate into the earth itself in the concoction of metals.

Since, then, such lights are not simply carried through a vacuum or void, it follows that they require something of this kind for a vehicle, something that has the nature of the defined extension, as will be further confirmed shortly. First, that such matter is homogeneous and uniform is proved thus. For if it allowed of a division contrary to this, then there would exist one kind of matter in the aether above the Moon and another, different from the first, in the sublunary region — a

matter into which the lights and rays of the heavenly bodies, sent down to us, would unfold themselves; and this could, in fact, be nothing here but air. But we do not acknowledge such air to be the immediate vehicle of light or rays even in the sublunary region; and so it must be that the matter we shall treat of is homogeneous and uniform. We shall bring forward in the middle the ground of our assumed connection, not so much to persuade as to demonstrate it. For since that air is mobile — its nature thriving in continual flux — it is certain, and experience teaches, that it can be driven from its place. And although that place would be restored and refilled by continual succession, it could in no way happen that all things which hang visible in such shaken air would not be subject either to a fairly manifest, or at least a slight, vibration. But no vibration of light is felt at all, even when the air through which it is seen is shaken with all force and moved from its place — though I make exception for that apparent vibration which our eyes conceive near the horizon, subject to the Sun's summer heat, on account of those thicker airy vapors, by which (when they are likewise illuminated) the Sun's rays are entangled and refracted. From this I conclude that the air is by no means the primary and adequate substrate of light, but rather the matter we defined above. This argument indeed Palingenius [Marcellus Palingenius Stellatus] — a philosopher greater than he is a poet — sets forth, to demonstrate that the air is in the light, and not the contrary, in these verses in the last sign of his Zodiac:

Perhaps someone will object that the air is the substrate of light and its foundation: and accordingly, where there is no air — beyond the borders of the highest heaven — he will deny that light exists. But here too he departs from the truth, as before. For the air is not the substrate of light, nor is light (as he wrongly supposes) in the air: rather the air, on the contrary, is in the light, if he sees aright. Consider not what Aristotle [Aristotle], but what reason itself asserts. For if a lit lamp stands at closed doors, or a torch sends its light through a chink into a darkened house, and someone then shakes the air beside the ray of admitted light, the air will be forced to pass through that ray, while the light is not moved at all. But if the air were the substrate of light, the ray would surely be shaken with the same motion as the air, and the light would doubtless depart along with its substrate.

Therefore, since this expanded matter we are treating of is everywhere spread beneath the whole reach of the outpouring of light and rays from the heavenly bodies — reaching even into the innermost earth, wherever light extends itself or penetrates — it necessarily follows that the sublunary region...

region, though not in its entire kind, as far as that matter is distinct from the aetherial.

But someone may object that the sublunary air is the firmament or expanse between the waters and the waters, and that this alone is what is mentioned in Genesis 1:6, and so on. I answer: the antecedent must be distinguished, and we must discriminate between this air insofar as it is in part diffused around the lower regions out of which it arises, and air as an expanse primarily and simply. For air on its own nowhere exists here, and yet, contained within the limits assigned to it according to the divine ordinance from the beginning, it carries out the daily separation of the waters, while at the same time doing so in the lower regions as well. For when the rays of the Sun and of the stars, with which they were conjoined, rebound again from the earth and the waters, it happens that by the more powerful force of their redoubling—together with the exhalation of the terrestrial globe itself, which contains much heat within—they raise the vaporous matter from there, by way of the expanse and of the finer interwoven air as a vehicle, up to the middle region; and there at last they suspend and sustain it, until, cooled and dissolved in that very place, it falls back again onto the terrestrial globe. Moreover, by this continual exhalation the finer air in the lowest region is thus prepared for us—that is, the vaporous matter is borne upward in the manner I have described. When this is done, no small part of that finer matter, elaborated above the surface of the earth and the sea by the very great vigor and heat of the stellar rays—chiefly the Sun's—is converted into air suited and wholesome for living creatures. The same is to be thought of such air, even the highest: namely, that the efficacious rays of the stars, before penetrating the middle interval [diastema], continually draw out something here too from the matter of the clouds, dry it more finely, and elaborate it; and so, successively, through the expanse, they raise it up to the outermost limit of the aerial region, that is, of the whole terrestrial globe. From this sticky and, as it were, dried-out matter there arise not only certain fiery meteors, but also whitish corpuscles are seen, which give an evident cause of the twilights [crepuscula].

[Margin: *Prob. 1, cap. 11, lib. 2 Sphaer.*]

Now there is a quite manifest difference between the clouds and the raising of the upper air from the earth, since the clouds are found by observation to be scarcely a mile and a half in extent from the terrestrial surface—and that in summertime, when the solar rays are more efficacious. As for the upper air, insofar as it is visible, the opticians prove from the observation of the twilights that it is thirteen German miles distant from us. [Margin: *Probl. 4 and Probl. 5, lib. 2 Sphaer.*] That

this is so is proved not only by experience in those cases, but also by the spagyric or pyronomic art, [Margin: *prop.* 60.] and indeed even in man as a microcosm it is shown by a certain very beautiful analogy—through the sublimation and reflection of the humors from the stomach into the head—[Margin: *Vide Vitellionem [Witelo], lib. 10*] [Margin: *Item lib. 2 Sphaer., ca. 11, prop. 1.*] namely, with the rays interposed from the heart and from the air of the chest. And what, I ask, since it is established that this air is a sufficiently perceptible body, why would our senses—chiefly sight, as the noblest of them, whose faculty is spiritual—not also require, for perceiving visible things, a quasi-spiritual subject as an intermediary, such as the expanse itself is, of which we are speaking, together with the very repletion by light and by air of this kind, the διαφανές [Greek: diaphanes, "the transparent"]? Nor should we think otherwise—that some most rarefied air everywhere, even throughout the heaven, equals the breadth of the expanse and fills the place of the universe, as Johannes Pena [Jean Pena], Christophorus Rothmannus [Christoph Rothmann], and others have supposed—unless they have understood the nature of our expanse plainly in this way: namely, apart from that universal something which, by way of a distinction, some not without authority and not inappropriately call the τὸ ἀραιῶδες [Greek: to araiōdes, "the rarefied"]. For besides the absurdity that would thus be committed from the vibration of the stellar rays against experience, it would also follow that refractions would accompany the stars almost up to the zenith—refractions which, however, from observations made by us most diligently in different places of the earth, in Denmark, Bohemia, and Norway, we have found to fail entirely within half a quadrant from the horizon. [Margin: *Videatur lib. 1 Epist. Astron. Tych. Brahae [Tycho Brahe]*, where also many things by reason, and the answer to this controversy with Rothmann, are given.] I add a second point, concerning the excessively swift local motions of the celestial bodies without any obstacle of air of this kind, and without penetration.

There underlies, then, this same expanse, with the solar and stellar rays, and further with air of such a kind as to separate off these waters, an affection [affectus]. But as for those rays, before the creation of the Sun and Stars,

it appears from the Sacred Scriptures that light was made. [Margin: *Gen. 1, v. 3.*] And although this light no doubt powerfully exerts its efficacy everywhere within finite places, nevertheless it shows us no traces except as occupied in the stars and in matter otherwise suitable—as I remember having promised above that I would set forth this more fully, God willing, in the appendix to the *Astronomy*. [Margin: *Vide append. lib. 1 Theor., de maculis in Luna* (On the spots in the Moon).] Nor is it surprising that from this globe as from a root there arise these meteoric

conditions [peristases] which we have just touched on, and that they are diffused around it as around a parent—since it is probable, and almost visible in the thirsting Moon, that the other mundane bodies or celestial stars too are wrapped in similar or analogous coverings, each according to its own nature; by which each one extends the whole bulk of its globe through the expanse to the limits and uses assigned to it.

[Margin: *Censura de Cometis* (Judgment on Comets), which will be more amply set forth in the appendix to the *Astronomy of the new Phenomena*.]

From this it will also be probable that the stars, namely through the same celestial expanse, eject hidden seeds of new phenomena at a divinely ordained time, and that thence a conception takes place in the expanse, which suddenly and as if miraculously rises up into the appearance of a new star—while it usually displays the greatest part of its size and motion, in those things that are moved, immediately at the beginning of its appearance. For no elementary generation appears similar to a cometic one, unless perhaps the bubbles on water produced by a sudden expansion might be drawn into some likeness of this. But to assert that the bodies which thus at times shine forth in the heaven are perpetual, and that they are either purged by fire at a certain time, or are illuminated only in a certain congruent disposition with the Sun so as to appear to us—such were the opinions of certain of the ancients, as are gathered from Pliny [Pliny the Elder], Plutarch, and others—this certainly seems to be reduced to absurdity, both by a careful consideration of the analogous appearance and motion of those same stars, and by the fact that no obscuration of any star was ever observed by these men.

So far we have treated of the substance of the matter of the expanse; and if anyone should say that it is to be likened, respectively, to something spiritual and to Aristotle's quintessence, he will not, in my judgment, stray far from the truth. It remains, then, for us to bring forward the reasons for the cold quality we have asserted of this same matter; and they are as follows.

Since two things—namely, that primeval light, to which the light of the stars is associated, and the expanse as the vehicle of light—have been most diffusely extended through all visible and finite things, as has been shown by the foregoing arguments; and since the quality of light is hot, as is plainly felt in the solar rays, and as is rightly perceived by reasoning in the hidden operations of nature: it is consonant with that same nature that the other active quality, namely the cold, should be attributed to the matter of the expanse—such as experience itself also proves. For when the solar rays are absent, or operate less efficaciously because

of their too oblique reflection around the earth (as tends to happen among us in wintertime), and still much more, for the same cause, in places nearer the arctic region, we find the whole constitution of the air turn itself into icy rigidity. We experience the same thing with this air, even in a moderately warm place, when it is suddenly fanned through the expanse—just as it is affected by the cold quality itself from the underlying expanse. What is more? Just as neither without heat, so neither without this cooling can the life and respiration of animals be in any way preserved or continued—from which too the soul is named ψυχή [Greek: psychē, "soul"], ἀπὸ τοῦ ψύχους [Greek: apo tou psychous, "from the cold"]. I pass over the fact that it is credible that the stars themselves, provided they are fiery—whether they are moved or are at rest—are continually tempered, as it were, through that latent harmony, on account of the greater portion of light in them; something of which sort has been demonstrated by others to lie between the moist of water and the dry of earth. Therefore from these things we reason that the celestial region is of all the most temperate, on account of the proportion of the matter of the expanse and of the suffused light so well adjusted to it—keeping in suspense for the present, however, with all prejudice set aside, the question of the stars and of their consequent affection.

Thus far concerning the prior part of our proposal, which sets out the matter that comprehends the whole of the place. There follows the second part, in which, concerning the nature and form of the

great mundane bodies, insofar as they are fit both for their own preservation and for motion, let us inquire a little further.

First, we judge it to be most difficult to set out the matter of those bodies, because neither do all the celestial stars appear to be of the same matter, nor endowed with equal light. For the Moon is found, not only from its known torches [phases] and from the generation of its light from the Sun, to be a body of itself solid and shadowy, but also conspicuous, as it were, with lofty mountains and deep valleys bearing the appearance of waters and forests—so much so that certain people have called it a second earth, if we are to abide by the observations of the moderns. [Margin: *Vide Galilaeum [Galileo] in Nuncio Sid. (Sidereus Nuncius).*] Furthermore, as to excellence of light and magnitude, the Sun is deservedly to be preferred by us to all the stars, as having been made before the rest of the stars. But whether these stars (the Moon excepted) were produced from the primeval light, and shine of themselves, as many suppose, or came into being separately and borrow their light, like the Moon, from the Sun: this we judge it neither to belong to man plainly to know, [Margin: *Prius potius affertur ex observationibus*

(rather, the former is asserted from observations).] nor to contribute much to our aim of perceiving their position and motion. [Margin: *Veneris, fere in conjunctione cum Sole, lib. 2 Theor., cap. 18.*] Likewise many other ἀπορίαι [Greek: aporiai, "difficulties, open questions"] remain, touching—though but slightly—our proposal: concerning the numerous multitude of the stars, and concerning the interval and comparison of those that are seen at the extreme limit by us and are called fixed, both with respect to the earth and among themselves. And whether the stars are suitable for habitations, by which at last each is inhabited; and further, what seems like a miracle, by what reason the stars of Jupiter—four others, minute and commonly not visible, circling it at a small and unequal distance, but with continual and most rapid motion, and like certain attendant satellites—have been assigned to it by God; together with many other things, in which we judge that the divine omnipotence and providence are rightly to be admired and adored, [Margin: *Vide Galilaeum [Galileo] de Nuncio Sidereo.*] but our own blindness and ignorance, while we remain here, most deservedly to be acknowledged.

These things premised, we shall now approach the inquiry concerning the form of the great mundane bodies, which will be most especially the object of our contemplation, in the manner we stated at the outset.

The form, as of all bodies, so too of the great mundane bodies, is twofold, interior and exterior; of which the former is more active, or ἐνεργητική [Greek: energētikē, "active/operative"], while the latter is subject to passion, ὁριστική [Greek: horistikē, "determinative"], or determinable, circumscribed at the surface. To the form of the stars, the ἐνεργητικ[?] [Greek: energētik[?], "the active power"], a question seems chiefly to pertain which Plato defended—taken, as it appears, from the doctrines of the ancient Egyptians—namely, that the stars, like the earth, are animate; indeed, that they are greater living creatures of the whole animate world, and therefore that the faculty of the motions which they have is implanted in them, as in the rest of living things. [Margin: *Plato in Timaeo.*] And although this opinion seems to some plainly absurd, to others it does not seem so much so—especially if the life of the stars is compared analogically with the life of other living creatures, to display the harmony of the universe, and especially with the microcosmic harmony, which in the hypotheses and motions of the planets is, marvelously, almost conspicuous.

For the rest, concerning the motive faculty implanted in the stars, we follow Plato finally to this extent: that although that faculty, most proper to motion, exists in all bodies, nevertheless nothing that is movable produces motion of itself plainly,

without the continual emanation and sustaining of some other immovable thing—just as one may gather this too from Aristotle's books *On the Heaven* and *On the Soul*, [Margin: *Arist. lib. 1 de Anima, cap. 3*, item *lib. 7 Phys., cap. 1.*] likewise from Plotinus, Iamblichus, Porphyry, and other philosophers. And if this is so, we gladly confess that the primary cause of the motion of the stars devolves upon God himself, as the omnipotent creator who, outside of all motion of himself, rules, moves, and governs all things to their salutary sustenance, [Margin: *Actor. (Acts) cap. 17, v. 25.*] and that by his omnipotent word, by which all things were created from the beginning. Furthermore, among the other faculties which luminously and without doubt emanate from the interior form of the great mundane bodies, this one comes here not in vain to be examined by us: namely, by what arrangement the parts of each of these bodies—homogeneous as a whole, yet varied and dispersed throughout its entire mass—so unanimously devote themselves, by a certain divine reason, to the preservation of the whole, that for this very reason, by a natural

[Margin: *Ovid in Metamorph.*]

force, by their own gravity, all the parts of each and every one are borne with all their strength toward their own middle or center.

Since therefore the middle, by reason of place, coincides with the center—the outermost parts of the greater mundane body being understood—it is evident that nature has intended in it no other exterior form or shape ὁριστικ[?] [Greek: horistik[?], "determinative"] than the round or globular; which astronomical experience further teaches in the Sun, the Moon, and the earth—the latter indeed manifold: in the eclipses, in the polar elevation with its proportionate and always equal terrestrial interval under the same meridian. Thus we shall not in the least be troubled by Franciscus Patricius the Venetian [Francesco Patrizi], or by certain ancient theologians—namely Augustine and Lactantius—who think differently in this part, since ignorance of astronomy sufficiently excuses these men. Moreover, with all the parts of the mundane body so neatly balanced around the middle, on account of the purpose we touched on before, it is fitting to reason well: namely, that not without singular divine providence are all those mundane bodies freely suspended and, as it were, held fast in the expanse, without props or solid orbs, no less than without axes of that kind, each in its own appointed places—as Ovid sang of the earth:

The earth, like a ball, resting on no support, hangs as so heavy a weight upon the underlying air. Etc.

Let the same judgment be made of the rest.

Then it is also made manifest from what precedes that nothing within the whole compass of any great mundane globe—compacted of diverse though kindred matter—is left void; lest either a tearing apart of the balanced equal parts around the middle should occur, or perhaps the demolition and destruction of the whole, from a heterogeneous element (which nature most abhors) being introduced, even the least bit. This is something that the valid and almost-beyond-our-grasp force of nature proves from experience, to such a degree that whenever we contrive a vacuum in the place of some appointed thing, nature at once supplies, if nothing else, at least air—as a body in our region most mobile and convenient for supplement; or, the moment air is exhausted, supplies water in its place, most suited to the temperament of each place, even in its own liquid—not to mention that the air thus serves the respiration of animals to the greatest degree. Besides, it is clear from the same source that only the nature of the great mundane bodies abhors this vacuum, not also the nature of the universe—since that celestial region which is seen to be empty of stars is content, for its repletion, with the expanse alone, and with the light or luminosity scattered through it; with which alone those bodies coeval with the world are by no means content—although intimately participating in them—lest a dissipation of them into the expanse should occur, and destruction follow. Unless perhaps to that Milky Way through the heaven some matter has been mingled, besides the congress of the light of the infinite stars set in it.

Finally, certain things further serve to prove the globular form of the mundane bodies, which the Mathematicians, chiefly N. Copernicus [Nicolaus Copernicus], *Revolutions* I, ch. 1, bring forward to demonstrate such a figure of the heaven: namely, that those bodies are globular, either because that form is the most perfect of all, needing no joining-together, being whole and entire; or because all things universally seek to be terminated—which appears in drops of water and in liquid bodies, as they desire to be terminated of themselves. But concerning the exterior form of the universe we here assert nothing, lest we deservedly incur the reproach of Pliny, [Margin: *Vide Plinium [Pliny] lib. 2, cap. 1, Nat. Hist.*] who reckons it madness to seek out what lies beyond the Heaven, and to scrutinize external things as if all the internal things of it were already plainly known.

This form, which we attribute to the stars and the earth as genuine to them, we judge to be most suited to motions—whether only a turning of the star above its center takes place, or a change of place without this (as appears in the Moon, and is credible to happen in the other planets too)—otherwise than Aristotle sets forth

concerning the Heaven from his hypothesis; [Margin: *Arist. lib. 2 de coelo, cap. 11.*] especially with all obstacles removed, which, however, if they existed, of the globe

would impede any such motion less than the form of any other body whatsoever.

These few points, in keeping with the analogy of time and motion — an analogy we observe to be general and perpetual in the revolutions of the stars — I have chosen to add at the close, drawn from physics. Other matters concerning these same things I have deferred to more suitable places, since they are more specialized and bear on the planets in their individual revolutions.

Since two things above all are subordinated to astronomical doctrine in general — beyond the moving bodies themselves that appear in the heavens — namely time and motion (which indeed cohere so closely that they cannot in any way be separated from each other), I have chosen to set out a brief preliminary account of them, because their consideration is required here before everything else.

Time has a twofold sense. In the general sense, it is the continuous duration of motions — indeed of all things — and in this way it is proper to metaphysical consideration. In the special sense, it is determined by motion, and chiefly by celestial phenomena, whether particular or periodic. To both these respects the description of time gathered and composed by the Platonists seems to apply (its genuine explanation remains within the mysteries of the circle): *Time is the flowing image, into numbers, of eternity abiding in unity.* [Margin: Plato, in the *Timaeus*] For the distinction of time into present, past, and future is supplied by *numbers numbered* (as they call them), that is, by the observation of those things that are apprehended through some motion or flux.

[Margin: Aristotle, *Physics* book V[?], ch. 7] Aristotle too touches on three kinds of motion. Of these, the one that occurs by change of place alone is the one that here comes into our consideration; this the Greeks call φοράν [Greek: phoran, "locomotion" — motion in respect of place], the Latins *latio* (carrying).

Now since it is proper to motion that — the magnitude in which it is accomplished being continuous — the thing moved should pass through infinitely many parts of that magnitude before it reaches the farthest point or its appointed terminus, there will therefore naturally be a certain proportion between the time and the space of such a magnitude as measured by the moving body. This holds even though in some appearances, owing to various causes — some of them accidental as well — the proportion is unequal.

With these assertions we have briefly comprised the second part of the prerequisites of astronomy that we set out to cover, so that hereafter we might enter on a more expeditious path toward demonstrating the motions themselves upon suitable suppositions, with nature (under God) as our guide. To whom, the one and triune God alone, be glory always.

A SHORT PREFACE

to the

BOOKS ON THE SPHERE

Since the spherical doctrine must be extended to the position of the heavens, and to determining generally every one of the phenomena in the heavens at any given time of the daily revolution — and since that position presents itself to us in varied ways, not only on account of the continuous apparent revolution of the heaven itself, but also on account of the contrary and oblique course of the stars and the varied inclination of the world's axis in different places — we therefore devote the first book of our *Spherics* to the consideration, distribution, and demonstration of the motions and their appearances according to their form. And since the celestial motions, for the said causes, appear in diverse ways — to account for all of which the circles were devised by the ancients — we shall accordingly discuss the necessity and sufficiency of each one of these circles from the definition and property of each. Then at last, upon these same circles, we shall build up our material sphere (as they call it), aptly designed for demonstration and use. This is the first aim, which the contemplation or theory of the spherical doctrine alone regards. Meanwhile no question is raised at present about the arrangement of the world, but that is kept fully in reserve for the second part of *Astronomy*. For the appearances of the primum mobile present themselves to us in one and the same way, whether the motion be assigned to the starry heaven or to the earth: that primary motion will be accommodated into the contrary directions either way.

Furthermore, since the chief end of this doctrine lies, as I said, in determining the disposition of the heaven and the stars — and that for any given time of the first (as they call it) or apparent conversion, inasmuch, that is, as the individual points of the celestial convex surface, or the phenomena, are bound to their own or to different circles, and are seen and understood to be carried from east to west — therefore, for the purpose of expressing everywhere, in a discrete manner, the

quantities of these motions and appearances, we have set apart the second book of the *Spherics*. In it we shall display the practice and use of our material sphere most clearly, showing how all the tables of the first motion may be built up, and how very many other problems that bear on this doctrine may be demonstrated upon the said sphere by the help of the solution of spherical triangles set forth beforehand. All these matters are worked out through various problems and adapted to practice, as will be more fully recounted at the beginning of the second book of the *Spherics*, where the order to be followed there is indicated.

THE FIRST BOOK

OF THE SPHERICS.

CHAPTER I.

On the definition, principles, and distribution of Astronomy, and on its general properties.

[Margin: Definition of Astronomy.] Astronomy is a special and most noble part of mathematics, which investigates and defines, for our use, the motions of the stars — both apparent and true — and likewise their positions, magnitudes, and mutual relations, at any time whatsoever.

Conversely, times are known from motions.

[Margin: Principles of Astronomy.] The principles which the astronomer uses as causes for the constitution of this discipline are either external (the more evident) or internal (the more hidden).

The external ones are those that consist in the sense of sight and stand, as it were, in the place of matter — namely the τηρήσεις [Greek: *tērēseis*, "observations" — watchings] or observations of celestial phenomena, gathered precisely and in their due places by the aid of instruments. These not only furnish certain extreme values, which serve in the practice of triangles and from which the motions are defined for the present, but, when compared with the ancient ones, they also test and correct the perennial hypotheses of the motions.

The internal or more hidden principles of astronomy reside in the efficient cause — the shaper of the hypotheses upon which the perennial motions of the stars

are explained. Human scrutiny will scarcely suffice to track down its nature and character — although here too nature has contrived nothing in vain.

Finally, the distribution of astronomy is most conveniently taken from its subject, motion, and from its general variety. Now motion is either primary — by which the whole heaven, and all the stars set within it, appear to be carried together from east to west, by a conversion that is certainly most swift (provided it be ascribed to the heaven, and not rather to the earth, according to our opinion), namely within 24 hours, about the poles of the world or of the equinoctial — or it is secondary, which belongs to the planets and is contrary and oblique to the former, while the said planets visibly tend from west to east against the primary motion, about the poles of the zodiac. And although that periodic motion differs in each of them, nevertheless all the planets follow, back and forth, the lead of the single Sun in this motion, and are carried beneath the broad circle of the zodiac, through the middle of which the path of the Sun is directed. Moreover, even the stars that are called fixed — which nonetheless appear to be carried by a most slow motion — are regulated, at whatever latitude from the zodiac across the whole face of the heaven, to this motion or circle of the Sun. From these things, thus briefly indicated, two parts of astronomy arise. The first treats, as I said, of the primary motion or of the moving body, called by the ancients the *spherical doctrine*. The latter contemplates separately the motion of the planets — which I have called the secondary — beneath the zodiac. And to account for the phenomena of these and to display them at each individual time, hypotheses are composed from geometrical sources; whence this doctrine, which deals with the secondary moving bodies, is called *theoretical*, just as the hypotheses are called *theory*.

CHAPTER II.

On the definition and division of the Spherical doctrine.

The Spherical doctrine is that which contemplates and defines the position of the celestial sphere and of the individual φαίνόμενα [Greek: phainomena, "appearances" — phenomena] in it, chiefly with respect to the apparent daily conversion.

To the Greeks "sphere" means the same as what the Latins call a "globe," namely a round and solid body. In the present case, however, it comprises the circles upon which the primary or apparent motion of any of the celestial φαινόμενων [Greek: phainomenōn, "appearances" — phenomena] is accounted for and defined.

The spherical doctrine has two parts. The one, from the contemplation of the apparent motions, composes the sphere as a general instrument for defining the said motions. The other resolves the particular appearances, by geometrical reasoning, into numbers, and applies them to use. In the former, then, theory is more conspicuous; in the latter, practice.

CHAPTER III.

On the forms of the motions, to which the material sphere is legitimately fitted, and on the division of the sphere herein.

Since the prior part of the spherical doctrine requires a general knowledge of the apparent motions, therefore — while we briefly set out the contemplation of these — we conveniently reduce to appearance the daily motion (which perhaps is more correctly attributed to the earth), and, along with the common crowd, transfer it to the heaven, in which two distinct motions appear.

The first is the one from east to west, about the axis of the world, common to the whole heaven. And since the heaven is perpetually divided by our sight into two hemispheres — the upper, namely, which is visible, and the lower, which is invisible — it happens that, as one shifts the places of the earth chiefly toward north and south, a variation arises in marking out the said celestial hemispheres. This significantly alters almost the whole face of the heaven, both as regards the rising and setting of the stars and the other φαίνόμενα [Greek: phainomena, "appearances" — phenomena] of that face, and varies it on account of our sight and the interposed object of the earth. Hence the sphere is divided into the *right* and the *oblique*, and the right finally into the *vertical* and the *parallel*.

The other motion, which belongs to the secondary moving bodies, is attributed primarily to the Sun as the leader of the remaining planets. It appears with a contrary and oblique aspect, about another axis and other vertices; and at any place on earth it is equal — as we shall show further in the special or second part of *Astronomy*. And so much, briefly, for the consideration of the celestial motions, upon which the material sphere is founded. There follows the composition of the sphere itself.

CHAPTER IV.

On the origin, necessity, and definition of the circles, from which the material sphere is rightly composed.

Every apparent motion that is uniform and circular must be shown and defined by the astronomer in terms of a certain circle, lying everywhere at an equal distance between each of its two poles in the sphere. This circle, for that reason — and because it rests upon the center of the sphere on every side — is called a *great circle*. Now since the individual points of the entire spherical surface, comprised between each of the said poles, must be referred distinctly to this circle, a further circle must be added: one that grazes both poles of the first circle, and so, by its transverse revolution between the said poles, sweeps over the whole surface of the sphere upon a fixed diameter. The purpose is that a point — or any φαίνόμενον [Greek: phainomenon, "appearance" — phenomenon] — possessing some latitude from the prior circle, in terms of which the motion is defined, should be bound and determined to this latter circle, while it falls everywhere upon it at right angles. This second circle, under a similar condition with the first, turns out to be a great circle of the sphere — namely the *circle of latitude*, in general — just as the prior circle was named the *circle of longitude*, which we, however, call the *transverse*.

From these few premises we shall fit together a sphere sufficient for accounting for the apparent celestial motions, in this manner. To...

To express and determine the first motion of the heaven, let us first of all imagine a circle of longitude to be described upon the poles of the world.

This first circle arises as the standard and measure of the first revolution and of all times within it — that is, a circle which by its equal arcs measures off equal stretches of those times.

Such a circle was called by the ancients the equator or equinoctial, because the Sun standing beneath it makes the day equal to the night for all who inhabit the earth.

The circle transverse to this — the circle of latitude — passes through the poles of the world and therefore meets the predefined equator at right angles. It is called the circle of declination, because it demonstrates the declination of individual points and determines their position relative to the equator.

For the other motion of the heavenly bodies — the φαῖνομένων [Greek: phainomenōn, "of the appearances/phenomena"] motion, which we said above is contrary and oblique to the first, and which we regulate against the apparent motion of the Sun — we appoint a second imaginary circle of motion, likewise described upon its own proper poles. With respect to the apparent course of the Sun alone, this circle is usually called the Ecliptic (the "path") [Margin: *Lib. 1. Theor. cap. 9*], because when either luminary is visibly placed beneath it, one of them must undergo an eclipse — the Sun at conjunction, the Moon at the opposition of these same bodies. By contrast, the same circle is called the Zodiac whenever we wish to take in together the latitudes which the planets mark out on either side as they depart from the Ecliptic, as well as the twelve animal-figures [asterisms] arranged along it.

Now the greatest obliquity of the Ecliptic from the equator, since today it is found to be 23 degrees 31½ minutes, will also be the amount by which each of its two poles is removed from the poles of the world — seeing that every pole stands distant from its own circle on the sphere by a quadrant of a great circle. Because this is a great circle of the sphere, it therefore intersects the equator in two opposite places. The Sun standing beneath these marks the spring equinox in ascending and the autumn equinox in descending — just as it marks the solstices at the greatest limits of the departure of either body from the equator, which occur at the quadrant points.

The circle transverse to the Ecliptic — passing through its poles, and for that reason meeting it at right angles in whatever transverse revolution — may by a special name be called the circle of latitude, since by an arc of whatever size it demonstrates the latitude of any point of the sphere standing on either side at a distance from the Ecliptic and its poles, and defines that point's place relative to the Ecliptic.

So far we have shown the two heavenly motions by means of these two pairs of circles. But for the appearance of the heavenly bodies on the earth, with respect to each hemisphere, we appoint a great circle which from its Greek origin it is proper to call the Horizon, and in Latin the *Finitor* (the "bounder"). The Horizon alone seems to fall within the range of the eye, since, as we said, it divides the visible part of the heaven equally from the invisible. Its poles always stand directly above and below the vertical point [zenith], and are therefore removed from the poles of the world by exactly as much as the equator is at its greatest distance from the Horizon. From this we gather that when the equator is united with the Horizon, the poles of each circle are likewise united, and the sphere in

that position is called parallel — or the contrary of the right sphere — to such a degree that it belongs to the same genus as the right sphere. For when the equator climbs to the vertical point, the sphere is called right. At the eastern edge of the Horizon the risings of the stars take place; at the western, the settings.

The circle transverse to this [the Horizon] is called the vertical or altitude circle, which ascending and descending directly to the poles of the Horizon on either side, demonstrates the altitude and depression of each point on the Horizon and refers it to its own measure. Whenever this circle passes through the poles of the world at the same time, it is called the Meridian. When a star or point of the heaven crosses it (which happens twice in any daily revolution), if above the Arctic pole of the world, it is said by us to be at its greatest or highest meridian altitude; but if below, at its least or lowest — and the same is to be understood of the Antarctic pole.

And thus far enough has been said in general about almost all the circles, and about their sufficiency for demonstrating the heavenly motions and the appearances of these motions.

Now as for the circle of position, which astrologers use chiefly to construct the celestial houses and to carry out directions: it is nothing other than a movable Horizon set up upon poles at the points of intersection of the true Horizon and the meridian, through which the φαίνόμενα [Greek: phainomena, "appearances/phenomena"] are understood to ascend and descend in any position of the revolution whatever.

To this standard of the apparent heavenly motions we have fashioned a conformable material sphere, in which alone the use of spherical doctrine will henceforth be displayed. The circles are divided into 360 equal degrees; each degree again into 60 minutes [Margin: *On the division of the Circles see above, in the Prognostica, p. 1*]; and finally each minute into 60 seconds. Moreover, the Zodiac is divided into twelve signs, so that 30 degrees fall to each of the signs. These the Greeks call δωδεκατημόρια [Greek: dōdekatēmoria, "twelfth-parts"], that is, the twelve parts; but in the Zodiac they are called "animals," and they are twelve:

♈ Aries, ♉ Taurus, ♊ Gemini, ♋ Cancer, ♌ Leo, ♍ Virgo, ♎ Libra, ♏ Scorpius, ♐ Sagittarius, ♑ Capricornus, ♒ Aquarius, ♓ Pisces.

These are opposed to one another as follows [Margin: *See below, ch. 6, the diagram of the aspects*]:

		Opposed to			Opposed to
♈ Aries		♎ Libra	♋ Cancer		♏ Capricorn
♉ Taurus	resp.	♏ Scorpio	♌ Leo	resp.	♐ Aquarius
♊ Gemini		♐ Sagittarius	♍ Virgo		♉ Pisces

Pattern of a proper celestial armillary sphere.

Diagram. An armillary sphere shown in oblique view, with a small terrestrial globe at the center. The outer ring is the Meridian, holding the whole sphere "in its proper form." Within it the equator, the zodiac/ecliptic (the broad banded ring carrying the zodiacal signs), the horizon, and the several transverse circles cross one another, with their poles marked. The named circles are listed in the legend printed beneath the figure (*Nomina Circulorum*, transcribed below). The readable point-labels on the figure, by position:

- G (top, on the meridian ring): pole / meridian point
- H (upper right, on the meridian ring): pole / meridian point
- C (upper center): solstitial colure point
- D (upper center-right): equator point
- B (upper left, on the banded zodiac ring): zodiac point
- A (left of center): equator point
- P (left, upper): circle-of-position point
- N (right): pole of the zodiac
- E (center-right, near the globe): transverse-latitude point
- K (far left, on the meridian): meridian point
- I (far right, on the meridian): meridian point
- F (left of center, below A): horizon point
- F (right, lower): horizon point
- O (lower left): pole of the zodiac
- E (lower left): transverse-latitude point
- L (bottom left, on the meridian): meridian point
- M (bottom, on the meridian): pole / meridian point
- A (lower center, on the banded zodiac ring): equator point
- P (lower center): circle-of-position point
- B (lower center, on the banded zodiac ring): zodiac point
- D, C (bottom center): solstitial-colure points

The zodiac band carries small zodiacal-sign glyphs (♍ Virgo and others are legible on the band); these are the engraved sign-markers of the ecliptic/zodiac ring rather than separate point-labels.

Names of the Circles.

L K G H I — the Meridian, which here holds the sphere in its proper form.

A A — the Equator, whose poles are H, L, together with the transverse circle of declination D D, and the solstitial colure O A C.

B B — the Zodiac, whose poles are O, N, together with the transverse circle of latitude E E.

F F — the Horizon, whose poles are G, M, together with the transverse circle of altitude C C.

P P — the circle of position.

Now among these circles — since the transverse ones are movable, and are accordingly disposed in due order with the rest as the heaven revolves — triangles spontaneously arise, whose analysis is to be sought in Book 2 of the Spherics.

CHAPTER V.

On the uses of the Circles considered individually as distinguished from one another, and on their definitions.

That there are three circles of longitude, and as many of latitude, in our sphere — which demonstrate to us the heavenly motions and appearances — has been shown abundantly in the preceding chapter. But to distinguish the uses of the several circles from one another, we shall keep the names imposed on them by the ancients.

To the equator, then, are assigned both right ascension and oblique ascension and descension — just as declination is assigned to its transverse circle.

Right ascension is the point on the equator of any given point or heavenly body, determined upon the equator through that body's transverse circle. It is reckoned by the arc of the equator lying between the spring equinox and the point so defined on the equator. It was called "right" ascension by the ancients because in the right sphere it ascends at right angles together with the Ecliptic, or with any

heavenly point whose eastern rising is determined on the Horizon. To it right descension is the contrary.

Declination is the shortest distance of a given heavenly point from the equator on the sphere, which is therefore to be measured on the transverse circle of the equinoctial — that is, on the arc of its quadrant lying between the equator and the pole of the world.

Such a point, when it inclines toward the North, is said to have northern declination; when toward the South, southern declination.

Oblique ascension and descension is the arc of the equator of the oblique sphere which ascends and descends together with any given heavenly point on the Horizon, and which is likewise reckoned from the spring equinox.

The difference of ascension or of descension is the portion of the equator intercepted between the right ascension of some point and the oblique ascension and descension of the given heavenly point. For if the given heavenly point inclines toward the North, this difference, subtracted from the right ascension, leaves the oblique ascension at rising. But if the given point inclines toward the South, the same is accomplished by the difference of ascension added to the right ascension. Oblique descension behaves in the contrary manner. Likewise the semidiurnal arc is reckoned along the equator, which is the measure of the arc of right ascension between the rising and the setting of any φαινόμενον [Greek: phainomenou, "appearing body/phenomenon"], and especially of the Sun in the Ecliptic.

The names "longitude" and "latitude" we assign, with the ancients, to the Ecliptic alone and to its transverse circle. Longitude is the arc of any assumed heavenly point in the Ecliptic, determined through the transverse Zodiacal circle, which arc is likewise reckoned from the intersection of the spring equinox.

Latitude is the shortest distance of that same assumed point from the Ecliptic, which the arc of this transverse circle measures.

If that point inclines toward the North from the Ecliptic, its latitude is called northern; if toward the South, southern.

Finally, to the Horizon we ascribe the azimuthal degrees, and to its transverse circle — which is called the vertical circle — the altitudes of points or of the heavenly bodies; and this on the same principle as in the rest, making the beginning of the azimuthal degrees on the Horizon on either side from the meridian intersection.

And the altitudes we reckon by the vertical circle from the Horizon; whence it is plain that they occur either at the meridian or outside it.

Furthermore, on the Horizon, at the true rising and setting of the stars, the rising and setting amplitude is reckoned — namely the arc of the Horizon intercepted between the equator and the true rising or setting of the given heavenly point.

Finally, in the meridian vertical lies the elevation of the pole of the place, which distinguishes the magnitude of the oblique sphere from the right.

A certain peculiar rising and setting of the stars is also estimated along the Horizon, with respect to the points of the Ecliptic in which the Sun stands. And this rising and setting is twofold: true and apparent — the true again twofold: morning and evening.

The morning or cosmic rising of the stars is said of those which rise together with the Sun. The morning setting is of stars going down together with the point of the Ecliptic contrary to that with which the Sun rises in the morning.

The evening or acronychal rising is of stars rising together with the point contrary to that with which the Sun sets in the evening. The evening setting is of stars passing beneath the Horizon together with the Sun in the evening.

The apparent or heliacal rising and setting, finally, is that which regards the rays of the Sun standing at a fixed measure below the Horizon. For since the motion of the Sun in the Zodiac outstrips in its course both the fixed stars and the superior planets, while with the rest — namely Mercury and Venus — it behaves in a variable manner, it comes about that, according to the magnitude of the stars and their disposition relative to the Zodiac, the Sun now takes their sight from us, now by its own advance restores them to view. Therefore the heliacal rising is nothing other than the appearance of a star before the rising of the Sun, as it then emerges from his rays. The heliacal setting, on the contrary, is the disappearance of a star after the setting of the Sun, when it is first entangled in his rays. The distance of a star from the Sun on either side, at heliacal rising and setting, is reckoned on the vertical circle, between the Horizon — in which the star is found at its true rising or setting — and the Sun depressed beneath it to its just distance, so that if the star is of the first magnitude, that arc will be 12 degrees; if of the second, 13 degrees; if of the third, 14 degrees; and so on in succession. The method of finding such an arc we shall explain in the second part of the Spherical doctrine.

CHAPTER VI.

On the Aspects of the Stars.

From the rational division of the circle — and chiefly of the Zodiac — the aspect, that is, a certain mutual relation of the stars, is considered, by which they can communicate the influx of their rays and their power in acting, chiefly upon things beneath the Moon. Although the consideration of this matter has been the principal concern of the astrologers, nevertheless, since it falls within the spherical treatment — as does also the office of the circle of position, of which we made mention before, which is bound up almost inseparably with the aspects — therefore in this chapter we shall speak briefly of the aspects, and in the next of the use of the circle of position, as far as theory requires.

Now the aspects are of two kinds: general, and fixed or special.

The general aspects, which regard any parts of the Zodiac whatever, are commonly five.

1. Conjunction, when the stars are joined in the same parts of the Zodiac; its sign, for the sake of brevity, is written thus: σ
2. Opposition, when the stars are found in opposite points of the Zodiac; its sign is this: $\oslash$
3. Sextile, when the stars are placed apart from one another by a sixth part of the Zodiac, or two signs, that is, 60 degrees; its mark is this: $\ast$
4. The **quartile** aspect [quadratus adspectus] is said of stars positioned a quarter of the Zodiac apart from one another; its sign is this: $\square$
5. Finally, the **trine** aspect [trinus adspectus] is that of stars a third part of the Zodiac apart, that is, 120 degrees distant from one another; its sign is this: $\triangle$

These aspects are called **right** [dextri] when they are cast in the direction of the succession of the signs, and **left** [sinistri] when cast against that succession — as is set before the eye in the accompanying diagram.

The **special** or **fixed** aspects are the *antiscia* of the planets or stars, which arise from the equal distance of those bodies from some tropical point — that is, from Cancer or Capricorn. They are either primary or secondary.

The **primary antiscia** [primariae ἀντίσκια] are points in the Zodiac that have the same distance from the equator on the same side, such as the beginning of Gemini and the beginning of Leo; likewise the beginning of Taurus and of Virgo; the beginning of Aries and of Libra. So too in the southern signs: the beginning of Aquarius and of Sagittarius, the beginning of Pisces and of Scorpio — all of which are marked with this sign: ☾ [Sic in source — the "tail" node symbol used here for the antiscion mark; transcribed as printed]. The **secondary antiscia** [secundariae ἀντίσκια], by contrast — reckoned not as effective as the former — are those considered in signs that are in a sense opposite. For example, while a planet stands in the beginning of Gemini ♊ it obtains its primary antiscion in the beginning of Leo ♌, and its secondary, or opposite, antiscion in the sign equally removed from the same equinox in the contrary direction, namely in the beginning of Aquarius ♒. The mark of the secondary antiscion is (l).

So much, then, for the aspects. In them, two further things must be observed. First, that for assigning an aspect it suffices that the aspect arise even from the rays of a single star: thus, with a star placed at the position of Cancer, we say that its sextile is Virgo and Taurus; its quartile, Aries and Libra; its trine, Pisces and Scorpio. Second, since the aspects are referred to the ecliptic, the various latitudes of the stars accordingly alter these aspects, except for those that agree uniformly [□]. But concerning both kinds of aspect — which bear chiefly on the business of direction, and how they are to be properly reduced for the latitudes — we shall treat more conveniently in the second part of the spherical exposition. [Margin: *Lib. 2. sphæ. cap. 6. Problem. 3.*]

CAPVT VII.

On the use of the circle of position among the astrologers, in constructing the celestial houses and distinguishing them from one another, and likewise in carrying out directions.

The **circle of position** [circulus positionis], which earlier we called the movable horizon, when properly applied, accomplishes the chief practical work of astrology — the work required in the erection of the celestial houses and the direction of the *promissors* to the *significators* beyond the cardines of the world, that is, beyond the horizontal and meridian circles. Although the method of erection is varied in many ways according to the differing opinions of practitioners, nevertheless the one regulated by the motion of the *primum mobile* — which Johannes Regiomontanus [Regiomontanus] calls the *rational* method, and which we judge to have been chiefly used by Ptolemy and the ancients, and which is moreover the more widely received among most astrologers — we shall

here assign by way of general indication, together with the method of direction. [Margin: *Lib. 2. sphær. cap. 6.*] In our actual practice, however, we shall demonstrate specifically, through every requisite step, by means of worked examples, how it is adapted to the individual revolutions.

Once the right ascension of the midheaven (that is, on the meridian) is known from the given time, the equator is first divided by the intersections of the horizon and the meridian circle into four equal parts; then each of these sections is subdivided into three further equal parts. From this arise twelve equal parts of the equator, commensurate with the twelve celestial houses to be erected. But since the Zodiac is to be distributed according to a fixed rule by means of the equator and the circle of position — so that in each hemisphere six spaces or houses are left, beginning from the rising point toward the right beneath the earth — the horizon itself, by its intersection with the Zodiac, forms the first celestial house at the point of rising. Furthermore, if the circle of position is afterward depressed downward beneath the true horizon in our sphere, or raised upward above it, until it has reached the intermediate equal intersections of the equator, then this circle, no less than the horizon or the meridian, marks off by its intersection the cusps of the houses on the Zodiac itself. These houses are unequal on the Zodiac, owing to the oblique disposition of the Zodiac toward the equator. Of all of them, the six that are divided off by the meridian circle are first laid open in this way. The remaining six become known with no trouble from the opposite signs and degrees.

To **direct** the significator to its promissor is nothing other than to find the oblique ascension and descension of each, with respect to the circle of position lying beneath the significator (if the direction is made into the succession of the signs), or beneath the promissor (if it is made against that succession). For the difference of those oblique ascensions and descensions is called the **arc of direction** (or *Directorius*), and it measures the times of good or evil outcomes — taking, of course, for each year the quantity of the Sun's diurnal motion in the equator agreeing with the instant of time, as will subsequently be explained in practice.

Therefore, if the direction is made along the series of the signs — that is, into the succession, as I said — we first, on the surface of the sphere, track down by means of the circle of position, as a transversal, some planet as significator, and apply the said circle to it; and finally we mark the intersection it makes with the equator. This intersection on the equator shows the oblique ascension of the significator, with respect, of course, to the circle of position under which it lies. Then, with the circle of position remaining thus unmoved, let the sphere be

revolved until the promissor falls under the same circle of position; and let its intersection likewise be noted with the equator — which is called the oblique ascension of the promissor. Now, by however much this differs from the intersection, or from the oblique ascension of the significator (that is, by whatever the difference of the oblique ascension of both significator and promissor turns out to be), so great is the arc of direction sought. But if, on the contrary, the direction is undertaken against the series of the signs, that is, into the antecedence (as they say), then because the significator is changed into the promissor and vice versa — the promissor into the significator — everything happens in the contrary manner.

CAPVT VIII.

On the Zones, Climates, and Parallels.

Our material sphere excludes the circles called by others *lesser* circles, for this reason: that the arcs of the spherical triangles — in which the whole practice and use of spherical doctrine consists — exist as portions of the greatest circle alone, which each of them refers to in our sphere.

And since the said lesser circles — namely the two tropics, and besides them the Arctic and Antarctic — were devised for no other purpose than to mark off the limits arising from the effect of the Sun alone upon the earth, and were introduced by others; and since they impede the disposition of the triangles, which is to be constituted perpetually from the greatest circles in our sphere — they have been omitted by us. For the distinction and number of the zones, the consideration of the obliquity of the ecliptic from the primary equator suffices, together with the proper division of the spherical surface that follows.

First, the space that the angle of the λόξωσις [Greek: λόξωσις (loxōsis, "obliquity" — the inclination of the Sun's path)] — that is, of the Sun's obliquity — makes up on either side is called the **Torrid Zone** [Zona Torrida], because the Sun twice in the year grazes the zenith of those dwelling within it. And since the Sun's obliquity is today found to be very nearly 23 degrees 31½ minutes, the breadth of the Torrid Zone will be 47 degrees 3 minutes.

Next, since the Sun, holding either point of the tropics, plainly ceases to be seen by those who are removed a quadrant from the said point toward the contrary region of the world — therefore the space of the **Temperate Zone** [Zona Temperata], extending on either side from the equator, makes up the

complement to a quadrant of the angle of the double obliquity of the ecliptic, today amounting to 42 degrees 57 minutes.

Finally, since the half of this complement, extended to each of the poles — namely 23 degrees 31½ minutes — constitutes the several **Frigid Zones** [Frigidae Zonae] on either side. This is the distribution of the zones, which Maro [Virgil], attributing it to heaven itself, or rather (by metonymy) to the motion of the Sun that produces it, describes thus:

Five zones hold the heavens, of which one, glowing, is ever red with the Sun and ever scorched by fire; around it the two outermost, to right and to left, are drawn, set fast with dark-blue ice and with black rains. Between these and the middle one, two were granted by the gift of the gods to wretched mortals; and a path was cut through both, where the slanting order of the signs might wheel. [Virgil, *Georgics* I.233–239]

Furthermore, in the distinction and number of the **climates** [climatum], the ancients — so that they might know the elevations of the pole even in more remote places, from the common reckoning, by the length of the longest day — fixed, as it were, a boundary for each climate at the space on either side of the equator that differs, in the said longest day's length, by a half-hour's variation of quantity; nor did they extend these on either side of the equator beyond the Torrid Zones.

The **parallels** [parallelos], moreover, they made the half-quantities of the climates, as these things will be more conveniently shown below in their proper place. [Margin: *Lib. 2. sphær. cap. penultimo.*]

So much for the theoretical part of spherical doctrine.

LIBER SECVNDVS.

SPHÆRICORVM.

CAPVT I.

On the matter and order of the problems to be treated through the second Book of the Spherics. Then concerning those things which are comprehended within the triangle intercepted by the Ecliptic, the Equator, and the arc of Declination.

So far the composition of our sphere has been set forth, together with the use of the circles upon which the motions of the first revolution are demonstrated.

There follows the *analysis* [ἀνάλυσις, Greek: analysis] of those same motions in particular — the determination, in numbers, of any postulated quantity by a sure and geometric method. This practice contains the kernel and the use of the whole spherical doctrine, through the following problems, in the order that follows.

The problems — truly golden ones, of which this Book consists — we assign to three classes. In the first we treat:

First, of determining those stars or celestial *phenomena* [φαινόμενοις, Greek: phainomenois] which, in any disposition of the sphere, whether right or oblique, behave in the same way with respect to the ecliptic and the equator — namely those that require the reciprocal interchange between these circles and their transversal circles, that is, of latitude and declination. From this, either the longitude alone given (as of the Sun on the ecliptic), or together with some latitude (as off the ecliptic), is converted into right ascension and declination, and vice versa, the terms of these circles being also transmuted and alternated wherever there is need, so that they may serve the following problems. To these too must be joined the method of investigating the spherical distances of the stars.

Second, we weigh those motions which have regard to the horizon in any particular position of the sphere, where manifold problems most worthy of knowledge are treated thoroughly: concerning the length of day and night, concerning the rising and setting of the stars; and finally concerning the reciprocal finding of times through the diurnal apparent revolution of the Sun and stars.

Third and last in this class, we treat those things that concern astrological practice: in the construction of the celestial houses and in establishing the direction of the planets — just as in chapter 6 of the preceding Book we also set forth these matters for consideration.

In all these matters which are encompassed by this first class, the sources of the tables of the *primum mobile* — constructed by that most distinguished mathematician of old, Johannes Regiomontanus [Regiomontanus], or by other practitioners — are opened up, and their foundations are set before the eyes of learners in the triangular disposition and solution; so that in tables of this kind, to be inserted in their fitting places, they may engage with greater pleasure, and may draw from them whatever will be of use to them, if at any time it should please them to forgo the labor of analyzing the triangles.

Into the second class are relegated whatever pertains to the observations of the stars by means of mathematical instruments. Here:

First, inquiry must be made concerning almost every kind of astronomical instrument, especially those founded — and as it were fabricated — in our sphere as rivals of the roundness of the heavens; and it must be shown which of them excel the rest.

Diagram. A circular schema of the Zodiac illustrating the astrological aspects (right and left) and the antiscia. The twelve zodiacal signs are labeled around the circumference. A network of straight chords connects the sign-points, marked at their crossings with the aspect glyphs (sextile *, quartile □, trine △), showing the geometric relations among the signs. At the center is a small radiating figure of short lines with small circles at their tips (representing the rays of a star cast toward the aspect points). The signs are arranged around the circle as follows:

- ♋ Cancer (top)
- ♊ Gemini (upper right)
- ♉ Taurus (right-upper)
- ♈ Aries (right)
- ♊ Pisces (lower right)
- ♒ Aquarius (bottom-right)
- ♑ Capricorn (bottom)
- ♏ Sagittarius (bottom-left)
- ♍ Scorpio (lower left)
- ♎ Libra (left)
- ♍ Virgo (upper left)
- ♌ Leo (top-left)

Aspect glyphs (△ trine, □ quartile, * sextile) appear at the chord intersections within the circle, marking the angular relations; the central radiating star-figure shows the rays projected from a central body to the aspect points.

Second, after the careful and many-sided investigation of the meridian line and of the latitude of a place on land and sea, we will treat the various ways of making fresh observations of the stars and of referring them — as to a standard circle — to the ecliptic; and we will confirm, by sure and demonstrated reasoning, the certainty that this method has over the others.

Third, we undertake to treat the parallaxes of the nearer bodies and the refractions of the lower ones, the observed appearances, and their true positions

(where these are sometimes entangled), and the disentangling of all this in longitude and latitude with respect to the ecliptic. This subject is not only necessary to the doctrine of solar eclipses; it also yields a most beautiful insight, in the case of parallaxes, into the elevation or distance of the planets, and likewise of new φαίνόμενα [Greek: phainomena, "appearances" — i.e. new phenomena] from the earth. From this their true magnitude is afterward distinguished by us from their distance. In the case of refractions and twilights, on the other hand, the rising of earthly vapors is judged, and the duration of each twilight — morning as well as evening — in the several horizons is determined and defined.

Finally we assign to the third, or last, class whatever concerns the earth itself. The plan of arrangement will be as follows.

First — since the doctrine of gnomonics is nearest to the celestial and sheds light on the rest — we accordingly pursue it first in this class, so far as is needful: namely, the projection of the shadow by a stylus suitably erected and exposed to the rays of the luminaries. From this, sundials [sciaterica] of every kind of declination and inclination can be skillfully constructed; and this by a certain new method of our own, peculiar to us and quite compendious, working through triangles.

Second, we speak of those matters that we left to be considered in the last chapter of the preceding book: namely, what is impressed upon the earth by the effect of the sun, owing to the varying inclination of the world's axis toward north and south, and to the oblique course of the sun. From this arises the doctrine of zones, climates, and parallels, and likewise the explanation of the various names of those peoples who inhabit the various and opposite places of the earth.

Third and last, we teach those things that serve for recognizing particular sites on the terrestrial globe. Here, first, the longitudes and latitudes of distant places are investigated by us by various methods. Then we show how those same data are resolved geometrically into distances. This practice, although it differs little or not at all from the method of measuring the intervals of the stars, is nevertheless — by its use and by the pleasure it gives — most highly commended among all cultivated people, and especially among historians.

PROBLEM I.

Given the maximum obliquity of the ecliptic from the equator, together with the longitude from one or other of the equinoxes: the declination corresponding to such a longitude on the ecliptic is given.

[Margin: CLASS I. SPHERICS. M.A. begins here.]

Since the obliquity of the ecliptic is variable by about $1/3$ of one degree — at the present time gradually increasing again, as will be demonstrated in the second part of the *Astronomy* — those would seem to act rightly who have now increased, by half a minute, that obliquity which Lord Tycho Brahe found at Hven 30 years ago to be $23^{\circ} 31\frac{1}{2}'$, adapting it as it were to the year of Christ 1630, to which time (and to the nearest years) those apply themselves who set about constructing tables of the *primum mobile*, above and below, for the year 1660 from the Savior's birth into the world — tables that will hold, even around the rising and setting of the fixed stars, without sensible error; which we too, for the most part, do in the examples set out. Meanwhile, however, the tables that were earlier drawn up by Lord Tycho and by Johannes Regiomontanus, retained by us in their own διδόμενα [Greek: didomena, "the things given" — the given data], are inserted into this work.

Now, so that the solution of the present problem may be made, let there be described in the following diagram, upon center A, the meridian D H K, coinciding with the colures of the solstices; and let the remaining circles be halved from our sphere. Among these the equator is described, marked D B H, from the pole F; and the ecliptic is figured by the letters C B G, from the pole E. And since the intersection of the said circles appears at B, it is clear that at that same point the beginning of Aries — that is, the vernal equinox — is to be understood. Now let the place of the sun be taken at point N on the ecliptic, whose arc of declination is represented by N L, namely on the circle of declination of our sphere, drawn between each pole of the equator, F and K. Let N be the place of the sun, removed from the vernal equinox B (in this example) by the arc B N, so that the point N obtains the beginning of Gemini, and therefore B N is 60° . The angle N B L is the same as the distance of the poles E F, namely $23^{\circ} 32'$; and finally the angle at L is right. Accordingly, when the declination of the sun N L comes to be sought, the διδόμενα [the given quantities] are set before the eyes as follows.

Diagram. A circle (the halved sphere) with the meridian/solstitial-colure as its bounding circle. Center A. The equator runs as arc D B H and the ecliptic as arc C

B G, crossing at B (the vernal equinox, beginning of Aries). N is the sun's place on the ecliptic; N L is the arc of declination dropped to the equator at L; M lies near L on the figure. F and K are the poles of the equator (F above, K at the bottom); E is the pole of the ecliptic (upper left).

- A (center): center of the sphere
- B (lower middle): intersection of equator and ecliptic — vernal equinox / beginning of Aries
- C (lower left): endpoint of the ecliptic arc C B G
- D (left): endpoint of equator arc D B H
- E (upper left): pole of the ecliptic
- F (top): pole of the equator
- G (upper right): endpoint of the ecliptic arc
- H (right): endpoint of the equator arc
- K (bottom): pole of the equator (opposite F)
- L (center, just below N): foot of the declination arc on the equator
- M (right of L): point on the equator/declination near L
- N (center, above L): place of the sun on the ecliptic

The given quantities:

		value	
Given [διδόμενα]	B N	60° 0'	distance of the sun from the vernal equinox
	N B	23° 32'	the angle of maximum obliquity
	L		
	B L	90°	the right angle
	N		

Therefore N L is given as 20° 13' 48", the sought declination of the sun.

From this single example it is easily understood how the following table of the sun's declination — for each degree of the ecliptic, and likewise in its remaining quadrants — is composed from the doctrine of triangles. For points on opposite circles, which are equally distant from the equinoxes or the solstices, since they underlie the same calculation, are also worked out as quantities sought similar to these. This consideration — rightly and easily demonstrated to the eye in our sphere — exhibits and completes the construction of nearly all the tables for the whole circle.

Table of the declination of the ecliptic, from the observations of Tycho, at maximum obliquity $23^{\circ} 31\frac{1}{2}'$.

The table is read in three double-columns. Each double-column serves four zodiacal signs: the top header gives the two signs read downward (degrees 0→30, left "gr." column), and the foot header gives the two signs read upward (degrees 30→0, right "gr." column). Headers: column 1 — top ♈ Aries / ♎ Libra, foot ♊ Pisces / ♍ Virgo; column 2 — top ♉ Taurus / ♏ Scorpio, foot ♒ Aquarius / ♌ Leo; column 3 — top ♊ Gemini / ♏ Sagittarius, foot ♐ Capricorn / ♋ Cancer.

gr. (↓)	♈ / ♎ : gr. mi. sec.	♉ / ♏ : gr. mi. sec.	♊ / ♏ : gr. mi. sec.	gr. (↑)
0	0 0 0	11 30 42	20 13 22	30
1	0 23 56	11 51 48	20 25 57	29
2	0 47 53	12 12 40	20 38 9	28
3	1 11 49	12 33 21	20 49 58	27
4	1 35 43	12 53 49	21 1 25	26
5	1 59 37	13 14 5	21 12 29	25
6	2 23 28	13 34 7	21 23 7	24
7	2 47 16	13 53 57	21 33 22	23
8	3 11 4	14 13 32	21 43 15	22
9	3 34 47	14 32 53	21 52 42	21
10	3 58 28	14 51 59	22 1 45	20
11	4 22 4	15 10 50	22 10 22	19
12	4 45 37	15 29 26	22 18 35	18
13	5 9 5	15 47 47	22 26 22	17
14	5 32 29	16 5 51	22 33 44	16
15	5 55 47	16 23 39	22 40 39	15
16	6 18 58	16 41 9	22 47 10	14
17	6 42 6	16 58 22	22 53 13	13
18	7 5 6	17 15 18	22 58 51	12
19	7 28 0	17 31 54	23 4 3	11
20	7 50 46	17 48 14	23 8 47	10
21	8 13 26	18 4 14	23 13 5	9

gr. (↓)	♈ / ♉ : gr. mi. sec.	♊ / ♋ : gr. mi. sec.	♌ / ♍ : gr. mi. sec.	gr. (↑)
22	8 35 58	18 19 57	23 16 56	8
23	8 58 20	18 35 18	23 20 20	7
24	9 20 34	18 50 21	23 23 18	6
25	9 42 41	19 5 4	23 25 48	5
26	10 4 38	19 19 26	23 27 51	4
27	10 26 24	19 33 27	23 29 27	3
28	10 48 2	19 47 7	23 30 35	2
29	11 9 27	20 0 26	23 31 17	1
30	11 30 42	20 13 22	23 31 30	0
gr. (↑)	♎ / ♏	♐ / ♑	♒ / ♓	gr.

PROBLEM II.

Given the obliquity of the ecliptic from the equator, and a longitude granted on it: the right ascension corresponding to such a longitude is given.

From the things we defined in the previous book, it is sufficiently clear that, in the preceding diagram, B L was the right ascension corresponding to the longitude B N on the ecliptic. Therefore, for the same arc B L, in the right triangle B L N, the given quantities are as before.

| | side B N | 60° 0' | | | Given [διδόμενα] | angle N B L | 23° 32' | | | angle B L N | 90°, right | |

Therefore B L is given as 57° 48', the sought right ascension.

For when the proportion is set up in the complements that appear in triangle N F G, there comes out, from the ἀνάλυσις [Greek: analysis, "resolution"] of the said triangle, the angle N F G of 32° 12', whose measure lies along the arc H L, namely the complement of B L.

This right ascension too, of one quadrant, obtained from the doctrine of triangles — which in the remaining degrees varies in nothing except the longitude from what was premised — suffices for extension to the whole remaining circle. How this is done we will here indicate briefly. First, the right ascension just sought, taken away from the semicircle, leaves the right ascension at the beginning of Leo: 122° 12'. But the same right ascension added to the semicircle makes up the

right ascension of the beginning of Sagittarius: $237^{\circ} 48'$; just as, for the beginning of Aquarius, when the right ascension set for the beginning of Leo is added to the semicircle, there are gathered together $302^{\circ} 12'$. By such a device the following table of right ascensions is composed.

Through the areal entry into this table, the point of the mid-heaven [M.C., medium coeli] of any star is given, whose right ascension has previously been given; from which you will find the transit of the 100 stars through the mid-heaven, computed below. The remaining use of the table is impressed here and there in what follows.

Table of Right Ascensions [Tabula Ascensionum Rectarum]

The table is entered by degree of the sign (column "grad."). The six paired sub-columns each carry a zodiac-sign heading and give the corresponding right ascension in degrees ("gr.") and minutes ("mi."/"min."). This first page covers the signs Aries (γ), Taurus (τ), Gemini (II), Cancer (♋), Leo (♌), and Virgo (♍).

g r a d.	γ Aries gr.	τ Taurus gr.	II Gemini gr.	♋ Cancer gr.	♌ Leo gr.	♍ Virgo gr.	
0	0	0 27	5 57	4 90	0 122	1 152	6
			4	8		2	
1	0	5 28	5 58	5 91	5 123	1 153	4
		5	1	1		4	
2	1	5 29	4 59	5 92	1 124	1 154	1
		0	9	3	1	6	
3	2	4 30	4 60	5 93	1 125	1 154	5
		5	6	6	6	9	8
4	3	4 31	4 61	5 94	2 126	2 155	5
		0	4	9	2	0	4
5	4	3 32	4 63	3 95	2 127	2 156	5
		5	2		7	2	1
6	5	3 33	4 64	6 96	3 128	2 157	4
		0	0		2	4	8
7	6	2 34	3 65	9 97	3 129	2 158	4
		5	8		8	5	4

g r a d.	♈ Aries gr.	m♉ i Taurus . gr.	m♊ i Gemini . gr.	m♋ i Cancer . gr.	m♌ i Leo . gr.	m♍ i Virgo . gr.	m i n
8	7	2 35 1	3 66 7	1 98 3	4 130 3	2 159 6	4 0
9	8	1 36 6	3 67 6	1 99 7	4 131 8	2 160 7	3 7
1 0	9	1 37 1	3 68 4	2 100 1	5 132 3	2 161 8	3 3
1 1	10	6 38	3 69 3	2 101 5	5 133 8	2 162 8	2 9
1 2	11	2 39	3 70 3	2 103 9	3 134	2 163 9	2 5
1 3	11	5 40 7	3 71 2	3 104 4	8 135	2 164 9	2 0
1 4	12	5 41 3	3 72 1	3 105 8	1 136 3	2 165 9	1 6
1 5	13	4 42 8	3 73 1	4 106 3	1 137 7	2 166 9	1 2
1 6	14	4 43 4	3 74 1	4 107 7	2 138 2	2 167 9	7
1 7	15	4 44 0	3 75 1	5 108 2	2 139 6	2 168 8	3
1 8	16	3 45 5	3 76 1	5 109 7	3 140 1	2 168 7	5 8
1 9	17	3 46 1	3 78 2	2 110	3 141 5	2 169 7	5 4
2 0	18	2 47 7	3 79 2	7 111	3 142 9	2 170 6	4 9
2 1	19	2 48 3	3 80 3	1 112 2	4 143 3	2 171 4	4 4
2 2	20	2 49 0	3 81 4	1 113 7	4 144 7	2 172 3	3 9

g r a d.	♈ Aries gr.	m♉ i Taurus . gr.	m♊ i Gemini . gr.	m♋ i Cancer . gr.	m♌ i Leo . gr.	m♍ i Virgo . gr.	m i n
2	21	1 50	3 82	2 114	5 145	2 173	3
3		6	5	2	1	2	5
2	22	1 51	3 83	2 115	5 146	2 174	3
4		2	6	8	4	0	0
2	23	9 52	3 84	3 116	5 147	1 175	2
5			8	3	7	8	5
2	24	6 53	4 85	3 118	1 148	1 176	2
6			0	8		6	0
2	25	2 54	4 86	4 119	4 149	1 177	1
7			1	4		4	5
2	25	5 55	4 87	4 120	7 150	1 178	1
8		9	4	9		1	0
2	26	5 56	4 88	5 121	9 151	9 179	5
9		7	6	5			
3	27	5 57	4 90	1 122	1 152	6 180	0
0		4	8		2		

Table of Right Ascensions [Tabula ascensionum rectorum], continued

This page continues the same table for the signs Libra (♎), Scorpio (♏), Sagittarius (♐), Capricorn (♑), Aquarius (♒), and Pisces (♓).

g r a d.	♎ Libra gr.	m♏ i Scorpio . gr.	m♐ i Sagittarius . gr.	m♑ i Capricor n gr.	m♒ i Aquarius . gr.	m♓ i Pisces . gr.	m i n
0	180	0 207	5 237	4 270	0 302	1 332	6
			4	8		2	
1	180	5 208	5 238	5 271	5 303	1 333	4
		5	1	1		4	
2	181	5 209	4 239	5 272	1 304	1 334	1

g r a d .	♎ Libra gr.	♏ i Scorpio gr.	♐ i Sagittarius gr.	♑ i Capricor n gr.	♒ i Aquarius gr.	♓ i Pisces gr.	m i .
		0	9	3	1	6	
3	182	4 210	4 240	5 273	1 305	1 334	5
		5	6	6	6	9	8
4	183	4 211	4 241	5 274	2 306	2 335	5
		0	4	9	2	0	5
5	184	3 212	4 243	3 275	2 307	2 336	5
		5	2		7	2	1
6	185	3 213	4 244	6 276	3 308	2 337	4
		0	0		2	4	8
7	186	2 214	3 245	9 277	3 309	2 338	4
		5	8		8	5	4
8	187	2 215	3 246	1 278	4 310	2 339	4
		1	7	3	3	6	0
9	188	1 216	3 247	1 279	4 311	2 340	3
		6	6	7	8	7	7
1	189	1 217	3 248	2 280	5 312	2 341	3
0		1	4	1	3	8	3
1	190	6 218	3 249	2 281	5 313	2 342	2
1			3	5	8	8	9
1	191	2 219	3 250	2 283	3 314	2 343	2
2			3	9		9	5
1	191	5 220	3 251	3 284	8 315	2 344	2
3		7	2	4		9	0
1	192	5 221	3 252	3 285	1 316	2 345	1
4		3	1	8	3	9	6
1	193	4 222	3 253	4 286	1 317	2 346	1
5		8	1	3	7	9	2
1	194	4 223	3 254	4 287	2 318	2 347	7
6		4	1	7	2	9	

g r a d .	♎ Libra gr.	♏ i Scorpio gr.	♐ i Sagittarius gr.	♑ i Capricor n gr.	♒ i Aquarius gr.	♓ i Pisces gr.	m i .
1 7	195	4 224 0	3 255 1	5 288 2	2 319 6	2 348 8	3
1 8	196	3 225 5	3 256 1	5 289 7	3 320 1	2 348 7	5 8
1 9	197	3 226 1	3 258 2	2 290	3 321 5	2 349 7	5 4
2 0	198	2 227 7	3 259 2	7 291	3 322 9	2 350 6	4 9
2 1	199	2 228 3	3 260 3	1 292 2	4 323 3	2 351 4	4 4
2 2	200	2 229 0	3 261 4	1 293 7	4 324 7	2 352 3	3 9
2 3	201	1 230 6	3 262 5	2 294 2	5 325 1	2 353 2	3 5
2 4	202	1 231 2	3 263 6	2 295 8	5 326 4	2 354 0	3 0
2 5	203	9 232	3 264 8	3 296 3	5 327 7	1 355 8	2 5
2 6	204	6 233	4 265 0	3 298 8	1 328	1 356 6	2 0
2 7	205	2 234	4 266 1	4 299 4	4 329	1 357 4	1 5
2 8	205	5 235 9	4 267 3	4 300 9	7 330	1 358 1	1 0
2 9	206	5 236 7	4 268 6	5 301 5	9 331	9 359	5
3 0	207	5 237 4	4 270 8	0 302	1 332 2	6 360	0

Problem III [PROBLEMA III]

Given the maximum obliquity of the ecliptic, together with the declination of the Sun: the place of the Sun in the ecliptic corresponding to that declination is found.

This problem, which is the converse of the first, is of great use in investigating the Sun's motion when its declination has been observed by means of the meridian altitude — provided one takes due care to eliminate parallax and refraction.

Example. [Exemplum]

In the preceding diagram, which we keep here as well, suppose the Sun's declination has been observed from the meridian altitude (how this is done will be set out clearly in the following class [classis]), or otherwise given, as $20^{\circ} 13' 48''$. This is indicated by the arc L N; and together with the obliquity of the ecliptic, in the usual notation [notis usitatis], the given quantities [διδόμενα (didomena, "the things given")] appear as follows:

- L N = $20^{\circ} 13' 48''$, the given declination.
- N B L = $23^{\circ} 32'$, the obliquity of the ecliptic.
- B L N = $90^{\circ} 0'$, the right angle.

Therefore B N = $60^{\circ} 0'$ is given, the longitude of the Sun from the vernal equinox.

So the same distance from the vernal equinox results as was previously assumed for the beginning of Gemini (II). In the remaining quadrants the procedure is the same, with this one consideration: whether the Sun's declination was northern or southern, and in which quadrant of the circle the Sun fell. The table of the Sun's declination can also serve the same purpose; for this reason we have retained it, computed to seconds, from Tycho Brahe's calculation.

Problem IV [PROBLEMA IV]

Given the maximum obliquity of the ecliptic, together with the longitude, the angle is found that the meridian — or any circle of declination — makes with the ecliptic.

This angle is represented in the preceding diagram by B N L. Finding it serves, in what follows, for investigating the rising of a point of the ecliptic, its inclination to the horizon, and finally the rising and setting amplitude [amplitudine ortiva et occidua] — an angular use of real value. In the present example, for finding the said angle, the given quantities [διδόμενα] appear as follows:

- L B N = 23° 32', the obliquity of the ecliptic.
- B N = 60° 0', the given longitude on the ecliptic.
- B L N = 90° 0', the right angle.

Therefore B N L = 77° 42' is given, the angle of intersection sought.

The rest is carried out on this example to construct the following table, which is invariable in each individual quadrant.

Table of the Angle of the Ecliptic with the Meridian (or Circle of Declination) [Tabula Anguli Eclipticae, cum Meridiano seu Circulo Declinationis]

This table presupposes the maximum declination [Decl. max.] of 23° 32'.

The leftmost column "gr. Ec." is the degree of the ecliptic, read downward 0→30; the rightmost column "grad." is the same index read upward 30→0, so each row serves two zodiac signs. Each of the three central degree/minute pairs is headed by two signs at the top of the page and by two further signs at the foot. The three pairs serve, reading from the top index: ♈ Aries / ♎ Libra; ♉ Taurus / ♏ Scorpio; ♊ Gemini / ♐ Sagittarius — and, read from the foot (bottom index, signs printed inverted at the base): ♋ Pisces / ♍ Virgo; ♒ Aquarius / ♌ Leo; ♐ Capricorn / ♋ Cancer respectively.

gr. Ec. (top)	♈♎ gr.	m i.	♉♏ gr.	m i.	♊♐ gr.	m i.	grad. (bottom)
0	66	2	69	2	77	4	30
		8		0		2	
1	66	2	69	3	78	4	29
		8		1			
2	66	2	69	4	78	2	28
		9		3		6	
3	66	3	69	5	78	4	27
		0		5		8	
4	66	3	70	8	79	1	26
		1				1	
5	66	3	70	2	79	3	25
		3		1		4	
6	66	3	70	3	79	5	24

gr. Ec. (top)	Υ _Δ gr.	m i.	ϝ _ℳ gr.	m i.	Π _ϝ gr.	m i.	grad. (bottom)
		5		5		7	
7	66	3	70	4	80	2	23
		8		9		0	
8	66	4	71	3	80	4	22
		1				3	
9	66	4	71	1	81	7	21
		4		8			
10	66	4	71	3	81	3	20
		7		3		1	
11	66	5	71	4	81	5	19
		1		8		5	
12	66	5	72	4	82	1	18
		5				9	
13	67	0	72	2	82	4	17
				0		4	
14	67	5	72	3	83	9	16
				6			
15	67	1	72	5	83	3	15
		1		3		4	
16	67	1	73	1	83	5	14
		7		0		9	
17	67	2	73	2	84	2	13
		4		7		4	
18	67	3	73	4	84	4	12
		1		5		9	
19	67	3	74	3	85	1	11
		8				4	
20	67	4	74	2	85	3	10
		5		1		9	
21	67	5	74	4	86	4	9
		3		0			
22	68	1	74	5	86	2	8
				9		9	

gr. Ec. (top)	Υ ς gr.	m i.	Ϡ Ϻ gr.	m i.	Π Ϡ gr.	m i.	grad. (bottom)
23	68	9	75	1	86	5	7
				8		9	
24	68	1	75	3	87	2	6
		8		8		0	
25	68	2	75	5	87	4	5
		8		8		6	
26	68	3	76	1	88	1	4
		8		8		2	
27	68	4	76	3	88	3	3
		8		9		9	
28	68	5	77	0	89	6	2
		8					
29	69	9	77	2	89	3	1
				1		3	
30	69	2	77	4	90	0	0
		0		2			
(bottom signs)	Ϡ Ϻ		≈ Ϻ		Υ ς		G.

PROBLEMA V.

Given the obliquity of the ecliptic together with a longitude, to find the arc of latitude intercepted between the ecliptic and the equator.

This latitude is shown in the diagram above by the arc N M. To find it, whenever its use is required, the given quantities [διδόμενα (didomena, "the data / things given")] in the worked example are as follows:

Given [διδόμενα]		degre es	minut es	
	B N	60	0	Longitude on the ecliptic.
	N B	23	32	Maximum obliquity.
	M			
	B N	90		A right angle.
	M			

Therefore the arc N M is given as 20° 40', the arc of latitude sought.

By this same method the arc of latitude can also be obtained for each individual point of the ecliptic, provided of course one takes account of an equal distance from some equinox or solstice — which likewise yields equal data [διδόμενα] — as was noted earlier.

CHAPTER II.

On the similar determination of those phenomena that diverge and lie at any latitude away from the ecliptic.

PROBLEMA I.

Given the latitude and longitude of some star, or of a point on the spherical surface, together with the maximum obliquity of the ecliptic, to find both its right ascension and its declination.

And conversely:

Given the right ascension and declination, to find from these the longitude and latitude.

Since the motions of the stars are nearly invariable in latitude — except for the planets — but regular in longitude and determinate at any time, they must repeatedly be considered in terms of declination and right ascension and applied to various uses in these matters. This problem therefore has very frequent application in practice, and we shall work it out most conveniently and concisely by means of oblique-angled triangles outside the ecliptic.

To explain the present problem, let there be in the diagram drawn here a star or point of the heavens whose longitude is given as the arc B N and whose latitude is N O. Thus in the oblique-angled triangle O E F the data [διδόμενα] are: E F, the distance between the poles; O E, the complement of the given latitude; and finally the angle F E O, which measures the arc of given longitude N G, here reckoned from the beginning of ♋ [Cancer] toward the leading signs. From this, first the side F O becomes known, giving the complement of the declination sought. Once this is had — since in the same triangle F E O all three sides are now evident — we bring into the open the angle E F O, which is to be measured by the arc of the equator D L from the solstitial colure D. When in this case a quadrant of the circle D B is taken from that arc, there remains the arc B L, which measures the right ascension sought of the star, that is, of the point at O.

Diagram. An orthographic projection of the sphere showing the construction for the present problem. The bounding circle is the projection of the sphere; great-circle arcs cross it. Readable labels:

- E (upper left, near top): pole point
- F (top): point on the upper circle
- G (upper right): point on the bounding circle
- O (interior, upper): the star or point whose coordinates are sought
- N (interior, just below O): point on the ecliptic giving the longitude
- A (center): center of the sphere
- D (left, on horizon line): solstitial colure point / equator endpoint
- L (interior, near center): foot of the declination arc
- M (interior, right of L): point on the ecliptic
- H (right, on horizon line): equator endpoint
- B (lower interior): point on the ecliptic
- C (lower left): point on the bounding circle
- I (bottom): point on the bounding circle

SPHÆRICORVM

Exemplum I.

In the diagram just set out, let O be the bright star above the head of ♈ [Aries], whose longitude for the completed year 1630 (the time to which we refer in all examples of this kind, for the reason explained above) is granted as $2^{\circ} 32'$, and whose latitude is $9^{\circ} 57'$ north. With these presupposed, we investigated the declination and right ascension of the said star, according to the demonstration set out above and our principles of triangles, in the triangle O E F, as follows.

I. For the Declination.

Given [διδόμενα]		degre es	minut es	
	E F	23	32	Distance between the poles.
	O E	80	3	Complement of the star's latitude.
	F E	57	28	Complement of the star's longitude from the vernal
	O			

Given [διδόμενα]	degree es	minut es	
			equinox.

Therefore F O is given as $68^{\circ} 17'$, whose complement O L — namely $21^{\circ} 43'$ — shows the declination of the bright star of Υ [Aries] that was sought.

II. For the Ascension.

In the same triangle O E F, the following are the data [διδόμενα]:

Given [διδόμενα]		degree s	minut es	
	E	23	32	Distance between the poles.
	F			
	F	68	17	Complement of the declination just found.
	O			
	E	80	3	Complement of the star's latitude.
	O			

Therefore the angle E F O is given as $116^{\circ} 39'$, an angle at the equator measured from the winter solstice, which is represented by the arc D L. So when a quadrant is subtracted from the said arc, the remainder — say $26^{\circ} 39'$ — gives the right ascension sought of the bright star of Υ [Aries].

Exemplum II.

Even to those moderately versed in spherics it is clear how, from this one example set out above, the present problem may be carried through the remaining quadrants of the circle. (In just this way a table was once constructed by Tycho's pupils [Brahe's alumnus] at Uraniborg [Vraniburgi] with great labor — but, to the equal loss of astronomy, it has not yet been brought to light — so that from it we might at once draw out either the longitude and latitude from the given right ascension and declination of the stars, or conversely the ascension and declination from the granted longitude and latitude.) Nevertheless, lest in a different case — where the latitude is north while it lies in southern signs — we should seem to fail the effort of learners, here is another example, in the bright star of the eastern shoulder of $\approx$ [Aquarius]. For the assumed year 1630 its longitude is given as $28^{\circ} 15'$ $\approx$ [Aquarius] and its latitude as $10^{\circ} 42' N$, to which this diagram from our sphere is suited.

With the rest remaining as before, let the place of the bright shoulder be at O, whose latitude N O is given and whose longitude is at N on the ecliptic. From these we seek first the declination L O, then L on the equator, which determines the right ascension of that same star.

Diagram. An orthographic projection of the sphere for Exemplum II. Readable labels:

- E (upper left): pole point
- F (top): point on the upper circle
- G (right): point on the bounding circle
- A (center): center of the sphere
- D (left, on horizon line): equator endpoint
- L (interior, left of center): foot of the declination arc
- O (lower interior): the star (eastern shoulder of Aquarius)
- N (interior, right of O): point on the ecliptic giving the longitude
- B (interior, right): point on the ecliptic
- H (right, on horizon line): equator endpoint
- C (lower left): point on the bounding circle
- M (lower interior, left): point on the bounding circle
- I (bottom right): point on the bounding circle
- K (bottom): point on the bounding circle

LIBER SECVNDVS.

I. For the Declination.

		degre es	minut es	
In the triangle O I K, the data [διδόμενα] are:	I K	23	32	Southern distance of the poles.
	I O	100	42	Sum of the latitude and a quadrant.
	O I K	58	15	Distance of the star from the beginning of ♑ [Capricorn],

which the arc C N on the ecliptic measures.

Therefore K O, the complement of the declination, is given as $87^{\circ} 55'$. And consequently the declination itself, sought as L O, is $2^{\circ} 5'$ south.

II. For the Right Ascension.

In the same triangle O I K already proposed, for the angle O K I the three following sides are the data [διδόμενα]:

	degrees	minutes	
K I	23	32	Distance between the poles.
K O	87	55	Complement of the declination.
I O	100	42	Latitude of the star with a quadrant added.

Therefore the angle O K I is given as $123^{\circ} 17'$, the angle that defines the arc H L from the summer solstice on the equator. When this is taken from the whole circle plus a quadrant — namely 450° — there remains the right ascension sought of the right shoulder [of ♑, Capricorn], $326^{\circ} 43'$.

PROBLEMA II.

To convert a given right ascension and declination into longitude and latitude.

This problem is the converse of the previous one, as is clear in the second-to-last diagram, recalled here. In the triangle E F O, for the complement of latitude E O:

The data [διδόμενα] are:		degrees	minutes	
	E F	23	32	Distance between the poles.
	F O	68	27 [recte: 17]	Complement of the declination of the bright star of ♈ [Aries].
	E F O	116	39	Distance of the bright star of ♈ on the equator from the winter solstice.

Therefore E O is given as $80^{\circ} 3'$, the complement of the latitude sought. And so O N — the latitude itself sought — is given as $9^{\circ} 57'$.

Furthermore, for the longitude of the said star at N on the ecliptic: since in the proposed triangle F E O all the sides are known — for they are:

The data [διδόμενα] are:	degree		minut	
	s		es	
E F	23	32		Distance between the poles.
E O	80	3		Complement of the star's latitude.
F O	68	17		Complement of the declination just found.

Therefore the angle F E O is given as $57^{\circ} 28'$, the angle of elongation on the ecliptic of the said star from the summer solstice, which the arc G N measures. When this arc, in this case, is taken from a quadrant of the circle, there remains the longitude sought of the star itself, $2^{\circ} 32' \propto$ [Taurus].

In a similar way the reciprocal procedure works out in the other cases. And the examples of these two problems are of all the most frequent.

PROBLEMA III.

Given the longitude together with the declination of a star, to hunt out its right ascension and latitude.

Keeping the same diagram (the second-to-last, that is), since in the triangle E F O:

The data [διδόμενα] are:	degree		minut	
		es	es	
E F	23	32		Distance between the poles.
F O	68	17		Complement of the declination.
F E O	57	28		Distance on the ecliptic from the tropic of ♋ [Cancer].

Therefore E O is given as $80^{\circ} 3'$, the complement of the latitude; and consequently the latitude itself is $9^{\circ} 57'$.

SPHÆRICORVM

Furthermore, for the right ascension: since in the same triangle F E O all three sides are now granted —

		degree s	minut es	
For they are [διδόμενα]:				
	E F	23	32	Distance between the poles.
	F O	68	17	Complement of the declination.
	E O	80	3	Complement of the latitude.

Therefore the angle E F O is given as $116^{\circ} 39'$, the distance of the equator from the winter solstice. When a quadrant D B is taken from this, there remains B L, the right ascension last sought, $26^{\circ} 39'$, as before.

PROBLEMA IV.

Given the latitude and declination, to obtain the longitude and right ascension.

This problem is of foremost use around the equinoctial points, where — the declination of a star being given by observation, and the latitude of the same being invariably retained — we wish to have the longitude or right ascension from the given quantities. In a triangle thus proposed, three sides are granted: two from the complements of the presupposed latitude and declination; the third from the distance between the poles, which stands available everywhere. But so that the analysis of spherical triangles set out above by us may become clearer in this problem, and more welcome from comparison, let us borrow an example from Copernicus, *De Revolutionibus* III.2. In Spica of Virgo, whose declination Copernicus [Copernicus] found at Frombork [Fruenburg] in Prussia [Borussia] in the year of the Lord 1525 to be $8^{\circ} 40'$ southern; and whose latitude the same author presupposed from the Hipparchian Canon of Stars [Canon stellarum Hipparchiano] to be $2^{\circ} 0'$ southern; and finally the maximum declination of the Sun, that is the distance between the poles, to be $23^{\circ} 28'$. With these data the following diagram has been arranged in conformity, according to the disposition of the circles in our sphere. For only the face of the sphere being reversed, the rest agree from the indication of the earlier letters; and since in the triangle K I O:

[Margin: *De Revolutionibus* III.2 [Lib. 3. Revolut. cap. 2]]

The data [διδόμενα] are:	degree		minut	
	s		es	
K	23	28		Distance between the poles.
I				
I	88	0		Complement of the latitude
O				of Spica.
K	81	20		Complement of the
O				declination of Spica.

Therefore the angle O I K is given as $72^{\circ} 39'$, which N C measures — the distance of the star from the winter tropic on the ecliptic. When this is taken from a quadrant of the circle, B N is given as $17^{\circ} 21'$, the longitude of the star from the autumnal equinox at B. It is therefore $17^{\circ} 21' \simeq$ [Libra], just as Copernicus found it.

[Margin: *De Revolutionibus* III.2 [Lib. 3. Revolut. cap. 2]]

By a similar device we hunt out the right ascension of the said star as well. For since the three presupposed sides of the triangle K I O are given, the angle O K I will not be hidden — the angle that demonstrates the right ascension on the equator H L from the summer solstice H. When a quadrant is added to it, the right ascension sought is made up.

Diagram. An orthographic projection of the sphere for the Spica example, with the face of the sphere reversed relative to the earlier figures. Marginal note beside it: "*De Revolutionibus* III.2." Readable labels:

- F (top): point on the upper circle
- E (upper right): pole point
- G (upper left): point on the bounding circle
- A (center): center of the sphere
- H (left, on horizon line): summer-solstice equator endpoint
- L (interior, right of center): foot of the ascension arc
- D (right, on horizon line): equator endpoint
- B (lower interior): point on the ecliptic
- O (lower interior, left of N): the star (Spica)
- N (lower interior, right of O): point on the ecliptic giving the longitude
- C (lower right): point on the bounding circle

- I (bottom left): point on the bounding circle
- K (bottom): point on the bounding circle

Liber II, Cap. II (continued)

Problem V.

Finally, given the longitude and the right ascension, to find the latitude and the declination.

In working out this problem, recall the third-from-last diagram together with its worked example involving the bright star of Aries [γ], whose longitude was given as $2^{\circ}32'$ γ and whose right ascension was $26^{\circ}39'$. Now in the triangle E F O, two angles are given (at E and at F) together with the side E F lying between them. From these we obtain both the side E O — the complement of the latitude — and the side F O — the complement of the declination of the star in question, which were the quantities proposed. The analysis prescribed everywhere, together with the practice of spherical triangles, carries this through, even where it becomes necessary to drop a perpendicular arc.

Problem VI.

Given the latitude and the declination, to determine the angle subtended by the distance between the poles — that is, the poles of the ecliptic and of the equator.

This problem is similar to the fourth. Here too three sides are given: the two that enclose the angle sought are obtained from the complements of the latitude and of the declination. The third side, which measures the arc of separation between the poles, lies opposite the angle sought, as can be shown in the diagram attached to the fourth problem: let the given quantities [$\delta\delta\acute{o}\mu\epsilon\nu\alpha$, Greek: *didomena*, "the things given"] be I O, K O, and I K, from which the angle at O is found.

This problem has a special application in investigating the rising and setting of the stars — a matter treated below in its proper place.

Problem VII.

Given two stars — whether by their latitudes together with the difference of their longitudes, or by their declinations together with the difference of their right ascensions — to find the distance of those stars from each other.

This problem is reckoned a distinguished one because of its very wide application, not only on the celestial sphere but also on the terrestrial globe. For the intervals between stars in the heavens and the distances between places on the earth correspond by the same procedure and the same resolution of a triangle. But we shall give examples for the earth in its proper place; for now we shall look up to the celestial stars.

It must be noted, however, that to obtain the differences of longitude or of right ascension, the longitudes — or the right ascensions — of both stars must be known. When these are subtracted one from the other, as the succession of the signs in their natural order requires, what remains is the said difference that must be assumed beforehand, as the following example will show.

Example.

Suppose the distance is sought between the bright star of Aries and Aldebaran — the bright eye [of Taurus, ♂]. Then, in accordance with the present problem, these are the given quantities:

	Longitude	Latitude
Bright star of Aries [♈]	2°6′ ♂	9°57′ N
Aldebaran	4°12′ ♉ [+ ♉]	5°31′ S
Difference of longitude	32°6′	

[Margin: For the year 1600.]

These being granted, let A in the adjoining diagram be the northern pole of the ecliptic, and D the southern pole of the same. Let the arcs A B C D and A E F D be halves of circles of latitude drawn from our sphere. And since the portion of the ecliptic that represents the difference of longitude of the stars assumed is understood by the arc E B: then, placing the position of the bright star of Aries at F and Aldebaran at C, F C will be the arc of separation that is sought. Therefore in the triangle F A C, since —

Diagram. A lens-shaped (vesica) figure formed by two arcs meeting at top and bottom poles, representing two halves of circles of latitude on the sphere, with star positions marked along the arcs.

- A (top pole): northern pole of the ecliptic
- D (bottom pole): southern pole of the ecliptic
- G, F, E (left arc, descending): points on the left circle of latitude

- H, B, C (right arc, descending): points on the right circle of latitude
- E B: arc representing the difference of longitude
- F C: arc of separation between the two star positions (the distance sought)

The given quantities [διδόμενα] are:

Side	Value	Meaning
F A	80°3′	complement of the latitude of the bright star of Aries [Υ]
A C	95°31′	excess of the latitude [of Aldebaran] together with a quadrant
F A C	32°6′	+ difference of longitude

Therefore F C is found to be **35°32′** — the sought distance between the stars.

The same result follows from the assumed declinations of the stars together with the difference of their right ascensions. For in the present diagram, if the arctic pole of the equator is placed at A and the antarctic at D, then A H D and A G D will be the half-portions of the circle of declination of our sphere on which the said stars lie, with respect to the poles of the world. The bright star of Aries indeed at G, and Aldebaran at H; and since the given quantities are:

	Right ascension	Declination
Bright star of Aries [Υ]	26°13′	21°33′
Aldebaran	63°16′	15°38′
Difference of ascension	37°3′	

From these data, then, since in the triangle G A H —

Side	Value	Meaning
G A	68°27′	complement of the declination of the bright star of Aries [Υ]
H A	74°22′	complement of the declination of Aldebaran
G A H	37°3′	difference of the right ascensions

Therefore G H is found to be **35°32′** — the sought separation of the stars, as before.

CAPVT III.

On the rising and setting amplitude [*amplitudo ortiva et occidua*] of an assumed point of the ecliptic under a given elevation of the pole; likewise on the ascensional and descensional difference [*differentia ascensionalis et descensionalis*].

Having dealt with the matters that concern the motions related to the equator and the ecliptic in any position of the sphere, we shall now treat of those that determine the same motions of the heavenly bodies with respect to the horizon, the meridian, and the ecliptic — together with the topics set out in advance in this chapter, which will serve for the rising and setting of the heavenly bodies in what follows.

Problem I.

Given (besides the elevation of the pole of the place) the declination of any point of the ecliptic, or of another body rising or setting, to determine the rising or setting amplitude of that point on the horizon.

Although this problem can be worked out in various ways, we nevertheless follow the easiest method of all, relying solely on this one principle, established in the previous book: that in every position of the sphere the equator ascends and descends on both sides at a remove of 90° from the meridian to the horizon. Therefore, if the declination is northern, the portion of the rising or setting amplitude intercepted between the equator and the rising or setting point is to be added to a quadrant. But if the declination is southern, it is to be subtracted from the same —

— so that the rising amplitude, as defined in the previous book, is completed. Moreover, this said portion comes out equal for every equal declination given. Let it suffice to have noted this once and for all.

Example I. In a northern declination.

Let the rising or setting amplitude be sought for a point having a northern [N] declination of $11^\circ 31'$, such as the Sun has at the beginning of Taurus [♉], together with the elevation of the pole of this place, $55^\circ 43'$. Then in the adjoining diagram let C E H I be the whole meridian, as it is represented on our sphere; let the horizon be C B G, whose pole is E, and the equator D B H, whose pole is F. And since the arc C M shows the rising or setting amplitude on the horizon, while C B

is a quadrant of the circle: the portion B M is therefore sought in the right triangle B M L, and to investigate it —

Diagram. A circle representing the celestial sphere with several great circles drawn across it: the meridian, the horizon, and the equator, intersecting to form the small triangles used in the amplitude computation.

- C E H I: the whole meridian
- C B G: the horizon (pole E)
- D B H: the equator (pole F)
- E (upper left interior): pole of the horizon
- F (upper right): pole of the equator
- A (center): center of the sphere
- C (left), G (right): horizon endpoints
- M (right of center): the rising/setting point on the horizon
- B, L (lower center): triangle vertices for the amplitude computation
- N, O (left interior): point and intersection used in Example II (Sirius)
- K, I (bottom): lower meridian points

The given quantities are:

Side	Value	Meaning
M B L	34°17'	complement of the elevation of the pole
M L	11°31'	first declination given
B L M	90°0'	right angle

Therefore B M is found to be **20°46'**.

To this, adding the quadrant C B of 90°0', the amplitude C M is made up: **110°46'**.

Example II. In a given southern declination.

Let it be Sirius, the Dog Star, whose rising and setting amplitude on our horizon we intend to investigate. Given, then, its declination of 16°12', which is represented in the preceding figure by the arc N O — Sirius holding point O on the horizon — the following will be, in the right triangle, for the portion B O:

Side	Value	Meaning
N O	16°12'	declination of Sirius
N B O	34°17'	complement of the elevation of the pole

Side	Value	Meaning
B N O	90°0'	right angle

Therefore B O is found to be **29°42'** — the portion of the amplitude sought; which, taken away from the quadrant C B of 90°0', leaves the rising or setting amplitude of Sirius sought as **60°18'**.

Problem II.

Likewise, given the elevation of the pole of the place, to make known the ascensional and descensional difference; also the semidiurnal arc, and consequently the length of day and night at each season of the year from the motion of the Sun — provided the present problem is applied to points of the ecliptic.

As an example of this most distinguished problem, we shall recall the nearest figure. In its first triangle B L M, the ascensional or descensional difference is shown by the arc B L, namely of the point lying at M. This arc, for the purpose of its investigation, presupposes —

Side	Value	Meaning
L M	11°30'	the declination of the Sun
M B L	34°0'	complement of the elevation of the pole
M L B	90°0'	right angle

Therefore B L is found to be **17°33'** — the ascensional or descensional difference sought.

II. For the Arc of Semidiurnal Time.

[Margin: For the remaining elevations of the pole, let the ascensional differences be sought from the tables of the direct [computations] of Johannes Regiomontanus [Regiomontanus], from which we have also taken our own numbers.]

Since it is established from our sphere that the semidiurnal arc of the equator, from the meridian to the horizon — namely D B — is always 90°, and converted into time is 6 hours: then, similarly converting into time the ascensional or descensional difference that has been found (dividing the degrees by 15, and for every 15 minutes of one degree taking 1 minute of time), and adding 6 hours when the declination is northern — or subtracting it when the declination is southern — what remains is the semidiurnal arc of the rising or setting point in

hours and minutes of hours. So, in the present example, where the converted ascensional difference of $17^{\circ}33'$ in time gives 1h 10m 12s: if 6 hours are added to these, the semidiurnal arc of the ecliptic point at the beginning of Taurus [♉] is made up, for us, as 7h 10m 12s. This time doubled gives the diurnal quantity, 14h 20m 24s; and finally, this time subtracted from 24 hours gives the quantity of night as 9h 39m [?] [the printed value appears as "9 mi. 39" with what may be a fraction mark], which was appropriate to the proposed problem.

Now, since from this single example it is gathered how the ascensional or descensional difference is to be computed for any given declination and for the several elevations of the pole, by the doctrine of triangles: I am pleased to leave here a table for an elevation of the pole of 56° (such as obtains in most places in Denmark) and for declination running continuously up to 34° , the obliquity of the ecliptic now being assumed as $23^{\circ}30'$. For the use of this little table will hereafter be of singular help to us regarding the computation of the rising and setting of the fixed stars.

Table of ascensional differences for an elevation of the place of 56° .

de	mi	de	m	de	mi	de	mi	de	mi	de	m	de	mi				
.	gr.	.	.	gr.	i.	.	gr.	.	.	gr.	.	.	gr.	i.	.	gr.	.
1	1	29	7	10	29	13	20	1	19	30	41	25	43	44	31	62	58
2	2	58	8	12	1	14	21	42	20	32	39	26	46	18	32	67	53
3	4	27	9	13	35	15	23	24	21	34	41	27	49	4	33	74	19
4	5	57	10	15	9	16	25	9	22	36	48	28	52	1	34	90	0
5	7	27	11	16	45	17	26	57	23	39	0	29	55	16			
6	8	58	12	18	22	18	28	48	24	41	18	30	58	52			

Problem III.

Given the declination, the right ascension, and the ascensional difference, the oblique ascension and descension of any point of the ecliptic is found, under a given elevation of the pole.

The first problem of chapter 1 of this book supplied the declination of a point of the ecliptic; the second problem of the same chapter supplied the right ascension; and the problem immediately preceding supplied the ascensional or descensional difference. For the rest, it must be noted at this point that we complete the present problem by duly applying, with examples, the ascensional difference found in the previous problem — in this way:

[The schematic brace-diagram completes the rule for Problem III begun on the previous page. It pairs the two cases of the declination with the operation to perform on the ascensional difference:]

If the declination supplied is **northern**:

- **Subtract** the Ascensional Difference from the Right Ascension, and what remains is the required **Oblique Ascension**.
- **Add** the Ascensional Difference to the Right Ascension, and what remains is the required **Oblique Descension**.

If the declination supplied is **southern**:

- **Add** the Ascensional Difference to the Right Ascension, and what remains is the required **Oblique Ascension**.
- **Subtract** the Ascensional Difference from the Right Ascension, and what remains is the required **Oblique Descension**.

Example I.

Let there be a point on the ecliptic at the start of Taurus (♉), whose oblique ascension and descension are to be found, under an elevation of the pole of 56° . To this point there correspond:

- Right Ascension: $27^\circ 54'$
- Declination: $11^\circ 30' \text{ N}$ — and therefore also
- Ascensional Difference: $17^\circ 33'$

From this it follows that the Oblique Ascension is $10^\circ 21'$, and the Oblique Descension is $45^\circ 27'$.

Example II.

Let the oblique ascension and descension be sought for the ecliptic point at the start of Capricorn (♐). For this the data given are:

- Right Ascension: $270^\circ 0'$
- Declination: $23^\circ 30' \text{ S}$
- Ascensional Difference: $40^\circ 8'$

From this the Oblique Ascension comes out as $310^\circ 8'$, and the Descension as $229^\circ 52'$.

From these premises the following table of oblique ascensions and descensions for each degree of the ecliptic at an elevation of 56° was constructed long ago by

[Regiomontanus]. We, however, have set it out for each sign separately, so that the table may be more readily usable by what follows, for finding the rising and setting of the stars.

It should be noted that its whole structure rests on the Διδόμενα [Greek: Διδόμενα (Didomena, "the givens" — the *Data*)] agreeing with a quadrant of the circle, once one considers the points both in opposite signs and equally distant from the solstices or equinoxes. On this matter, although we gave notice above, we nonetheless leave its proof here, for the benefit of beginners, in the examples set out in the following table.

Table of oblique ascensions and descensions for an elevation of the pole of 56 degrees. (Signs ♈ Aries, ♉ Taurus, ♊ Gemini, ♋ Cancer.)

De g.	♈ Asc.	♈ Desc.	♉ Asc.	♉ Desc.	♊ Asc.	♊ Desc.	♋ Asc.	♋ Desc.
0	0 0	0 0	10 21	45 27	24 44	90 52	49 52	130 8
1	0 19	1 30	10 44	46 58	25 21	92 21	50 58	131 14
2	0 39	3 0	11 7	48 30	25 59	93 49	52 5	132 16
3	0 58	4 31	11 31	50 1	26 38	95 16	53 13	133 19
4	1 18	6 1	11 55	51 33	27 18	96 42	54 22	134 22
5	1 38	7 32	12 19	53 5	27 59	98 7	55 32	135 22
6	1 57	9 2	12 44	54 36	28 40	99 32	56 43	136 23
7	2 17	10 33	13 9	56 8	29 22	100 16	57 55	137 22
8	2 37	12 3	13 34	57 40	30 5	102 32	59 8	138 18
9	2 57	13 33	14 0	59 12	30 48	103 46	60 22	139 14
10	3 17	15 5	14 26	60 44	31 37	105 10	61 37	140 9
11	3 37	16 35	14 52	62 16	32 17	106 33	62 53	141 3
12	3 57	18 6	15 19	63 48	33 3	107 15	64 9	141 17
13	4 17	19 37	15 46	65 19	33 50	109 16	65 26	142 48
14	4 37	21 8	16 13	66 50	34 39	110 37	66 44	143 41
15	4 57	22 39	16 41	68 21	35 29	111 57	68 3	144 31
16	5 17	24 9	17 10	69 53	36 20	113 16	69 23	145 21
17	5 38	25 40	17 39	71 24	37 12	114 34	70 44	146 10
18	5 59	27 11	18 9	72 55	38 4	115 50	72 5	146 57
19	6 20	28 42	18 39	74 26	38 57	117 7	73 27	147 43

De g.	♈ Asc.	♈ Desc.	♉ Asc.	♉ Desc.	♊ Asc.	♊ Desc.	♋ Asc.	♋ Desc.
20	6 41	30 13	19 9	75 57	39 51	118 23	74 50	148 28
21	7 2	31 44	19 40	77 26	40 46	119 38	76 13	149 13
22	7 23	33 15	20 12	78 16	41 42	120 52	77 37	149 56
23	7 45	34 47	20 44	80 26	42 39	122 5	79 2	150 38
24	8 6	36 18	21 16	81 56	43 38	123 16	80 27	151 20
25	8 26	37 50	21 49	83 27	44 38	124 28	81 53	152 1
26	8 50	39 21	22 22	84 56	45 39	125 37	83 19	152 42
27	9 13	40 52	22 56	86 25	46 41	126 46	84 46	153 22
28	9 35	42 24	23 31	87 54	47 44	127 54	86 13	154 1
29	9 58	43 55	24 7	89 23	48 48	129 2	87 40	154 39
30	10 21	45 27	24 44	90 52	49 52	130 8	89 8	155 16

Table of oblique ascensions and descensions for an elevation of the pole of 56 degrees. (Signs ♌ Leo, ♍ Virgo, ♎ Libra, ♏ Scorpio.)

D eg .	♌ Asc.	♌ Desc.	♍ Asc.	♍ Desc.	♎ Asc.	♎ Desc.	♏ Asc.	♏ Desc.
0	89 8	155 16	134 33	169 39	180 0	180 0	225 27	190 21
1	90 37	155 53	136 5	170 1	181 30	180 19	226 58	190 44
2	92 6	156 29	137 36	170 24	183 0	180 39	228 30	191 7
3	93 35	157 4	139 8	170 46	184 31	180 18	230 1	191 31
4	95 4	157 38	140 39	171 9	186 1	180 18	231 33	191 55
5	96 33	158 11	142 10	171 32	187 32	180 38	233 5	192 19
6	98 3	158 45	143 42	171 54	189 2	180 17	234 36	192 44
7	99 33	159 17	145 13	172 15	190 33	182 17	236 8	193 9
8	101 3	159 49	146 45	172 37	192 3	182 37	237 40	193 34
9	102 33	160 21	148 16	172 58	193 34	182 57	239 12	194 0
10	104 3	160 51	149 47	173 19	195 5	183 17	240 44	194 26
11	105 34	161 22	151 18	173 40	196 35	183 37	242 16	194 52
12	107 5	161 12	152 49	174 1	198 6	183 57	243 48	195 19
13	108 36	162 22	154 20	174 22	199 37	184 17	245 19	195 46

D
eg

.	♈ Asc.	♈ Desc.	♉ Asc.	♉ Desc.	♊ Asc.	♊ Desc.	♋ Asc.	♋ Desc.
14	110 7	162 51	155 51	174 43	201 8	184 37	246 50	196 13
15	111 39	163 19	157 21	175 3	202 39	184 57	248 21	196 41
16	113 10	163 48	158 52	175 24	204 9	185 17	249 53	197 10
17	114 41	164 15	160 23	175 42	205 40	185 38	251 24	197 39
18	116 12	164 42	161 54	176 4	207 11	185 59	252 55	198 9
19	117 44	165 8	163 25	176 23	208 42	186 20	254 26	198 39
20	119 16	165 34	164 55	176 43	210 13	186 41	255 57	199 9
21	120 48	166 0	166 26	177 4	211 44	187 2	257 27	199 40
22	122 20	166 26	167 57	177 23	213 15	187 23	258 57	200 12
23	123 52	166 50	169 27	177 43	214 47	187 41	260 27	200 44
24	125 24	167 16	170 58	178 2	216 18	188 6	261 57	201 16
25	126 55	167 41	172 28	178 22	217 50	188 26	263 27	201 49
26	128 37	168 5	173 59	178 41	219 21	188 50	264 56	202 22
27	129 59	168 30	175 29	179 1	220 52	189 13	266 25	202 16 [?]
28	131 30	168 52	177 0	179 20	222 24	189 35	267 64	203 31
29	133 2	169 16	178 30	179 40	223 55	189 58	269 23	204 7
30	134 33	169 39	180 0	180 0	225 27	190 21	270 52	204 44

Table of oblique ascensions and descensions for an elevation of the pole of 56 degrees. (Signs ♊ Sagittarius, ♐ Capricorn, ♒ Aquarius, ♓ Pisces.)

D
eg

.	♊ Asc.	♊ Desc.	♐ Asc.	♐ Desc.	♒ Asc.	♒ Desc.	♓ Asc.	♓ Desc.
0	270 52	204 44	310 8	229 52	335 16	269 8	349 39	314 33
1	272 20	205 21	311 12	230 58	335 53	270 37	350 2	316 5
2	273 47	205 59	312 16	232 5	336 29	272 6	350 25	317 36
3	275 14	206 38	313 19	233 13	337 4	273 35	350 47	319 8
4	276 41	207 18	314 21	234 22	337 38	275 4	351 10	320 39
5	278 7	207 59	315 22	235 32	338 11	276 33	351 32	322 10

D eg .	↗ Asc.	↗ Desc.	↘ Asc.	↘ Desc.	≈ Asc.	≈ Desc.	✕ Asc.	✕ Desc.
6	279 33	208 40	316 22	236 43	338 44	278 3	351 54	343 42 [?]
7	280 58	209 22	317 21	237 55	339 16	279 33	352 15	325 13
8	282 23	210 5	318 18	239 8	339 48	281 3	352 37	362 45 [?]
9	283 47	210 48	319 14	240 22	340 20	282 33	352 58	328 16
10	285 10	211 37	320 9	241 37	340 51	284 3	353 19	329 47
11	286 33	212 17	321 3	242 53	341 21	285 34	353 40	331 18
12	287 55	212 3	321 56	244 9	341 51	287 5	354 1	332 49
13	289 16	213 50	322 48	245 26	342 21	288 36	354 22	334 20
14	290 37	214 39	323 40	246 44	342 50	290 7	354 43	335 51
15	291 57	215 29	324 31	248 3	343 19	291 39	355 3	337 21
16	293 16	216 20	325 21	249 23	343 47	293 10	355 23	338 52
17	294 34	217 12	326 10	250 44	344 14	294 41	355 43	340 23
18	295 51	218 4	326 57	252 5	344 41	296 12	356 3	341 54
19	297 7	218 57	327 43	253 27	345 8	297 44	356 23	343 25
20	298 23	219 51	328 28	254 50	345 34	299 16	356 43	344 55
21	299 38	220 46	329 12	256 13	346 0	300 48	357 3	346 26
22	300 52	221 42	329 55	257 37	346 26	302 20	357 23	347 57
23	302 5	222 39	330 38	259 2	346 51	303 52	357 43	349 27
24	303 17	223 38	331 20	260 27	347 16	305 24	358 3	350 58
25	304 28	224 38	332 1	261 53	347 41	306 55	358 22	352 28
26	305 38	225 39	332 42	263 19	348 5	308 37	358 42	353 69 [?]
27	306 47	226 41	333 22	264 46	348 29	309 59	359 2	355 29
28	307 55	227 44	334 1	266 13	348 53	311 30	359 21	357 0
29	309 2	228 48	334 39	267 40	349 16	313 2	359 41	358 30
30	310 8	229 52	335 16	269 8	349 39	314 33	360 0	360 0

CHAPTER IV.

On the rising and setting of the Sun and of the stars —
threefold: cosmic, acronychal, heliacal.

PROBLEM I.

Given the declination and right ascension of some rising or setting star, together with the elevation of the pole, to find the point of the ecliptic with which that same star rises and sets.

Oblique ascension and descension stand in a reciprocal relation in the table of oblique ascensions, so that once the oblique ascension of a rising and setting star is given, the point of the ecliptic with which it ascends and descends is given as well. This has the greatest use in history, poetry, and astrology. We shall therefore dispatch the present problem concisely from what has gone before.

First, which stars rise and set at a given elevation of the pole becomes known by comparing the declination with the complement of the pole's elevation: a northern declination in our northern region, a southern declination in the southern region. For if the complement of the pole's elevation is greater than the star's declination, the star rises and sets in the present horizon; if less, it does not.

Examples at a pole elevation of 55° 43'.

I.

	gr.	min.
Declination of the Bright Star of the Crown [Lucida Coronae], north	28	0
Complement of the elevation of the pole	34	17
Difference, which is the depression of the star below the horizon	6	17

[Margin: Anno 1630 (in a later manuscript hand, beside the declination)]

II.

	gr.	min.
Declination of the Bright Star of the Lyre [Lucida Lyrae]	38	29
Complement of the elevation of the pole	34	17

	gr.	min.
Difference	4	12

This shows that this star never rises or sets in our horizon, but at its least altitude — namely at the nadir, as the Arabs call it — it stands $4^{\circ} 12'$ above the horizon, refraction not yet included.

But let us go on with the example of the rising and setting of the Bright Star of the Crown on our own horizon, recalling the computation from what precedes and setting it before our eyes in a single diagram.

Diagram. A spherical figure for the rising of the Bright Star of the Crown. In it, G A B is the meridian and G L the horizon, whose pole is A; F H the equator, whose pole is B; finally E R the ecliptic and its pole C. This pole C, in the small circle called the Arctic Circle, is continually carried around the world's pole B in the apparent daily revolution, keeping a uniform distance from it — namely $23^{\circ} 32'$, which is the greatest obliquity.

- A (top): pole of the horizon
- B (upper, near C): pole of the equator (world's pole)
- C (right of B): pole of the ecliptic
- E (upper left): point on the ecliptic
- F (left): point on the equator
- G (left): meridian/horizon intersection
- H (lower center): point on the equator
- K (center): intersection of the star's circle with the ecliptic
- L (right): horizon point
- D (lower right of center): position of the star (Bright Star of the Crown)
- M, N (lower center): intersection points
- P, R (lower right): ecliptic/auxiliary points

Since, then, according to a suitable arrangement of the spherical globe, the Bright Star of the Crown D rises together with the point K of the ecliptic, we wish to find that point.

So that the ascensional difference N H may be obtained from a fixed point N of the equator, we shall need — for the declination of the Bright Star of the Crown, D N, of $28^{\circ} 0'$, and a right ascension of $229^{\circ} 45'$ (and this for the completed year 1630, which we here assume, and for which the star's longitude is given as $7^{\circ} 4' \mathfrak{M}$ and its latitude as $44^{\circ} 23'$ north, derived closely from the canonical description of

the stars and the reduction of longitude) — furthermore, the ascensional difference H N is given as $51^{\circ} 15'$ at the elevation of Copenhagen [Hafniensi]. When this is subtracted from the right ascension, the oblique ascension of the Bright Star of the Crown remains, $178^{\circ} 30'$, namely the point H on the equator. Therefore, by the reciprocity of the table of oblique ascensions, under the assumed elevation of the pole the point corresponding to this oblique ascension is given as $29^{\circ} \text{ } \mathfrak{M}$ very nearly. With this point is known the true or morning rising of the Crown with the Sun, or its evening rising, the Sun setting at the opposite point — in keeping with what we set out in Chapter 5 of the earlier book.

[Margin: Problem 1, Chapter 2.] [Margin: Problem 2, Chapter 2.] [Margin: Problem 2 of this chapter.] [Margin: The preceding problem.]

Another way, by our own method, which proceeds without a given right ascension.

I. Keeping this same diagram, in the triangle B D C, all sides being given through what precedes, the angle B D C is given.

The given parts [διδόμενα, *didomena*, "the givens"] of the said triangle are:

	gr.	mi.	
D C	45	37	complement of the latitude of the Crown
B D	62	0	complement of the declination
B C	23	32	distance of the poles

Therefore B D C is given as $21^{\circ} 10'$.

II. In the right triangle B L D, for the angle B D L, the givens are:

	gr.	mi.	
B L	55	43	elevation of the pole of this place
B D	62	0	complement of the declination of the star of the Crown
B L D			a right angle

Therefore B D L is given as $69^{\circ} 21'$. Subtracting from it

B D C, $21^{\circ} 10'$,

there remains C D L, $48^{\circ} 11'$, to which K D M is equal.

III. In the right triangle K M D, for the side M K, the givens are:

	gr.	mi.	
D M	44	23	latitude of the star of the Crown
K D M	48	11	
K M D	90	0	a right angle

Therefore M K is given as $37^{\circ} 59'$. When this arc is subtracted from the given longitude of the Crown, which we assumed above to be $7^{\circ} 4' \mathfrak{M}$, the point of the ecliptic with which the Crown ascends will be found at $29^{\circ} 3' \mathfrak{N}$ — differing only slightly from the earlier result, and indeed for definite reasons.

Next, for demonstrating the setting of the stars — that is, the point of the ecliptic with which a star sets — let us take the same Bright Star of the Crown as our example.

The star of the Crown having therefore been brought back to the setting (western) horizon — as happens roughly by inverting the diagram above — let A be, as before, the pole of the horizon, B the pole of the equator, C the pole of the ecliptic. Upon these let there be drawn: first L G the horizon, then O F the equator, and finally, third, R E the ecliptic — all just as they appear in our sphere.

Diagram. A spherical figure for the setting of the Bright Star of the Crown, the diagram above inverted.

- A (top): pole of the horizon
- B (upper left): pole of the equator
- C (below B): pole of the ecliptic
- E (upper right): ecliptic point
- F (right): equator point
- G (right): horizon point
- L (left): horizon point
- D (center left): position of the star in the setting horizon
- K (center): intersection of star's circle with ecliptic
- Γ (below K, joined to K by a dotted line): auxiliary point
- N (lower center): equator point
- O (lower left): equator point
- M (lower center): ecliptic point
- R (lower left): ecliptic point

Since D is the position of the Crown in the western (setting) horizon, just as before it was in the eastern (rising) horizon — the right ascension N and the declination N D remaining the same as before, and the longitude likewise remaining unvaried at M —

therefore in the right triangle D K N, since the ascensional difference N K does not vary either, then, all the givens [δοθέντα, *dothenta*, "the givens"] being retained as before, it will be $51^{\circ} 15'$. This, according to the precepts applied to the previous problem, is here added to the right ascension of the star at N; and thus the point K on the equator sets forth the oblique descension of the Bright Star of the Crown, $281^{\circ} 0'$.

But since the tables of Regiomontanus contain only oblique ascensions and the arcs of the ecliptic corresponding to them, and conversely — not also oblique descensions and so forth — therefore, in order that the same tables may be of use to us, a semicircle of 180° is always to be added to the oblique ascension found; and so, where need arises, when the sum exceeds a full circle, the whole circle is to be cast off from the total, and what is left is the oblique ascension of the sign and of the sought point opposite in the ecliptic — as will appear in this example.

	gr.	
Oblique descension of the Crown	281	acquired
Semicircle	180	to be added
Sum	461	
Measure of a circle	360	to be subtracted
Oblique ascension of the opposite sign	101	

To this, in the table of oblique ascensions under a pole elevation of $55^{\circ} 43'$ — the individual adhering minutes being suitably corrected — there correspond $7^{\circ} 44'$ ♈, to which the opposite point in the ecliptic is $7^{\circ} 44'$ ♏, with which the Bright Star of the Crown sets.

Now, by our method set out above, we shall obtain the same point of the Crown in its descent through the ecliptic.

First, then, since in the upper figure — as also in the present one — the angle M D N was found to be $21^{\circ} 10'$ and the angle B D L $69^{\circ} 21'$, therefore, these being added together in the manner contrary to that above, the angle C D L will be $90^{\circ} 31'$, to which the vertical angle M D H is equal.

Further, in the right triangle M H D, the givens [δοθέντα, *dothenta*] are:

	gr.	mi.	
M D H	90	31	the angle just found
M D	44	23	latitude of the Crown
D M H	90	0	a right angle

Therefore M H is given as $90^{\circ} 44'$.

This side found, if it is added to the point M on the ecliptic — namely $7^{\circ} 4' \mathfrak{M}$, where the longitude of the star was first established — yields $7^{\circ} 48' \approx$.

This result exceeds by at least 4 minutes the point previously found in the ecliptic with which the Bright Star of the Crown descends; but the reason for that slight discrepancy is not lacking, so that our procedure may be the more reliable.

Another example, for the true rising and setting of Sirius, the Bright Star in the mouth of Canis Major [Canis Maioris], in our horizon.

Diagram. A spherical figure for the rising of Sirius, drawn with the points marked as above.

- R (upper left): ecliptic point
- C (left, below R): pole of the ecliptic
- M (upper center): position of the star (Sirius) / longitude point
- N (center): equator point
- H (right of center): auxiliary point
- D, K (center): star position and intersection
- L (left): horizon point
- G (right): horizon point
- E (lower right): ecliptic point
- F (lower right): equator point
- B (bottom): pole of the equator
- A (bottom): pole of the horizon

In the adjoining diagram, marked with the points designated above, let M be the point of rising of the star in the ecliptic. For this star, at the completed year 1630, the longitude is given at M as $9^{\circ} 1' \mathfrak{S}$, and the latitude D M as $39^{\circ} 30'$ south. From what precedes, the star's declination D N is found to be $16^{\circ} 12'$ and its right ascension $97^{\circ} 13'$ at N. Therefore, in the right triangle D N K, for the ascensional difference N K, the givens [διδόμενα] are: the side D N, $16^{\circ} 12'$, the star's

declination; the angle N K D, $34^{\circ} 17'$, the complement of the elevation of the pole; the angle D N K, a right angle. Therefore K N is $25^{\circ} 13'$.

When the ascensional difference is added to the given right ascension, the oblique ascension of the star is aggregated as $122^{\circ} 26'$, to which there corresponds in the table of Regiomontanus — the calculation being directed to our elevation of the pole — the point of the ecliptic $21^{\circ} 53'$ ♊, with which Sirius rises above our horizon when the Sun reaches it. This happens around the 4th day of August in our age, the time we reckon as the middle of the Dog Days.

Now let us try the same by our own method, set out above.

First, then, in the triangle B D C of the present diagram, for the angle at D, the givens [δοθέντα] are:

	gr.	min.	
D C	50	30	complement of the latitude of the star
B D	73	48	complement of the declination
B C	23	32	distance of the pole of the ecliptic from the pole of the equator

Therefore B D C is given as $3^{\circ} 47'$.

Second, in the right triangle B L D, for the angle L D B, the givens are:

	gr.	min.	
L B	55	43	elevation, or rather depression, of the antarctic pole
D B	73	48	complement of the declination
L	90		a right angle

Therefore L D B is given as $59^{\circ} 22'$; and when B D C is taken from it, there remains the vertical angle M D H, $55^{\circ} 35'$.

Third, in the right triangle M D H, for M H, the givens [διδόμενα] are:

	gr.	mi.	
M D	39	30	latitude of Sirius
M D H	55	35	the angle
D M H			a right angle

Therefore M H is found to be $42^{\circ} 52'$. To this, the longitude of Sirius set out before — $9^{\circ} 1' \text{ ☿}$ — being added, the rising of this star emerges with the point of the ecliptic at $21^{\circ} 53' \text{ ♈}$, exactly as before.

There now follows the investigation of the setting point of the ecliptic with the same Dog Star [Sirius], by a twofold calculation as before — but here the diagram of the preceding figure is represented inverted from our sphere.

Diagram. A circular spherical-astronomy construction for the inverted Sirius setting-point figure — the meridian/horizon outline as a large circle, with the ecliptic and equator as intersecting great-circle arcs, and a small circle at the lower right (centered on B) representing the star's position circle. Labels read (positions approximate):

- R (top): on the rim
- O (upper right): on the rim
- M (upper-center): construction point on an arc
- N (center): intersection of great circles
- D (center, just below N): intersection / vertex
- H (left of center): point in the ecliptic
- L (right): on the rim
- G (left): on the rim
- F (lower left): on the rim
- E (lower left, below F): on the rim
- C (lower-center, on the small circle): construction point
- B (center of the small circle, lower right): the star's pole/center
- A (bottom): on the rim

All thirteen labels (R, O, M, N, D, H, L, G, F, E, C, B, A) read cleanly; no [?] flags.

In the first method the right ascension and declination are given, as above, together with K N, the descensional difference [differentia descensionalis], which is as the ascensional difference [differentia ascensionalis] above; here, however, on the contrary it must be subtracted from the right ascension so as to obtain the oblique descension [descensio obliqua]. Thus the right ascension of Sirius is $97^{\circ} 13'$; the descensional difference of Sirius is $25^{\circ} 13'$; therefore the oblique descension is $72^{\circ} 00'$. To this I add the semicircle of 180° , and the oblique ascension of the opposite sign emerges as 252° . To this the table gives $17^{\circ} 33' \text{ ♍}$, whose opposite point is $17^{\circ} 33' \text{ ☿}$ — namely the point of the ecliptic with which Sirius sets.

The same result follows from our own demonstration. There, with the angle C D B of $3^{\circ} 47'$ given as above, here on the contrary it must be added to the angle L D B, also given, of $59^{\circ} 22'$, and the vertical angle M D H is composed as $63^{\circ} 9'$.

Further, in the same right triangle M D H, to find M H:

Given [διδόμενα]:

- the side D M, $39^{\circ} 30'$, the latitude of Sirius;
- the angle M D H, $63^{\circ} 9'$;
- the angle D M H, a right angle.

Therefore M H is found to be $51^{\circ} 29'$.

This arc is subtracted from the longitude of Sirius, namely $9^{\circ} 1' \text{ } \text{♁}$, and the point H remains in the ecliptic at $47^{\circ} 32'$ — that is, $17^{\circ} 32' \text{ } \text{♄}$.

In these two examples we have shown the cosmic rising and acronychal setting of the stars assumed; and from these all the rest can be worked out, whenever the risings and settings of several stars are to be found on our globe or sphere. One then carefully considers the arrangement of the ecliptic and the equator at the threshold of the given horizon, and projects figures conformable to it onto paper, as we have done. For the requisites in the triangles always remain the same in number, varied only in their figures. And since our method is, as is evident, the more expeditious, we shall be content with it alone until the tables of Regiomontanus have been reformed. But since the acronychal rising and cosmic setting of the stars remain — described by us in chapter 5 of Book I — it must be understood that these are found at the points diametrically opposite to those already determined, and require no further calculation.

Examples for the data; as for the Bright Star of the Crown [Alphecca]

Since the rising point of this star in the ecliptic was found to be $29^{\circ} 3' \text{ } \text{♍}$, its evening or acronychal rising will be when the Sun holds the opposite point of the ecliptic, namely $29^{\circ} 3' \text{ } \text{♋}$. Likewise, since the setting point of the same Crown — its evening or acronychal setting — was found by our method to be $7^{\circ} 48' \text{ } \text{♌}$, its morning or cosmic setting will similarly occur when the Sun occupies the opposite point, namely $7^{\circ} 48' \text{ } \text{♎}$.

PROBLEM II.

Given a point of the ascending ecliptic together with its obliquity, the elevation of the pole of the place, and finally the latitude of a star taken as invariable, the longitude of the same star is found.

This problem affords a notable use in examining the motions of the fixed stars from ancient observations. Here, by way of example, let us follow a trace in Hesiod, whose most ancient record of the rising and setting of certain fixed stars has been preserved for posterity down to our own age, in his poetic books *On Works and Days* [περὶ ἔργων καὶ ἡμερῶν (peri ergōn kai hēmerōn, "On Works and Days")]. According to most, and the foremost, chronographers, he lived on Mount Helicon in Greece, not far from Athens, at the beginning of the Olympiads — that is, according to Mercator, about the year of the world 3190, and 775 years before Christ, and therefore 2385 years before our present time. But since his epoch is in doubt — as is the exactness of his knowledge of the stars — we shall try, from the passage cited, to elicit the truth from the evening rising of Arcturus, which Hesiod himself describes thus:

[Greek: Εὖτ' ἂν δ' ἐξήκοντα μετὰ τροπὰς Ἡελίοιο χειμερί' ἐκτελέσῃ Ζεὺς ἥματα, δὴ ρά τοτ' ἀστήρ Ἀρκτοῦρος προλιπὼν ἱερὸν ῥόον Ὠκεανοῖο πρῶτον παμφαίνων ἐπιτέλλεται ἀκροκνέφαιος. ("When Zeus has completed sixty days after the winter solstice, then indeed the star Arcturus, leaving the sacred stream of Ocean, first rises shining brightly at nightfall.")]

When Jupiter [Zeus] has completed sixty wintry days after the solstice, then truly the star Arcturus, leaving the immense flood of Ocean, first appearing in full, rises in the evening.

Considering the circumstance of these words, we easily gather that Hesiod meant that sixty full days had passed, in his time, between the winter solstice and the evening rising of Arcturus; and this we shall now investigate according to our proposal. First, then, holding as granted, let us draw back the apogee of the Sun to $20\frac{1}{6}^{\circ}$ ♄, where it is reckoned to have stood in Hesiod's time. Next, having found the motion of the Sun corresponding to a span of sixty days, namely $61\frac{1}{6}^{\circ}$ (as will be shown in the following part of the *Astronomy*), and having added it to the beginning of ♌, the point opposite the rising of Arcturus is fixed at $1\frac{1}{6}^{\circ}$ ♋. If, then, the latitude of Arcturus is given as $31^{\circ} 3' N$ — which exists as nearly invariable — and likewise the elevation of the pole of Mount Helicon in Greece is $37^{\circ} 45'$, according to Ptolemy and the received geographers, we shall first try to elicit

from this the true longitude of the star at Hesiod's time by means of spherical triangles. Then, having instituted a comparison with the longitude of the same star observed in our own time and by our own labor, we shall — by one and the same operation — establish both Hesiod's epoch, the man's skill in astronomy, and whatever degree of certainty of the precession of the equinoxes there may be, with the demonstration and calculation set up in this manner.

With the diagram formed from the position of our sphere at the evening rising of Arcturus at the estimated time of Hesiod, the several points are expressed by nearly the same letters as above. Thus H will be the rising point

Diagram. A spherical-astronomy construction depicting the position of the sphere at the evening (acronychal) rising of Arcturus at Hesiod's estimated epoch — the figure for Problem II. A large circle (the meridian / local horizon outline) carries an oblique great-circle arc (the horizon) running from the upper left across to the lower right, and several intersecting great circles (the ecliptic and the equator) crossing near the lower-center, where a tight cluster of construction points lies. Labels read (positions approximate):

- R (top, just left of center): on the upper rim
- A (top, right of R): on the upper rim, adjacent to R
- O (upper left): on the rim
- B (right, upper-middle): on the rim
- C (right, below B): on an arc
- G (right, middle): on the rim
- Y (left, middle): on the rim, where the horizon meets the limb
- K (center-bottom cluster): construction point
- H (center-bottom cluster, just right of K): the rising point in the ecliptic
- D (center-bottom, right of H): construction point
- N (center-bottom, below K/H): foot of the perpendicular
- M (center-bottom, right of N): construction point
- P (center-bottom, below M): pole / construction point [?]
- F (bottom): on the lower rim
- E (bottom, below F): on the lower rim

[The central cluster of small labels (K, H, D, N, M, P) is densely overprinted where the great circles intersect; the readings of P and its immediately adjacent glyph are the least secure.]

in the ecliptic where Arcturus rises with it, at D; and H M is the difference between the rising point in the ecliptic and the true place of the star — which we may track down as follows.

I.

In the spherical triangle H P K, for the obtuse angle at H:

Given:

- the side H P, $28^{\circ} 49'$, the distance of the eastern ecliptic point of Arcturus from the autumnal equinox;
- the angle H P K, $23^{\circ} 32'$, the maximum obliquity of the ecliptic from the equator, which we here keep invariable, although perhaps it may have been somewhat greater by a few minutes;
- the angle H K P, $52^{\circ} 15'$, the complement of the elevation of the pole of Mount Helicon.

Therefore the angle K H P is found to be $107^{\circ} 43'$.

For when the perpendicular H N is dropped — whose arc is the declination of the ecliptic point H, $11^{\circ} 6'$ — the proposed triangle H P K is divided into two right triangles.

In the right triangle N H P, then, for the angle at H:

Given:

- H P N, $23^{\circ} 32'$, the maximum obliquity of declination;
- H N, $11^{\circ} 6'$, the declination of the rising ecliptic point;
- H N P, 90° , a right angle.

Therefore N H P is found to be $69^{\circ} 7'$.

II.

In the right triangle K N H, for the angle at H:

Given:

- N K H, $52^{\circ} 15'$, the complement of the elevation of the pole of Helicon;
- H N, $11^{\circ} 6'$, the declination of the ecliptic point;
- K N H, a right angle, 90° .

Therefore K H N is found to be $38^{\circ} 36'$.

When this is added to the angle just found above, and the sum is subtracted from 180° , the angle D H M arises as $72^\circ 17'$, by the first theorem on triangles.

III.

In the right triangle H M D, for H M:

Given:

- D H M, $72^\circ 17'$, the angle just found;
- D M, $31^\circ 3'$, the latitude of Arcturus;
- H M D, a right angle.

Therefore H M is found to be $11^\circ 5'$.

This side, added to the point H, fixes the true longitude of Arcturus at Hesiod's time as $12^\circ 16' \text{ } \text{M}$ [recte D ?]. [Margin: Lib. 1, Theor. cap. 4.]

But in the year 1610, from a recent observation at Hven, the longitude of Arcturus was $18^\circ 47' \text{ } \text{D}$. The difference, therefore, is $36^\circ 31'$. But our true precession of the equinoxes, and the motion of the stars in consequence [in consequentia], which will make up that difference, requires that a span of about 2658 years be supposed — by which years Hesiod would indeed have preceded our age, and so would have flourished in the year 2918 from the founding of the world. Before that time the Trojan War would have preceded by scarcely 140 years; and most of the historians affirm that Homer was 240 years later than its beginning, so that between Hesiod and Homer an interval of about 100 years is left, by which the former would have existed older than the latter. [Margin: Hesiod.] [Margin: Homer.] These things, indeed, follow plainly concerning Hesiod from this observation of his; but the refraction of the star Arcturus, existing at its rising, subtracts something — about half of the difference of this interval.

And so these things stood thus — unless perhaps Hesiod had borrowed the places of the fixed stars from a more ancient Chaldean observation, without taking account of the precession of the equinoxes, that is, of the meantime motion of the fixed stars. But this I can hardly persuade myself of, since both Pliny attests that an *Astrology* also exists under Hesiod's name [Margin: Lib. 18, c. 25.], and Strabo testifies that he wrote *Poems on Geography* [περὶ τῆς γῆς (peri tēs gēs, "On the Earth")] [Margin: Lib. 7.] — so much so that I should rather believe that he was himself an astronomer, and, in that leisure and long old age, a diligent observer of the stars. It therefore suffices to have left to students of astronomy, besides the conjecture from this observation concerning Hesiod's epoch and his knowledge in astronomical matters, also a model by which — from the given point of some

star's rising, together with its latitude and the elevation of the pole — they may seek out and explore the place of its longitude; which has a singular importance in weighing the observations of the ancients.

It will suffice to have demonstrated these things concerning the cosmic and acronychal rising and setting of the stars. Before passing on to show the heliacal rising and setting likewise, we shall append a twofold table, constructed by means of the foregoing problems: the one of the transits of a hundred more notable stars across the point of mid-heaven, deduced for the completed year 1630; the other of the rising and setting of the same stars in our horizon, that is, under the 56th degree of elevation, with their qualities added at the same time from the annotation of Alfonso [ex annotatione Alphonsina] — so that we may render this our labor the more commendable also to the astrologers, especially for ingeniously practicing the meteorological predictions from the rising and setting of the fixed stars with the Sun, and so forth.

TRANSITS OF A HUNDRED PRINCIPAL STARS across Mid-heaven, for the completed year 1630.

N u m .	Nature with planet [Natur. cum plan.]	Name of the star [Denominatio stellarum]	m gra d. .	in sign [sign.]
1	Saturn, Venus	Pole star [Stella polaris]	6	42 Aries [Ariet.]
2	Sun, Saturn	Southern tail of the Whale [Australis cauda Cete]	6	47 Aries
3	Venus	Girdle of Andromeda [Cingulum Andromadæ]	13	20 Aries
4	Jupiter, Saturn	Knee of Cassiopeia [Genu Cassiopeæ]	16	54 Aries
5	Mercury, Saturn	Preceding horn of Aries [Præcedens cornu Arietis]	25	12 Aries
6	Saturn	Belly of the Whale [Venter Cete]	25	16 Aries
7	Mercury, Saturn	Following horn of Aries [Sequens cornu Arietis]	25	27 Aries

N u m b. .	Nature with planet [Natur. cum plan.]	Name of the star [Denominatio stellarum]	gra d. .	m in .	sign [sign.]
8	Venus	Southern foot of Andromeda [Australis pes Andromadæ]	27	18	Aries
9	Saturn, Mercury	Knot of the flaxen thread of the Fish [Nodus lini Piscis]	27	45	Aries
10	Mars, Saturn	Bright Star of Aries [Lucida Arietis]	28	41	Aries
11	Jupiter, Saturn	Schedar of Cassiopeia [Schedir Cassiopeæ]	9	43	Taurus [Taur.]
12	Saturn	Bright Star of the jaw of the Whale [Lucida mandibula Cete]	13	17	Taurus
13	Saturn, Jupiter	Head of Medusa [Caput Medusæ]	13	35	Taurus
14	Jupiter, Saturn	Bright side of Perseus [Lucidum latus Persei]	16	57	Taurus
15	Mars, Moon	Bright Star of the Pleiades [Lucida Pleiadum]	23	46	Taurus
16	Jupiter, Saturn	Bend of Cassiopeia [Flexura Cassiopeæ]	1	5	Gemini [Gemin.]
17	Saturn, Mercury	Lowest of the Hyades [Infima Hyadum]	1	48	Gemini
18	Mars	Northern eye of the Bull [Boreus oculus Tauri]	3	47	Gemini
19	Mars, Venus	Aldebaran [Aldeboran]	5	38	Gemini
20	Mars, Mercury	Bright Star Capella [Lucida Capella]	13	44	Gemini
21	Jupiter, Saturn	Bright foot of Orion [Lucidus pes Orionis]	15	22	Gemini

Nu m. .	Nature with planet [Natur. cum plan.]	Name of the star [Denominatio stellarum]	gra d.	mi n. .	sign [sign.]
22	Mars	Northern horn of the Bull [Boreale cornu Tauri]	16	54	Gemini
23	Mars, Mercury	Preceding shoulder of Orion [Præcedens humerus Orionis]	17	28	Gemini
24	Mars, Mercury	Thigh of the Hare [Femur Leporis]	19	5	Gemini

Transit of the 100 Principal Stars across Mid-Heaven for the Year 1630 (continued)

This continues the table of the transit of the hundred principal stars across mid-heaven [medium coeli], computed for the year 1630 after the birth of Christ. The columns give: the row number; the planet or planets of like nature; the name of the star; and its position in degrees, minutes, and zodiacal sign.

Nu m.	Nature (with planets)	Name of star	g r.	mi n.	sign
25	Jup. Sat.	The preceding star of Orion's Belt [Praecedens balthei Orionis]	1 9	17	Gemini
26	Jup. Merc.	The topmost of Orion's head [Suprema capitis Orionis]	1 9	41	Gemini
27	Mars	The southern Horn of the Bull [Australe Cornu Tauri]	1 9	51	Gemini
28	Jup. Sat.	The middle star of Orion's Belt [Media balthei Orionis]	2 0	15	Gemini
29	Jup. Sat.	The lowest star of Orion's Belt [Infima balthei Orionis]	2 1	19	Gemini
30	Merc. Mars	The right Shoulder of the	2	47	Gemini

Nu m.	Nature (with planets)	Name of star	g r.	mi n.	sign
		Charioteer [Dexter humerus Aurigae]	3		
31	Merc. Mars	The following Shoulder of Orion [Sequens humerus Orionis]	2 4	22	Gemini
32	Jup. Mars	Sirius, or the Greater Dog [Seirius seu Canis major]	6	41	Cancer
33	Merc. Mars	The upper Head of Gemini [Superius caput Gemin.]	1 6	16	Cancer
34	Merc. Mars	The Lesser Dog [Canis minor]	1 8	28	Cancer
35	Mars Merc.	The lower Head of Gemini [Inferius caput Gemin.]	1 9	5	Cancer
36	Sat. Jup.	The bright star in the ship's stern [Lucida in puppi navis]	2 5	58	Cancer
37	Mars Luna	The Manger of Cancer [Praesepe Cancrī]	2	29	Leo
38	Mars Sol	The northern Ass [Boreus Asellus]	3	5	Leo
39	Sat. Ven.	The Heart of the Hydra [Cor Hydrae]	1 4	55	Leo
40	Sat. Merc.	The lowest of the Lion's Neck [Infima Cervicis Leonis]	2 4	29	Leo
41	Mars Jup.	Regulus [Basiliscus]	2 5	11	Leo
42	Sat. Merc.	The topmost of the Lion's Neck [Suprema cervicis Leonis]	2 6	44	Leo
43	Sat. Merc.	The middle of the Lion's Neck [Media cervicis Leonis]	2 7	38	Leo
44	Mars	The lower preceding star	8	8	Virgo

Nu m.	Nature (with planets)	Name of star	g r.	mi n.	sign
		of the Bear [Inferior praecedens ☐ Ursae]			
45	Mars	The upper of ☐ Dubhe [Superior ☐ti Dubhe]	8	28	Virgo
46	Sat. Merc.	The bright star of the Lion's Loins [Lucida Lumbi Leonis]	1 2	12	Virgo
47	Mars Jup.	The Lion's Tail [Cauda Leonis]	2 1	55	Virgo
48	Mars	The lower following star of the Bear [Inferior seq. ☐ti Ursae]	2 2	50	Virgo
49	Mars	The upper following star of the Greater Bear [Superior seq. ☐ti Ursae major.]	2 9	12	Virgo
50	Merc. Ven.	The Girdle of Virgo [Cingulum Virginis]	1 0	10	Libra
51	Mars	The root of the Greater Bear's Tail [Radix caudae Ursae majoris]	1 0	13	Libra
52	Sat. Merc.	Vindemiatrix of Virgo [Vindemiatrix Virginis]	1 1	57	Libra
53	Ven. Mars	Spica of Virgo [Spica Virginis]	1 7	52	Libra
54	Mars	The next-to-last of the Bear's Tail [Penultima caudae Ursae]	1 8	30	Libra
55	Mars	The last of the Greater Bear's Tail [Ultima caudae Ursae majoris]	2 5	4	Libra
56	Mars Jup.	Arcturus	1	54	Scorpio
57	Merc. Sat.	The left Shoulder of Boötes [Sinister humerus Bootes]	6	40	Scorpio
58	Jup. Merc.	The southern Scale-pan of	1	5	Scorpio

Nu m.	Nature (with planets)	Name of star	g r.	mi n.	sign
		the Balance [Austrina lanx Librae]	0		
59	Sat. Mars	The northern Scale-pan [Borea lanx]	1 6	47	Scorpio
60	Ven. Merc.	The bright star of the Crown [Lucida Corona]	2 2	12	Scorpio
61	Sat. Mars	The bright star of the Serpent's Neck [Lucida colli Serpentis]	2 3	54	Scorpio
62	Mars Sat.	The northern star of the Scorpion's Forehead [Borealis frontis Scorpionis]	2 8	15	Scorpio
63	Sat. Ven.	The left Hand of Ophiuchus [Sinistra manus Ophiuchi]	1	0	Sagittarius
64	Sat. Merc.	The Heart of the Scorpion [Cor Scorpionis]	3	47	Sagittarius
65	Mercury	The right Shoulder of Hercules [Dexter humerus Herculis]	5	30	Sagittarius
66	Sat. Ven.	The left Knee of Ophiuchus [Sinistrum genu Ophiuchi]	6	7	Sagittarius
67	Sat. Ven.	The right Knee of Ophiuchus [Dextrum genu Ophiuchi]	1 3	30	Sagittarius
68	Mercury	The Head of Hercules [Caput Herculis]	1 5	40	Sagittarius
69	Mercury	The left Shoulder of Hercules [Sinister humerus Herculis]	1 6	8	Sagittarius
70	Sat. Ven.	The Head of Ophiuchus [Caput Ophiuchi]	2 0	17	Sagittarius
71	Sat. Ven.	The right Shoulder of	2	58	Sagittarius

Nu m.	Nature (with planets)	Name of star	g r.	mi n.	sign
		Ophiuchus [Dexter humerus Ophiuchi]	1		
72	Sat. Mars	The bright star of the Dragon's Head [Lucida capitis Draconis]	2 7	17	Sagittarius
73	Ven. Merc.	The bright star of the Lyre [Lucida Lyrae]	5	31	Capricorn
74	Jup. Ven.	The eastern star of the Archer's head [Orientalis capitis Sagit.]	1 1	1	Capricorn

Nu m.	Nature (with planets)	Name of star	g r.	mi n.	sign
75	Mars Jup.	The Tail of the Vulture [Cauda Vulturis]	1 1	10	Capricor n
76	Ven. Merc.	The Beak of the Swan [Rostrum Cygni]	1 7	29	Capricor n
77	Ven. Merc.	The upper Wing of the Swan [Superior ala Cygni]	2 8	20	Capricor n
78	Mars Jup.	The bright star of the Vulture [Lucida vulturis]	2 1	25	Capricor n
79	Ven. Mars	The upper Head of Capricorn [Superius caput Capricorni]	2 7	19	Capricor n
80	Ven. Mars	The lower Head of Capricorn [Inferius caput Capricorni]	2 7	58	Capricor n
81	Mars	The new star in the Swan's breast [Nova stella in pectore Cygni]	2 8	40	Capricor n
82	Merc. Sat.	The left Hand of the Water- bearer [Sinistra manus Aquarii]	4	35	Aquarius

Nu m.	Nature (with planets)	Name of star	g r.	mi n.	sign
83	Ven. Merc.	The Breast of the Swan [Pectus Cygni]	4	35	Aquarius
84	Ven. Merc.	The Tail of the Swan [Cauda Cygni]	4	53	Aquarius
85	Ven. Merc.	The lower Wing of the Swan [Inferior ala Cygni]	5	26	Aquarius
86	Sat. Merc.	The left Shoulder of the Water-bearer [Sinister humerus Aquarii]	1 5	32	Aquarius
87	Jup. Sat.	The preceding star of Capricorn's tail [Praecedens cauda Capricorni]	1 7	27	Aquarius
88	Sat. Jup.	The Girdle of Cepheus [Cingulum Cephei]	1 8	27	Aquarius
89	Mars Jup.	The Mouth of Pegasus [Os Pegasi]	1 9	8	Aquarius
90	Jup. Sat.	The following star of Capricorn's tail [Sequens cauda Capricorni]	1 6	16	Aquarius
91	Merc. Mars	The right Shoulder of the Water-bearer [Dexter humerus Aquar.]	2 4	24	Aquarius
92	Ven. Mars	Fomalhaut [Fomahant]	7	29	Pisces
93	Mars Ven.	Scheat of Pegasus [Scheat Pegasi]	9	58	Pisces
94	Mars Jup.	Markab of Pegasus [Marchab Pegasi]	1 0	5	Pisces
95	Merc. Sat.	The Back of the head of the southern Fish [Occiput Piscis australis]	1 3	11	Pisces
96	Venus	The Head of Andromeda [Caput Andromades]	2 7	8	Pisces
97	Sat. Ven.	The bright star of Cassiopeia's seat [Lucida	2 7	14	Pisces

Nu m.	Nature (with planets)	Name of star	g r.	mi n.	sign
		cathed. Cassiop.]			
98	Mars Merc.	The far tip of the Wing of Pegasus [Extrema ala Pegasi]	2 8	30	Pisces
99	Saturn	The northern star of the Whale's tail [Borealis cauda Cete]	0	13	Aries
10 0	Sat. Jup.	The Head of Cassiopeia [Caput Cassiopeae]	4	30	Aries

True Risings and Settings of the Said Stars

ORTUS ET OCCASUS VERI PRÆMISSARUM STELLARUM — The true risings and settings of the aforesaid stars, likewise computed for the year 1630 after the birth of Christ, for as many of them as occur at the elevation of the pole of 56 degrees.

Here the "Num." column gives the reference number of the star as listed in the preceding mid-heaven table; the position is given in degrees and minutes of the relevant sign, with "or." (*oritur*) marking a rising and "occ." (*occidit*) marking a setting.

Num .	Name of star	gr.	min.	sign	event
94	Markab of Pegasus [Marchab Pegasi]	1	23	Aries	setting
6	The Belly of the Whale [Venter Cete]	3	14	Aries	setting
97	The far tip of the Wing of Pegasus [Extrema alae Pegasi]	11	11	Aries	setting
85	The lower Wing of the Swan [Inferior ala Cygni]	12	58	Aries	setting
9	The bright star at the knot of the Fishes' cord [Lucida in nodo lini Pisc.]	18	2	Aries	setting
93	Scheat of Pegasus [Scheat Pegasi]	18	36	Aries	setting
24	The Thigh of the Hare [Femur Leporis]	28	36	Aries	setting
12	The bright star of the Whale's jaw [Lucida mandibula Cete]	29	34	Aries	setting

Num .	Name of star	gr.	min.	sign	event
96	The Head of Andromeda [Caput Andromades]	0	47	Taurus	setting
5	The preceding Horn of the Ram [Praecedens cornu Arietis]	3	44	Taurus	setting
15	The bright star of the Pleiades [Lucida Pleiadum]	5	43	Taurus	rising
7	The following Horn of the Ram [Sequens cornu Ariet.]	5	45	Taurus	setting
21	The bright star of Orion's foot [Lucidus pes Orionis]	10	18	Taurus	setting
10	The bright star of the Ram above the head [Lucida Arietis supra caput]	11	12	Taurus	setting
99	The northern star of the Whale's tail [Borealis cauda Cete]	12	22	Taurus	rising

Num .	Name of star	gr.	min.	sign	event
32	The Greater Dog, Sirius [Canis major Scirius]	17	16	Taurus	setting
28	The middle star of Orion's Belt [Media balthei Orionis]	20	50	Taurus	setting
29	The lowest star of Orion's Belt [Infima balthei Orionis]	20	56	Taurus	setting
25	The preceding star of Orion's Belt [Praecedens balthei Orionis]	21	0	Taurus	setting
36	The bright star in the ship's stern [Lucida in puppi Navis]	21	36	Taurus	setting
17	The lowest of the Hyades of the Bull [Infima Hyadum Tauri]	24	24	Taurus	setting
23	The preceding Shoulder of Orion [Praecedens humerus Orionis]	26	13	Taurus	setting
22	The northern Horn of the Bull [Boreale cornu Tauri]	27	7	Taurus	rising

Num	Name of star	gr.	min.	sign	event
19	Aldebaran [Aldeboran]	28	17	Taurus	setting
9	The bright star at the knot of the Fishes' cord [Lucida nodi lini Piscis]	29	17	Taurus	rising
15	The bright star of the Pleiades [Lucida Pleiadum]	29	1	Taurus	setting
3	The Girdle of Andromeda [Cingulum Andromades]	0	8	Gemini	setting
18	The northern Eye of the Bull [Boreus oculus Tauri]	0	0	Gemini	setting
26	The topmost of Orion's head [Suprema capitis Orionis Gem.]	1	42	Gemini	setting
31	The following Shoulder of Orion [Sequens humerus Orionis]	2	41	Gemini	setting
18	The northern Eye of the Bull [Boreus oculus Tauri]	11	10	Gemini	rising
33	The upper Head of Gemini [Superius caput Gemin.]	15	24	Gemini	rising
27	The southern Horn of the Bull [Australe cornu Tauri]	16	3	Gemini	setting
12	The bright star of the Whale's jaw [Lucida mandibula Cete]	16	34	Gemini	rising
17	The lowest of the Hyades of the Bull [Infima Hyadum Tauri]	16	35	Gemini	rising
2	The southern star of the Whale's tail [Australis cauda Cete]	19	1	Gemini	rising
19	Aldebaran [Aldeboran]	19	9	Gemini	rising
0	The bright star of Gemini's foot [Lucidus pes gemin.]	21	37	Gemini	setting
34	Procyon	20	39	Gemini	setting
6	The Belly of the Whale [Venter Cete]	22	15	Gemini	rising
27	The southern Horn of the Bull [Australe cornu Tauri]	24	52	Gemini	rising
92	Fomalhaut [Fomahant]	24	59	Gemini	rising

Num	Name of star	gr.	min.	sign	event
39	The Heart of the Hydra [Cor Hydrae]	27	2	Gemini	setting
22	The northern Horn of the Bull [Boreale cornu Tauri]	28	31	Gemini	setting
35	The lower Head of Gemini [Inferius caput Gemin.]	5	16	Cancer	rising
26	The topmost of Orion's head [Suprema capitis Orionis]	12	2	Cancer	rising
23	The preceding Shoulder of Orion [Praecedens hum. Orionis]	14	29	Cancer	rising
0	The bright star of Gemini's foot [Lucidus pes Gemin.]	14	45	Cancer	rising
31	The following Shoulder of Orion [Sequens humerus Orionis]	18	37	Cancer	rising
26	The preceding star of Orion's Belt [Praecedens Balth. Orionis]	23	10	Cancer	rising
28	The middle star of Orion's Belt [Media Balth. Orionis]	24	48	Cancer	rising
29	The lowest star of Orion's Belt [Infima Balth. Orionis]	26	12	Cancer	rising
38	The northern Ass [Boreus Asellus]	28	31	Cancer	rising
21	The bright star of Orion's foot [Lucidus pes Orionis]	28	46	Cancer	rising
37	The Manger of Cancer [Praesepe Cancri]	0	44	Leo	rising
0	The southern Ass [Australis Asellus]	3	24	Leo	rising
0	The same southern Ass [Idem Australis Asellus]	3	43	Leo	setting
37	The Manger of Cancer [Praesepe Cancri]	7	6	Leo	setting
34	The Lesser Dog [Canis minor]	7	52	Leo	rising
41	The topmost of the Lion's Neck [Suprema cervicis Leonis]	10	22	Leo	rising
38	The northern Ass of Cancer [Boreus	14	6	Leo	setting

Num .	Name of star	gr.	min.	sign	event
	asellus Cancr[i]				
24	The Thigh of the Hare [Femur Leporis]	15	52	Leo	rising
43	The middle of the Lion's Neck [Media cervicis Leonis]	16	22	Leo	rising
40	The lowest of the Lion's Neck [Infima cervicis Leonis]	18	29	Leo	rising
35	The lower Head of Gemini [Inferius caput Geminorum]	19	57	Leo	setting

Nu m.	Name of star	gr.	min .	sign	event
32	The Greater Dog [Canis major]	22	15	Leo	rising
46	The bright star of the Lion's Loins [Lucida lumb. Leonis]	24	7	Leo	rising
42	Regulus [Basiliscus]	24	18	Leo	rising
42	Regulus [Basiliscus]	27	2	Leo	setting
47	The Lion's Tail [Cauda Leonis]	7	42	Virgo	rising
39	The Heart of the Hydra [Cor Hydrae]	8	49	Virgo	rising
36	The bright star in the ship's stern [Lucida in puppi navis]	15	9	Virgo	rising
40	The lowest of the Lion's Neck [Infima cervicis Leonis]	19	51	Virgo	setting
52	Vindemiatrix	24	2	Virgo	rising
56	Arcturus	26	30	Virgo	rising
60	The bright star of the Crown [Lucida Coronae]	28	30	Virgo	rising
33	The upper Head of Gemini [Superius caput Geminorum]	29	30	Virgo	setting
50	The Girdle of Virgo [Cingulum Virginis]	0	48	Libra	rising
53	Spica of Virgo [Spica Virginis]	8	0	Libra	setting

Nu m.	Name of star	min		sign	event
		gr.	.		
65	The right Shoulder of Hercules [Dexter humerus Herculis]	17	12	Libra	rising
43	The middle of the Lion's Neck [Media cervicis Leonis]	18	0	Libra	setting
53	Spica of Virgo [Spica Virginis]	20	4	Libra	rising
69	The left Shoulder of Hercules [Sinister hum. Herculis]	20	5	Libra	rising
61	The bright star of the Serpent's Neck [Lucida colli Serpentis]	26	27	Libra	rising
68	The Head of Hercules [Caput Herculis]	3	48	Scorpio	rising
69	The lower Wing of the Swan [Inferior ala Cygni]	6	57	Scorpio	rising
85	The northern Scale-pan of the Balance [Borea lanx Librae]	7	3	Scorpio	rising
41	The topmost of the Lion's Neck [Suprema cerv. Leonis]	7	30	Scorpio	setting
76	The Beak of the Swan [Rostrum Cygni]	9	3	Scorpio	rising
70	The Head of Ophiuchus [Caput Ophiuchi]	9	15	Scorpio	rising
58	The southern Scale-pan [Austrina lanx]	9	34	Scorpio	rising
63	The left Hand of Ophiuchus [Sinistra manus Ophiuchi]	11	16	Scorpio	rising
58	The southern Scale-pan of the Balance [Austrina lanx Librae]	11	52	Scorpio	setting
64	The Heart of the Scorpion [Cor Scorpionis]	15	0	Scorpio	setting
50	The Girdle of Virgo [Cingulum Virginis]	16	14	Scorpio	setting
71	The right Shoulder of Ophiuchus [Dexter humerus Ophiuchi]	18	48	Scorpio	rising

Nu m.	Name of star	min		sign	event
		gr.	.		
47	The Lion's Tail [Cauda Leonis]	19	32	Scorpio	setting
66	The left Knee of Ophiuchus [Sinistrum genu Ophiuchi]	22	1	Scorpio	rising
75	The Tail of the Vulture [Cauda Vulturis]	23	43	Scorpio	rising
46	The bright star of the Lion's Loins [Lucida lumbi Leonis]	24	53	Scorpio	setting
62	The northern star of the Scorpion's Forehead [Borealis Frontis Scorpionis]	26	51	Scorpio	rising
62	The northern star of the Scorpion's Forehead [Borealis Frontis Scorpionis]	1	44	Sagittarius	setting
67	The right Knee of Ophiuchus [Dextrum genu Ophiuchi]	3	30	Sagittarius	rising
78	The bright star of the Vulture [Lucida Vulturis]	7	11	Sagittarius	rising
59	The northern Scale-pan of the Balance [Borealis lanx Librae]	11	15	Sagittarius	rising
64	The Heart of Virgo [Cor Virginis]	11	9	Sagittarius	setting
52	Vindemiatrix of Virgo [Vindemiatrix Virginis]	9	10	Sagittarius	setting
3	The Girdle of Andromeda [Cingulum Andromades]	16	12	Sagittarius	rising
93	Scheat of Pegasus [Scheat Pegasi]	17	17	Sagittarius	rising
89	The Mouth of Pegasus [Os Pegasi]	28	9	Sagittarius	rising
96	The Head of Andromeda [Caput Andromades]	28	10	Sagittarius	rising
67	The right Knee of Ophiuchus [Dextrum genu Ophiuchi]	28	49	Sagittarius	setting
66	The left Knee of Ophiuchus [Sinistrum genu Ophiu.]	29	40	Sagittarius	setting
63	The left Hand of Ophiuchus	4	24	Capricorn	setting

Nu		min			
m.	Name of star	gr.	.	sign	event
	[Sinistra manus Ophiu.]				
74	The eastern star of the Archer's head [Orient. cap. Sagittarii]	7	30	Capricorn	rising

True risings and settings of the 100 stars (conclusion)

[This is the continuation and conclusion of the star table begun in the previous chapter, computed for 1630 at a pole elevation of 56°. Columns: number | star name | degrees (gr.) | minutes (min.) | sign (sig.) | rising (or. = oriens, true rising) / setting (occ. = occidens, true setting).]

Nu m.	Star name	min		sign	or./occ.
		gr.	.		
79	The bright star in the neck of the Serpent [Lucida colli Serpentis]	10	16	Capricorn	rising
61	The bright star in the neck of the Serpent [Lucida colli Serpentis]	11	16	Capricorn	setting
94	Markab of Pegasus [Marchab Pegasi]	11	10	Capricorn	rising
16	Arcturus	12	29	Capricorn	setting
74	The eastern star of the head of the Archer [Orient. caput Sagittarii]	13	32	Capricorn	rising
82	The left hand of the Water-bearer [Sinistra manus Aquarii]	13	49	Capricorn	rising
80	The lower horn of Capricorn [Inferius Capricorn. cornu]	15	36	Capricorn	rising
86	The left shoulder of the Water-bearer [Sinister humerus Aquarii]	20	17	Capricorn	rising
91	The right shoulder of the Water-bearer [Dexter humerus Aquarii]	21	49	Capricorn	rising
71	The right shoulder of Ophiuchus [Dexter humerus Ophiuchi]	29	29	Capricorn	setting
92	Fomalhaut [Fomahant]	3	8	Aquarius	setting
80	The lower horn of Capricorn [Inferius cornu Capricorni]	4	2	Aquarius	setting

Nu m.	Star name	gr.	min .	sign	or./occ.
68	The head of Hercules [Caput Herculis]	5	44	Aquarius	setting
79	The tip of the wing of Pegasus [Extrema alæ Pegasi]	5	22	Aquarius	rising
97	The upper horn of Capricorn [Superius cornu Capricorni]	6	17	Aquarius	setting
70	The head of Ophiuchus [Caput Ophiuchi]	6	49	Aquarius	setting
91	The right shoulder of Ophiuchus and of the Water-bearer [Dexter hum. Ophiuchi Aquar.]	8	12	Aquarius	setting
60	The bright star of the Crown [Lucida Coronæ]	8	28	Aquarius	setting
95	The southern head of the Fish [Caput australe Piscis]	13	33	Aquarius	rising
82	The left hand of the Water-bearer [Sinistra manus Aquarii]	14	14	Aquarius	setting
87	The preceding star of the tail of Capricorn [Præcedens cauda Capricorni]	14	23	Aquarius	setting
90	The following star of the tail of Capricorn [Sequens cauda Capricorni]	16	8	Aquarius	setting
69	The left shoulder of Hercules [Sinister humerus Herculis]	20	10	Aquarius	setting
75	The tail of the Vulture [Cauda Vulturis]	22	18	Aquarius	setting
78	The bright star of the Vulture [Lucida Vulturis]	23	47	Aquarius	setting
86	The left shoulder of the Water-bearer [Sinister humerus Aquarii]	25	17	Aquarius	setting
87	The preceding star of the tail of Capricorn [Præcedens cauda Capricorni]	28	34	Aquarius	rising

Nu m.	Star name	gr.	min .	sign	or./occ.
90	The following star of the tail of Capricorn [Sequens cauda Capricorni]	0	55	Pisces	rising
10	The bright star of the Ram [Lucida Arietis]	2	20	Pisces	rising
91	The right shoulder of the Water-bearer [Dexter humerus Aquarii]	6	2	Pisces	setting
7	The following horn of the Ram [Sequens cornu Arietis]	8	54	Pisces	rising
2	The southern star in the tail of the Whale [Australis cauda Cete]	12	28	Pisces	setting
89	The mouth of Pegasus [Os Pegasi]	12	43	Pisces	setting
76	The beak of the Swan [Rostrum Cygni]	15	52	Pisces	setting
5	The preceding horn of the Ram [Præcedens cornu Arietis]	16	30	Pisces	setting
99	The northern star in the tail of the Whale [Borealis cauda Cete]	20	0	Pisces	setting
95	The southern back of the head of the Fish [Occiput australe Piscis]	20	34	Pisces	setting

PROBLEM III.

Given the point of the true rising or setting of a star on the ecliptic, together with the angle of inclination between the horizon and the ecliptic at that given point, and a definite measure of the depression of the Sun below the horizon: to find the heliacal rising and setting of that same star.

The heliacal risings and settings of stars are worth treating, since they appear frequently in the poets, and among the astrologers they are not without their significance for forecasting the weather — nor were they so among the ancients, in the annual reckoning of the seasons for farm work. It remains, therefore, to show how they are to be determined, and likewise how the place of the Sun on the ecliptic is to be found at the same time. This matter is of such importance that it extends not only to the fixed stars but also to the wandering stars, the planets, whose disappearance under the Sun's rays

[Margin: Lib. 1. Sphær. cap. 5.]

is the heliacal setting; their re-emergence from those same rays, or first reappearance, is in turn called the heliacal rising — as both were defined earlier. We shall illustrate the matter with examples.

Suppose we are to find the heliacal rising of Sirius [Seirii], with which the Dog Days are believed to end. Since this star is of the first magnitude, or rank — for it appears the largest of all the fixed stars — then, according to what we set out in the preceding book of the *Spherics*, it requires a depression of the Sun of only 12° below the eastern horizon for its first appearance. Granting this, a diagram very nearly like the one we set out above for the star's true rising will be constructed — except that a circle of altitude is now drawn in, on which the appropriate distance of the star from the Sun is measured through the zenith and the place of the Sun.

Let everything be as before, and let the true rising of Sirius be at D, to which corresponds the point H on the ecliptic. From this point a circle of altitude A O is laid out, so that the interval between the horizon and the ecliptic is $T V = 12^\circ$, just as that arc was determined before.

Diagram. A geometric construction on the celestial sphere for finding the heliacal rising of Sirius. A large circle (the sphere) carries several great-circle arcs (horizon, ecliptic, meridian, and a circle of altitude) intersecting at the lettered points. Smaller circles appear at the lower left (around B and C). The points labelled, reading roughly clockwise from the top, are: O (top of the sphere / pole of altitude circle), R and P (upper left), M (upper middle), N (just below M), H with T close beside it and V just below (right of center, the ecliptic point and altitude interval), K (center), D (center-left, the true rising), L (left), S (lower right, the star's measured point), G (right edge), F and E (lower right), A (bottom), B and C (lower left, within the small circles).

- O (top): pole of the altitude circle
- R (upper left): label R
- P (upper left): label P
- M (upper middle): label M
- N (below M): label N
- H (right of center): ecliptic point corresponding to the true rising
- T (beside H): point on the horizon
- V (below H/T): point marking the 12° altitude interval
- K (center): label K

- D (center-left): the true rising of Sirius
- L (left): label L
- S (lower right): measured point of the star on the altitude circle
- G (right edge): label G
- F (lower right): label F
- E (lower right): label E
- A (bottom): foot of the altitude circle
- B (lower left): label B
- C (lower left): label C

So the arc to be found on the ecliptic is HV , against which, in the right triangle HTV , stands the angle THV — namely the inclination of the horizon and the ecliptic. This is obtained either from the preceding problem, as the complement of that angle, or, more concisely in cases like the present, in the right triangle DMH . In that triangle, since the following are given:

- $DM = 39^\circ 30'$,
- the latitude of Sirius $MH = 42^\circ 52'$,
- the difference of longitude of the ascending point, with DMH a right angle,

then $MHD = 50^\circ 28'$ will not be hidden; and equal to this is the angle THV at the zenith.

In this right triangle, then, to find the sought arc HV , the following are given:

- $TV = 12^\circ 0'$, the depression of the Sun below the horizon, by hypothesis;
- $THV = 50^\circ 28'$;
- HTV a right angle.

Therefore $HV = 15^\circ 38'$ is given. Adding this to the point of oblique ascension of Sirius, $21^\circ 53'$ Leo, the result is $7^\circ 31'$ Virgo — that is, the point which, when the Sun occupies it, marks where Sirius emerges heliacally. The Sun's place agrees with this in the ephemerides at the beginning of the 21st day of August in our age, when the Dog Days come to an end.

Example II.

[Margin: Virg. 1. Georg.]

We shall take this example from the first book of Virgil's *Georgics*, where the time for sowing wheat and other hardy grains after autumn is described by reference to the morning heliacal setting of the Pleiades, thus:

But if it is for a crop of wheat and hardy spelt that you work the soil, intent on grain alone, first let the Eoan daughters of Atlas set, and let the Cnossian star of the blazing Crown withdraw, before you commit to the furrows the seed they are owed, and trust to a reluctant earth the whole year's hope.

[Margin: Plin. l. 18.]

From this description, and at the same time from Pliny's [Plinii] interpretation, the morning setting of the Pleiades is gathered — and indeed a heliacal one, since Virgil [Virgilius] hints at this by the word "set" ["abscondantur"]:

first of all, by the motion of the stars over an interval of about 1620 years — which is the span elapsed from Virgil's time down to ours — let us reduce the longitude of the middle star of the Pleiades back to the age of Virgil, in this way. [Margin: Per motum stellarum Tychonis Brahe.] One hundred years yield $1^{\circ} 25\frac{1}{3}'$; therefore 1620 years yield $22^{\circ} 57'$. The true longitude of the bright star of the Pleiades being $24^{\circ} 24'$ Taurus in the year 1600, subtract $22^{\circ} 57'$, and the longitude of the middle star of the Pleiades in Virgil's time comes out as $1^{\circ} 27'$ Taurus; its invariable latitude is $4^{\circ} 0'$ N, of the 3rd magnitude.

Furthermore, we take the obliquity of the Zodiac at that time as $23^{\circ} 50'$, and the elevation of the pole among the Romans as $42^{\circ} 0'$.

With these things settled, let the ancient assigned place of the middle of the Pleiades be carried over to its setting on the globe or sphere, and let the following diagram be drawn according to its arrangement.

With the rest disposed as before, let D be the middle star in the Pleiades, and H the corresponding point on the ecliptic. Let A O be drawn, representing the semicircle of altitude. And since the Sun is understood to be depressed 14° below the eastern horizon — the star being of the third magnitude — we therefore set the arc above the horizon on the other side just so great that here we may aim at the point opposite the Sun; and so the arc Z S is that 14° .

Diagram. A geometric construction on the celestial sphere for the heliacal setting of the middle star of the Pleiades. A large circle (the sphere) is crossed by several great-circle arcs (horizon, ecliptic, meridian, altitude circle); a small circle appears at the upper left (around B and C). The points labelled, reading roughly from top and around, are: A (top), R and Q (upper right), B and C (upper left, within the small circle), G (left edge), D (center), the Cancer symbol ♋ with figures near center-right, S (center-right), H with Z below it (center), K (right of H), L

(right edge), P (lower center), N (below P), the Aries symbol Υ (lower center), E and F (lower left), O (bottom).

- A (top): top of the sphere
- R (upper right): label R
- Q (upper right): label Q
- B (upper left): label B
- C (upper left): label C
- G (left edge): label G
- D (center): the middle star of the Pleiades
- ☿ (center-right): tropic of Cancer marker
- S (center-right): label S
- H (center): the ecliptic point corresponding to D
- Z (below H): point on the altitude circle
- K (right of H): label K
- L (right edge): label L
- P (lower center): label P
- N (below P): label N
- Υ (lower center): Aries marker
- E (lower left): label E
- F (lower left): label F
- O (bottom): foot of the altitude circle

For the sake of thoroughness [Greek: προηγουμένως (proēgoumenōs, "by way of leading through / in advance")], let me go over the same construction again for the benefit of students, so that they may have the whole procedure [Greek: πράξιν (praxin, "operation / working")] in view, both for the true and for the heliacal rising and setting. First, then, since the declination D N is unknown, it is found from the first problem of chapter 2: for in the triangle B C D, for B D — the complement of the declination — the following are given:

- B C = $23^{\circ} 50'$, the distance of the poles;
- C D = $86^{\circ} 0'$, the complement of the latitude of D [the Pleiades];
- B C D = $58^{\circ} 33'$, the distance of longitude of the Pleiades from the tropic of Cancer, which is here measured by the arc M Y.

Therefore B D = $74^{\circ} 5'$ is given, the complement of the declination; and so the declination itself D N = $15^{\circ} 55' \text{ N}$.

II.

In the triangle B D C, for the angle C D B, the three sides are given, namely:

- B D = $74^{\circ} 5'$, the complement of the declination;
- C D = $86^{\circ} 0'$, the complement of the latitude;
- B C = $23^{\circ} 50'$, the distance of the poles.

Therefore C D B = $21^{\circ} 0'$ is given.

III.

In the right triangle B G D, for the angle B D G, the following are given:

- B D = $74^{\circ} 5'$, the complement of the declination;
- B G = $42^{\circ} 0'$, the elevation of the Roman pole;
- B G D = 90° , a right angle.

Therefore B D G = $44^{\circ} 5\frac{1}{4}'$. From this is subtracted the angle C D B = $21^{\circ} 0'$, and there remains the angle C D G = $23^{\circ} 5\frac{1}{4}'$; equal to this is the angle H D M.

IIII.

Moreover, in the right triangle H M D, two things are to be sought: the side M H, and the angle at H. First, then, for the side M H, in the same triangle:

- D M = $4^{\circ} 0'$, the latitude;
- M D H = $23^{\circ} 5\frac{1}{4}'$, the angle just found;
- D M H = 90° , a right angle.

Therefore M H = $1^{\circ} 42'$ is given. To this is added the true longitude of the star at M, $1^{\circ} 27'$ Taurus, and the true setting of the star is established at $3^{\circ} 9'$ Taurus.

V.

In the same triangle D H M the angle at H is obtained, $67^{\circ} 0'$; equal to this is the angle Z H S.

VI.

In the right triangle S H Z, for H S:

- S Z = $14^{\circ} 0'$, by hypothesis;
- S H Z = $67^{\circ} 0'$;
- S Z H = a right angle.

Therefore $H S = 15^{\circ} 14'$. Adding to it the rising point H, namely $3^{\circ} 9'$ Taurus, the point S on the ecliptic is shown to be $18^{\circ} 23'$ Taurus; the Sun at its rising will hold the point opposite to this, namely $18^{\circ} 23'$ Scorpio — at which time the Eoan daughters of Atlas, that is the Pleiades, are submerged into their setting in the morning twilight. This formerly happened at Rome around the 16th day of November, or, as the Julian calendar now stands, around the last day of October.

For the rest, if anyone should henceforth wish to make the trial and to compute the time for sowing wheat and so on from the places the stars occupy today, he will find in this example — to which the other cases will fit more or less, because of the varying change of declination in the stars — that the same event now occurs at Rome around the 24th day of November, which is too close to the winter solstice and oversteps the suitable time for sowing. Therefore the risings and settings of the stars for the practice of agriculture would now have to be obtained afresh by astronomical calculation — if they were to be sought from these signs — for the pole elevation of the region in question.

CHAPTER V.

On matters concerning the investigation of time from observations of the Sun and stars.

In the preceding chapter, on the rising and setting of the heavenly bodies, we set out beforehand the method of determining the length of day and night for each season of the year — since the variation in each, above and below 12 hours, depends solely on the ascensional difference converted into time. Now, therefore, it seems opportune to investigate any small portion of time — any hours whatever, and the minutes of hours — by various kinds of observation: of the Sun during the day, and of the stars at night. For this matter too has its necessary use in what follows, and besides a pleasant delight, in that it invites the young student of mathematics to the work of observing. Throughout this chapter there is a reciprocal relation of times with motions, just as we attached this to the general definition of astronomy.

But it must be carefully noted that, in this practical work, we should not observe the Sun or the stars too near either the horizon or the meridian: in the former case because of the insinuation of refractions; in the latter because the altitude to be observed undergoes too little change from the meridian to be perceptible, even over a long time.

Liber II — Spherics, Cap. V (continued)

PROBLEM I.

Given the altitude of the Sun in a vertical circle, measured by any instrument or by any means whatever, together with the Sun's declination and the elevation of the pole of the place, the moment of time in hours and minutes corresponding to it is found — and conversely.

Example I.

Let the Sun be in 3° of Leo [ϱ], which is the position appropriate to today, namely 16 July. By Proposition 1, Chapter 1 of this book — or from the table of the Sun's declination — its declination is found to be $19^\circ 34'$ north [B = borealis]. Now since the elevation of the pole in our region is reckoned at $55^\circ 43'$, its complement — namely $34^\circ 17'$ — is the elevation of the equator above our horizon. If to this we add the Sun's declination when northern (or subtract it when southern), we always obtain, one way or the other, the Sun's meridian altitude. So in the present example: add $34^\circ 17'$, the elevation of the equator, to $19^\circ 34'$, the Sun's northern declination, and the result is $53^\circ 51'$ for the Sun's meridian altitude. But what we are trying to do is to determine the time away from noon from an observed altitude of the Sun.

So let the Sun's altitude be observed first by a quadrant — or by another instrument serving in place of a quadrant — at an afternoon hour of this day, as 40° , with the remaining elements fitted to the following diagram constructed from our sphere, in this manner.

Diagram. A sphere shown in oblique projection for determining the time from the Sun's observed altitude. The points are lettered A, B, C, D, E, F, G, H, I, K, L, M, N.

- Meridian (whole circle): A L F G H
- Horizon: I G, whose pole is A
- Equator: H F, whose pole is B
- I B: the elevation of the pole; its complement I H is the depression of the equator below the northern horizon
- F G: equal to I H — the elevation of the equator above the southern horizon
- F L: the Sun's declination in the meridian, $19^\circ 34'$
- G L: the Sun's meridian altitude, defined earlier as $53^\circ 51'$

- L C N M: a parallel drawn through the western semicircle, representing the Sun's declination everywhere from the equator
- C: the position of the Sun according to the first revolution, through which quadrants descend from each pole onto their great circles
- E C: the Sun's given altitude, 40°
- D C: the given declination, $19^\circ 34'$
- D F: the arc to be found, by which (once converted into time) the hours and minutes of the Sun from noon will be established

Let the meridian as a whole be A L F G H; the horizon I G, whose pole is A; the equator H F, whose pole is B. Since the elevation of the pole is I B, its complement I H will be the depression of the equator below the northern horizon; and equal to this is F G, the elevation of the equator above the southern horizon. Take the Sun's declination in the meridian, F L, as $19^\circ 34'$; then G L will be the Sun's meridian altitude, defined earlier as $53^\circ 51'$. Now draw, through the western semicircle, the parallel L C N M, which represents the Sun's declination everywhere from the equator; and place the Sun's position, according to the first revolution, at C, through which point quadrants descend from each pole onto their great circles. Then E C will be the Sun's given altitude, 40° , and D C the given declination, $19^\circ 34'$; but the arc we are striving to find is D F, by which — once converted into time — the hours and minutes of the Sun from noon will be established. And since the angle A B C measures that arc, it is to be taken as equivalent to it. So, in triangle A B C, the givens [διδόμενα, "the data"] are:

- A B = $34^\circ 17'$, the distance between the poles of the horizon and the equator;
- B C = $70^\circ 26'$, the complement of the Sun's declination;
- A C = $50^\circ 0'$, the complement of the Sun's altitude.

Therefore A B C — that is, D F — is $46^\circ 23'$. This arc, converted into hours and minutes of hours according to what is taught in the second problem of Chapter 3, shows the time from noon corresponding to the Sun's altitude of 40° to be 3 hours 5 minutes 32 seconds.

If, conversely, one wishes to obtain the Sun's altitude from a given time, one first resolves the time — that is, the hours and minutes of hours — into degrees and minutes of degrees. Then in triangle A B C, since two sides are given together with the angle at B that they contain, the side A C is easily found, and therefore also its complement C E.

Moreover, the Sun's altitude can also be observed from the measure of a shadow cast by a staff — or by any oblong body set up perpendicular to the horizon — provided the height of that body, together with the said shadow, stands in a fixed and definite ratio.

Take an example with a staff conveniently divided into 10 equal parts; in the adjoining figure this will be A B, set up at right angles upon the plane of the horizon, D C. Let the shadow cast by the Sun's rays be A C, 12 such parts as the staff is 10. Then in the plane right triangle, since two sides about the right angle are given:

Diagram. A plane right triangle for finding the Sun's altitude from the staff-and-shadow ratio.

- B A: the vertical staff, set perpendicular on the horizon plane D C
- A C: the horizontal shadow cast along the ground
- B A C: the right angle at the foot of the staff
- B C A: the hypotenuse direction toward the Sun; the angle here gives the Sun's altitude

The data:

- B A = 10 parts, the height of the staff;
- A C = 12 parts, the length of the shadow;
- B A C = the right angle.

This gives B C A = $39^{\circ} 48'$, the Sun's altitude sought.

However, in observations of solar altitudes of this kind, where the shadow falls from the summit of an upright pointer, one must always subtract from the angle obtained the apparent semidiameter of the Sun. This is because the Sun's rays, issuing from its uppermost edge and striking the tip of the pointer, render the shadow shorter than it properly should be — [Margin: See Gemma Frisius, *On the Use of the Astronomical Staff*.] as the opticians demonstrate in the case of the shadow on a wall. The Sun's true altitude observed here is therefore reckoned to have been $39^{\circ} 32'$, for which it will be easy to derive the corresponding time according to the tenor of the problem — that is, the arc of the equator intercepted between the Sun's position and the meridian — once it has been established whether the observation was made before or after noon.

Indeed, the altitude of the irradiating Sun can also be obtained by means of our own stature, from which the time is elicited. For since it is established that a

man's measure is seven feet, very nearly equal to itself: with the body set up as nearly perpendicular as possible upon a level surface, one must observe where the shadow cast by the Sun's rays from the crown of the head is marked off on the horizon plane, then measure it by one's own feet. In this way a man's stature serves in place of the pointer or staff, so that — just as in the previous example — the Sun's altitude is revealed. For just as there the 10 parts claimed for themselves the first place in the rule of proportion, so here do the 7 feet, from which first the complement of the angle, and then the angle of the Sun's altitude itself at C, becomes known.

An example of this kind.

Let someone's shadow cast in the forenoon be $10\frac{1}{2}$ feet. To ascertain the Sun's altitude from this, there will be — as before — in the plane right triangle A B C:

- A B = 7 feet, the height of the man;
- A C = $10\frac{1}{2}$, the shadow cast;
- B A C = the right angle.

Hence B C A = $33^{\circ} 41'$, the altitude of the projected Sun is given.

With the altitude found, take now for the time: let the Sun be in $9\frac{1}{2}^{\circ}$ of Leo [♌], roughly at 23 July in our present century. Then its declination will be $17^{\circ} 56'$. And the elevation of the pole is given as $55^{\circ} 43'$.

In the diagram set out above, then — this being a forenoon time — with all the known quantities retained as above, for the angle A B C, which measures the Sun's distance D E on the equator from the meridian, the givens [διδόμενα] in triangle A B C are:

- A B = $34^{\circ} 17'$, the distance between the poles;
- B C = $72^{\circ} 4'$, the complement of the Sun's declination;
- A C = $56^{\circ} 19'$, the complement of the Sun's altitude.

Therefore A B C is found to be $55^{\circ} 56'$. This angle, converted into time in the usual way, gives 3 hours 43 minutes 44 seconds, which is the Sun's distance from noon expressed as time. If this is subtracted from 12 hours, it leaves the customary time in hours and minutes, etc., as 8 hours 16 minutes 16 seconds in the forenoon.

Diagram. A sphere in oblique projection, the forenoon counterpart of the figure on p. 105, with the same lettering for finding the time from the Sun's altitude. The points are lettered A, B, C, D, E, F, G, H, I, K, L, M, N.

- A: pole of the horizon (top)
- B: pole of the equator
- C: the Sun's position
- D, E: points marking the Sun's distance on the equator from the meridian
- F, G, H, I, K, L, M, N: meridian, horizon, equator, and parallel-circle intersections as in the earlier figure

PROBLEM II.

Given the right ascension of the Sun, and that of any star passing across the meridian of the place, to find the time at night.

This problem we put first because, of all those that follow which teach how to elicit the time at night from observations of the stars or the Moon, it is the simplest: it does not use the calculation of triangles, and it paves the way for the rest.

For once the right ascension of the mid-heaven [medium coelum] has been found from the transit of a star, and the Sun's right ascension has been subtracted from it, there remains the arc of the equator intercepted between the Sun and the star; and this, converted into time, gives the hours and minutes sought.

So if the meridian line of the place is exactly established, this practical procedure cannot fail in the least. But if not, draw — as best one can judge by eye — an arc from the Pole Star [Cynosura] through the zenith [verticem coeli], which will be the meridian, unless the Pole Star inclines too far to the side, namely around the meridian transit of the tropical points (Cancer toward the east and Capricorn toward the west), deviating slightly. The astronomer will easily meet this inconvenience by some limitation.

Example.

Let the time be sought from the transit of the bright star of the Vulture [lucida vulturis, Altair] across the meridian on 13 September, the following night. The Sun is then in 2° of Libra [$\underline{\text{♎}}$]. Therefore, by Problem 2 of Chapter 1, or from the table of right ascensions, the Sun's right ascension is found to be $181^\circ 50'$. The right ascension of the bright star of the Vulture — that is, of the mid-heaven — is $293^\circ 0'$. [Margin: E probl. 1. cap. 2.] The difference is $111^\circ 0'$. The time corresponding is 7 hours $24\frac{2}{3}$ minutes.

For the convenient aid of memory, however, we assist this procedure with the following table, in which we have contracted the right ascensions of the Sun and

of the principal stars, for the completed year 1630, into equinoctial times — that is, into equal hours and minutes — for every tenth day of each month. Once these are had and kept in memory, the remaining intermediate values can easily be estimated as far as the Sun is concerned, by adding here, namely, 4 minutes of time per single day — since the positions of the stars remain almost perpetually fixed.

So, when a star is seen on the meridian, and from its assigned equinoctial time the Sun's time is deducted, the time sought remains at once.

But one must remember that 24 hours are to be added to the time ascribed to the star whenever the subtraction cannot otherwise be made. This nautical practice by means of the nocturnal [nocturnale], using the Pole Star and the stars in Ursa Major, is the more reliable the larger the compass of sky over which it is exercised; and it is the more agreeable the less it needs that instrument. I therefore wish it to be held in its proper place and value by all people of good sense.

A scheme for inquiring into the hours and minutes at night, under any longitude and latitude.

Let the time be sought, in the night preceding 25 December of this present year 1617, when the bright star of the Ram [lucida ♈, the bright star of Aries] transits the meridian. The time from the Sun's right ascension, for 20 December, is found to be 18 hours 38 minutes; to this, for the roughly $4\frac{1}{2}$ days, is added 18 minutes, and they accumulate to 18 hours 56 minutes. But the time corresponding to the right ascension of the bright star of the Ram is 1 hour 46 minutes, or — with a whole revolution added — 25 hours 46 minutes. The difference, therefore, is 6 hours 50 minutes, which indicates the time of transit of the bright star of the Ram through the mid-heaven on that night — and conversely.

Diagram. A circular nocturnal volvelle for finding the time at night under any longitude and latitude.

Structure (from center outward):

- **Hub:** a central disk labeled **Polus Mundi** ("Pole of the World"), with a small center dot.
- **Inner ring (radial labels):** a band of named principal fixed stars, each written radially with its designation, arranged around the hub. This is the column headed in the wheel by *Denominatio Stellarum* ("Designation of the stars").

- **Middle/outer rings:** concentric bands giving, for each star and around the rim, the twelve months (named in capitals around the outermost band) divided at the 1st, 10th, and 20th of each month, with the corresponding equinoctial time (hours·minutes) entered for each division.
- **Rim band:** the month names run around the circumference; near the bottom are small marker letters keying the bottom segment.

Inner-ring fixed-star designations (reading around the wheel; epithets transcribed as printed, with the modern identification where settled):

- Spica ♍ [Spica Virginis — Spica]
- Vindemiatrix [ε Virginis]
- Arcturus
- Cauda ♌ [Cauda Leonis — Denebola; label reads "Cauda" with the Leo sign]
- Cor ♌ [Cor Leonis — Regulus]
- Basiliscus [Regulus / Basiliscus]
- Cor Hydrae [α Hydrae — Alphard]
- Praesepe ♊ [Praesepe, the Beehive in Cancer]
- Procyon [α Canis Minoris]
- Seq. Hum. Orionis ["the following (eastern) shoulder of Orion" — Betelgeuse]
- Praec. Hum. Orionis ["the preceding (western) shoulder of Orion" — Bellatrix]
- Capella
- Aldeboran [Aldebaran]
- Lucida Plejadum ["the bright star of the Pleiades" — Alcyone]
- Luc. Mandib. cete ["bright star of the jaw of the Whale" — Menkar, α Ceti]
- Cingulum ♈ [printed with the Aries sign; "the girdle"]
- Stella Polaris [the Pole Star]
- Cingulum Androm. [Cingulum Andromedae — the girdle of Andromeda]
- Caput Andromad. [Caput Andromedae — the head of Andromeda]
- Marcab Pegasi [Markab, α Pegasi]
- Os Pegasi ["the mouth of Pegasus" — ε Pegasi, Enif]
- Dext. Humer. Pegasi ["the right shoulder of Pegasus" — Scheat]
- Cauda Cygni [the tail of Cygnus — Deneb]
- Lucida Vulturis [the bright star of the Vulture — Altair]
- Lucida Lyrae [the bright star of the Lyre — Vega]

- Dext. Hum. Ophiuchi ["the right shoulder of Ophiuchus"]
- Dext. Genu Ophiuchi ["the right knee of Ophiuchus"]
- Cor M. ♎ [Antares — Cor Scorp̄ii; printed "Cor M." with the Scorp̄ius sign]
- (*continuing back to Spica ♏*)

Outer-ring months and selected equinoctial-time entries (month name in the outer band, then the readable hour·minute values at the 1, 10, 20 divisions; "L" marks the division printed with a long-s "1"-style "1st"):

Month (outer band)	day 1	day 10	day 20
IVNIVS	5·17	5·54	6·36
IVLIVS	7·21	7·57	8·37
AVGVSTVS	9·23	9·57	10·34
SEPTEMBER	11·17	11·49	12·26
OCTOBER	13·6	13·39	14·18
NOVEMBER	15·43	16·25	17·13 [?]
DECEMBER	18·38 [?]	19·32 [?]	20·10 [?]
IANVARIVS	20·52	21·40 [?]	22·15
FEBRVARIVS	22·54 [?]	23·27	23·59
MARTIVS	0·36	1·19 [?]	1·52
APRILIS	2·30	2·52 [?]	3·12 [?]
MAIVS	3·47	4·9 [?]	4·27 [?]

Additional readable inner-rim equinoctial-time figures running clockwise alongside the star band, by readable cell: "16·6", "14·57", "12·44", "11·30", "9·49", "9·9", "8·19", "7·24", "6·8", "5·35", "5·5", "4·49", "4·45", "3·25", "2·43", "2·15", "1·46", "0·49", "0·27" (several clear, a minority abraded). Bottom rim marker letters (small, beneath the December–January arc): "HM", "HWM", "D", "M" [?] — keying the lower equinoctial-time cells; partly abraded.

PROBLEM III.

Given the right ascension of the Sun, together with the right ascension and declination of some star, and likewise the altitude of that star from observation under a known elevation of the pole: the corresponding nocturnal time is found.

Let it be again for the 13th day of September. The altitude of the bright star of the Ram [lucida ♈, Hamal, the bright star of Aries], observed by a quadrant, is 25°, in

the eastern part of the sky. The star's longitude is given as $2^{\circ} 14'$ of Taurus [ϑ], its latitude $9^{\circ} 57'$ north [B]. Hence its declination is $21^{\circ} 36'$, and its right ascension $26^{\circ} 21'$.

Diagram. A sphere in oblique projection for finding the nocturnal time from a star's observed altitude, lettered as in the solar figures above. The points are lettered A, B, C, D, E, F, G, H, I, K.

- A: pole of the horizon (top)
- B: a point on the upper right
- C: the star's position
- D, E: intersections on the parallel/declination circle
- F, G, H, I, K: meridian, horizon, and equator intersections as in the earlier figures

The rest is granted from what precedes.

Therefore, in the figure set here (keeping the same notation as before), in triangle ABC, for the angle at B — that is, arc DF, the distance of the star's right-ascension point from the meridian — the givens [Greek: διδόμενα (didomena, "the givens, the data")] are:

- AB, $34^{\circ} 17'$, the distance of the poles;
- BC, $68^{\circ} 24'$, the complement of the declination of star DC;
- AC, $65^{\circ} 0'$, the complement of its altitude, namely EC.

From these the sought angle ABC is given as $76^{\circ} 56'$, whose measure lies along DF.

Subtract this angle from the right ascension of the bright star of the Ram [Iucida Υ] ($26^{\circ} 21'$), with a full circle added beforehand, and there remains the right ascension of the midheaven, $309^{\circ} 25'$. From this, in turn, subtract the right ascension of the Sun ($181^{\circ} 50'$), and there is left the arc between the midheaven and the Sun, $127^{\circ} 35'$. Converted into time, this gives 8 hours $30\frac{1}{2}$ minutes. If, on the other hand, the star's altitude is observed on the western side of the sky, then the sought angle must be added to the star's right ascension, so that the right ascension of the midheaven may be assembled, and so on.

PROBLEMA IV.

Given the elevation of the pole of the place and the right ascension of the Sun, and granted in addition two stars on one and the same vertical circle:

not only the nocturnal time is given, but also the altitude of either of the proposed stars.

This problem, although it gives the computer a good deal of work in solving several triangles, nevertheless has a distinguished and ready use whenever a lunar eclipse — or some other moment of time that must be caught precisely — has to be observed accurately at night, using nothing but a plumb-line (a hanging string) to mark out two stars on a single vertical. The procedure and demonstration [Greek: ἀπόδειξις (apodeixis, "demonstration"); πράξις (praxis, "practical operation")] in the following example is as follows.

For the observation of the bright star in the girdle of Andromeda [lucida balthei Andromedae] and the second star in the horn of the Ram [Υ], both on one rising vertical circle, the following figure is drawn. In it, with everything else as in the preceding figures, AE is the vertical, C is the bright star in the girdle of Andromeda, and D is the following star in the horn of the Ram.

Now, since our aim is to investigate the angle ABC or ABD — which measures along the equator the arc from the midheaven, FL or FM — and since in triangle ABC at least two sides are given:

- AB, the distance of the poles here, $34^{\circ} 17'$;
- likewise BC, $56^{\circ} 25'$, the complement of the declination of the girdle of Andromeda;
- or BD, $71^{\circ} 8'$, the complement of the declination of the following star in the horn of the Ram.

Therefore, in triangle CBD, from the given BD (complement of the declination of Υ , $71^{\circ} 8'$), likewise BC (complement of the declination of the girdle of Andromeda, $56^{\circ} 25'$), together with the angle CBD (the difference of right ascension of the bright star of the girdle of Andromeda and the following star in the horn of the Ram, $12^{\circ} 20'$), there is given DC, the distance of those same stars, $18^{\circ} 23'$.

Diagram. A sphere shown in plane projection, lettered as in the preceding solar/stellar figures of this chapter. The outer circle is the meridian. N marks the top (zenith/north), with a dotted arc descending from it. The points and their positions:

- N (top): the zenith point, with a dotted line
- A (upper, just left of center top): a point on the vertical
- B (upper right): a point on the meridian/vertical
- F (left, upper): a point on the circumference

- C (center-upper): the bright star in the girdle of Andromeda
- L (center-left): an intermediate point
- D (center): the following star in the horn of the Ram
- M (center-left, below L): an intermediate point
- G (left, on the horizontal diameter): left end of the horizontal great circle
- I (right, on the horizontal diameter): right end of the horizontal great circle
- E (lower center): a point near the bottom
- K (lower center, beside E): a point near the bottom
- H (lower right): a point on the circumference

[II.]

[The margin-rubric "I I." printed between the figure and the next paragraph appears to be a section/step marker.]

Further, in triangle BDC, for the angle at D — since all three sides are given: namely:

- DC, $18^{\circ} 23'$;
- DB, $71^{\circ} 8'$;
- BC, $56^{\circ} 25'$.

Therefore the angle CDB is given as $34^{\circ} 23'$.

[I I.]

[Margin: Vide Analysin Orthogoniorum. (See the Analysis of Right-Angled Triangles.)]

In triangle ABD, for the angle at B — which measures the distance of the bright star of the Ram [γ] from the midheaven [M.C.] — three things are given, namely two sides:

- AB, $34^{\circ} 17'$, the distance of the poles;
- BD, $71^{\circ} 8'$, the complement of the declination of the following star in the horn of the Ram;

and finally the angle just found at D. From these the proposed angle at B is given. For when the perpendicular arc BN is dropped — falling outside the triangle, since the angle BAD is obtuse — the triangle NBD is divided into two right triangles, BND and BNA.

First, for the perpendicular NB, it comes out $32^{\circ} 18'$. Next, in the right triangle BND, the angle at B is given as $77^{\circ} 31'$. Finally, in the right triangle BNA, the angle NBA is found, $21^{\circ} 59'$. Subtracting this from the angle previously obtained, NBD (namely $77^{\circ} 31'$), leaves the angle ABD of $55^{\circ} 32'$, which measures the distance of the following star in the horn of the Ram [Υ] from the midheaven. Now the right ascension of this star is supposed to be $23^{\circ} 18'$; adding a full circle to it gives $383^{\circ} 18'$, from which, subtracting the angle ABD of $55^{\circ} 32'$, leaves in the remainder the sought right ascension of the midheaven, $327^{\circ} 46'$. If from this right ascension of the midheaven the right ascension of the Sun appropriate to any given time is taken away, the arc of the equator will be established — to be converted into hours and minutes of time according to the method repeated several times above.

It now remains for us to carry out the other part of the present problem: the determination of the altitudes of the said stars. The demonstration of this rests on the preceding problem.

For here too, for side DA, two sides are given together with the included angle:

- AB, $34^{\circ} 17'$, the distance of the poles;
- BD, $71^{\circ} 8'$, the complement of the declination of the following star in the horn of the Ram;
- ABD, $55^{\circ} 32'$, the angle of distance from the meridian.

Therefore DA is given as $55^{\circ} 20'$, whose complement — namely $34^{\circ} 40'$ — is the altitude of the star in the horn of the Ram. And if to this you add the distance of the stars found earlier, say $18^{\circ} 23'$, the altitude of the bright star in the girdle of Andromeda will be $53^{\circ} 3'$. Moreover, from this example it is clear how one may, conversely, recover or investigate the altitude in the Sun and the stars from a given time from noon, together with the declination and the elevation of the pole — a thing that often has its use, especially in computing lunar eclipses, where the altitudes of the bodies must be derived from calculation beforehand, for the sake of knowing the time and the parallaxes.

CAPVT VI.

On the Foundation of the rational method of constructing the celestial houses for any given time, of establishing the determinations of aspects and the direction of a significator to a promissor (and conversely), and of working these out geometrically in numbers: where the kernel of the practice of astrology is set forth demonstratively.

PROBLEMA I.

Given the time and the elevation of the pole of the place, the cusps of the 12 celestial houses on the ecliptic are given according to the rational method.

Having completed, according to our intention, spherical astronomy — which considers the face of the heavens as anciently depicted with the constellations — we shall now subject to practice the use of the circle of position, which properly serves astrology, in the way we examined it in chapter 7 of the prior book [Margin: Probl. 2, cap. 1]. We shall show demonstratively (as in everything else) how, by both the rational method (the one ascribed to Johann Regiomontanus) and the other method that some prefer more — that of Campanus and Gazulus — the erection of a celestial chart is accomplished for any given time, and the direction of each significator to its promissor is carried out. Once these things are adduced, at a single glance, from a geometric source (a thing we have scarcely yet seen achieved by anyone), the understanding of the tables of directions — those that survive in the works of Johann Regiomontanus and others, and those that a more correct posterity will one day produce — will be grasped here and there, and their use will be more readily available.

Let us take, as a paradigm or example of this practical treatise [Greek: *πραγματεία* (*pragmateia*, "treatise, systematic working-out")] to be demonstrated in numbers, a certain notable celestial revolution which occupies the midheaven at $5^{\circ} 54'$ of the Ram [Υ], and that under a pole elevation of 56° less two sixtieths. For this time the place of the Sun is found from the corrected tables to be $29^{\circ} 50'$ of the Ram [Υ], whence the corresponding right ascension is given as $27^{\circ} 45'$.

With these things found and assumed for the given time and place, the first thing sought of all is the right ascension of the midheaven [M.C.], which — according to what was handed down in the preceding chapter — is offered as $5^{\circ} 25'$. For to this right ascension there corresponds, by the conversion [*anastrophe*] of Problem 2 of chapter 1 (or of the table of right ascension, as we have said) [Margin: Probl. 2, cap. 1], the longitude $5^{\circ} 54'$ of the Ram [Υ], the beginning of the tenth house. And so, with the standing of the heavens given, and with the beginning of the tenth house, we now reason demonstratively in the following way, for expressing the rest conveniently by the figure below.

First, for the proper representation of the figure in the plane, let the meridian ABED be described, and let DE be the horizon, FG the half of the equator, and YI the half of the ecliptic — so applied to the present case that through the beginning Υ both equinoxes are indicated. Further, let the half of the equator be divided

into six equal parts (three above and as many below the earth) by the marks L M K N R, with the meridian FG determining the figure.

Then let the circle of position in our sphere be so raised from the horizon itself, and so depressed (according to what was handed down above), that it grazes each of the sections of the equator; and where it intersects the ecliptic, let the cusps of the houses be marked. Among these, the tenth house occupies the midheaven at Y, the eleventh at O, the twelfth at S, the first at T, the second at V, the third at Σ , the fourth (or the lower midheaven [imum coeli]) at I.

Diagram. A plane figure of the celestial sphere set up for the rational division of houses. The outer circle is the meridian ABED (A at top, B at upper right, E at right, D at left, C at bottom). The horizontal diameter D–E is the horizon; the great-circle arc F–G is the half of the equator; the arc Y–I is the half of the ecliptic. Crossing arcs are the position circles dividing the houses. Readable labels with positions:

- A (top): top of the meridian
- B (upper right): point on the meridian
- E (right end of horizontal diameter): the horizon point
- D (left end of horizontal diameter): the horizon point
- C (bottom): bottom of the meridian
- H (far left): a point on the circumference
- Y (upper left, with "10"): cusp of the 10th house (midheaven), labelled "10"
- Υ (left, below Y): the beginning of the Ram, near the equinox
- F (left): end of the equatorial half-arc
- L (left-center): equator-division mark
- O (center-upper, with "11"): cusp of the 11th house, labelled "11"
- M (center-left): equator-division mark
- K (center): the central point (intersection)
- S (center-right, with "12"): cusp of the 12th house, labelled "12"
- P (right-center): a point on a position circle
- W (upper right, between B and S): a point on a position circle
- T (center-right, with "1"): cusp of the 1st house, labelled "1"
- N (lower center): equator-division mark
- V (lower-center, with "3"): cusp of the 3rd house, labelled "3" [the "2" appears beside T/V region]
- Σ (lower left): a division/cusp mark

- X (lower left): a point on a position circle
- R (lower right, with "Q"): point near Q
- Q (lower right): a point on a position circle
- G (lower right): end of the equatorial half-arc
- "4" (lower right, near I/G): cusp of the 4th house (lower midheaven), labelled "4"

With these things thus arranged in the eastern hemisphere, all the rest follow of their own accord in the averted (or western) hemisphere, in the opposite arrangement of signs and so on — since indeed every great circle falling upon another great circle divides it into two opposite points, as is shown plainly enough in our sphere, and as will later be indicated further in the present example.

For the practice, however, in finding the houses — although one may proceed in various ways — we shall nonetheless first adopt the one that best serves for explaining the tables of Regiomontanus. And since the circle of position is nothing other than a movable horizon, as was shown above, the first care will be, in the upper hemisphere, to acquire — after the elevation of the world, above the circle of position — the twelfth and eleventh houses, the former of which is represented by the arc BP, the latter by BW. Equal to these, beneath the earth, are the arcs CX and CZ.

Now we shall fit the calculation to the investigation of BP.

So, in the right triangle MFD, for the angle M — to which the complement of the sought arc BP is equal, like FKD, the complement of the elevation of the pole BE — the givens [Greek: διδόμενα (didomena, "the givens")] are:

- DF, $34^{\circ} 2'$, the complement of the elevation of the pole;
- DM, $60^{\circ} 0'$, the arc on the equator, that is, the right ascension;
- MFD, a right angle, corresponding to the 12th house by hypothesis.

Accordingly the angle DMF is given as $37^{\circ} 57'$, whose complement — namely $52^{\circ} 3'$ — is the sought arc BP, that is, the elevation of the pole above the circle of position [Margin: Pag. 109] of the 12th house, which same value is found in Regiomontanus's tables of directions.

In the same way, in the right triangle LFD, for the angle at L — by whose complement the arc BW is measured, namely the elevation of the pole above the circle of position of the eleventh house — the givens [Greek: διδόμενα (didomena, "the givens")] are:

- DF, $34^{\circ} 2'$, the complement of the elevation of the pole;
- FL, $30^{\circ} 0'$, the arc of the equator, that is, the oblique ascension, corresponding to the eleventh house by hypothesis;
- LFD, a right angle.

Therefore the angle FLD is given as $53^{\circ} 29'$, and its complement is BW, $36^{\circ} 31'$, the sought elevation of the pole at the cusp of the eleventh house, which is likewise found in Regiomontanus's tables. With the elevations of the pole above the circles of position granted, the remaining practice is for the investigation of the arcs of the ecliptic at the beginning of the houses — which we now reach in knowledge of the sought quantity (by means of the analysis [Greek: ἀνάλυσις (analysis)] of triangles) from the supposed oblique ascension of each house and the given elevation of the pole for the same. Here, although the solution could be given by a dropped perpendicular arc and the right triangles that follow from it, the more expeditious one is that which our theorems on oblique-angled triangles supply — and since these have not yet been practiced by others with equal ease, it is pleasing to assign these examples to them as well.

So let it be, for the first example, that the point of rising, or of the horoscope [horoscopus], is to be sought on the ecliptic at T. On account of this, all the angles of triangle TKQ will first be had; then the sought side QT will emerge.

For in the same triangle TQK, for the obtuse angle at T, the givens [Greek: διδόμενα (didomena, "the givens")] are:

- angle TQK, $23^{\circ} 32'$, the greatest obliquity of the ecliptic;
- angle TKQ, $34^{\circ} 2'$, the complement of the elevation of the pole;
- side QK, $84^{\circ} 35'$, the complement of the semicircle on the equator, that is, arc HK.

Therefore the sought angle KTQ is given as $137^{\circ} 27'$.

Now that the three angles of the proposed triangle KTQ are given, the sought side TQ is given. The givens of the angles are:

- TQK, $23^{\circ} 32'$, the obliquity of the ecliptic;
- KTQ, $137^{\circ} 37'$, the angle just found;
- TKQ, $34^{\circ} 2'$, the complement of the elevation of the pole, which angle is opposite to the sought side TQ.

Therefore...

Therefore the sought side itself, TQ, is given: $55^{\circ} 45'$. If this is subtracted from six signs, it leaves the ascendant, or horoscope, at $15^{\circ} 0'$ of Leo (ϱ). Almost the same value is found in Regiomontanus's tables of oblique ascensions, under an elevation of the pole of 56° , from the given oblique ascension of $95^{\circ} 25'$.

II. For the cusp of the twelfth house.

We proceed in the same way for the sides HS and HO. For in triangle SHM two angles are granted: angle SHM, $23^{\circ} 32'$, the obliquity of the ecliptic; and angle HMS, $142^{\circ} 3'$, the supplement of angle HMD found earlier. Together with the side HM, $65^{\circ} 25'$, derived by hypothesis from the oblique ascension of the twelfth house, this yields angle HSM, $34^{\circ} 24'$.

With the three angles now given as above —

Angle	Value
SHM	$23^{\circ} 32'$
HSM	$34^{\circ} 24'$
HMS	$142^{\circ} 3'$

— the side HS is therefore known: $98^{\circ} 10'$, measured in the ecliptic from the vernal equinox. For that reason it terminates at $8^{\circ} 10'$ of Cancer ($\mathfrak{C}$), and this is the point of the ecliptic that the cusp of the twelfth house occupies. We obtain the same value very nearly in Regiomontanus's tables of oblique ascensions, at an elevation of 52° and the assumed oblique ascension of $65^{\circ} 25'$.

III. For the cusp of the eleventh house.

We acquire the cusp of the eleventh house by a not dissimilar procedure. For in triangle HLO, first, for the angle at O:

The data given are —

Element	Value	Note
OHL	$23^{\circ} 32'$	obliquity of the ecliptic
OLH	$126^{\circ} 31'$	supplement of FLD to a semicircle
HL	$35^{\circ} 25'$	oblique ascension of the eleventh house, by hypothesis

Therefore angle HOL is given: $36^{\circ} 11'$.

For the side HO:

The data given are —

Element	Value
OHL	23° 32'
HOL	36° 12'
HLO	126° 31'

Therefore HO is given: 52° 3', the elongation from the vernal equinox. When one whole sign — namely Aries — is subtracted from this, the remainder shows the sought cusp of the eleventh house at 22° 3' of Taurus (♉); this is likewise to be found, very nearly, at the elevation of 36° 31' in Regiomontanus's tables.

By exactly the same procedure, the beginnings of the second and third domiciles below the earth are found. First, for the cusp of the second house — namely, V — in triangle VQN, since:

The data given are —

Element	Value	Note
angle VQN	23° 32'	obliquity of the ecliptic
QNV	37° 57'	equal to angle FMD
side NQ	54° 35'	the distance in the equator from the autumnal equinox

This distance arises from the complement of the oblique ascension of the second house; and that complement, in turn, arises from four signs, or 120°, added to the right ascension of the M.C. Therefore, as above, we obtain:

First, the angle QVN, 125° 30'. Then the side QV, 38° 0'; and if this is subtracted from a semicircle, it leaves the cusp of the second house marked at 22° 0' of Leo (♌) — the very same value that is offered in the tables of oblique ascensions at an elevation of the pole of 52° 3', the reckoning begun from the vernal equinox.

Finally, last of all, for the cusp of the third house.

Since, in triangle SQR:

The data given are —

Element	Value	Note
angle SQR	23° 32'	obliquity of the ecliptic
SRQ	53° 29'	equal to the angle FLD above

Element	Value	Note
side RQ	24° 35'	complement of the oblique ascension to a semicircle

Therefore we obtain: angle RSQ, 104° 42'; and side SQ, 20° 13', the complement of the sought cusp of the third house to a semicircle, as before. The said cusp therefore falls at 9° 47' of Virgo (♍), likewise to be drawn from the tables very nearly at an elevation of the pole of 36° 31'.

Now that six domiciles have been found by means of the triangles — as a foundation — or else from the tables, whether of oblique ascensions or of houses in the ephemerides, the remaining six present themselves of their own accord in the opposite signs, with the same degrees and minutes, as follows:

Order of domiciles found	10	11	12	Horosc.	2	3
Cusp of the domicile	5° 54' ♈	22° 3' ♉	8° 10' ♊	4° 15' ♋	22° 0' ♌	9° 47' ♍
Opposite domicile	4	5	6	7	8	9
Cusp of the domicile	5° 54' ♏	22° 3' ♐	8° 10' ♑	4° 15' ♒	22° 0' ♓	9° 47' ♈

Once the cusps of the twelve domiciles have been found, they are inscribed in order into the figure customary among astrologers. For the sake of beginners, it has seemed good to set it out here too.

Diagram. The horoscope figure ("Theme of the twelve celestial domiciles") in the standard early-modern square-in-circle form: an outer circle enclosing an inner square (set on its corners) and four arcs, dividing the chart into twelve houses, with the cusp longitude inscribed in each sector. The center carries the pole-elevation legend. The inscribed cusps are read directly from the engraving below; in several sectors the engraved minutes differ from the worked-text table (e.g. 8°16' vs 8°10', 22°8' vs 22°3').

- Center: "Elevat. Poli gr: 55: m: 58" (Elevation of the pole, 55° 58')
- Upper left sector: ♊ 8° 16' (Cancer, twelfth-house cusp region; engraved "8.16" — differs from the table's 8°10')
- Upper, left of meridian: ♈ 22° 8' [the eleventh cusp; engraved as Aries glyph beside "22.8" — differs from the table's 22°3']

- Top center: γ 5° 54' (engraved as Taurus glyph beside "5.54"; the 10th-cusp region — the table gives this cusp as γ 5°54')
- Upper right of top: ♍ 9° 47' [engraved near "36 9.47" — the third-house value mirrored]
- Upper right sector: $\approx$ 22° 2' [Aquarius, "22.2"]
- Left, above center: ♌ 4° 15' (Leo)
- Right, above center: $\approx$ 4° 15' (Aquarius)
- Left lower: ♌ 22° 2' (Leo)
- Right lower: ♋ 8° 16' (Cancer; engraved "8.16" — differs from the table's 8°10')
- Lower left sector: ♍ 9° 47' (Virgo)
- Bottom center: ♎ 5° 54' (Libra / Imum Coeli)
- Lower right sector: ♏ 22° 8' (Scorpio, "22.8")

PROBLEMA II.

παράλληλα(?) — [Greek: δεδομένοις (dedomenois, "the things given / data")], as before, are presupposed: the cusps of the twelve celestial domiciles in the ecliptic are given, according to the method of Campanus and Gazulus and of other more recent authors.

Here the prime vertical circle, at its rising and setting, is divided into twelve equal parts — that is, each quadrant into three equal parts, just as the equator was before. As the circle of position passes across each of these single sections, one must observe where it simultaneously cuts the ecliptic; and there the beginnings of the domiciles are to be fixed.

Although this method of dividing the heaven into its twelve domiciles has been less commonly used, it is nevertheless more general than the preceding one — since, by that other method, the heaven cannot be so divided when the pole of the world stands at the zenith. For then the equator is so joined with the horizon that it cannot be cut from it by any circle of position. Nevertheless, the cusps of the four principal domiciles agree with those obtained before — namely the ascendant, the imum coeli, the medium coeli, and the descendant.

The point of the medium coeli, found in the ecliptic at 5° 54' of Aries (γ), has its declination sought, which in the accompanying figure is PK, 2° 21'. From this, the side OK is established as 36° 23'. For OP is 34° 2', the complement of the elevation of the pole; next, the angle KOL is known, 30°, by hypothesis, whose measure is AD. Besides this, the angle LKO must be ascertained, which is the complement of

angle HKP; and this is found to be $66^{\circ} 35'$. Therefore the said angle OKL is $113^{\circ} 25'$.

Diagram. An astronomical sphere drawn in plano, used for the Regiomontanus house construction. The outer circle is the meridian, with A at the top, B at the bottom, and the diameter ADEMFGIB running vertically through the center. A horizontal line runs from O (on the left circumference) to C (on the right circumference). Several oblique great-circle arcs (the circles of position and the ecliptic/horizon) are drawn from O to C, cutting the central vertical at the lettered points and bearing the house numbers along the right-hand arc. Marginal references key the construction to earlier problems.

- A (top of meridian): A
- D (on vertical, below A): D
- E (on vertical, with subscript "12"): E — twelfth-house point
- M (on vertical, below E): M
- F (center, on horizontal OC): F
- G (on vertical, below F): G
- I (on vertical, below G): I
- B (bottom of meridian): B
- O (left end of horizontal): O
- C (right end of horizontal): C
- K (upper left, on an arc, marked "10"): K — tenth-house point
- 11 (label beside K, upper left): 11
- L (just below K): L
- H (far upper left): H
- P (below L): P
- Z (below P): Z
- N (right of center, on an arc): N
- 1 (near N): 1
- 2 (lower right, on arc): 2
- R (lower right): R
- S (below R): S
- 3 (beside R/S): 3
- T (below, on arc): T
- W (lower right, outside): W
- 4 (lowest arc-point on right): 4

- X (below 4): X
- Right margin (upper): "Per 1 Probl. cap. 1, vel a tabula decl. solis." (By Problem 1, chapter 1, or from the table of the sun's declination.)
- Right margin (lower): "Per Probl. 4, cap. 1." (By Problem 4, chapter 1.)

Now for the side KL: since in triangle KOL —

The data given are —

Element	Value	Note
KOL	30° 0'	by hypothesis
LKO	113° 25'	the angle just found
KO	36° 23'	the side previously acquired

— there is given, first, the angle at L, 44° 26'; then the sought side KL, 25° 4'. When HK is added to this, HL is composed: 0° 58' of Taurus (♉), the sought cusp of the eleventh house.

From the data —

Element	Value	Note
KO	36° 23'	
OKM	113° 25'	
KOM	60° 0'	by hypothesis

— if the M.C., 5° 54', is added, the cusp of the twelfth house is disclosed at 16° 25' of Gemini (♊); but the cusp of the first house remains as before.

Furthermore, the same procedure is to be used below the earth, for acquiring the third and second houses. For CX is equal to KO, 36° 23'; the angle CXS is equal to angle HKP, 67° 9'; and the angle RCX is 60 parts. From these, the cusps of the third and second houses are easily known, observing those things, in the solution of the given triangles, which we have so often repeated here.

It now remains for us to disclose, by a single example, Regiomontanus's method for constructing the table that is entitled *Table of Houses according to Campanus and Gazulus*; and then to apply the canon to its use in this business.

Two things are required here: the elevation of the pole above the circle of position of the cusps of the domiciles, and the quantity of the equator enclosed within each domicile. Once these are had, the cusps of the individual domiciles are sought from the table of houses under the appropriate pole, as before. As a

present illustration, we shall give the example of the establishment of the eleventh house.

In the right-angled triangle OPZ, then, for the arc of the equator PZ — with OP set as the whole sine — PZ will be the tangent of the angle at O lying opposite; therefore:

	OP		POZ
as 90°	34° 2'	so 30°	to PZ
S.T. 100000	S.R. 55967	T. 57735	T. 32313 → res. 17° 54'

Next, the angle PZO is sought by the first rule of prosthaphaeresis [prosthaphaeresis], for:

	compl. PZ	POZ	compl. Z
as 90°	55° 58'	30°	to 41435 → resp. 24° 29'
S.T. 100000	S.R. 82871	S.R. 50000	(this very value is the elevation of the pole)

Moreover, if we now enter the table of oblique ascensions with the oblique ascension of 23° 19' (for only that much is gathered from HP and PZ together), likewise with the elevation of the pole of 24° 29', the cusp of the eleventh house corresponds at 0°

0' minutes — all being adjusted for the adhering minutes; which does not differ so very much from our own earlier finding, yet leaves a certain discrepancy for definite reasons.

And from all this we judge it sufficiently shown how the erection of the domiciles is carried out from the foundation of triangles. We now proceed to the directions themselves.

PROBLEMA III.

Given the latitude or declination of a planet, or of a star, the limit of longitude for an aspect with respect to the ecliptic, or to the equator, is found.

[Margin: See Problem 31, Table of Directions of Regiomontanus.]

Since it greatly concerns astrologers, for the practice of their art, to know how the stars, casting their rays, affect these lower things — and also which latitudes need

to be brought within a limit — it has seemed good to insert this problem before the doctrine of directions.

It must be understood that, although the stars everywhere cast and diffuse their light orbicularly together with their efficacy, they nevertheless operate by certain hidden, as it were, ratios — chiefly according to the rational division of the circle: namely in the sextile ($\star$), opposition ($\oslash$), trine ($\triangle$), square ($\square$), and [the aspects] as set forth above to the eye, and determined by definite measures [Margin: Liber I, cap. 6]. But in the conjunction ($\odot$), opposition ($\oslash$), and square ($\square$), the departure of a planet into latitude introduces nothing that would either detract from or add to its longitude.

For the opposition is diametrical, and falls into the contrary latitude; the conjunction holds fast; and finally the place of radiation in the square always stands a quadrant of a circle distant from the place of the star's longitude. But in the remaining aspects — namely the sextile ($\star$) and the trine ($\triangle$) — to determine the longitude according to the departure in latitude, one must proceed as follows.

An example in each case, in the sextile and the trine, with a latitude of any size whatever — namely, 20° .

Let A be some star, whose rays, diffused orbicularly, extend to the sextile aspect through the described circle BC; and let BDC be the ecliptic or equator, and AD the latitude or declination of the star at A. Now take AC on the spherical surface, by hypothesis, here for the sextile — namely 60 parts; and AD, the latitude or declination, 20° , likewise by hypothesis.

Diagram. A circle with a horizontal diameter BDC: B at the left, D at the center, C at the right. From a point A inside the upper half, two lines descend — one to D (the latitude AD, perpendicular to the diameter) and one to C — forming the small right triangle ADC.

- A (upper interior): A — the star
- B (left end of diameter): B
- D (center of diameter, foot of the perpendicular): D
- C (right end of diameter): C

So in the spherical right triangle ADC, for DC, the arc bounded in the ecliptic or equator:

Element	Value
AC	$60^\circ 0'$

Element	Value
AD	20° 0′
ADC	right

Therefore DC is given: 57° 51′, for the sextile in the ecliptic or equator, according as the latitude or declination is given.

But to bound the longitude — or the right ascension — in the trine, the subtraction of the sextile thus found is made from 180°, or a semicircle, and the remainder is reckoned from the place of the planet's longitude or ascension, for the triangular reduction. Thus in the present example, with the longitude given at the beginning of Aries (♈):

	Value	
	180° 0′	semicircle
	57° 51′	sextile aspect
Difference	122° 9′	

This corresponds in the ecliptic to 2° 9′ of Leo (♌); or, in right ascension in the equator, 122° 9′. From this the rest is estimated.

Liber II — Spherics, Cap. VI (continued)

Problem VI

Given the right ascension and the declination of both the significator and the promissor, there is found — upon the circle of position of the significator — the difference of the oblique ascension of each, which is called the directorial arc [arcus directorius], by which the times of events are customarily measured.

We will set out the doctrine of direction — which is the very kernel and guarantor of the whole of judicial astrology — through a few examples drawn from the present illustrative figure, in such a way that its foundations may be unfolded along the way from the tables of Johannes Regiomontanus. [Margin: *Lib. 1, c. 7.*] As for what direction is, since we described its full circumstances in the preceding book, we will now devote ourselves only to the practical procedure.

Example I. Where the circle of position of the significator is given — as, for instance, the cusp of some house, or a planet standing on a cusp.

In the figure set out above, let the Midheaven [M.C.] serve as the significator, to be directed toward the place of the Sun as the promissor. Since the Midheaven has no elevation of the pole, only its right ascension is to be used; this is found to be $5^{\circ} 25'$.

Next, the right ascension of the Sun was likewise found above to be $27^{\circ} 45'$. The difference, then — which is the directorial arc — comes out as $22^{\circ} 20'$. Now, since by the measure of the Sun's daily motion along the equator one estimates an equal number of years from experience, we seek the Sun's daily motion in the equator; it is found to be $0^{\circ} 58'$. From this, by a quick method — because in each degree there is an excess of $2'$ — we must add $44'$ for the 22 degrees. When these are added to $22^{\circ} 20'$, the total comes to $23^{\circ} 4'$. From this we judge that the time of some notable event will fall at the beginning of the 24th year.

Example II.

Let the horoscope — namely $4^{\circ} 15'$ of Leo (ϱ) — be directed to the right trine aspect (Δ) of Mercury, which will be placed in the tenth house at $13^{\circ} 35'$ of Aries (Υ), according to the Copernican foundation.

First, then, since the horoscope here is the significator whose circle of position is given — its elevation of the pole being $55^{\circ} 58'$, and its oblique ascension having earlier been established at $95^{\circ} 25'$ by adding 90° to the Midheaven — the only thing we have to find is the oblique ascension of Mercury, which falls at $13^{\circ} 35'$ of Leo (ϱ). That oblique ascension, under the elevation of the pole of $55^{\circ} 58'$, is yielded as $109^{\circ} 25'$, both by the third problem of Cap. 3 of this book and by the tables of Regiomontanus. But the oblique ascension of the horoscope was $95^{\circ} 25'$. The difference, therefore, is 14° , which is also called the directorial arc; this, extended into years according to the Sun's daily motion, gives roughly $14\frac{1}{2}$ years.

Example III.

Let the body of the Moon be directed into the left square ($\square$) of Saturn, outside the cusp of the houses.

Moon's	longitude	$20^{\circ} 38' \varrho$
Moon's	latitude	$5^{\circ} 10'$ south

Here, since the significator is outside the cusp of the houses — specifically, near the end of the first house — we will work the direction through the resolution of spherical triangles, applying to the following figure everything required for it: where C is the significator and N the promissor. Having worked out each item in full for the benefit of our beginners, we have first, for the declination and right ascension of the significator:

Diagram. A large circle (the celestial sphere in stereographic projection) with several intersecting great-circle arcs — the horizon, the equator, the meridian, and circles of position — used to work the directional problem of Example III. A small circle near the top, around the points A and B, marks the region of the significator. Labels read (positions approximate):

- H (upper left, outside the main circle): endpoint of an arc
- ♃ (Jupiter, just below H): label on the upper-left arc
- B (top, within the small circle): point
- A (top, within the small circle, the significator region): point
- F (left, on the horizontal diameter): endpoint
- K (left of center, on the horizontal diameter): intersection point
- G (right, on the horizontal diameter): endpoint
- S (just left of center): intersection point
- C (center, the significator): point
- D (just right of center): point
- ♄ (Saturn, right of center): label on the inner arc
- N (right of center, the promissor): point
- E (below center, near W): intersection point
- W (below center): point
- R (below center): point
- Q (below center, right): point
- L (lower right): point on the main circle
- M (bottom): point on the main circle

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Given in triangle ABC:

- $CB = 95^{\circ} 10'$, the excess of the Moon's latitude
- $AB = 23^{\circ} 32'$, the distance of the poles
- $ABC = 50^{\circ} 38'$, the distance of the Moon from the tropic of Cancer (☿).

Therefore $AC = 80^{\circ} 14'$, the complement of CE . And $BAC = 128^{\circ} 37'$, the distance in the equator from the tropic of Capricorn (♋) in the direction of decreasing degrees [in antecedentia]. When this is subtracted from a third of the circle — that is, from 270° — there remains the right ascension of the significator, $141^{\circ} 23'$.

II. For the angle ACG.

Since in triangle CAG the following are known:

- AC , the complement of the declination
- $AG = 55^{\circ} 58'$, the elevation of the pole
- the angle $CAG = 44^{\circ} 2'$, which is measured by EL , and which is known once the right ascension of the significator is subtracted from the right ascension of the I.C. at L . (Now the right ascension of the I.C. is $185^{\circ} 25'$ — namely, $5^{\circ} 25'$, the right ascension of the Midheaven, with a semicircle added.)

There is therefore given, first, the side $GC = 46^{\circ} 59\frac{1}{2}'$.

Next, in the aforesaid triangle ACG, since all the sides are given, the sought angle at C is given as $51^{\circ} 58\frac{1}{2}'$.

III.

Furthermore, in the right triangle SCE, since

- $SCE = 51^{\circ} 58\frac{1}{2}'$
- $CE = 9^{\circ} 46'$, the complement of AC found earlier
- CES is a right angle —

Therefore $CSE = 39^{\circ} 4\frac{1}{2}'$ is given.

Likewise $SE = 12^{\circ} 14'$, the ascensional difference, which, subtracted from the right ascension of the significator ($141^{\circ} 23'$), leaves the sought oblique ascension of the significator, $129^{\circ} 9'$.

IV.

Next, for the declination and right ascension of the promissor, there are given, in the right triangle NRQ: the angle $NQR = 23^{\circ} 32'$; NRQ a right angle; and $NQ = 26^{\circ} 43'$. Therefore there are given $NR = 10^{\circ} 20'$, the declination of the promissor; and

the arc QR = $24^{\circ} 47'$. When this is subtracted from the semicircle, there remains the right ascension of the promissor, $155^{\circ} 13'$.

V.

Finally, in the right triangle NRW, for the side RW, the given quantities [*diaomena sunt*] are:

- NR = $10^{\circ} 20'$
- NWR = $39^{\circ} 4\frac{1}{2}'$, equal to CSE
- NRW a right angle —

Therefore RW is given as $12^{\circ} 59'$, the ascensional difference of the promissor, to be subtracted from the right ascension of that same body; so there also remains the oblique ascension of the promissor, $142^{\circ} 14'$. But the oblique ascension of the significator was $129^{\circ} 9'$. [*Continues below.*]

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(continuation from p. 102) And accordingly the difference — which is the directorial arc — will be $13^{\circ} 5'$, to which correspond about $13\frac{1}{2}$ years.

In this way we have also sought out, within the doctrine of triangles, the foundation of directing, and have set it forth with sufficient examples. For when a direction has to be set up against the order of the signs, the promissor and the significator are interchanged — and the converse — and so the circle of position is sought under which the promissor lies. Nor is there anything otherwise different from the rest of what has already been demonstrated, just as Johannes Regiomontanus expounds it more fully. [Margin: *Cap. 26 T. Direct.*]

CHAPTER VII

On the principal astronomical instruments, with their use briefly assigned.

[Margin: *Classis secunda.*]

Now that we have dealt with those matters which can or should be inscribed on the surface of the celestial sphere or globe, we judge it to follow that we should bring forward how the phenomena are to be drawn toward observation by means of mathematical instruments — especially since, without this knowledge, no one turns out to be a craftsman in our astronomy. Whether the phenomena of the present age are to be rightly assessed; or those handed down by the ancients

are to be examined and weighed in a just trial (so that the most excellent of all disciplines may not be made liable to the perpetual errors of insufficient and childish observations); or, finally, whether new portents appearing in the sky from time to time — especially in these days of ours — are to be measured in their motion, distance, and magnitude: none of this can be done without instruments. Otherwise these very things mislead the philosophers, who would then judge about the whole nature of the heavens through such appearances otherwise than the truth demands. This is what once happened to Aristotle concerning the nature and generation of comets. His opinion, though erroneous, carried such authority right down to our own age that it dragged the mathematicians into error with it — until our own Atlas, Lord Tycho Brahe, through the skillful construction and sufficient magnitude of his instruments, and finally through a timely and proper program of observing, grasped the unconquerable truth in this matter, and demonstrated that comets are celestial. This he made known to posterity in the books of the *Progymnasmata*, and proved with luminous demonstrations against the Peripatetics. As for us — although the *Astronomia Mechanica*, once elegantly adorned with engravings by Tycho himself and brought to light, sets out the construction and use of many such instruments [Margin: *Vide Astron. Mech. T. B.*] — nevertheless, since nearly all those things which rival the heavens in their roundness are found in the dissection of our material sphere, we will therefore deal briefly, in the first place in this class, with certain of them whose use is foremost.

Among the astronomical instruments that rival the roundness of the heavens, some are fixed to their own poles, upon which they are balanced — such as the two armillae that follow shortly, and any others there may be that keep watch on the horizontal poles together, like the *torquetum* — in which, too, a center, like a pointer, is fixed in the middle of the axis. But others are free and portable, attached only to centers and circumferences, like the quadrant or the sextant, and likewise the *triens*, the *octans*, and any other solitary instruments built from the section of a circle. Further, those that rest upon their own poles are composed of two or three sections of circles, while those that are free are content with a single section of a circumference. To carry out observations rightly with these, two observers are for the most part required.

I. On the Equatorial Armillae.

When the elevation of the pole of the place is given, with the axis of the sphere passed through both poles, let the center be marked in the very middle. Then,

with the circles remaining in place — the equatorial one being movable, but the declination one able to be carried around the poles — movable

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and necessary sights [dioptrae] are fitted to both. And in this way not only are the times and right ascensions given in the equator, but also the declinations in their own declination circle. [Margin: *The occasion of the refractions in the Sun and stars is usually detected through the equatorial armillae. T. B.*] By the benefit of this instrument, first of all, with Tycho Brahe, the refractions of the Sun, the Moon, and the stars near their rising and setting were discovered.

II. On the Zodiacal Armillae.

With the poles of the place likewise fixed in the meridian circle, or at any of its points, and with the distance between the poles of the equator and of the zodiac given on both sides, and finally with the sights applied as before: through the circle of the ecliptic the longitudes of the stars themselves are given, and through the latitude circle their latitudes, the sighting being made toward a fixed center in the middle. [Margin: *From these two the ancient Alexandrian armillae were composed; Ptolemy makes use of them here and there in his Almagest.*]

III. On the Torquetum.

In the same way, finally — once the elevation of the pole is given, and a vertical circle together with a horizontal one is assumed — there are given on the horizon the azimuthal degrees from the meridian, and on the vertical circle the altitudes themselves from the horizon. But if the remaining circles — namely the equator and the zodiac, with their transversals — are attached, the old torquetum is constructed from this, an instrument more remarkable for its craftsmanship than for its use, since the torquetum twists violently under its own weight.

IV. On the Quadrant.

So far concerning the instruments that are in some way disposed and fixed to the elevation of the pole of the place; there follows the topic of the free and portable instruments, among which the quadrant holds the first place, on account of its convenient use so far — whether you wish to measure the altitudes of the stars on either side, from the vertex or from the horizon, or the distances comprised within a quarter of the heavens. The construction of this is easy; for whether it is made of solid material, or worked out openwork and latticed, the circumference of the quadrant circle has its measure marked off in degrees and minutes, like the

others, while the two sides toward the center each represent the radius of the circle. [Margin: *Vide Astron. Mech. Tych., on the rotatable quadrant.*] Now if the quadrant simply extends its pointer, set rightly toward the horizon on every side, it will be quite suitable for investigating parallaxes — which are usually traced out by taking the degrees of altitude and the azimuthal degrees together — so much so, indeed, that the parallactic instrument of Ptolemy and Copernicus readily yields to it, since the latter, among other drawbacks, is also burdened too heavily by its own weight.

V. On the Sextant Instrument, *which is most familiar to us on account of its construction and use.*

First, as for what concerns the finished construction of the sextant: no astronomical instrument is built more easily than the sextant. For since it is established that the side of a hexagon inscribed in a circle is equal to the radius, the side of the hexagon is easily traced out on every side by means of the compass.

Next, although the use of this instrument is simple, it is most conspicuous in taking the distances of the stars. Yet if the sextant is skillfully prepared and made ready — such as the one we took care to have crafted by our own diligence, having indeed five cubits in its radius — it suffices not only for measuring all manner of stellar distances within 150 degrees, but also for any altitudes whatever, by means of sights, that is, skillfully disposed on it. Indeed, it even takes the place of a geometric instrument, without any triangular calculation, since we have brought the tangents at the same time onto its limb. And this instrument, since it stands for us as practically the equal of all the rest, we will not begrudge to display here a little more fully and demonstratively as to its construction. First, as for the construction of the sextant: this is easy, as I said. For once the size of it — that is, its semidiameter — has been granted, the compass (which, with straight and oblong wood,

oblong wood and with two iron pointers — one fixed at the end, the other sliding to whatever measure is wanted — will, by an equal extension, test all its sides. As for the framing, the following figure can serve as a guide; but the sextant itself can be examined more fully and more exactly with me, together with its ingenious division between the transverse lines — a method of division that Tycho Brahe regularly employed in his finer instruments.

Diagram. A sextant: a sixty-degree arc of a circle braced by an open triangular framework of struts. The two straight sides meet at the apex A at the top; the

graduated arc runs along the bottom from B (left) through D to C (right). A central medallion within the frame carries the date and maker's initials. The graduated limb is numbered in tens.

- A (apex): vertex of the sextant; the center of the circle
- B (lower left, on the arc): position of the fixed sighting-vane (dioptra)
- C (lower right, end of the arc): one end of the limb
- D (lower middle, on the arc): point on the limb
- H (within the frame, left of center): position of an alternative vane
- Arc graduations: 10, 20, 30 (at center, near D), 40, 50, 60 (near C) — degrees of the limb
- Central medallion: "1607 / F F / C.S.L." [the year 1607; the initials, presumably for the maker — Christen Sørensen Longomontanus]

Since one sighting-vane (dioptra) is fixed at B and another at D on the movable rule, it is clear that the angle B A D measures the distance between two stars, for the angles at the vertex are equal.

Now, although the limb of the sextant extends no further than 60° , by the following device we have extended its use so that it serves well even for measuring distances up to 150° . Place one vane at C and the other at H. Then, when the line of sight runs from C through H — taken by one observer — and from B through the center A — taken by the other — it is certain that the distance of the two stars so observed amounts to a quadrant, that is, 90° . Moreover, the nearer the rule D, with its vane, is moved toward C, the more is subtracted from the quadrant, so that in this way every measurement between 90° and 60° is carried out.

Again, once the sighting is set up the other way — back from H toward C — it will likewise follow that the quadrant is completed for the observer sighting from B through the center A. But since this same observer uses the vane on the rule D, then, as that vane moves toward C, the angle A H C is widened — so much so that, when D has reached C, the sextant is added to the measure of the quadrant, and a distance of 150° is produced. Furthermore, with the sight-vanes fixed and immovable at B and C, one can measure on the limb of the instrument that part which falls between 150° and 90° , or the quadrant of the circle — as the geometrical demonstration itself shows in the measurement of the circle.

So much for obtaining distances by means of the sextant.

On Altitudes by the Sextant [De Altitudinibus per sextantem]

[Margin: See the construction and use of the sextant in Tycho Brahe's *Progymnasmata*, Book I, p. 248.]

The sextant is also well suited to the altitudes of the stars, both meridian altitudes and any others. For if it is hung from its stand in such a way that its upper side answers exactly to the horizon, it will suffice for observing an altitude of 60° from the horizon. But if a greater altitude is required, the level is so adjusted that the upper side, through the center, is raised 30° from the horizon; and when the sextant is fastened to its stand as before, it is found suited to taking an altitude all the way up to the zenith — which follows easily from the hypothesis of the instrument's construction set out above. On the model of the sextant, the *trient* (a third) and the *octant* (an eighth) are also built — each, of course, with the appropriate portion of the circle for that instrument. Their use is much the same as that of the sextant, in obtaining the distances and altitudes of the stars.

VI. On the Geometric Square [De Quadrato Geometrico]

Furthermore, so that the sextant might be reserved chiefly for drawing out the distances of the stars, I had another instrument made for the altitudes of the stars — by which the time is known — in a square form and of suitable size. Its two sides, which receive the divisions, represent the tangent lines, and so, given a number on these (whether direct or reverse), the corresponding arc is at once obtained from the canon of tangents — and this is the altitude of the star, reckoned either from the horizon or from the zenith.

Besides being especially suited to measuring terrestrial things and to directing practical mechanical work, this instrument is also so assembled and divided by means of its screws that it can be enclosed in an oblong case and carried to various places as need requires.

VII. On the Construction and Use of the Astronomical Radius, and a Certain Insufficiency of It [De Radii Astronomici fabrica, usu, et quadam ejus insufficientia]

So far concerning the astronomical instruments, which for the most part rival the heavens because of their roundness. There follows the radius [*radius* — the cross-staff], a very ancient instrument; for of it Virgil sang as follows:

[Margin: Eclogue 3.]

Who was that other Who with a rod (*radio*) described for the nations the whole globe?

[Margin: Aeneid.]

And the courses of the heaven they will trace with the rod (*radio*), and tell of the rising stars.

On the Construction of the Radius [*De Fabrica Radii*]

The radius, as it is described by Gemma Frisius, Petrus Ramus [P. Ramus], and others, consists of a staff (*baculus*) and a cross-piece (*transversale*), whose convenient proportion will be 1000 parts of the staff to 200 parts of the cross-piece — once the space of the sight-vanes on either side of the cross-piece, and the standing-point on the staff, have been allowed for. Then let the cross-piece be made movable on the staff, with a hole fitted at its middle. So, when each rod has been divided into the numbers of equal parts I have stated (a use it may also have in geometry), each is, by the following demonstration of triangles, distributed into degrees and minutes, so that the radius becomes suited to observing the heavenly phenomena.

Diagram. The radius (cross-staff): a long vertical staff with the eye at the foot B and two cross-pieces shown at successive positions on the staff. The upper cross-piece runs from C through A to D; the lower one runs from G through A to H. Thin lines converge from the ends of the cross-pieces down to the eye at B.

- B (foot of staff): the eye / standing-point
- Upper cross-piece, left half C to A: "100"; right half A to D: "100" — each half divided into 100 equal parts
- C (upper left end), D (upper right end): ends of the first cross-piece
- F, E (small marks at the upper cross-piece): the sight-vanes
- A (center of each cross-piece): hole through which the staff passes
- G (lower left end), H (lower right end): ends of the second cross-piece
- Staff scale: "1000" — the staff A B divided into 1000 parts

So, in the present figure let the staff be A B, and the cross-piece C D, movable on the staff at A; its segment C A (or D A) will be divided into 100 equal parts, of which A B is 1000. Then, with the cross-piece kept at A, its first station, we first proceed geometrically as follows for the individual degrees. Take, for example, the angle A B E of one degree.

[Greek: Διδόμενα (didomena, "the givens")], therefore:

- A B, parts 1000
- A B E, degrees 1
- E A B, a right angle

Therefore A E sought is 17 parts, 45 minutes. For, putting A B as the whole sine, E A will be the tangent of the angle at B. Hence, when the last four figures are dropped in the table of triangles, the tangent for each single degree is at once obtained from the table of tangents. From this, A C comes out as 5 degrees, 43 minutes — to which A D is equal. Therefore C D, in the station of the staff at A, is 11 degrees, 26 minutes.

From here, for the division of the staff A B into degrees and minutes, we proceed thus. When the distance A B is made nearer toward the eye B, the given isosceles triangle C B D is divided into two equal right triangles C B A and D B A. Since, then, A C B is the complement, to the quadrant, of half the assumed angle, A B sought is easily obtained: for, putting A C as the whole sine, A B will be the tangent of the angle C, whose complement is C B A.

So, with the equal angles C B A and D B A combined, the measure A B is in the middle [i.e. is read off at the staff's midline], and so on.

Now let the distance on B A corresponding to 30° be sought — so that with these two examples we may settle at once both the construction and the use of the radius proposed — the measure which is contained within the angle G B H. Having divided the proposed angle into two equal parts, 15° fall to each; and the tangent of the complement of this, 75° , namely $373\frac{1}{2}$ parts, indicates the measure on the staff A B.

From this it can readily be understood that the radius, divided primarily into equal parts as we have shown, can be used without this secondary distribution, using the table of tangents alone.

Example [*Exemplum*]

Two stars have been observed by means of the angle G B H, where the measure B A is indicated on the staff as 577 parts. Now since A H, 100 parts, is to A B, 577 parts, so is the whole sine to the tangent of the angle A H B, therefore, when the whole sine is set at 10000000, the tangent of the angle A H B becomes 5770000. To this there corresponds in the table of tangents $80^\circ, 10'$, very nearly. Its complement is the angle A B H, $9^\circ, 50'$. The double of this, then, H B G, $19^\circ, 40'$, is the angle of sight at B, that is, the sought distance of the stars. And so in other examples: where only the distance B A, presented from the observation, is supplied, the angle A H B arises through five figures without further calculation (in a radius so divided), and so on.

So much, then, for the construction and use of the radius — except for what may be further learned in practice.

The insufficiency of the radius lies chiefly in practice: namely, that the eye, sighting from both ends — especially over large distances — does not, because of the meanwhile-shifting motions of the stars and the variable center of vision, suffice for precision of alignment. Sailors, too, use this instrument to take altitudes of the Sun and stars, since they readily dispense with the exact precision the astronomer requires. So much for the astronomical instruments.

CHAPTER VIII

On the Various Ways of Investigating the Meridian Line and the Elevation of the Pole of a Place [*De variis modis investigandi lineam Meridianam et elevationem poli loci*]

Now that the more select astronomical instruments have been described and prepared, and are most fit for making observations, two things are required here first of all — as a certain and given starting point — for their use in acquiring observations: namely, the meridian line and the elevation of the pole. The problems of this chapter set out, in order, the manifold investigation of each.

On the Meridian Line of a Place [De Linea Meridiana Loci]

PROBLEM I

With a gnomon erected on a level surface, and with the equal length of the shadow observed before and after noon, the meridian line of the given place is found.

This is the ancient and simple way to investigate the line of noon — the line which, climbing through the poles of the world and the zenith of the place, divides the upper hemisphere of the heaven equally into north and south. Namely: on a level horizontal surface, upon a fixed tablet, let a stylus or gnomon be erected at right angles; and from its center let circles (one or more) be described. Then, in the forenoon, in a very clear sky, observe most carefully where the tip of the shadow of the erected stylus or gnomon cuts the circumference of the nearest of the described circles as it enters — for, as the Sun rises, the shadow contracts. Likewise in the afternoon, where the tip of the shadow falling from the same gnomon grazes the circumference of the same circle as it goes out — for, as the Sun descends, the shadow lengthens.

The midpoint of this intercepted arc, explored and exactly determined by the compass, will be the point of noon.

For afterward the sought line is found — the straight line, namely, drawn from this point to the middle of the foot of the stylus on the plane — as you see in the adjoining figure.

Diagram. A square tablet with corners B (upper left), A (upper right), C (lower left), D (lower right). A gnomon or stylus stands at O near the top; an arc of a circle (part of the inscribed circle, H K) sweeps across the lower part, passing through G and F. The meridian line is marked running across the figure.

- B (upper left corner), A (upper right corner)
- C (lower left corner), D (lower right corner)
- M (left side), L (right side): midpoints of the left and right edges
- O (top center): top of the stylus
- "Stylus." (vertical label, upper left): the stylus / gnomon
- H (upper center, on the arc): forenoon shadow-tip intersection
- "Lin: merid:" (horizontal label, center): the meridian line, running to K
- K (right, on the meridian line), P (left of center): points on the meridian line
- G (center), F (lower center, on the arc): F is the afternoon shadow-tip intersection; G the midpoint region

PROBLEM II

Given the observed altitude of the Sun, and given besides its declination and the elevation of the pole of the place, the meridian line is found by this single observation, made before or after noon.

This meridian line, then, will be had at one time without delay, by employing a quadrant or other instrument by which the altitude of the Sun can be observed precisely — and in this manner.

Let a circle drawn on a tablet be divided into 360° , and each of these into 60 minutes; or at least let a part of the circle be exactly distributed by this standard, so that a measure can be taken from the remainder as need requires. Then, with any intersection of the stylus's shadow noted on the circumference, and with its opposite point, at any clear time — whether before or after noon, and at a moderate remove from noon, yet not too near the Sun's rising and setting on account of the refractions of the Sun — at which moment the altitude of the Sun is observed exactly, and its declination from the given place of the Sun being known, and applied together with the elevation of the pole of the place: then,

through the solution of the spherical triangle, there is made manifest the arc of the horizon between the meridian and the point opposite to the point of the shadow's intersection marked earlier — as will be further shown by the following demonstration and example.

Diagram. A spherical-projection figure: a large outer circle (the meridian A B C D L) with the horizon A L drawn across it and the equator B M. Within it are drawn intersecting great-circle arcs and a smaller circle H K about the center O. A stylus G O is erected. Points are lettered around the construction.

- A (left, on the horizon), L (right, on the horizon): the horizon A L
- B (upper left), C (top), D (upper right): the meridian arc A B C D
- M (lower right): the equator B M
- E (upper center): the position of the Sun, elevated above the horizon through arc F E
- F (lower left, on the outer circle): point where the shadow-line runs out to the celestial horizon
- N (left of E): point on the construction
- O (center): center; top of the stylus G O
- G (just right of center): foot of the stylus
- H (upper center, on the small circle), K (right of G): points of the small circle H K
- I (below O, on the small circle): the point opposite the shadow
- R (left of G): point on the horizon line

Let it be, in the adjoining figure: A B C D L the meridian, A L the horizon, B M the equator. Let the circle H K be drawn as before, and the stylus G O erected. With the Sun raised above the horizon through the arc F E, let the shadow of the stylus from the Sun at E be marked at H, and its opposite point I — which line runs out to the celestial horizon at F. Then, with the quadrants C F and D N given from each pole, namely the vertical pole and the pole of the world, which intersect each other at the place of the Sun at E, all the sides of the spherical triangle E C D are given. For C D is the complement of the elevation of the pole of the place to 90°, that is, the distance of the poles

[Margin: it will be known by the following example.]

of the world and the vertical circle, which is equal to the arc A B, the elevation of the equator above the horizon. E C, by contrast, is the complement of the Sun's altitude F E; and finally E D is the complement of the Sun's declination. The angle

E C D will therefore not be hidden, for it is measured in the horizon by F L, whose complement to the semicircle is F A — and this arc reveals the distance between the given point opposite [the Sun] and the meridian of the place. Thus the straight line drawn from A through the center G to L shows the meridian line that was sought.

Example.

At Rostock [Rostochium], where the elevation of the pole is about 54° , on 30 August, when the Sun was in $15^\circ 40'$ of Virgo [♍] and its declination was $5^\circ 46'$ north: the Sun's altitude was observed by a quadrant at the same time at which the stylus's shadow was marked on the tablet — its opposite point falling at some definite degree toward H — and it was $20^\circ 0'$. Therefore, in triangle E C D:

	given	as the complement of:
C D	$36^\circ 0'$	the elevation of the pole
E C	$70^\circ 0'$	the altitude of the Sun
D E	$84^\circ 14'$	the declination of the Sun

(The "given" quantities are marked διδόμενα [Greek: didomena, "the given/data"].)

Therefore E C D = $108^\circ 36'$.

And so F A = $71^\circ 24'$, by which the point F found in the horizon is distant from the meridian.

I. This same thing is accomplished on the said horizontal tablet without a stylus, using the quadrant alone, when the Sun or a star is observed by placing the quadrant perpendicularly above the center of the circle of section. For here too there remains the same opposite point that the stylus's shadow marks out during the observation.

II. The same can also be done when the length of the stylus and of the shadow on the tablet is given. For from these the Sun's altitude is first determined, then the hour, and consequently the distance from the meridian.

III. Furthermore, the same is found from the rising and setting of the Sun or a star, marked on the said horizontal circle, between which the meridian line passes midway; or from the altitude taken equal on both sides of the meridian, together with the azimuthal degrees on the same circle of section.

IV. Likewise, the same is accomplished from the observed elongation of the circumpolar stars — and especially of the polar star — to their greatest extent on either side, on the horizon circle. Sir Tycho Brahe judged this method the safest of all for obtaining an accurate investigation of the meridian line on the azimuthal quadrant.

V. Finally, given the elevation of the pole of the place, the meridian line is shown by means of two stars observed on the same vertical circle; and likewise by many other methods of induction, which are everywhere available to the practiced astronomer. For if one resource here is removed, opportunity and skill will substitute and supply a second, a third, and more — so that he need not, where precision is required, place his trust, with the common crowd, in the mariner's compass as an instrument that, for many reasons, is rather unreliable. [Margin: Consult the following Problems.]

On the Elevation of the Pole of the Place

PROBLEM I.

Given the observed altitude of the Sun or of a star at the meridian, together with its declination: the elevation of the pole of the place is found.

The elevation of the pole of the place is investigated either below the meridian or outside it. Below the meridian — as in the present problem — it is found from the altitude of the Sun, or of stars positioned at the meridian. For given the meridian altitude of a phenomenon and its declination,

the elevation of the pole becomes known. For if at the meridian the altitude has been taken, and a northern declination is subtracted from it (or a southern one added to it), there is shown — in the former case by the remainder, in the latter by the sum — the complement of the elevation of the pole of the place that was sought.

Example.

[Margin: A.D. 1599.]

On 31 August at Rostock the Sun's meridian altitude was observed by quadrant of $1\frac{1}{2}$ ells [$1\frac{1}{2}$ uln.] as $41^{\circ} 2'$. The place of the Sun was $17^{\circ} 0'$ of Virgo [♍], its declination $5^{\circ} 8'$ south. When this declination is subtracted from the observed altitude, there remains the complement of the elevation of the pole — that is, the height of the equator itself above the southern horizon — $35^{\circ} 54'$. Therefore the elevation of the pole is $54^{\circ} 6'$.

The same can, and should, be obtained in the same prescribed way from the meridian altitude of any star whose declination is known.

Closely related to this Problem are also these methods.

For, first, in the case of circumpolar stars, the same is accomplished even without a given declination, from the highest and the lowest meridian altitude of the same star. For when half the difference of these observed altitudes is added to the smaller (or subtracted from the larger), the remainder shows the elevation of the pole that was sought.

Next — especially on a clear winter night — let a thread or cord be hung perpendicularly between two supports, and let the star passing across the zenith of the place be marked off against that perpendicular. For the declination of this star shows the elevation of the pole that was sought. The same can be found against a wall or support erected exactly true.

PROBLEM II.

Given two stars observed on the same vertical circle, whose right ascensions and declinations are known, together with the altitude of one of these two stars: the elevation of the pole of the place is found.

The elevation of the pole from observations outside the meridian is derived, in the first place, from two stars seen on a single vertical circle, together with the altitude of one of them measured by a quadrant at the same time, by means of spherical triangles, in this way:

Diagram. A circle (the vertical circle of the place) with a horizontal diameter M A K F running through its center A; M at the left, F at the right, K just right of center. D is the vertical pole, near the top; E is the pole of the world, at the upper right on the circle. Two stars lie on the same vertical: B (inside, right of center) and C (above B, toward D–E). Auxiliary points H and G lie between B and the center; I lies on the circle at the upper left; L lies on the circle at the lower right, where the line through E and K meets the circumference. Arcs connect D, C, B, E and the circle.

- D (top): the vertical pole (zenith)
- E (upper right, on circle): pole of the world
- B (inside, right of center): the lower of the two stars on the vertical
- C (above B): the upper star on the vertical
- A (center): center of the figure

- K (on the horizontal diameter, just right of center)
- M (left end of diameter), F (right end of diameter)
- H, G (between B and the center)
- I (upper left, on circle)
- L (lower right, on circle)

In the adjoining figure, let D be the vertical pole, E the pole of the world, and let the stars on the same vertical be at B and C; and let the altitude of the one at B be known, through its arc A B. Then, in triangle C E B: from the given declinations of each star, and afterward from their complements represented in the arcs C E and B E, together with the difference of right ascension — which the angle C E B measures — the distance B C between the stars observed on the same vertical becomes known. Second, in triangle C B E, from the three given sides the angle C B E is found. Last, in triangle D B E: since two sides are granted, together with the angle they contain — namely B E, the complement of the declination of the star at B; D B, the complement of the altitude of the observed star at B; and the angle C B E just obtained — there emerges the arc D E, the distance between the poles of the world and of the vertical, which is the complement of the elevation of the pole that was sought.

But if the altitude of the upper star at C is observed, then the angle at C in triangle B C E is sought, whose complement to 180° is the angle D C E. In this triangle, the two sides D C (the complement of the altitude C A) and C E (the complement of the declination C H) are given, together with the contained angle C; and so D E, which was sought, is found.

PROBLEM III.

Given the right ascension of the midheaven M.C. [medium caeli], and besides this two stars outside the meridian on the same vertical, together with the right ascension and declination of those same stars: the elevation of the pole of the place is found.

Recalling the construction of the immediately preceding figure: with the right ascension of the midheaven M.C. given, two angles are given in triangle D E B — namely the one at B, by the preceding problem, and the one at E, through the difference of the right ascensions of M.C. and of the star at B. There is given besides the side B E, the complement of the declination of the star at B. Consequently D E, the complement of the elevation of the pole that was sought, will not be hidden — and therefore neither will the elevation of the pole of the

place itself. The same is done when the angles D E C and D C E are given, by carrying out the procedure through the star at C.

PROBLEM IV.

Given two pairs of stars observed at night together on two vertical circles, with the declinations and the mutual distances of these stars known by the preceding methods: the elevation of the pole of the place is found.

Since this problem (like the one immediately preceding) teaches how to investigate the elevation of the pole of the place quite precisely by careful attention alone — through the straight extension of a thread between the stars on the same vertical and the eye, without any other instrument, whether one is on land or at sea — it is therefore, in my judgment, to be greatly valued, especially as it has not, so far as I know, been discovered or brought forward by anyone before. Moreover, the use that the present problem affords is outstanding, and it will make the induction of triangles, and the rather laborious analysis, nothing but pleasant.

In this practice there is a twofold case, since such observations are for the most part made on verticals outside the meridian. (For those that occur on the meridian can be referred back to the preceding problem.) For either both verticals fall into the same quadrant of the sky, toward the rising or toward the setting; or one falls toward the rising and the other toward the setting, with the meridian lying between.

Diagram of the former case, with its demonstration [διαπόδειξις / ἀπόδειξις [Greek: apodeixis, "demonstration"]].

Diagram. A large sphere drawn in projection. The meridian E B D forms the bounding circle, with B at the top (pole of the horizon at one end) and D, E its other extremities; the pole of the horizon E D is B, and the pole of the equator F G is C. F is at the left, G at the lower right, on the equator arc F G. The first pair of stars H and K lie on the vertical B H K P; the later pair I and L on the vertical B I L O. Numerous arcs cross the figure connecting these points; A is near the center, and M, N, R, Q, P, O are intermediate points around the lower left and bottom.

- B (top): pole of the horizon (one end of E D)
- C (upper right): pole of the equator
- F (left), G (lower right): ends of the equator arc F G
- H, K (upper area): the first pair of stars, on vertical B H K P
- I, L (right of center, below C): the later pair of stars, on vertical B I L O

- A (center)
- M, N (upper left, near F), K (center-left)
- R, Q (lower left), P (left), O (lower center)

From the center let the meridian $E B D$ be described, in which the pole of the horizon $E D$ is B ; and the pole of the equator $F G$ is C . The rest are known. For $H K$, the first pair of stars, are marked on the vertical $B H K P$; while $I L$, the later pair, are on the vertical $B I L O$. And since the declinations, as well as the distances of the stars, are known from the foregoing: in triangle $K C L$, with its sides given, the angles at K and at L are given. Then, from the upper triangles $H K C$ and $C L I$ — whose

[angles,] this one $[I L C]$ being subtracted and that one $[H K C]$ on the contrary being added to $C K L$, there are left, in triangle $B K L$, two known angles at K and L , together with the retained side $K L$. Therefore from these $B L$ is given. Once this is had, since in triangle $B L C$ two sides are granted together with the contained angle at L , $B C$ — the complement of the elevation of the pole that was sought — will not be hidden.

Demonstration of the latter case.

Diagram. A sphere in projection, with all the same lettering as in the preceding figure, but with the points arranged for the case in which the two verticals lie on opposite sides of the meridian. B is at the upper right; C at the right; F, K, H, M, N at the upper left; A near the center; R, Q at the left; E at the lower left; D at the right; O and G at the bottom. The stars K and L lie on the verticals; arcs cross the figure as before.

- B (upper right), C (right edge)
- F, H, K, M, N (upper left)
- A (center)
- I, L (center)
- R, Q (left of center)
- E (lower left), D (right)
- O (bottom center), G (bottom)

In this latter diagram everything stands under the same letters as in the former, with this one consideration alone: that each angle — namely $I L C$ and $H K C$ — is *subtracted* from the angles at L and K of triangle $L C K$. From this there afterward remains the triangle $K B L$, with known angles at K and L , and with the contained side $K L$, the distance between the stars at K and L . From these "givens"

[Διδομένοις [Greek: didomenois, "given things, data"]] the side B L is first given; then, in triangle B L C, from the granted sides B L and L C together with the contained angle B L C, B C is acquired — the complement of the elevation of the pole that was sought. It should be noted, moreover, that in this and the preceding problem the altitudes of the stars and the times of the observations made, if anyone desires them, may afterward come to light through the preceding methods.

PROBLEM V.

Given two stars rising or setting together, with their declinations and right ascensions: the elevation of the pole of the place is found.

This problem too is carried out without any instrument, from two stars observed rising or descending equally above the horizon, whose declinations are of the same denomination, northern or southern. It is likewise done through the doctrine of spherical triangles, by this method of induction:

Diagram. A circle with a horizontal diameter C K F (the horizon) through its center K; C at the left, F at the right. E is at the top, D at the upper left on the circle; G at the lower right; H at the bottom. Two stars marked with small star-glyphs lie on the horizon line on each side: a pair B and A to the left of center (between C and K), and another pair A and B to the right of center (between K and F). Curved meridian-like arcs run from E down to H through these star positions.

- E (top): one celestial point (toward which the stars rise)
- D (upper left, on circle)
- C (left end of horizon), F (right end of horizon)
- K (center)
- A, B (left of center, on horizon, star-marked): the two stars in one position
- A, B (right of center, on horizon, star-marked): the same two stars in the other position
- G (lower right)
- H (bottom)

In the figure set alongside, let there be two stars, emerging or setting, at A and B. Then, in triangle A E B: since two sides are given — A E and B E, the complements of the declinations of the stars — together with the angle A E B, the difference of the right ascension of each, the distance of the stars on the horizon is sought, and likewise the angle E A B. Once these are found, there are granted in triangle E A F:

E A as before, E A B (that is, E A F), the angle just found, together with the right angle E F A. Therefore the arc E F, which measures the elevation of the pole that was sought, will not be hidden. This procedure has the less error, inasmuch as the refractions of the stars are for the most part equal. But of an observation of this kind [the chance is] rare...

such an opportunity is rare, because of the vapors — indeed, the clouds — that almost continually surround the horizon.

Moreover, the elevation of the pole is made plain from the observed magnitude at rising or setting, together with the given declination. For when the arc on the horizon A F is given, along with the complement of the declination E A, the arc E F becomes known in the same right-angled triangle E A F already proposed. But this route is mistaken and treacherous, because of the refractions of the Sun and the stars.

CHAPTER IX.

On the various ways of observing celestial phenomena afresh and of resolving them in respect of ecliptic longitude and latitude; and on the comparison of these observations and their greater certainty.

Two ways present themselves for searching out from the heavens the place of an unknown star. One is quicker, and in part also more reliable, carried out with sufficient astronomical instruments — those already explained. The other proceeds without these, namely by stretching a thread, and beyond that by mere eyesight. But this must be understood — and the astronomer must weigh it most carefully — that not all observations, even those carried out with instruments of proper size, deserve equal certainty. (For we cast out entirely from this field, which is to be cultivated and cleared, those ancient instruments, small and childish as they were.) Observations belonging to the first revolution of the heavens, made with a fixed moment of time recorded — as when the altitude of a phenomenon [Greek: φαίνομένου (phainomenou, "of the appearing thing," i.e. the celestial object)] is taken together with its azimuth and the recorded instant of time — are in truth judged to be very slippery and uncertain for fixing the star's place. For the slightest error in the elapsed time, by which one easily goes astray — whether in clocks, as most often happens, or in carrying out instantaneous observations — turns out to be all too fertile of error. Although this method is subtle, and better suited to the demonstrations by which parallaxes are worked out, yet for the reasons stated it has deceived nearly all the astronomers of an earlier age, as many as have attempted anything by it — whether to establish the

places of stars contemporary with the world, or to search out the positions of more recent phenomena. [Margin: *Lib. 1 Progymn. de fundamento restit. Stellarum Fixarum.*] This is just as Master Tycho Brahe [Tycho Brahe] proves and shows from experience. But so that lovers of learning will not be left in the dark concerning the methods of Regiomontanus and the ancients in these matters, we shall first set out briefly a demonstration of this procedure.

PROBLEM I.

Given a fixed moment of time, together with the altitude of a new phenomenon above the horizon and its azimuth — both to be observed at once — its right ascension and declination are given, and therefore also its longitude and latitude.

In the diagram added here, let M K N be the horizon, M L A B the meridian, and L H K the quadrant of the equator, whose pole is at B, while the pole of the horizon is at A. Now let there be given, from observation by the quadrant C G, the altitude of some appearing object [Greek: φαινόμενον (phainomenou)] situated at C; likewise M G, its distance on the horizon from the meridian; and finally the point L, namely the midheaven on the equator, which — once the Sun's right ascension is applied — is known from the lapse of time between the noon of the preceding (or following) day and the moment the observation was made. You do this by resolving the hours and minutes of the stars, known from a clock or from the altitude of some known star, into degrees and minutes of the equator, just as we have shown in several preceding examples.

Diagram. A semicircle drawn on the horizontal diameter M—N, with G marked between M and N (G to the left of center). A is at the top of the semicircle (the zenith / pole of the horizon), B is on the arc to the right of A. Inner arcs connect across the figure: a chord/arc runs from A down toward K (near the diameter), and another arc passes through L, C, and toward B. The point C lies inside, below A and left of B; L is on the arc at the left; H and F lie inside, below L and C respectively, near the line descending toward K.

- M (left end of diameter): horizon point
- N (right end of diameter): horizon point
- G (on diameter, left of center): foot of the star's azimuth on the horizon
- A (top of semicircle): pole of the horizon (zenith)
- B (on arc, right of A): pole of the equator
- L (on arc, left): midheaven point on the equator
- C (interior, below A): the observed phenomenon (star)

- H (interior, below L): intersection point
- F (interior, below C): intersection point
- K (on lower line, right of G): point on the equatorial quadrant

So in the triangle A B C, the following are given: A B, the distance of the poles — namely of the world and of the vertical — which is the complement of the elevation of the place, as we have often said; likewise C A, the complement of the altitude of the observed phenomenon, together with the angle C A B, that is, G N, the complement of the observed azimuth M G to the semicircle. From these, C B is made plain — the measure giving the complement of the sought declination — and hence also the declination H C itself. Then, since all the sides of triangle A B C are now known, the angle A B C will not escape us — that is, L H, the distance of the sought phenomenon from the point M.C. (midheaven), revealed at L. When this arc is added to the right ascension of the M.C. (already given from the time) — if the phenomenon was found in the eastern region — or subtracted, if it lay in the part west of the meridian, the right ascension of that unknown star too is brought into the open. [Margin: *Probl. 1. c. 2.*] Once these have been obtained, the longitude and latitude of that same star are determined from what precedes.

PROBLEM II.

Given a meridian point, or some fixed star outside the meridian, in the equatorial armillary circle, and given moreover the declination of the phenomenon in the declination circle of the same armillary: its right ascension is also given along with the declination.

Likewise, in the zodiacal armillae rightly set up, given such a point — whether the M.C. or a known star outside the meridian — the longitude of the unknown phenomenon is given in the longitude armillary circle, and its latitude in the latitude circle.

There follow the observations to be made with armillary instruments, namely the equatorial and the zodiacal with their colures, set out in some fashion earlier in our sphere. Certainly they are to be reckoned in nearly the same standing as those things the immediately preceding problem taught, and of no much greater certainty. For although armillae of this kind are gathered to have been very much in use among Hipparchus [Hipparchus], Ptolemy of Alexandria [Ptolemy], and the ancients for taking the positions of stars — because the places of the Sun and stars could be had with them immediately, in longitude and latitude, without the laborious computation of triangles, which for them was far more intricate than for us — yet, on account of the more difficult combination and arrangement of

those circles (which require the most exact workmanship), found troublesome both in construction and in use and application, these means of observing among the ancients are not free from suspicion and fault. Accordingly, although our Tycho Brahe once procured himself instruments of this kind, conspicuous in size and moreover wrought with the greatest industry and skill and at very great expense, nevertheless, when the utmost precision was to be pressed in observing, he in the end set them aside and all but entirely made them obsolete — just as he did the astrolabe and the torquetum, fashioned in the plane after their likeness.

As for this twofold problem, the things accomplished by it will be most correctly known in practice alone, with the armillary instruments duly set to the meridian. These can also be most conveniently demonstrated in our material sphere, which represents both with its circles.

PROBLEM III.

Given the meridian altitude, and the distance of an unknown star from some known fixed or wandering star: the place of the unknown is likewise given.

Among astronomical instruments, then, the quadrant chiefly for altitudes, and the sextant — which Tycho devised in place of the insufficient cross-staff, and conveniently substituted for it, to observe the intervals or distances of the stars, from which afterward all that we desire and wish can be drawn out — are best suited.

Whether both of these, or only one — since the other can also duly discharge this office — will suffice for the astronomer who wishes to explore the positions and motions of the stars, will appear in this and the next following problems. For by the meridian altitude of an unknown star, conveniently to be found on a quadrant or a square, and afterward its declination; and by the distance of that same star from some known star on a sextant — the rest, for tracking down its place, we shall most easily and correctly determine by the following method.

An example, for the place of the bright star of the Lyre [Vega] obtained, brought here from Book 2 of the Progymnasmata of Tycho Brahe.

The distance of the bright star of the Lyre from the upper preceding star in the Square of Pegasus, which is called Scheat, observed by the Tychonic sextant, was about $55^{\circ} 31'$.

The declination of the bright star of the Lyre, obtained either from the meridian altitude or in the circle of declination in the year [15]77, was $38^{\circ} 26'$ N.

Now Scheat of Pegasus is known: { right ascension $340^{\circ} 52'$; declination $25^{\circ} 50'$ N. }

From these given quantities [Greek: διδομένοις (didomenois, "given things")] the following triangle is constructed in this way.

Diagram. A spherical triangle. A is at the top apex (pole of the world). Two arcs descend from A — quadrants of the colure of the equinoxes (circles of declination of our sphere): one running to D at lower left, one running to B at lower right. C lies on the left arc, between A and D. A short arc of the equator, the portion D E, runs along the bottom between D (left) and E (right). The arc C B crosses the interior from C down to B.

- A (top apex): pole of the world
- C (on left arc, below A): the bright star of the Lyre [Vega]
- B (lower right): Scheat of Pegasus
- D (lower left, on equator): foot of the colure / equator point
- E (lower right, on equator, below B): equator point (right ascension of Scheat)

In the appended figure, from the pole of the world A let there be drawn quadrants of the colure of the equinoxes — that is, of the circles of declination of our sphere; and let D E be the portion of the equator. Accordingly, in triangle A B C the following are given: B A = $64^{\circ} 10'$, the complement of the declination of Scheat of Pegasus; A C = $51^{\circ} 34'$, the complement of the declination of the Lyre; C B = $55^{\circ} 31'$, the observed distance of the given stars from one another. Therefore the angle C A B emerges as about $65^{\circ} 14'$ — this is called the angle of the ascensional difference of the right ascensions, whose measure lies along D E. Since, then, the right ascension of Scheat of Pegasus, placed at B, has been determined through the point E on the equator at $340^{\circ} 52'$, it follows that, subtracting from this the arc D E found ($65^{\circ} 14'$) — since the Lyre is situated in advance (to the west) — there remains the sought right ascension of the bright star of the Lyre: $275^{\circ} 38'$.

This, together with its observed declination of $38^{\circ} 26'$ N — if it be presupposed — [Margin: *Per Probl. 1. cap. 2 conversionem.*] lets us determine from these its latitude as $61^{\circ} 43'$ N, and its longitude as 9° [Margin: *Probl. 1.*] $20'$ ♊ (Capricorn), in the same year [15]77. And by this method the positions in the heavens of all unknown appearing objects [Greek: φαινομένων (phainomenōn)] can be tracked down, once one is first given that will hold the place of a foundation — just as Tycho Brahe substituted the bright star of Aries [Υ , i.e. the first point of Aries / a

reference star], by means of the Sun and the star of Venus. [Margin: *T. B. lib. 1. Progym. Capit. 2.*]

For the rest, it must be noted here that two stars to be explored by distance in this way should obtain a disposition toward the equator not too oblique; otherwise the practice will scarcely have settled certainty.

Furthermore, for the sake of examination and of further certainty, the place of an unknown star may be discerned by exploring its distance from two other known stars on either side, and weighing them by this same standard, until the right ascension drawn from them coincides as closely as possible in one and the same point. This can be done, if the instruments employed for observing are of sufficient size and structure.

PROBLEM IV.

Given the distances of an unknown phenomenon from two stars whose longitudes and latitudes are known: the longitude and latitude of the unknown are likewise given.

The fourth and last way, by means of instruments, of observing the stars whose places we aim to fix at definite points of the heavens and to know in the same way in longitude and latitude, dispatches the business from the distances alone of the unknown appearing object [Greek: φαινομένου (phainomenou)] from two stars known in latitude and longitude. By the immediately preceding method — if the observations have been duly carried out, and those three stars to be observed have formed a moderately well-shaped triangle — it is no less reliable, although it is more laborious in the deduction and calculation of triangles. As an example of this, let us take the famous comet of the year [15]77, observed on the 13th of November by the triangular instrument, that is, the sextant, of Tycho Brahe, in its distances from these two fixed stars, as follows. [Margin: *Vide lib. 2. Prog. pag. 35.*]

I. The comet was distant from the bright star of the Vulture [Altair] $26^{\circ} 48'$. Then, at the same time, the distance of that same comet from the lower horn ♎ (of Capricorn) was observed as $21^{\circ} 19'$. The longitude of the bright star of the Vulture is $25^{\circ} 52' ♎$ (Capricorn); but its latitude is $29^{\circ} 19' \text{ N}$. The longitude of the lower horn ♎ (of Capricorn) is $28^{\circ} 16'$; and its latitude $4^{\circ} 37' \text{ N}$.

From these given quantities [Greek: διδομένοις (didomenois)] the adjoining figure is constructed in this way, and resolved into numbers for the things sought.

Diagram. A spherical figure. A is at the top apex (pole of the ecliptic). From A three quadrant-arcs descend toward the ecliptic: A C E, A B F, and A D G. The arc

E F G along the bottom represents a portion of the ecliptic, with E (left), F (middle), G (right). B lies in the interior (on the middle arc, the bright star of the Vulture); C lies at lower left (on the left arc, the comet); D lies at lower right (on the right arc, the lower horn). The three star-places C, B, D are connected by great-circle arcs B C, B D, and C D.

- A (top apex): pole of the ecliptic
- B (interior, on middle arc): the bright star of the Vulture [Altair]
- C (lower left): the comet
- D (lower right): the lower horn ♑ (of Capricorn)
- E (bottom left, on ecliptic): foot of arc A C E
- F (bottom middle, on ecliptic): foot of arc A B F
- G (bottom right, on ecliptic): foot of arc A D G

Let A be the pole of the ecliptic, from which the described arc E F G represents a portion of the ecliptic. Let the bright star of the Vulture also be at B, the lower horn ♑ at D, and the comet at C; and let three quadrants A C E, A B F, A D G descend through these three places to the arc of the ecliptic. Let the three places of the stars be connected by arcs of great circles, namely B C, B D, and C D. With the figure thus arranged, let us consider it, and resolve the triangles in order for C — that is, for the sought place of the comet — both as to its latitude E C and its longitude E [F]. So in triangle A B D the following are given:

	degrees	minutes	meaning
A B	60	41	complement of the latitude of the bright Vulture
A D	85	23	complement of the latitude of the horn ♑
B A D	2	24	difference of longitude of the Vulture and the horn ♑

Therefore B D = 24° 48', the distance of those same stars.

II. In triangle A D B, from all the sides granted —

	degrees	minutes
A D	85	23
B D	24	48
A B	60	41

— the angle A D B emerges as 4° 59'.

III. Likewise in triangle B D C, since the following are given:

	degrees	minutes	meaning
B D	24	48	distance of the stars previously found
C D	21	19	distance of the comet from the horn ♄
C B	26	48	distance of the comet from the Vulture

therefore C D B = $72^{\circ} 4\frac{1}{2}'$ is given; and adding to it the angle A D B, the whole angle A D C = $77^{\circ} 3\frac{1}{2}'$ is composed.

IV. In...

IV. In the triangle described at C, then, two sides about the angle found at D are given — namely:

Side / angle	Degrees	Minutes
A D	85°	23'
C D	21°	19'
A D C	77°	$3\frac{1}{2}'$

Therefore C A = $81^{\circ} 1'$, the complement of the comet's latitude; and so the latitude sought is itself $8^{\circ} 59'$ north.

And C A D = $21^{\circ} 1'$, which measures the arc E G — that is, the difference in longitude of the comet from the lower horn of Capricorn ♄.

Since the comet was ahead of that star, this arc of $21^{\circ} 1'$, subtracted from the star's longitude of $28^{\circ} 16' \text{ ♄}$, leaves the comet's longitude sought at $7^{\circ} 15' \text{ ♄}$. By this method, then — or even from the distances alone of unknown stars from two known stars — we can hunt down their positions. Here, however, it must be noted that the three stars to be taken in this way should be fitted to our procedure so as to form a triangle, referred to the ecliptic, that is not too distorted or slanted toward a single straight line — a point I warned of earlier as well. [Margin: See Tycho Brahe's *Progymnasmata* I, p. 551, in the Maestlin example.]

Anyone wanting more examples should seek them near the end of Chapter 2 of Book I of Tycho Brahe's *Progymnasmata*, where, by a similar induction of triangles, the stars of Cassiopeia — twenty-three in number — were restored from observations.

PROBLEM V.

When an unknown star is arranged, along with two pairs of others whose positions are known, on straight lines that cross — these lines being detected merely with a ruler or a stretched thread — the longitude and latitude of the unknown phenomenon are given.

Among those methods that bring the positions of the stars into our knowledge without an elaborate instrument, the procedure carried out with straight lines crossing toward four stars — explored by means of a ruler, or rather a stretched thread — deserves the greatest reliability of all. For that reason I will set it out here demonstratively, in a worked example. We had an observation of this kind, among others, in the year of the Savior 1610, on December 6, around 9 o'clock in the evening, of the planet Mars [in stella Martis], which at that time was found on a straight line passing as nearly as possible through both the bright star of Aries [lucida Arietis Υ] and the star behind it on the back of the Whale [Cetus]; and then on another straight line through the tip of Pegasus's wing [extrema ala Pegasi] and the star at the point of the Whale's nostrils, first in number.

Once these things have been noted, a corresponding figure can be assigned on a globe or sphere in this way:

Let A and C be the poles of the ecliptic, B D the portion of the zodiac running between them through the middle, and K the star of Mars. The fixed stars, with their positions, are marked with these characters:

Char.	Star	Longitude	Latitude
E	tip of Pegasus's wing	3° 46' Υ	12° 35' N
F	back of the Whale	6° 20' Υ	16° 55' S
H	bright star of Aries	2° 14' X	9° 57' N
G	snout of the Whale	9° 39' X	7° 50' S

Diagram. A pointed-oval (vesica) projection of the celestial sphere onto the plane of an ecliptic meridian, with the two poles of the ecliptic A (top) and C (bottom) joined by the curving meridians that bound the figure. A horizontal line through the middle represents the zodiac/ecliptic B–D. Several great-circle arcs are drawn from pole A down through the field to pole C. A cluster of intersection points (K, L, M, N, P) lies in the dense central region just left of the midline, where the construction lines cross.

- A (top vertex): pole of the ecliptic

- C (bottom vertex): pole of the ecliptic
- E (left, on the zodiac line): tip of Pegasus's wing
- H (upper middle): bright star of Aries
- D (right of center, on the zodiac line): point on the zodiac
- G (lower middle, below D): snout of the Whale
- F (lower left): back of the Whale
- K, L, M, N, P (central cluster of intersections): auxiliary points of the construction

Procedure.

I. In triangle E A G, for E G the givens [διδόμενα, "the data"] are:

		Degrees	Minutes
E A	complement of the latitude of Pegasus's wing	77°	25'
G A	excess of [the latitude of] the Whale's snout	97°	50'
E A G	difference of longitude	35°	53'

Therefore G E is given = 41° 5'.

II. In the same triangle, for E G A, all the sides being given, E G A is given = 60° 30'.

III. In the right triangle G D L, for D L G and D L, the givens [διδόμενα] are:

		Degrees	Minutes
D G	7°	50'	latitude of the Whale's snout
L G D, that is, A G E	60°	30'	just found
D	(right angle)		

Therefore there is given:

	Degrees	Minutes
D L G	30°	26'
D L	13°	33'

IV. Similarly in triangle F C H, for F H and H F C — or rather its supplement to a semicircle, H F A — the givens [διδόμενα] are:

		Degrees	Minutes
F C	73°	5'	complement of the latitude of the Whale's back
C H	99°	57'	excess of the latitude of the bright star of Aries Υ beyond 90°
F C H	25°	54'	difference of longitude of these stars

Therefore there is given:

	Degrees	Minutes
F H	37°	7'
H F A	45°	29'

V. In the right triangle M F N, for M N F and M N, the givens [διδόμενα] are:

		Degrees	Minutes
F M	16°	55'	latitude of the Whale's back
M F N	45°	29'	the same as the H F A just found
M	(right angle)		

Therefore there is given:

	Degrees	Minutes
M N F	46°	59'
N M	16°	29'

Furthermore, let the sides D L and M N be added, and the sum subtracted from M D; the side N L = 3° 17' is left.

VI. In triangle N K L, for N K, the givens [διδόμενα] of the angles at the side are:

	Degrees	Minutes
N	46°	59'
L	30°	26'
N L	3°	17'

Therefore N K is given = 1° 38'.

VII. And finally, in the right triangle N K P, for K P and N P, the givens [διδόμενα] are:

		Degrees	Minutes
K N P	46°	59'	
K N	1°	38'	
K P N	(right angle)		

Therefore there is given:

	Degrees	Minutes
K P	1°	12'
N P	1°	10'

To this last is added M N = 16° 29', producing the arc M P = 17° 39'. If this is added to the longitude of the star at F — which is alongside M, at 6° 20' Υ — there emerges at the same time the longitude [of Mars] = 23° 59' Υ .

PROBLEM VI.

Given a straight line of two stars together with an unknown phenomenon, and at the same time the distance of one known star from the unknown star: its longitude and latitude are given here.

Moreover, from an observation in a single straight line, with at least two stars, together with the phenomenon's [φαινομένου] distance from one of those same stars, we can investigate its position more compactly — as in the preceding diagram, where the straight line E K is given through the position of Mars at K, with the distance E K. First the whole distance E G — that is, of the stars — is sought; then the angle A E K; and finally, in the triangle A E K, since two sides about the given angle at E are given, A K (the complement of the latitude of Mars) will not stay hidden, nor will E A K (the difference of longitude from the star at E).

For the remaining methods of observing and investigating the unknown position of a moving star without instruments, this can be referred to here: as often as we see it conjoined, more or less partilely [partiliter, "degree-for-degree"], with some fixed star, or with another wandering star whose position in the sky is known — or, when as nearly as possible, we estimate how far it falls short of a partile conjunction, as precisely as can be done — and afterward, mechanically, by means of a globe or of figures of the asterisms (above all those of the distinguished Johann Bayer [D. Ioannis Baijerii]), we fit this for the position of the unknown body. [Margin: See the distinguished work of star-figures of Johann Bayer, V.I.D., drawn up from the verification of Tycho Brahe.]

And so much for observations.

CHAPTER X.

On the parallaxes of a phenomenon, arising from the perceptible size of the Earth's semidiameter relative to the orbit and position of the phenomenon; together with the principal methods of the parallaxes to be observed in altitude — whether the phenomenon be fixed or have a motion of its own — from which the phenomenon's distance from the Earth is afterward given. Finally, by which methods in particular it is fitting to separate and resolve the said parallaxes of altitude into longitude and latitude.

In the preceding chapter we showed, by various methods, how the stars are commonly to be observed and reduced to order. In the present one, divine helper [D.A., *Deo Adiuvante*, "God helping"], we are about to treat of the most ingenious part of all astronomy — namely, the way in which, through observations to be precisely and opportunely undertaken in their own manner, and afterward bound together by the reckoning of triangles, the parallax (or aberration of sight) in the marking of a phenomenon — on account of the distance of the Earth's center from the observer's place, if any be perceptible in comparison with the phenomenon's distance — may be known. From this are deduced very many things in astronomy as necessary as they are delightful, and fitted, as it were, especially to establishing the truth of physics — namely, concerning the distances of phenomena from the Earth and their magnitudes. These bear not only on the calculation of the eclipses of the luminaries, but also uniquely on the generation and situation of comets and of other new phenomena, against the opinion of the old Peripatetics.

But so that the cause of parallaxes may be established at the outset, the following lemma must be set down first. In it, let A be the Earth, B the observer's station on its surface, and C L the quadrant of the sphere of the fixed stars, which — because of their too-great distance from the Earth — lack all parallax, so that the lines from the Earth's center A and from B on the surface are understood, for that reason, to converge. Let the orbit of a phenomenon that does have a parallax be D M, since at its semidiameter A M a perceptible magnitude of the side A B is found. The angle of parallax in the horizon is A M B, to which H M K is equal; but in the position where the phenomenon is elevated — namely at E — the angle of parallax is A E B, to which F E G is made equal.

For the rest, when the phenomenon is at D, the angle of parallax wholly vanishes, because A, B, D, C constitute a straight line. It is also readily perceived that the

angle BMA turns out the greatest of all the parallactic angles, while the smallest is the one nearest to D , on account of the relation of the angle at B . For when that angle is right, as in the horizon, each of the angles at A and M will be greater than when the same angle at B is found obtuse — and this successively, as in the remaining positions of the parallactic phenomenon above the horizon. These things being so established, we will now proceed to the demonstration of parallaxes, generally to be indicated from observations.

Diagram. A quadrant of the celestial sphere illustrating the parallax lemma. The right angle is at the lower-left vertex A (the Earth's center); the horizontal base AL and the vertical side AC bound the quadrant, whose arc runs from C (top) to L (lower right). B lies on the vertical above A (the observer's station). The phenomenon's orbit is an inner arc; M and E are positions of the phenomenon, with sight-lines drawn from A and B through them out to the labeled points (F, G near the top; H, K on the right side) on the fixed-star quadrant.

- A (lower-left vertex, right angle): center of the Earth
- B (on the vertical above A): observer's station on the surface
- C (top of the vertical side): point on the quadrant of the fixed stars
- L (lower-right end of the base): point on the quadrant
- D (left, on the horizontal): position where parallax vanishes
- M (inner, lower): position of the phenomenon (parallax angle BMA)
- E (inner, upper): elevated position of the phenomenon (parallax angle AEB)
- F, G (upper region of the quadrant arc): points cut on the fixed-star sphere
- H, K (right side): points cut on the fixed-star sphere

PROBLEM I.

Given both meridian altitudes of a phenomenon that is always visible around the pole of the world: the parallax aggregated in both altitudes (if it has any), or their difference, is given.

A celestial phenomenon whose parallax we are at pains to inquire into — if it has any — will either be immobile relative to the fixed stars, or will move. We will treat of both, in this problem and in those that follow. For if the phenomenon is in motion, it is reduced to immobility by observing the change of its declination in the diurnal motion and accommodating it proportionally.

Now there are two cases of the present problem. For either the phenomenon goes around the pole of the world in such a way that it is always seen below the vertex

of the place, toward the north, in both meridians; or else in the upper meridian it surpasses the vertex toward the south. The adjacent diagram explains the former case. In it — keeping for the most part the earlier letters, and the axis of the world halved at A N P — with a circle of the phenomenon described from the center A, this phenomenon's highest meridian altitude is set at E, its lowest at M, and that with respect only to the first conversion [the diurnal rotation], the phenomenon being otherwise immobile. Therefore, the elevation of the pole at N or P being given, it is easily gathered from both meridian altitudes of the phenomenon by how much the arc N M exceeds the arc N E.

Now for the aggregate of the parallaxes: the semicircle G Q is drawn from the pole of the world P; and since the arcs P F and N E are equal, P Q is made equal to P G. But the parallax of the phenomenon's lowest altitude, at M, is at the angle H M K (or B M A); if to this is added the arc Q H, which is equal to F G, the parallax of the highest altitude, then the aggregate of both parallaxes of the phenomenon at that point emerges in the arc Q K.

In the other case, we extract no less the difference of the parallaxes, if there be any — though account must be taken of refraction, if the phenomenon in the lowest meridian is near the horizon. In the present diagram, let the phenomenon be in the upper meridian at E, surpassing the vertex E D (or F C). Since it is established from the foregoing that the parallactic angle in the upper meridian is at B E A, but in the lowest meridian B M A, then — drawing the semicircle F Q from the pole P — K Q will be the difference of the parallaxes, that is, of the said angles B E A and B M A, which we proposed to obtain.

Diagram. A semicircular (quadrant-plus) diagram for the first case of Problem I, a phenomenon circling the pole below the vertex. The base runs from R (lower left) through A (center) to L (lower right); A is the foot of the world-axis. Concentric arcs are drawn from A: an outer quadrant from C (top) and an inner circle of the phenomenon. The pole region is at upper right (P, Q). E and M are the phenomenon's highest and lowest meridian positions (each drawn with a small radiant burst), with sight-lines to the cut points F, G (top), H, K (right).

- A (center of base): foot of the axis of the world / Earth's center
- R (lower-left end of base): point on the base line
- L (lower-right end of base): point on the base line
- C (top): top of the outer quadrant
- D (left of center): point on the construction
- B (on the vertical above A): observer's station

- E (upper inner): highest meridian altitude of the phenomenon
- M (lower inner): lowest meridian altitude of the phenomenon
- N (center-right): point on the phenomenon's circle
- P (upper right): pole of the world
- Q (right, below P): point of the parallax semicircle
- F, G (top): points cut on the fixed-star quadrant
- H, K (right side): points cut on the fixed-star quadrant

Diagram. A semicircular diagram for the second case of Problem I, the phenomenon in the upper meridian surpassing the vertex toward the south. The base runs from the lower left through A (center) to L (lower right); the outer semicircle rises to C (top). A dotted arc runs from the lower right up to the upper left (F, G region), marking the refracted/displaced path. E lies on the inner field with a small radiant burst near D; B is on the vertical near A; the parallax points H, K, Q appear on the right.

- A (center of base): foot of the axis of the world / Earth's center
- L (lower-right end of base): point on the base line
- C (top): top of the outer semicircle
- F, G (upper left): points cut on the fixed-star quadrant
- D (center): point on the construction
- E (inner, with radiant burst): position of the phenomenon in the upper meridian
- B (on the vertical above A): observer's station
- N (center-right): point on the construction
- P (upper right): pole of the world
- Q (right): point of the parallax semicircle
- H, K (right side): points cut on the fixed-star quadrant

PROBLEM II.

Given a phenomenon positioned together with two fixed stars on the same vertical circle — the positions of those stars being known, along with the elevation of the pole for the place — and given also the altitude of the phenomenon taken at the same moment as that of one of the fixed stars; then, in a second observation, taking no account of the verticals, given the distances of the phenomenon from those same fixed stars together with its own altitude and likewise the altitude of one of the stars: to determine the parallaxes of the phenomenon proper to each of the two times, if it has any.

This problem is a general one for finding all parallaxes — whether the phenomenon is continuously visible or whether it rises and sets. It was devised most ingeniously by Digges the Englishman [Thomas Digges], but given a new dress by Thaddaeus Hagecius [Tadeáš Hájek]; we now set it out concisely here. In the first observation we require only the altitude, without the distances from the fixed stars being observed, as in the following diagram. With the rest known in themselves, let the apparent place of the phenomenon in the first observation be at I, in which vertical its true position is also to be understood at D, and the fixed stars at C and B. Therefore, given the altitude of the phenomenon and likewise that of one of the stars, its distance from each of them is obtained by one and the same operation through the preceding problems. Then, in the second station, with the phenomenon shifted to H and the fixed stars appearing as companions to it, since here the data [Greek: δεδομένα (dedomena, "the things given," i.e. the givens)] are sufficiently present by the terms of the problem, its parallax at last follows from these, at each altitude, and so on.

[Margin: See problem, T. B., book 1, p. 668.]

Diagram. A hemisphere (celestial sphere drawn in perspective) with the zenith A at the top and the bounding circle as the horizon. Within it, great circles cross near the center to mark the apparent and true positions of the phenomenon and the two fixed stars across the two observations.

- A (apex): zenith
- I (left of center): apparent place of the phenomenon in the first observation
- D (below I): true position of the phenomenon
- B (upper center): fixed star
- C (two occurrences, upper center): fixed star (the lettering C appears twice as the circles cross)
- H (below center): position of the phenomenon in the second observation
- K (left): point on the horizon circle
- L (lower right): point on the horizon circle
- G (bottom): point on the bounding circle

PROBLEM III.

Given, over an interval of time, the distance of a phenomenon from two fixed stars positioned above or below it, together with the altitude of the phenomenon at each occasion — from the lowest that can be obtained up to the highest: the

track of the phenomenon's orbit is gathered, and from it the difference of parallax, if it has any.

This appears to be the most general method for unfolding parallax in those phenomena that admit it. In the adjoining figure, let A B C be the meridian and A D C the horizon, whose pole is B. Let the true path of the new phenomenon be E F, its apparent path D G, and let there also be two fixed stars from which, over the interval, its distance is taken — at M and N — at the first altitude at H, at the second at K, and at the third and last at L. Since the greater parallax is always nearer the horizon, the distances from the fixed stars come out wider here and narrower the higher the phenomenon has ascended — provided it is placed below the stars, as in the present figure. The reverse holds if the same phenomenon is higher than the stars in the daily revolution: so much so that, if you try to trace the track of a new phenomenon possessing parallax by means of Thaddaeus's [Thaddaeus Hagecius] observations, you would never manage to keep to the vestiges of a great circle in the sky.

[Margin: A fuller account of unfolding the parallaxes of new phenomena may be read in Thomas Digges, *De stella anni 72* [the new star of 1572], and in Thaddaeus Hagecius in his *Dialexis*.]

And so concerning the inquiry into the parallaxes of a new phenomenon that admits them, we have given certain problems that seemed to us best suited to this business. Whoever requires more and varied methods should consult Regiomontanus, Digges, Thaddaeus [Hagecius], Tycho [Brahe], and other astronomers for himself.

Diagram. A quarter-sphere (the upper half of the meridian) with the horizon A C as the base and the pole B at the top. Several great-circle arcs and chords cross within it, marking the true and apparent paths of the phenomenon and the points of distance-measurement at the three altitudes.

- B (top): pole of the horizon (zenith)
- A (lower left): point on the horizon
- C (lower right): point on the horizon
- F (upper left): point on the true path
- G (upper left, below F): point on the apparent path
- N (upper left): measured-distance point
- M (three occurrences, descending toward the right): the fixed-star/distance point at successive altitudes
- L (left of center): point

- K (center-left): point on the path at the second altitude
- H (lower center): point on the path at the first altitude
- D, E (lower center, near the horizon base): points where the paths meet the horizon
- R, V, Q (along the horizon base, left of center): points
- [Margin labels alongside this figure, transcribed under Problem III above.]

PROBLEM IV.

Given the parallax of a phenomenon's altitude: its distance from the earth is obtained.

For the demonstration of the present problem, let us call back here the first diagram of this chapter, upon which we explained the cause of the parallax that arises. Let the parallactic angle at the horizon first be given, B M A. Since, then, in the same plane triangle A B is given — the semidiameter of the earth, always remaining 860 German miles, as is gathered from Geometry — and likewise the angle A B M is right, and besides this the parallactic angle B M A is given, and therefore also the angle at A: the side B M is thereby obtained, or else A M, that is, the distance of the phenomenon from the surface or from the center of the earth, as one pleases.

The same is also accomplished in the triangle B E A, given the complement of the apparent altitude of the phenomenon D B E (or of the true altitude D A E) together with the parallactic angle B E A. For thus the three angles will be given along with the side B A, as was demonstrated before; and therefore either the side B E or A E will be given as well.

But since it helps to illustrate the proposed problem by examples — which are therefore most conveniently drawn from the Moon, as she suggests to us parallaxes that are, as it were, familiar and well marked — let it be so. From the restitution of the lunar hypothesis, let the horizontal parallax of the Moon, when she is at her mean distance from the earth, be $1^{\circ} 0' 51''$, which the angle B M A measures. And since the angle at B here is right, the angle B A M will be $88^{\circ} 59' 9''$.

In the said right triangle A B M, then, for the side A M as the distance of the Moon from the center of the earth, we have:

M	A B	B	A M
as $1^{\circ} 0' 51''$	to 1 semidiameter of the earth	so 90°	to $56\frac{1}{2}$ semidiameters of the earth

M	A B	B	A M
S.R. 176998	10000000		

Diagram. A quarter-circle parallax construction (the first diagram of the chapter, recalled here). The earth's semidiameter A B forms one side; arcs and radii within mark the apparent and true altitudes and the parallactic triangles A B M and B E A.

- C (top): apex of the quadrant
- F (upper right of center): point
- G (right of F): point
- D (left, mid-height): point
- E (center): vertex of the parallactic triangle
- B (lower left): center of the earth
- A (bottom left, below B): point
- M (lower center): apparent place / foot of the parallactic angle
- H (right): point on the arc
- K (lower right): point on the arc
- L (bottom right): point on the horizon

Having found the semidiameters of the earth by which the Moon is distant from it, it will be easy to extend the result into miles. For:

Semidiameters of the earth	mile s	Semidiameters of the earth	mile s
as 1	860	$56\frac{1}{2}$	4859 0

Let a second example be taken, likewise drawn from the Moon, in the triangle B E A, at her mean distance from the earth as before. Let the true altitude of the Moon be 45° , which the angle B A E measures; and let the parallactic angle A E B be $43' 35''$. Therefore, by the complement of the sum of these to a semicircle — by which the working must proceed — there is found $45^\circ 43' 35''$. And so:

Semidiameters of the earth		Semidiameters of the earth
as $43' 35''$	1 $45^\circ 43'\frac{7}{12}$	$56\frac{1}{2}$ — nearly as before
S.R. 126776	S.R. 7160143	

Furthermore, as to the given distance of the phenomenon from the earth, it is clear — by the inverse of the solution of the triangle A B M, or A B E — how a table of parallax from the horizon up to the vertex may be constructed: namely, with the sides A M (or A E) and A B known, so that no further labor need be undertaken for the demonstration of that matter. And once the distance of the star from the earth is given, together with its apparent magnitude, its true magnitude in comparison with the earth is easily obtained — an example of which is to be left by us to about the beginning of Book I of the *Theory*.

[Margin: Book 1 of the *Theory*, chap. 1.]

PROBLEM V.

To distribute a given altitude parallax from the horizon into longitude and latitude.

Among the various methods of this procedure handed down by others, we will bring forward two: one discovered by Lord Tycho [Brahe] and inserted in Book 2 of the *Progymnasmata* — which is the more general, and does not differ much from the following diagram — and another, which is more special and best suited to a solar eclipse, devised by ourselves.

[Margin: Book 2 of the *Progymnasmata*, p. 38.]

The method produced by Tycho is demonstrated as follows. In the adjoining figure, let A B E be the meridian of the place, A the pole of the world, B the pole of the place's vertex, and finally C the pole of the ecliptic — this last set within the arctic circle around the pole of the world. From these poles are described the horizon D E, whose pole is B, and the ecliptic F H from its own pole C. Now, with the quadrant B R drawn, the parallax is given by hypothesis in the circle of altitude M Q. Then, with C Q and C P drawn from the pole of the ecliptic C, and with Q P dropped perpendicular at P, it is plain that in the small right triangle Q P M every species of parallax is to be found. For Q M is of altitude, as we defined above; while here M P is of latitude, and Q P of longitude — differing almost imperceptibly from the arc L K marked off on the ecliptic.

For the knowledge of these latter quantities, one must proceed by this route. First, in the triangle A B C, for the side B C: since A B, the distance of the poles of the world and of the vertex, is given, and A C, the distance of the pole of the world and of the ecliptic — which is always equal to the maximum obliquity of the equator and the ecliptic — is given; and besides, the angle B A C is known from the remoteness of the ecliptic's pole C from the point of mid-heaven at N (for

which task the right ascension of the Sun and the time are required, to be presupposed in the daily revolution). Therefore, from these three:

Diagram. A sphere shown in perspective with a small circle (the arctic circle around the pole of the world) at upper left, and the horizon D E drawn as the long chord across the middle. Within it the spherical triangle A B C and the small right triangle Q P M of parallax are constructed.

- B (top): pole of the place's vertex
- N (top, left of B): point of mid-heaven (where C is reckoned from)
- A (within the small upper-left circle): pole of the world
- C (below A): pole of the ecliptic
- H (right): point on the ecliptic
- K (center-right): point
- M (center-right): apparent place in the circle of altitude
- L (center): point on the ecliptic (arc L K)
- P (right of center): foot of the perpendicular Q P
- Q (below K/M): vertex of the small parallax triangle
- D (left, on the horizon line): point on the horizon
- E (right, on the horizon line): point on the horizon
- F (lower left): point on the ecliptic
- R (lower right, on the horizon): foot of the quadrant B R

(Problem V, continued)

the first sought arc C B is obtained. Then in the triangle C B M, for the angle at M: since the said triangle has its sides given — namely B M, the complement of the star's altitude above the horizon; C M, the arc determined from the pole of the ecliptic at M, which here exceeds a quadrant by the arc K M; and finally C B, just now found — the angle C M B is thereby obtained from these, to which Q M P is equal. At last, in the small right triangle Q P M, from the data — besides the altitude parallax Q M, the two angles, namely Q P M right and Q M P just found — there are obtained the very things we sought: M P, the parallax of latitude, and Q P, of longitude.

This procedure suffices well enough for middling parallaxes such as the Moon's. But if larger ones could be conceived, to be drawn out into longitude and latitude according to the position of the ecliptic, the operation may be more correctly carried out in the oblique-angled triangle C M Q. In it, since the two sides C M and Q M, together with the contained angle at M, are known from what precedes, the

side C Q — measuring the longitude parallax beyond C M — will not be hidden, nor the angle M C Q, which rightly contains within itself the longitude parallax. An example of this problem is to be sought in Tycho [Brahe] at the place cited.

Diagram. A sphere in perspective with a small circle at upper left (around the pole) and the long horizon chord D E across the middle, repeating the construction of the previous figure for the continued solution.

- B (top): pole of the vertex
- N (top, left of B): point of mid-heaven
- A (within the small upper-left circle): pole of the world
- C (below A): pole of the ecliptic
- H (right): point on the ecliptic
- K (center): point (arc K M)
- M (center-right): apparent place
- L (center): point
- P (right of center): foot of perpendicular
- Q (below center): vertex of the small parallax triangle
- D (left, on the horizon line): point on the horizon
- E (right, near the horizon): point
- F (lower center): point
- R (center-right, on the horizon): point

PROBLEM VI.

Given a point on the ecliptic that bisects the sky [i.e. is at mid-heaven], together with its declination and the angle of intersection of the meridian and the ecliptic; given also the place of the luminaries on the ecliptic: the true altitude of these is obtained, to which — by the preceding problem — the corresponding parallax is also distributed into longitude and latitude.

We devised this method of disentangling and distributing parallaxes as best suited, if I am not mistaken, to a solar eclipse. For when we have first obtained the altitude of the luminaries above the horizon (which is required in this business), the angle running between the vertex and the ecliptic, and finally the altitude parallax, it becomes quite easy to distribute this parallax through longitude and latitude.

Let there be a diagram in which A B D M is the meridian, A M the horizon from the pole D, E N the equator from the pole Q, and finally E P the zodiac. Let the

place of the Sun be at F on the ecliptic, whose right ascension is known at R; and the time by which the Sun is distant from mid-heaven is measured by B R. Further, since A B is given — namely the elevation of the equator above the horizon — its complement B D is also given; likewise B C, the declination of mid-heaven, from what precedes; and similarly C F. And because C D is also granted from the induction of the preceding, the angle D C F is acquired —

[Margin: Problem 1 of this chapter; Problem 4, chap. 1 of this book.]

Diagram. A sphere in perspective with the meridian A B D M as the bounding circle, the horizon A M as a chord, and the equator E N and zodiac E P drawn across, with the small triangle of declination and parallax near the right.

- E (top): point where equator and zodiac meet (Aries symbol ♈ printed beside it)
- D (top, right of E): pole of the horizon
- C (upper center): point (declination of mid-heaven)
- Q (upper right): pole of the equator
- B (upper left of center): point on the meridian
- R (center): right-ascension point
- F (center): place of the Sun on the ecliptic
- T (two occurrences, center): points of the small triangle
- S (center): point
- M (right): point on the meridian / horizon
- A (left): pole of the horizon / point on the meridian
- G (lower left): point
- L (lower center): point
- N (lower right): point on the equator
- P (lower right): point on the zodiac

or rather is given from its own table. And so in triangle D C F, for D F (the complement of the Sun's altitude) and the angle C F D, the given quantities [διδόμενα (didomena, "the things given")] are:

C D D F C F therefore from / are given \ D C F these are given C F D

If, however, the angle C F D turns out to be obtuse, this will be a sign that the luminaries have passed the ninetieth degree of the ecliptic, and that the Moon is inclining toward the west by the parallax of longitude. Now let the arc, or line, S T be drawn. It then becomes clear that in the small right triangle F S T all the

parallaxes are given: that of altitude by S F, that of latitude by S T, and finally that of longitude by F T. And because this triangle is so cramped that its longest side scarcely spans a single degree, we err imperceptibly if we treat it as a plane triangle rather than a spherical one. In this triangle all the angles are given — for besides the one at F (given against the ecliptic), the angle constituted at T is always right — and likewise the side S F that measures the parallax of altitude. So S T, the parallax of latitude, and T F, the parallax of longitude, will easily be found, once the parallax of altitude has first been determined by S F.

This method seems to me the most concise and convenient of all for a solar eclipse, especially where the Moon has no appreciable latitude — quite apart from the fact that it supplies the foundation of Reinhold's tables (p. 99 and following). When, however, the Moon has northern latitude, one can take account of the augmentation of altitude when the calculation of the eclipse is set up. Anyone who wants a worked example will find it in Book I of the *Theorica*, in the solar eclipse computed for the year 1612, the 20th day of May. [Margin: *Theorica* Book I, Cap. 8; also *Theorica* Book I, Cap. 9.]

CAPUT XI.

On the Refractions of the Heavenly Bodies, on the Twilights, and on the Raising-up of the Meteors from the Surface of the Earth.

The doctrine of the refraction of the heavenly bodies, although it was in a manner already indicated in the optics of Witelo and of Alhazen [Ibn al-Haytham], was nevertheless in our own age first discovered from celestial experience by our Tycho Brahe, and its causes were brilliantly debated with Rothmann. [Margin: *Tomo 1 Epistolarum Astronomicarum*. Johannes Kepler, in his *Optics*.] John Kepler afterward attempted to explain these, binding them to numbers in a demonstration, but in a way not sufficiently general, since in maritime places and those nearer the Arctic pole the refractions vary immensely on account of the denser constitution of the air, so much so that they become there twice or even three times greater than in the interior of Germany, or elsewhere in places that have their air purified by the more effective rays of the Sun. Two things, therefore, present themselves for indicating the variety in the refraction of the heavenly bodies: the nature of the soil, and the twofold slanting of the Sun's rays as they strike the surface of the earth — one slant from the approach of the pole, the other from the horizon in each individual place. These things considered — and since beyond a half-quadrant from the horizon scarcely any sensible refractions are detected in any of the places where I have so far

resided, in Norway and elsewhere — I shall set out its causes briefly, together with Lord Tycho, by means of an attached diagram. In it,

Diagram. A spherical figure illustrating the resolution of solar-eclipse parallax into components of longitude, latitude, and altitude. A large circle (the sphere) carries several great circles intersecting near the top-left, where a tiny triangle of the parallax components is clustered. At the top is a small cluster of letters (the ecliptic point near the constellation symbol ♈ Aries and E); the great circles run down across the sphere to the lower labels.

- ♈ E (top left): the spring equinoctial point / start of the ecliptic, with E
- B, C, D (upper left, across the top): points where the great circles cross
- T, F (just below, in the tiny triangle): F = the Sun's place on the ecliptic; T = foot of the latitude/longitude resolution
- R, H, S, T (small cluster, upper left): the cramped right triangle F S T of the parallaxes (S = altitude foot)
- A (left): point on the horizon circle
- G (lower left): point on a great circle
- L (center): intersection point
- K (lower center): point on a great circle
- M (right): point on a great circle
- ♍ / ♌ N (lower right): zodiac/equator points with N
- Q (upper right): point on a great circle
- P (bottom right): point on the ecliptic or equator

three orbs are described about center A: the outermost, the celestial, in which the heavenly bodies are understood to be fixed, B C D; the terrestrial globe, H N; and the aerial sphere M K L, poured out everywhere around the earth. Now let the ethereal region — that which is contained between this sphere and the outermost orb — be of so subtle a matter and essence (just as that was shown by us in the preliminary matters, and is shown more fully by the doctrine of parallaxes and of refractions) that it causes by itself no refraction or vibration of the heavenly bodies. It is therefore established that the cause of refractions is to be considered as residing in the aerial sphere alone, in the manner I described above — so much so that, both by reason of the denser transparent medium and of the more slanting projection of the rays, the heavenly bodies are refracted more powerfully, whether through our own sight, or as they insinuate their own rays to us. Hence it happens that a star existing in the most sloping position is subject to the greatest refraction — as in the case of a star existing at L on the horizon G:

on account of the more slanting incidence of the ray at L, and the denser transparent medium H L, which at P nearly vanishes.

Diagram. A quarter-circle (quadrant) figure with center A at the bottom, illustrating the three concentric orbs and the refraction of a star's ray near the horizon. The outermost arc (celestial orb) runs from B (bottom left) up through C, D (top), to the right edge; two smaller concentric arcs below it represent the aerial sphere and the terrestrial globe. The horizontal base B–A–N–G is the horizon line; D–A is the vertical.

- D (top): zenith point on the celestial orb
- C (left): point on the celestial orb at the horizon level
- E (upper right): point on the celestial orb
- F (right): point where the horizontal meets the celestial orb
- M (center, on the horizontal): point on the middle (aerial) sphere
- P (center, right of M): point on the aerial sphere
- K (center, below P): intersection of the ray with an arc
- H (center, below M): point on the terrestrial globe / inner arc
- L (center, right of K): the star's apparent place near the horizon
- A (bottom center): the center of the earth (and of all three orbs)
- N (bottom, right of A): point on the horizon base
- B (bottom left): terminus of the horizon base, on the celestial orb
- G (bottom right): terminus of the horizon base (the horizon point of the star)

[Margin: See the *Prognosticon Astronomicum*.]

PROBLEM I.

Given the depression of the Sun below the horizon of a place, at the moment when the morning or evening twilight wholly ceases: the elevation of the outermost vapors from the earth is given.

We touched on the cause of the twilights in the preliminaries of *Astronomy* — namely the little corpuscles among the vapors raised on high above the clouds, which, because they are illuminated by the Sun's rays, incline toward whiteness. As for the heights of these corpuscles, let us inquire, from the last proposition of Alhazen's *Optics* and the 60th proposition of Witelo's first book, into the highest limit at which (beyond doubt) refractions begin to come about, so that the difference between the side H L and the side H P — as it were the refracting transparent medium in the preceding figure — may become known to us, and

that afterward the limits of these twilights may be ascertained for each individual place on earth and at each season of the year. [Margin: For Alhazen, the last [proposition]; and Witelo, proposition 60.]

Now on account of the discrepancy that is found between the observations of Tycho and Rothmann — book I of the *Astronomical Letters* — of 7 degrees, by which Rothmann increases the depression of the Sun below the horizon beyond Tycho's figure (all twilight then ceasing), while the former sets that depression at 24 degrees and Tycho at only 17; and furthermore because we in Norway have found the twilights to be more lasting, even in the winter season — we do not doubt that, for the final fading of these twilights in general, the Sun is placed 20 degrees below the horizon. Hence the demonstration of the proposed problem, upon the appended diagram, follows in this manner.

Let A (in the following figure) be the center of the earth; let B C E F G be the circle of the Sun, C H D that of the Sun [recte: C H D the circle of the body of the Sun, drawn from center B], and K T L P [recte: K T L R] that of the earth. And let the visible horizon be L M, above which straight line alone there is sight. Let the shadow of the earth cast by the Sun be R P T Q, while the part of the earth illuminated by the Sun is P K T, 180 degrees 28 minutes, because the diameter of the Sun contains the diameter of the earth...

about $5\frac{1}{2}$ times over, with the Sun raised to its mean distance from the earth — namely 1200 earth-semidiameters, according to Ptolemy, and even a little more according to our recent emendation. And so the angle P A B (namely half the circumference P K T) will be 90 degrees 14 minutes. But the angle G A B, by hypothesis of the Sun's depression below the horizon (the twilight, that is, wholly failing), was 20 degrees. Therefore the angle P A G is given as 70 degrees 14 minutes. And since L A G is a right angle, L A P will be 19 degrees 46 minutes. Moreover, since the twilight closes at point M — where those whitening corpuscles, which from the terrestrial vapors cause the twilights, are no longer visible — let the angle L A P be bisected at point M, at the point of contact of the circle where the sides M P and M L are equal; and M A P will be 9 degrees 53 minutes. But the proposal is to inquire into A M. Therefore, for this purpose, in the plane right triangle M P A — since all the angles are given, together with the side A P, the semidiameter of the earth, as 860 German miles — A M will not lie hidden: 873. And so the difference of A L and A M, namely N M, of nearly 13 miles, defines the exaltation of the highest vapors from the earth, of which we have been treating.

Diagram. A large circle (the Sun's circle, B C E F G) with center near A, within which a smaller circle (the earth, with center A) sits at the lower left. A narrow shaded cone (the earth's shadow, R P T Q) extends up to the upper left toward Q, R. The horizontal line E–A–G is the visible horizon; the vertical F–A–C runs through the figure. A second small circle (lower) carries B, C, D, H.

- F (top): top point on the great Sun-circle, on the vertical
- Q, R (upper left): the apex region of the shaded shadow-cone
- M (center, in the cone): point where the twilight closes
- L (center left): the star's apparent / horizon place
- N (center): point near A on the horizon
- P (center right of N): point on the small earth-circle
- T (left of A): point on the earth-circle
- A (center): the center of the earth
- K (below A): point on the earth-circle
- G (right): horizon terminus on the great circle
- E (left): horizon terminus on the great circle
- H (lower center): point on the lower small circle
- D (lower right): point on the lower small circle
- B (bottom): point on the lower small circle
- C (bottom left): point on the lower small circle

PROBLEM II.

Given the distance of the highest vapors from the earth, to bring into the open the quantity of the refracting transparent medium in each case — at the maximum and at the minimum apparent refraction — and to distinguish them from one another.

For the explanation of this problem, let the second-to-last diagram (or the one placed on the page that next follows) be recalled, with straight lines drawn from the center of the earth A to L and to P. So in the first right triangle A H L — the right angle being at H (because the starry heaven at G excludes sensible parallaxes, as was shown above concerning the fixed stars) — for the sought H L we proceed thus:

First.

as	A L	is to	H	so	A H	is to	H L
	873		90°		860		sine R. 98516

Answer: 80 degrees 7 minutes [the angle H L A].

Then in the same triangle:

a s H	is to	A L	s o H A L	is to	H L
90° (sine T. 100000)		8 7 3	9° 53'		150 — [German miles]

(sine R. 17164)

Further, at the altitude of the phenomenon of 45 degrees — where we said before that the refraction of the heavenly bodies nearly ceases — in the triangle H A P, since the angle P H A is given as 135 degrees (45 degrees of P H L, that is, being added to the right angle), together with A H and A P, first, for the angle at P, it will be:

a A s P	is to	A H P	s A o H	is to	H P A
8 7 3		135° (sine R. 70710)	8 6 0		(69658, to which the sine R. corresponds)

Answer: 44 degrees 9 minutes [the angle H P A], add:

gr. 44 mi. 9 add. gr. 135 mi. 0 gr. 179 mi. 9 gr. 179 mi. 60 (semicircle) H A P gr. 0 mi. 51 difference: the sought H A P.

Then:

a s A H P	is to	A P	s o H A P	is to	H A
135° (sine R. 70710)		8 7 3	51' (sine R. 1483)		18½ (nearly) [miles]

150 — the upper measure [from above].

Therefore, the sought difference between the densest transparent medium and that in which the refractions cease is 131½ German miles.

[Margin: On the twilights, let there be read Petrus Nonius [Pedro Nunes], Johannes Kepler, Lord Ambrosius Rhodius, etc., and Lord Petrus Laurenbergius [Peter Lauremberg].]

Diagram. A quarter-circle (quadrant) figure with center A at the bottom, repeating the structure of fig-p126-1: the outermost arc (B–C–D–F, the celestial orb), two inner concentric arcs (aerial sphere and terrestrial globe), the horizontal base B–A–N–G as the horizon, and the vertical D–A. Rays are drawn from A out to E, K, L on the arcs.

- D (top): zenith point on the celestial orb
- C (left): point on the celestial orb at horizon level
- E (upper right): point on the celestial orb (the phenomenon at altitude 45°)
- F (right): point where the horizontal meets the celestial orb
- M (center, on horizontal): point on the aerial sphere
- P (center, right of M): point on the aerial sphere
- K (center, below P): intersection of a ray with an arc
- H (center, below M): point on the inner arc
- L (center, right of K): the star's apparent place
- A (bottom center): the center of the earth
- N (bottom, right of A): point on the horizon base
- B (bottom left): terminus of the horizon base
- G (bottom right): terminus of the horizon base

From this comparison one may see how much a transparent medium denser in quantity near the horizon contributes to augmenting the refractions — beyond the two causes that concur together: the one, namely, always similar to itself, concerning the unequal obliquity of the incidence of the rays; the other concerning the change of the air with the change of place on earth. And we find by experience that so great a variety inheres in this last cause that we cannot bring ourselves to measure the refractions except by that same experience — imitating in this matter Lord Tycho Brahe, who had published tables of refractions separately for the Sun, the Moon, and the stars, in the first book of the *Progymnasmata*, for the horizon of Denmark.

As for the observation of the Hollanders in Nova Zembla in the year of our Lord 1596 and following — where the Sun returned to their view 17 days before its lawful time — what else shall we say, except that so great a density of the air was accumulated, through the Sun's perpetual absence below the horizon from the

3rd of November to the 11th of February (new style), that it produced this whole discrepancy?

PROBLEM III.

Given the elevation of the pole and the declination of the Sun, the moment of the hour of twilight — whether morning or evening — that ends at the horizon is given.

But what twilight is, is gathered from the foregoing causes: namely the doubtful light that precedes the rising of the Sun and follows its setting, and is not wholly closed off by the horizon until the Sun has been depressed 20 degrees below it, according to the primary motion. Moreover, in the first place, before the declaration of the problem, by comparing the complement of the pole's elevation and of the northern declination of the Sun (together with the given depression of the Sun) on the near side, or of the meridional declination beyond the equator, it will be explored when and in what places the twilight is continuous. For when and where the aggregate — formed from the Sun's declination at any given time and its depression of 20 degrees below the horizon (by hypothesis) — is greater than the elevation of the equator, that is, than the complement of the pole, perpetual twilight reigns, as is easily recognized in our material sphere.

[recognized]. We now turn to the demonstration, by means of an example fitted to the present time. On this present day, when the Sun at dawn was at $6^{\circ} 14'$ of [Pisces] [Margin: 6 D. 14 Febr.], its declination is $9^{\circ} 11'$, the complement of the elevation of the pole of this place is $34^{\circ} 17'$, and finally the depression of the Sun below the horizon is 20° , by hypothesis. We shall fit the following diagram to our sphere.

Diagram. A celestial sphere shown in the plane of the meridian, for the dawn/twilight problem. The meridian circle is F E H G; the horizon is the line F H; its lowest pole (nadir) is A; the equator is E G, whose antarctic pole is B. C is the position of the Sun below the horizon at the moment dawn first appears. From C, arc L C is the Sun's southern declination ($9^{\circ} 11'$), and arc M C is the Sun's depression below the horizon (20°). Triangle A B C is the figure to be solved.

- I (top): pole/zenith point at the top of the meridian
- E (upper left): equatorial point on the meridian
- F (left): horizon point (west/east endpoint of F H)
- K (center): center of the sphere
- M (upper right, on horizon): endpoint of the horizon line; foot of the Sun's depression arc

- M [recte: μ] (just left of M, near center): the small point marking the depression measure on the Sun's circle [?]
- L (below M, center-right): point on the equator/declination arc
- M (right, on horizon): horizon endpoint labelled M
- G (lower right): equatorial point G
- C (center, below L): the Sun's position below the horizon
- B (lower left): antarctic pole of the equator
- A (bottom): lowest pole of the horizon (nadir)

The triangle A B C is solved for the angle at B, which is "the one being sought" [Greek: διδόμενα (didomena, "the things given"); here printed Διδόμενα referring to the data of the triangle]. The given quantities are:

- A B = $34^{\circ} 17'$, the distance between the poles;
- B C = $80^{\circ} 49'$, the complement of the Sun's declination;
- A C = $70^{\circ} 0'$, the complement of the Sun's depression — continuously.

From these, angle C B A is found to be $67^{\circ} 48'$, which is measured by the arc G L of the equator. Converted into time, this gives 4 hours 31 minutes 12 seconds — the first morning dawn. From this the end of the evening twilight is likewise found, by subtracting the same time already determined from 12 hours: namely, 7 hours 28 minutes 48 seconds.

Another example.

We seek the moment at which the evening twilight ends on 24 March of the year 1611, on which the solemnity of Easter falls. Around this time the Sun is at $13\frac{1}{2}^{\circ}$ of [Aries], to which corresponds a solar declination of $5^{\circ} 21' N$.

Diagram. A second celestial sphere in the plane of the meridian, of the same construction as the first, for finding the angle C B A in triangle A B C. The vertical dotted line is the axis through K.

- A (top): pole/zenith point at the top of the meridian
- B (upper left): antarctic pole
- G (upper right): equatorial point
- D (left, on horizon): horizon endpoint
- L (center-left): point on the declination arc
- M (center-left, right of L): point marking the depression measure
- K (center): center of the sphere

- C (center, below M, marked with an asterisk-like crossing): the Sun's position below the horizon
- H (right, on horizon): horizon endpoint
- I (lower left): equatorial/meridian point
- F (bottom): lowest pole (nadir)

In triangle A B C the given quantities are: A B = $34^{\circ} 17'$, the distance between the poles; B C = $84^{\circ} 39'$, the complement of the Sun's declination; A C = $110^{\circ} 0'$, the excess of the Sun's depression below the horizon (20°) beyond a quadrant. Therefore the oblique angle A B C is found to be $138^{\circ} 20'$, to which there correspond in time 9 hours 13 minutes 20 seconds. Subtracting these from 12 hours leaves 2 hours 46 minutes 40 seconds — and this time also indicates the first morning dawn on the same day.

PROBLEM IV.

Given the direct distance on the earth between two observers who observe the same small cloud at [the same] altitude above the horizon, the elevation of that cloud above the surface of the earth is found.

This problem investigates for us the distance of the middle region of the air, through the elevation of clouds. Here it should be noted in passing — something we have seen confirmed by experience even in this matter — that in the winter season, on account of the weaker effect of the Sun's rays, clouds and meteors of this kind are far nearer to us, almost by half, than in summer; this being in proportion to how high the Sun itself is raised above the horizon of a place in its daily revolution. With us, therefore, in winter they are raised scarcely $\frac{3}{4}$ mile, whereas in summer about $1\frac{1}{2}$ mile from the surface of the earth.

But to add a demonstration to this problem: let the horizon G B be drawn, below the apparition of the suspended cloud at C; and at the two stations A and D, whose interval can be ascertained, let the altitude of the same cloud be taken by both observers at once to a given mark, measured from the horizon on the same side — so that meanwhile the cloud's motion or breadth does not impede this operation. Then, lines having been drawn from A and D to the small cloud C, and also the perpendicular C B, it is established that these measure the sought elevation of the cloud from the earth, in this way:

Diagram. A woodcut of a cloud suspended above a horizon line. The horizon is the line G B ... E (drawn across the lower part). Below the cloud's apparent

position C, lines run from the two ground stations A and D up to the cloud; the perpendicular C B drops from the cloud to the horizon.

- G (lower left): left endpoint of the horizon line
- A (lower center): the first observer's station
- D (lower center, right of A): the second observer's station
- B (lower center, right of D): foot of the perpendicular from the cloud
- E (lower right): right endpoint of the horizon line
- C (within the cloud, above): the apparent position of the cloud

First, in triangle A D C, since all the angles are given together with the side A D — inasmuch as the angle A is measured by the altitude of the prior [observation], while D is measured by the complement of the altitude of the later observation to a semicircle, and the side A D is granted by hypothesis — the side A C is therefore found. From this, in the right triangle A B C, with the angles given together with the side A C, the sought elevation of the cloud, C B, is found.

PROBLEM V.

Given the distance between the shadow of a cloud cast perpendicularly by the Sun's rays onto the surface of the earth or water, and the Sun's altitude at the same time from the place of the observer, the elevation of the observed cloud above the earth is found.

This problem builds upon the previous one (on ascertaining the altitude of clouds) with the experience of Mr. David Pastor of Resterhave in East Frisia [Ostfriesland], an excellent cultivator of the renewal of astronomy in our time. It is to be demonstrated by the following diagram.

Diagram. A woodcut showing a cloud above the earth's surface with the Sun's rays. D F E is the meridian; from the center, namely the earth A, the cloud is seen at C. Through its nearer, facing part A, the line A F is drawn, leading the Sun's rays from F (the Sun). The shadow of the cloud C H falls directly on the earth's surface below, namely B G.

- D (lower left): left endpoint of the meridian/horizon line
- F (upper, on the arc): the Sun
- H (within/below the cloud): the point of the cloud casting the shadow
- A (lower center): the earth / observer's center
- B (lower center, right of A): the foot of the cloud's shadow

- K (lower center, between B and G): the alternative observation station mentioned in the text
- G (lower center, right of B): edge of the shadow region
- E (lower right): right endpoint of the meridian/horizon line
- C (above, in the cloud): the observed cloud

Since the whole region H F above the clouds is illuminated by the rays of the same Sun, the shadow of the cloud C H directly besets the surface of the earth below — namely B G — and so the angle at B is made a right angle. The angle C A B measures the very altitude of the Sun above the horizon, whose complement the angle A C B exhibits; and the side A B is likewise known, from the distance of the shadow (at a definite place on the surface of the earth or water) as it appears from the observer's station A. Therefore C B itself, the sought elevation of the cloud, is found.

Moreover, this method of ours too was born of the occasion, among these reflections: namely, if the observer's station is fixed beneath the shadow of the cloud, between B and G — say at K — and a line is drawn from K to C, through whose end at C the altitude of the cloud above the horizon is observed (with a quadrant or other instrument) from K; and at the same time it is marked where the Sun's ray first lets itself fall perpendicularly from C, that is, from the same cloud, at B — something that can often be noticed in open country. Therefore, in the right triangle K B C, with all the angles likewise given together with the side K B, B C is found as before.

CHAPTER XII.

On the doctrine of Gnomonics in the construction of sundials.

[Margin: CLASSIS TERTIA. ("Third class/division.")]

I do not here take up the doctrine of Gnomonics for Altimetry — that is, the measurement of things — as the ancients did, who for the most part measured the heights of things from the direct and reverse shadow of the Sun (since we more correctly use, in geometry, a quadrant, sextant, or some other sufficient instrument in place of a gnomon). Rather I direct the chief part of this doctrine to the skillful construction of sundials [sciatherica], and indeed of every kind, upon the sphere — by methods more compendious than those hitherto found out by others — about to show, in this part, what I can contribute. These methods I discovered in the castle of Rosenholm in the year of the Lord 1601, in that ample leisure which then fell to me — when I had first returned from Bohemia —

through the kind invitation of that noble man, Senator of the realm, and pillar of all solid learning and of the learned, Mr. Holger Rosenkrantz [Holger Rosenkrantz]; before, in the same year, I set out with that most noble man of learned refinement, Mr. Steno Bille of Billesholm [Steno Bille], into Norway, as far as Nidaros [Trondheim].

This Gnomonic doctrine is so called from [Greek: γνώμων (gnōmōn, "gnomon," the pointer/style)] — that is, the style — because it embraces a sure account of the shadow of the style cast by the Sun's rays onto a surface, chiefly a plane. Such a shadow is marked either from the root of the style fixed to the surface, or from its extremity. Next, the thing itself — [Greek: σκιαθηρικόν (skiathērikon, "shadow-catcher," sundial) from σκιά (skia, "shadow")], that is, "shadow," — the said instrument is very ancient, since it was chiefly destined by the ancients for marking out times in hours and in the sixtieths of hours [Margin: 2 Reg. c. 20. (2 Kings 20)]. It can be fashioned in different forms: either it will be adjusted to the plane of the horizon, whence it is called horizontal; or it will be erected at right angles upon a wall facing the Sun, and is thence called mural. Each kind, however, either indicates the times simply in hours and their minutes; or, with more elaborate apparatus — namely with an inscription of the signs of the zodiac — by which the Sun's course through the whole year, likewise the length of day and night, together with the fixed feasts, the twilights, and the rest, are known for each single day.

Lastly, it should be indicated generally that sundials can be constructed for any plane, and likewise for a spherical or even concave surface. And although others have completed this doctrine mechanically, through many windings, we nonetheless shall in what follows demonstrate it more correctly and with greater profit by the doctrine of triangles, briefly — first in sundials that are to be constructed on the plane of the horizon, then in those for the vertical (the wall). [Margin: Exstant inter alia multa hujus generis scripta octo libri Clavii de Gnomonicis. ("Among many other writings of this kind there exist the eight books of Clavius on Gnomonics.")]

PROBLEM I.

Given the elevation of the pole of a place, there are found — both in the plane of the horizon and on the vertical — the intersections of the equal hours, together with the arcs intercepted between the equator and the horizon, and likewise between the equator and the vertical of the place.

[Margin: Lib. 1. Sphær. cap. 4. (Book 1 of the Spherics, chapter 4)]

What we recalled in the previous book about the equator — namely that it is the regulator of all equal times, as was afterward shown by several examples in this book — will now likewise be called especially to mind, since we are dealing with sundials.

But to come to the exposition of the problem:

Diagram. A circle representing the sphere for the horizontal and vertical sundial construction. A B O H is the meridian; E H is the horizon, whose pole is B; K B is the principal vertical (prime vertical), at the ninetieth degree from the meridian. At K, in the east or west, is O G, the equator, with pole A. The quadrant O K is divided into six equal parts for the six hours equally taken on either side of noon, with the style erected from K toward A (so that it always stays conformable to the world-axis). From the world-pole A, circles (the style's shadows) are drawn that divide the equator into six equal parts at O, C, D, M, N, P; on the horizontal quadrant K H and the vertical B K these fall at the points Q, R, S, T, V, P.

- B (top): pole of the horizon (zenith)
- A (upper left): pole of the equator (world-pole), source of the shadow-circles
- Q, R, S, T, V (descending the left side toward center): the hour-intersections on the vertical B K
- O (right): equatorial point on the prime vertical
- C, D, M, N, P (descending on the right side toward center): the hour-divisions of the equator
- E (left, on horizon): horizon endpoint
- K (center): center of the sphere; foot of the style
- V, T, S, R, Q (along the horizon line to the right of K): the hour-intersections on the horizontal quadrant K H
- H (right, on horizon): horizon endpoint
- G (lower left): equatorial point
- G (lower right): equatorial point

It is necessary that these intersections — which, when they are on the equator (as I said) are equal — be unequal on the horizontal quadrant K H and on the vertical B K, understood by the same marks which we have applied on either side (just as in our material sphere all this is seen by eye). So, lines having been drawn from the world-pole A as proportional arcs of circles (like the shadows of the style), which divide the equator into six equal parts O, C, D, M, N, P — reckoning 15° for each single interval, as agreeing with one hour — it must be seen how much

space the individual intervals claim for themselves, both on the horizon and on the principal vertical, corresponding to the equal spaces of the equator. The doctrine of spherical triangles makes this manifest as follows.

First, on the horizon, for the eleventh hour before noon and the first after noon: since in the right triangle Q C K there are given K C = 75° by hypothesis, the angle Q K C = $34^\circ 17'$ (the elevation of the equator of this place), together with the right angle at C — there is therefore found from these K Q = $77^\circ 31'$. Further, in the same way, supposing K D = 60° by hypothesis, and retaining the angle at K, there emerges K R (or rather H R), the interval of two hours from noon. And so in the rest, diminishing each time the arc of the equator intercepted between the meridian and the east or west by 15° .

For the vertical sundial, in just the same way, both B Q and the rest of the hour-intersections on the wall (the principal vertical) become known, the angle B K O being here set as the elevation of the pole itself — as in this example. Let the interval on the wall K B agreeing with three hours from noon be given. Since the demonstration is enclosed within the right triangle K M S, we have for K S the following:

M	{ K compl.	{ K M compl.	{ K S compl.	B S.
grad. 90	ad gr. 34 m. 17	sic gr. 45	ad. T	
S. T. 100000	S. R. 56329	T. 1000000	56329	Resp. gr. 29 m. 24.

That is: as M (90°) is to K compl. ($34^\circ 17'$), so K M compl. (45°) is to K S compl., giving B S = $29^\circ 24'$. In modern terms: as the sine total (radius) 100000 is to the sine of $34^\circ 17'$ (S. R. 56329 [recte: this is the sine of the complement / co-function value used]), so the tangent of 45° (T. 1000000) is to the tangent T. — yielding 56329 — answer $29^\circ 24'$.

And so in the rest, until tables are drawn up for definite pole-elevations, and the intervals from the meridian up to the sixth hour — equal on either side on the equator — have come out. Those that we obtain on the horizontal arise by contrary intersections. But the hours that lie beyond this on the horizontal are indicated by the opposite intersections.

Now, with a quadrant of a circle prepared — on which both degrees and minutes can be had — it will be easy to delineate a sundial of either kind by its hours, whether it be large or small. For since all circles are equally divided, if several

circles are described from the same center, a rule marked off by degrees and minutes on the quadrant cuts equal portions in all of them,

to each individual hour — which is a mechanical matter, and so is best left to the ingenuity of each practitioner.

Finally, as to the placement of the stile: since the stile itself takes on the arrangement of the world's axis, in a horizontal sundial it will have an inclination toward the north corresponding to the elevation of the pole above the plane of the horizon; whereas in a wall (mural) dial the stile will stand off from the wall toward the south by an angle corresponding to the elevation of the equator at the place. In either case the conformity of its positioning can be checked with a small quadrant, and the length of the stile here is purely arbitrary. This brief treatment will suffice for the simple sundials laid out along the axis of the world (that is, of the earth), without any declination.

Last of all comes the investigation of the intercepted arcs — which was the second part of the problem — and this too is easily handled in the preceding diagram. For in triangle C K Q (taking as our example the eleventh, or first, hour from the meridian): K C is given; likewise the angle C K Q, the elevation of the equator; and besides, the angle at C in the equator is a right angle. Thus C Q is given for the horizontal dial, and likewise C Q for the mural dial together with its complement A Q — that is, when arc B O on the meridian is supposed for the angle at K, namely the elevation of the pole at the place. The use of these arcs is required later, in connection with the skillful inscription of the signs of the zodiac on the dial, where worked examples of finding these arcs in numbers are set out more fully. And although tables for this problem will be found below, drawn up for nearly every elevation of the pole, from which sundials can be quickly constructed, we nonetheless leave this particular one — fitted to the elevation of Copenhagen [Hafnia] — in this place, at least for the hour-intervals on both kinds of dial.

Small table for the hour-intervals, with their differences, at an elevation of the pole of 55 deg., 43 min., on the Copenhagen [Hafnia] dial, both horizontal and mural.

Hours		In		In		In mural:		In mural:	
from		horizontal:		horizontal:		Arc on wall		Difference	
meridian		Arc on		Difference		Arc on wall		Difference	
		horizontal							
A.M.	P	grad.	m	grad.	m	grad.	m	grad.	m
	.		i		i		i		i

Hours from meridian	In horizontal: Arc on horizontal		In horizontal: Difference		In mural: Arc on wall		In mural: Difference	
	M		n		n		n	n
	.		.		.		.	.
12	1	0	0		0	0		
	2							
11	1	12	2	13	1	8	3	2
			9				6	5
10	2	25	3	14	4	18	1	2
			0					3
9	3	39	3	15	3	29	2	5
			4		0		4	4
8	4	55	4	16	5	44	1	1
					8		8	6
7	5	72	2	17	5	64	3	2
					8		4	6
6	6	90	0			90	0	

PROBLEM II.

Given the inclination of a wall to the rising and setting from the meridian: the hour-intersections on a mural dial are also given.

[Margin: Probl. 1, c. 8 [of] this [book].]

For the mural dial there is this special case: that the walls or surfaces to which dials are fixed are rarely set exactly opposite the point of the meridian. This inclination is determined either by the compass, or by a meridian line, or — better — from celestial observation, since it is to be examined from the shadow of the Sun, or the appearance of a star, against the flatness of the wall. Once it has been found, proceed as follows. In the added diagram, on center A, let there be the wall K E D, inclining toward the rising or setting by 30 parts from the principal vertical circle F H L C, drawn beforehand about B, the antarctic pole of the world. From here let the line E A D descend perpendicular to the wall. From this the aforesaid inclination is indicated both by the arcs K F and H I, and by the distance B of the pole of the equator from that perpendicular line. And because the arc E B D C, lying superficially upon the said perpendicular, represents the

meridian, it is plain that the point of the meridian, that is, of the twelfth hour, resides at point D C. Now let it be granted that the wall here inclines toward the west; then the rays of the Sun will eventually appear upon it in the morning at F, but will likewise depart at H so much the later in the evening — for the mural plane can receive the Sun's rays for only 12 hours, as the half of the equator H L F demonstrates. But before all else, so that the angle D F C and the arc C F may be known, proceed as follows.

Diagram. A full circle on center A, representing the declining mural dial. A horizontal diameter runs K (left) — A (center) — to the right; a vertical diameter runs E (top) downward. B, the antarctic pole of the world, sits below center, and from B a fan of straight hour-lines radiates down toward the lower rim. Around the lower circumference the hour-points are marked, with Arabic morning-hour numerals on the lower-left arc (9, 10, 11, 12) beside Roman hour-labels (IX, X, XI, XII), the meridian/twelfth-hour point at D and C near the bottom, and Arabic afternoon numerals up the right-hand arc (1, 2, 3, 4, 5, 6, 7, 8, 9) with their Roman counterparts. F lies on the lower-left rim where the morning illumination begins; H and I lie on the upper-right arc; L and V appear on the right side. A dotted line runs through the figure marking the meridian E B D C.

- E (top of vertical diameter): the zenith / top of the principal vertical circle
- A (center): center of the dial
- K (left end of horizontal diameter): left terminus
- B (below center): the antarctic pole of the world, origin of the hour-lines
- F (lower-left rim): point of first morning illumination
- H (upper-right arc): point of evening departure of the rays
- I (upper-right arc, above H): inclination-arc point H I
- L, V (right side): rim points on the half-equator H L F
- D, C (bottom): the meridian / twelfth-hour point
- 9, 10, 11, 12 (lower-left arc, Arabic): morning hour-numbers, with Roman IX, X, XI, XII alongside
- 1, 2, 3, 4, 5, 6, 7, 8 (right arc, Arabic): afternoon hour-numbers, with their Roman hour-labels

In the right triangle D F C — since the given quantities [διδόμενα, the data] are D C, the elevation of the Copenhagen pole itself, 55 deg., 43 min.; the angle F D C, falling short of a right angle by exactly as much as the wall inclines on the other side, so that in this example it is 60 deg.; and the angle at C is right — therefore D F C is given as 60 deg., 48 min., and F C as 55 deg., 3 min. Further, once F C and the

angle at F have been found, that arc C F of the equator must be divided, as far as can be done, into equal hour-parts. In this example arc C F, 55 deg., 2 min., makes 3 hr., 40 min., 12 sec. From this it is understood that the first entry of the solar rays upon this declining dial takes place at F, at about 8 hr., 20 min., the time in the morning at which the first illumination of that same dial occurs.

In the same way the operation can also be set up for the eastern side. For when, between the end of the quadrant at L and the 3rd hour P.M. — by reducing the hour-arc on the equator into hours and so on, and by its resolution [ἀνάλυσις, analysis] — there are gathered 10 deg., 3 min., the consequence is that arc H falls short of the quadrant on the equator by exactly that amount, up to the 3rd hour. Thus H on the equator is removed from the 3rd hour P.M. by 79 deg., 57 min.; and the following points likewise, for the remaining hours down to the last, and so on. With these properly divided and distributed across the equator, the found angle D F C, 60 deg., 48 min., is taken as if for the elevation of the pole; and then, for the hour-arcs and the rest, the procedure goes by the immediately preceding problem.

[Margin: Prob. 1, cap. [of] this [book].]

PROBLEM III.

Given the declination of the Sun and the arc intercepted between the horizon and the equator on a horizontal dial — or the arc intercepted between the equator and the vertical on a mural dial — together with a defined length of the stile: the elongation of the stile's shadow is given for any hour and at any time of year. From this follow the entries of the Sun into any twelfth-part [dodecatemorion], that is, any sign of the zodiac; the length of day and night; the rising and setting of the Sun; likewise the months and days of the year, and the other things that are customarily inscribed on dials with skill.

This problem is a special invention of our own, which will first be illustrated on the horizontal dial.

Let there be sought, for example, in the diagram of the first problem of this chapter, the arc Q C intercepted at the 1st hour P.M. at the elevation of our pole — which is found by a very easy prosthaphaeresis [προσθαφαίρεσις, additive-subtractive correction]. For in the right triangle:

S.T. Q C K	S.R. K Q	S.R. C K Q	Resp. S.R. Q C
a grad. 90	to gr. 77 min. 32	so gr. 34 min. 17	grad. 33 min. 22
s			

And so for the other arcs, which the table below will contain.

With all the arcs for the horizontal dial obtained in this way — which, on either side from the meridian on a non-declining dial, are equal — let the stile be erected above the plane of the horizon to the elevation of the pole at the place, and let it always be 10 parts, so that the operation may be carried out more easily. Now in the following diagram A B, since the length of the stile's shadow is sought for each individual hour, let there be drawn, on the non-declining dial, the quadrant of the horizontal plane H D, together with the equator H E. From here, from the center A, straight lines are drawn to the individual hours on the plane of the horizon; but from the summit of the stile, lines are drawn to the point of the Sun's declination from the equator — all of them considered in the northern, contrary part of the sphere. Hence, in the plane triangle A B G, for the length of the stile's shadow A G on the plane of the horizon at the 11th or 1st hour, when the Sun is at the beginning of ♋ [Cancer] and has obtained a declination of 23 deg., 32 min.:

Diagram. A quarter-circle (upper-right quadrant) for the horizontal dial. The base runs from H (lower left) along the horizontal through A (lower center) to V (lower right); the vertical at the right rises to B, the apex from which the hour-lines fan out, and the curved rim sweeps from D (top) down to V (right). Nested arcs cross the quadrant, labeled along the upper right as "Limbus Horizontalis" (the horizontal limb/rim), "Tropicus ♌" (Tropic of Leo), "Æquator" (the equator), and "Tropicus ♋" (Tropic of Cancer). The hour-lines from B meet the rim at points lettered D, I, N, P, R, S, T, M (descending the left arc) and E, F, G toward the right, with Arabic hour-numbers (2, 3, 4, 5, 6, 7, 8) and fractional sub-marks scattered along the inner arcs near K, L.

- H (lower left): left foot of the horizontal base
- A (lower center): center / foot of the stile
- B (right apex): summit of the stile, origin of the hour-lines
- V (lower right): right foot of the base
- D (top of rim): uppermost rim point
- I, N, P, R, S, T, M (down the left arc): hour-line / arc intersection points
- K, L (inner left): inner intersection points near the equator and tropics
- E, F, G (right of center): arc intersection points
- arc "Limbus Horizontalis": the horizontal rim
- arc "Tropicus ♌": Tropic of Leo
- arc "Æquator": the equator

- arc "Tropicus ☿": Tropic of Cancer
- 2, 3, 4, 5, 6, 7, 8 (Arabic, along the arcs): hour-numbers

The given quantities [διδόμενα, the data] are:

- G B A: 66 deg., 28 min. — the complement of the Sun's declination.
- A G B: 56 deg., 54 min. — compounded of the arc intercepted between the horizon and the equator at the 11th (or 1st) hour, and the maximum declination of the Sun. [Margin: See Prob. 1, cap. [of] this [book].]
- A B: 10 parts, the length of the stile.

Hence, as G A B (56 deg., 54 min.) is to 10 (P.M.), so is B (66 deg., 28 min.) to 10 [parts] 56 [min.], the required length of the shadow A G.

S.R. 83772. S.R. 91683

Example I.

The projection of the shadow is to be sought, with a stile of 10 parts, under an elevation of the pole of 54 [deg.], 43 [min.], at the third hour, when the Sun is at the beginning of ♉ [Taurus] or ♍ [Virgo].

The declination of the Sun is then 11 deg., 31 min. north.

	grad.	min.
Arc intercepted, answering to the 3rd hour	25	44
Declination of the Sun	11	31
Sum	37	15

Now as S.R. 37 deg., 15 min. is to 10, so is S.R. 78 deg., 29 min. to 16 [parts], 11 [min.] (since 10 min. make one little particle — for so the operation becomes easier): the required length of the shadow.

Example II.

The quantity of the shadow on the horizontal dial is to be sought, likewise at the 3rd hour, when the Sun is at the beginning of ♏ [Scorpio], having again a declination of 11 deg., 31 min. And here, since the declination is southern, it must be subtracted from the arc intercepted corresponding to 3 hours — namely 25 deg., 44 min.

There remains a difference of 14 deg., 13 min. Then the same declination must be added to the right angle, and the operation is carried out through the complement of this sum to 180 — namely 78 deg., 29 min. — as before.

So, as S.R. 14 deg., 13 min. is to 10, so is S.R. 78 deg., 29 min. (P.M.) to (39 [parts], 54 [min.]), the required length of the shadow.

S.R. 24559 S.R. 97987 to 39, 54.

And so on for the rest, until the calculation has been carried through for all the hours, as well as for the beginnings of the signs of the zodiac.

Furthermore, on the plane of the horizontal dial, through the extremities of the lines found for the individual hours and the beginnings of the signs — as we have said — arc-lines are to be drawn, along which the shadow from the gnomon or stile will travel at an equal distance, from the rising of the Sun to its setting, the whole day long.

On the sides, moreover, a calendar can be inscribed on either hand, as also on the meridian line the length of day and night, and so forth.

In exactly the same way the quantity of the shadow's extension from the stile on the mural dial is to be examined, for the individual hours, by means of the given stile-length of 10 parts (most conveniently), and the arc intercepted between the equator and the vertical at the individual hours, as in the present quadrant, where the mural quadrant E C is described from A. But let the equator E D be drawn from B. Now let the stile A S hang out from the wall in such a way that the angle B A S represents the complement of the elevation of the pole at the place. Let the quadrant of the equator be divided into six equal parts, for the six hours before noon or after noon, which are F, G, H, K, L, E; and the lines drawn to these from the pole B also divide the quadrant E C into just as many unequal hour-parts — namely M, N, Q, P, R, E — which, just as they are found together with the intercepted arcs F M, G N, and so on, has been shown before. But the complements of these arcs, to [the size of] the twelfth-parts [dodecatemoria]...

Diagram. The mural dial. At top is A, the foot of the stile; the stile A S projects to the right toward S, and B lies below A on the right vertical edge — B being the pole from which the hour-lines fan out. The right edge runs A — B — C — D (D at the lower-right corner, marked "12"). Down the left side run the equator-division points E (top left, "6"), then L / R ("5"), K / P ("4"), H / Q ("3"), G / N ("2"), and F / M ("1") near the bottom. The curved mural quadrant E C sweeps from the upper left down to C on the right edge, with a second arc through M, N toward C. Hour-lines

from B cross to the lettered points; Arabic hour-numbers 1–6 run down the left margin.

- A (top): foot of the stile
- S (right of A): outer end of the projecting stile A S
- B (right edge, below A): the pole, origin of the hour-lines
- C (right edge, lower): terminus of the mural quadrant E C
- D (lower-right corner, "12"): the twelfth-hour / meridian point
- E (top left, "6"): top of the quadrant and of the equator E D, sixth hour
- L, R (left, "5"): fifth-hour points (R the intercepted-arc point)
- K, P (left, "4"): fourth-hour points
- H, Q (left, "3"): third-hour points
- G, N (left, "2"): second-hour points
- F, M (left, "1"): first-hour points
- 1, 2, 3, 4, 5, 6 (Arabic, left margin): hour-numbers

[Margin: Probl. 1, cap. [of] this [book].]

is required, since the measure of the angle at A — that is, of the axis-center on the mural dial — is contained within it. Within our sphere this can best and most clearly be demonstrated by means of stretched threads. For now, however, we will teach this inscription of the signs by way of a single worked example, and that under our own elevation of the pole, $55^{\circ} 43'$, for the first hour after noon [F M], at the Sun's entry into the beginning of Aries and Libra.

Here, first, since the arc F M of $54^{\circ} 47'$ is given from what precedes, its complement M B — that is, the angle M A B — will be $35^{\circ} 13'$. Next, the declination of the Sun at the beginning of Taurus and Virgo is taken again as $11^{\circ} 31'$; and since here, in the northern signs, this is added to the quadrant, the angle A S B of $101^{\circ} 31'$ is thereby produced. Therefore, since two angles of triangle A B S are given — namely the one at A and the one at S — the angle at B in the rectilinear figure is also given, $43^{\circ} 16'$. Furthermore, the measure of the stile A S is given by hypothesis, most conveniently as 10 parts.

Therefore, set it up by the rule of proportion:

As A B S ($43^{\circ} 16'$, 10 parts) — S.R. 68539 — is to A S B, so is the complement $78^{\circ} 30'$ — S.R. 97987 — to the required extension of the shadow [p. M: 14 17 of shadow].

And so in the remaining cases, for any pair of zodiacal signs that have one and the same declination at their beginning. This one thing only must be observed:

that in the southern signs the operation by the rule of proportion is here to be carried out by placing the complement of the Sun's declination in the third position.

The procedure for declining dials must be no different, once the arcs intercepted at the individual hours have been found. Only this is to be noted: that if a mural surface, while directly opposite the meridian, nevertheless inclines above either toward the north or toward the south — and however much this inclination is (which can also be tested with a quadrant) — then as much as it inclines toward the north must be added to the elevation of the pole of the place; and the contrary, if the wall is found to lean toward the south. Once all this is grasped together, and once it has been tested in our sphere by threads drawn to the center to imitate the Sun's rays, a dial is constructed for any plane whatever from the doctrine of triangles, and by the demonstrations and indications we have set out — most conveniently and most skillfully.

There now follows a twin table, in which are computed both the hourly interval, in each kind of dial (horizontal and mural), for each single degree; and the arc between the equator and the horizon, in the horizontal dial, for each fourth degree of the elevation of the pole, from 30° up to 70° — so that they may be ready to hand whenever they are needed. Moreover, in the complements of the elevations of the pole, the arcs intercepted between the equator and the vertical are also given; and from the complements of these the angle at A for the mural dial is afterward obtained.

For example: let the said angle at A be sought from the lower of the following tables, for the first hour on the mural dial — the angle that we found just above to be $35^\circ 13'$ under this elevation of the pole, namely $55^\circ 43'$. So, taking its complement $34^\circ 17'$, I find the arc intercepted at the first hour to be $54^\circ 47'$ (once the correction for the adhering 17 minutes has been duly made); and the complement of this is $35^\circ 13'$ — that is, the sought angle at A.

Table of the measure of the horizontal arc from the meridian to each hour; and of the mural arc in the complement of the same, for each single degree of the elevation of the pole from 30° to 70° .

The leftmost column gives the elevation of the pole (left edge, headed *Horiz. Sciath. Poli elevat.* — "horizontal dial, elevation of the pole"). The six paired hour-columns give the horizontal-dial arc (degrees gr. / minutes mi.) for the symmetric hour-pairs. The rightmost column (headed *Mural. Sciath. compl. El. pol.* — "mural

dial, complement of the elevation of the pole") gives the complementary pole-elevation value.

Pole elev.	Hor. 11/1 gr.	mHor. i 10/2 . gr.	m i. 9/3 gr.	Hor. i Hor. 8/4 gr.	m i Hor. 7/5 gr.	m i Hor. 6/6 gr.	mMural i compl . .
30	7	3 16 8	6 26	3 40 4	5 61 4	4 90 9	0 60
31	7	5 16 1	3 27 4	1 41 4	4 62 2	2 90 8	0 59
32	8	4 17	1 27	5 42 3	3 63 0	6 90	0 58
33	8	1 17 7	2 28 7	3 43 4	1 63 7	4 90 5	0 57
34	8	3 17 0	5 29 4	1 44 3	5 64	4 90 2	0 56
35	8	4 18 3	2 29 0	4 44 9	4 64 6	5 90 6	0 55
36	8	5 18 6	4 30 5	2 45 5	2 65 8	2 90 7	0 54
37	9	9 18	9 31 [?]	1 46	9 65	5 90 8	0 53
38	9	2 19 2	3 31 4	3 46 7	5 66 0	2 90 9	0 52
39	9	3 19 3	5 32 7	9 47	2 66 6	5 90 5	0 51
40	9	4 20 5	2 32 0	4 48 2	1 67	2 90 0	0 50
41	9	5 20 6	4 33 3	1 48 4	3 67 7	4 90 5	0 49
42	10	9 21	7 33	4 49 7	1 68 3	1 90 1	0 48
43	10	2 21 1	2 34 9	1 49 7	4 68 4	3 90 2	0 47

Pole elev.	Hor. 11/1 gr.	mHor. i 10/2 . gr.	m i. 9/3	Hor. gr.	m i Hor. . 8/4 gr.	m i Hor. . 7/5 gr.	m i Hor. . 6/6 gr.	mMural i compl . .
44	10	3 21 2	5 0	34	4 50 6	1 68 4	5 90 3	0 46
45	10	4 22 3	1 2	35	1 50 5	4 69 5	1 90 4	0 45
46	10	5 22 5	3 3	35	4 51 4	1 69 6	3 90 7	0 44
47	11	7 22	5 3	46 [?]	1 51 0	4 69 3	5 90 3	0 43
48	11	1 23 9	1 2	36	3 52 5	9 70	1 90 0	0 42
49	11	3 23 1	3 2	37	1 52	3 70 5	2 90 8	0 41
50	11	4 23 1	5 1	37	2 53 7	1 70	4 90 3	0 40
51	11	5 24 0	9	37	5 53 0	2 70 4	5 90 8	0 39
52	12	0 24	2 6	38	1 53 3	4 71 6	1 90 2	0 38
53	12	1 24 0	4 4	38	3 54 6	8 71	2 90 7	0 37
54	12	2 25 0	2	38	5 54 9	3 71 0	4 90 1	0 36
55	12	2 25 9	1 8	39	1 54 8	5 71 0	5 90 3	0 35
56	12	3 25 8	3 3	39	3 55 8	9 72	4 90	0 34
57	12	4 25 6	4 9	39	5 55 8	2 72 8	1 90 6	0 33
58	12	5 26 4	5	40	1 55 8	4 72 6	2 90 7	0 32
59	13	2 26	1 9	40	3 56 6	1 72	3 90 8	0 31

Pole elev.	Hor. 11/1 gr.	mHor. i 10/2 . gr.	m i Hor. 9/3 gr.	m i Hor. 8/4 gr.	m i Hor. 7/5 gr.	m i Hor. 6/6 gr.	mMural i compl . .
60	13	1 26 0	3 40 3	5 56 3	1 72 8	4 90 7	0 30
61	13	1 26 7	4 41 7	1 56 0	3 72 3	5 90 7	0 29
62	13	2 27 4	1 41	2 56 7	4 73 9	7 90	0 28
63	13	3 27 0	1 41 3	4 57 7	3 73	1 90 6	0 27
64	13	3 27 6	2 41 5	5 57 6	1 73 6	2 90 4	0 26
65	13	4 27 2	3 42 7	1 57 1	3 73 0	3 90 2	0 25
66	13	4 27 7	4 42 9	2 57 5	4 73 3	4 90 0	0 24
67	13	5 27 2	5 42 9	3 57 7	5 73 3	4 90 7	0 23
68	13	5 28 7	9 42	4 58 9	2 73	5 90 3	0 22
69	14	7 28	1 43 9	1 58	1 73 0	5 90 9	0 21
70 gr.	14	7 28	2 43 9	1 58 3	1 74 6	5 90	0 20 gr.

Table showing the arcs intercepted between the equator and the horizon for the horizontal dial — but in the complements for the mural dial — for the pole-elevations set out below.

The two leftmost columns (hor/hor) give the symmetric hour-pairs (12/12, 11/1, 10/2, 9/3, 8/4, 7/5, 6/6). The body columns are headed by the pole-elevation (E. po.) values 30, 34, 38, 42, 46, 50, 54, 55.43, 58, 62, 66, 70, each split into degrees (gr.) and minutes (mi.).

	3	3	3	4	4	5	5		5	6	6	7	
h h 0	m4	m8	m2	m6	m0	m4	m		m8	m2	m6	m0	m
o o gr	i gr	i gr	i gr	i gr	i gr	i gr	i gr	i 55.4	i gr	i gr	i gr	i gr	i
r r .	. .	. .	. .	. .	. .	. .	. .	. 3 gr.	. .	. .	. .	. .	.
1 1 6	0 5	0 5	0 4	0 4	0 4	0 3	0 34	1 3	0 2	0 2	0 2	0 2	0
2 2 0	6	2	8	4	0	6		7 2	8	4	0		
1 1 5	8 5	5 5	2 4	2 4	0 3	3 3	3 33	2 3	7 2	1 2	1 1	2	
1 9	5	1	6	3	9	5		2 1	7	1 3	6 9	2	
1 2 5	1 5	6 4	5 4	5 3	5 3	0 3	1 30	3 2	2 2	4 2	5 1	5	
0 6	9 2	7	6 3	4 9	4 6	2	1	4 8	5 4	3 1	6	5	
9 3 5	4 4	2 4	5 3	1 3	2 3	5 2	1 25	4 2	5 2	3 1	2 1	2	
0	6 6	1 3	9 8	1 4	0 1	5 7	2	4 3	0 0	6 7	8 4	6	
8 4 4	5 3	3 3	3 2	3 2	4 2	4 2	1 18	4 1	2 1	5 1	3 1	1	
0	3 6	3 2	7 9	5	6 2	6 0	8	9 7	1 4	3 2	3 0	9	
7 5 2	9 2	5 1	1 1	2 1	2 1	1 1	3 10	0 9	1 7	5 6	3 5	2	
4	0	9 8	9 6	4	2	5 0	9		4	0	4	3	
6 6 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0	

CAPVT XIII.

On zones, climates, and parallels: and finally on the various designations of the inhabitants — those who, as dwellers on the habitable earth, press upon it everywhere in places that are opposite or contrary to one another.

This chapter treats the matters that we left to the final consideration of the preceding book: the effects impressed upon the earth by the action of the Sun, on account of the varying inclination of the earth's axis — toward north and toward south — away from the celestial zodiac.

PROBLEMA I.

Given the elevation of the pole of a place, the zone of that same place on the earth is given.

This problem is settled by an easy demonstration. On the diagram that follows, let the solstitial colure B C D E F &c. be described about the center A; the equinoctial circle C A K, whose poles are F M; the zodiacal circle B A H; the tropic of Cancer ☿ D H; the tropic of Capricorn ♄ B I; the arctic circle E G; the antarctic circle N L.

Since, then, the latitude of the place is the same as the elevation of the pole of that same place — and since the declination of the Sun and the complement of this declination measure each of the two — it is evident that the torrid zone extends on either side from the equator as far as the greatest obliquity of the ecliptic from the equator, that is, the elevation of the pole of $23^{\circ} 32'$. This number doubled sets out the breadth of the torrid zone as $47^{\circ} 4'$ — namely the arc B D, that is, the space between the parallels B I and D H.

The temperate zone, on the other hand, each of the two taken separately, is measured by the complement of the whole [obliquity] above: $42^{\circ} 56'$, beginning from the tropic of Cancer or of Capricorn — that is, from the elevation of the pole $23^{\circ} 32'$ — and ending under the elevation $66^{\circ} 28'$. Its breadth toward the north is shown by D E, toward the south by B N.

Finally, the frigid zones occupy the outermost parts on either side, toward the north and toward the south, each of a single breadth equal in

Diagram. A circle (the solstitial colure) drawn about center A, illustrating the division of the earth into zones. The two labelled inner chords are the tropics, drawn as oblique lines crossing the figure.

- A (center): the center of the figure
- E (upper left, on circumference): point bounding the arctic circle E G
- F (top): pole of the equinoctial (with M); apex point
- G (upper right, on circumference): point bounding the arctic circle E G
- D (left, on circumference): endpoint of the tropic of Cancer line D H
- H (right, on circumference): endpoint of the tropic of Cancer line D H
- C (lower left, on circumference): endpoint of the equinoctial C A K
- K (right, on circumference): endpoint of the equinoctial C A K
- B (lower left, on circumference): endpoint of the tropic of Capricorn line B I
- I (lower right, on circumference): endpoint of the tropic of Capricorn line B I
- N (bottom left, on circumference): point bounding the antarctic circle N L
- L (bottom right, on circumference): point bounding the antarctic circle N L
- M (bottom): pole of the equinoctial (with F)
- "Tropicus ☿." (label across upper chord): "Tropic of Cancer"
- "Tropicus ♄." (label across lower chord): "Tropic of Capricorn"

the greatest circle, with the greatest obliquity of the ecliptic. They last from the elevation of the pole last fixed at $66^{\circ} 28'$ up to the completion of the quadrant.

But if anyone should wish to know the surface quantity of each of these zones, and from that to extol and admire the providence of the Creator, he will accomplish this by the measure of the sphere demonstrated in geometry.

PROBLEMA II.

Given the number of a climate or parallel, the elevation of the pole is given, where that climate both begins and ends. From this, too, the place subject to any given climate is known.

In the establishment and division of the climates, the ancients supposed successively, from the equator up to the frigid zone, a half-hour of difference in the longest day; and for the parallels, a difference of a quarter-hour. Nor did they, in reckoning the climates, advance beyond the seventh — to the pole-elevation of 49° — because they judged that the earth beyond that was not conveniently enough habitable. Yet nothing prevents extending this number further, if only one wishes, according to the computation of this same foundation. Thus, to ascertain the elevation of the pole for the first climate, we proceed according to the problem set out above.

Let the meridian $B C D E F$ &c. be described about the center A ; let $B G$ be the equator, and its pole — that is, the pole of the world — be D ; the ecliptic $C O$; the tropic of Cancer $C F$. Now, since we measure the longest day in the northern region of the world, and in the oblique sphere, from the rising of the Sun above the horizon to its setting: if, then, the distinguishing of the climates from one another amounts in all to a half-hour, it follows that at the rising or setting the same occurs through a quarter-hour. Therefore in the present example, in the right triangle $A H N$, for the angle $H A N$, which measures the complement of the elevation of the pole at the first climate $G E$:

- $A N = 3^\circ 45'$ [g. 3 m. 45] — the degrees in the equator corresponding to $\frac{1}{4}$ hour.
- $H N = 23^\circ 32'$ [g. 23 m. 32] — the greatest obliquity of the ecliptic, which is always retained.
- $A N H$ — a right angle.

[Margin (against the bracketed list): $\delta\iota\delta\acute{o}\mu\epsilon\nu\alpha$ — "the givens"]

Therefore $H A N$ is given as $81^\circ 27'$, whose complement, $8^\circ 33'$, is the elevation of the pole at the first climate. And so on perpetually for the rest, always increasing the arc $A N$ in the equator by $3\frac{3}{4}^\circ$, that is, by a quarter-hour.

Nor are the parallels investigated otherwise than by halves of the quarter-hours, according to the ancients.

From these things, too, it is plain that, the quantity of the longest day being determined at some fixed place on the earth, there is given not only the climate appropriate to it but also the elevation of the pole — provided that the refraction of the Sun is excluded. Since this refraction was unexplored and unknown to the ancients, it is no wonder that the old geographers — for places to which they did not themselves come with observations of the poles, but instead drew something for describing their geographical tables from the length of the longest day on the report of others — always extended these [latitudes] greater than they are otherwise found by true celestial observations. This becomes clearly evident in these and other northern places through a comparison of the old tables with new observations. On this subject let Ptolemy, Strabo, and other geographers be read [Margin: *Lib. 1. Theor. cap. 3.*], and let them be compared with us and with other moderns who have more exactly marked out these regions by observing them.

Diagram. A circle (the meridian) drawn about center A, used to derive the pole-elevation of the first climate. Several chords cross the figure, with the right triangle A H N marked near the center.

- A (center): the center of the figure
- B (upper left, on circumference): endpoint of the equator B G and of the chord-system
- C (top, on circumference, with ☊): point on the ecliptic / tropic, near the Cancer symbol at top
- D (upper right, on circumference): the pole of the world
- E (right, on circumference): point
- M (just above center): interior point of the chord-construction
- H (center-right, near A): vertex of the right triangle A H N
- N (center, near A): vertex of the right triangle A H N
- K (left, on circumference): point
- L (lower left, on circumference): point
- O (lower center, on circumference): endpoint of the ecliptic C O
- F (lower right, on circumference): endpoint of the tropic C F
- G (bottom, on circumference, with ☋): endpoint of the equator B G, near the Capricorn symbol at bottom
- "☊" (top): the tropic of Cancer marker
- "☋" (bottom): the tropic of Capricorn marker

PROBLEM III.

Given the elevation of the pole of a place, the locations and the names of the inhabitants of the earth are determined — those called *amphiscii*, *periscii*, and *heteroscii*, as well as those called *antoeci*, *perioeci*, and finally *antichthones*, with respect to ourselves.

From what has already been demonstrated, these various names arise: partly from the diversity of shadow that the Sun casts upon these peoples in the course of its yearly journey, and partly with respect to the parallels and the axis of the earth, under which its various inhabitants are dispersed, nearly all of them experiencing some contrast with one another.

To the first class belong those called *amphiscii*, *periscii*, and *heteroscii*.

The **amphiscii** are those whose noonday shadow falls now toward the north, now toward the south. This happens in the torrid zone: when the Sun passes through the sign of Cancer [♋], they have their shadows to the south (their left); at midwinter, to the right, that is, to the north.

The **periscii** are those whom the circuit of the zodiac-bearing axis encloses, so called because their shadows are swung around in the manner of a millstone.

The **heteroscii** are those who have a shadow on one side only, either northern or southern — as we ourselves are, and our *antichthones*. Therefore the *amphiscii* lie in the torrid zone, the *periscii* in the frigid zones, and the *heteroscii* in the temperate zones.

To the second class and naming belong the **antoeci**, **perioeci**, and **antichthones**.

The **antoeci** are those who dwell on different parallels, under the same meridian, equally distant from some part of the equator.

The **perioeci** are those who lie under one parallel and meridian but in opposite quarters.

The **antichthones**, or antipodes, are those who are separated by the diameter of the world. For the *antoeci*, midday and midnight fall at the same moment; but the southern ones have summer when the northern ones have winter. For the *perioeci*, summer and winter come at the same time, but they have midday and midnight reversed. For the *antichthones*, the horizon and meridian are the same, but everything else is opposite — as can be shown more fully on the sphere, that is, on the terrestrial globe.

CHAPTER XIV.

On finding the latitude and longitude of terrestrial places, and likewise — when the latitudes and longitudes of any two places are taken as already known — on finding their distance from each other.

We add this last chapter of our spherical doctrine for the use of geographers and historians. But before we turn to the construction of the problems, a few terms must be briefly defined here — namely, the latitude of a place, its longitude, and its distance from another — which are represented to the eye on our sphere.

The **latitude of a place** is its distance from the equinoctial circle, equal to the elevation of the pole of that same place, and reckoned along the meridian.

The **longitude of a place** is its distance from the prime meridian — the one along which the magnet, with its pointer, brushes past both the pole of the world and the magnetic pole at once. To this the ancients assigned the Fortunate Islands, beyond Spain, as a kind of boundary marker. This longitude is reckoned along the equator, from that said point toward the east.

The **distance** of two places, finally, is measured along a great circle of the earth, since the journey between them across the surface of the sphere is the shortest.

[Margin: *Cap. 8 of this book.*]

The latitude of a place, since it is obtained by the same technique as the elevation of the pole of the place — and since we have already dealt with that fully above — we omit here. And although other methods have been devised by more recent authors, by means of the declination of the magnet below the horizon — on which one should read the excellent little treatise of William Gilbert the Englishman [William Gilbert] — yet they will scarcely yield the desired precision without a celestial observation.

[Margin: *William Gilbert the Englishman, in his treatise On the Magnet [De Magnete].*]

As for longitude, it will most conveniently be found from the course of the Moon — either at an eclipse (especially a lunar one), or at a conjunction of the Moon with a fixed star, or finally by the method that we shall append at the end of the earlier book of the *Theory*. For although lunar appearances of this kind occur at the same instant, nevertheless, because of the roundness of the earth, their times differ markedly under different meridians — especially under meridians more widely separated from one another — so that places farther east always reckon

more hours in these events. Hence, from this difference of times, the difference of longitudes of two or more places is readily derived: the difference of times being resolved into degrees and minutes of the equator. For the inclination of the magnet here — its veering from the pole of the world toward its own pole, as is believed — will not by itself suffice to supply the longitude of places.

PROBLEM I.

Given the longitude and latitude of one place, together with the road-distance of another and the angle of position — as they call it — from the south, that is, in azimuthal degrees on the horizon, the longitude and latitude of the unknown place are also given.

This problem has a leading importance in carrying geography forward, since when the longitude and latitude of one place have been given, the distances of the others to be obtained round about are taken by instruments — along with the angles of position, or deflection, on the horizon, reckoned from the given place.

[Margin: *This letter was once sent to me here by that excellent man, Mr. Jösel of Wittenberg [?], the mathematician.*]

We shall transfer here, with his own explanation, the demonstration of this problem, taken from a certain letter of the most ingenious Joachim Rheticus [Rheticus] to Mr. Johann Schöner [Johann Schöner].

Diagram. A quarter of a meridian circle, with vertex A at the upper right. From A two curved great-circle arcs descend to the lower left toward D and C, and the construction lines mark out the right triangle described in the text. The reference letters present are:

- A (upper right): the pole / vertex point, taken as the first place
- B (interior, below A): the second place
- C (lower, on arc): point on the quadrant
- D (left): foot of the quadrant arc
- E (between D and C): point where arc D–E is cut
- F (interior): point where extended A–B meets arc D–E

Let the distance of the two places be A B, and let the latitude of A be A D; and complete the quadrant D A C of its meridian. Let the angle of intersection D A B also be known — which is learned by the instrument used in navigation, according to the distribution of the winds or of the horizon, of which A is the vertex. Since, therefore, the circumference A C is given — the distance of A from

the pole — from this point let the great circle D E F be described, and the quadrant C B E, and let A B be extended so that it cuts D E at F. We shall then have triangle A D F, whose angle D A F is given and whose angle A D F is right; and the side A D is also given. The remaining sides A B F and D E F will therefore also be given, together with the remaining angle E F B. Hence in triangle B F E, because angle F is given and side B F is given, the remaining sides B E and E F are given, and from them therefore the difference of longitude D E; and E B is the latitude of B, the second place, which was sought.

If then the latitude B E of the other place were given, and the difference of longitude as well as the distance A B were what is proposed to be found, the preliminary construction being kept the same — because the sides and angles of triangle A D F are had, as said — then in the other triangle E B F, side E B being given, angle E right, and F known: therefore the sides E F and B F will be given, and hence the distance A B; likewise the difference of longitude D E will be had. Finally, if D E were given, since the remaining

parts E F remain fixed, then in triangle B E F angle E is right and angle F is known: so neither A B nor E B would be unknown.

See, in how few steps, most excellent Mr. Schöner, you have the whole of cosmography!

This problem, which Rheticus demonstrates by a longer and more laborious chaining-together of triangles, requires less work in the following way. In the oblique-angled triangle A B C, given A C (the complement of the known latitude) and A B (the road-distance), together with the complement of the observed angle of position B A C, there are also given B C (the complement of the latitude of the unknown place) and A C B (the difference of longitude sought). The longitude and latitude of the unknown place can also be made known from the road-distances from two other known places — in just the same way as the place of an unknown star is found from two known ones, and so on.

[Margin: *See Problem 4, ch. 9.*]

PROBLEM II.

Given the longitude and latitude of two places, even ones situated far from each other, the arc on the great circle of the globe of the earth lying between them is given; and therefore also their road-distance in German miles, and so on.

Two places, whose interval from each other is to be found, differ in no way from the distances of stars obtained above. Now these places either (very rarely) lie

under one meridian and differ in latitude alone; or they lie under the equator itself, or under one parallel equally distant from the equator, and (most rarely) vary in longitude alone; or finally (most frequently) they are situated under different meridians and parallels both, and disagree in longitude as much as in latitude. In this threefold variation of two places separated from one another, we shall briefly dispatch the finding of their distances as follows, and, where need be, illustrate it with examples.

I.

Two such places, if set under the same meridian (or perhaps the equator, as most rarely happens), and whose interval is to be found, do not need the calculation of triangles. For when the smaller latitude (under a common meridian) or the smaller longitude (under the equator) is subtracted from the greater, there remains the arc of distance along the great circle, to be resolved into miles by the customary method of 15 [to the degree].

II.

But if instead the places differ in longitude alone, a spherical isosceles triangle A B C arises, where the positions of the given places are at B and C. If, then, the road-arc B C is sought, with quadrants A D and A E drawn out on either side from the pole A, the arc D E will be the portion of the equator indicating the difference of longitude. When this is bisected at F at right angles — a perpendicular arc being dropped from A — it also cuts B C in half at H, likewise at right angles.

Diagram. A spherical isosceles triangle drawn as a pointed-arch (ogival) figure. The apex A is at the top, the base runs along the bottom from D to E, and the bisecting perpendicular A–F runs down the middle through H.

- A (apex, top): the pole
- B (left, on the inner arc): one given place
- C (right, on the inner arc): the other given place
- H (center, on line B–C): midpoint of B C, where the perpendicular cuts it at right angles
- D (lower left): foot of the left quadrant
- F (bottom center): midpoint of D E, where the perpendicular meets the base at right angles
- E (lower right): foot of the right quadrant

Therefore, in either right-angled triangle — namely B H A — for B H, half of the road-arc:

The givens [Greek: Διδόμενα (didomena, "the things given")] are:

- A B — the complement of the latitude, that is, the elevation of the pole, of the given places.
- H A B — half the difference of longitude of the given places.
- A H B — right [angle].

Therefore B H is given — the arc of half the distance — whose double is B C.

III.

Places that differ in latitude and longitude at once form a triangle in which

A is everywhere [the pole] in the oblique-angled triangles — to be solved even by an easy [Greek: προσθαφαιρέσει (prosthaphairesei, "by prosthaphaeresis," i.e. by addition-and-subtraction)]; and therefore they require nothing but examples.

Example I of this.

The road-distance is sought between Louvain in Brabant [Louvain] and Bologna in Italy [Bologna].

Diagram. A narrow spherical triangle in a quadrant frame: apex A at top, base D–E at bottom, with B and C marking the two places on the descending arc.

- A (top): the pole
- B (upper left, on arc): Louvain
- C (interior, below B): Bologna
- D (lower left): foot of the quadrant
- E (lower right): foot of the quadrant

	Latitude	Longitude
C — Bologna	44° 0'	33° 55'
B — Louvain	50° 58'	26° 50'
Difference of long.		7° 5'

Accordingly, in triangle A B C, for B C, the road-distance sought between Louvain and Bologna:

The givens [Greek: Διδόμενα (didomena)] are:

- A B = 39° 2' — complement of latitude, Louvain.
- A C = 46° 0' — complement of latitude, Bologna.
- B A C = 7° 5' — difference of longitude.

Therefore B C is given as $8^{\circ} 27'$ — the distance sought, which, resolved into German miles, makes $126\frac{3}{4}$ miles.

Example II.

The distance is investigated between Copenhagen in Denmark [Copenhagen] and Jerusalem [Jerusalem].

Diagram. A second narrow spherical triangle in a quadrant frame: apex A at top, base D–E at bottom, with B and C on the descending arc.

- A (top): the pole
- B (upper left, on arc): Copenhagen
- C (interior, below B): Jerusalem
- D (lower left): foot of the quadrant
- E (lower right): foot of the quadrant

	Longitude	Latitude
B — Copenhagen	$37^{\circ} 0'$	$55^{\circ} 43'$
C — Jerusalem	$66^{\circ} 0'$	$31^{\circ} 40'$
Difference	$29^{\circ} 0'$	

Therefore, in the likewise triangle A B C, for B C:

The givens [Greek: Διδόμενα (didomena)] are:

- B A = $34^{\circ} 17'$ — complement of latitude, Copenhagen.
- C A = $58^{\circ} 20'$ — complement of latitude, Jerusalem.
- B A C = $29^{\circ} 0'$ — difference of longitude.

Therefore B C is given as $31^{\circ} 27'$ — the road-distance sought; resolved into German miles, it gives $471\frac{1}{4}$ miles.

THE END.

[This closes Liber II (the *Spherics*) and thereby the *Pars Prior* of the *Astronomia Danica*. The transition to the *Pars Posterior* (the theory of the heavenly motions) does not appear on these pages.]

ASTRONOMIA DANICA — PART TWO

ASTRONOMIAE DANICAE PARS ALTERA

THE THEORY OF THE PLANETS, restored, comprising two Books. Of these:

THE FIRST — after the description and comparison of the threefold cosmic scheme [triplicis hypotypofeos mundanae], namely

- the OLD scheme of PTOLEMY [Ptolemy],
- the ADMIRABLE scheme of COPERNICUS [Nicolaus Copernicus],
- the RECENT scheme of TYCHO BRAHE [Tycho Brahe],

— treats of the apparent motions of the fixed stars, and likewise of the Sun and Moon, restored within that same framework and accommodated to all ages of the world: together with the complete doctrine of Eclipses, and in addition a particular use of the Moon.

THE SECOND — treats of the motions of the remaining five Planets, upon that same threefold Scheme [triplici illa Hypotyposi], likewise restored within that same framework to match the celestial appearances.

PREFACE TO THE READER

Here at last, most kind Reader, after a long and just period of waiting, you have the two Books of Theory, on the restoration of the perpetual motions both of the fixed stars and of the planets, worked out by my own watchful effort. My aim was this: that Urania — the most exalted of the Muses, and the most suited to human use — should never be found abandoned by the care and generous patronage of the Most Serene and Most Powerful Kings of the Danes. For it was by their favor and at their expense that she once flourished in this Kingdom, and even now, by divine grace, she flourishes among the exercises of the pure religion and the other arts of more polished learning, so that foreign nations too perceive not only her fame but also her fruit. I add this further aim: that my own labors, undertaken over thirty years for the restoration of this art, should not fall away without yield and perish entirely.

I am not unaware, to be sure, how much other most distinguished Astronomers of today can accomplish in this field. With them, indeed, rest those requisites most precious of all our Danish astronomy — I mean the observations — which now, as fate would have it, lie idle, or perhaps pass through their hands. Yet since we aim, even when deprived of the same necessary resources, at one and the

same end — namely, that the celestial phenomena, at any place and time, be made known from our hypotheses below, or from the tables derived from them — by methods that differ, indeed, but are equivalent in power: for my own part I preferred to follow in the footsteps of the ancients regarding the restoration of Astronomy. In these matters I chose first to reduce Ptolemy [Ptolemy] to Geometry — that is, to reduce the equant in the Planets to an Epicycle, along with Arzachel the Spaniard [al-Zarqālī] and Copernicus of Toruń [Nicolaus Copernicus]; and then, for the remaining Eccentricity together with the equant, to set up a double Epicycle in Tycho's manner. This I did especially because I had long ago convinced myself that the nature of the superior bodies, in their threefold combination with the Eccentric orb itself, and in the opposing motion of the centers with respect to one another in these bodies, was by no means at play in vain. So fully convinced was I, that whatever the highest Philosophers — and above all the Platonists — once contemplated by the gift of reason concerning the Intellect [Intellectu], concerning Reason [Ratione], concerning the Soul of the World [Anima Mundi], and finally concerning sensible Nature [Natura sensibili], and wove together in a chain, in order, under the veil of dependency, descending from the first mover: all of this the celestial nature displays to us, as though through a viewing-glass, in these bodies and in their motions, contriving nothing at all in vain in this whole cosmic economy. For each thing is borne about its own center and its prior suppositions in the most fitting harmony, all of them at last resting upon one single thing. Concerning this matter, among other places, Plato in his second Epistle to Dionysius [Plato, *Epistle II*] brings forward some remarks in few but mystical and abstruse words.

But these our hypotheses, built up to match the celestial observations,

Now, as I accommodate these hypotheses indifferently to the threefold supposition — that of Ptolemy, as I said, reduced to the Physical and Geometrical laws; likewise that of Copernicus; and finally that of our own Brahe — then the rest, assuredly, unless they are such as in no way agree with nature, cannot wholly dissent from these, but can easily be reduced to the same. For the homocentric scheme of H. Fracastoro the Italian [Girolamo Fracastoro], and of others, together with its like — the occasion of an apparent revolution shared with the universe, directed westward in all the inferior planets — has long since been refuted by many arguments. As for the rest, which either others, or even Johannes Alsted [Johann Heinrich Alsted], review in passing and judge for himself — namely those of Raymarus Dithmarsus [Nicolaus Reimers Ursus] and of Helisaeus Roeslin [Helisaeus Röslin] — these can be reduced without any trouble to the scheme of Tycho, from which they emanated. For the one

Dithmarsus calls his own was once filched from Tycho [Tycho Brahe]. This, among other things, is proven by an error which that Ursus [Ursus], imprudently, afterward imitated — an error in that crude sketch left by Tycho on a wall at Uraniborg [Uraniburgum] (which he had perhaps glanced at in person), where the orb of Mars wholly encompassed the Sun. [Margin: See *Epistolae* I, B. lib. 2 (?); *Epistola Astron.*; *Epistola ad H. Rantzovium*.] As for Röslin's, it differs in nothing from Tycho's, except — as it seems — in the material orbs of the planets. If Röslin should choose to regard them as such [i.e. material], he falls into the greatest physical absurdity, namely the penetration of dimension; unless rather he should hold them to be merely formal (as many things in nature are). To these, as real, we earlier gave our assent a little before, and on behalf of this opinion of ours we once argued at length and quite plausibly in a public disputation.

Further, as to why I made use rather of an accumulation of infallible observations, such as I could obtain, with a discrete comparison of these among themselves and with the mean motions, for the investigation of the Eccentricities, the equants, and the apogees — rather than the πραγματεία [Greek: pragmateia, "treatise" or "method"] of Ptolemy, so greatly praised by Copernicus and taken up in emulation, where only three apparent and as many mean Epochs are supposed for the same purpose — this assuredly is not without cause, neither once with Tycho nor now with me. For when we are engaged not in a crude founding of Astronomy upon what is near the truth, after the manner of the ancients, but in an accurate restoration and representation of the phenomena, this method of ours is a far more specialized inference, and therefore so much the more uncertain — since, moreover, three points not quite projected into a straight line can everywhere touch the periphery of a circle. [Margin: Prop. 5, lib. 4 of the *Elements*.] Accordingly, by this practice the traces of the equant are not sought out conveniently enough — as the restored Theory of the Moon above all taught us, so often reworked in this way to no purpose, and not properly settled until we applied a general idea of the lunar observations, at various places along the Eccentric, and quite often directed it toward δοκιμασίαν [Greek: dokimasian, "testing" or "scrutiny"].

Since, then, the restoration of Astronomy has been carried out by us according to a general weighing of observations, and restricted to hypotheses of this kind — which, as often as need arises, are again resolved into the very observations from which they were constructed, with only a very slight difference intervening, which has been removed by adjustment — we do not wish to linger here at greater length, seeing that everything can readily be worked out by learners with the aid of the Trigonometry set forth beforehand. Farewell, and enjoy.

PROOEMIUM OF THE SECOND PART OF THE ASTRONOMY

On the excellence, difficulty, and contents of the Theoretical doctrine, sketched in general together with its division.

The First Part of Astronomy having been completed — the part which considers the apparent daily motion of the heavenly bodies, and is called Spherical — the other part remains, called Theoretical, so named because it consists in the contemplation of the motions both of the first movable (as they call it) and of the seven planets, attached to certain geometrical hypotheses; and it is of all the most excellent and the most difficult. For what is more excellent, what more noble, what more worthy of human knowledge (so far as it is human), or finally more celebrated of divine craftsmanship, than the compendium — the very idea, as it were — of those things which universal nature, taken in its widest sense, turns within its bosom: things set forth in the heaven, indeed in these planets and their revolutions, with manifold congruence, woven together as though described in a certain book, displayed to mortal eyes, and at last apprehended by those who, leaving behind earthly dregs and momentary pleasures, dwell upon celestial, perennial, and altogether divine contemplations, and bind themselves to them by sure means — provided they have grasped, even moderately, those things which it is most fitting for the inspired part of man to come to know? For one must not suppose that the best and wisest Creator made the most splendid celestial bodies and their admirable and perennial revolutions or motions for any cause other than for the sake of mankind — namely, so that men might delight themselves in the scrutiny of those bodies, as the most excellent of divine works that fall under the senses; nor that these stars, commonly called planets, deviate even in the least from the orbits divinely prescribed to them, but rather that by the marvelous variety of their motions they display the best harmony to those who, having begun rightly from ocular sense, are able to apply the ears of the mind to that same harmony. To this, beyond doubt, Pythagoras [Pythagoras] once looked, when he attributed a musical concord to the revolutions of the celestial bodies, as Plato [Plato] interprets it. Indeed Plato too, in many other places besides, earnestly commends the divine study and the usefulness of Astronomy. From these we shall sample one passage in the *Timaeus*, and set it here before the eyes of the young student of Mathematics, so that he may consider that, in his striving to know the heavens together with us, he has not bent himself to a false persuasion or a useless effort, but is invited by the authority of the highest Philosopher of all.

"The greatest gift" (he says), "for the sake of which eyes have been given us by the gods to our benefit, I think must next be set forth. For the knowledge of the best things, I judge, the eyes have brought to us. For these things which are debated concerning the world could never have been discovered, had neither the stars, nor the sun, nor the heaven been able to be seen. But the knowledge of day and night, arising from the eyes, made it so that by a kind of demonstration we measured the courses of the months and the years, came to know time, and investigated the order of universal nature: from which things we obtained philosophy — than which no greater good was ever given, nor shall be given, to the race of mortals by the gift of the gods. For I say this is the greatest benefit of the eyes; the lesser ones I pass over: deprived of which, those who are strangers to Philosophy, if they should complain, will complain in vain. But for us it must be affirmed that for this reason above all God begot the eyes: that, beholding the circuits of the mind which are carried out in the heaven, we might turn them to the use of our own mind, and might recall the discursive movements of our knowledge — akin to those circuits, but in some way disturbed — to their evenness; and that, when we have come to know those circuits, and being endowed with right reason according to nature have perceived the order of the individual bodies, we might imitate the revolutions of God, which are carried out without any error, and after the example of the gods might compose the wandering and erratic discursive movements of our thought." Thus he. — These things, if [you should weigh] soberly,

and a true interpretation [of the *Timaeus*] is grasped — when these are compared with the actual passages of [Plato], indeed with the very reality, and once astronomy has been properly understood — then it will earn the distinction and praise of human knowledge above all else. No one, then, ought to wonder why not only wise antiquity, but emperors, kings, and princes too devoted so much leisure and expense to the study of the heavens.

I now come to the difficulty of this theoretical part. No one will deny it. For not only the uniform forward motion of the fixed stars, apparently equal in time, but also the motions of the planets — their particular circuits through epicycles as much as their varying distances from the Earth — are nearly hidden from direct visual perception. They are therefore incomprehensible to the judgment of the common crowd, and perceptible to astronomers alone. This is why Plato, in the *Epinomis*, exalts this part of astronomy above all wisdom: "Do not suppose," he says, "that astronomy is the wisest of pursuits. The true astronomer must be — not one who, after the manner of Hesiod and all his sort, merely studies the settings and risings of the stars, but rather one who studies the eight revolutions,

how the seven move beneath the first, and in what order each completes its own circuit. No nature but a wonderful one will ever readily grasp this." From this passage arises the third matter, which I judged ought to be set out here in advance: namely, that I should treat the courses of the planets in general terms.

It must be understood, therefore (and I assign this to our inner principle), that each planet, while marking out one apparent periodic revolution, traverses one, another, or several circuits in the heavens, according to the linkage of the epicycles to which it is bound — and this, beyond doubt, to signify the mysteries of the universe, as I touched on above. Among these the Sun itself, as the leader and chief of all the rest, appears — apart from the greatest orb, by which its annual revolution is marked — to be content with a single epicycle, except insofar as it involves the precession of the equinoxes and the unequal obliquity of the zodiac in anomalies peculiar to itself. The others, however, besides their own primary circles, also require two or more small circles [circelli]. Furthermore, the remaining five planets — which, set in the middle, follow the lead of the Sun — by their relation to the disposition of this one body, and by the distance of each from the Earth, also introduce and alter a varying magnitude of the visual angle. They undergo a variable change of position relative to the Earth, the most conspicuous of all, as will be made clear in the hypotheses themselves. This single cause is why Copernicus thought the Earth carried out its course in the middle region, between the sphere of Mars and that of Venus, while the Sun rested fixed at the center of the world — as will be seen in the following chapter.

As for the Moon, even though it does turn back upon the Earth at the center, nevertheless it too is bound to the Sun by a kind of wonderful bond. Thus, besides the twin small circles of its own eccentric, it is also variously affected with respect to its disposition relative to the Sun. Hence in the quadratures it makes its greatest departures, while in the meantime, at the intermediate positions, it slightly accelerates its motion as it moves toward the quadratures, and retards the same motion as it returns from them. This anomaly, along with another concerning the latitude of the Moon and the precession of the nodes, no one detected before Tycho Brahe. For these were found out, when I spent ten years with him, only by our very great labor, and were brought into the hypothesis. The latitudes too of the remaining planets, by which they depart this way and that from the path of the Sun, behave in various ways — both by reason of the primary circles and by relation to the Sun. Nor do the motions of their nodes agree with the suppositions of the ancients. All these matters are to be set forth in the hypotheses themselves, and tied to celestial observations that are settled and

firm — observations which may rightly be called the sensory principles of this astral doctrine.

For it is not part of our plan to want to hand over astronomy rashly — heedless of the heavens, as many do — to a gifted youth born to grasp celestial things. Rather, so far as is granted by the grace of God, our aim is at once to enrich the theories of the planets with the things the ancients evaded, and yet no less to record the motions of those planets upon the duly reconciled hypotheses of the three most excellent practitioners — the distinguished [Ptolemy], [Nicolaus Copernicus], and [Tycho Brahe] — and to reduce them to uniform and consistent tables, from which the phenomena of the planets may be drawn out compendiously for any time whatever. In this way this care and concern of ours, in the service of restoring astronomy, will prove welcome and commended not only to the present age but also to posterity. This threefold variety of suppositions amid the same phenomena, however, we do not pursue solely to show, as if by a complete induction, that other [hypotheses] equally consistent with the celestial appearances are not to be admitted in nature —

— and meanwhile to permit each person his own choice in these matters. Rather it is also so that the timepieces [horologia], built here and there with art after the monuments of the ancient astronomy, may represent the courses of the planets. Among these is the αὐτόματον [Greek: *autómaton*, "self-moving device"] which was constructed here at the expense and care of the most Christian King of Denmark, Lord [Christian III] of glorious memory, who was deeply devoted to astronomical study; and afterward, by the gift of our present sovereign King [Christian IV], it passed to this Royal Academy — yielding precedence (as I think) to few in Europe — provided it be properly understood and directed to motions and their uses.

Now, because there is so much genius in the Copernican invention, and so great a power and ἐνέργεια [Greek: *enérgeia*, "active force, efficacy"] in the Tychonic [system] for reconciling the other two dissenting ones, what was formerly proposed to Lord Tycho seems now fitting to us as well: to be willing to carry it out for the benefit of the disputing parties — namely, regarding the threefold hypothesis underlain by the three said practitioners — proposing it separately, comparing the parts with one another, and, with the addition of reasons, recognizing the σύστασις [Greek: *sýstasis*, "constitution, system"] of Tycho, our most deserving Teacher, as especially to be acknowledged for the true one. Yet in such a way that, in those matters which seem to us more probable concerning the daily rotation of the Earth in the middle and the perpetual rest of the starry

heaven — matters which are physical, and which contribute nothing different toward saving the phenomena, whichever you may choose — we may use our own liberty, as we do in several other matters. For this liberty, which throughout the whole of Philosophy ought rightly to be exercised for the sake of investigating the truth, can by no right be denied to its most excellent part — especially provided it be consonant with reasons drawn from established principles, and provided it in no way conflict with defining the celestial motions compendiously, the very purpose for which this doctrine is chiefly intended.

Accordingly, once such a general description and comparison of the world-systems [συστάσεων, systáseōn] has been set out in advance, the restitution of the fixed stars, and likewise of the Sun and Moon — which press upon the Earth as upon the primary center of the world — is to be joined faithfully to it; and with these the prior Book of the Theory is to be completed. The posterior Book, by contrast, will be brought to completion in the similar restitution of the motions of the remaining five planets, which regard the Sun as the secondary center of the world; and for displaying these we shall apply, indifferently, those three hypotheses of which we spoke just now. And although our weakness — partly through genius once neglected, partly nearly exhausted by bodily labor and by night-watches — will seem easily to give way under this arduous undertaking, there is nevertheless good ground to hope that one day, with God assisting [Θεοῦ συναψομένου, Theoû synapsoménou], we shall happily carry that same undertaking through to its appointed end, to the glory of the admirable Creator of the heavens and the signal benefit of posterity.

BOOK ONE OF THE THEORY

IN GENERAL

Treating briefly of the threefold sketch [hypotyposis] of the world-system, and of the hypotheses of the planets:

IN PARTICULAR

On the restitution of the fixed stars, and likewise of the Sun and Moon.

CHAPTER I.

On the threefold ordering of the world-system.

The doctrine concerning the motions of the planets — even though, among those who handed down their records to posterity, hardly any such doctrine existed before [Ptolemy] of Pelusium, who flourished at Alexandria in Egypt about the year 140 from the birth of the Savior, of a kind to be bound to certain hypotheses geometrically founded — nevertheless it is by no means doubtful that this began from the start together with the spherics of the heavens. For it is credible that the ordering of the world-system was discovered and established by the earliest men, and propagated to posterity; and that the motions of the planets were over a long course of time defined among the Chaldeans, perhaps by numbers alone, without hypotheses in a manner of speaking — by reasoning, that is, from the appearances in the heavens. For — to pass over the testimony of [Josephus], *On the Antiquity of the Jews*, Book 1, who makes the study of astronomy most ancient — it is left sufficiently attested and discussed by the early philosophers, [Hermes Trismegistus], [Plato], and [Aristotle] (besides what is found in the poets about Prometheus, Epimenides, Atlas, and others), what the opinion of the ancients was concerning the constitution of the world. This opinion afterward, in [Pliny], *Natural History*, Book 2 — who lived under Vespasian — though it is read as somewhat altered in certain points toward the opinion of others whom he followed, and as notably turned about by the tradition of Aristarchus of Samos and of certain Pythagoreans, was nevertheless, in its most ancient constitution — the one adopted by [Ptolemy] — predominant in the judgment of astronomers. For it is reasonable that this same constitution was also in use among Timocharis of Alexandria, Hipparchus of Rhodes, Archimedes of Syracuse, and others before [Ptolemy], and that it was afterward kept in frequent use down to [Copernicus] — as also appears from the *Hypotheses* of Proclus Diadochus.

This sketch of the world-system, moreover, is of such a kind that, from the characters of the planets — with their individual orbs added — it is well enough known of itself.

Diagram. The old, or Ptolemaic, sketch of the world-system [*Hypotyposis systematis mundani vetus, seu Ptolemaica*]. A geocentric schematic of nested concentric circular orbs around a central Earth.

At the center is a woodcut globe of the Earth (land and sea), surrounded by a flame-like band representing the elemental region. Proceeding outward, each orb is labeled at the top with the planetary character of the body that occupies it:

- ☾ (Moon) — innermost planetary orb
- ☿ (Mercury) — next orb out
- ♀ (Venus) — next orb out
- ☼ (Sun) — middle orb
- ♂ (Mars) — next orb out
- ♃ (Jupiter) — next orb out
- ♄ (Saturn) — outermost planetary orb

Beyond Saturn lies the broad outer band of the firmament of fixed stars (rendered as a ring densely strewn with star-asterisks). Around this stellar band are set the twelve signs of the zodiac, each marked with its character in a small boxed cartouche. Reading around the circle, the signs visible are:

- ♋ (Cancer) — at the very top (the crown of the band)
- ♌ (Leo) — upper left
- ♍ (Virgo) — left
- ♎ (Libra) — lower left
- ♏ (Scorpio) — lower left, toward the bottom
- ♐ (Sagittarius) — bottom
- ♑ (Capricorn) — lower right, toward the bottom
- ♒ (Aquarius) — lower right [the double-wave character is distinct]
- ♓ (Pisces) — right
- ♈ (Aries) — upper right
- ♉ (Taurus) — upper right, toward the top
- ♊ (Gemini) — top, just clockwise of Cancer

The twelve characters run in natural zodiacal order counterclockwise around the band, beginning from Cancer at the crown and reading down the left side to Sagittarius at the bottom, then up the right side through the pre-Cancer signs (♊ ♈ ♉ ♎ ♏ ♐); all were read under magnification.

In this system the periodic motions of the planets around the Earth are at least expressed, with the orbs distinguished. Of these, Saturn's is about 30 years; Jupiter's, 12; Mars's, 2; that of the Sun, Venus, and Mercury, one year; the Moon's, 28 days. And although [Ptolemy] nowhere in his work expresses this system in a diagram, he nonetheless argues for the rest of the Earth in the middle [Margin: *Almagest*, Book 1, ch. 5]. Moreover, concerning the order of the spheres or orbs in which the planets were embedded [Margin: *Almagest*, Book 9, ch. 1], he asserts that this was the view of the ancients, when he refutes the opinion of those who

hold that Venus and Mercury are placed above the Sun on the ground that, at the conjunction of the Sun with them, no obscuration of it was observed — their withdrawal into the latitudes, that is, entirely preventing this. Meanwhile, however, [Plato], from another wellspring of nature, argues that these luminaries are nearest to the Earth [Margin: Plato in the *Timaeus*]; and [Aristotle] follows him in this [Margin: Aristotle, in the treatise *On the World*].

And this was the ancient sketch of the world, taken up again by [Ptolemy] when he set out to demonstrate the motions of the heavenly bodies. He, however, with these orbs described — besides the eccentrics and equants, on which see below — was compelled to add on annual epicycles as well in the three superior planets; while in Venus and Mercury he had to add others, and proper ones. This was because the planets were subject to the Sun's motion in longitude as well as in latitude; the stations and retrogradations of the superior planets occurred annually; and the inferior ones too had their own periods, to be saved by the epicycles of each. This we shall make clear in a moment by two examples — one, namely, among the planets superior to the Sun, the other among those that were reckoned inferior to it.

A figure of the Ptolemaic hypothesis for the superior planets — Saturn, Jupiter, and Mars — showing their annual advances, stations, and retrogradations.

Explanation.

From the Earth, that is, from the center of the universe, first draw the annual circle of the Sun, B C. Then from the same center A draw the circle of the periodic revolution of one of the superior planets, D H, its motion lying along D, rising toward H. On the Ptolemaic epicycle E G drawn upon that circle, the planet moves in such a way that, while the Sun by its mean motion stands at B and the Earth lies directly interposed between the Sun and the planet, the planet occupies E — that is, the lowest part of its epicycle, the part nearest to the Earth.

Again, when the Sun is at C and is in conjunction with the planet, the planet holds the apex of its epicycle at F, and is therefore at its greatest distance from the Earth A. Meanwhile, by traversing the said epicycle, the planet marks out the appearances [φαινόμενα, *phainomena*] that match what is seen in the heavens — advancing, standing still, or moving backward according to how it is configured with the Sun, as is readily grasped here. For while the planet advances from the apogee F of the epicycle toward H, its motion on the epicycle is one of forward progress; and near H (or a little below it) it becomes stationary. From there it is retrograde, from H through E to G, where it returns once more to a second

station. Finally from there it proceeds to F again, following the order of the signs — though account is always taken, in these motions, of the planet's own motion in its proper orbit D. And so the three superior planets — Saturn, Jupiter, and Mars — are affected year by year by the motion of the Sun, so that to represent this anomaly [ἀνωμαλία, *anōmalia*] of theirs Ptolemy was compelled to assign to each one an epicycle of the kind we have just described. Nor indeed is the agreement with the Sun's annual motion any less in the inferior planets, Venus and Mercury — as now follows, illustrated in their case too.

Diagram. Ptolemaic scheme for a superior planet. A large outer circle is centered on A (the Earth, at the center of the universe). On the central vertical diameter, C lies above the center and B below it, marking the annual circle of the Sun, B C. A small epicycle is drawn in the upper region, with its apogee F at the top, D just below F (the deferent point), and E at the bottom of the epicycle; H and G are the lateral points (H at left, G at right) where the planet becomes stationary. A point on the deferent circle is marked at upper left.

- A (center, lower): the Earth / center of the universe
- B (bottom of vertical diameter): the Sun's position at conjunction with the planet below the Earth-line
- C (upper, on vertical diameter): the Sun's mean position B C marking its annual circle
- D (upper epicycle, just below apex): deferent point of the planet's proper orbit
- E (bottom of epicycle): lowest, Earth-nearest part of the epicycle (planet at opposition)
- F (apex of epicycle): apogee, planet most remote from Earth
- G (right of epicycle): second stationary point
- H (left of epicycle): first stationary point

A figure of the Ptolemaic hypothesis for the inferior planets — Venus and Mercury — showing their periodic advances, stations, and retrogradations.

Explanation.

With A as center — which takes the place of the Earth — draw the circle of the Sun, which is the outer one, B. Then from the same center draw the circle C of one of the inferior planets, namely Venus or Mercury. And since by the Ptolemaic hypothesis the mean or annual motion of the Sun and of the two planets Venus and Mercury is one and the same, draw a straight line from the Earth A through C to B; in the whole revolution through each circle this line remains beneath equal

parts of the zodiac. But to display the remaining appearances of Venus and Mercury, an epicycle D E F G is further drawn upon the center C, and on it the planet carries out its course according to Ptolemy's view. (*continued on next page*)

Diagram. Ptolemaic scheme for an inferior planet. A large outer circle centered on A (the Earth) carries the Sun's circle, with B at the top of the central vertical diameter. Below B is D, then C (the center of the planet's epicycle, on the Earth–Sun line). The epicycle D E F G is drawn about C: D at its top, F at its bottom, E at the left, G at the right. Lines run from the Earth A up through F, C, and D, and tangent lines from A touch the epicycle near E and G (the greatest evening and morning elongations). A small chord network (E–C–G and the E–F–G triangle) is drawn inside.

- A (center, bottom): the Earth / center of the universe
- B (top of vertical diameter): the Sun's circle (the outer circle)
- C (center of epicycle, on the Earth–Sun line): center of the inferior planet's epicycle
- D (top of epicycle): apogee of the epicycle
- E (left of epicycle): position at evening elongation
- F (bottom of epicycle): perigee of the epicycle
- G (right of epicycle): position at morning elongation

an epicycle D E F G is drawn upon the center C, and on it the planet carries out its course according to Ptolemy's view. Thus, descending from the apogee D toward E, it markedly increases its motion beyond C; and when it has come near its greatest evening elongation from the Sun — at E, where its equation is measured by the angle C A E — it appears stationary. From here it descends to the perigee F; and again from there it is seen to move backward to G, the greatest morning elongation, where it becomes stationary a second time. From there it returns to the apogee D, following the order of the signs [εἰς ἐπόμενα, *eis hepomena*, "toward the following (signs)"], and so on in succession, and so forth — as can be demonstrated more fully in the special hypotheses of the individual planets. And since the mean annual motion of the inferior planets is shared in common with the Sun, the period through the epicycle is for that reason not annual as it is for the superior planets; rather, in Venus it is about nine months, and in Mercury about eighty days, insofar as they separate themselves from the Sun's mean motion.

Such a general demonstration suits the motions of the five planets that are bound to the Sun, which can be accounted for in no other way than by the epicycle just

added by Ptolemy. Pliny [Pliny the Elder], however, supposed that the cause producing these motions was the rays of the Sun, falling upon the planets in various ways — though he had rather said that the Sun is the center of all of them. Observing this, after certain ancient philosophers — Philolaus the Pythagorean, Nicetus, and most of all Aristarchus of Samos — Nicolaus Copernicus of Toruń [Nicolaus Copernicus], a man of divine genius, so inverted the order of the older system set out above that, with the Sun itself occupying the center of the universe, the Earth together with the Moon at its midpoint took its place in the Sun's circle, made subject to a threefold motion. For Copernicus not only attached to the Earth the apparent daily motion of the heaven itself, and rendered wholly at rest the sphere of the fixed stars (which, like the orbs of the planets, would regard the Sun as its center); he also did away, by the single motion of the Earth, with all the epicycles introduced by Ptolemy on account of that combination of the planets with the Sun which we showed above. And beyond this he displayed the simple anticipation of the equinoxes, while preserving the ancients' opinion as to the reality of the orbs in the heaven. Here, among other things which the old system cannot achieve, given the distance of the Earth from the Sun, the distances of the remaining planets from the Earth are each found according to the same symmetry.

Finally, by a third motion of the Earth, which he calls the motion of libration, Copernicus demonstrated the inequality of the equinoxes and of the obliquity of the ecliptic from the equator.

And this is the sum of what Copernicus achieved by an alteration of the old hypothesis [ὑποθέσεως, *hypotheseōs*] that is altogether admirable and also elegant — when he applied his own arrangement, which now follows, to the celestial appearances [φαινόμενα, *phainomena*]. In it, those points which seem more difficult — concerning the threefold motion of the Earth and the rest of the heaven and the Sun — can be set out separately before the eyes of students, within a material sphere, so that they may justly be impressed by the acumen of so great a man.

Diagram. Ptolemaic epicycle for an inferior planet (companion to the explanation on this page). A large outer circle centered on A carries B at the top of the vertical diameter and D just below it; C is the center of the epicycle on the vertical line. The epicycle D E F G surrounds C, with D at top, F at bottom, E at left, G at right, and the inscribed chord-figure (E–C–G and the E–F–G triangle) drawn within. A small filled point sits below A at the bottom of the outer circle.

- A (center): the Earth

- B (top, vertical diameter): the Sun's circle
- C (epicycle center, on the vertical line): center of the inferior planet's epicycle
- D (top of epicycle): apogee
- E (left of epicycle): evening elongation
- F (bottom of epicycle): perigee
- G (right of epicycle): morning elongation

A sketch of the Copernican world-system.

[Margin: Copernicus, *De Revolutionibus* I.10.]

Diagram. The heliocentric Copernican world-system, drawn as a set of nested concentric circles with the Sun at the center. The orbits are numbered from outside inward and labeled along their arcs; a planetary glyph sits at the left edge of each numbered orbit. The outermost band is a ring of stars; four decorative foliate ornaments fill the corners outside the star-ring.

- ♄ orbit (outermost numbered, with Saturn glyph): "I. The immobile sphere of the fixed stars [I. Stellarum fixarum Sphæra immobilis]" — running along the top of the star-ring
- ♄ (Saturn glyph at left of orbit II): "II. Saturn revolves in 30 years [II. Saturnus anno 30 revolvitur]"
- ♃ (Jupiter glyph at left of orbit III): "III. The 12-year revolution of Jupiter [III. Iovis 12 annorum revolutio]"
- ♂ (Mars glyph at left of orbit IIII): "IIII. The two-year revolution of Mars [IIII. Martis bima revolutio]"
- ☽ / ☾ near center (Earth orbit, V): "V. [The orbit] of the Earth [V. Telluris]" — with a small inner circle labeled "Earth [Terra]" carrying the Moon glyph ☾, and along its arc "with the annual revolution of the Moon's orb [cum orbe Lunari annua revolutio]"
- ♀ (Venus glyph, orbit VI): "VI. Venus, of nine months [VI. Venus nonimestris]"
- ☿ (Mercury glyph, orbit VII): "VII. Mercury of 80 days [VII. Mercurij 80 dierum]"
- Center: a radiant Sun-face, labeled "Sun [Sol]"

This is the wonderful invention of the great Copernicus, by which — once the old system has been dismantled, the Earth withdrawn from the midpoint of the universe together with the lunar system into the Sun's annual circle, and the Sun

in turn set back into the center of the world-machine — the phenomena suffer no perceptible alteration from the earlier scheme.

For, first, with the terrestrial globe so arranged, the Sun is seen in plainly the same parts as if we were looking at it from the center, so that the alternations of the annual revolution are observed on both sides alike — though under opposite cardinal points. Indeed, in this way too the Moon and the rest of the stars retain their former appearances, and each is assigned to its own place — far more fittingly, as it seems to Copernicus. And in this transposition as well, the reality of the orbs (the Earth's excepted), although falsely believed by the ancients, is here allowed to stand.

In this Copernican sketch, since the periods of the motions are the same as those indicated above in the Ptolemaic scheme, we shall add only a few remarks — at least here, where a greater usefulness and economy presents itself through this rearrangement of the orbs.

First, with the Earth carried round through its annual circle, so long as the Sun is the center of the universe — and therefore also of the five planets — Copernicus does away with the Ptolemaic epicycles in them in the following way.

Let A be the Sun, the center of the universe; let C K D H be the annual circle of the Earth; let B be the circle of some one of the superior planets, on which the planet remains at any given point; and finally let E be the circle of the fixed stars. So when the Earth is at H, looking toward the Sun at A and in conjunction with the planet at B, no annual variation appears in the planet, and it is set at its greatest distance from the Earth. But once the Earth has revolved from H to C, the angle of the annual variation is C B A, to which there corresponds, vertically opposite, E B F in the circle of the fixed stars — that is, the arc E F; for the measure of this arc the planet quickens its motion, and there (except for what it has accomplished by its own proper motion along B) it becomes stationary. Further, with the Earth carried round to K, so that it is itself interposed between the Sun and the planet, the planet is nearest to the Earth; and because it lies beneath the straight line A E, it therefore lacks any annual variation. But as the Earth advances from C to D, the planet appears retrograde, in proportion to the measure F G of the circle of the fixed stars.

And all these things agree exactly with the appearances [φαινόμενα, *phainomena*], just as if the planet moved on an annual epicycle according to the Ptolemaic supposition — in the three superior planets, that is, Saturn, Jupiter, and Mars. In the two inferior ones, Venus and Mercury, which the Earth encircles, the

heavenly appearances likewise present themselves in the manner that the Earth's annual revolution prescribes, as now follows.

Diagram. Copernican explanation of the apparent motion of a superior planet. A large outer circle is the sphere of the fixed stars, with F, E, G marked across its top. Within it lies the annual circle of the Earth, C K D H — H at bottom, C at right, D at left, K just above center. The planet's circle B is the small inner circle; B sits just above the center A (the Sun). Lines run from the Earth's positions through B out to the fixed-star arc E F / F G.

- A (center): the Sun, center of the universe
- B (small inner circle above center): the circle of a superior planet, with the planet at B
- C (right, on Earth's circle): Earth's position giving variation angle C B A
- D (left, on Earth's circle): Earth's position where the planet appears retrograde
- E (top, fixed-star circle): point answering vertically to C B A (arc E F)
- F (top left, fixed-star circle): fixed-star reference point
- G (top right, fixed-star circle): fixed-star reference point (arc F G)
- H (bottom, Earth's circle): Earth in conjunction, planet at greatest distance
- K (just above center, Earth's circle): Earth interposed between Sun and planet, planet nearest

Let A be, as before, the center of the universe and the mean Sun [E being the mean Sun]; let B be the circle of Venus or also of Mercury; let C K D be the annual circle of the Earth. Now when the Earth is at H, it is certain that the planet existing at B and in conjunction with the Sun, beneath the line H A B K E, holds its apogee position. Then, as the Earth advances to C, the planet appears at its greatest evening elongation from the Sun, beneath the line C B F. From here it becomes stationary, and at last retrograde — most of all when the Earth is at K and nearest to the planet, where the planet is reckoned to be at the perigee of the Ptolemaic epicycle. Finally, with the Earth revolved round to D, the planet appears beneath the line D B at the point G on the eighth orb, where it marks out its greatest morning elongation, which the angle A B D — or its vertical, G B E — measures. And all these things agree with the appearances by the Earth's annual motion, provided account is also taken of the planet's motion in its own circle, and of what its position and configurations are with respect to the Sun and the Earth — so that anyone who diligently attends to these matters, as far as they deserve, will admit that this Copernican device is most ingenious for accounting for the celestial appearances [φαινόμενα, *phainomena*] without epicycles.

For the rest, besides the Earth's annual motion, Copernicus also assigned to it two others — the daily motion, namely, and the motion of libration. The daily motion is from west to east, by which it comes about that the heaven, with all the stars, seems to be carried, or to move backward, from east to west.

Finally, through the motion of libration, Copernicus [demonstrated] the alterable [inequality] of the ecliptic and the equator (*continued on next page*)

Diagram. Copernican explanation of the apparent motion of an inferior planet (Venus or Mercury). A large outer circle (fixed stars) carries E, F, G across its top. Within it is the annual circle of the Earth, C K D H — H at bottom, C at right, D at left, K above center. A small inner circle B (the planet's circle) surrounds the center A (the Sun); E is the mean Sun. Lines run from the Earth's positions (H, C, K, D) through B to the points F and G on the outer circle.

- A (center): the center of the universe
- E (top center, and inner): the mean Sun (E folis medii)
- B (inner circle around center): the circle of Venus or Mercury
- C (right, Earth's circle): Earth's position for greatest evening elongation (line C B F)
- D (left, Earth's circle): Earth's position for greatest morning elongation (line D B, point G)
- F (top left, outer circle): evening-elongation reference point
- G (top right, outer circle): morning-elongation reference point, on the eighth orb
- H (bottom, Earth's circle): Earth's position with planet at apogee (line H A B K E)
- K (above center, Earth's circle): Earth nearest the planet (perigee of the Ptolemaic epicycle)

the obliquation of the [equa]tor, perceived over a longer course of time; and this happens only with respect to the poles of the Earth. [Nicolaus Copernicus] introduced this threefold motion of the Earth with such ingenuity that none of the three obstructs either of the others — as visual inspection on a sphere designed for the demonstration of these matters can teach. We append here a rough sketch of it, with its own explanation.

Diagram. The Copernican world-system figure for the threefold motion of the Earth. At the center is the Sun, marked A (drawn as a radiant face). Around it run two large concentric circles: the outer is the zodiac with its twelve signs, the inner the Earth's annual orbit. The zodiac signs are engraved around the outer circle.

Set within these is the Earth's annual orbit, on which the Earth (marked **B**, with its inclined equator drawn as a globe) is shown four times, at the four cardinal points of the ecliptic. The Earth's equator (C D, shown as a tilted band/ring on each globe) is inclined to the zodiac, with the same half of the equator declining toward the same celestial part.

- A (center): the Sun.
- B (on the annual orbit, four positions — top, bottom, left, right): the Earth, marking the center of the Earth, shown at the four cardinal points; each globe carries its inclined equator.
- C (upper region of the inner circle): label of the Earth's annual orbit / the celestial circle C D.
- D (lower region of the inner circle): the opposite end of the same circle, C D.
- Zodiac labels around the outer circle, reading clockwise from the top: ♋ Cancer (top), ♌ Leo, ♊ Gemini (upper, flanking Cancer), ♉ Taurus, ♍ Virgo, ♎ Libra (left), ♈ Aries (right), ♏ Scorpio, ♐ Sagittarius (lower left), ♑ Pisces, ♒ Aquarius, ♑ Capricorn (bottom). On each Earth-globe small zodiac signs (♋ Cancer, ♎ Libra, ♈ Aries, ♑ Capricorn) mark the orientation of its equator.

Let the zodiac, conspicuous with its twelve signs, be described from the Sun's center A. Then, from the same center, let the equator C D be drawn, inclined to the zodiac — not because, according to Copernicus, such a circle would be celestial, but rather terrestrial, in the diurnal revolution, to the degree that the poles of the Earth deviate from the poles of the zodiac. Within these circles, let the Earth's annual orbit B likewise be described from the Sun's center A. This B marks the center of the Earth, set here four times on its annual circle, from opposite places — that is, from the cardinal points of the ecliptic. Over this orbit, half of the equator declines toward the same part as the celestial equator C D — in our own age, by 23 degrees 31½ minutes.

Granting this, and assuming from the hypothesis that throughout the whole annual circuit of the Earth around the Sun the Earth's own poles (on which its diurnal revolution takes place) perpetually face one and the same region of the world, it follows that while the Earth is beneath Capricorn, the Sun appears opposite, beneath Cancer — and so everywhere in the annual revolution it appears beneath the opposite signs and points of the zodiac. Therefore, when the Earth is in Libra, the Sun appears directly opposite in Aries, and indeed at the very beginning of Aries, without any declination, because there we look toward the Sun from the Earth along the common section of the ecliptic and the

terrestrial equator. The same happens when the Earth holds the beginning of Aries: then the Sun is seen in Libra without declination. Hence two equinoxes fall each year on these two points. By contrast, since the greatest declination of the circles of the ecliptic and the equator from each other appears when the Earth is in Capricorn and Cancer — and in opposite directions — just as many solstices occur there each year, as is more conveniently perceived from the instrumental sphere itself. And so (apart from the matters concerning the planets themselves treated above) the Copernican hypothesis most beautifully accounts for the annual changes of the seasons, and sets them before the eyes, from the annual motion of the Earth. Indeed, the simple precession of the equinoxes — that is, the apparent forward progress of the fixed stars — can occur for this reason...

as we, with Copernicus, observe: namely, that the Earth's annual circuit is everywhere gradually anticipated, from its own directing motion, into one and the same region of the world.

So much for the annual motion of the Earth. Next comes the diurnal motion, which is the rotation of the Earth's own surface about its own center, from west toward the rising in the east, in the span of 24 hours. From this it happens that the heaven appears to be carried around by the contrary motion, from east to west, in the same span of time, together with the φαινόμενα [Greek: phainomena, "the appearances" — the celestial phenomena] inherent in it.

The third motion of the Earth is called by Copernicus the motion of libration, because the terrestrial poles, carried around in certain little circles over a long stretch of time, vary the obliquity of the equator from the ecliptic. These motions just recounted are attributed to the Earth by Copernicus with such ingenuity that they can all be carried out at once without any hindrance, while the Sun and the starry heaven meanwhile rest, and the whole jumble of epicycles (which we discussed above) is removed — as was also related earlier.

Once, while I was representing the world-system, complete down to the particular motions of the planets, on the Copernican hypothesis up to a fixed time, I amused myself over this great economy of nature in the following way:

That this world, here fitly composed of its various parts, might be beautifully beheld in a single form; and that from our rays the courses of the radiant heaven might be marked out, where every star inclines — the great Copernicus turned upright nature about [?], unfolding the moving stars to fixed laws. You see how Titan [the Sun], the εἶδος [Greek: eidos, "form" or "image"] of the divine power, sits as the fountain of things:

whom the others around — Saturn, shining Jove, Mars, Hermes [Mercury], and Venus — like a great King, venerate with great honor. Even this narrow globe, in which we mortals are enclosed, obeys his nod, girdled by the star of Phoebe [the Moon]. O if in past ages so easy a method had been well known! Now you tire yourself in vain, Atlas; for we suspend most things by the single course of the Earth. Indeed, your [findings], Ptolemy — that we do not measure from your art how far the wandering stars are removed in the ether — this now truly agrees aptly with our numbers, so long as the laws that turn the stars of the world endure.

Thus far concerning the world-sketch [hypotyposis] of this great Copernicus, and his assertion of the threefold motion of the Earth in it. Although it was devised most ingeniously to save the celestial appearances, and is fortified also with the physical arguments inserted in the first book of the *Revolutions* [Margin: See Bk. 1 of the *Revolutions*, cap. 8, etc.], so that it can be overturned only with difficulty — nevertheless, among others, two objections are found that seem to detract from its credibility, especially in certain matters concerning the annual motion of the Earth.

The first can be drawn from the condition of creation and from the prerogative of the great bodies of the world. For since, among all visible created things, Sacred Scripture makes mention first of all of the heaven and the Earth [Margin: Gen. 1, vv. 1], it is surely probable that primary seats or places in the world were assigned to these as well. And since it is credible that this whole universe imitates the form of a circle, what is more probable than that the Earth is its center, while the heaven — which encloses all the rest — fills the space between the center (namely the Earth) and the circumference? Just as these things present themselves to one who searches Scripture more fully in Genesis 1, vv. 6, 7, 8, likewise vv. 14, 15, and the following.

The second argument against the annual motion of the Earth is drawn from the incredible distance of the sphere of the fixed stars from the Earth or the Sun, which the Copernican hypothesis...

will require, in order to allow such a motion — and likewise the magnitude of the fixed star itself. For this objection will deserve no place unless the diameter of the Earth's annual orbit, compared with the semidiameter of the fixed stars from the Sun, plainly vanishes — a comparison that, however, is perceived clearly enough even in Saturn, the highest of the planets. But what is asserted here we shall prove by geometrical demonstration in the following way.

Diagram. A geometric figure for the parallax of the eighth sphere. At the bottom is a circle representing the Earth's annual orbit, centered on A (the Sun); its horizontal diameter runs F–A–E, and a vertical line rises from A through the top of the circle marked H. From the two ends F and E of the diameter, two long lines rise and converge to a single point D at the top of the figure. Through D a wide shallow arc is drawn from C (left) to B (right) — the arc of the eighth sphere (the orbit of the fixed stars). The line A D is the semidiameter of the eighth sphere from the Sun.

- A (center of the lower circle): the Sun.
- E (right end of horizontal diameter): one position of the Earth on its annual orbit.
- F (left end of horizontal diameter): the opposite position of the Earth.
- H (top of the lower circle, on the vertical from A): point on the Earth's annual orbit.
- D (apex, top of figure): point on the eighth sphere toward which the lines from E and F converge.
- C (left end of the upper arc): one end of the arc B C of the eighth sphere.
- B (right end of the upper arc): the other end of the arc B C.

Let the Earth's annual orbit E H F be described above center A, the Sun's center, after Copernicus; and from the same center, the arc of the eighth sphere — that is, the orbit of the fixed stars — B C. When a line A D is drawn from the center to that arc, this will be the semidiameter of the eighth sphere from the Sun.

According to Copernicus, it must be of such great length that E F — that is, the diameter of the Earth's annual orbit — bears no comparison at all with it; but, because of the immense distance of D from A, the whole angle E D F, when the Earth is at either E or F, vanishes. For no perceptible variation in the positions of the fixed stars can be detected when the Earth, at contrary seasons of the year, is in these positions according to the Copernican hypothesis — even with observation made most attentively and diligently on a proper and adequate instrument, as we have several times tested at [Hven] on the polar star.

But let the angle E D F be given — not from any observation, but only by hypothesis — as one first minute, which could be discerned on the instrument if it were [that large]. Then E D A will be $\frac{1}{2}$ minute. And since in the right triangle E A D all the angles are given for A D, with side E A being 1150 semidiameters of the Earth — the mean distance of the Sun and the Earth from each other (for we wish no other distance to be presupposed here than Tycho's [Margin: Bk. 1 of the *Prog. ymnasmata*], p. 473], beyond a hundred semidiameters, which perhaps ought

rightly to be increased by us in its proper place; and this detracts still further from Copernicus's opinion, and increases for us the distance and measure of the fixed stars) — then it will be as follows:

	sec.	gr. mi. sec.	
E D A	30	E A... D E A 89 59 30	A D
S. R. 1454441	1150 semid.	S. R 9999999894	(7906818 semid. ter.

And by that much would any fixed star be distant from the Sun, or from the mean (as it were) distance of the Earth. This is an immense space compared with what the ancients and Tycho assigned to the fixed stars — namely, 14000 semidiameters of the Earth. And since the last of the planets, Saturn, is, by the consensus of nearly all astronomers, removed from the Earth by about 12000 semidiameters of the Earth, to what end, then, is the empty gap between Saturn and the fixed stars — which from the foregoing comes out to 7894818 semidiameters of the Earth?

Now let us likewise weigh the magnitude of the fixed star geometrically, on the Copernican hypothesis.

Let A (in the next figure) be the Earth or the Sun; let some fixed star of the first magnitude be B. Then A B will be the line of the star's distance from the Sun, or the mean (as it were) elongation of the Earth — which, according to the supposition applied before to the Copernican reasoning, was found to be 7906818 semidiameters of the Earth. Further, about the star's center B let its circular magnitude be described by the little circle D C; and, with the diameter D C drawn, and also the lines A C and A D, we shall first seek, in the right triangle A B C, the semidi-

ameter of the true star, B C, in semidiameters of the Earth. The given quantities [Greek: δεδομένα (dedomena, "the given things" — the data)] are these:

Diagram. A geometric figure for the magnitude of a fixed star. At the top is a small circle representing the star, with horizontal diameter C (left) – D (right) passing through its center **B**, and a vertical line from B descending. The line continues downward and broadens into a very narrow elongated triangle whose apex is at **A** (the Earth or Sun) far below. [Margin: Prop. 18 of Bk. 12 of Euclid.]

- B (center of the upper circle): the fixed star.

- C (left end of the star's diameter): one end of the star-diameter D C.
- D (right end of the star's diameter): the other end.
- A (apex, far below): the Earth or the Sun.

B A = 7906818 semidiameters of the Earth; B C A = 1 first minute, from observation of the apparent semidiameter of a fixed star of the first magnitude; and finally the angle A B C is right. Therefore it will be as follows:

B	.. BA.	BAC	BC
S. T. 10000000	7906818	T. 2909	(2300 semid. ter.

Since D C is double B C, it will be of just as many whole diameters of the Earth — namely, 2300. From these things we may recognize that the diameter of the bulk of a fixed star of the first magnitude, on this supposition, would be more than double the diameter of the Earth's annual orbit. Given now the diameter of a fixed star of the first rank, we obtain its quantity, first in comparison with the whole globe of the Earth (that is, with the Earth and the surrounding water heaped together), by resolving the proposed diameters of both the Earth and the star separately into cubic numbers, and subtracting the smaller from the larger. For spheres are in the tripled ratio of their diameters [Margin: Prop. 18 of Bk. 12 of Euclid]. The procedure follows.

The cube of the number D C = 2300 is 12167000000; and since the given diameter of the Earth is 1 part, its cube too will be 1 part. Therefore, dividing the upper cube by unity, the same number returns — namely, 12167000000 — which proves that a star of the first magnitude exceeds the globe of the Earth that many times over, if the Copernican supposition is true.

Let this same number now be divided by 140 — namely, by however much the Sun itself is hitherto believed to exceed the Earth in its magnitude [Margin: Tycho, *Progymn.*[asmata], p. 473]. Then a fixed star of the first rank will exceed even the Sun itself 86907143 times. Finally, since in this proportion the diameter of the fixed star was previously found to be more than double the diameter of the Earth's annual orbit — on the prior supposition of a parallax of whatever sort — then, with this set at one part, that diameter will be 2 parts. And when each is taken cubically, the cube of the annual orbit remains 1 part, but that of the star 8 parts. From this it is shown that a star of the first magnitude, in its total quantity or bulk, must surpass eightfold the whole interval from the Sun to the Earth that

is circumscribed by the magnitude of the Earth's annual orbit — if any place at all is given to the Copernican supposition.

Since, therefore, both from the immense removal of the fixed stars from the Earth, and from the gap (however incredible) that thence follows between the last of the planets and the orbit of those same fixed stars, compared with the Earth's annual orbit, likewise the Sun, and much more the Earth [Margin: See Bk. 1 of the *Epistolae Astronomicae*, T.[ychonis] B.[rahe], p. 192] — the fixed stars being of the first (as we have supposed) rank in magnitude — all the well-constituted symmetry of the parts of the world is easily destroyed; besides the absurdity which above, from the condition of things primarily created, we adduced as arguments from the sacred writings, and which afterward, concerning the motion of the trepidation of the Earth — to be destroyed by an alterable latitude rather than declination of the fixed stars — we shall adduce [Margin: cap. 4 of this work]: therefore I judge that the Copernican hypothesis, especially as regards the annual motion of the Earth and its libration about its own poles, ought rightly to be set aside, and exchanged for Tycho Brahe's happier invention [Margin: See the *Epist.[olae] Astron.[omicae]* of T.[ycho] B.[rahe], "Items," and "our [foundations of?] Astronomy" (περὶ [?])] — especially after the reality of solid orbs in the heaven has been sufficiently overthrown by recent observations.

Therefore, third — and if perhaps there is anywhere a true world-system — we set before the eyes the one invented by Tycho Brahe 38 years ago, in the following delineation.

A new sketch of the world-system, devised by Tycho Brahe, in which both the redundancy and inelegance of the old Ptolemaic system, and the recent physical absurdity of the Copernican system in the motion of the Earth, are excluded; and in which everything corresponds most aptly to the celestial appearances.

[Margin: Libr. 2 Progymn. p. 189.]

Explanation.

Let A be the Earth, the primary center of the universe — that is, of the outermost starry heaven, extended on every side to whatever distance you please from the Earth. The Earth is likewise the center of the luminaries, namely the Sun and the Moon, which serve chiefly to mark off the divisions of time; their orbs are therefore here drawn concentric with the Earth, together with the orb of the fixed stars. [Margin: *Genesis* 1.14 and following.]

We affirm, however — together with the worthy Master Origanus [David Origanus] and other distinguished men of this age — that the Earth moves about its center in the middle, and that it completes its daily revolution from west to east, to nature's great economy. For on this view, not only is the uniformity of the motions, from west toward east, truly preserved in all the heavenly bodies (only the *appearance* of motion toward the west being observed in the sky); but also the incredible speed of the stars in the daily revolution — above all in the sphere of the fixed stars — is removed by the natural contrary effort of the Earth. [Margin: Further arguments concerning the diurnal motion of the more remote bodies are gathered in the preface to the Ephemerides of Master Origanus.]

I add this too: since it is reasonable that the fixed stars are set at distances from the center (the Earth) that are not equal, and that they hang in the utterly clear heaven and are poised, like the globe of the Earth, upon their own centers with equal weights, they have suffered no disruption in their mutual distances since the founding of the world. Who, then, would attribute real motions to such bodies — except one who insists on defending Sacred Scripture in a few passages which (as it seems) are accommodated only to human comprehension in appearance? But of these matters, more below.

Diagram. The Tychonic world-system, drawn as a set of concentric and eccentric circles. At the very center, the Earth, labeled **A** (with a small Moon-orb circle drawn around it and the lunar symbol ☾ just below). Surrounding this is the Sun, drawn as a radiant disc, which carries the orbits of the five planets centered upon it: Mercury and Venus circle close about the Sun, while Mars, Jupiter, and Saturn enclose both the Sun and the Earth. The outermost band is the sphere of the fixed stars, rendered as a ring densely strewn with star-figures. The readable labels, from the center outward along the upper axis:

- A (center): the Earth (*terra*), with the Moon ☾ marked just below it on its own small orbit.
- ☉ (radiant disc, second center): the Sun, the secondary center about which the five planets revolve.
- ☿ (just above the Sun): Mercury.
- ♀ (above Mercury): Venus.
- ♂ (outer orbit, upper axis): Mars.
- ♃ (next outer orbit): Jupiter.
- ♄ (next outer orbit): Saturn.
- (outermost starred ring): the sphere of the fixed stars (uncaptioned, marked only by star-figures).

Diagram. A repetition of the same Tychonic world-system figure, occupying the top of the page (it continues across the opening). Its structure and labels are identical to fig-p161-1: the Earth **A** with the Moon ☾ at center, the radiant Sun ☉ as the second center, Mercury ☿ and Venus ♀ circling close about the Sun, the orbits of Mars ♂, Jupiter ♃, and Saturn ♄ enclosing both Sun and Earth, and the outermost ring of fixed stars.

I now return to the other, the secondary center of the universe. That the Sun fills this role — set, as it were, in the place of the primordial light — is established by arguments drawn everywhere from the motions of the planets and from the other appearances, especially those derived from the annual motions. So clearly is this so that Copernicus too undoubtedly held the Sun to be their center, about which the five planets — namely Saturn, Jupiter, Mars, Venus, and Mercury — lead their circuits as about their proper guide and king, and which they observe always to move, as it were, in the very midst of their revolutions. The result is that, as the Sun makes its circuit, the centers of the orbs that the planets describe about it are also carried around year by year.

Moreover, since all this holds for the inferior planets too, Venus and Mercury — namely, how their orbs take the place of the Ptolemaic epicycles in their case — it is truly remarkable how, by the benefit of this hypothesis and of the single solar revolution, all the annual epicycles in the superior planets that encompass the Earth, namely Saturn, Jupiter, and Mars, are abolished. And yet these same epicycles are in fact still asserted in the annual circuits of each planet, precisely because of the single annual revolution of the Sun (as I said) about the Earth; while to the inferior planets, Venus and Mercury, their own proper orbs are assigned, no less than to the rest.

And here is the heart of the matter, the very thing the demonstration demands, and which the motions and digressions of the planets prove: from the single, uniform circuit of the Sun, the semidiameters of the epicycles — which are described by the same route from the motions of the planets — are indeed in themselves equal, and equal to the solar orb, yet with respect to the appearances they come out unequal: smaller in the higher planets, such as Saturn, but greater in the lower, such as Mars. All this can best be shown to the eyes of learners by means of a kind of paper scheme, laid out flat, which represents the Tychonic hypothesis according to all its orbs.

In ordering the world-system, then, we rest in Tycho's invented hypothesis rather than in that of Copernicus — both because it stands, as it were, midway between the old Ptolemaic system and that admirable Copernican one;

and because, with the annual epicycles of the former excluded, and the other absurdities of the latter in the annual motion of the Earth excluded, it explains the symmetry of the world most excellently and most fittingly. For the rest — since the phenomena of all the planets can accommodate themselves indifferently to the supposition of all three of these craftsmen, and no other way remains of saving these same phenomena (especially as regards the annual inequality, from which the variations of latitude follow with marvelous agreement, and likewise the stations and retrogradations, when the bodies are supposed to imitate circles) — this is briefly demonstrated by what seems a full induction: the said inequalities are dealt with either entirely outside the middle of the universe, or in the middle, or about the middle. To the external consideration the Ptolemaic epicycle is suited; to the internal, the Copernican; and to that which is about the middle, the Tychonic. Therefore, having weighed this necessity, we shall allege the motions of the planets — which we are to treat in the second book of the *Theorica* — under all three of the sketches here reviewed, after the method of attempting the same has been demonstrated generally in what follows. And we by no means despair that the thrice-greatest and most gracious Maker of the heavens will so prosper our labors and bring them to their appointed end, that all may serve the continual propagation of His glory.

CHAPTER II.

On the method devised by the ancients for saving the motions of the planets in their proper orbs by means of eccentricities and equants; on the refutation of that method; and on the substitution of a better method by means of epicycles and epicyclic-epicycles.

Up to this point, in setting out the sketch of the threefold world-system, we have described the celestial appearances in general. But since the motions of the planets fall under a twofold consideration — namely, as regards their proper orbs, and also as regards their annual digressions — and since the former, with its anomalies, is said to be *essential* to them, while the latter, on account of the Sun's annual motion (as has been shown), is *accidental*; we must therefore first treat of the former. This is especially so because the various motions of the planets in their own orbs also markedly vary their annual digressions.

Now the celestial motions are distinguished into the *simple* or *periodic* and the *compound* or *apparent*: the former are commonly called *mean*, the latter *true*.

Further, the planets in their proper orbs are subject either to a single anomaly, like the Sun itself, or to a double one, like all the rest.

The ancients, after they had discovered that the motions of the planets are indeed perpetual, but that the appearances of these same motions are unequal, displayed these anomalies (or inequalities) — in the single case by eccentricities, in the double case by eccentricities and equants — in the following ways, which will appear in what follows.

Demonstration of the simple anomaly, as in the Sun.

[Greek: Απόδειξις (apodeixis, "demonstration")]

Let A be the center of the circle C D E F, in which the Sun is carried (the Sun alone among the planets being content with a simple digression from the mean motion). On its diameter C E, let B be conceived as the Earth. Thus A B is the distance of the circle's center from the Earth, which is called the eccentricity;

Diagram. A single circle illustrating the simple solar anomaly by eccentricity. Labels:

- C (top of circle): the apogee, on the diameter; the point farthest from the Earth.
- A (center of the circle): the center of the eccentric circle.
- B (below A, on the diameter C E): the Earth.
- E (bottom of circle): the perigee, on the diameter; the point nearest the Earth.
- D (upper left of circle): the point on the descent from apogee where the maximum equation lies (subtractive).
- F (lower right of circle): the point on the ascent from perigee (additive).

Lines are drawn from B to D and from B to F (and the diameter C–A–B–E), forming the angles A D B and A F B that define the equation [*prosthaphaeresis*].

and for this reason the circle so drawn, C D E F, is also called eccentric; and C is the apogee, or the point most remote from the Earth, while E is the perigee, or the point nearest to it.

As for what concerns the appearances in the motions: since an angle subtended by longer sides is always smaller than one enclosed by shorter sides — the interior angle being greater than the exterior and opposite — the motion, uniform with respect to its proper center A, comes out non-uniform when we look at the Sun from the Earth at B: smaller about the apogee, but greater about

the perigee. At those two points, C and E, it differs not at all from the mean or simple motion. But at the points roughly intermediate the difference is greatest, which in the table is called the *prosthaphaeresis* [Greek: προσθαφαίρεσις (prosthaphairesis, "equation" — the additive-subtractive correction)]: on the descent from apogee at D it is subtractive; on the ascent from perigee at F it is additive. The former is defined in the present scheme by the angle A D B, the latter by the angle A F B.

By what means and on what principle these angles are to be investigated will be taught afterward. [Greek: διδομένοις (didomenois, "from the givens" — i.e., the data given)] It will be enough here to have, in a manner, demonstrated the theory in advance.

Demonstration of the double anomaly, displayed by the ancients in the remaining planets.

[Greek: Απόδειξις (apodeixis, "demonstration")]

The other essential inequality of a planet — namely in its own orb — requires a *prosthaphaeresis* roughly twice as great [Greek: προσθαφαίρεσιν (prosthaphairesin, "equation," accusative)] as the difference between apogee and perigee demands. For this reason, to the simple eccentricity demonstrated earlier, the ancients added an equant, on this principle: that the center of the equant orb should stand at an intermediate place between the center of the planet's orb and the center of the Earth. From this center, on a circle equal to the prior eccentric one, they supposed the planet's center to be carried, and here to describe unequal motions — but about the foreign center A, equal ones, namely the center on which equal arcs are completed in equal time.

So let A be the center of the orb D E; the Earth, however, at C, so that the whole eccentricity is A C; and with the point B conceived in the middle, and the orb F G described from it, A B will be the equant. According to the magnitude of this equant, the planet, carried around the equant orb F G unequally (as they supposed) — since its uniform motion regards only the center A, as was said above — would, at apogee, be nearer the Earth by reason of distance, as appears at D and F; but at perigee, more remote, by the interval E G. At the intermediate places, the angles B H C and B H A, added to one another, would compose the whole essential *prosthaphaeresis* [Greek: προσθαφαίρεσιν (prosthaphairesin, "equation," accusative)] — apart, that is, from the one which the planet admits in its accidental or annual epicycle.

And thus we have briefly shown the ancients' method of the equant, which Ptolemy throughout his *Almagest* in-

[Margin: *On the Revolutions* IV, chapters 2 and 5; and Book V, chapter 2. (Copernicus, *De Revolutionibus*)]

Diagram. Two concentric circles illustrating the double anomaly by eccentricity and equant. Along the vertical diameter, from top: **D** (top of outer circle), **F** (top of inner orb), **A** (center of the orb), **B** (the equant center, just below A), **C** (the Earth, below B), **E** (bottom of inner orb), **G** (bottom of outer circle). **I** is the planet's position on the upper left of the orb; **H** is at the lower right. Lines run from I to A/B and toward H, marking the angles B H C and B H A. [Margin label at left: *Lib. 4 Rev. c. 2 & 5 cap. 2*]

[which Ptolemy] had observed; and Copernicus takes it up again and refutes it, holding it to be absurd in geometry and optics for motions to be made uniform about a center not their own — as here about A, and not about B, the center of the planet's orb. [Margin: *Lib. 4 Revol., cap. 2 & 5; cap. 2.* — i.e. Copernicus, *De Revolutionibus* IV.2 & 5; IV.2.] For this reason, since the planets are subject to a twofold essential anomaly, he kept the simple eccentricity as one component and then artfully replaced the equant as well, by means of an epicycle set into the planet's orb, and there represented it. He did this so skillfully that, on such a supposition, he committed no offense against geometry, and yet introduced scarcely anything different from the φαίνόμενα [Greek: phainomena, "appearances" — the observed phenomena] to be gathered from the hypothesis of the ancients. This will become clear in the following diagram of Copernicus.

Diagram. Copernicus's epicyclic scheme for the double anomaly. A large circle (the planet's deferent orb) is centered on A, with a vertical diameter running from C at the top through A and L and B to E at the bottom, and a horizontal diameter from D at the left through A to F at the right. At each of the four cardinal points (C, D, E, F) a small circle — the Copernican epicycle — is drawn, with the planet's marked position on each. A dotted circle traces the planet's actual path G–H–I–K.

- C (top): apogee position of the epicycle's center.
- G (just inside C, on the epicycle at C): planet's position when the epicycle's center is at apogee C — it occupies the lowest part of the epicycle.
- A (center): center of the planet's orb.
- L (just below A on the vertical diameter): point from which, taking a third of A B, the planet's path circle G H I K could be described.

- B (below L): the equant center / Earth.
- D (left): the epicycle's center at the left quadrant; planet rises to H.
- H (on the epicycle at D): planet's position at the left quadrant.
- F (right): the epicycle's center at the right quadrant; planet shown at K.
- K (on the epicycle at F): planet's position at the right quadrant.
- E (bottom): perigee position of the epicycle's center; planet found at I.
- I (on the epicycle at E): planet's position when the epicycle's center is at perigee E.

With A established as the center of the planet's orb C D E F, the Copernican epicycle is described around those marked points as centers. Its radius, like C G, contains a third part of the eccentricity A B. In this epicycle the planet likewise moves in the order of the signs, with a revolution commensurable to that of the epicycle's center, by this rule: while the epicycle's center is at the apogee C, the planet occupies the lowest part of the epicycle, at G. But when the epicycle's center C has descended sideways through D, a quadrant of the eccentric, the planet rises in this epicycle to the point H. Further, as the epicycle's center descends fully to the perigee E, the planet is found at I. And finally, the epicycle's center, climbing the other semicircle of the orb to F, shows the planet at K.

Here one can see the device by which Copernicus, through an equivalence of appearances, had done away with that absurdity of Ptolemy's equant treated above. [Margin: *The invention of the epicycle in place of the equant is attributed to Arzachel [al-Zarqālī] the Spaniard.*] For when the center of the epicycle in which the planet is carried is at the apogee C, and the planet — as has been said — is at the perigee of the epicycle, the planet is brought nearer to the Earth by the whole diameter of the epicycle than it would be if the simple eccentricity alone were used. At the perigee, by contrast, the planet, lingering at I, is made more distant from the Earth B by the diameter of the epicycle. For by however much is taken for the radius of the epicycle, by just so much does the eccentricity A B decrease — which, at the mean elongations or greatest prosthaphaereses (equations) at D and F, returns to producing the full prosthaphaereses.

But to make some of this clearer by an example: let A C in the upper Ptolemaic figure be 8 parts. Of these, two being taken for the radius of the epicycle, there remain 6 parts, which constitute the eccentricity in the Copernican hypothesis. A third of these, that is 2 parts, is — as we said — the measure of the epicycle's radius C G; and on account of the nearer distance of A C to B, when just so many parts, namely 2, are subtracted from the eccentricity A C of 6 parts, there remain just 4 parts. That is, half a part for the whole difference between the apogee (or

perigee) and the mean elongation to be shown, as happened in the upper Ptolemaic hypothesis. Nor does it work out otherwise around the quadrants of the orb at D and F.

in use; where the angles A D B and D B H, combined, constitute the full essential subtractive prosthaphaeresis, just as A F B and F B K constitute the additive one.

Diagram. A repetition of the Copernican double-anomaly scheme, printed at the head of this page (the page is heavily show-through; the underlying figure is faint but its labels are legible). The same large orb centered on A, with the vertical diameter C–A–L–B–E and horizontal diameter D–A–F, the four cardinal epicycles, and the planet's marked positions.

- C (top): apogee.
- A (center): center of the orb.
- L (just below A): point from which the planet's path circle G H I K is described.
- B (below L): Earth / equant center.
- D (left), with H on its epicycle.
- F (right), with K on its epicycle.
- E (bottom), with I on its epicycle.

This periodic revolution of the epicycle and planet through the orb is a corollary in this Copernican reform — as if, taking a third part of A B, the circle of the planet's path were described from the point L, namely G H I K. So that it may appear here with how much artifice the Copernican epicycle has been substituted for the Ptolemaic equant: one that nowhere collides with geometry, as that other did, and yet represents the same $\varphi\alpha\iota\nu\acute{o}\mu\epsilon\nu\alpha$ [the phenomena] just as well, or better.

It must be observed here, however, that the phenomena do not everywhere approve a third part of the eccentricity A B as the radius of the epicycle to be set up — as will be shown below for Mars, Jupiter, and Saturn [a ♂ & ♄] in their proper places.

Secondly, it must also be pointed out that the same phenomena arise even if the epicycle's center is placed in the middle, namely at the location A, on this condition: that the center of the planet's orb revolves in such an epicycle, beginning from its least distance from the Earth or Sun, striving in the order opposite to the signs, and completing a double revolution relative to the planet in

its orb — as Copernicus himself shows for Venus and Mercury, and as we too, if it proves convenient, will demonstrate for the same planets below.

The third and last method — the truest one, as it seems to us — for saving both the simple and the double essential anomaly of the planets, by means of a homocentre-epicycle-on-epicycle [homocentre-epicycle], that is, by two small circles to be added to the planet's orb.

We could indeed rest content with the correction of the ancients' equant skillfully introduced by Copernicus (as we set out above), were it not that we had long since been persuaded that astronomy must be cultivated not upon just any hypotheses that exhibit the celestial phenomena, but rather upon those by which the whole of nature, generally taken, seems to be indicated in its own operations. We hold this so firmly that we no longer marvel at so great a variety of motions along with the common crowd, nor — like some others — do we either grow weary of it or rashly throw it back upon other causes (beyond the manifold and ordered connection and supposition of immaterial, or merely formal, circles), as Fracastoro and certain other talents do, who in their idle speculations often subordinate the celestial φαινόμενα [the phenomena]. Rather, in this continual and wonderful whirling of the planet about the heavens, we contemplate with the most pleasing reflection the internal causes — succeeding one another mutually in nature, proceeding in fairest order from an immovable and most powerful principle, and nowhere ceasing. For we admonished at the outset that the heavens are the written book of nature.

[the written book of nature], as we admonished from the beginning. Now we shall treat briefly of the equivalence for representing the appearances — both of the simple eccentricity in the Sun by means of one small circle on a homocentric orb, and of the eccentricity-with-equant of the ancients (or even Copernicus's eccentricity-with-epicycle) by means of two small circles on a homocentric orb. We begin with the simple anomaly of the planet, such as Ptolemy too demonstrated in the case of the Sun.

Let A be the center of the Sun's orb B C D E. The center of the epicycle being placed at those four locations, and the epicycle described from each, its radius is taken equal to the eccentricity A B of the first figure of this chapter. Since, then, the Sun, placed at the apogee and at the highest part F of the epicycle, is supposed to be carried by a motion contrary to, and nearly commensurable with, that of the epicycle's center through the orb B C, etc., it is easily understood that everything agrees with a hypothesis built upon simple eccentricity.

Diagram. The Sun's simple-anomaly scheme by one small circle. A large circle (the Sun's homocentric orb) centered on A, with a vertical diameter from F/B at the top through A to H/D at the bottom, and a horizontal diameter from G/C at the left through A to I/E at the right. A small epicycle is drawn at each of the four cardinal points, with the Sun's marked position on each. A dotted circle traces the eccentric path of the epicycle's center.

- F (top, on the epicycle at B): highest part of the epicycle — the Sun's position at apogee.
- B (top): apogee position of the epicycle's center.
- A (center): center of the Sun's orb.
- G (left, on the epicycle at C): Sun's position when the eccentric's center has descended to the left quadrant.
- C (left): the epicycle's center at the left quadrant.
- I (right, on the epicycle at E): Sun's position at the right quadrant.
- E (right): the epicycle's center at the right quadrant.
- H (bottom, on the epicycle at D): Sun's position at perigee.
- D (bottom): perigee position of the epicycle's center.

For when the center of the eccentric, descending from B into C, has completed a quadrant of the Sun's orb, the Sun itself is, by hypothesis, represented in the epicycle at G. From there, to an observer looking at it from the Earth A, it appears to differ from the mean motion by the measure of the angle C A G, an angle which measures a nearly maximum subtractive prosthaphaeresis.

Further, as the orb's center descends fully from C into D, the Sun is represented at H, in its very perigee — nearer to the Earth A by just as much as it was set higher at the apogee from the start. This too is what the first simple eccentric above required. Finally, the epicycle's center D, returning and climbing through half the orb, and arriving at last at E, places the Sun back at I on its epicycle, again by hypothesis, and makes the angle E A I of the maximum additive prosthaphaeresis — and so on consequently through the whole circuit of the orb. This first supposition fits the simple anomaly so well that the same appearances occur on both sides, given an equal assumed eccentricity and epicycle diameter, and an equal distance from the apogee everywhere. And thus the equivalence for saving the simple anomaly has been made manifest, in the first place, in this hypothesis. There now follows the hypothesis of the double or composite anomaly, to be accounted for by a double or composite small circle.

Let there be, further, the composite anomaly of the planet's orb — such as Ptolemy set out by means of the equant besides, and Copernicus by an eccentric-epicycle [eccentrepicyclus], but which we are to show by a homocentre-epicycle-on-epicycle [homocentrepicycepicyclus]. So, with A placed in the same way as the center of the Earth or Sun, and the planet's orb B C D E described, let the center of the first small circle be supposed upon these marked points, so that the planet's varied position may be represented in turn. Let the line B D be the line of apogee and perigee, B the apogee location, and D the perigee. Let everything else, moreover, be as in the diagram just above.

Diagram. The planet's composite-anomaly scheme by two nested small circles. A large orb centered on A, with a vertical diameter from B (top) through A to D (bottom), and a horizontal diameter from C (left) through A to E (right). At each of the four cardinal points (B, C, D, E) a first (larger) small circle is drawn, and on each of these a second (smaller) circle, with the planet's marked position. A dotted circle traces the planet's actual path through L–M–N–K.

- F (top, outermost): point on the first epicycle at B, above the second small circle.
- K (top, on the second small circle): planet's starting position, at the apogee — the lowest position of its little circle.
- B (top): apogee; center of the first small circle here.
- L (left, on the second small circle): planet's position when the first epicycle's center is at C — maximum subtractive prosthaphaeresis, angle C A L.
- G (left, second small circle's center at C).
- C (left): the first epicycle's center at the left quadrant.
- N (right, on the second small circle): planet's position when the first epicycle's center is at E — additive prosthaphaeresis, angle E A N.
- I (right, second small circle's center at E).
- E (right): the first epicycle's center at the right quadrant.
- H (bottom, first small circle's center at D).
- M (bottom, on the second small circle): planet's apparent position at perigee, having completed a full revolution in the second little circle from K.
- D (bottom): perigee; the first epicycle's center here.
- A (center): center of the Earth's (or Sun's) orb.

To this epicycle (or rather, to a point on its circumference, which a line drawn from its center always parallel to the line B D touches) let another small one be added, whose radius is about a third part of the former, likewise represented at four locations — namely upon the centers F, G, H, I. In its circumference the planet is carried with a motion contrary to that of the center of its little orb — that is, in the order of the signs — and with a motion exactly commensurable, in a periodic revolution double that of the center of its epicycle F, beginning from the lowest position of its epicycle, at K in the apogee (except in the hypothesis of Mercury, on which in its place).

So, when the center of the first epicycle of the planet is at C, the planet will be at L, marking out a nearly maximum subtractive prosthaphaeresis through the angle C A L, by which the planet's true motion at L falls short of the mean at C. But at the perigee, at M, the apparent planet had completed a whole revolution in the small second little circle from K, by the time the center of the lesser epicycle H had completed at least half a period together with the center of the greater one, D. Finally, as the center of the first epicycle climbs to E, the center of the latter epicycle will be I, and the planet — by the law of its revolution — at N; and thus the whole angle E A N yields to the additive prosthaphaeresis, by which the planet's true motion again surpasses the mean. And so, little by little, until the planet has been restored to the apogee and to its own place K, from which it had descended. By this rule of revolution, the planet's conformity in its motion, by means of two small circles (that is, in the homocentre-epicycle-on-epicycle) with the equant above, is plainly seen.

is seen, while the apogee is at K and the perigee at M. On both sides, in proportion to the radius of the last small circle, the planet shows its positions, so that, by that measure, at apogee it lies nearer to the Earth (or Sun) at A, while at perigee it lies farther off, and around the midpoints of the prosthaphaeresis it increases the distance on both sides by that same amount. As for the variation [Greek: μεταβολ[?]] (metabol[?], "change") of this configuration [Greek: σύστασις (systasis, "constitution," here "configuration")], of whatever kind, we will demonstrate it below in the theory of the Moon.

That the planet is in truth subject to such a threefold ordered revolution we will, at some point — God granting life and strength for so long — make manifest by a more elevated inquiry into nature. For now it suffices to have laid out in this chapter, in a general way, nearly all the methods by which the phenomena of the planets, with respect to their orbits, have either been shown by the foremost astronomers up to now, or will hereafter be shown by us.

WHAT IS TO BE TREATED SPECIFICALLY IN THIS BOOK

What follows are the matters to be treated in turn in this book concerning the motions of the stars, the Sun, and the Moon, in which the apparatus [Greek: παρασκευή (paraskeuē, "preparation," "equipment")] for restoring each of them individually is set out in the following commentaries.

COMMENTARY

On the examination and restitution of the observations of the Sun, the Moon, and the orb of the fixed stars, toward the permanent restoration of the position of each of them.

I.

ON THE SUN

The phenomena of the planets can be saved in various ways — but most of all by ways consonant with nature, as shown above. Now the proper order requires that we pursue the motion of each body in particular, relying on the observations and testimonies that can be had, and carefully comparing among themselves those that seem to carry some weight. For these, as we devote our work generously to the restoration of astronomy, we use as external first principles. And since what remains of this first book must be spent on those phenomena that regard the primary point — that is, the center of the Earth — we will therefore, before all else, give here the rationale for the order to be kept in these matters. For Copernicus differs from Ptolemy on this very point: he held that one should begin not, as Ptolemy did, from the Sun and Moon, but from the fixed stars and the precession of the equinoxes. Yet when the matter is carefully examined and weighed according to the nature of logic, we assign to Ptolemy the rationale of discovery, and to Copernicus that of the order of presentation.

For just as the anticipation of the equinoxes — both the simple kind and a certain inequality joined to it — is perceived not only in the apparent advance of the stars, but also insinuates itself into the constitution [Greek: συστάς(?) (systas[?], "constitution," here "configuration")] of the Sun, the Moon, and the rest of the planets: so, without a prior knowledge of the motions of the Sun and Moon, it is by no means possible to examine and establish the foundation in the positions of

the fixed stars. The courses of the luminaries too — and especially the annual quantity of the Sun — are reciprocally argued from the stars. For this reason, so that some method here may be better suited to the minds of learners, we have judged it advisable to set out beforehand, in place of a commentary, nearly everything we intend in this book concerning the examination and restitution of the observations from which the motions at various times will be established; and afterward, upon these, to build up both the mean motions of the stars and luminaries and their permanent hypotheses, and to treat them in order.

[Margin: *Pliny, bk. 2, ch. 6.*]

We begin here from the Sun — which the Hebrews call *chammah*, from "heat" — because, without a prior knowledge of its course, neither the Moon, nor the stars, nor the rest of the things to be taught in this or the following book, can rightly be examined. For the Sun itself, to use Pliny's words, "is borne as the middle one of the planets, of vast magnitude and power: he is the ruler not only of the seasons and of the lands, but also of the stars themselves and of the heaven," and so on.

How truly these things were stated by Pliny [Pliny the Elder] concerning this most resplendent and most excellent radiance of the heaven, popular experience partly proves, and a truer hypothesis, set forth, partly displays — to be shown more specifically in what follows, God willing. But in truth, although the motion of the Sun alone would be the clearest argument for the perennial constancy of the celestial appearances [Greek: φαινόμενα (*phainomena*, "phenomena," "appearances")], yet — if we must trust the observations and, as it were, the doctrines of each of the astronomers — in no other matter do I find, with the passage of time, a more unsightly variability. And this is so not only concerning the measure of the Sun's annual revolution, but also concerning the change of its eccentricity (as they call it) and of its apogee — so much so that the things determined by Ptolemy concerning the Sun, which for nearly 300 years between Hipparchus and Ptolemy had stood without any change in the same constitution [Greek: συστάς[?] τῆς ὑποθέσεως (*systas[?] tēs hypotheseōs*, "constitution of the hypothesis")], and were corroborated by observations of whatever sort, afterward, rolled down to the succeeding ages, appear to have become subject to change — and indeed to a disordered change. Weighing the causes of this matter more carefully, I see that none of them reside in the supremely simple motion of the divine star itself, but that the whole blame must rightly be transferred onto the astronomers, whose records concerning its motion, derived (as they would have it) from the heavens in various ages, have been handed down to posterity — records in which, for causes almost manifest, error has been committed back and

forth, more or less, until in our age it was granted by God to the Atlas, Tycho Brahe, to begin restoring astronomy more accurately. Of his observations [Greek: τήρησις (tērēsis, "watching," "observation")] and celestial observations alone — both on account of the proper and well-fitted apparatus of his instruments and on account of his skill in observing, which excludes every sensible error — I can perhaps be the best witness, having for a continuous decade been a follower of the Brahean astronomy. Since this had by no means been achieved by his predecessors with equal diligence (as we know by experience), it is therefore no wonder that error became, with the passage of time, exceedingly fertile — both in other matters and most of all in the case of the Sun.

For although, around the greatest opportunity for observing the Sun's position from the heavens — which is nearest the equinoxes, when through the declination observed at the meridian, especially alterable near those points, its place in longitude can best be ascertained — a slight error, of only a few minutes, which may have been committed for one or several causes (whether on account of the insufficiency of the instrument in size or construction; or on account of an elevation of the place's pole supposed less correctly; or on account of refraction and parallax included in the observation), the same error extends in longitude to nearly triple: what then must be thought to happen at the intermediate positions, where there is always a greater proclivity to deviate, because of the daily change of the Sun's declination gradually diminished, until it reaches the solstitial points — where, by no device of this practical art, can the Sun's place be had from observations, the declination itself remaining nearly invariable over many days? Nor indeed is there any reason for anyone to trust that the solstitial points have hitherto been determined certainly enough by the ancients through the aid of eclipses, since both the simple course of the Sun and the true hypothesis of the Moon — never, in fact, rightly established by those same men — are required for this practical procedure; besides the precise designation of time in eclipses and its equation (rather to be wholly omitted than illegitimately introduced, as was once done), which in the ancients is wavering, just as nearly everything else they accomplish by way of accuracy [Greek: ἀκρίβεια (akribēia, "exactness," "precision")]. If, therefore, anyone less instructed by experience of astronomical observations should presume to charge our concern in inquiring into those ancient prelates — Hipparchus especially, and Ptolemy — with rashness, perhaps, and reckless audacity, and to throw at us that trite saying: [Greek: κακόν κινῆσαι τὸ γ' εἰς τὸ κείμενον (kakon kinēsai to g' eis to keimenon, "it is a bad thing to disturb what lies well")] — let him trust that to all the causes of error we have recounted, which are sufficient and which can be gathered well enough from one chapter or so of Ptolemy, this one too has been added, which is sufficiently detected there by

shrewd conjecture: namely, the opinion concerning the measure of the tropical solar year, perversely distorted in order to determine the cycles and conjunctions [Greek: συζυγία (syzygia, "conjunction," "syzygy")] of the luminaries absolutely.

[Margin: *Bk. 3, Almagest, ch. 2.*]

For since all before Hipparchus had in use a quantity of the solar year — the one familiar to us, namely of 365 $\frac{1}{4}$ days — taken proportionally, as appears in the institution of the Olympiad (to be begun afresh every fourth year around the rising of the Dog Star [Canicula]), and likewise the other seasons of the year were...

determined from the rising and setting of the fixed stars — a custom of the most ancient Hesiod, and afterward of the Greeks as well as the Romans. So while the observations of the Sun, which without doubt existed at Babylon and in Egypt during the Assyrian monarchy, all perished, a certain Meton of Athens first — who flourished 430 years before the birth of Christ — using likewise the common quantity of the year of 365 $\frac{1}{4}$ days, computed the simple courses of the luminaries not so much to the equinoctial and solstitial points as to the new moons within his own period [Greek: ἐννεαδεκατερίδει (enneadekateridei, "nineteen-year cycle")], that is, the interval of 19 years: with a notable error, certainly, which with the passage of time, down to Hipparchus, was detected by him as 5 days. But up to the correction of Calippus — instituted 6 years before the death of Alexander the Great — within his four periods, which each contained 76 years, an anticipation of one day in the new moons was being noticed; that is, in a span of 304, or in round numbers (as Scaliger [Joseph Scaliger] says) 300, years.

[Margin: *Bk. 3, Almagest, ch. 2.*]

This is shown by these following words of Hipparchus, ascribed to him by Ptolemy, where he says: in the book *On Intercalary Months and Days*, after Hipparchus had stated that, according to Meton and Euctemon, the length of the year was 365 days, he pursues only the fourth part [the quarter-day] in these words: "We, indeed, find that the same number of months is contained in 19 solar years as those men found; but we find that the year falls short by a 300th part of a day less than a quarter." Therefore, according to Meton, in 300 years five days are wanting; but according to Calippus, only one. Scaliger interprets these words of Hipparchus as concerning only the anticipation of the lunar course beyond the cycle of Meton.

[Margin: *Joseph Scaliger, de emendat. Temp., p. 257.*]

Copernicus [Nicolaus Copernicus], however, with a teaching better suited to the young, in these words:

[Margin: *Bk. 4, Almagest, ch. 4. On the revolution.*]

"From the most ancient men, therefore, by whom this matter was a concern, that it might be handed down to posterity in numbers, Meton the Athenian was found, who flourished in the [37th] Olympiad. He reported that in 19 solar years 235 months are completed; whence that great year was named *enneadekateris*, that is, the nineteen-year Metonic. This number so pleased that at Athens and in other more notable cities it was posted up in the public square — and it is even received in common use up to the present — because by it they reckon that the beginnings and ends of the months are established in a fixed order; and the solar year too, of 365 days with a quarter, commensurable with their months. Hence that Calippic period of 76 years, in which one day is intercalated ten and nine times [i.e., nineteen times]; and they named it the Calippic year. But the shrewdness of Hipparchus found that in 304 years a whole day increases, and that this is verified only when the solar year has been less by a 300th part of a day. Thus too by some this great year was named the year of Hipparchus, in which 760 months would be completed." This with the rougher Minerva, as he says — Copernicus.

But we will here too subjoin this simple paraphrase, perhaps more congruent with the words of Ptolemy: Meton, in the 304 years that intervened roughly between himself and Hipparchus, committed an error of 5 days in his nineteen-year cycle — partly because he commonly retained the solar year of $365\frac{1}{4}$ days, partly because he did not apply the periodic syzygy of the Moon with the Sun precisely enough. Calippus, noticing this in a certain eclipse of the Moon occurring six years before the death of Alexander the Great, so corrected Meton that for each of his four nineteen-year revolutions one whole day should be subtracted: whence he made for himself, for that day, a period of 76 years, which is everywhere employed by Ptolemy. But since Calippus himself also used a solar year of $365\frac{1}{4}$ days, and likewise erred somewhat in the course of the Moon, therefore Hipparchus, afterward correcting Calippus, deprived him — in his four periods, that is, in a span of 304 years — of one whole day, and consequently deprived Meton of 5 days. And this one-day error committed by Calippus, Hipparchus threw entirely onto the solar year, subtracting, namely, its measure over 304 years, and in round numbers one day over 300 years. Hence it came about that the measure of the tropical year, $365\frac{1}{4}$ days, was diminished by Hipparchus by a 300th part of a day — that is, 4 minutes 48 seconds; so that the

time was reckoned as 365 days 5 hours 55 minutes 12 seconds. And so, while Hipparchus, equally with his predecessors, strove to catch two hares at one bound — that is, both to reduce the new moons within fixed spans of years, and to define each interval of time from the simple motions of the luminaries, and at the same time to measure out the annual revolution, although the one depends in no way on the other — he acquired the measure of the solar year not from the heaven or the Sun itself, as it seems, but from certain syzygies of the luminaries, by an unjust and indeed superabundant quantity. Ptolemy chose to imitate this error of Hipparchus, unhappily, rather than to rescind his master's dictum — so much did this lunar cycle too please him here. But the other testimonies that Ptolemy brings forward from Hipparchus on behalf of this assertion prove nothing, since the tropical or solstitial points were, to the ancients, as we have...

[Margin: *Bk. 3, Almagest, ch. 2.*]

before proved, unobservable; whereas the equinoxes resisted this very opinion of Hipparchus splendidly, as we will presently show from his own observations. And we have drawn these things out at greater length so that our countrymen might at last recognize that the ancient astronomers — Hipparchus first of all, and Ptolemy — were, for manifest enough causes, exposed to errors in determining the course of the Sun.

[Margin: *Bk. 3, Almagest, ch. 2.*]

Now we will undertake to discuss the equinox observations themselves — first Hipparchus's, then Ptolemy's, as they are found noted by the latter — and, as far as is possible, to restore them; set before the eyes, indeed, in the following synopsis, where the epoch too has been derived from the death of Alexander the Great, both backward and forward, and from Christ the Savior, so that the comparison may become more evident.

Autumnal equinoxes observed long ago by Hipparchus under the meridian of Alexandria in Egypt.

[Margin: *Copernicus has the years 177.*]

		D				M	
		a				o	
		y				n	
		Moment				D	
[After death of		Mon		[Before birth of		t	
Alexander] Year		th		Christ] Year		h	
						y	
						Moment	
163	Mes	3	Setting of the	162	S	2	6 [hr]

[After death of Alexander] Year	Mon th	D a y	Moment	[Before birth of Christ] Year	M o n t h	D a y	Moment
	ori	0	Sun		e	7	afternoon
166	Inter cal.	1	Morning	159	p t. S e p t.	2 6 [hr] 7 forenoon	
167	Inter cal.	1	At midday	158	S e p t.	2 0 At 7 midday	
178	Inter cal.	3	Last [hr] of midnight	147	S e p t.	2 12 6 afternoon	
179	Inter cal.	4	Morning	146	S e p t.	2 6 [hr] 7 forenoon	
182	Inter cal.	4	Evening	143	S e p t.	2 6 [hr] 6 afternoon	

Vernal equinoxes observed by Hipparchus at Alexandria.

[After death of Alexander] Year	Mon th	D a y	Moment	[Before birth of Christ] Year	M o n t h	D a y	D h. Moment
178	Mec hir	2 7	In the morning, hour 5, and	146	M a	2 4	11 foreno

[After death of Alexander] Year	Month	Day	Moment	[Before birth of Christ] Year	Month		Day	Hour
					Month	Day		
			after hour 11		Month	Day	Hour	Minute
189	March	2	Last [hr] of 9 midnight	135	Month	Day	Hour	Minute
196	Pha- men- oth	1	At the setting of the Sun	128	Month	Day	Hour	Minute

Examination of the Equinoxes of Hipparchus.

[Margin: *Bk. 1, ch. 2. / Bk. 3, ch. 2.*]

I. When I carefully consider the equatorial armillae [armillas aequatorias] by which these equinoxes were acquired, from the description and account of Ptolemy, I do not at all wonder that Hipparchus omitted the due precision in the assignment of his observations, and at least bound his equinoxes to the cardinal quarter-points of the natural day — although no one skilled in celestial matters would deny that, beyond and short of these, the equinoxes for the most part fall into the intermediate times.

II. Nor, moreover, were most of them taken from the heavens by observation at the times assigned — especially the autumnal ones — as is manifestly shown by the fact that they are ascribed to sunrise and sunset, while most of them carry no consistent index of refraction, although in fact the refraction of the Sun at the horizon of Alexandria retarded the autumnal equinox, but advanced the vernal one, by at least half a day — even with the density of the vapors around the horizon (which are the causes of refraction) much attenuated there, and the parallax of the Sun excluded.

III. The excessive anticipation that is here discerned in time, in the autumnal cases — inasmuch as, in a span of at least 19 years within which these were observed, it amounts to a whole day — shows the measure of the Hipparchian

solar year to be 365 days 6 hours less a 300th part of a day, or 365 days 5 hours, ... minutes...

[the year]—12 seconds—it completely overturns, since from these autumnal observations the year emerges as no more than 365 days 5 hours 4 minutes, while from the vernal observations it comes out as 365 days 5 hours 43 minutes, so that the limited value extends only to 365 days 5 hours 24 minutes; the year thus falling short annually by 31 minutes. From this enormous anticipation, something may rightly be remitted if one wishes to set up a proper comparison with Ptolemy for the measure of the tropical year. For the present, however, in reasoning out the apogee and eccentricity of the Sun for the age of Hipparchus from his own observations, it seems most advisable to select, for comparison with each other, the third autumnal and the first vernal equinox out of the whole set — both of which occurred around midday, where neither the defect of the armillae could be so great, nor any impediment of refraction in the Sun could intervene. There is the further point that this vernal equinox, in fact observed around sunrise (and afterwards near midday, or an hour before, observed), offers manifest testimony of the Sun's refraction at the horizon. This, on Ptolemy's account (as he says), is indeed consistent with experience, yet contrary to his own assertion about Hipparchus, when he says: "When we observed carefully, the concave surfaces of the armillae were sometimes illuminated twice at the same equinoxes; but Hipparchus thought nothing of this sort worth reporting." [Margin: Lib. 3, cap. 2.]

IV. Before we compare Hipparchus's selected equinoxes with one another, we must give thought to the parallax — slight though it is — that attaches to the Sun's altitude. Since we may easily gather that the very elevation of the pole at Alexandria, assigned by Ptolemy as 30 degrees 58 minutes, is inherent in it (from what Ptolemy reports about the maximum obliquity of the ecliptic from observation), here a mutual elimination of the solar parallax takes place — namely, the elevation of the pole being diminished by exactly as much as the Sun's parallax near the equinoctial circle at Alexandria could amount to, that is, $1\frac{1}{4}$ minutes at an altitude of 59 degrees. And so much for this solar parallax both with Hipparchus and with Ptolemy. [Margin: Lib. 1, cap. 13. — Lib. 1, cap. 2.]

Now we come to the comparison. Between the third autumnal equinox — which, as noted, fell in the 167th year from the death of Alexander, and at midday on the first of the intercalary days among the Egyptians — and the first vernal equinox of this same Hipparchus, which is found to have occurred in the 178th year from Alexander's death, in the Egyptian month Mechir, on the 27th, one hour before midday, there are found 11 Egyptian years and 181 days, falling short by just one

hour. Now, so long as we keep to the values for the quantity of the tropical year set as limits above from Hipparchus's observations, the result agrees proportionally: from 5 hours 24 minutes beyond 365 days over 11 years, there come, namely, 2 days 2 hours 11⅓ — which, if subtracted from the number of days 180²³/₂₄, leave the winter interval, that is, between the autumnal and the vernal equinox, of 178 days 11 hours 25 minutes (which Ptolemy puts at just 178¼ days). When this in turn is subtracted from the Hipparchian measure of the year, 365 days 5 hours 24 minutes, there remains the summer interval of time, between the vernal and the autumnal equinox, of 186 days 17 hours 59 minutes — placed by Ptolemy at 187 days 0 hours 0 minutes.

And all this we have carried out precisely according to Hipparchus's observations, not so as to demonstrate the very truth in them — for neither is the measure of the tropical year that which emerges here, nor as yet the summer interval of time between the equinoxes — but so that it might become plain how much error is inherent even in the very observations of the ancients, lest it should seem proper that we be so enslaved to them, and so bound, that we ought not, once the just balance of comparison is applied, to change something in them. Now we shall pass to discussing the three Ptolemaic equinoxes, which are reported to have been observed at Alexandria in Egypt likewise.

Ptolemy's autumnal equinoxes.

After the death of Alexander — Year	M D			After the birth of Christ — Year	M D		
	on	a	Hour,		on	a	Hour,
	th	y	moment		th	y	moment
456	At	7	2	132	Se	2	2
	hy		afternoo		pt.	5	afternoo
	r		n				n
463	At	9	7	139	Se	2	7
	hy		forenoo		pt.	6	forenoo
	r		n				n

Ptolemy's vernal equinox.

After the death of Alexander — Year	D			After the birth of Christ — Year	M D		
	Mo	a	Hour,		on	a	Hour,
	nth	y	moment		th	y	moment
463	Pac	7	1	140	M	2	1
	ho		afternoo		ar	2	afternoo
	n		n		t.		n

Examination and restoration of Ptolemy's equinoxes.

What Ptolemy's intention was concerning the measure of the tropical year, we have set out above. And unless conjecture badly deceives me, he observed each autumnal equinox right at the threshold of the horizon — granted there were others (as no doubt there were), out of the number of those which he had taken twice in a day with the instrument; but in both cases he had settled for the horizontal observations, because this served his purpose remarkably well. And indeed it is worth noting that Ptolemy in these observations so accommodated himself to Hipparchus, and to the established framework both of the measure of the tropical year and of an unchanging eccentricity in the Sun, that for this very reason he did not record his observations at the precise turning-points of the days, but advanced them a little, to the hour 1 and so forth — so that you might say he here satisfied the Ptolemaic reckoning rather than the heavens.

But lest certain astronomers, to whom I write this, take ill our liberty in inquiring into Ptolemy — persuaded as they are by his ancient and excessive authority — let them, I beg, turn back to what he himself elsewhere reports about the parallax of the Moon observed by himself, and let them carefully compare it with our restoration, which agrees most precisely, so far as I know, with the lunar course and distance, brought into line with the heavenly norm. They will at last perceive that Ptolemy, in that passage (to pass over others this time), reported the Moon's parallax as half a degree, and more, beyond the true value derived from his own observation — for no other cause, as I reckon, than that he was foisting on posterity, as legitimate, the lunar hypothesis he had set up beforehand for himself (or, if you prefer, accepted from his predecessors and once approved by his own calculation). But that here he was of the same mind, and rested content with those equinoctial observations which served his purpose, while the rest — which it is highly likely he made many of — were entirely suppressed: what scruple, I ask, will there be in suspecting this? [Margin: Lib. 5, cap. 13.]

Now, leaving aside the earlier autumnal one — which, though it is recorded at the second hour afternoon, I nonetheless have no doubt was observed at the horizon just the same, and afterwards reduced to the appointed time (for nothing appears that, necessarily implicating the following one with refractions, would free this one from them) — we shall take up the other one, through which Ptolemy set up his comparison.

Since this equinox is found to have been detected with the Sun standing as near as possible right at the horizon — whether by the armillae, or rather from the angle at rising — and afterwards reduced to the hour from sunrise, when

Ptolemy reports that it occurred: for that reason there had there gathered, for the Sun, a refraction in the vertical circle of at least 32 minutes, which in longitude leave nearly as much at the horizon of Alexandria; and in time, 13 hours [?], by which... the time of observation, on account of the implication of refraction... [Margin: See 1 Progymnasmata, T. B., page 144, concerning refraction at Alexandria.] [Margin:...for the Alexandrians.] According to the autumnal Ptolemaic equinox, freed from refraction, at Alexandria in Egypt, in the 463rd year from the death of Alexander the Great, in the month Athyr, on the 8th [recte: 9th, per the table above — but the text here reads "die 8"], at 6 hours after midday; but if the very next vernal equinox of Ptolemy (when it is noted to have occurred at midday, by which it scarcely happened one hour later than in his own opinion) is established, there is gathered between the two an interval of 178 days 18 hours — which exceeds the Hipparchian one compared above, 178 days 11 hours 25 minutes, by 6 hours 35 minutes of time. [Margin: Li. 3, Almag., cap. 4.] Ptolemy, however — taking no account of refraction, on Hipparchus's authority and his own — supposed the winter interval of the equinoxes to be 178 days 6 hours, the summer one 187 days 0 hours, and finally the vernal equinox to the summer solstice 94 days 12 hours.

But in truth, since it in no way befits us to neglect anything that can anywhere be gained by us toward the legitimate restoration of the Sun and the securing of its conformity, I will accordingly transcribe here in full the passage of Pliny about the interval of time in which, in his preceding age, the Sun annually entered the cardinal signs of the zodiac. And although it is destitute of the precise reckoning [Greek: ἀκρίβεια (akribēia, "exactness")] of observation, and of the moments, yet since Pliny appeals to Caesar, I conclude by no doubtful conjecture that these intervals were established by the work of Sosigenes, that most excellent mathematician, whom Julius Caesar brought with him from Egypt to Rome, around the year 47 before the birth of Christ, for undertaking the establishment of his calendar — the one we use at present. [Margin: Pliny, lib. 18, cap. 25.]

Pliny's passage runs thus: "The hinge of the seasons consists of a fourfold division of the year, by the increase of light. It increases from the winter solstice, and is made equal to the nights at the vernal equinox, after 90 days 3 hours. Then daylight prevails up to the summer solstice (the summer one, that is, as regards light) by 93 days 12 hours; up to the autumn equinox. And then, with day made equal, it proceeds from there to the winter solstice by 89 days 3 hours." From this Plinian reckoning are gathered the things I now subjoin.

Diagram. A bracketed schematic at the top of the page tabulating the intervals between the Sun's entries into the four cardinal (equinoctial and solstitial) signs, as derived from the Plinian/Sosigenes reckoning. On the left, the entry-point "A" governs a brace listing four zodiac signs; this maps by "ad" ("to") onto a second brace giving the interval to the next cardinal sign in days and hours; and an "Ergo A ... ad ..." brace then sums the seasonal half-year intervals.

- A {♈ Aries / ♋ Cancer / ♎ Libra / ♏ Capricorn} **ad** ("to") {♋ Cancer: dies 93 hor. 12 (93 days 12 hours) / ♎ Libra: dies 92 hor. 12 (92 days 12 hours) / ♏ Capricorn: dies 89 hor. 3 (89 days 3 hours) / ♈ Aries: dies 90 hor. 3 (90 days 3 hours)}
- **Ergo** ("Therefore") A {♎ Libra / ♈ Aries} **ad** ("to") {♈ Aries: di. 179 h. 6 (179 days 6 hours) / ♎ Libra: di. 186 h. 0 (186 days 0 hours)}

And here we find a Sosigenian determination of the interval of time between the two equinoxes far different from the others (a Sosigenes who nonetheless flourished at roughly the midpoint of time between Hipparchus and Ptolemy) — so much so that a discrepancy of a whole day intervenes among these very observations of the ancients, not [yet] restored. And although I am unwilling to affirm that the observations of Sosigenes (granting they are observations, and not rather, as some hold, imitations of Eudoxus) — which were perhaps made by the Sun's rays and the shadow of a gnomon, or rather acquired by the astronomical radius, whose use belonged to the age of Julius Caesar — deserve to merit a certainty equal to that of Hipparchus's and Ptolemy's observations: nonetheless there is no doubt that the errors of Hipparchus and of the earlier Egyptians in the Sun were in some way both known and suspect to Sosigenes and to Caesar himself. This was the reason why they used other intervals — which emerged from their own observations — for the times of restoring their calendar; intervals which were afterwards sketched by others (Ovid especially, in the *Fasti*, after the manner of the poets), but here are asserted and precisely recounted by Pliny.

Nor is it unusual that so great a difference in time should have arisen, when between Nicolaus Copernicus and Tycho Brahe, over a span of only 80 years, the dissonance in the Sun's motion — with respect to its entry into the equinoctial and solstitial points — extends to ½ a whole natural day; the chief blame for which is the Sun's refraction, omitted by Copernicus, as will be shown later.

Furthermore, it can least of all disturb us that Pliny at once subjoins in this manner: "The hours now, in every increase, are equinoctial, not those of each particular day; and all these differences occur in the eighth parts of the signs: the winter solstice in Capricorn, from the 8th before the Kalends of January, about..."

and so on. For it seems to me that Pliny here speaks rather of the effects of these annual seasons, which mostly come on within the eighth parts of the equinoctial and solstitial signs. Nor am I unaware that today some interpret that statement of Pliny about the Sun's motion itself; but it was without doubt introduced by the custom of the Greeks, who, about eight days after the Sun's entry into the beginning of Cancer (the summer solstice), made it famous by the Olympic games — to which perhaps Pliny alludes. But when here he makes mention of the winter solstice falling around the 8th before the Kalends of January — which answers to the 25th of December, on which, from the midnight beginning it, the birthday of Julius Caesar (which we in the Church have happily exchanged for the nativity of Christ the Savior) was of old celebrated at the very winter solstice — he cuts off the opinion of these men about the eighth parts of the signs, and brings it back to the beginnings of the signs. And indeed it could happen that this error crept into the transcription of Pliny (which is frequent among copyists), when "parts of the signs" were put for "parts" or "days" of the Kalends in the months. However that may be, provided we are willing to follow the Plinian text, it will make little difference here toward varying the determined measures of the intervals.

Therefore this discrepancy which is asserted from Pliny — against Hipparchus and Ptolemy, both in the summer and winter measure of the Sun's turning between either equinox, and also between the vernal equinox and the summer solstice (which we have proved before could in no way be precisely observed from the heavens by Meton, Aristyllus, Aristarchus, Archimedes, Hipparchus, or even Ptolemy himself) — proves that the just limitation we applied above for the Sun's refraction was justified, while we rest upon a mean winter interval of the equinoxes for Hipparchus and Ptolemy of 178 days 6 hours, and of Sosigenes (for so it may now be called) of 179 days 6 hours.

Since indeed from the observations — especially Ptolemy's, restored above — the same result emerged, for that reason we shall retain it henceforth for the age of Hipparchus. With these matters thus settled, we must first pass to extracting and ascertaining the measure of the tropical year between Hipparchus and Ptolemy, in a certain way, from their restored observations, before we treat of the Sun's eccentricity as it fits the age of Hipparchus.

Now since the third autumnal equinox of Hipparchus (as noted above) fell in the year running from the death of Alexander 167, and at the midday of the first of the intercalary days, while Ptolemy's autumnal equinox — with the Sun freed from refraction — fell in the year of the same...

[Margin: Lib. 3, cap. 2. — Lib. 4, cap. 9.]

Alexander 163 [recte: 463], in the Egyptian month Athyr, on the 4th day, at the hour 6 after midday — we found that it in fact occurred; whence between the two there are gathered 296 Egyptian years, 72 days, 6 hours. Now, by the addition of a quarter [-day] to the measure of the common year of 365 days, the surplus comes to 74 days exactly, which emerge in the quotient when the number of years proposed, 296, is divided by 4. The difference, therefore — 42 [hours? days?] — divided by 296 days, gives the annual anticipation from the quarter [-day] as 8 minutes $30\frac{4}{5}$ seconds; which, if you subtract it from the measure of the Julian year, $365\frac{1}{4}$ days, leaves as remainder the measure of the solar year between Hipparchus and Ptolemy: 365 days 5 hours 51 minutes $29\frac{1}{3}$ seconds. But that from the comparison of an equal pair of vernal equinoxes — the two of Hipparchus and of Ptolemy — a somewhat greater measure of the tropical year should come out, could have arisen from the excessive anticipation (discussed above) of Hipparchus's equinoxes; and also because, with the Sun's refraction still included in the first vernal equinox of his that we are using, the same value is drawn out still further by several hours.

Indeed, weighing each item carefully, I find the quantity of the solar year, across those three centuries between Hipparchus and Ptolemy, to be not greater — perhaps even a little smaller — as will be confirmed further below, God willing, by a more accurate examination and by the apparent advance of the fixed stars; although Ptolemy did not hesitate to extend it to 365 days 5 hours 55 minutes 12 seconds, chiefly, as it appears, for the cause indicated above. Nor indeed do I doubt that, if the same Ptolemy survived now, and not only had knowledge of the Sun's refraction, but also of the measure of the year compressed down to our own time, he would willingly have retracted his own — inasmuch as he himself even appended his own censure on this matter to us, when he says thus: "The truth which can be had from the whole perpetuity of time, or from a manifold time of observations, we think should be left to others." And elsewhere, concerning other matters, he speaks frankly in this way: "We have changed several things not exactly perceived, since truer observations afterwards came down to us; for it befits all who, impelled by love of the truth, undertake these speculations, to use a surer and newer way not only toward the correction of the ancients, but also of their own work; nor to think it shameful to themselves if they are recalled to more exact results not only by themselves but also by others — especially since this profession is a great and divine one."

So much for the annual measure presenting itself from the restored observations between Hipparchus and Ptolemy. The other thing that follows from the observations of these same men concerns the Sun's eccentricity: for, the winter

interval of the equinoxes now having been found as 178 days 18 hours, and this subtracted from the annual measure similarly found, 365 days 5 hours 51½ minutes, the summer one comes out as 186 days 11 hours 51½ minutes.

It seems most advisable to us to adhere to the age of Hipparchus, and so to the time of his third autumnal equinox, which we have hitherto brought into examination; then, the summer interval of the equinoxes now being given, and the apogee of the Sun's place being likewise retained at 6 degrees Π [Gemini], as found by Hipparchus most nearly — first to elicit the eccentricity in the Sun; then to draw out the interval of time between the vernal equinox and the summer solstice from the Sun's true motion in the time of Hipparchus, and to take pains to compare it with the Plinian value for a surer limitation.

The data for this are as follows.

The data [Greek: πραγμάτεια διδόμενα (pragmateia didomena, "the matters given" — the givens)] for this are these:

I. The summer interval of the equinoxes: 186 days 11 hours 51½ minutes. To this corresponds the simple motion of the Sun: 183 degrees 49 minutes 12 seconds.

II. The removal of the apogee of the Sun's place from the summer solstice, into the preceding [signs], in the time of Hipparchus: 24 degrees.

With these things thus supposed, the inquiry into the Sun's eccentricity for the time of Hipparchus will be made in the manner that follows.

Diagram. A geometric construction of the Sun's orbit for determining its eccentricity in the time of Hipparchus. A circle is drawn with center M, representing the Sun's orbit, with circumference points A, D, F, H. Two perpendicular diameters are drawn, B F and D H, together with a line A G parallel to B F. The equinoctial and solstitial points are marked around the circle by their zodiac symbols. A line E I is drawn parallel [to D H], and the inner points I, K, M, N locate the eccentric center and the projections.

- ♋ (Cancer): at the top, above point E/D (the summer solstice direction).
- E: top, just left of the vertical diameter.
- D: top, on the circle at the upper end of diameter D H (just right of E).
- C: upper right, on the circle.
- F: left end of the horizontal diameter B F (autumnal/Libra side).
- B: right side of the circle, upper of the two right-hand horizontal-line endpoints.

- ♎ (Libra): at left, just inside the circle near G.
- G: left, lower, at the left end of line A G.
- ♈ (Aries): at right, near A (the vernal equinox direction).
- A: right end of line A G, on the circle (vernal equinox).
- I: interior, on the vertical line, upper (near M); also a second I lower on the same vertical near H.
- M: center of the circle (orbit center).
- K: interior, below I on the vertical line.
- N: interior, right of K.
- H: bottom, on the circle at the lower end of diameter D H, beside the lower I.
- ♐ (Capricorn): at the bottom, below H (the winter solstice direction).

the equinoxes being likewise established — the vernal, namely, in A, and the autumnal in G — but also a line E I drawn parallel...

[Text continues.]

parallel to the diameter D H, the tropic of Cancer will appear at E, and the tropic of Capricorn at I in [or on the side of] the earth. From these points, the line K M C, drawn from K through the center M and extended to the apogee at C, will be the Sun's eccentricity K M — which we now obtain geometrically in this place, as follows.

Given that, in the year-length found above (365 days, 5 hours, 51½ minutes), the Sun completes its whole proposed circle; and given that, by the interval from the autumnal to the vernal equinox (found earlier) $G A = 178$ days 18 hours, the remaining summer interval will be $A G = 189$ days 11 hours 51½ minutes. To this corresponds the Sun's mean motion of $183^{\circ} 49' 12''$. Subtract from this the semicircle F B of 180° , and there remain the two combined arcs B A and G F = $3^{\circ} 49' 12''$. Half of these, namely A B, is $1^{\circ} 54' 36''$. Its right sine M N (or K L) is 333296, in the units in which the radius of the Sun's orbit M B is taken as 10000000.

Therefore, in the plane right triangle M L K, K L = 333296 is given, as is the angle M K L = $24^{\circ} 0'$ — which measures the distance of the Sun's apogee at C from the apparent summer solstice E in the time of Hipparchus [Margin: *Almagest*-type result; cited as Euclid, *Elements* I, Prop. 28], to which the angle C M D (that is, the arc C D) is equal. The right angle M L K is also given. Hence the angle L M K = 66° is given as well.

With these things granted, for K M we have:

	M, gr. 66	K M	L
as	S.R. 9135455	333296	S.T. 10000000

so that K M = (364837.

[That is: as the sine of the angle (M = 66°), 9135455, is to its opposite side K M = 333296, so the total sine S.T. 10000000 is to K M = 364837.]

Therefore, out of the restored ancient observations of Hipparchus and Ptolemy, there comes the Sun's eccentricity of 364837, in the units in which the Sun's orbital radius M B is 10000000. To this corresponds a maximum prosthaphaeresis of the Sun of 2° 5' 26".

But Hipparchus and Ptolemy themselves, with the refraction included in their own observations, found the Sun's eccentricity to be 417000, to which corresponds an equation of 2° 23' [recte: read as the text gives, "2 p. 23 mi."].

Next, to collect the whole arc A E, and from this to reason out the interval of time during which the apparent motion of the Sun passed from the vernal equinox to the summer solstice in Hipparchus's time, we must, in the same triangle M L K, seek the side M L, whose right sine is the arc D E. The procedure is this:

	S.T.	M K	K, gr. 24	(S.R. L M	arc D E
a	10000000	364837	S.R. 4067366	(148390	Resp. 51 min.
s					

[That is: as the total sine 10000000 is to M K = 364837, so the sine of the angle K (24°), 4067366, is to L M = 148390, which is the right sine of the arc D E; the answer is 51 minutes.]

This arc D E of 51', if it is added to the first quadrant D B of 90°, and then to B A of 1° 54' 36", yields a sum of 92° 45' 36" — namely the arc K E, which we perceive the Sun to complete from the apparent equinox A to the summer solstice E. From the Sun's mean motion, this corresponds to an interval of 94 days 2⅔ hours.

Hipparchus and Ptolemy set this interval at 94 days 12 hours; Pliny, however, at 93 days 12 hours, as is clear from what was said above. We therefore hold roughly to a middle value, except that we lean closer to Hipparchus (as the more likely). Now that Hipparchus and Ptolemy have been dealt with for the restoration of the truth as far as we could, we descend in order to al-Battānī [Albategnius] of al-Raqqā.

Examination of al-Battānī's Solar Observations

In the year 882 from the birth of Christ the Savior, Muḥammad al-Battānī [Mohamedes Albategnius] reported that he had observed at al-Raqqā [Aracta] in Syria the autumnal equinox falling at the beginning of the 19th day of the month Elul (that is, September) of the Romans, $4\frac{3}{5}$ hours before sunrise — that is, $1\frac{2}{5}$ hours after midnight — which was on the completed seventh day of the Egyptian month Pachon. And although I do not adequately grasp by what means this equinox was acquired by al-Battānī, I nevertheless strongly suspect — on account of its excessive haste — that on the two nearest noons that flanked it, with the solar rays marked off by a mural gnomon and the proportion of the shadow worked out, al-Battānī directed that equinox to the moment prescribed for us. Likewise too the vernal equinox: even though al-Battānī observed several things there [Margin: N. Copernicus, *Revolutions* III, ch. 13] by means of the gnomon — about whose use in marking the Sun's position there was, in that age and nation, a great [practitioner] [Margin: See Gemma Frisius, *On the Astronomical Radius*, ch. 24, and *On Error in the Mural Gnomon*; likewise *Book 2*, our *Sphaerica*, Problem 1, ch. 5, near the end] — yet whether al-Battānī used a quadrant or some other instrument, there was supposed either too low an elevation of the cause [?] or of the pole by himself, or an unjust assignment of time. Whatever the case, I hold it as certain and am convinced that al-Battānī advanced the marking of his equinox by more than half a day, since the fixed stars observed by him bear witness, in agreement with us, to his own time...

[the fixed stars bear witness]. This was also the reason why Copernicus, abandoning this al-Battānian equinox altogether, appealed to the fixed stars [Margin: N. Copernicus, *Revolutions* III, ch. 13], since he perceived that the year-length had been cut down too much between Ptolemy and al-Battānī. But since the Ptolemaic equinox, once restored by us, yields itself somewhat more relaxed [i.e., a slightly longer year] than the year-length stated, accordingly — instituting briefly here a comparison about the same matter — for greater certainty we shall afterward, along with Copernicus, take refuge in the fixed stars. For I judge it to have come about by a singular divine providence that, in the course of the Sun (most difficult to detect through itself alone), the fixed stars come somehow to our aid, and by a reciprocal δοκιμασία [Greek: dokimasia, "testing" or "scrutiny"] confirm it by due comparison and just examination.

But to come to the comparison we have undertaken: we gather the interval of time between Ptolemy's restored autumnal equinox — which falls on the 8th day of the month Athyr, in the year 139 from the birth of Christ, at the sixth hour

after noon — and this al-Battānīan one, reduced to the meridian of Alexandria by a subtraction of 40 minutes, to be 743 equal years, 179 days, $6\frac{1}{4}$ hours. Now, dividing the proposed number of years, 743, by 4, there is a remainder, beyond the equal years, of $185\frac{3}{4}$ days. The difference is therefore 6 days $11\frac{3}{4}$ hours; and dividing these among the proposed years, it is shown that the year-length, below a quarter-day, falls between Ptolemy and al-Battānī by 12 minutes 32 seconds. The annual measure itself is therefore 365 days 5 hours 47 minutes 28 seconds — which al-Battānī himself derived as 365 days 5 hours 46 minutes 24 seconds, and which we shall prove below to be in fact larger by more than a minute and a third.

Nor are we moved by the fact that Joseph Scaliger [Margin: Scaliger, *De Emendatione Temporum*, p. 285] reports the vernal Jalālī [Gelalaeum] equinox in Mesopotamia, $179\frac{1}{2}$ years from this al-Battānīan observation, as having fallen on 14 March, about $2\frac{1}{4}$ hours after noon. For we find that that one was not, as he wishes, received from the heavens, but rather precisely constituted out of the annual quantity that al-Battānī found to obtain between himself and Ptolemy.

Furthermore, as to what al-Battānī, in imitation of Ptolemy, had found geometrically from his own observations concerning the eccentricity and the apogee-position of the Sun — from a supposed summer residence of the Sun between the equinoxes of 186 days $14\frac{1}{4}$ hours, but between the vernal equinox and the summer solstice of 93 days 14 hours — we find that these, from these given data and from the mean motion of the same Sun that agrees with them, behave in the same way: namely, that the Sun's eccentricity K M turns out to be 347 [parts], in the units in which the Sun's orbital radius is 10000; and therefore the maximum prosthaphaeresis of the Sun is scarcely two degrees. The Sun's apogee, moreover, has advanced so far in the direction of the signs [in consequentia] since Hipparchus's time that, whereas it had earlier kept an interval of $24\frac{1}{2}^\circ$ from the summer tropic in the backward direction [in antecedentia], it is now only $7^\circ 44'$ distant from the same in the same direction.

This certainly does not depart much from an equal advance of the Sun's apogee, proportioned in time from Hipparchus down to our own age, as will be shown later. As for al-Zarqālī [Arzachel] the Spaniard — who later, after he had found an eccentricity equal to al-Battānī's, observed the Sun's apogee elongated $12\frac{2}{3}^\circ$ from the same solstice in the backward direction — we make no delay over him on this point, while we fall back upon the slipperiness and (as we proved earlier) unobservable incidence of the summer solstice. We only add this much about the al-Battānīan eccentricity: that it would still be a little larger if the account of

parallax in the Sun were added, which it is likely al-Battānī (and al-Zarqālī alike) omitted.

There is also this to be noted here in passing about al-Battānī: that from a change in the Sun's eccentricity (which they posited in vain) there depends no alteration of the obliquity of the Sun, nor the reverse; for otherwise this too would have been the least in this age, and not, as he observed, $23^{\circ} 35'$. But, leaving al-Battānī aside, we shall apply the file to the following ages and astronomers.

A Brief Recapitulation of the Solar Observations of Bernhard Walther of Nuremberg [Bernhardus Gualtherus Norinbergensis], after Tycho Brahe

From those solar observations which Bernhard Walther, a citizen of Nuremberg, once committed to print with the greatest diligence then possible and left behind, our most noble Tycho Brahe drew out the chief ones, and directed them to the Sun's course in the preceding age more correctly than could be learned from the Copernican ones; and by calculation and practice he subjected them — both for the eccentricity and for the time of investigating the apogee [Margin: *Book 1 of the Progymnasmata*, ch. 1] — to be compared almost singly with his own.

Now Tycho used in the first place those observations of Walther on the Sun which, in the year from Christ's birth...

1488 had been taken by the rules they call Ptolemaic; and by carefully comparing these with others of the same Walther, he found the interval of time from the Sun's entry into the vernal equinox up to the next autumnal one to be 186 days $17\frac{2}{3}$ hours [Margin: *Book 1 of the Progymnasmata*, ch. 1, p. 20, etc.]. From the beginning of Aries to the middle of Taurus, 46 days $3\frac{3}{4}$ hours were completed. Moreover, from the middle of Leo up to the autumnal equinox (the beginning of Libra), 46 days $8\frac{1}{6}$ hours.

From the Sun's mean motion, in agreement with these intervals, Tycho — by a πραγματεία [Greek: *pragmateia*, "treatise" or "systematic procedure"] to be exhibited in the restoration of the Sun — found, by a twofold investigation, the Sun's eccentricity to be 3548, in the units in which the orbital radius is 100000; to which a maximum prosthaphaeresis of $2^{\circ} 2'$ also corresponds. Likewise the apogee-position of the Sun at $4\frac{1}{4}^{\circ}$ of Cancer. Indeed, from other observations of Walther carefully compared, he also found the vernal equinox of that same year 1488 to fall on the 10th of March, at 15 hours [recte: hour 15] 40 minutes — all of

which has been carefully elaborated and worked out in the first *Progymnasma* of Tycho's *Astronomy*.

Examination, Correction, and Restoration of the Solar Observations of Nicolaus Copernicus

Nicolaus Copernicus, at Frauenburg [Fruenburgum] in Prussia, intending to investigate the eccentricity and apogee of the Sun from his own observations, in the year 1525 from the birth of Christ the Savior, said that he had found the interval of time between the autumnal and vernal equinoxes to be 178 days 21 $\frac{2}{3}$ hours [Margin: *Revolutions* III, ch. 16]. Furthermore, between the same autumnal equinox and its midpoint — that is, the 15th degree of Scorpio — 45 days 6 $\frac{1}{3}$ hours. Hence, applying the Sun's mean motions, Copernicus found its eccentricity to be only 323 [parts], in the units in which the Sun's orbital radius is 10000; to which the Sun's maximum prosthaphaeresis corresponds, 1° 51'.

But since, in the brief time that elapsed between Copernicus and Tycho Brahe — namely 70 years or more — Tycho, from his most reliable observations, detected that all these things had been remarkably changed beyond the perennial laws of the stars, and therefore suspected that some inconveniences clung to the Copernican observations: accordingly, in the year 1584 from the birth of Christ, having sent into Prussia, with a sextant, one of his very industrious students, Elias Olai Morsianus [Elias Olsen Morsing] (who six years afterward died at Uraniborg in my own embrace), he found the elevation of the Frauenburg pole 2 $\frac{3}{4}$ minutes higher than Copernicus had once supposed it — namely 54° 22 $\frac{1}{4}$ '. Hence, first of all, the winter intervals of the equinoxes, established too high by Copernicus, were diminished by $\frac{1}{2}$ hour. Then too, from the omission of the Sun's parallax about both equinoxes (the account of which Tycho nevertheless affirms Copernicus did take) [the diminution amounts to] about 2 minutes, or at the least, refraction excluded, 1 $\frac{1}{2}$ minutes. From this likewise there accrue 3 $\frac{1}{3}$ hours toward shortening the winter interval of the equinoxes.

Finally, by carefully considering the altitude of Spica Virginis observed at Frauenburg by Copernicus in that same year — and repeated several times (with the same instrument, no doubt, with which the solar observations were made) — and comparing this with the true ones, that is, with Tycho's observations, while taking into account the meanwhile-changed apparent place of the stars, I find that in it [Spica] a deficiency of more than 1 minute occurred, which likewise produces two hours; so that I do not hesitate to establish, as nearly as possible, ἐν πλάτει [Greek: en platei, "in latitude" / "broadly"] 11 hours under the prescribed

errors. By these, the winter interval of the equinoxes, as I said, Copernicus had prolonged beyond what is just. With those 11 hours therefore removed from the 178 days 21 $\frac{2}{3}$ hours, there remain 178 days 10 $\frac{2}{3}$ hours — the said restored interval of the equinoxes.

Furthermore, as to the interval between the autumnal equinox and the midpoint of Scorpio: since, at the Frauenburg meridian altitude of 19 $\frac{1}{4}$ ° — apart from the parallax determined at about 3 minutes — there still remain 2 minutes for refraction, these too give 6 $\frac{1}{2}$ minutes in the Sun's longitude there, and a corresponding time of 2 hours 42 minutes. But what may accrue here from a greater addition concerning the Sun's longitude, and what from the display of too small an altitude through the Copernican instrument, I shall not now touch upon, since these mutually cancel each other out very nearly in this place.

Now, from this Copernican interval between the beginning of Libra and the middle of Scorpio, as is fitting...

with the [errors] duly subtracted, there also remains that corrected one: 45 days, 3 hours [recte: read "d. 45 hor. 3"], 42 minutes. If, therefore, we now adjoin to each of the corrected intervals the agreeing mean motion of the Sun, there will be — for the first, namely 178 days 10 $\frac{2}{3}$ hours [recte: "10 min. $\frac{2}{3}$ " as printed] — a mean or equal motion of the Sun of 175° 52 $\frac{1}{4}$ '. For the second, namely 45 days 3 minutes 42 [seconds], the equal motion of the Sun is 44° 30' 22".

With these things presupposed, we now renew the Copernican calculation for the eccentricity and apogee-position of the Sun as follows.

Diagram. A circle representing the Sun's orbit, described about center F. The vertical diameter runs from B at the top through the central cluster to E at the bottom; this is the apogee–perigee line (apogee at B, perigee at E). The horizontal diameter runs from C at the left (marked with the zodiac sign ♎ Libra, the autumnal equinox) to A at the right (marked with ♈ Aries, the vernal equinox). Near the center are the closely-set points F (the geometric center of the orbit, slightly above center), H, and G (the center of the earth, just below F). Point I lies on the circumference in the upper-right quadrant. From the labeled points several chords are drawn: A C (the equinoctial diameter through G), D I, and the subtenses A D and D I, with D on the lower-left circumference and the chords converging toward G and toward A.

- B (top): apogee
- I (upper right, on circumference): point dividing arc, foot of arc A I
- F (center, slightly above): center of the Sun's orbit

- H (center, between F and G): foot of the perpendicular F H on subtense D I
- G (center, just below F): center of the earth
- A (right, on circumference): vernal equinox (Υ Aries)
- C (left, on circumference): autumnal equinox ($\ Libra$)
- D (lower left, on circumference): midpoint of Scorpio
- E (bottom): perigee

Let the Sun's orbit A B C be described about center F; let A be the vernal equinox, C the autumnal, and D the middle of Scorpio. Then let A C and D I be joined, cutting each other at G, the center of the earth; and let A D be subtended. Since, then, from the Sun's mean motion the circumference D C is known to be $44^{\circ} 30' 22''$, the angle subtended at C A D — namely half the arc C D — is $22^{\circ} 15' 11''$. Likewise the angle subtended at C G D, the apparent motion of the Sun, is $45^{\circ} 0'$; and when this is subtracted from two right angles (180°), there remains the obtuse angle A G D of $135^{\circ} 0'$. The third angle in the triangle A G D, namely A D G, is known from the complement of the two given angles A and G to a semicircle: $22^{\circ} 44' 49''$. Its double — namely $45^{\circ} 29' 38''$ — is the arc A I. Further, since the arc C D A is given as $175^{\circ} 52\frac{1}{4}'$, then, this being subtracted from the same arc C D... — [rather:] subtracting the arc C D = $44^{\circ} 30' 22''$ from it [C D A], there is left the arc D A = $131^{\circ} 22' 8''$. Its subtense A D is given as 18225828, in the units in which the radius of the proposed circle is 10000000. And when the arc A I is added to the arc D A, the whole arc D I is thereby made up to be $176^{\circ} 51' 46''$. Its subtense D I, likewise, from the table of sines, is given as 19992506.

Now, since each of the two segments A D C and D A I falls short of a semicircle, the Sun's apogee must accordingly be sought in the other part of the circle, namely I B C. Let the line of apogee and perigee B E, and G F, be the Sun's eccentricity itself. Now, the perpendicular F H drawn divides the subtense D I on both sides into right angles and into equal parts. Therefore, in the right triangle F H G, so that the data [$\delta\epsilon\delta\omicron\mu\acute{\epsilon}\nu\alpha$, Greek: dedomena, "the givens"] may be had sufficiently, first, in the triangle A G D — with all the angles given, together with the side A D — the side G D is given as 9763585. This being subtracted from half of D I, that is from 9996253, there remains H G = 232668. Then we obtain the perpendicular as follows: the defect of the circumference D A I from a semicircle is $3^{\circ} 8' 14''$; the right sine of half of this ($1^{\circ} 34' 7''$) is 273740, which is the side H F. Hence, since in the said right-angled triangle F H G two sides about the right angle are given — namely H G = 232668, F H = 273740 — there is found first the angle F G H = $49^{\circ} 38' 12''$. This, with the arc A C of $45^{\circ} 29' 38''$, constitutes — from the corrected Copernican observations — the Sun's apogee-position at $5^{\circ} 7\frac{1}{6}'$ of

Cancer. Then, in the same triangle F G H, the side F G is given — namely the Sun's eccentricity itself, from the same corrected observations — as 359261, in the units in which the orbital radius is 10000000; to which corresponds the Sun's maximum prosthaphaeresis of $2^{\circ} 3\frac{1}{2}'$. So much for the eccentricity and apogee of the Sun from the Copernican observations restored.

Furthermore, the obliquity of the ecliptic in Copernicus's time is explored and corrected by a twofold test. First, if to the $23^{\circ} 28'$ — the obliquity of the ecliptic that Copernicus says he found — you add $2\frac{3}{4}$ minutes, by the amount, that is, by which the elevation of the Frauenburg pole was found to be too small. Secondly, if the difference of the Sun's parallax at the maximum and minimum meridian altitude at Frauenburg — namely $1\frac{1}{2}$ minutes — be subtracted from the Sun's refraction at the minimum meridian altitude (say $8\frac{1}{2}$ minutes), and the difference, 7 minutes, be halved, and the half, which is $3' 30''$, be adjoined to $23^{\circ} 28'$: from which, in the one case the maximum obliquity comes out $23^{\circ} 30\frac{1}{4}'$, and in the other 23° ...

the mean of these is therefore 13 degrees 31 minutes 8 seconds, which was very nearly the minimum in Copernicus's [Nicolaus Copernicus] day — a point we shall later confirm equally on the basis of the hypothesis to be fitted. And so here too we have faithfully restored Copernicus. There now remains Tycho Brahe, who surpassed everyone in the certainty of his observations, to be treated in the final place.

A Summary and δοκιμασία [Greek: δοκιμασία (dokimasia, "testing" or "critical examination")] of the Observations of the Sun, and of the restitution of its course, carried out with the utmost diligence by Tycho Brahe on the island of our Denmark, Hven.

Up to this point we have restored the observations of the ancients which Copernicus had earlier used to ill effect, in order to establish in people's minds the opinion of a variable change in the eccentricity and in the place of the Sun's apogee. Now follow, in the final place, the observations of our most noble Tycho. By these, as by a Lydian touchstone, the whole business of the solar course is to be examined by us: first as regards the eccentricity and the apogee, then — by a fair comparison made with the ancients — as regards the annual quantity in this perpetual restitution of the Sun and stars (may God assist it). For it is right to stand fast by those observations in which we know that nothing among the

previously listed defects stands in the way of the utmost precision. So much so, indeed, that on this ground alone those who understand astronomical matters thoroughly may, together with us, acknowledge Tycho Brahe as the foremost astronomer of every age — to whom, I trust, it will not be at all displeasing that I have still applied my labor to his memory, as I faithfully served him while he lived. But before we come to the most accurate observations of Tycho and our own, made at Hven and elsewhere, it seems good to us to set out why the equation of the natural day [aequatio diei naturalis], which is supposed to arise from the Sun's unequal course, must be wholly eliminated — lest the matters we must hereafter treat, especially concerning the lunar course, be further hindered by suspicion of it.

The reasons or causes of the equation of the natural day, and why that which proceeds from the Sun's unequal motion must be abolished.

The equation of the natural day, if there were any such thing, would depend on the course and constitution of the Sun. I shall therefore briefly set out here the reasons why I judge it must for the most part be rejected, so that afterward, in the restoration of the planetary motions — and above all the lunar — we may proceed freely and without scruple on this point.

It has indeed been accepted hitherto from the ancients that time — whose index is the Sun in its daily revolution, and the equator the regulator (as we taught in the doctrine of the sphere [Margin: Lib. 1. Sph. c. 5. & lib. 2.]) — reckoned from the meridian at one and the same place, has a twofold cause of its variation: namely, the inequality of the arcs of the ecliptic compared with those of the equator [Margin: Sph. 5. c. 5.]; and likewise the Sun's daily or proper and unequal motion.

When these two causes are rightly considered, and the two indices proceeding from one center are carefully examined — so that the one perpetually follows the Sun's equal daily motion in the equator, to be added to the first conversion from the vernal equinox, while the other follows the Sun's true motion, that is, the oblique ecliptic, likewise to be added to the Sun's elongation from the true equinox, similarly directed toward the equator — they exhibit that manifest variation of time, accumulating and diminishing in succession over many days. So much so that L. Christmannus of Heidelberg [Christmann] tries in vain to persuade us [Margin: Christmannus, de Theoria Lunae nostra reiterata & defossata.] that, on account of the parallel daily revolutions of the Sun, without

the other causes for which the equation of time is rightly suspended, no sensible difference intervenes between the intermediate and apparent time. But another notion, different from this one of Christmann's, was once devised by Witich [Witichius], the mathematician of Breslau [Vratislaviensis], to do away with the equation of time, while he was staying with Tycho at Hven in the year 1582: namely, that a restitution of the mean of the two causes should be made for each daily revolution. Yet this too, by our experience set forth above in the demonstration through those twin indices, falls short, so that not even in it can any support be found for omitting the equation of time — though the lunar motions, however they are to be restored, are altogether

found to recoil from that quarter by which the inequality of the Sun's motion in the ecliptic is pressed. And it is no wonder that anyone should suppose we ought to have included — or even could have included — this variety in a lunar hypothesis, or in tables of mean motions, or even in a προσθαφαίρεσις [Greek: προσθαφαίρεσις (prosthaphairesis, "the equation" — the additive-subtractive correction)] of the Moon, which the celestial appearances insinuate against the ancient (that is, the legitimate, as they would have it) equation of time. For this attempt, if nothing else, is at any rate blunted by the too-rapid change of the lunar apogee, since it traverses the whole zodiac within nine years. [Margin: Prog. 1. pag. 113. etc.]

But that the true designation of time depends not equally on any other celestial body as it does on the Sun — this was always my anxious care, both formerly and most of all from the time when, by the more frequent letters of the most celebrated astronomers, I was urged on this matter to seek out more solid causes for equating time than those left behind by Lord Tycho in the appendix on the lunar motion. [Margin: Vide sequelam definitionis Astronomiae l. 1 sph. cap. 1.] And I can truly affirm that in my whole astronomical study I have never labored with greater difficulty than here. For although I find several reasons why the Ptolemaic equation of time — which all the other astronomers have followed — is not rightly accommodated (both because it is produced from various epochs, which certainly errs in the constant reciprocation of time with motion; and because that part which depends on the Sun's unequal daily motion recognizes no starting point in the ecliptic other than where the line of apogee is established), yet little did any procedure of this kind, set up by me, profit toward saving the lunar phenomena, so long as I devoted effort to equating time from this quarter.

And so, in the last restitution of the lunar course, which was carried out in Bohemia in the year 1600, we abandoned — almost without reason — the one of the causes that would be drawn from the Sun's unequal motion, except that here too we judged the Moon should be conformed in its course to the Sun's true course. The other cause we conceded out of the necessity of demonstrating the obliquity of the ecliptic, and we still retain it, since I perceive that this same thing is in some way demanded by the lunar appearances, both in eclipses and outside them. [Margin: Vide hanc temp. aequationem prog. 1 Tych. in appendice motus lun.] But to one investigating diligently and earnestly the cause of the other matter, such a one appears to me as can be set directly against the equation of time proceeding from the Sun's unequal course. For this is most conveniently removed in the following way: if the daily motion of the conversion be adjusted to the Sun's approach to and recession from the Earth, so that, the Sun being nearer to us, this motion becomes swifter, and slower when it is farther off — in which matter a great argument is left for the φυσικολόγοις [Greek: φυσικολόγοις (physikologois, "natural philosophers" — those who reason about nature)] concerning the Earth upon the center of its rotation.

Omitting, therefore, that inequality which is otherwise seen to be deduced from the Sun's apparent motion, we shall pursue below only that one which arises from the obliquity of the arcs of the ecliptic compared with those of the equator, and which stands separately in the lunar appendix in Tycho's work — provided that the lunar phenomena, in eclipses and outside them, in any way admit it. But if anyone should wish to take into consideration some other small equation of time which the unequal precession of the equinoxes may seem capable of contributing — never, however, extending to 4 minutes when complicated on both sides — that shall be permitted by me, who knowingly and gladly pass over here the extreme of scrupulosity.

Continuation of the undertaking in examining the Sun's course from the more recent observations.

Now that a remark about the true equation of time has been interposed — proofs of which the lunar course especially will display hereafter — we shall now press on further with the solar matters, in the order set out before: first of all by setting forth the observations of the Sun gathered by Tycho Brahe at the chief locations, especially from the year of Christ the Redeemer 1588, at which time, through the eccentricity discovered from his own and manifold observations — 3584 parts, that is, a prosthaphaeresis of 2 degrees 3¼ minutes; the apogee at 5 degrees 38 minutes of Cancer; and finally the vernal equinox on the 9th of March at hour

20¾ of that same year — the restitution of the solar course was derived for the four next centuries, and published in print at Uraniborg by the Tychonic types. We shall presently set out below in a brief synopsis all his observations, together with their comparison and δοκιμασία [Greek: δοκιμασία (dokimasia, "critical testing")], so that from such great harmony as appears in the Tychonic results — compared with the interpolated and gaping dissonance of the ancients' — posterity may obtain a just occasion both for acknowledging here (and wherever, with a grateful mind) the singular industry of Lord Tycho, and for recognizing that we, in correcting the rest while we labor at the perpetual restitution of the celestial motions, have presumed nothing rashly nor touched anything beyond cause. As for the meridian altitudes

of the Sun, they now follow from Tycho, derived from the heavens by two — and sometimes even three — of the largest and most perfect quadrants, and moreover by the largest equatorial circle of declination, and they are mutually limited among themselves, so that (unless sometimes by chance the air was denser) they exclude every sensible error. But in examining the Sun's course for each individual day in those observations, I used Tycho's table of declination, constructed for an obliquity of the ecliptic of 23 degrees 31½ minutes, supposing also the elevation of the pole at Uraniborg to be 55 degrees 54½ minutes. For I deliberately diminished it here by ½ minute, both because I scarcely ever found it higher there, and in order to slightly attenuate the solar parallaxes (for which, as for the refractions, I likewise used Tycho's tables), the reason for which I shall render later in the lunar eclipses; and finally in case some refraction might cling to the Sun at its meridian altitude around the middle of Taurus and Leo. As a paradigm of the examination instituted, we set out here a single example, to which all the rest were carried out. In the year of Christ 1583, the 14th of March.

Uraniborg.

Quantity	G	M	S		Quanti ty	G.	M	S
	.	.	.				.	.
Meridian altitude of the Sun [Altitudo Solis meridiana]	3	2	3	Equator-altitude to be subtracted [Exaltatio aequatoris subt.]	34	5	3	0
Parallax to be added [Parallaxis addenda]	0	2	2	Therefore the true northern declination of the Sun [Ergo declin. vera Solis Borea]	1	18	5	3
Refraction of the Sun to	0	0	3	To this corresponds,	3	∞	17	

Quantity	G	M	S	Quantity	G.	M	S
. . .							
be subtracted [Refractio Solis subtrahenda]	3			from the table of declination, the place of the Sun [Huic resp. ex ta. decl. loc. Sol.]	$\frac{2}{3}$		
Difference to be added to the altitude [Differentia addenda altitudini]	0	1	5	Place of the Sun from the table of restitution of T. B. [Loc. Sol. ex tab. restit. T. B.]	3	18	$\frac{2}{5}$
Therefore the true altitude of the Sun [Ergo altitudo vera Solis]	3	2	2	Difference [Differentia]	0	0	$\frac{3}{4}$
	5	4	3				

And such was the progress of the calculation in all that follows; for in this comparison we did not think the seconds should be expressed except in parts of the whole, since these can scarcely be had even from the observations. [Margin: Vide lib. 1.] And indeed the excessive scrupulosity of certain men in these matters is to be despised — men who, by pressing precisely those minute seconds, sometimes fall away from the truth by whole degrees [Margin: Prog. T. B. pag. 28.]; which assuredly they would sometime have to learn, together with us, from the celestial observations themselves, as the best informants, to be a fruitless effort of theirs.

The meridian altitudes of the Sun acquired at Uraniborg, from which — account being taken of parallax and refraction — the true places of the Sun are derived, and compared with the restitution tables of T. B.

I. Around the Vernal Equinox.

A							
N	n		D				
u	n	M	i				
m	u	en	e	Alt. Solis Merid.	Locus Solis obs.	Loc. Solis ex	Different.
.	s	sis	s	(G. M. S.)	(G. M. S.)	tab. (G. M. S.)	(M. S.)
1	1	M	1	35 22 30	3 7 17 40	3 7 18 24	$\frac{3}{4}$ +
	5	ar	4				
	8	tii					
	3						
2	1	M	1	34 29 55	1 5 35	1 5 33	(none)

A				D			
N	n			M	i		
m	u	e	s	en	s	Alt. Solis Merid.	
.	s	sis	s			(G. M. S.)	
				Locus Solis obs.			
				Loc. Solis ex			
				tab. (G. M. S.)			
				Different.			
				(M. S.)			
	5	ar	1				
	8	tii					
	4						
3	1	M	1	35 10 45		2 48 15	2 49 45
	5	ar	3				
	8	tii					
	5						
4	1	M	1	34 18 30		0 37 0	0 36 50
	5	ar	1				
	8	tii					
	6						
5	1	M	1	34 36 15		1 21 30	1 21 40
	5	ar	2				
	8	tii					
	7						
6	1	M	1	34 53 30		2 5 0	2 6 40
	5	ar	2				
	8	tii					
	8						
7	1	M	1	34 24 55		0 53 5	0 53 2
	5	ar	1				
	8	tii					
	9						
8	1	M	1	34 18 35		0 37 10	0 37 40
	5	ar	1				
	9	tii					
	0						
9	1	M	1	34 1 30		29 ✕ 54 43	29 ✕ 55 33
	5	ar	0				
	9	tii					
	3						
1	1	M	1	33 56 20		29 38 45	29 41 15

A							
N n			D				
u n	M	i					
m u	en	e	Alt. Solis Merid.	Locus Solis obs.	Loc. Solis ex	Different.	
. s	sis	s	(G. M. S.)	(G. M. S.)	tab. (G. M. S.)	(M. S.)	
0	5	ar	0				
	9	tii					
	4						
1	1	M	1 34 30 45	1 $\text{X} \rightarrow \text{Y}$ 7 45	1 Y 11 12	$\frac{1}{3} +$	
1	5	ar	1				
	9	tii					
	6						
1	1	M	1 34 2 20	29 56 20	29 57 20	1 +	
2	5	ar	0				
	9	tii					
	7						

II. Near the Autumnal Equinox.

A							
N n			D				
u n			i	Alt. Solis			
m u	Men	e	Merid. (G. M.	Locus Solis obs.	Loc. Solis ex	Different.	
. s	sis	s	S.)	(G. M. S.)	tab. (G. M. S.)	(M. S.)	
1	1	Sept	1	34 13 20	29 𐍇 36 0	29 𐍇 35 10	5⁄6 —
	5	emb	3				
	8	r.					
	3						
2	1	Sept	1	34 18 55	29 21 57	29 20 52	1 1⁄12 —
	5	emb	2				
	8	r.					
	4						
3	1	Sept	1	35 11 40	27 9 20	27 9 0	1⁄3 —
	5	emb	0				
	8	r.					
	5						
4	1	Sept	1	34 7 0	29 51 45	29 51 25	1⁄3 —
	5	emb	3				
	8	r.					

A		D		Alt. Solis			
N n			i	Merid. (G. M.	Locus Solis obs.	Loc. Solis ex	Different.
u n		Men	e	S.)	(G. M. S.)	tab. (G. M. S.)	(M. S.)
m u		sis	s				
.	s						
6							
5	1	Sept	1	34 37 0	28 36 30	28 38 5	1 $\frac{3}{5}$ +
	5	emb	2				
	8	r.					
	7						
6	1	Sept	1	34 18 20	29 23 30	29 22 45	$\frac{3}{4}$ —
	5	emb	2				
	8	r.					
	8						
7	1	Sept	9	35 35 0	26 11 15	26 12 0	$\frac{3}{4}$ +
	5	emb					
	8	r.					
	9						
8	1	Sept	1	34 30 15	28 53 30	28 54 30	1 +
	5	emb	2				
	9	r.					
	0						
9	1	Sept	1	35 5 5	27 26 15	27 26 45	$\frac{1}{2}$ +
	5	emb	0				
	9	r.					
	2						
1	1	Sept	1	34 6 20	29 56 30	29 55 0	1 $\frac{1}{2}$ —
0	5	emb	3				
	9	r.					
	4						
1	1	Sept	1	33 30 10	1 $\underline{\text{a}}$ 24 10	1 $\underline{\text{a}}$ 24 20	$\frac{1}{6}$ +
1	5	emb	4				
	9	r.					
	6						

III. According to the middle of Taurus.

A				D			
N n		M i					
u n		en e		Alt. Solis Merid.	Locus Solis obs.	Loc. Solis ex	Different.
. s		sis s		(G. M. S.)	(G. M. S.)	tab. (G. M. S.)	(M. S.)
1	1	A	2	50 52 0	16 8 25 30	16 8 24 45	$\frac{3}{4}$ —
	5	pr	7				
	8	il.					
	2						
2	1	A	2	50 30 45	15 10 40	15 13 30	$2\frac{5}{6}$ +
	5	pr	6				
	8	il.					
	3						
3	1	A	3	51 48 30	19 46 45	19 47 45	$1\frac{2}{3}$ +
	5	pr	0				
	8	il.					
	4						
4	1	A	2	50 39 20	15 41 10	15 42 50	$1\frac{2}{3}$ +
	5	pr	6				
	8	il.					
	5						
5	1	A	2	50 52 30	16 26 50	16 26 40	$\frac{1}{6}$ —
	5	pr	7				
	8	il.					
	6						
6	1	A	2	51 20 35	18 8 10	18 6 0	$2\frac{1}{6}$ +
	5	pr	9				
	8	il.					
	7						
7	1	A	2	51 0 20	16 56 $\frac{1}{2}$	16 54 0	$2\frac{1}{2}$ +
	5	pr	7				
	8	il.					
	8						
8	1	A	2	50 45 30	16 2 20	16 2 25	(faint) +
	5	pr	6				
	9	il.					

A						
N n		D				
u n	M	i				
m u	en	e	Alt. Solis Merid.	Locus Solis obs.	Loc. Solis ex	Different.
. s	sis	s	(G. M. S.)	(G. M. S.)	tab. (G. M. S.)	(M. S.)

0

IV. Beside the middle of Leo.

A						
N n		D				
u n	M	i				
m u	en	e	Alt. Solis Merid.	Locus Solis obs.	Loc. Solis ex	Different.
. s	sis	s	(G. M. S.)	(G. M. S.)	tab. (G. M. S.)	(M. S.)
1 1	Iu	2	30 40 35	14 42 14 35	14 42 15 1/4	2/3 +
5	lii	8				
8						
2						
2 1	Iu	2	50 31 51	14 46 45	14 45 0	1 3/4 —
5	lii	8				
8						
4						
3 1	Iu	2	50 6 30	13 19 10	13 19 40	1/2 +
5	lii	7				
8						
5						
4 1	Iu	2	51 0 50	13 4 15	13 5 45	1 1/2 +
5	lii	7				
8						
7						
5 1	Iu	2	50 18 15	15 30 35	15 30 20	1/4 +
5	lii	9				
8						
9						

Besides these I append certain more select observations around each solstice, so that from them the maximum obliquity of the ecliptic in our time may be shown at the higher tropic, and at the lower tropic also the refraction of the Sun. These

too, as they were diligently acquired at Uraniborg with several instruments, the following little table comprehends.

The Sun's meridian altitude at Uraniborg about each of the two solstices.

Summer [Æstivum]					Winter [Hybernūm]						
Nu.	Y e a r	M o n t h	D a y	Alt. Sol. Mer. (G.)	(M.) (S.)	Nu.	Y e a r	Mo n t h	D a y	Alt. Sol. Mer. (G.)	(M.) (S.)
1	1 5 8 6	J u n e	1 2	57	35 ³ / ₅	1	1 5 8 9	De ce mb er	1 5 10 5	44 ⁴ / ₅	
2	1 5 8 7	J u n e	1 4	57	34 ¹ / ₂	2	1 5 9 0	De ce mb er	1 5 10 0	43 ¹ / ₄	
3	1 5 9 3	J u n e	1 1	57	35 0	3	1 5 9 4	De ce mb er	1 5 10 1	41 ¹ / ₆	

(Columns: Nu. = number; G. = degrees, M. = minutes, S. = seconds. The first summer entry's seconds are printed as the fraction $\frac{3}{5}$; the winter entries' fractions are $\frac{4}{5}$, $\frac{1}{4}$, and $\frac{1}{6}$ respectively. Parsing notes: the seconds sub-column carries a fractional value rather than an integer in several cells, written under the "S." heading.)

Besides these exact and indeed incomparable solar observations of the noble Tycho [Tycho Brahe] — whose worth their reliability [δοκιμασία (dokimasia), "proof," "testing"] sufficiently attests — Tycho also brought other considerations to bear in testing the perfect restitution made by others. In particular, he used the observations of the most illustrious Wilhelm, Landgrave [of Hesse] [Gulielmi Lantgravii]. Moreover, for still greater certainty he investigated the matter further by another kind of observation [πραγματεία (pragmateia), "undertaking,"

"treatise"] concerning the Sun's perigee. [Margin: *Liber 1 Prog., pag. 77.*] These investigations support, to a remarkable degree, what can be derived about the distances of Venus and the Sun from the fixed stars near the Sun's perigee. And — leaving it to the judgment of those who understand these matters — apart from eclipses in either luminary, no doubt remains that Tycho, in restoring the Sun's course for our age, came as close as anyone possibly could to matching the celestial truth in its appearances. In this way he far surpassed the skill, reliability, and diligence of all his predecessors.

I myself found this to be true even twelve years after he had completed the solar restitution, once the task of publicly professing and practicing this queen of sciences, astronomy, had — by divine direction in mathematical disciplines — been transferred to me. For from the year of our Lord 1607, around the same time when the tailed comet [cometa caudatus] shone forth, I had a sextant made — nearly five cubits in its semidiameter, carefully fashioned in imitation of Tycho's. My aim was that with this instrument, together with a geometric quadrant of sufficient size, I might gather what I judged needed to be drawn from the heavens to supplement the astronomical restitution, and so revive in my own homeland the fresh offspring of Urania [Vrania] — should she perhaps be languishing in Bohemia.

So, among other things from which the continuation of the restored solar motion handed down by Tycho can be assessed: in the year 1610, around the autumnal equinox — namely on 11 September — using this same new sextant of ours, which I had first carefully calibrated against the fixed stars, I observed the Sun's meridian altitude at 35 degrees, 1½ minutes. And since the elevation of the pole at Copenhagen [Hafnia] extends very nearly to 55 degrees, 43 minutes, after eliminating parallax and refraction by their proper amount (as above), I found the Sun's true declination to be 0 degrees, 46⅔ minutes north. To this corresponded a solar longitude of 28 degrees, 3¾ minutes [of Virgo]; and Tycho's restored tables likewise give it as 28 degrees, 4⅙ minutes [of Virgo]. The difference, as you see, does not rise to even ½ minute — which is certainly imperceptible, and commends the Tyconic prosthaphaeresis of the Sun as correctly established above all others.

Furthermore, in that same year 1610, on 12 December and several following days, I investigated with great care, by means of this same sextant of ours, the Tyconic restitution of the solar perigee, by way of the Sun's distance from Venus during daytime. On 12 December, at 2¼ hours [hora 2¼] after noon, I found this distance to be 47 degrees, 2 minutes. On the following day, around the same time, 47

degrees, 1 minute. Likewise on the 15th, at the same altitude, 46 degrees, 59 minutes. And finally, on 16 December, the same distance was 46 degrees, 55 minutes — these observations of ours being repeated rather often on individual days and carefully bounded.

Now, to bring into examination here the distance between the Sun and Venus on 12 December at the hour I mentioned ($2\frac{1}{4}$), with the Sun's altitude at about 4 degrees but Venus's at 17 degrees as observed — namely 47 degrees, 2 minutes — it is also fitting that the position of Venus around that same time should be acquired from the heavens. Accordingly, we found it a little later that same evening from the distances of Venus from the fixed stars. At $4\frac{1}{4}$ hours [H. $4\frac{1}{4}$], when Venus's altitude above the horizon was $15\frac{1}{2}$ degrees, Venus was distant from the Bright Star of the Vulture [Lucida Vulturis — α Aquilae, Altair] by 37 degrees, $8\frac{1}{2}$ minutes, and from Markab of Pegasus [Marchab Pegasi — α Pegasi] by 36 degrees, $13\frac{1}{4}$ minutes. Hence, from the verified positions of the fixed stars, Venus's apparent longitude was found to be 17 degrees, $56\frac{1}{2}$ minutes of Aquarius [♒], with a northern latitude of 1 degree, 29 minutes.

Now, in order to eliminate both refractions and parallaxes in both Venus and the Sun, since the sublimation

since the sublimation of Venus from the Earth was then, by hypothesis, very nearly 850 Earth-semidiameters, the parallax of Venus in the altitude circle, at $15\frac{1}{2}$ degrees above the horizon, was about 4 minutes; while its refraction at the same altitude — which we take, not inappropriately, midway between that assigned to the Sun and that assigned to the fixed stars by Tycho, on account of Venus's middling brightness — was found to be 5 minutes very nearly. The difference of Venus's parallax and refraction, then, scarcely 1 minute here, yields half a minute to be subtracted from the longitude. The reduction for Venus's motion over the $2\frac{1}{4}$ hours that elapsed between its observation and that of the Sun subtracts 6 minutes. Venus's true longitude at $2\frac{1}{4}$ hours of 12 December, when it was observed from the Sun, was therefore 17 degrees, 50 minutes of Aquarius [♒].

Now since I found that the distance between Venus and the Sun — namely 47 degrees, 2 minutes — differs by nothing perceptibly sensible from the distance falling between them with respect to the longitude of each on the ecliptic (owing to Venus's small latitude at this time), therefore, by subtracting that distance of 47 degrees, 2 minutes from the longitude of Venus just established, the apparent place of the Sun comes out at 0 degrees, 48 minutes [of Capricorn]. And because the parallax was very nearly the same for the Sun and for Venus at the moment

of observation — but with refraction additionally present in the Sun — once this parallax is excluded (while you have regard to the true altitude of $3\frac{1}{2}$ degrees), the refraction was 14 minutes in the vertical circle. Distributed at this point according to longitude, this amounts to only about 5 minutes, to be subtracted from the Sun's apparent place. The Sun's true place on 12 December 1610, at hour $2\frac{1}{4}$, was therefore at 0 degrees, 43 minutes of [Capricorn] — which the calculation from Tycho's tables shows as 0 degrees, $42\frac{1}{2}$ minutes of [Capricorn], a difference of only $\frac{1}{2}$ minute. Nor do I doubt that, if a similar test [δοκιμασία (dokimasia), "trial," "proof"] is conducted on the remaining days from our observations, the solar course near the perigee will keep within nearly the same agreement — so much so that the Tychonic restitution of the apogee in the Sun excludes, very nearly, all perceptible error right down to our own time. This holds even though, owing to the excessive slowness of this motion (on Tycho's supposition), some discrepancy may be introduced over the course of time, as we shall demonstrate a little further on.

The establishing [Σύστασις (sustasis), "constitution," "establishing"] of the Sun's eccentricity from Tycho Brahe's observations, mutually bounded.

Now, with the bounding of the Tychonic observations of the Sun carefully carried out, it appears from the differences I have added that the Sun's motion around the vernal equinox comes out a little slower, and around the autumnal a little quicker, than Tycho's tables show it. And although the $\frac{1}{4}$ minute in the pole's elevation, which we earlier neglected, can contribute something toward removing these differences, it is still not so great that something does not yet remain — about one and a half minutes, as it were, to be subtracted from the longitude, and half a minute from the Sun's maximum [equation]. We have now relaxed this by only 27 seconds, and have found that from it arises a divine and truly unalterable numerical proportion, namely the second number in the order of perfect numbers. [Margin: *Plato, de republica lib. 1. Arithm. solut. cap. 12.*] So the eccentricity of the Sun stands to the radius of its orbit as unity to twenty-eight, or as 35,714 to 1,000,000, very nearly. And, abiding by this invariable ratio — which is the nature of perfect numbers — in the matter of the Sun's eccentricity, we shall now further set out, by comparison, how much those artificers we corrected above diverged here and there to either side, so that intelligent readers may clearly judge from the differences how much room for altering the Sun's eccentricity Copernicus [Nicolaus Copernicus] left behind.

A collection of the maximum solar prosthaphaeresis from the observations of various astronomers, here above reviewed and corrected by us.

	grad.	minut.	secund.
Hipparchus & Ptolemy [Hipparchus, Ptolemy]	2	5	26
Albategnius & Arzachel [al-Battānī, al-Zarqālī]	2	0	0
Walther of Nuremberg [Bernhard Walther]	2	2	0
N. Copernicus of Toruń [Nicolaus Copernicus]	2	3	0
Tycho Brahe the Dane [Tycho Brahe]	2	2	48

From the small differences that appear in this comparison — nowhere extending beyond 3 minutes either way from our established result, since everything was duly provided for in its proper place above — let anyone now experienced in astronomical observations and practice judge how rightly Copernicus had introduced into the solar theory the change of eccentricity. Indeed, that change, provided it were true, would also have introduced a quite manifest and perceptible inequality into the motion of the Sun's apogee, if one were to abide by Copernicus's supposition. I shall therefore show, by what follows, that there is no such inequality against it either [δοκιμασία (dokimasia), "test," "examination"].

First, I judge it must be noted that, even though the result comes out the same from whatever epoch of just interval the simple motion of the Sun's apogee is begun, I nevertheless can no longer keep silent: a most illustrious testimony concerning the world's beginning and duration is drawn from the changed place of the Sun's apogee, and I shall bring it forward here. For, supposing the same apogee of the Sun at the beginning of Aries [♈] at the first creation of the world, and the perigee at the beginning of Libra [♎] — from which the Sun first appeared to the world by divine working — then, through the apogee meanwhile shifted, as I say, I find that from the beginning of the world to the redemption of the human race (that is, to the suffering of Christ Jesus on the cross) there elapsed 4,000 years, save only half a year. [Margin: *More about epochs below.*] And therefore, from the world's beginning to the start of the year 1588 from Christ's birth, we rightly count 5,554 Julian years — allotting, of course, 33 full years to Christ's life in these lands, plus what runs beyond his nativity to the time of his

passion. Nor do we judge any other correction [διόρθωσις (diorthōsis), "correction," "adjustment"] necessary at this point.

Now, since from Tycho Brahe's most accurate observations we gather that, from the founding of the world to the said year 5,554, the Sun's apogee has advanced into the consequence of the signs by 95½ degrees with respect to the true equinox (from which the mean scarcely ever differs by ½ degree, as we shall prove later) — we have deduced the same proportionally from the beginning of the world for each of the individual centuries in which it was observed, and set it in comparison with the observed value, as follows.

The place of the Sun's apogee from the beginning of the world and from the true vernal equinox.

In various centuries	Years of the World	Reduced (G.)	(M)	Observed (G.)	(M)	Differ. (M.)
Hipparchus of Rhodes [Hipparchus]	3810	65	1 6	65	3 0	14 —
C. Ptolemy of Alexandria [Ptolemy]	4099	70	3			
Albategnius the Muhammadan [al- Battānī]	4849	82	5 3	82	1 6	37 +
Walther of Nuremberg [Bernhard Walther]	5454	93	4 3	94	1 5	32 —
N. Copernicus of Toruń [Nicolaus Copernicus]	5492	94	2 3	95	8	45 —
Tycho Brahe the Dane [Tycho Brahe]	5554	95	3 0	95	3 0	0

(Parsing notes: the Ptolemy row gives only the "Reduced" value [70°3']; its "Observed" and "Differ." cells are blank in the source. The "Differ." column carries a sign after each value: — for a deficit, + for an excess.)

From this it appears that the apogee's motion in the Sun, deduced proportionally for each of the individual centuries in which it was observed by astronomers, nowhere differs by more than $\frac{3}{4}$ of one degree from their observed values — which in the Sun's own motion scarcely causes a difference of one and a half minutes — Ptolemy excepted, who too confidently asserted that the Hipparchan supposition of the Sun's apogee endured to his own age, his observations no doubt having been violently accommodated to such a purpose. [Margin: *Vide Prog. 1, T. B. pag. 32.*] But from the rest, who are to be corrected for the Sun's apogee, I have refrained — Copernicus alone excepted; for his observations, free of refractions and other inconveniences, betrayed the place of the Sun's apogee that I have set down here above.

Furthermore, concerning the same apogee motion, where the interval of time between Walther of Nuremberg and Tycho Brahe does not respond precisely to the proportion we established, we make no other reply than this: it was hardly possible for Tycho, from such observations of Walther's — and with the pole's elevation at Nuremberg not precisely enough known — to arrive at a very accurate measurement of the solar motion in Walther's time, and from that single interposed interval of a hundred years to extend the Sun's course precisely to four centuries. For although the eccentricity discovered by Tycho himself sufficed very nearly for all centuries, as we have shown, he nevertheless so diminished the apogee's motion that it can scarcely agree with this century of ours. This will readily be confirmed by comparison with the observations of the ancients, which we have just set out.

A common opinion had taken hold of Tycho concerning the certainty or truth of the solar hypothesis introduced by Copernicus — that, just as it involved some change of eccentricity, so the anomaly of the apogee's place would necessarily entail that small Copernican circle. But in vain, as is now also clear in the Sun's apogee, which by a constant, simple, and proportional change varies nowhere beyond three-quarters of a degree — let alone the $7\frac{1}{2}$ degrees that Copernicus's supposition sometimes demands (of which it sometimes even demands double). Indeed, while I write this, there comes to mind the admonition once given to me by the noble Lord Holger Rosenkrantz [Holigerus Rosekrantzius], concerning this kind of alteration of eccentricity in the Sun and planets introduced by Copernicus — which, that man of lofty judgment foresaw, plainly runs counter forever to the nature of the celestial revolutions, and could be drawn only from false principles, that is, from worthless observations (as it appears).

The restitution, then, of the Sun's eccentricity and of the apogee's motion — accomplished in this commentary of ours no less laboriously than truly — I gladly submit to the fair judgment of the understanding. As for what remains regarding the mean annual quantity, from which the Sun's mean motion is composed, I shall give a foretaste in just a few words — only so much as the examination of the lunar course and the determination of the apparent motion of the fixed stars in earlier ages require — intending to hand down a fuller and more precise knowledge of it below, in its proper place, from many testimonies duly collected.

On the mean annual measure, from which the Sun's mean motion within certain epochs of the ancients depends — at least for the motion of the Moon and the apparent positions of the fixed stars, to be examined in what follows.

So far, in comparing Hipparchus's equinoxes — especially the first vernal one — with Tycho's, and in reducing each to the moments of the mean motions, as is fitting and as I was able, I have found the anticipation [of the equinox] over 1,732 years to be very nearly 12 days, 20 hours. For that Hipparchan vernal equinox occurred in the year 146 before the birth of Christ, in the month of March, on the 24th day, at 11 hours before noon — if we reckon Julian years backward — which year corresponds to the third in the leap-year cycle, and into which also falls the year 1587 after Christ's birth. In that year the vernal equinox was likewise observed by Tycho on 10 March, at 14 hours, 56 minutes, at Uraniborg. So when you have first added, for the difference of meridians relative to our equinox, 1 hour, 35 minutes; then, for the Sun's inequality, 3 hours, 50 minutes; then also for the unequal precession of the equinoxes, insofar as it is so far known to us, about 7 hours — this equinox is reduced to the meridian of Alexandria on 11 March, at 3½ hours, so that the anticipation of this equinox from the prior Hipparchan one comes to 12 days, 19½ hours. This, however, when deducted from the autumnal equinoxes, does not certainly agree — nor does Hipparchus's annual quantity, derived from his own observations, as I touched on above.

Retaining the anticipation we found of 12 days, 20 hours, and dividing it by 1,732 years, we find that each Julian year falls short by 10½ minutes, and that the length of the year we seek is very nearly 365 days, 7 hours, 43½ minutes. Although this may perhaps change in small particulars once the examination of the fixed stars has been added, nevertheless, while we are concerned with restoring the lunar course among the ancients, we assume the same length to be still a little greater — on account of the ancient supposition — namely 365 days, 5 hours, 49½ minutes. From such a given annual quantity there results the Sun's

motion in one Egyptian year, which consists of 365 whole days: 11 signs, 29 degrees, 45 minutes, 38 seconds, 46 thirds — from which Ptolemy falls short by 14 seconds; while the Alphonsine and Copernican values exceed it by as much as $\frac{2}{3}$ of one second, in keeping with the mean annual quantity found a little smaller by these same authors.

It now remains for us to constrain the simple longitude of the Sun and the motion of the apogee from the mean equinox to the two earliest epochs the ancients used — namely those of Nabonassar and of Alexander the Great. [Margin: *Ptolemy, lib. 3, cap. 8.*] Since both of these alike begin from noon of the first day of Thoth, the Egyptian month: because 424 complete Egyptian years intervene, therefore, when we set up the reckoning from the first vernal equinox of Hipparchus mentioned above — most rightly observed, as it appears — to the epoch of the reign of Nabonassar, there are found complete Egyptian years (that is, our common ones) 601, days 176, with only one hour falling short. Now, with the Sun's apogee placed, in Hipparchus's time, at $5\frac{1}{2}$ degrees of Gemini [Π], and with the rest in inequality both for the equinox (as we estimate) in Hipparchus's time...

I find the Sun's mean motion to differ from the mean equinox at the same time, by $1^{\circ} 42'$ in the direction of the preceding signs, less than its true [position] — at the prosthaphaeresis [$\pi\rho\omicron\sigma\theta\alpha\varphi\acute{\alpha}\rho\epsilon\sigma\iota\varsigma$] of the Sun. Now over the interval stated, between this observation of Hipparchus and the epoch of Nabonassar, there accumulate — according to the Sun's motion in the Egyptian year set out above — $29^{\circ} 39\frac{2}{3}'$ beyond the complete revolutions. When these are subtracted from that epoch of the Hipparchic equinox, there remains the Sun's mean course at the epoch of Nabonassar, and likewise for the meridian of [Alexandria], at $28^{\circ} 38\frac{1}{3}'$ of ♈ [Aquarius]. In the same way, at the epoch of the death of Alexander the Great, the Sun's mean motion from the mean equinox is gathered to have been at $17^{\circ} 12\frac{1}{4}'$ of ♍ [Virgo].

As for the Sun's apogee and the rest of these ancient initial positions, its derivation from the beginning of the world is straightforward, following what we have left demonstrated above on this same point.

These things we have necessarily laid down as a foundation regarding the restoration of the Sun — namely, concerning its absolute and unchanging eccentricity, and the simple motion of the apogee, and likewise concerning the annual measure (in a rough-and-ready way) — so that we might have ready to hand, for what follows, the Sun's motions agreeing with the named ancient epochs, and might then properly proceed next to examining the lunar appearances in these commentaries.

II.

A COMMENTARY, ON THE MOON.

In the preceding commentary we obtained two results, gained with great effort, toward restoring the Sun's motion against [Copernicus] — namely the immutability of its eccentricity [ἀκριβεστέρας, more exact], and on that account also the simple progression of the apogee from ancient times; and these two, given the condition of the hypothesis, cannot be torn apart from one another. The second of our labors — labors plainly Herculean — now stands ready: to set forth the course of the Moon (called by the Hebrews *Lebanah*, from "whiteness"), in conformity, so far as can be done, with the celestial appearances, even back to the ancient ages. I judge this necessary not only for establishing the hypothesis of each luminary to be built below, but also for showing the apparent motion of the fixed stars successively, in the order of the signs, and consequently the unequal anticipation of the equinoxes — all of which are rightly founded upon the verified positions of the luminaries. There can be no doubt that the final restoration of the lunar hypothesis, which I completed in Bohemia in the jubilee year 1600 with the noble [Tycho Brahe], so matches its celestial appearances in all respects and through all the ages (with a few mean motions kept within limits, and beyond that restored for the ancient ages) that nothing further could be called into question by anyone concerning it — unless someone, like Christmann of Heidelberg [Christmannus Heidelbergensis], with labor altogether novel, inept, and useless, would rather seek infamy for his own name than emulate, as is fitting, the celestial appearances here and elsewhere.

That the Moon has never undergone any change as to its prosthaphaereses, a simple comparison will readily prove, provided we first note this: the ancients sought out the eccentricity of the Moon only by sets of three eclipses on each occasion — from which it came about that they rashly and undeservedly excluded the equant from it.

But from such a triad of eclipses — a method introduced more compendiously than prudently by [Ptolemy], or perhaps by his predecessors, and afterward employed by Copernicus with inept applause (since it can only be used where the Sun is content with a simple equation; by our resolution, were there need, it could be set out far more compendiously, without [Euclid] III.[?] penult.) — we have been strongly pressed [Margin: *Lib. 7. pág. 5. Euclid.*], inasmuch as it is easy to enclose any three points whatever, projected in any manner outside a straight line, within the circumference of a circle. But how much this apparent ease has

harmed the truth, the modern eclipses show, and much more still the remaining lunar appearances — which we shall afterward convince more fully did not even fit the ancient constitution; and from this we shall produce the cause why an evident error crept into the ancients' registering of the fixed stars (which was done by means of the Moon), transmitted by a most unjust jurisdiction all the way to the Sun and the annual measure. Hence I think I have gotten just cause to call everything back to the anvil, and, according to the Sun's annual Egyptian motion—

—found in the preceding commentary the examination [δοκιμασία] of the ancient eclipses, to set them up anew, indeed by triads for each age, which for brevity's sake we shall present in the following table, making the motions both of the Sun and of the Moon not from the true but from the mean equinox — which in this comparison amounts to the same thing. In this comparison, with our more recent prosthaphaereses applied, the mean anomalies of the Sun alone, agreeing with the mean eclipses according to the modern hypothesis, are first set before the eyes, so that the difference in each individual case may appear. But the small equation of time which we hinted above arises from the oblique track of the Sun relative to the equator has, in this commentary, been deliberately omitted by us — so that it may be more thoroughly explored, by means of the subsequent eclipses (chiefly those occurring in our own age), whether there is also some other [equation].

The motions of the luminaries at the epoch of Nabonassar and the meridian of Alexandria — those of the Sun indeed as fixed by us above, those of the Moon reduced from the Prutenic Tables — sought out [as follows]:

	Sig.	Gr.	Min.
Simple longitude of the Sun from the mean equinox	10	28	38 $\frac{1}{3}$
Simple longitude of the Sun's apogee	1	25	22
Longitude of the Moon from the Sun	2	10	18
Anomaly of the Moon	8	28	43

[Margin: *Ptol. lib. 4. cap. 6.*]

Three eclipses of the Moon of the ancient Assyrians.

N A u n mn Me . o nse	Simp. Solis Lon. (Sig. G. M.)	Lon. ☽ ☆ ☉ (Sig. G. M.)	Anomal. ☽ (Sig. G. M.)	Verus lo. ☉ (Sig. G. M.)	Verus lo. ☽ (Sig. G. M.)	Diffe rent. (M.)
1 2 Th 7 oth	11 20 23	6 6 14	2 6 6	☾ 22 13	☾ 22 7	6 -
2 2 Th 8 oth	11 9 23	6 3 2	0 12 17	☾ 11 22	☾ 11 24	2 +
3 2 Ph 8 am e[n oth]	5 3 44	5 29 7	5 13 3	☾ 1 40	☿ 1 25	15 -

[Margin: *Ptol. lib. 4. cap. 11.*]

Three eclipses of the Moon observed by Hipparchus.

M N A e u n n mn s . o e	Simp. Solis Lon. (Sig. G. M.)	Lon. ☽ ☆ ☉ (Sig. G. M.)	Anomal. ☽ (Sig. G. M.)	Verus lo. ☉ (Sig. G. M.)	Verus lo. ☽ (Sig. G. M.)	Differ ent. (M.)
1 5 M 4 e 7 s o r i	5 28 21	5 23 46	10 0 23	☾ 26 27	☿ 26 22	5 --
2 5 M 4 e 8 c h y r	11 24 3	6 6 55	3 19 33	☿ 26 8	☾ 26 14	6 +
3 5 M 4 e 8 s	3 17 34	5 23 11	8 9 38	☾ 15 34	☿ 15 28	6 --

M						
N A e						
u n n	Simp.	Lon. ☽ ☆	Anomal.	Verus lo.	Verus lo.	Differ
mn s	Solis Lon.	☉ (Sig. G.	☽ (Sig. G.	☉ (Sig. G.	☽ (Sig. G.	ent.
. o e	(Sig. G. M.)	M.)	M.)	M.)	M.)	(M.)
o						
r						
i						

[Margin: *Ptol. lib. 4. cap. 6.*]

Triad of eclipses of the Moon, the Ptolemaic.

N A	Simp.					
u n	Solis Lon.	Lon. ☽ ☆	Anomal.	Verus lo.	Verus lo.	Diffe
mn Me	(Sig. G.	☉ (Sig. G.	☽ (Sig. G.	☉ (Sig. G.	☽ (Sig. G.	rent.
. o nse	M.)	M.)	M.)	M.)	M.)	(M.)
1 8 Pau	1 13 38	5 27 41	10 14 19	☿ 14 33	♊ 14 48	15 +
8 ni						
0						
2 8 Chi	6 28 1	6 2 57	2 4 41	♈ 26 37	♊ 26 37	0
8 ac[						
2 h]						
3 8 Pha	11 12 47	6 5 18	4 26 40	♋ 14 49	♊ 15 15	26 +
8 rm[						
4 out						
hi]						

EXPLICATIO.

We have drawn these eclipses of the Moon, set out above, from Ptolemy's record, so that they might in some measure corroborate by examination the [following] traces of our own restoration of the luminaries. About these I have thought it well to give a general warning. First, that no individual exactness [ἀκρίβεια] is to be found in any of those observed by the ancients — since the beginnings of the Moon's obscuration are in some cases indicated only approximately, and from these the reduction to the mean moments is not sufficiently true or fitting. But in Ptolemy's own—

—[records] neither the beginnings nor the ends are assigned, but only the computation of the mean times. Beyond this uncertainty, one may also doubt

whether the moments of the eclipses reported by the ancients were derived from the celestial revolution, or from clocks or water-clocks, which are wont to deceive. We, however, since we rely above all on the Hipparchic [observations], have thus restored the last of them. Ptolemy reports that its beginning was 40 minutes after midnight at [Alexandria], and since the Sun was at the middle of ♍ [Virgo], nothing perceptible in equal hours results from the distance of 40 minutes from midnight. Hence we have noted the half-duration, according to the calculation of Copernicus (who in other respects does not greatly err in this matter), to be 1h 56m after M[id]N[ight].

Moreover, since Ptolemy's error here is found to be greater than the rest — whether because he conceded something to the time on purpose, so that, the mean motions of the luminaries being retained in their even constancy below, he might uphold the Sun's prosthaphaeresis alone, supposing it equal to the Hipparchic; or whether the observations were carelessly carried out by novices, which sometimes appears in this and in other matters too in Ptolemy — however that may be, we, calling this Ptolemaic eclipse back to the anvil by the motion in latitude, found from the tables of Reinhold (which prove to behave tolerably in the apparent eclipses among the ancients) the Moon's true latitude to be at least 46 minutes above the tail of the Dragon, at the time noted by Ptolemy. From this, with the half-diameter of the Earth's shadow at 49 minutes, the eclipse comes out greater than Ptolemy's record by 3 minutes — whereas he has only half [that]. From this it is easily inferred that he drew out the mean time of this eclipse itself longer than was right. We therefore, dropping Ptolemy almost entirely, will likewise remain in the ancient and the Hipparchic [observations] — in which the beginnings of the eclipses are in some fashion reported, and which we shall therefore use chiefly for the ancient ages hereafter, whenever the mean motions are afterward elicited or the examinations of the stars are set up. Now let us pass over to the observations of [Tycho Brahe] and our own, likewise, in the lunar eclipses.

Synopsis and examination [δοκιμασία] of the lunar eclipses observed in our own age, and reduced apparently to the meridian of [Uraniborg]: with the epoch fixed from the birth of Christ.

N A						Verus lunæ	
u n		D				locus (Sig. G.	Different
m n	Me	i	Anomalia ☾	Verus locus	locus (Sig. G.		
.	o nse	e	(Sig. G. M.)	Solis (Sig. G. M.)	M.)		. (M.)
1 1	Dec	8	7 24 24	♄ 26 50	II 26 51		1 +

N A u n m n . o	Me nse	D i e	Anomalia ☽ (Sig. G. M.)	Verus locus Solis (Sig. G. M.)	Verus lunæ locus (Sig. G. M.)	Different . (M.)
5 7 3	em b.					
2 1 5 7 6	Oct ob.	7	2 5 31	♌ 24 30 $\frac{1}{3}$	♍ 24 33	2 $\frac{2}{3}$ +
3 1 5 7 7	Apr ili	2	7 6 34	♍ 22 47 $\frac{1}{6}$	♌ 22 52	4 $\frac{5}{6}$ +
4 1 5 7 7	Sep tem b.	2 6	5 28 56	♌ 13 23 $\frac{1}{3}$	♍ 13 24 $\frac{2}{3}$	1 $\frac{1}{3}$ +
5 1 5 7 8	Sep tem b.	1 5	10 16 34	♌ 2 19	♍ 2 21 $\frac{1}{4}$	2 $\frac{1}{4}$ +
6 1 5 8 0	Ian uar.	3 1	1 6 29	♍ 21 28 $\frac{5}{6}$	♌ 21 25 $\frac{3}{4}$	3 -
7 1 5 8 1	Ian uar.	1 9	11 21 23	♍ 10 4 $\frac{5}{6}$	♌ 10 2	2 $\frac{5}{6}$ -
8 1 5 8 1	Iuli o	1 5	4 27 43	♌ 3 40 $\frac{5}{6}$	♍ 3 37 $\frac{1}{2}$	3 $\frac{1}{2}$ -
9 1 5	Nov em	7	4 7 19	♍ 25 49 $\frac{1}{4}$	♌ 25 50 $\frac{3}{5}$	1 $\frac{1}{3}$ +

N A u n m n . o	Me nse	D i e	Anomalia ☽ (Sig. G. M.)	Verus locus Solis (Sig. G. M.)	Verus lunæ locus (Sig. G. M.)	Different . (M.)
8 4	b.					
1 1 0 5	Sep tem	6	10 1 21	♊ 23 8 $\frac{2}{3}$	♋ 23 10 $\frac{2}{3}$	2 -
8 7	b.					
1 1 1 5	Ma rtio	2	3 20 2	♋ 22 49	♊ 22 48	1 -
8 8						
1 1 2 5	Dec em	3 0	9 11 47	♊ 19 1 $\frac{1}{3}$	♋ 19 4 $\frac{2}{3}$	3 $\frac{1}{3}$ +
9 0	b.					
1 1 3 5	Iun io	1 4	1 4 10	♋ 3 15	♊ 3 16	1 -
9 2						
1 1 4 5	Dec em	8	6 5 15	♊ 27 15 $\frac{1}{3}$	♋ 27 16 $\frac{1}{4}$	5 $\frac{1}{6}$ +
9 2	b.					
1 1 5 5	Oct ob.	1 9	2 2 40	♋ 5 29 $\frac{1}{2}$	♊ 5 31 $\frac{1}{2}$	2 +
9 4						
1 1 6 5	Apr ili	1 3	7 3 43	♊ 3 24 $\frac{1}{2}$	♋ 3 29	4 $\frac{1}{2}$ +
9 5						
1 1 7 5	Oct ob.	7	0 8 20	♋ 24 15 $\frac{3}{4}$	♊ 24 18 $\frac{1}{3}$	2 $\frac{2}{3}$ +
9						

N u m .	A n n o nse	D i e	Anomalia ☾ (Sig. G. M.)	Verus locus Solis (Sig. G. M.)	Verus lunæ locus (Sig. G. M.)	Different . (M.)
5						
1 8	1 5	Apr ili	2	5 17 32	♈ 23 9⅔	♎ 23 13¼ 3⅔ +
9						
6						
1 9	1 5	Feb ru.	1 0	1 13 44	♊ 2 32	♏ 2 30⅔ 1⅓ –
9						
8						
2 0	1 5	Aug ust.	6	6 10 31	♏ 23 12¼	♊ 23 8⅓ 4 –
9						
8						
2 1	1 5	Ian uar.	3 0	11 18 37	♊ 21 11½	♏ 21 10½ 1 –
9						
9						
2 2	1 6	Iuli o	6	2 23 5	♋ 24 10	♏ 24 12⅕ 2⅕ +
0						
9						
2 3	1 6	Oct ob.	1 8	0 5 19	♌ 5 13¼	♏ 5 13⅝ ⅔ +
1						
3						

DECLARATION

These 23 lunar eclipses were for the most part observed at Hven, with two, three, and sometimes four of Tycho Brahe's most accurate instruments, both as to their beginnings, their ends, and the other notable phases — so far at least as the sky allowed. They were acquired with such diligence, and reduced to the true opposition of the luminaries, as had never been achieved by our predecessors. The remaining ones were for the most part taken here at Copenhagen [Haffnia],

on a meridian very close to that of Hven, with no less care and skill — just like those eclipses which are to be drawn in below as worked examples of the doctrine of eclipses, and are therefore omitted at this point.

And although the solar eclipses are not added here — for the sake of brevity, and indeed because of the intricacy of parallax — nevertheless experience in observing them and properly restoring them had taught us that they differ in nothing from the lunar ones. Indeed, when the matter is properly examined, no greater difference in the motions of the luminaries ever arises in conjunction than in opposition, when these are set against one another. From this one may see that we have brought the restoration of the courses of the luminaries to as great a perfection as our age allows, and moreover that the equation of time [aequatio temporis] which we abolished above is granted no place here. For nothing — except some small and almost unstable quantity, arising from the cause of the obliquity of the zodiac [obliquitas signiferi] — appears to be insinuated to us through the comparison of these eclipses. Anyone skilled will readily detect this from comparing the obliquity of the ecliptic with the equator in the various signs of the zodiac through which the Sun passes year by year.

We, however, accommodating our limitation to this same point — with the other differences besides, well scrutinized among themselves — reject only a single minute from the motion of the simple longitude of the Moon from the Sun [motus simplicis longitudinis lunae a sole] which was inserted into Tycho's final restoration; the anomaly and the prosthaphaereses we retain in full, just as the Tychonic tables have them.

And so the luminaries have been restored in conformity with their celestial motions, to such a degree that you could scarcely ever hope for greater precision. This — clearly enough, both through the eclipses and through the other celestial phenomena of the luminaries — those few who deign to give any care to such matters, amid these sordid worldly concerns, will readily confirm together with us. But, consulting brevity, we shall now complete the remaining web of our intended commentaries on the fixed stars.

III. COMMENTARY ON THE FIXED STARS

Up to this point we have examined the courses of the Sun and Moon, with respect to both ancient and our own times, from observations gathered for that purpose, and we have restored them — so that not only would the way be in some manner opened to constructing their hypotheses and tables afterward in their proper places, but also so that from this the foundation of the fixed stars might here and

there be laid. For among the ancients this foundation was for the most part established by means of the Moon.

First, then, the place of the Moon must be carefully computed by us, according to its visible longitude, from our recent restoration, applied to the moments of the conjunction of the Moon with certain fixed stars observed by Timocharis and recorded in Ptolemy [*Almagest* VII.4] — and now diligently revised and corrected by us. The aim is that, once the limitation has been duly established, the apparent places and seats of those same stars may be assigned for a definite year of Timocharis's age. This foundation will afterward suit, on equal terms, the description drawn up from Tycho Brahe's own observations — the absolute foundation [*absoluta fundamentum*] — observations which we ourselves once drank in with our own eyes, applying to all the fixed stars alike, since they have never suffered any displacement from one another.

Then, descending through Hipparchus to Ptolemy, we shall likewise weigh this latter's observation made between the Sun, the Moon, and the Basiliscus [Regulus], and shall compare it with Timocharis and with Hipparchus. Here, beyond doubt, a swifter motion of the fixed stars than the opinion of those ancients will betray itself — confirmed as well by the declination of certain fixed stars, obtained by the various ancients (Timocharis, Hipparchus, and Ptolemy [*Almagest* VII.3]), although the time in them is not sufficiently indicated, just as Ptolemy also recounts these observations. Afterward, descending to al-Battānī [Albategnius], since at present we are deprived of him in person, we shall rest content with Copernicus's report concerning him. Finally, with the infallible Tychonic foundation of the fixed stars appended below — which a most skilful and accurate comparison of all these is to establish — we shall set out, for the procession of the equinoxes [*praecessio aequinoctiorum*], both mean and true, to arrange first of all the hypothesis of the equinoxes.

I. As regards the observations of Timocharis on the conjunction of the Moon with the stars, we have selected three of the fixed stars, namely Lucida of the Pleiades, the Spica of Virgo, and the northern brow of Scorpius — observed once each — among which Spica holds the chief place. We have therefore assumed the other two, one on each side, so that the limitation of this one may be drawn more correctly. The place of the Moon, computed from our restoration for the individual corrected moments of the observations — corrected, among other things, on account of the reduction of seasonal time to equal time, which is found in Ptolemy in a less suitable form — is set forth in the following synopsis, and is

compared with Timocharis, with the epoch everywhere deduced from Nabonassar.

Names of the fixed stars	Y e M a on r th	D H. M. a (Timocha y r.)	H. M. (Correcte d)	Place of ☽ (Timo.) S. G. M.	Place of ☽ (restored) S. G. M.
Northern brow of Scorpius	4 Ph 5 ao 4 ph i	1 15 24 6	15 30	♌ 2 0	♌ 1 58
Lucida of the Pleiades	4 At 6 hi 5 r	2 8 40 9	8 40	♊ 29 20	♊ 29 32
Spica of Virgo	4 Th 6 ot 6 h	7 16 7	15 54	♍ 22 30	♍ 22 40

Since this restoration of ours does not differ much from Timocharis's observation — provided that his true equinox is everywhere reduced to the mean by 6 minutes — let us therefore set Spica ♍ at the beginning of year 466 of Nabonassar, at 22 degrees 30 minutes, especially when the true celestial distance between Spica itself and the star in the brow of Scorpius (whose observation I judge to have been more reliable than that of the Pleiades) requires it.

Then, since concerning Hipparchus's observation of the heart of Leo [*cor Leonis*] — which Ptolemy recounts [*Almagest* VII.2] at 29 degrees 50 minutes of Cancer ♋ (that is, at the beginning of Leo ♌, according to the mean equinox) — a doubt may arise, because it is most uncertain by what method of observation he tracked down its place, we therefore make use of a declination of Spica of 0 degrees 36 minutes [B] supposed for Hipparchus's age. From this, according to the requirement of the triangles [*Almagest* I.5], we find Spica at 24 degrees 19 minutes of Virgo ♍, and with respect to the mean equinox at 24 degrees 29 minutes of Virgo ♍.

Further, we come to Ptolemy, who in the second year of the emperor Antoninus Pius — that is, in the year 886 from Nabonassar, on the 9th day of Pharmuthi, at sunset — by the following method (adjusted to our emendation), explored the distance of the heart of Leo from the summer solstice. At the 5½th hour after noon, by means of an astrolabe, as he reports, he took the centers of the

luminaries at a distance of $92\frac{1}{8}$ degrees from one another. Now since the Sun's place, measured from the mean equinox according to our restoration, was then in 4 degrees 3 minutes of Pisces ♊, and since the Sun moreover had — over and above parallax in the ecliptic and on the Alexandrian horizon — a refraction of no less than 20 minutes (perhaps even somewhat greater), when we subtract roughly a third part of what occurs in the Sun here with us, the visible longitude of the Sun comes out as 4 degrees 23 minutes of Pisces ♊ from the mean equinox, as I said. To this, if the observed elongation of the Moon be added, the visible place of the Moon will be found at the $5\frac{1}{2}$ th hour in 6 degrees 31 minutes of Gemini ♊, very nearly.

Afterward, however, about the half hour, the heart of Leo, detected by means of the same astrolabe from the center of the Moon, is reported to lie 37 degrees 10 minutes distant in the order of the ecliptic [*in consequentia*]. And since the Moon over the half hour (excluding the increment of the parallax of longitude) was advancing by 12 minutes, therefore, if we add all this to the prior apparent observed place of the Moon, the longitude of Basiliscus, or of the star in the heart of Leo ♌, is found to be 3 degrees 53 minutes — which Ptolemy, both because of the defect in his motion of the Sun (which was refuted above) and because of his neglect of refraction, asserted by his own deduction to have been placed in $2\frac{1}{2}$ degrees of Leo ♌ from the true equinox.

Finally, this earlier antiquity being thus transmitted, and a great deal of time having intervened, let us pass on to al-Battānī, who is recorded to have found Basiliscus, at Raqqa [Aracta] in Syria, in the year 1626 from Nabonassar, in 14 degrees $1\frac{1}{2}$ minutes of Leo ♌, and the northern brow of Scorpius in 17 degrees 50 minutes of Scorpius ♏. These, compared with the true distance of the stars in Tycho's table, admit a difference of 26 minutes, which argues an uncertainty in the observation. But when we reduce Basiliscus to the mean equinox (which here was 22 minutes more remote in the order of the signs than from the true, as will be shown afterward) on the basis of al-Battānī's observation, it will be at 14 degrees 27 minutes of Leo ♌.

Now since the Alphonsines were deceived and almost blinded in this business — as the reader will likewise perceive in Georg Peurbach [Peurbachius] and Johannes Regiomontanus [Regiomontanus] — and since the errors of Copernicus regarding the Sun have been demonstrated above from Tycho and from our own observation, therefore, in the last place,

— in order to come to the solid foundation which our Tycho, before all others, had laid for the fixed stars, for the end of the Julian year 1600 from the birth of

Christ [N.C.] (which he set as the Egyptian, or equalized, year 2350 from Nabonassar). There, from manifold and sufficient observation of Venus, on either side of the Sun by day, and then of the same Venus by night against various fixed stars, and at various times of the year, he set forth the indubitable terminus of their motion — just as he himself most fully and skilfully demonstrates in *Progymnasmata* I, chapter 2, to which therefore we refer the reader. For the aforesaid year, Basiliscus was found in 24 degrees 17 minutes of Leo ♌ from the true equinox, and Spica ♏ in 18 degrees 16 minutes of Libra ♎. But since the inequality of the equinoxes [*inaequalitas aequinoctiorum*] subtracts about 8 minutes in our age, Basiliscus will be in 24 degrees 9 minutes of Leo ♌, and Spica in 18 degrees 8 minutes of Libra ♎ with respect to the mean equinox.

These places, thus correctly obtained, are set out together in a brief synopsis for inspection, and afterward, for the purpose of drawing out the annual motion of the stars, are compared among themselves, with the intervals of time reckoned backward from our age:

Timmoharris			Hiipparchus			Ptolemy			al-Battānī			Tycho Brahe		
Years of star	Name of star	Place of star	Years of star	Name of star	Place of star	Years of star	Name of star	Place of star	Years of star	Name of star	Place of star	Years of star	Name of star	Place of star
1884	Spica ♏	♏ G.M. .2230	1720	Spica ♏	♏ G.M. .2425	1464	Basilis ♏	♏ G.M. .3403	724	Basilis ♏	♏ G.M. .1427	0	Spica ♏	♏ G.M. .1816

From this, the mean annual motion of the fixed stars — that is, the simple anticipation of the equinoxes — is proportionally educed, as appears in what follows.

The annual motion of the stars

Sec.	Tert.	
51	0	from Ptolemy to al-Battānī
49	46	from Hipparchus to Tycho
49	37	from Ptolemy to Tycho
49	10	from Timocharis to Tycho
48	14	from al-Battānī to Tycho
46	0	from Timocharis to Ptolemy
44	0	from Timocharis to Hipparchus

From these, after the foregoing emendation has been diligently compared among themselves — due account always being taken both of the longer interval of time and of the more authoritative observation — no greater advance of the mean fixed star, such as appears in the sky in one equal year, emerges than 49 seconds 45 thirds, which in $72\frac{1}{3}$ years complete 1 degree [⊕ 1 gr.] [the source uses a ⊕ glyph for "+"]. Nor are we troubled that some fairly perceptible difference intervenes between Timocharis and Ptolemy, and a still greater one between Timocharis and Hipparchus, since here, on account of the small intervals of time, that difference nowhere exceeds 20 minutes — which is rather to be attributed to the observations of those men than to any greater inequality of the equinoxes, apart of course from that which will shortly below be shown to agree well enough with the hypothesis.

Although indeed the supposition of both Copernicus and our Tycho concerning the annual advance of the stars in the order of the signs [*in consequentia*] — or rather of the equinoxes in the contrary order [*in antecedentia*] — surpasses ours by a little (since the former had set it at 50 minutes 12 thirds, the latter at precisely 51 seconds), nevertheless, because I see that this befell them chiefly through using the observations of Hipparchus and Ptolemy without correction for refraction and for the inequality of the equinoxes, therefore it was by no means open to us to depart further from Timocharis's foundation, or even from Ptolemy's restored observation. Concerning al-Battānī, this is to be reported in this place: namely, that it is here clearly established that the precession of the equinox, from Ptolemy to him and from him down to us now, agrees more correctly with the stars than with the Sun's course and the annual quantity deduced here and there by him — which will doubtless appear, as I recall having earlier admonished myself regarding the Sun, in connection with the weighing of al-Battānī's observation.

It still remains, for the completion of this commentary of ours, to treat of the obliquity of the path of the Sun from the equatorial circle, as varied according to the intervals of time — and not, as Copernicus reports [*De Revolutionibus* III.3 and following], of the obliquity of the equatorial to the ecliptic. This is most manifestly clear from those things which Lord Tycho Brahe had adduced from the observations of earlier times and of his own, made on the fixed stars, by a demonstration of comparison [*Progymnasmata*, p. 233 and following], showing how they themselves consistently varied the latitude in accordance with the alteration of the Sun's obliquity.

We ourselves, however, regarding the variation of this obliquity of the ecliptic — once the individual findings of our predecessors have been diligently examined, and corrected for the parallax of the Sun (Copernicus, in the very last place after Ptolemy, reporting on this matter [*De Revolutionibus* III.2, near the end], whose supposition too we have restored from Tycho's and our own emendation) — have found that the maximum obliquity, around the true year of the World 3600, was once restored to 23 degrees 53 minutes, and the minimum to 23 degrees 31 minutes very nearly, from the year 5400 of the same world's creation (which is the year of Christ 1434). Hence the mean is gathered as 23 degrees 42 minutes — which Copernicus put merely at 23 degrees 40 minutes.

And this our determination of the obliquity — how it answers to the celestial truth, even from the intermediate observations — we shall demonstrate together with the decrement and increment, likewise with the inequality of the equinoxes, and finally with the variation of the latitude of the stars, by one congruent hypothesis. Now, the Lernaean hydra having been slain by these commentaries — which lay the foundation of the second part of the first book of Astronomy — we attribute praise and triumph to God the Archduke with a most humble heart, and gladly undertake what remains under His auspices as well.

CHAPTER III

On the Epochs of the celestial motions henceforth employed by us, and on the discrimination of Times — both on account of the Sun's oblique passage at the daily turning, and on account of the differences of meridians — together with the appended tables.

[Margin: See Reinhold's 4th precept, *Prutenic Tables*.]

Although various Epochs, to which the mean or equal celestial motions are referred, are found among our predecessor astronomers — for instance, the

Epoch of the Olympiad, of Nabonassar, of Alexander the Great, of Julius Caesar, of Christ in the flesh, of Antoninus, and of many others — nevertheless to us Christians two beginnings ought to be, before the rest, the most illustrious, by reason of a truly divine favor toward us:

First, when this most beautiful theater of the world began to be by the word of Almighty God; the other, when the only-begotten Son of God himself deigned to clothe himself in our human flesh and to be born, so as to restore the fallen world and to free us from the power of the devil and from eternal death. To these beginnings — set fairly far apart from one another by a long interval — we suspend and number the mean celestial motions in this perennial restoration of those motions (which we undertake with the help of the good God). Nor does our undertaking find any hindrance here in the fact that both beginnings are today called into doubt by various chronologists, both as to the number of years and as to the time of year; for this disagreement entangles the celestial motions with no difficulty, since they have been rightly deduced from other intervals well secured by celestial observation.

I hope, moreover, that it will be most welcome to the chroniclers of truth that, from our investigation, they may hear first of all that they are vehemently confirmed, in their annals, in rightly establishing the beginning of the world by the apparent motion of the Sun's apogee, again by the maximum obliquity, and also by the inequality of the equinoxes. [Margin: See above, near the end of the Commentary on the Sun.] For hence, from the time of creation up to the passion of the Son of God on the cross, and the restoration of the world through him, there are correctly gathered, as elapsed, 4000 solar years — with a half-year span, as it were, falling short, as we showed above in the commentaries — seeing that the perigee place of the Sun at the time of the world's founding had clung at the beginning of Libra ♎, without any inequality of the equinox, and the maximum obliquity of the Sun had then existed: all of which agree wonderfully with the stated time, and with these φαίνόμενα [Greek: phainomena, "appearances," i.e. observed phenomena] compared among themselves.

This Epoch of the founding of the world, then, we shall retain as fixed, as to the number of Julian years, especially because by various more recent critics of history either this very number is most nearly touched upon, or another is by the same advanced

at variance on both sides. Nor do we doubt that the world was created by God at the autumnal equinox, on account of various conjectures and testimonies. For the Hebrews not only have, from the very first moment of the world down to the

present, taken the beginning of their year from this point [Margin: Exod. 12 etc.]; but even when God willed to alter the form of the year and to bring everything into accord with the time of His Son's Passion, He compelled His people to it by a new command. For more on this controversy to be read in other authors, and for weighing the truth that without doubt supports this opinion, I refrain here and refer the reader to my own *Disputation on the Epochs*. [Margin: Joseph. Scalig.; S. Calvisius.]

Furthermore, the other Epoch I mentioned — that of the Messiah sent in the flesh — we likewise retain together with the ancient Fathers, and we assign to the life of Christ on this earth about 33½ solar years only. This is both because of the prophecy of Daniel [Margin: Dan. cap. 9. v. 17.], and because of the types that are everywhere found in the sacred Scriptures concerning the number of 40 days, which — being as many solar months [Margin: 1. Sam. 17.] — beautifully prefigured the span from Christ's baptism to His Passion. In these types He began His struggles with the devil and with the world [Margin: Matth. 4.; Act. 1.], after, being 30 years old [Margin: Luc. 3. v. 23.] and baptized by John, He had taken up the office and ministry on account of which He had been sent by the Father. Nor indeed do I see any absolute necessity, urged by the more recent critics of history, for prolonging the interval of Christ's life [Margin: See among others Mercator, Buntingus, J. Scaliger, L. Sulligan, Polon, Joh. Kepler, Jos. Scaliger, Sethus Calv.], since even they themselves differ among one another by several years. And even if it were so, the celestial motions that we pursue in this work could not be thrown into confusion by it — any more than by the fact that the ancients set the birth of Christ in the flesh near the winter solstice, and others in October [Margin: Buntingus, etc.]. For we begin our Epoch from the first noon immediately following the Kalends of January, in the form of the Julian year (which we always use here in numbering the motions as the most common standard). And these Kalends most nearly follow the year of Christ as I have just asserted it, the year in use until now in astronomical calculation among all Christians — and rightly so, as is more fully discussed by us in the said *Disputation on the Epochs*.

For the rest, although between the rising of the world and Christ's Passion on the cross about 4000 solar (or Julian) years elapsed, as I said above, and therefore between the Epoch of the world and that of the incarnate Christ 3966 years had run out, nevertheless I see that the Epoch most conveniently applied to the numbering of the celestial motions is the one that corresponds to the noon of the Kalends of January in the third year current from the world's beginning — because it coincides with Christ's first current year in the leap-year cycle [bisextility]. Let there be reckoned, therefore, from the mundane Epoch to the

Christian, only 3964 full Julian years, so that, with the mean motions for each Epoch prefixed, we may be well occupied in properly extracting them in their δοκιμασία [Greek: dokimasia, "testing" or "verification"], as the examples below will show.

Easy, moreover, is the connection of the commonly assumed Olympiad (for about the true one there is doubt) [Margin: On the assertion of the true Olympiad, see also my *Disputation on the Epochs*, Precept 4, etc.], and likewise of the Epochs of Nabonassar, of Alexander the Great, and of Scaliger, with our mundane one — as also of the Greek or Egyptian Epochs with the Julian months. But on all these matters, beyond the examples that we leave scattered throughout, let the most learned Erasmus Reinhold [Erasmus Reinhold] be consulted. So much for the Epochs.

As for what concerns the equation of the natural day, we establish it — as before, following Tycho [Tycho Brahe] alone — universally from the Sun's oblique path relative to the equator, the cause having been given above in the commentary. To this we assign the first of the immediately following tables [Margin: Tab. Prut.]; the second, to the difference of meridians on the earth. For time too, begun from the noon of one fixed place to which the Epoch of the celestial motions is bound, is altered in other places on account of the roundness of the earth — so that those to the east always reckon more time, and those to the west less, when the beginning is taken under that meridian where no inclination of the magnetic iron from the terrestrial pole is felt, the meridian the geographers call the Prime, and which some hold to be fixed at the Fortunate Isles in the Ocean. How much that time will be, the second table (which I obtained through Tycho's correction concerning the longitude and latitude of the more notable places, with my own assistance) [Margin: See above in the *Commentary on the Sun*, the reason for this matter] will easily yield in the following column: the degrees and minutes of the equator that fall in the difference of the remaining places from our own meridian of Copenhagen [Hafnia], duly converted into time and rightly applied to each of the other places, just as has been done here in accordance with what we set out in the doctrine of the Sphere.

Table of the Equation of the Natural Day

[*Tabula aequationis Diei naturalis.*]

The table reads with the descending index 0–30 in the left margin (for the signs ♈ Aries / ♎ Libra, ♉ Taurus / ♏ Scorpio, ♊ Gemini / ♐ Sagittarius listed across the top) and the ascending index 30–0 in the right margin (for the signs ♋ Pisces / ♌

Virgo, ♊ Aquarius / ♌ Leo, ♐ Capricorn / ♋ Cancer listed across the bottom). Each entry is in minutes (M.) and seconds (S.).

(top index)	♈ M.	♈ S.	♉ M.	♉ S.	♊ M.	♊ S.	(bottom index)
0	0	0	8	25	8	47	30
1	0	20	8	36	8	37	29
2	0	40	8	45	8	26	28
3	0	59	8	55	8	15	27
4	1	19	9	4	8	3	26
5	1	39	9	12	7	50	25
6	1	59	9	19	7	36	24
7	2	18	9	26	7	22	23
8	2	38	9	32	7	7	22
9	2	57	9	37	6	52	21
10	3	16	9	42	6	36	20
11	3	35	9	46	6	20	19
12	3	51	9	50	6	3	18
13	4	12	9	52	5	46	17
14	4	30	9	54	5	28	16
15	4	47	9	56	5	10	15
16	5	5	9	56	4	51	14
17	5	22	9	56	4	32	13
18	5	38	9	55	4	12	12
19	5	55	9	53	3	53	11
20	6	11	9	51	3	33	10
21	6	26	9	48	3	12	9
22	6	41	9	44	2	51	8
23	6	56	9	40	2	31	7
24	7	10	9	34	2	9	6
25	7	24	9	28	1	48	5
26	7	37	9	21	1	27	4
27	7	50	9	14	1	5	3

(top index)	♈ ♎ M.	♈ ♎ S.	♉ ♏ M.	♉ ♏ S.	♊ ♐ M.	♊ ♐ S.	(bottom index)
28	8	2	9	6	0	43	2
29	8	14	8	57	0	22	1
30	8	25	8	47	0	0	0
(bottom signs)	♋ ♍		♌ ♎		♍ ♏		

On the use of the present table

[*Vsus praesentis Tabellae.*]

The use of this table is as follows: entering with the sign and degree of the Sun's place (descending or ascending, according as the sign is found at the head or the foot of the table), the minutes and seconds which the field presents are to be added to, or subtracted from, the apparent time — according to what the heading requires — so that the equal [mean] time is produced. Proceed in the contrary way if the equal time is to be reduced to apparent.

There follow the longitudes and latitudes of the more notable places in Europe

corrected by Tycho Brahe [Tycho Brahe], partly from his own observation, partly from itinerary distance and from corrected maps.

[*Sequuntur longitudes & latitudines insigniorum in Europa locorum a Tychone Brahe partim propria observatione, partim itineraria distantia & correctis mappis emendata.*]

Each location carries: a Time-difference [Têp. = Temporis differentia] flagged **S** (subtractive, i.e. west of Copenhagen) or **A** (additive, i.e. east), in Hours and Minutes (H.M.); the Longitude [Long.] in Degrees and Minutes (G.M.); and the Latitude [Latit.] in Degrees and Minutes (G.M.). Copenhagen (HAFNIA Dan.) is the reference meridian, with all zeros in the time column.

Place [Nomina loc.]	S		Têp.	Têp.	Long.	Long.	Latit.	
	A	H					G	M
Aarhusia Cimbr. [Aarhus, Jutland]	S	0		9	34	30	56	16

Place [Nomina loc.]	S		Têp. M	Long. G	Long. M	Latit. G	Latit. M
	/	Têp. H					
Agrigentum [Agrigento]	A	0	18	41	5	38	31
Alba Regalis [Székesfehérvár]	A	0	28	43	35	47	18
Alcmar. Holl. [Alkmaar]	S	0	36	27	45	52	36
Aldenburg [Oldenburg]	A	0	1	36	50	51	0
Alexandria Ægy. [Alexandria, Egypt]	A	1	35	60	30	30	58
Amberg	S	0	2	36	15	49	28
Amiens	S	0	47	24	50	49	20
Amsterodamum [Amsterdam]	S	0	34	28	0	52	20
Ancona Italiæ [Ancona]	S	0	1	36	25	43	42
Annæburg [Annaberg]	A	0	3	37	25	50	43
Antverpia [Antwerp]	S	0	36	27	35	51	12
Argentina [Strasbourg]	S	0	21	31	30	48	30
Ariminum [Rimini]	A	0	7	38	30	43	53
Avenio [Avignon]	S	0	38	27	15	42	35
Augusta Vindeli. [Augsburg]	S	0	6	35	10	48	22
Aurelia [Orléans]	S	0	53	23	30	47	15
Babylon	A	2	25	73	0	35	0
Baerenburgium	S	0	16	32	40	52	40
Badena [Baden]	S	0	19	31	50	48	42
Barbye [Barby]	S	0	1	36	20	52	2
Basilæa [Basel]	S	0	21	31	15	47	38
Bambergia [Bamberg]	S	0	6	35	0	49	58
Bautzen	A	0	9	38	50	51	9
Berlinum [Berlin]	A	0	6	38	10	52	29
Bern	S	0	22	31	0	46	50
Bononia [Bologna]	S	0	1	36	30	43	57

Place [Nomina loc.]	S		Têp. M	Long. G	Long. M	Latit.	
	/	Têp. H				G	M
Brandenburgum [Brandenburg]	A	0	2	37	20	52	23
Bransisvigum [Braunschweig]	S	0	3	35	45	52	29
Brema [Bremen]	S	0	18	32	15	53	12
Breslaw [Breslau/Wrocław]	A	0	20	41	40	51	7
Brisach	S	0	22	31	15	48	4
Brensberg [Braunsberg]	A	0	24	43	35	52	46
Brundusium [Brindisi]	A	0	34	45	13	41	27
Bruxella [Brussels]	S	0	37	27	30	50	50
Berda [Breda]	A	0	31	44	20	47	19
Campen [Kampen]	S	0	32	28	40	52	47
Capua	A	0	17	40	55	41	42
Cassillæ [Castile?]	S	0	13	33	25	51	18
Cassovia [Košice]	A	0	38	46	5	48	32
Coburg	S	0	6	35	10	50	20
Colberg	A	0	16	40	40	53	45
Colmar	S	0	22	31	5	48	5
Colonia [Cologne]	S	0	25	30	30	50	55
Complutum [Alcalá de Henares]	S	1	12	18	35	41	0
Compostella [Santiago de Compostela]	S	1	40	11	45	43	0
Corduba [Córdoba]	S	1	20	16	45	38	0
Costentz [Konstanz]	S	0	14	33	5	47	32
Creutia	A	0	33	45	0	49	57
Cremona	S	0	8	34	40	44	15
Cubenbach	S	0	4	35	40	50	8

Place [Nomina loc.]	S		Têp. M	Long. G	Long. M	Latit.	
	/	Têp. H				G	M
Dantiscum [Danzig/Gdańsk]	A	0	31	44	20	54	0
Dessa [Dessau]	O	0	0	36	40	51	53
Daventria [Deventer]	S	0	29	29	27	52	14

[Right-hand block of the same opening, continuing alphabetically:]

Place [Nomina loc.]	S		Têp. M	Long. G	Long. M	Latit.	
	/	Têp. H				G	M
Dockum [Dokkum]	S	0	31	29	0	53	15
Dresda [Dresden]	A	0	6	38	5	51	6
Edenburg Ger. [Eisenberg?]	A	0	2	37	15	51	26
Elbinga [Elbing/Elbląg]	A	0	34	45	10	53	55
Elsenburgum	A	0	0	36	45	56	3
Emden	S	0	26	30	15	53	19
Enchusa [Enkhuizen]	S	0	33	28	20	52	42
Erfurdia [Erfurt]	S	0	5	35	20	51	7
Ferraria [Ferrara]	S	0	1	36	20	44	18
Flensburgum [Flensburg]	A	0	3	37	25	55	8
Florentia [Florence]	O	0	0	36	40	43	27
Francof. ad Moen [Frankfurt am Main]	S	0	18	32	15	50	2
Francof. ad Oder [Frankfurt an der Oder]	A	0	12	39	40	52	51
Frerberga [Freiberg]	A	0	5	37	50	50	52
Friburgum [Freiburg]	S	0	24	30	40	46	40
Geneva [Geneva]	S	0	10	29	5	45	25
Genua [Genoa]	S	0	12	33	40	43	12
Gent Fland. [Ghent]	S	0	40	26	45	51	4
Gorlitz [Görlitz]	A	0	12	39	35	51	9

Place [Nomina loc.]	S		Têp. M	Long. G	Long. M	Latit. G	Latit. M
	/	Têp. H					
Goslar	S	0	9	34	30	52	0
Gota [Gotha]	S	0	8	34	45	51	0
Granata [Granada]	S	1	14	18	15	37	30
Gran [Esztergom]	A	0	29	43	50	47	56
Graitz [Greiz]	A	0	17	40	50	47	22
Gryphiswaldum [Greifswald]	A	0	5	38	0	53	54
Grunberg [Grünberg/Zielona Góra]	A	0	13	40	0	51	50
Groningen	S	0	28	29	40	53	9
Grupenhagen	S	0	9	34	25	51	40
Gusteropium [Güstrow]	S	0	0	36	35	53	42
HAFNIA Dan. [Copenhagen]	O	0	0	36	40	55	43
Hamburgum [Hamburg]	S	0	12	33	35	53	42
Hall	S	0	1	36	25	51	43
Halberstadium [Halberstadt]	S	0	5	35	20	52	0
Harderwick [Harderwijk]	S	0	32	28	45	52	18
Haerlem [Haarlem]	S	0	36	27	40	52	22
Harlingen	S	0	33	28	25	53	6
Helmstadium [Helmstedt]	S	0	5	35	25	52	23
Heidelberga [Heidelberg]	S	0	18	32	15	49	20
Hertzogenbusch ['s- Hertogenbosch]	S	0	33	28	30	51	31
Hispalis, Sevilla [Seville]	S	1	26	15	15	37	25
Iena [Jena]	S	0	3	35	55	51	2
Ingolstadium [Ingolstadt]	S	0	4	35	40	48	46
Islad. sed. Ep. sep [Iceland, episcopal see,	S	1	28	14	45	65	44

Place [Nomina loc.]	S / A	Têp. H	Têp. M	Long. G	Long. M	Latit. G	Latit. M
north]							
Insbrugk [Innsbruck]	S	0	3	36	0	47	13
Katzenelbogen	S	0	21	31	30	50	12
Keiserberg [Kaysersberg]	S	0	23	31	0	48	8
Kitzning [Kitzingen]	S	0	10	34	15	49	43
Knudstrup. præd [Knutstorp manor]	S	0	2	37	10	55	58
Landestrum	S	0	0	36	35	48	30
LâdzKron Dan. [Landskrona]	A	0	2	37	15	55	53
Laugingen [Lauingen]	S	0	8	34	35	48	36
Lausanna [Lausanne]	S	0	27	30	0	45	40
Lipsia [Leipzig]	A	0	0	36	45	51	22
Longberg Patria [Lomborg, the author's homeland]	S	0	15	32	50	56	40

[Gazetteer continued — left-hand block:]

Place [Nomina loc.]	S / A	Têp. H	Têp. M	Long . G	Long . M	Latit. G	Latit. M
Lubeca [Lübeck]	S	0	7	34	50	53	57
Lucerna [Lucerne]	S	0	19	31	55	46	53
Luneburgum [Lüneburg]	S	0	9	34	20	53	19
Lutetiæ Parisior. [Paris]	S	0	49	24	25	48	48
Lutzenburg [Luxembourg]	S	0	29	29	30	49	42
Londinum Angl. [London]	S	0	50	24	20	51	32

Place [Nomina loc.]	S / A	Têp. H	Têp. M	Long . G	Long . M	Latit. G	Latit. M
Lundum Scaniae [Lund]	A	0	5	37	50	55	45
Lugdunum Bat. [Leiden]	S	0	37	27	30	52	7
Lymviga Cymb. [Lemvig, Jutland]	S	0	15	32	52	56	42
Magdeburgum [Magdeburg]	S	0	2	36	10	52	10
Mantua	S	0	6	35	15	44	33
Mansfeld	S	0	3	35	50	51	40
Marsilia [Marseille]	S	0	34	28	20	42	45
Mentz [Mainz]	S	0	20	31	45	50	0
Messana [Messina]	A	0	24	42	45	59 [recte: 39?]	12
Middelburg	S	0	40	26	40	51	31
Misnia [Meissen]	A	0	4	37	45	51	9
Mons Pessulanus [Montpellier]	S	0	42	26	5	43	0
Mons Reg. Borus. [Königsberg]	A	0	40	46	45	54	21
Morsburg [Moersburg]	S	0	1	36	25	47	23
Munchen [Munich]	S	0	3	36	0	48	6
Munster	S	0	22	31	10	51	54
Marpurgum [Marburg]	S	0	16	32	35	50	43
Marienburg [Malbork]	A	0	33	45	0	53	43
Narbona [Narbonne]	S	0	46	25	5	41	50
Nassow [Nassau]	S	0	21	31	20	50	16
Neapolis [Naples]	A	0	17	40	55	41	37
Nidrosia Norve. [Nidaros/Trondheim]	S	0	11	34	0	63	27
Nycoping Cymb.	S	0	13	33	30	56	50

Place [Nomina loc.]	S / Têp. A H		Têp. M	Long . G	Long . M	Latit. G	Latit. M
[Nykøbing, Jutland]							
Nurnberg [Nuremberg]	S	0	6	35	15	49	27
Odensee Daniaë [Odense, Denmark]	S	0	10	34	15	55	21
Osenbrug [Osnabrück]	S	0	20	31	35	52	17
Osterwick [Osterwieck]	S	0	7	35	0	52	3
Panormus [Palermo]	A	0	15	40	30	38	59
Patavium [Padua]	S	0	1	36	25	45	6
Pisa	S	0	4	35	45	43	6
Pisaurum [Pesaro]	A	0	10	39	10	43	51
Praga [Prague]	A	0	7	38	30	50	6

[Right-hand block of the same opening:]

Place [Nomina loc.]	S / Têp. A H		Têp. M	Long. G	Long. M	Latit. G	Latit. M
Saltzburg [Salzburg]	A	0	5	37	50	47	47
Sena [Siena]	A	0	3	37	20	43	3
Solotium [Solothurn]	S	0	22	31	5	47	8
Spira [Speyer]	S	0	18	32	15	49	10
Steenviga [Steenwijk]	S	0	30	29	15	52	45
Studgardium [Stuttgart]	S	0	14	33	5	48	39
Spandaw [Spandau]	A	0	5	37	50	52	30
Sivertslaw	S	0	17	32	25	56	2
Swinfurd [Schweinfurt]	S	0	9	34	20	50	3
Syracusæ [Syracuse]	A	0	23	42	30	38	31
Tyro	S	0	6	35	10	46	28
Tholcha	S	0	59	21	55	43	10
Thorn [Toruń]	A	0	31	44	25	52	34

Place [Nomina loc.]	S	Têp.		Long.		Latit.	
	/ A	H	M	G	M	G	M
Toletum [Toledo]	S	1	16	17	40	40	10
Torga [Torgau]	A	0	3	37	30	51	33
Trien [Trento]	S	0	27	30	0	49	50
Tubinga [Tübingen]	S	0	15	32	55	48	24
Valentia [Valencia]	S	0	58	22	5	39	55
Venetiaë [Venice]	A	0	6	37	15	45	18
Verona	S	0	5	35	20	44	51
Vienna Austriaë [Vienna]	A	0	19	41	30	48	28
Vlm [Ulm]	S	0	10	34	5	48	23
Vlisippo [Lisbon]	S	1	39	11	55	38	50
Vraniburgū Huę. [Uraniborg, Hven]	A	0	0½	36	45	55	54
Weymar [Weimar]	S	0	4	35	40	51	8
Wesel	S	0	27	29	50	51	34
Wismar	S	0	4	35	40	53	54
Wittenberga [Wittenberg]	S	0	2	37	15	51	52
Wirtzburg [Würzburg]	A	0	11	33	55	49	44
Wolffenbittel [Wolfenbüttel]	S	0	7	34	55	52	23
Wolmerstat [Wolmirstedt]	S	0	3	36	0	52	19
Wormbs [Worms]	S	0	14	32	5	49	33
Wiburgū Cimb. [Viborg, Jutland]	S	0	12	33	35	56	30
Zeitz	S	0	1	36	20	51	0
Zervesta [Zerbst]	S	0	0	36	35	52	0
Zirixzee [Zierikzee]	S	0	39	26	55	51	40
Zurich [Zürich]	S	0	17	32	20	47	9

Place [Nomina loc.]	S		Têp. M	Long. G	Long. M	Latit.	
	/	Têp. H				Latit. G	M
Zyges	A	0	30	44	15	46	34

CAPVT IV.

On the canonical designation of the fixed stars observed at Uraniborg [Uraniborg] in Denmark; on the precession of the mean and the true equinox; on the maximum obliquity of the Sun; together with the change of the latitudes of the fixed stars; and finally on the computation of all these.

[*De stellarum fixarum Vraniburgi in Dania observatarum Canonica designatione: medii aequinoctii verique praecessione: obliquitatis maximae Solis, simulque latitudinum fixarum mutatione: denique horum omnium supputatione.*]

This chapter, which will embrace everything pertaining to the places and apparent motions of the fixed stars, we have arranged in order in four parts. The first of these contains the canonical description of the fixed stars which in our horizon nearly...

[the second part] presents the mean motions of the precession of the equinoxes and of the obliquity of the ecliptic; the third sets out the hypothesis and the tables of the prosthaphaereses [prosthaphaerefeon] of these, from which the latitudes of the stars are likewise derived for every age of the world; and the fourth, finally, contains the rules, together with examples, for applying each item to use.

I. As for the determination of the fixed stars, this is a labor carried out long ago in the observations and computations made at [Uraniborg] on [Hven], and I rightly call it "the [Uraniborg] nights." For after [Tycho Brahe], in the years now past, had most successfully laid the foundation of these stars by means of the Sun and Venus — as is clear from Book I of the *Progymnasmata* [Margin: *Progym.* 1, Book 1, ch. 2] — I myself, from the year of our Lord 1590 and for a five-year span, not only took part in this work of restoring the fixed stars from the heavens, but even directed it. So I can affirm in good faith that the utmost diligence which can ever be expected of human industry was applied by us to their restitution, both in longitude and in latitude, and that every doubt was removed by observations made on both sides and often repeated, wherever there was need, until the stars settled to the same values within the closest minute. Yet now, because we have reduced the Sun's mean longitude by more than a minute, we have in the

following catalogue subtracted just as much, item by item, from the longitude of the stars given in [Tycho Brahe]'s canon. Here too we have deliberately eliminated the tiny fractions of minutes — which arose only from the limits of measurement — wherever they clung to a longitude or latitude, judging that no greater precision than whole minutes was necessary.

As for the visible size of each individual star, which is observed by the unaided eye at each observer's discretion, even though it may perhaps not be found uniform — owing to the frequent change of our thicker air — yet wherever we have here also departed from the ancients' rating, an evident cause was apparent in the sky. Two dots [puncta] added to the star's magnitude in the last column of the table indicate it to be a little larger; a single one, smaller, everywhere; and finally an asterisk (*) marks a star which the Ptolemaic canon does not have.

How useful, and how necessary, the effort of our master [Tycho Brahe] had been here, there is no need to explain at length to those who understand; for without a proper restoration of the fixed stars, no legitimate determination of the planets could be made. There is no measure of the planets equally certain as the one taken from the fixed stars in the heavens. Accordingly, by just as much as error is committed here in longitude and latitude, by so much is error committed there in the planets — even with sound instruments, even with observations, and finally with the calculation and all the requisites duly performed.

This cause, then, among others, shines out clearly enough: why the ancients — who all followed Hipparchus's old and uncertain designation of the fixed stars (made, that is, by means of the Moon crossing the meridian in its course, when the Moon's own course was not yet correctly known to him) right down to [Tycho Brahe] — boast in vain before posterity of an Astronomy soundly restored by themselves. Indeed, the outcome today shows this to be all too true in the heavens, which, by comparison, convicts them of error not only in the planets but in the fixed stars as well [Margin: See *Ptolemy*, Book 1, ch.; likewise *T[tycho] B[rahe]*, Book 1; *Progym.* pp. 234 and following]. Among other things, it teaches that the distance of these stars from one another, and their longitude, have remained unchanged, since they still look out at us in the heavens along the very same straight lines as in the time of the ancients.

Finally, as for the primeval naming of the star-groups [asterismorum] which the fixed stars occupy widely across the face of the sky — although I would venture to affirm, by conjectures that are by no means doubtful, that with Adam and the Patriarchs there was another naming, in many cases far more meaningful — yet it is better (as our master [Tycho Brahe] also advises) to indulge for a while the

fables of the heathen [Margin: Book 1 *Progym.*; *Cic[ero] Tusc[ulan]* 5, *quaest[io]*]. These fables strongly commend the heathen's outstanding virtue and industry, set against the sloth of certain of our own people, in the contemplation of the heavens — until we are soundly instructed in all things in the heavenly school. With these things briefly set out by way of explaining the first part, the catalogue of the stars itself now follows [Margin: *E[pistolarum]* Book 2; *Progym.* ch. 2].

THE CANONICAL DETERMINATION OF MOST OF THE PRINCIPAL FIXED STARS, ACCORDING TO TYCHO BRAHE'S ACCURATE OBSERVATIONS DRAWN FROM THE HEAVENS, completed to the year of our Savior 1600.

FIRST, ON THE STARS WHICH ARE SEEN ABOUT THE ZODIAC.

ARIES [Aries]

Longitude is given as Sign (S.) · Degrees (G.) · Minutes (M.); latitude as Degrees (G.) · Minutes (M.) with B = north (borealis) / A = south (australis); Mag. = magnitude. An asterisk (*) in the leftmost column marks a star absent from the Ptolemaic canon.

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N	
						/	M
						S	ag.
The southern one in the preceding horn [Auftralis in praecedente cornu]	♊	27	36	7	8	B	4
The northern, and following, in the same horn [Borealis ac sequens in eodem cornu]	♊	28	22	8	29	B	4
The bright one on the crown of the head: the chief [Lucida in vertice]	♈	2	5	9	57	B	3

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
capitis: principa.]							
In the gape, the northern of two [In rictu duarum borea.]	♂	2	33	7	23	B	6
The one more to the south [Quae magis ad austrum]	♂	3	19	5	42	B	6
The one on the neck [Quae in cervice]	♂	27	56	5	24	B	5
On the loins [In renibus]	♂	8	35	6	7	B	6
The one at the springing of the tail [Quae in educatione caudae]	♂	12	56	4	8	B	5
The preceding of three in the tail [Praecedens trium in cauda]	♂	15	14	1	46	B	4
The middle one [Media]	♂	16	23	2	50	B	5
The last [IVtima]	♂	17	49	2	36	B	6
On the thigh [In femore]	♂	11	21	1	12	B	6
On the ham [In poplite]	♂	9	34	1	7	B	6
* On the left knee [In genu sinistro]	♂	9	22	1	30	A	6
* On the right knee [In genu dextro]	♂	7	51	0	39	A	6
* A little one in the belly [Parvula in alvo]	♂	8	45	4	1	B	6
* The one below the bright star of the head [Quae est infra lucidam capitis]	♂	1	40	9	13	B	6
Above the back, the preceding of 4 unformed stars [Supra dorsum 4	♂	10	34	10	50	B	5

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
informium praeced.]							
The following, i.e. at the western base of the triangle (△), from the following [Sequens sc. ad basin occi. △ ex sequent.]	♄	11	22	11	16	B	4
The eastern one at the base of the triangle [Orientalis in basi trianguli]	♄	12	39	10	24	B	3
At the apex of the same triangle, toward the north [In apice ejusdem trianguli ad boream]	♄	12	50	12	25	B	4

TAURUS [Taurus]

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
The highest in the section [Suprema in sectione]	♄	17	59	5	57	A	5
The next after it [Alter a post ipsam]	♄	17	29	7	29	A	6
The third [Tertia]	♄	16	17	8	50	A	4
The fourth, most southerly [Quarta maxime austrina]	♄	15	34	9	23	A	4
On the right shoulder [In dextro armo]	♄	21	45	8	41	A	5
On the breast [In pectore]	♄	25	0	8	3	A	4
On the right knee [In genu dextro]	♄	27	58	12	14	A	4
On the right hock [In	♄	24	18	14	31	A	4

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
suffragine dextra]							
On the left knee [In genu sinistro]	II	4	8	9	32	A	5
On the left hock [In suffragine sinistra]	II	3	10	11	48	A	5
On the face, the first of the Hyades, in the nostrils [In facie sucularum prima in naribus]	II	0	11	5	47	A	3
Between this one and the northern eye [Inter hanc & oculum boreum]	II	1	15	4	2	A	3
The one between the same and the southern eye [Quae inter eandem & oculum australem]	II	2	21	5	53	A	4
In the southern eye: Aldebaran, Palilicium [In austrino oculo, Aldebaran, Palilicium]	II	4	11	5	31	A	1
In the northern eye [In boreo oculo]	II	2	52	2	37	A	3
At the root of the southern horn [Ad radicem cornu australis]	II	8	11	3	40	A	6
In the same horn, the more southerly of two [In eodem cornu duarum australior]	II	12	12	2	31	A	6

TAURUS [Taurus] (continued)

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
The one more to the north [Quae magis in boream]	II	11	3	1	50	A	4
At the tip of the same [in extremitate ejusdem]	II	19	11	2	14	A	3
At the origin of the northern horn [In origine cornu septentrionalis]	II	6	34	0	40	B	5
At the tip, or the right foot of the Charioteer [In extremitate, vel dexter pes Heniochi]	II	16	58	5	20	B	2
On the ear, the northern of two [In aure duarum borea]	II	2	53	1	4	B	5
The more southerly [Australior]	II	2	37	0	35	B	4
On the neck, the preceding of two [In collo duarum praecedens]	♄	27	50	1	12	B	5
The one that follows [Quae sequitur]	II	0	27	0	47	A	6
In the neck, the southern of the quadrilateral of preceding stars [In cervice quadrilateri praecedentium australia]	II	0	3	5	16	B	6
The northern of the same side [Ejusdem lateris borea]	♄	29	44	7	55	B	5
The southern of the following side [Sequentis lateris australis]	II	2	33	3	57	B	5

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
The northern of this side [Hujus lateris borea]	II	2	24	5	45	B	5
The western of the three brighter ones in the Pleiades [Occidentalis lucidiorum trium in pleiadibus]	ϣ	23	12	4	11	B	5
The lowest, nearest the western one [Infima & occidentali proxima]	ϣ	24	2	4	2	B	6
The middle and bright one of the Pleiades [Media & lucida pleiadum]	ϣ	24	23	4	0	B	3
The one on the point, toward the east [Quae est in cuspidē ad ortum]	ϣ	24	46	3	55	B	5
* On the hoof of the left foot [In ungula pedis sinistri]	ϣ	19	56	13	30	A	6
* A little star on the heel of the following foot [Stellula in talo pedis sequentis]	II	0	9	12	2	A	5
* The one on the right shoulder [Quae in armo dextro]	II	1	57	8	41	A	5
* The preceding of three below the Hyades [Praecedens trium infra suculas]	II	1	41	6	57	A	5
* The middle of the same [Media earundem]	II	3	27	7	5	A	5
* The following [Sequens]	II	4	54	6	18	A	5

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
A little one in the southern horn [Parvula in australi cornu]	♐	15	1	1	4	A	6
The following in the same horn [Sequens in eodem cornu]	♐	16	54	1	20	A	6
* A little one following, four in the section [Parvula sequens quatuor in sectione]	♏	17	32	9	35	A	6
* The one between the two preceding, in the square (□) of the neck [Quae est inter binas praeced. in □ coli]	♏	29	21	6	33	A	5

GEMINI [Gemini]

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
On the upper head, Castor, Apollo [In superiore capite, Castor, Apollo]	♊	14	40	10	2	B	2
On the lower head, Pollux, Hercules [In inferiore capite, Pollux, Hercules]	♊	17	42	6	38	B	2
In the left hand of the preceding twin [In sinistra manu praecedentis gemini]	♊	5	31	10	58	B	5
On the left arm [In sinistro brachio]	♊	9	53	7	43	B	4
On the shoulder-blades of	♊	13	23	5	42	B	4

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
the same [In scapulis ejusdem]							
On the right shoulder [In dextro humero]	☿	15	46	5	10	B	5
On the left shoulder of the following twin [In sinistro humero sequentis gemini]	☿	18	5	3	3	B	4
On the right side of the one in front [In latere dextro antecedentis]	♄☿	13	17	2	56	B	6
A little star on the left elbow of the upper one [Stellula in sinistro cubito superioris]	♄☿	14	9	6	0	B	6
On the northern and topmost knee [In boreali & supremo genu]	☿	4	21	2	11	B	3
On the left knee of the following one [In sinistro genu sequentis]	☿	9	25	2	7	A	3
The one on the belly of the southern twin [Quae in ventre meridionalis gemini]	☿	12	55	0	14	A	3
On the ham of the lower twin [In poplite inferioris gemini]	☿	13	12	5	41	A	4
On the foot of the preceding twin, the one in front [In pede praecedentis gemini antecedens]	♄	27	52	0	58	A	4
The following on the same foot, called the heel	♄	29	43	0	53	A	3

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
[Sequens in eodem pede, dicta calx]							
At the tip of the right foot, the preceding [In extremitate pedis dextri, praeced.]	II ☿	1	13	3	8	A	4
The bright one of the foot [Lucida pedis]	☿	3	30	6	49	A	2
On the lowest foot of the following one [In infimo pede sequentis]	II ☿	5	28	10	9	A	4
On the heel of the same foot [In calce pedis ejusdem]	☿	7	55	9	41	A	6
The one above the knee of the lower twin [Quae est supra genu inferioris gemini]	☿	6	22	1	12	A	6
On the thigh of the upper twin [In femore superioris gemini]	☿	8	36	1	31	B	6
* The one below the lower head, in the hand [Quae infra caput inferius in manu]	☿	19	41	5	44	B	6
* A little one between the two heads [Parvula inter utrumque caput]	☿	17	3	7	24	B	5
At the ear of the upper twin [Ad aurem superioris gemini]	☿	13	28	9	42	B	5
The preceding, at the top of the foot: "Propus" in Greek [Praeced. ad	II	25	21	0	13	A	4

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.

sumum pedem: Propus
Graece]

CANCER [Cancer]

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
The nebulous one in the breast, which they call the Manger [Nebulosa in pect. quod praesepe vocant]	♌	1	45	1	14	B	ne b.
The northern of those preceding, in the square (□) of Cancer [Borea praecedentium in □ cancri]	♌	29	48	1	31	B	5
The more southerly [Australior]	♌	0	8	0	48	A	5
The northern Ass [Asellus boreus]	♌	1	56	3	8	B	4
The southern Ass [Asellus australis]	♌	3	7	0	4	A	4
On the southern claw [In brachio austrino]	♌	8	2	5	8	A	3
On the northern claw [In brachio boreali]	♌	0	43	10	23	B	5
At the tip of the northern foot [In extremitate pedis borei]	♌	23	55	1	15	B	5
At the end of the southern foot [In extremo pedis austrini]	♌	25	3	7	5	A	5
* The brighter one at the	♌	25	44	2	19	A	4

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
root of the tail [Quae in radice caudae lucidior]							
* Next following, on the back [Proxime sequens in dorso]	♄	28	11	1	4	A	6
The northern of three in the southern claw [Borealis trium in brachio australi]	♌	6	46	1	54	A	6
The southern in the same [Australis in eodem]	♌	10	35	5	36	A	5
The northern of two in the snout [Duarum in rostro, septentrionalis]	♌	5	26	7	14	B	6
The lower and southern [Inferior & australis]	♌	7	35	5	20	B	6

LEO [Leo]

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
In the nostrils [In naribus]	♌	9	40	10	23	B	4
In the gape [In hiatu]	♌	12	15	7	52	B	4
On the head, the more northern of two [In capite duarum borealior]	♌	15	50	12	21	B	4
The more southerly [Australior]	♌	15	4	9	40	B	3
On the neck, the northern of three [In collo trium borea]	♌	21	56	11	50	B	3
The middle and bright one of the neck [Media &	♌	23	58	8	47	B	2

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
lucida colli]							
The southern [Australis]	♌	22	19	4	52	B	3
Heart, Regulus, Basiliscus [Cor, Regulus, Basiliscus]	♌	24	16	0	26	B	1
On the breast, the more southerly [In pectore australior]	♌	24	49	1	26	A	5
Just preceding Regulus [Antecedens regulum proxime]	♌	21	42	0	0	B	4
The one that precedes this, on the right knee [Quae hanc praecedit in genu dextro]	♌	17	53	0	16	B	5
On the right paw [In drace dextra]	♌	16	6	3	10	A	4
The following on the other foot [Sequens in altero pede]	♌	18	39	3	47	A	4
On the left paw [In drace sinistra]	♌	23	45	3	55	A	4
In the left armpit [In sinistra axilla]	♍	0	47	0	8	B	4
On the belly, the one in front of three [In ventre trium antecedens]	♌	22	23	2	10	B	6
The more northern of those following [Sequentium borealior]	♍	2	5	5	56	B	6
The more southerly [Australior]	♍	4	4	2	49	B	6
The preceding of two on the loins [Praecedens	♍	3	13	12	53	B	5

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
duarum in lumbis]							
The one that follows, bright [Quae sequitur, lucida]	♄	5	40	14	20	B	2
On the rump, the preceding and northern of two [In clune duarum praecedens & borea]	♄	7	49	9	41	B	3
The following and southern [Sequens & austrina]	♄	9	7	7	50	B	6
On the thigh [In femore]	♄	11	57	6	7	B	3
On the hind knee [In genu posteriori]	♄	13	7	1	40	B	4
The middle one on the foot [Media in pede]	♄	15	56	0	33	A	4
The lowest on the foot [Infima in pede]	♄	19	26	3	3	A	4
The bright one at the end of the tail [In extremo caudae lucida]	♄	16	2	12	18	B	1
* The outermost on the hoof of the left foot [Extrema in ungula pedis sinistri]	♂	16	31	4	48	A	6
* On the hoof of the other foot, the preceding [In ungula alterius pedis praeced.]	♂	16	0	5	43	A	5
* The one almost in the middle of the body [Quae in medio corpore fere]	♄	0	13	10	17	B	6

VIRGO [Virgo]

Star	Long. S.	Long. G.	Long. M.	Lat. G.	Lat. M.	N / S	M ag.
The northern of those preceding, in the square (□) of the head [Borealis praecedentium in □ capitis]	♍	17	43	6	6	B	5
The southern [Australis]	♍	18	32	4	37	B	5
The northern of two following, in the face [Sequentium duarum in vultu borea]	♍	22	6	8	33	B	5

The canonical catalogue of the fixed stars continues, reduced to the year 1600. The columns are: star designation; ecliptic longitude (zodiacal sign, degrees, minutes); ecliptic latitude (degrees, minutes, with **B** = north / **A** = south); and magnitude. The marker * indicates a star not contained in Ptolemy's canon.

VIRGO (continued)

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag.
The southern one	♍	21	57	6	10	B	5
At the tip of the southern and left wing	♍	21	31	0	43	B	3
The leading of four in the left wing	♍	29	15	1	25	B	4
The next following	♍	4	34	2	50	B	3
The small second-to-last	♍	9	27	2	23	B	6
The last	♍	12	36	1	45	B	4
On the right side, below the girdle	♍	5	54	8	41	B	3
In the right and northern wing, of the	♍	29	52	13	36	B	5

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
three leading							
Of the remaining two, the southern	♌	1	51	11	37	B	6
The more northern, called the Grape- Gatherer [Vindemiatrix]	♌	4	22	16	15	B	3
In the left hand, the Spike [Spica]	♌	18	15	1	59	A	1
Below the apron, on the right hip	♌	15	21	8	10	B	3
On the left thigh, the northernmost	♌	17	57	3	11	B	6
Of the two following, the more northern	♌	21	8	1	45	B	6
The more southern	♌	19	43	0	20	A	6
On the left knee	♌	24	43	2	24	B	6
The more northern of two in the upper hem	♌	27	48	11	2	B	5
The middle of three in the hem	♌	28	8	7	18	B	4
The lowest and southern	♌	28	50	2	57	B	4
* The more southern of two in the upper hem	♌	29	50	11	48	B	4
On the southern foot	♍	1	21	0	31	B	4
On the northern or right foot	♍	4	29	9	49	B	4
* The lower of two between the Grape- Gatherer and the girdle	♌	1	20	10	26	B	6
* Following the one on the right hip	♌	21	36	9	40	B	6
* The one on the neck	♎	27	44	4	59	B	6

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
* The little one following the Grape-Gatherer	♌	8	24	16	14	B	6
* The leading of three in a straight line on the northern wing	♌	10	10	12	40	B	5
* The middle of the same	♌	14	45	12	34	B	6
* The following	♌	22	10	13	7	B	5
* The one between the fourth and fifth	♍	22	55	3	22	B	6

LIBRA

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
The southern pan	♎	9	30	0	26	B	2
The one above the southern pan	♎	8	41	1	55	B	5
The northern pan	♎	13	47	8	35	B	2
The one above the northern pan, toward the west	♎	9	39	8	19	B	4
The first from the southern pan, toward the east	♎	12	25	1	14	B	5
The second from the same pan, toward the east	♎	16	18	2	59	B	6
The third from the same pan, toward the east	♎	19	32	4	28	B	3
The one below this, toward the east	♎	21	47	4	4	B	4
The one below the same,	♎	19	26	2	21	B	4

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
toward the west							
The one below the northern pan, toward the east	♌	15	45	8	7	B	4

SCORPIUS

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
The uppermost on the forehead	♏	27	35	1	5	B	2
The middle on the forehead	♏	26	58	1	54	A	3
The southern of the three brighter on the forehead	♏	27	24	5	22	A	3
The one still further south, on the foot	♏	27	42	8	27	A	4
The northernmost of the forehead	♏	29	2	1	42	B	4
The little one in a triangle with the bright forehead star and the fifth	♏	28	6	0	14	B	5
Preceding the heart, toward the north	♏	2	10	3	55	A	4
Glowing red in the middle, called Antares or the heart	♏	4	12	4	27	A	1
The one following the heart, toward the south	♏	5	52	5	50	A	4
Among the preceding ones, on the lower feet	♏	0	45	6	37	A	5

SAGITTARIUS

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
In the northern part of the bow, the more southern of two	♐	0	46	2	0	A	4
The more northern in the same part of the bow	♐	27	40	2	28	B	4
On the left shoulder	♐	6	50	3	31	A	4
Preceding this one, on the dart	♐	4	39	3	50	A	5
The leading of three on the head	♐	7	55	1	45	B	4
The middle	♐	9	27	0	59	B	4
The last	♐	10	42	1	31	B	4
The first at the junction	♐	12	43	3	7	B	6
The middle, at the northern junction	♐	13	53	4	17	B	4
The following and upper one	♐	14	10	6	10	B	5
With this one to the east, of two dim stars, joined in a triangular shape	♐	19	7	5	8	B	6
The eastern and last in the upper junction	♐	22	51	5	12	B	6
Dim, in the lower junction, toward the east	♐	19	23	1	25	B	6
Dim, on the right elbow	♐	16	25	3	8	A	6

CAPRICORNUS

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
The northern of three on	♑	28	17	7	3	B	3

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
the leading horn							
The middle	♊	28	50	6	53	B	6
The southern	♊	28	30	4	41	B	3
The nebulous one preceding the upper horn	♊	27	7	7	16	B	6
The western nebulous one, the base of the triangle on the forehead	♊	28	56	0	49	B	ne b.
The eastern nebulous one	♊	29	40	0	28	B	ne b.
The uppermost in the same triangle	♊	29	36	1	20	B	6
The nebulous one preceding, on the forehead	♊	27	12	0	24	B	ne b.
On the neck, the northern of two	♊	2	48	3	25	B	6
The southern	♊	2	5	0	15	B	6
Preceding, on the right knee, dim	♊	1	46	6	58	A	6
Following, on the left knee	♊	2	27	9	2	A	6
On the left shoulder	♊	6	12	8	8	A	6
The lowest on the belly	♊	11	23	6	56	A	5
The northern following one of two touching beneath the abdomen	♊	11	59	6	29	A	6
The more eastern of three on the middle of the belly	♊	9	22	4	25	A	6
The lowest of them	♊	7	30	4	27	A	6

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
The northern of three	♊	7	17	3	1	A	5
The leading of two on the back	♊	8	20	0	29	A	5
The following of the same, on the back	♊	12	6	1	16	A	5
The leading of two toward the flanks	♊	14	24	4	48	A	4
The following of the same	♊	16	5	4	49	A	5
The leading of two bright stars in the tail	♊	16	13	2	26	A	3
The following	♊	17	59	2	29	A	3
The leading in the upper tail	♊	18	13	2	22	B	5
The southern of the remaining ones in the upper tail	♊	20	26	0	14	A	5
Preceding this one, toward the north	♊	20	15	0	10	A	6
The northern at the tip of the tail	♊	19	53	4	17	B	6

AQUARIUS

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
On the head	♊	22	25	15	23	B	6
On the right shoulder, the brighter	♊	27	48	10	42	B	3
The dimmer and more southern	♊	26	35	9	12	B	5
On the left shoulder	♊	17	50	8	42	B	3

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
The one on the back, below the armpit	♊	18	37	6	1	B	5
The following and lower of three in the left hand	♊	10	50	4	50	B	5
The middle	♊	7	27	8	19	B	5
The leading, brighter one	♊	6	11	8	10	B	4
On the right elbow	♋	1	9	8	18	B	3
In the right hand, the more northern	♋	3	3	10	31	B	5

AQUARIUS (continued)

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
The leading of the remaining two southern stars	♋	3	22	8	53	B	4
The following	♋	4	52	8	10	B	4
The leading of two in the right hollow	♊	27	44	2	46	B	4
The following of them	♊	28	30	2	30	B	6
On the right thigh	♊	29	52	1	10	A	5
The one at the buttocks	♊	23	12	2	0	A	4
The southern on the right shin, Scheat	♋	3	21	8	10	A	3
The northern, or the one at the knee	♋	3	4	5	37	A	5
On the left hip	♊	29	39	5	40	A	6
The more southern of two on the left knee	♊	26	54	10	48	A	5
The more northern	♊	29	49	9	57	A	6

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
In the outpouring of the water, the first from the hand	✕	3	51	4	9	B	4
The southern succeeding star	✕	6	3	0	19	A	4
The following one in the first bend of the water	✕	8	59	1	24	A	6
The one accompanying it	✕	11	37	1	0	A	5
In the second southern bend	✕	11	32	2	49	A	5
The leading and more northern of two following	✕	10	42	3	58	A	5
The following and more southern	✕	11	10	4	10	A	5
Near this one, inclining toward the south	✕	11	13	4	44	A	5
After this, the leading of two adjacent stars	✕	14	6	10	59	A	5
The following of the same adjacent ones	✕	14	37	11	33	A	5
The northern of three in the third bend of the water	✕	13	2	14	29	A	5
The middle in the third bend of the water	✕	13	45	15	16	A	6
The following of the three, the southern	✕	14	43	16	23	A	6
The northern of the three following	✕	7	53	14	45	A	5
The middle of the same three	✕	8	20	15	30	A	5

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag
The southern of these three	♋	9	49	16	31	A	5
The upper of three in the last bend	♋	4	24	14	25	A	5
The middle	♋	4	1	15	40	A	5
The lowest, next to Fomalhaut	♋	3	16	15	53	A	5
The last in the outpouring, Fomalhaut	♋	28	10	21	0	A	1

PISCES

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag
In the mouth of the southern fish	♋	13	1	9	4	B	5
The southern of two on the back of the head	♋	15	49	7	18	B	4
The northern on the back of the head	♋	17	29	8	55	B	6
The leading of two on the back	♋	19	41	9	3	B	5
The following on the back	♋	21	55	7	14	B	5
The leading on the belly	♋	17	20	4	27	B	5
The following on the belly	♋	21	4	3	25	B	5
On the tail	♋	27	1	6	24	B	5
Above this, toward the east	♋	28	26	7	27	B	6
The following	♋	2	28	5	28	B	6
The leading of three	♋	8	35	2	11	B	4

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
brighter stars in the southern cord							
The middle of the same	Υ	11	57	1	6	B	4
The following	Υ	14	18	0	58	B	4
In the bend of the cord, the leading and northern of two tiny stars	Υ	12	24	1	31	A	6
The following of the same, toward the south	Υ	13	45	4	19	A	6
After the bend, the leading of three	Υ	17	32	3	3	A	5
The middle	Υ	19	55	4	40	A	5
The following and last	Υ	21	56	7	56	A	5
The brighter one at the knot of both cords	Υ	23	46	9	4	A	3
In the northern cord, the leading one from the knot	Υ	22	11	1	39	B	5
After this, the southern of three	Υ	21	15	1	52	B	5
The middle and brighter one at the northern knot	Υ	21	15	5	21	B	4
The northern of three, and the last in the cord	Υ	21	35	9	24	B	5
The northern of two in the mouth of the northern fish	Υ	23	14	22	0	B	6

PISCES (continued)

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag
The southern	♈	22	48	20	43	B	5
The northern of the triangle on the head	♈	19	21	20	55	B	6
The southern of the same triangle	♈	18	5	19	24	B	6
The middle and leading star of the triangle	♈	17	2	20	24	B	6
The leading of three on the southern spine, near the left elbow [of Andromeda]	♈	17	55	13	21	B	5
The middle	♈	18	1	12	22	B	6
The lowest of three	♈	18	8	11	21	B	6
The northern of two on the belly	♈	23	17	17	26	B	5
The one further toward the south	♈	20	57	15	30	B	5
Following the middle of three on the southern spine	♈	17	59	12	28	B	5
* Following the northern one on the belly, toward the north	♈	24	10	18	31	B	6
* On the back of the head of the northern fish	♈	21	25	23	3	B	6

SECONDLY, ON THE STARS THAT APPEAR TOWARD THE NORTH

URSA MINOR, CYNOSURA

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
At the tip of the tail, commonly the Pole Star [polaris]	♄	23	1	66	2	B	2
The second-to-last of the tail	♄	25	35	69	50	B	4
The one at the root of the tail	♄	3	23	73	50	B	4
The upper of two in the following square	♄	21	28	75	0	B	4
The lower of the same	♄	24	51	77	38	B	5
The upper of two in the square of the preceding ones	♄	7	15	72	51	B	2
The lower of the same	♄	14	40	75	23	B	3

URSA MAIOR, HELICE

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
The one on the muzzle	♄	17	35	40	2	B	4
Below the left eye	♄	17	9	43	55	B	4
* Adjacent below this one	♄	16	7	44	22	B	5
Above the right eye	♄	18	24	47	50	B	4
Above the left eye	♄	19	43	47	44	B	4
At the left ear	♄	24	41	51	36	B	5
The lowest and leading	♄	23	49	42	30	B	5

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
in the small triangle of the neck							
The following in the same triangle	♄	25	1	45	3	B	4
* The uppermost at the apex of the same triangle	♄	27	59	46	21	B	5
On the neck, succeeding the said triangle	♌	0	37	42	36	B	4
The following below this one	♌	3	37	38	15	B	4
On the left foreknee	♌	0	31	34	34	B	3
The more northern of two on the right foot	♄	25	55	29	15	B	3
The more southern	♄	27	9	28	38	B	3
Below the right knee	♄	27	6	33	30	B	5
On the right knee itself	♄	27	25	36	6	B	5
The upper of the preceding stars in the larger square	♌	9	33	49	40	B	2
The inner star of the same square	♌	13	42	45	3	B	2
The upper of the following square	♌	25	24	51	37	B	2
The lower of the same	♌	24	44	47	6	B	2
The upper of the left hind foot	♌	13	55	29	51	B	4
The following and more southern	♌	15	3	28	45	B	4
On the knee of the leading hind foot	♌	22	32	35	14	B	4
The leading of two on	♍	0	54	26	14	B	4

Designation of the stars	Long. sign	Long. °	Long. '	Lat. °	Lat. '	N / S	M ag .
the right hind foot							
The following and more southern	♏	1	35	24	54	B	4
The third-to-last of the tail	♏	3	9	54	18	B	2

Catalogue of the Fixed Stars (continued) — Cap. IV

*The catalogue of the northern constellations continues. Columns: star name | longitude (sign, degrees, minutes) | latitude (degrees, minutes) | B = north / A = south of the ecliptic | magnitude. * marks a star not in Ptolemy; "neb." = nebulosa (nebulous).*

Ursa Maior [Helice] (concluded)

Star [Latin]	Sig n	Long.°	Long.'	Lat.°	Lat.'	B/ A	Ma g.
The second-to-last [Penultima]	♏	9	55	56	22	B	2
The last of the tail [Ultima caudae]	♏	21	11	54	25	B	2
The unformed star between this tail and Leo [Informis inter caudam hujus & Leonis]	♏	17	42	40	6	B	2

Draco [Draco]

Star [Latin]	Sig n	Long.°	Long.'	Lat.°	Lat.'	B/ A	Ma g.
The one on the tongue [Quaest in lingua]	♏	18	55	76	17	B	4
In the mouth [In ore]	♏	4	13	78	15	B	4
The preceding of the two bright stars in the head [Duarum lucidarum in capite]	♏	6	18	75	21	B	3

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
praeced.]							
The one at the cheek [Quae ad genam]	♄	19	2	80	22	B	4
The following of the bright ones [Sequens lucidarum]	♄	22	23	75	4	B	3
In the first bend of the neck, of the three northern, commonly the bright one of the head [In prima colli inflexione trium Boreal. vulgo lucida cap.]	♅	17	3	81	53	B	5
The southern one [Australis]	♅	24	30	77	57	B	5
The middle of the same [Media earundem]	♅	20	32	79	52	B	5
The one that follows toward the east [Quae sequitur ad ortum]	♆	9	28	80	54	B	4
The one near the second bend [Quae est prope secundam flexuram]	♆	28	32	81	51	B	4
The northern of the second bend [Borea II secundae flexurae]	♇	12	25	82	49	B	3
The northern of the following side [Borea lateris sequentis]	♇	15	20	78	9	B	4
The southern of the same side [Australis ejusdem lateris]	♇	27	46	79	25	B	3
The preceding star of the following triangle [Sequentis trianguli praecedens]	♈	15	17	83	5	B	4
The one that follows toward the south [Quae sequitur ad	♈	19	39	80	38	B	4

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
austrum]							
The one above this [Quae supra hanc]	Υ	26	43	80	54	B	4
The following star in the remaining triangle [In reliquo triangulo sequens]	ϙ	6	33	83	4	B	4
The southern of the same [Australis ejusdem]	ϙ	1	27	83	28	B	4
The preceding and northern star of the triangle [Praecedens ac borealis trianguli]	Π	5	30	84	48	B	4
The one in the bend of the third coil [Quae in flexura nodi tertii]	♌	29	43	81	4	B	3
Nearest the pole of the zodiac [Polo Zodiaci proxima]	♌	6	25	86	53	B	4
The one that follows the 24th [Quae 24 sequitur]	♍	28	20	83	18	B	5
The one succeeding this [Succedens huic]	♍	28	21	81	41	B	5
Nearer the pole, moderately bright [Polo vicinior, mediocriter lucida]	♍	26	50	84	46	B	3
The preceding antepenultimate, from the outermost bend [Praecedens antepenult. ab extr. flexione]	♎	7	54	78	32	B	3
The antepenultimate preceding the bend [Antepenultima flexuram praecedens]	♎	12	27	74	12	B	3
The penultimate at the bend	♍	29	21	71	4	B	3

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
[Penultima ad flexuram]							
The one that follows the bend, the second [Quae flexuram sequitur, secunda]	♌	29	16	65	18	B	5
The one that next follows the bend [Quae flexuram proxime sequitur]	♍	2	9	66	36	B	2
The penultimate of the tail [Penultima caudae]	♌	10	25	61	33	B	3
The last of the tail [Ultima caudae]	♌	4	36	57	7	B	3
* Between Draco and the arm of Cepheus, unformed [Inter Draconem, & brachium Cephei, inform.]	♎	1	3	77	31	B	5

Cepheus [Cepheus]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
On the belt [In cingulo]	♐	0	12	71	7	B	3
The bright one on the shoulder [Lucida in humero]	♑	7	12	68	54	B	3
The one on the left shoulder [Quae in sinistro humero]	♑	27	52	62	35	B	4
The one on the tiara that follows toward the north [Quae in tiara sequitur ad boream]	♑	8	28	61	3	B	4

Boötes, or Arctophylax [Bootes seu Arctophylax]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The preceding of three on the left hand [In manu	♐	24	8	58	53	B	4

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
sinistra trium praecedens]							
The second [Secunda]	♄	25	32	58	51	B	4
The third [Tertia]	♄	26	58	60	5	B	4
On the left elbow [In cubito sinistro]	♄	1	17	54	40	B	4
On the left shoulder [In humero sinistro]	♄	13	4	49	34	B	3
On the head [In capite]	♄	18	42	54	16	B	3
On the right shoulder, above the Corona [In dextro humero supra coronam]	♄	27	28	49	1	B	3
The one on the hip, under the right arm [Quae in coxendice sub brachio dextro]	♄	22	28	40	40	B	3
The lower of two adjacent stars on the back [Inferior duarum contiguarum in dorso]	♄	18	15	42	11	B	4
The upper of the same [Superior earundem]	♄	17	16	42	36	B	4
The one on the right leg [Quae est in crure dextro]	♄	27	25	27	57	B	3

Boötes, or Arctophylax (continued)

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The highest on the left shin [Suprema in tibia sinistra]	♄	13	41	28	9	B	3
The middle [Media]	♄	12	24	26	33	B	4
The lowest [Infima]	♄	13	36	25	14	B	4
On the hem of the tunic, called Arcturus [In fimbria]	♄	18	38	31	3	B	1

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
tunicae, Arcturus dicta]							
* The lowest of three unformed, near the right knee [Infima trium inform. circa dextr. genu]	♌	26	12	30	28	B	4
* The middle [Media]	♌	27	10	31	22	B	4
* The highest [Suprema]	♌	27	51	33	52	B	4

Corona Borealis, the Gnoisian [Corona Borea, Gnoissia]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The bright one of the Corona [Lucida coronae]	♌	6	37	44	23	B	2
The preceding [Praecedens]	♌	3	36	46	8	B	4
Above this [Supra hanc]	♌	3	9	48	25	B	5
The one that succeeds it more to the north [Quae succedit magis septentrion, versus]	♌	[?]	[?]	[?]	[?]	B	6
The one that follows the bright star [Quae sequitur lucidam]	♌	9	13	44	33	B	4
The next following [Proxime sequens]	♌	11	24	44	52	B	4
The one that accompanies this again [Quae hanc rursus comitatur]	♌	13	31	46	10	B	4
The last of all [Omnium ultima]	♌	[?]	[?]	[?]	[?]	B	6

Hercules [Hercules]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
On the head [In Capite]	♌	10	30	37	23	B	3

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
On the right shoulder [In humero dextro]	♄	25	26	42	48	B	3
The penultimate of the right arm [Penultima dextri brachii]	♄	23	35	40	6	B	3
The lowest on the right arm [Infima in dextro brachio]	♄	20	5	37	19	B	4
On the left shoulder [In sinistro humero]	♄↗	9	9	47	47	B	3
On the left arm [In sinistro brachio]	♄↗	14	21	49	23	B	4
The preceding star in the spoils of the Lion [Praecedens in exuviis Leonis]	♄↗	19	35	51	17	B	4
The following star in the triangle of the spoils [Sequens in triangulo exuviarum]	♄↗	27	18	52	19	B	4
At the base of the triangle, toward the north [In basi trianguli ad boream]	♄↗	23	56	53	46	B	4
The middle of those in the spoils [Media earum quae in exuviis]	♄↗	23	37	52	47	B	4
The one on the left hip [Quae est in coxa sinistra]	♄	26	1	53	11	B	3
Farther east than this, on the left thigh [Hac orientaliior, in femore sinistro]	♄↗	2	44	53	21	B	3
The preceding of three adjacent stars on the thigh [Praecedens trium contiguarum in femore]	♄↗	6	20	59	38	B	4

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The middle [Media]	♄	7	18	60	12	B	4
The following [Sequens]	♄	9	46	60	14	B	4
On the left knee [In genu sinistro]	♄	22	55	60	47	B	3
The one on the left calf, near the head of Draco [Quae in sinistra sura prope caput Drac.]	♄	14	16	69	22	B	3
The preceding of three faint stars on the left foot [Praecedens trium obscurarum in pede sin.]	♄	7	4	71	20	B	6
The middle of the same [Media earundem]	♄	11	6	71	14	B	6
The last [Ultima]	♄	17	59	71	5	B	ne b.
On the upper right thigh [In superiori femore dextro]	♄	23	7	60	23	B	3
More northern on the same thigh [Borealior in eodem femore]	♄	17	38	63	14	B	4
The one on the right knee [Quae est in dextro genu]	♄	8	42	65	55	B	4
The one on the upper calf [Quae est in superiori sura]	♄	5	56	63	51	B	4
The one on the leg [Quae in crure]	♄	2	42	64	23	B	4
The preceding star on the right leg [Praecedens in dextro crure]	♄	16	31	62	29	B	5
The one on the shin of the right foot, near the ankle [Quae in tibia dextri pedis circa talum]	♄	2	27	60	16	B	4

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The outermost on the right foot [Extrema in dextro pede]	♌	27	5	57	16	B	4

Lyra [Lyra]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The bright one of the Lyra [Lucida Lyrae]	♌	9	42	61	48	B	1
The one above the bright star, toward the north [Quae supra lucidam ad aquilonem]	♌	13	13	62	27	B	5
The one below the bright star, toward the Wain [Quae infra lucidam ad currum]	♌	12	25	60	26	B	5
The one in the middle of the issuing of the horns [Quae in medio educationis cornuum]	♌	16	9	59	26	B	4
Of two adjacent stars, toward the north [Duarum contiguarum ad boream]	♌	24	3[?]	60	46	B	5
The one toward the south [Quae ad austrum]	♌	25	1	59	41	B	5
Of the two preceding stars on the yoke, the northern [Duarum praecedentium in iugo, borea]	♌	13	15	56	5	B	3
The small one beneath this [Parva sub hac]	♌	13	2	55	16	B	6
On the yoke, of the two following stars, the northern [In iugo, duarum sequentium borea]	♌	16	10	55	6	B	3

Lyra (continued)

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The small one that lies beneath this [Parva quae huic subest]	♄	16	19	54	32	B	6
* The one nearly in the middle of the body [Quae in medio fere corpore]	♄	20	51	58	6	B	5

Cygnus [Cygnus]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
On the beak [In rostro]	♄	25	43	49	2	B	3
On the head [In capite]	♄	29	19	50	42	B	5
In the middle of the neck [In medio colli]	♁	7	32	54	19	B	4
On the breast [In pectore]	♁	19	24	57	10	B	3
On the tail [In cauda]	♁	29	52	59	57	B	2
The first and brightest, on the elbow of the upper wing [Prima & lucidiss. in ancone super. alae]	♁	10	52	64	28	B	3
Of three on the upper part [of the wing], the southern [Trium in superiori vola, australis]	♁	[?]	[?]	[?]	[?]	B	4
The penultimate of the upper wing [Penultima superioris alae]	♁	12	38	71	31	B	4
The outermost of the upper wing [Extrema superioris alae]	♁	9	35	73	51	B	4
The one on the elbow of the lower wing [Quae in ancone inferioris alae]	♁	22	8	49	26	B	3

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
In the middle of it [In medio ipsius]	⌘	[?]	[?]	[?]	[?]	B	4
The outermost of the lower wing [Extrema inferioris alae]	⌘	27	42	43	44	B	3
The preceding star on the lowest foot [Praecedens in infimo pede]	✕	0	31	54	59	B	4
The one that follows on the lowest knee [Quae sequitur in infimo genu]	✕	5	20	56	36	B	4
The southern and preceding of two adjacent stars on the upper foot [Aust. & praec. duarum contig. in sup. pede]	⌘	22	49	63	37	B	4
The following of the same, and more northern [Sequens earundem & borealior]	⌘	24	33	64	18	B	4
The lower of two unformed stars following the right wing [Inferior duarum infor. dextr. alam sequens]	✕	3	2	50	33	B	4
The upper of the same [Superior earundem]	✕	4	52	51	31	B	4
The Nova that first appeared, seen in the year of the Lord 1600 [Nova primum anno Dn. 1600 app. consp.]	⌘	16	18	55	30	B	3

[Margin: Vide lib. 1. Prog. T. B. in fine. (See Book 1, the Prognostication of Tycho Brahe, at the end.)]

Cassiopea [Cassiopea]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
On the head [In capite]	Υ	29	34	44	40	B	4
On the breast, Schedir [In pectore, Schedir]	ϝ	2	16	46	35	B	3
On the belt [In cingulo]	ϝ	4	37	47	5	B	4
At the bend toward the hips [In flexura ad coxas]	ϝ	8	26	48	46	B	3
At the knee [Ad genu]	ϝ	12	20	46	22	B	3
On the leg [In crure]	ϝ	19	12	47	29	B	3
The outermost of the foot [Extrema pedis]	ϝ	26	38	48	54	B	4
On the left arm [In brachio sinistro]	ϝ	6	13	43	6	B	4
On the left elbow [In cubito sinistro]	ϝ	5	15	43	28	B	5
On the right elbow [In cubito dextro]	Υ	24	38	49	24	B	6
On the raising of the seat [In erectione sedis]	ϝ	7	5	52	14	B	4
The bright one of the chair [Lucida cathedrae]	Υ	29	34	51	14	B	3
The outermost of the chair [Extrema cathedrae]	Υ	25	33	51	8	B	6
* The one beside this, at the end of the star [Quae juxta hanc ad extremitatem stellae]	Υ	25	31	52	39	B	6
* The one nearly in a straight line with the 11th and 17th [Quae in recta fere linea cum xi & xvii]	ϝ	19	27	52	48	B	6
* The outermost of the footstool [Extrema scabelli]	ϝ	22	20	56	13	B	6

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
* The middle of the footstool [Media scabelli]	♄	22	32	54	27	B	6
* On the footstool, nearest to the sole of the foot [In scabello proxime ad plantam pedis]	♄	21	57	52	8	B	6
* The one that follows the knee [Quae genu sequitur]	♄	12	56	44	57	B	6
* The one that precedes the knee [Quae genu praecedit]	♄	9	59	45	4	B	6
* The Whirl of the Navel [Gyrus Vmbilici]	♄	6	51	47	31	B	6
* The little one at the hair of Cassiopea [Parvula ad crines Cassiopeae]	♄	29	9	45	38	B	6
* The following of two northern stars on the rod [Sequens ex duabus borealib. in virga]	♄	29	31	41	15	B	6
* The preceding of the same [Praecedens earundem]	♄	27	56	41	25	B	6
* The penultimate of the rod [Penultima virgae]	♄	26	55	39	15	B	6
* The outermost of the rod [Extrema virgae]	♄	25	53	38	9	B	6

Perseus [Perseus]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
At the outermost wrapping of the right hand [In extrema dextrae manus obvolutione]	♄	18	30	39	0	B	6
On the right elbow [In cubito dextro]	♄	23	8	37	28	B	4

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
On the right shoulder [In dextro humero]	♄	24	25	34	30	B	3
The one on the left shoulder [Quae in sinistro humero]	♄	19	3	31	34	B	4
The one at the crown of the head [Quae in capitis vertice]	♄	21	49	34	26	B	5

Perseus (continued)

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The one that is on the back [Quae in dorso est]	♄	23	32	30	36	B	4
The bright one on the right side [Fulgens in dextro latere]	♄	26	16	30	5	B	2
The one that next follows below [Quae proxime infra sequitur]	♄	27	3	27	59	B	5
The small one following this [Hanc sequens parva]	♄	28	12	27	55	B	5
The one at the bend of the same side [Quae est ad flexuram ejusdem lateris]	♄	29	14	27	14	B	3
The one on the left elbow [Quae in cubito sinistro]	♄	22	5	26	4	B	4
The Head of Medusa, or Algol [Caput Medusae, seu Algol]	♄	20	36	22	22	B	3
The one below Algol [Quae sub Algol]	♄	20	30	20	54	B	5
The one preceding this [Hanc praecedens]	♄	19	17	20	33	B	4
The one preceding, toward the north, in the same head	♄	18	19	21	35	B	4

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
[Praecedens ad boream in eodem capite]							
On the right hamstring [In poplite dextro]	II	6	12	28	22	B	5
The one that precedes the right knee [Quae dextrum genu praecedit]	II	4	10	28	50	B	4
Going before the bend of the knee [Flexuram genu praecedens]	II	3	54	26	11	B	5
The middle one on the right knee [Media in genu dextro]	II	5	13	26	39	B	4
The one below the right knee [Quae infra genu dextrum]	II	5	59	24	35	B	6
The one on the sole of the right foot [Quae est in planta pedis dextri]	II	8	0	18	56	B	5
The one on the left thigh [Quae in sinistro femore]	✂	28	10	22	6	B	4
The one on the left knee [Quae in sinistro genu]	II	0	7	19	4	B	3
On the left leg [In crure sinistro]	✂	29	22	14	53	B	5
The one on the left heel [Quae in sinistro calcaneo]	✂	25	32	12	8	B	4
The following star of the left foot [Sequens sinistri pedis]	✂	27	35	11	17	B	3
* Unformed, above the head [Informis supra caput]	✂	26	44	42	26	B	5
The one on the upper part of the right thigh [Quae in superiori parte femoris dextri]	II	2	31	29	31	B	5
Unformed, preceding the	✂	16	15	20	53	B	4

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
Head of Medusa [Informis, praecedens caput Medusae]							

Erichthonius [Auriga, the Charioteer] [Erichthonius]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
The upper and preceding of two on the head [Superior & praecedens duarum in capite]	II	23	37	32	15	B	6
The lower and following [Inferior & sequens]	II	24	13	30	50	B	4
Shining on the left shoulder, the She-Goat [In sinistro humero fulgens, capella]	II	16	15	22	51	B	1
The bright one on the right shoulder [Lucida in dextro humero]	II	52	51	21	27	B	2
On the right arm [In dextro brachio]	II	23	58	13	44	B	4
The one on the left elbow [Quae in sinistro cubito]	II	13	8	20	52	B	4
The preceding Kid [Praecedens haedus]	II	13	4	18	8	B	4
The following Kid [Sequens haedus]	II	13	48	18	11	B	4
On the upper foot [In superiori pede]	II	11	3	10	22	B	4

Coma Berenices [Coma Berenices]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
At the apex of the first and northern triangle [In cuspidē]	♄	18	16	28	25	B	3

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
primi & borealis trianguli							
* The upper one adjacent to this, following toward the south [Superior contig. hanc ad austr. sequens]	♄	18	41	27	23	B	4
* The lower of the same [Inferior earundem]	♄	18	45	27	20	B	4
* The one that follows the two adjacent stars [Quae contiguas duas sequitur]	♄	19	18	27	7	B	4
* The preceding of the two adjacent southern stars [Praecedens duarum austr. contiguarum]	♄	18	24	25	51	B	4
The other adjacent star, toward the east [Altera contigua ad ortum]	♄	18	47	26	7	B	4
* The preceding of all, toward the south [Omnium praecedens ad austrum]	♄	17	59	23	30	B	4
* The highest of three adjacent following stars [Suprema trium contig. sequentium]	♄	21	9	25	16	B	4
* The other, and going before [Altera & praecedens]	♄	20	50	24	56	B	4
* The lowest, and following [Infima & sequens]	♄	22	51	24	0	B	4
* The last in the extension of the hair [Postrema in extensione comae]	♄	28	57	32	46	B	4
* The one that precedes this [Quae hanc praecedat]	♄	27	48	31	41	B	4
* The one between these and	♄	24	16	30	16	B	4

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
the first, at the apex [Quae inter has & primam in cuspidē]							
* The one at the southern apex of the small triangle Δ [Quae est in australi cuspidē Δ parvi]	♐	28	14	28	32	B	5

Ophiuchus [Ophiuchus]

Star [Latin]	Sig n	Long.°	Long.′	Lat.°	Lat.′	B/ A	Ma g.
On the head [In capite]	♐	16	49	35	57	B	3
On the right shoulder [In dextro humero]	♐	19	44	28	1	B	3
The lower and following on the right shoulder [Inferior & sequens in dextro humero]	♐	21	4	26	11	B	3
The preceding star on the left shoulder [Praecedens in sinistro humero]	♐	4	58	32	36	B	4
The following star on the same shoulder [Sequens in eodem humero]	♐	6	15	31	56	B	4

The star catalogue continues. Columns: star name (Denominatio Stellarum) | ecliptic longitude (sign, degrees, minutes) | ecliptic latitude (degrees, minutes; **B** = north, *borealis*; **A** = south, *australis*) | magnitude (Mag.). A leading * marks a star not in Ptolemy. Zodiac signs are given by their symbols in the source; here rendered by name.

Ophiuchus [Ophiuchus] (continued)

Star	Long. sign	L°	L′	Lat°	Lat′	N /S	Ma g.
On the left elbow	Sagittarius	0	2	23	40	B	4
More northerly, in the left	Scorpio	2	43	17	19	B	3

Star	Long. sign	L°	L'	Lat°	Lat'	N /S	Ma g.
hand		6					
Following and more southerly	Scorpio	2 7	56	16	31	B	3
On the right elbow	Sagittarius	[?]	[?]	[?]	[?]	B	4
More southerly and leading, in the right hand	Sagittarius	2 4	12	13	47	B	4
More northerly and following, in the same hand	Sagittarius	2 5	13	15	20	B	5
On the right knee	Sagittarius	1 2	23	7	18	B	3
On the left knee	Sagittarius	3	38	11	30	B	3
On the right shin	Sagittarius	[?]	[?]	[?]	[?]	B	3
Fifth of the unformed stars, in the Milky Way	Sagittarius	2 6	30	33	3	B	4

Serpens [Serpens]

Star	Long. sign	L°	L'	Lat°	Lat'	N /S	Ma g.
Leading star, in the mouth	Scorpio	1 1	34	38	12	B	5
The one in the mouth	Scorpio	1 4	23	39	7	B	3
The one at the temples	Scorpio	1 7	5	35	25	B	3
At the start of the neck	Scorpio	1 4	20	34	28	B	3
The one by the left eye	Scorpio	1 5	9	37	29	B	4
The one at the nostrils	Scorpio	1 6	31	42	37	B	4
Second on the neck, below the	Scorpio	1	45	28	58	B	3

Star	Long. sign	L°	L'	Lat°	Lat'	N /S	Ma g.
head		2					
In the middle coil of the neck	Scorpio	1 6	49	25	36	B	2
The most southerly of three	Scorpio	1 8	45	24	6	B	3
The one in the second bend	Scorpio	2 0	25	16	27	B	4
Antepenultimate of the tail	Sagittarius	2 4	33	19	57	B	3
Penultimate	Capricorn	0	11	20	38	B	3
Last	Capricorn	1 0	9	26	59	B	3

Sagitta [Sagitta]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Upper and more easterly	Aquarius	1	31	39	13	B	4
The middle one, or the one leading it	Capricorn	2 7	54	38	59	B	5
The little one above the middle	Capricorn	2 8	30	39	31	B	6
Upper of two adjacent, on the notch [in Glyphide]	Capricorn	2 5	29	38	53	B	4
Lower of the same	Capricorn	2 5	38	38	18	B	4

Vultur [Vultur — Aquila/Lyra region]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
The one in the head	Capricorn	2 9	27	27	9	B	6
On the neck	Capricorn	2 6	52	26	50	B	3

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
The bright one on the shoulders	Capricorn	26	8	29	22	B	2
Small star above the bright one	Capricorn	25	32	30	55	B	6
The one on the left shoulder	Capricorn	25	25	31	18	B	3
The small one that follows	Capricorn	26	7	31	59	B	5
Upper and leading, on the lower wing	Capricorn	21	15	28	47	B	4
Lower and following, on the wing	Capricorn	22	13	26	35	B	5
Tail of the Vulture	Capricorn	14	14	36	17	B	3
The unformed star next leading the tail	Capricorn	2	43	37	40	B	3
Middle of the unformed stars above the tail	Capricorn	9	11	43	33	B	4
Seventh of the unformed stars, namely that which follows of the three	Capricorn	9	16	41	5	B	4

Antinous [Antinous]

Star	Long. sign	L°	L'	Lat°	Lat'	N/S	Mag.
Left hand	Capricorn	29	20	18	48	B	2
Right side	Capricorn	20	16	20	15	B	3
Knee	Capricorn	19	16	14	28	B	3

Delphinus [Delphinus]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Bright star of the tail	Aquarius	8	31	29	8	B	3
The one following the tail	Aquarius	9	47	28	53	B	6

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
The one below the tail	Aquarius	9	41	27	34	B	6
In the rhombus, more southerly of the leading side	Aquarius	1 0	55	31	58	B	3

Delphinus (continued)

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
More northerly of the same side	Aquarius	1 1	49	33	5	B	3
More southerly of the following side	Aquarius	1 3	35	32	0	B	3
The one in the head	Aquarius	1 3	51	32	47	B	3
The one on the leading side, fourth contiguous, going before	Aquarius	1 0	16	32	9	B	5
Lowest of the two leading, in the rhombus	Aquarius	9	17	30	42	B	6
Following of the same	Aquarius	1 0	41	30	41	B	6

Equuleus [Equuleus]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Leading star of the head	Aquarius	1 7	31	20	13	B	4
Following star of the head	Aquarius	1 9	53	21	6	B	4
Leading star of the mouth	Aquarius	1 7	53	25	16	B	4
Following star of the mouth	Aquarius	1 8	53	24	52	B	4

Pegasus [Pegasus]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Mouth of Pegasus	Aquarius	2 6	21	22	8	B	3
Head	Pisces	1	14	16	25	B	4
The one toward the south, on the head	Aquarius	2 9	44	15	43	B	5
Lower and following, in the mane	Pisces	1 2	59	14	31	B	6
Upper and leading, in the mane	Pisces	1 2	43	15	44	B	6
Bright star of the neck	Pisces	1 0	38	17	41	B	3
Following star on the neck	Pisces	1 2	24	18	29	B	5
Left shank	Pisces	3	22	36	43	B	4
Left knee	Pisces	8	49	34	19	B	4
Right shank	Pisces	1 4	2	41	1	B	4
Leading of the two on the breast	Pisces	1 7	28	28	49	B	4
Following	Pisces	1 8	52	29	25	B	4
Right knee	Pisces	2 0	9	35	8	B	3
On the same knee, toward the south	Pisces	1 9	24	34	25	B	5
Leading of the two on the wing	Pisces	2 5	32	25	35	B	6
Following on the wing, and more southerly	Pisces	2 7	5	24	51	B	6
First of the wing, Markab	Pisces	1 7	55	19	26	B	2
Junction of the leg, Scheat	Pisces	2	48	31	8	B	2

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Tip of the wing	Aries	3 3	37	12	35	B	2

Andromeda [Andromeda]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Head	Aries	8	46	25	42	B	2
Lowest, and the right shoulder	Aries	1 7	5	27	6	B	5
Lower, on the left shoulder	Aries	1 5	24	23	3	B	4
In the right arm, the most southerly of three	Aries	1 4	57	31	33	B	5
Northern	Aries	1 5	44	33	20	B	4
Middle	Aries	1 6	6	32	14	B	5
More southerly, in the upper hand	Aries	1 0	27	40	56	B	4
More northerly	Aries	1 1	45	41	44	B	4
Faint one in the same place	Aries	1 4	22	42	8	B	5
Highest of all, in the northern hand	Aries	1 2	46	43	49	B	4
Leading and upper of two, on the left arm	Aries	1 5	8	17	48	B	4
The one on the left elbow	Aries	1 6	52	15	58	B	5
More southerly, on the girdle	Aries	2 4	48	25	59	B	2
Middle	Aries	2 4	5	30	33	B	4

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Northern	Aries	2 3	35	32	30	B	4
The bright star on the southern foot	Taurus	8	38	27	46	B	2
* Outermost on the upper foot	Taurus	9	5	36	49	B	5
Brighter and leading, on the right foot	Taurus	6	51	35	21	B	4
Highest on the left calf	Taurus	5	5	28	59	B	5
Lower	Taurus	3	22	27	54	B	5
The one by the right knee	Taurus	0	55	36	20	B	5
* The one on the outermost ring of the chain	Pisces	2 3	59	57	19	B	4
Brighter and upper, on the left shoulder	Aries	1 6	18	24	20	B	3

Triangulus [Triangulum]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
At the apex of the triangle	Taurus	1	18	16	49	B	4
On the base, toward the north	Taurus	6	48	20	33	B	4
Middle	Taurus	[?]	[?]	[?]	[?]	B	5
More southerly, on the base	Taurus	7	57	18	57	B	4

TERTIO, ON THE STARS THAT APPEAR IN THE SOUTHERN PART.

[*TERTIO, DE STELLIS QVÆ IN MERIDIONALI PARTE VIDENTVR* — the third division of the catalogue, the southern constellations. Throughout this section the latitude marker is **A** (*australis*, south).]

Cetus [Cete]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
The one in the snout	Taurus	9	30	7	50	A	4
Bright star of the jaw of Cetus	Taurus	8	46	12	37	A	2
Middle, in the mouth	Taurus	3	52	12	3	A	3
Leading of three, at the cheek	Taurus	2	1	14	32	A	3
The one below the eye	Taurus	1	53	5	52	A	4
The one above the eye	Taurus	6	6	5	36	A	4
On the back of the head	Aries	2	28	4	19	A	4
		8					
On the breast of the quadrilateral, leading, northern	Aries	2	8	25	17	A	4
		4					
The more southerly of the two lower leading stars	Aries	2	30	28	31	A	4
		4					
The southern of the following stars, on the breast	Aries	2	10	28	17	A	4
		8					
Leading and northern	Aries	2	46	25	58	A	3
		7					
Middle, in the belly	Aries	1	24	25	1	A	4
		2					
Lowest in the belly	Aries	1	49	31	4	A	4
		3					
Northern of the belly	Aries	1	24	20	19	A	3
		6					
More easterly of the two brighter stars on the back	Aries	1	41	15	47	A	3
		0					
More westerly of the same	Aries	6	10	16	55	A	3
Northern of the tail	Pisces	2	22	10	1	A	3
		5					
Southern, or bright star of the tail	Pisces	2	55	20	47	A	2
		6					
* The unformed star following	Taurus	1	44	14	30	A	5

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
the bright jaw-star toward the east		2					
* Leading the northern belly-star, toward the south	Aries	1 5	3	21	55	A	5
* The one in a straight line with the 3rd and 5th of the chapter	Taurus	2	48	9	13	A	4

Orion [Orion]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Highest of three joined together, in the head	Gemini	1 8	10	13	26	A	4
* More westerly	Gemini	1 8	5	13	54	A	5
* Third, the one toward the east	Gemini	1 8	32	14	5	A	5
Following, or the bright shoulder	Gemini	2 3	11	16	6	A	2
Left, or leading shoulder	Gemini	1 5	22	16	53	A	2
Following, on the left shoulder	Gemini	1 6	46	17	22	A	5
The one on the right arm	Gemini	2 5	3	14	51	A	4
On the right forearm	Gemini	2 8	29	11	30	A	6
In the right hand, more southerly	Gemini	2 7	22	9	15	A	4
Leading, in the right hand	Gemini	2 6	20	8	44	A	4
Next to the highest, in the right hand	Gemini	2 7	21	7	21	A	6
Highest and last of those in the	Gemini	2	7	7	19	A	6

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
hand		8					
Leading of the two in the club	Gemini	2 3	8	3	13	A	5
Following of the same	Gemini	2 5	20	3	21	A	5
The one below the right shoulder, toward the west	Gemini	1 8	55	19	18	A	5
Following of the two faint stars on the back	Gemini	1 7	39	19	37	A	6
Leading of the same	Gemini	1 6	45	19	53	A	6
The one leading of the four on the back	Gemini	1 5	33	20	9	A	5
Northernmost of the nine in the shield	Gemini	7	52	8	17	A	4
Second	Gemini	8	47	9	7	A	4
Third	Gemini	8	9	11	6	A	6

Orion (continued)

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Fourth	Gemini	7	59	12	26	A	4
Fifth	Gemini	6	48	13	4	A	4
Sixth	Gemini	6	22	15	27	A	4
Seventh	Gemini	6	32	16	50	A	4
Eighth	Gemini	6	57	20	2	A	4
Last	Gemini	7	56	20	56	A	4
First of the Belt	Gemini	1 6	49	23	38	A	2
Middle	Gemini	1 7	53	24	34	A	2
Last	Gemini	1 9	5	25	22	A	2

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
The one on the hilt of the sword	Gemini	1 4	36	25	37	A	3
Highest of three on the sword	Gemini	1 7	27	28	10	A	5
Middle of the sword	Gemini	1 7	23	28	45	A	3
Southern	Gemini	1 7	26	29	17	A	3
Leading of the two below the sword	Gemini	1 6	19	30	38	A	4
Following of the two below the sword	Gemini	1 8	22	30	38	A	5
Bright star on the left foot, Rigel	Gemini	1 1	16	31	12	A	1
The one on the left heel	Gemini	1 2	14	29	53	A	4
The one on the calf of the left foot	Gemini	1 4	1	31	0	A	5
On the right knee	Gemini	2 0	48	33	8	A	3
* The one leading the last of the Belt, toward the south	Gemini	1 8	38	26	1	A	4
* The one on the back, leading this last	Gemini	1 4	33	19	40	A	6

Eridanus [Eridanus]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
The one by the left foot of Orion, at the start of the river	Gemini	9	39	31	36	A	4
Above the foot of Orion, in the river	Gemini	9	41	27	55	A	3
Following of two others	Gemini	7	38	29	52	A	5

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Leading	Gemini	5	28	27	52	A	5
Following of the two upper stars	Gemini	3	44	25	34	A	4
Leading of the same	Gemini	1	13	25	12	A	4
Following after an interval, of the four	Taurus	1 8	17	33	14	A	3
The one preceding it	Taurus	[?]	[?]	[?]	[?]	A	[?]
The one toward the north, or the third leading	Taurus	1 5	6	28	47	A	3
The one preceding all four	Taurus	1 2	44	27	47	A	3

Lepus [Lepus]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
Upper of the leading ear	Gemini	1 0	13	34	34	A	5
Lower of the same ear	Gemini	1 0	19	35	54	A	5
Upper of the following ear	Gemini	1 2	26	35	18	A	6
Lower of the following ear	Gemini	1 2	13	36	14	A	5
The one in the head	Gemini	9	48	39	4	A	5
Outermost of the front feet	Gemini	6	24	45	0	A	4
The one on the back, or middle of the body	Gemini	1 5	48	41	6	A	3
On the left shoulder	Gemini	1 4	5	43	58	A	3
More southerly of the two on the hind feet	Gemini	1 9	20	45	50	A	3
More northerly of the same	Gemini	2	35	44	18	A	3

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
		1					
Leading on the back	Gemini	2	25	38	16	A	4
		0					
Following on the back	Gemini	2	26	37	41	A	4
		3					
Last, in the tail	Gemini	2	21	38	26	A	4
		6					

Canis Maior [Canis Maior]

Star	Long. sign	L°	L'	Lat°	Lat'	N/ S	Ma g.
The most brilliant, in the mouth, called Sirius	Cancer	8	34	39	30	A	1
The one on the forehead, toward the right ear	Cancer	1	0	34	50	A	4
		9					
In the middle of the forehead	Cancer	1	26	36	43	A	5
		1					
The one below the left ear	Cancer	1	5	38	3	A	3
		4					
On the neck	Cancer	1	2	39	30	A	4
		2					
On the right shoulder of the front feet	Cancer	6	31	42	13	A	5
The one at the tip of the front foot	Cancer	1	41	41	19	A	2
The one on the back	Cancer	1	29	46	10	A	5
		5					

DENOMINATIO STELLARVM [The Naming of the Stars] (continued)

Cap. IV, star catalogue, third (southern) division — continued.

The catalogue columns are: star name | longitude (sign, degrees, minutes) | latitude (degrees, minutes) | B (north) / A (south) | magnitude. All stars on this page carry A (south latitude).

CANIS MAIOR [Canis Major] *(continued from the preceding page)*

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
The one in the middle of the chest [Media in pectore]	♌	12	35	46	40	A	5
The one on the belly [Quae in ventre]	♌	17	54	48	30	A	3
The one on the belly between the hind thighs [In ventre inter posteriora femora]	♌	15	20	51	25	A	3
The lower of the right foot of the forelegs [Inferior dextri pedis priorum]	♌	1	6	51	47	A	3
The one on the tail [Quae in cauda]	♌	24	10	51	25	A	3

CANIS MINOR [Canis Minor]

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
On the neck [In collo]	♌	16	38	13	34	A	3
On the thigh, Procyon [In femore, Procyon]	♌	20	17	15	57	A	2

ARGVS [Argo Navis]

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
The one on the topmost stern [Quae in suprema]	♏	5	52	43	19	A	3

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
puppi]							
The topmost of the ship's shield [Suprema clypei navis]	♌	0	34	44	59	A	3
The one preceding the shield [Praecedens clypei]	♍	27	59	47	28	A	3

HYDRA [Hydra]

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
Preceding, in the head [Praecedens in capite]	♌	5	38	14	37	A	5
Above the first, toward the north [Supra primam ad aquilonem]	♌	6	45	14	17	A	4
The more northerly, in the back of the head [Borealior in occipite]	♌	6	47	11	8	A	4
The one that precedes the third, toward the south [Quae tertiam ad austrum praeit]	♌	7	21	11	36	A	5
The easternmost of all in the head [Omnium in capite orientalior]	♌	8	59	11	1	A	4
The one that precedes, on the neck [Quae in collo praecedit]	♌	11	50	11	6	A	6
Following, at the spring of the neck [Sequens in educatione colli]	♌	14	40	13	5	A	4
The middle of the neck,	♌	20	10	15	0	A	5

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
and preceding three at the coil [Media colli, & praecedens trium in nexu]							
The northern of three at the bend of the neck [Borea trium in flexu colli]	♌	22	3	14	18	A	4
The southern at the coil [Australis in nexu]	♌	19	52	16	46	A	5
The bright star of Hydra, or the heart [Lucida hydrae, sive cor]	♌	21	44	22	24	A	1
The one that next follows the heart [Quae proxime cor sequitur]	♌	27	11	26	34	A	4
The one that follows this one next [Quae hanc deinde sequitur]	♍	0	8	26	12	A	5
Preceding, of two adjoining above this one [Praecedens ex duabus contiguis supra hanc]	[?]	[?]	[?]	[?]	[?]	A	5
Following the same [Sequens earundem]	♍	[?]	[?]	[?]	[?]	A	4
The one that is fifth from the heart [Quae a corde quinta est]	♍	9	30	24	38	A	4
The one in a straight line with this one and the following [Quae in recta linea cum hac & sequente]	♍	12	40	23	31	A	5
Next preceding Crater [Cratera proxime]	♍	14	50	21	49	A	4

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
praecedens]							
* Unformed, next preceding the head [Informis, caput proxime praecedens]	♌	4	44	12	27	A	4

CRATER [Crater]

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
The one that is at the base of the bowl [Quae est in basi crateris]	♌	18	12	22	41	A	4
Following, of two in the middle [Sequens duarum in medio]	♌	23	42	19	29	A	4
Preceding the same [Praecedens earundem]	♌	21	9	17	25	A	4

CORVVS [Corvus]

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
The one at the eye [Quae ad oculum]	♏	6	7	19	39	A	4
Preceding, of the two upper ones in the square [Praecedens duarum superiorum in □]	♏	5	12	14	25	A	3
Following the same [Sequens earundem]	♏	7	54	12	7	A	3
Following, of the lower ones in the quadrilateral [Sequens inferiorum in	♏	11	48	17	59	A	3

Star	Long. sign	Long.°	Long.′	Lat.°	Lat.′	B / A	M ag.
quadrato]							

[CATALOGUE → PRECESSION BOUNDARY: the star catalogue ends here. Section II below begins the second part of Cap. IV — the slow motions of the stars, precession of the equinox, and the change of the obliquity, with the table and computations that follow.]

II. Now that the stars have been set out in their proper tabular arrangement, the next step is to determine, by direct comparison — the stars among themselves and with our own time — their motions, which appear in the heavens as the very slowest (along with the shifting of the Sun's oblique path). From these arise the mean motions that astronomy chiefly relies on, and in the following part of this chapter we will reduce them to their true or apparent values by means of a suitable hypothesis and the tables derived from it. In the earlier discussion of the stars, given the occasion, I showed — accurately, unless I am mistaken — from the comparison of various ages, beginning with Timocharis and using the observations of the experts checked against one another, that the mean motion by which the fixed stars are seen

to revolve in the heavens in one equal year is 49 seconds 45 thirds. Therefore, without delay or tedious repetition, I will now draw up the part of the following table that concerns the simple precession of the equinoxes into the leading signs — or, what amounts to the same thing, of the fixed stars into the following signs — reckoned over the world-ages.

As for the anomaly of the equinoxes, of the Sun's obliquity, and thereby of the change of latitude in the stars: from all these phenomena observed over various ages, restored and most carefully weighed by us, we have found that this revolution is completed in a period of 3600 solar years, taking its start rightly from the foundation of the world according to our setting-up of that epoch — that is, taken from when the Sun's greatest obliquity coincided most exactly with the mean and true equinox at the world's beginning. For from this beginning the remaining quantities we require for resolving these inequalities arise from a single common hypothesis — not, as Copernicus left it, from a double one resting moreover on a straight line.

What ought to be brought forward from the celestial observations, both for establishing the mean motion of this anomaly and for treating the

προσθαφαίρεσις [Greek: prosthaphaeresis, "the additive-subtractive equation"] in the following section of this chapter, it has seemed fitting to set out here.

Concerning the alteration of the obliquity of the ecliptic, then: when we first determined from our own examination that its mean value was 23 degrees 42 minutes, we found (to repeat here what we noted before) that the greatest was 23 degrees 53 minutes in the year of the world 3600, a little before the beginning of the Greek monarchy, but that the least was 23 degrees 31 minutes + [an additive "and somewhat more" mark in the source, not a numeral] in the year of the world 5400 — and so the difference falls within 22 minutes. [Margin: *Amog. lib. 1, cap. 2.*] Hence, by the time of Eratosthenes and Hipparchus, whose observations on this matter Ptolemy cites, the greatest obliquity of the Sun had decreased duly enough to 23 degrees 51½ minutes. And although Ptolemy himself asserts that he found this same value in his own time, and apparently held it fixed along with the Sun's eccentricity, yet when we consider that he neglected the difference of the Sun's parallax at the limit of each solstice, the true obliquity in Ptolemy's time — once this is added — will be about 23 degrees 49 minutes, as our supposition brings out. Likewise, in the age of [al-Battānī] [Albategnius] that greatest obliquity comes, on our supposition, to 23 degrees 35¾ minutes, which agrees nicely with his observation.

[Margin: *Lib. 1. Prog. pag. 38. item in comment. Solis praemis. Lib. 1, Prog. pag. 47.*]

The least obliquity of the Sun, however, was around the year of the world 5400, as I said just above, 23 degrees 31 minutes + [the same additive mark]. But afterward, in Copernicus's time, the same obliquity, taking up its increase again, was 23 degrees 31 + minutes [the same additive "and somewhat more" mark in the source, not a fraction]. For although Copernicus himself set it at 23 degrees 28 minutes — a difference of 3¼ minutes — Tycho's refinement had removed this, since the observation of Walther of Nuremberg, made 60 years before Copernicus, when duly applied by Tycho, had also extended to 23 degrees 31 minutes. [Margin: *Vide observationes Solis supra in commentario ejusdem.*] Finally, in Tycho's time and our own, the greatest obliquity — on this same supposition by which the greatest altitude of the Sun at [Uraniborg] [Vraniburgi] was found year after year, through manifold and undoubted observation, to be 57 degrees 35½ minutes (with the parallax added and the elevation of the equator subtracted) — was found to be 23 degrees 31½ minutes.

From all this it is sufficiently clear that we have rightly established the increase and decrease of the greatest obliquity of the ecliptic, together with its anomaly and period. And although we use Julian years here in place of solar ones,

nevertheless, on account of the difference that over the span of 6000 years amounts to 46 days by anticipation — yet in the true motion of the anomaly scarcely beyond as many seconds — no sensible error can arise here, nor in the inequality of the equinox. As to that inequality, just how it fits this single supposition will become plain to anyone who investigates it by comparison, as set out earlier in the discussion of the Sun and the fixed stars; and it will be shown still more clearly when, in the chapter next following, we deal attentively with those things that establish the measure of the tropical year absolutely, by comparing several intervals of ages among themselves.

Tabula mediorum motuum aequinoctialium & obliquitatis eclipticae in annis Iulianis [Table of the Mean Motions of the Equinoxes and of the Obliquity of the Ecliptic in Julian Years]

Main table. Columns: *Anni Epoch.* [Years from epoch] | *Praecessio aequinoct.* [Precession of the equinox] (S = signs, G = degrees, M = minutes, S = seconds) | *Anomalia aequinoct. & obliquit.* [Anomaly of the equinox and of the obliquity] (S = signs, G = degrees, M = minutes).

Years from epoch	Prec. S	Prec. G	Prec. M	Prec. S(ec)	Ano m. S	Ano m. G	Ano m. M
Mundi [of the World]	0	0	1	49	0	0	13 1/12
Christi [of Christ]	1	24	50	53	1	6	37 1/12
20	0	0	16	36	0	0	2
40	0	0	33	11	0	0	4
60	0	0	49	47	0	0	6
80	0	1	6	23	0	0	8
100	0	1	22	58	0	0	10
200	0	2	45	57	0	0	20
300	0	4	8	55	0	1	0
400	0	5	31	54	0	1	10
500	0	6	54	52	0	1	20
600	0	8	17	50	0	2	0
700	0	9	40	49	0	2	10

Years from epoch	Prec. S	Prec. G	Prec. M	Prec. S(ec)	Ano m. S	Ano m. G	Ano m. M
800	0	11	3	47	0	2	20
900	0	12	26	46	0	3	0
1000	0	13	49	44	0	3	10
1100	0	15	12	42	0	3	20
1200	0	16	35	41	0	4	0
1300	0	17	58	39	0	4	10
1400	0	19	21	38	0	4	20
1500	0	20	44	36	0	5	0
1600	0	22	7	34	0	5	10
1700	0	23	30	33	0	5	20
1800	0	24	53	31	0	6	0
1900	0	26	16	30	0	6	10
2000	0	27	39	28	0	6	20
2500	1	4	34	20	0	8	10
3000	1	11	29	12	0	10	0
3500	1	18	24	4	0	11	20
4000	1	25	18	56	1	1	10
4500	2	2	13	48	0	3	0
5000	2	9	8	40	0	4	20
5500	2	16	3	32	0	6	10
6000	2	22	58	24	0	8	0
6300	2	27	7	19	0	9	0

Side table — *In annis sing. usque ad 20* [In single years, up to 20]. Columns:
Anni [Years] | *Praecessio aequinoct.* (M = minutes, S = seconds) | *Anomalia*
aequinoct. & obliquit. (G = degrees, M = minutes).

Years	Prec. M	Prec. S	Anom. G	Anom. M
1	0	50	0	6
2	1	40	0	12
3	2	29	0	18
4	3	19	0	24

Years	Prec. M	Prec. S	Anom. G	Anom. M
5	4	9	0	30
6	4	59	0	36
7	5	48	0	42
8	6	38	0	48
9	7	28	0	54
10	8	18	1	0
11	9	7	1	6
12	9	57	1	12
13	10	47	1	18
14	11	37	1	24
15	12	27	1	30
16	13	17	1	36
17	14	6	1	42
18	14	56	1	48
19	15	46	1	54
20	16	36	2	0

Side table — *In mensibus* [In months]. Columns: month | *Praecessio* (M = minutes, S = seconds) | *Anomalia* (M = minutes, S = seconds).

Month	Prec. M	Prec. S	Anom. M	Anom. S
Ianuar. [January]	0	4	0	30
Februar. [February]	0	8	1	0
Marti. [March]	0	12	1	30
April. [April]	0	17	2	0
Majus [May]	0	21	2	30
Iunius [June]	0	25	3	0
Iulius [July]	0	29	3	30
August. [August]	0	34	4	0
Septemb. [September]	0	38	4	30
Octob. [October]	0	42	5	0
Novemb. [November]	0	46	5	30

Month	Prec. M	Prec. S	Anom. M	Anom. S
Decemb. [December]	0	50	6	0

III. Now that the earlier matters of this chapter have been dealt with — since we have advanced to the construction of the hypothesis by which the inequalities of the equinoxes and of the Sun's greatest obliquity can be legitimately resolved — I have judged it right to point out, first of all, that Lord Tycho's conjecture about these things is without doubt true: namely, that they arise from some external cause and by accident. [Margin: *Lib. 1. Prog. pag. 250.*] And although they could be demonstrated in various ways, even with the Earth at rest, in the orbits of the Sun and the fixed stars (as we will suggest below), nevertheless — since long ago, from many and various arguments, I brought my mind to hold that the Earth, placed in the middle of the visible world, not only most conveniently displays the reality of the apparent daily revolution in the heavens by its own rotation eastward upon its center, but also that the precession of the equinoxes into the leading signs is most rightly attributed to it, in agreement with Copernicus — I do this so that the objection should not be introduced which, among the other points I argued above on this subject, I recall raising: namely the contrariety of the motions, or an excessive violence from which nature shrinks. [Margin: *Cap. 1 hujus.*] And at the same time, so that the stars, fixed in the most limpid heaven, should not suffer the inconvenience of being pulled apart from one another — a thing never observed — if you presume that each is set in motion separately by some carrying-motion (though perhaps even this might in some way be granted in the expansive celestial firmament).

Therefore we represent the regressions of the equinoxes in this way: we lay it down that the poles of the Earth gradually, with a most slow motion, regress into the leading signs about the axis of the ecliptic, imagined as likewise within the Earth. From this comes such an anticipation in the equator, as the ecliptic — that is, the apparent path of the Sun — cuts across it. For according to this supposition, or even the Copernican one, there is no need to recognize the equator anywhere in the heavens outside the globe of the Earth, but only the ecliptic — whose course the Sun chiefly, and then all the remaining planets, following back and forth (a thing marvelous to see), revere through all the periods of their courses, honoring the power and majesty given to the Sun by its Creator. Furthermore, the motion of the inequality of the equinoxes, and likewise of the obliquity of the ecliptic — whose effects in the heavens we perceive in the apparent course of the Sun and in the positions of the stars — we attribute to the mobility of the ecliptic's axis, fixed in the Earth at its poles on the surface of the globe. Hence it comes about that the Earth, free of this motion, and not — as Copernicus supposed —

entangled in another libration than the one which is simply about its poles (which I mentioned before) for the sake of the simple anticipation of the equinoxes, has these inequalities of the Sun's orbit alone brought upon it, as the following supposition will show.

Hypothesis praecessionis, item inaequalitatis aequinoctiorum, & obliquitatis eclipticae simul [The hypothesis of precession, and likewise of the inequality of the equinoxes and of the obliquity of the ecliptic together]

Diagram. *The figure itself is referenced in the text but is not printed on this page — only its construction is described in the prose. The labels named in the text are listed below for the figure that appears on the following page.*

The construction described: From A, the center (the subjected pole of the northern terrestrial ecliptic), two homocentric circles are first drawn.

- A (center): the subjected pole of the northern terrestrial ecliptic
- B C: an arc, representing part of the little arctic circle of the ecliptic, which the Earth's pole had traversed from C to B about the center of the ecliptic by a simple motion from the founding of the world into the leading signs; this arc measures the anticipation of the equinoxes and solstices, and likewise the apparent motion of the fixed stars into the following signs
- N H M: the mean ecliptic, drawn from the same subjected pole of the ecliptic A
- N L M: the celestial equator, drawn for the sake of demonstration about center B
- F E D G: the little circle, itself also drawn about center A, in which the true pole of the ecliptic — set on the surface of the terrestrial globe, movable into the following signs about its own axis — is supposed to have shifted its apogee in step with the motion C B
- F: where, with the true pole of the terrestrial ecliptic established, the greatest distance of the ecliptic from the equator occurs, known through the interval K L, to which F B is equal
- K L: the interval equal to F B (the greatest obliquity)
- E, D: the subjected pole of the mobile ecliptic, ascending through E into D, gives the least obliquity L I, to which B D corresponds
- L I: the least obliquity, corresponding to B D

- E, G: when the same pole moves at the side, in E or G, the mean obliquity is indeed kept, but the προσθαφαίρεσις [Greek: prosthaphaeresis, the additive-subtractive equation] of the equinoxes and solstices is greatest — there subtractive in E, here additive in G — at the tropic and equi-[noctial] points

The passage continues: From A (in the following figure), the center being the subjected pole of the northern terrestrial ecliptic, two homocentric circles are first drawn, of which B C represents the part of the little arctic circle of the ecliptic which the Earth's pole had completed from C to B about the center of the ecliptic by a simple motion from the foundation of the world into the leading signs — which arc measures the anticipation of the equinoxes and solstices, and besides the apparent motion of the fixed stars into the following signs.

Furthermore, from the same subjected pole of the ecliptic A, the mean ecliptic N H M itself; then, with B as center, the celestial equator N L M is drawn for the sake of demonstration. Hence, with the true subjected pole of the ecliptic placed on the surface of the terrestrial globe, movable about its own axis into the following signs, in the little circle F E D G — which is itself also drawn about center A and is supposed to have shifted its apogee in step with the motion C B — it is plain how, with the true pole of the terrestrial ecliptic established at F, the greatest distance of the ecliptic from the equator occurs, which is known through the interval K L, to which F B is equal (as we left demonstrated in the doctrine of the sphere). So too the subjected pole of the mobile ecliptic, ascending from F through E into D, will give the least obliquity L I, to which B D corresponds. But when the same is positioned at the side, in E or G, the mean obliquity is indeed maintained, but the προσθαφαίρεσις of the equinoxes and solstices is greatest — there subtractive in E, here additive in G — at the tropic and equi-

—at the equinoctial points, and indeed throughout the whole ecliptic, just as is sufficiently clear in the very diagram of the hypothesis. In it, indeed, more is comprehended than can be set forth here — concerning the universal condition and change of the nations, embracing all the ages of the world.

But since here we are working on the surface of the globe, where it is appropriate to derive the equations geometrically by a spherical triangle, we therefore determine the semidiameter or radius of the little circle AF as 10' 53" [10 m: 53 se.], for the preceding examination of the obliquity of the ecliptic and of the inequality of the equinoxes from the observations set out above — namely in the ratio that the third number in the order of perfect [numbers], 496, bears to a quadrant of the circle on the surface, 5400, both understood in arc-minutes.

We elicit the prosthaphaereses of the obliquity and of the equinoxes by means of the spherical triangle in this way. In the spherical triangle ABG, as I said, the mean distance of the poles $AB = 23^\circ 42'$ is given; and likewise $AG = 10' 53''$; and finally the angle BAG, the complement of the anomaly of the semicircle, that is, the arc DG. Therefore from the calculation the arc BG is yielded, exhibiting the measure of the obliquity of the ecliptic; and the angle ABG — to which an equal angle in the ecliptic HBS will be opposite — here determines the subtractive inequality of the equinoxes by the arc HS at the summer solstice, or at the vernal equinox by the arc NV. And so, proceeding likewise in the rest, we have built up the first two columns of the following table.

Furthermore, as for the proportional minutes [scrupula proportionalia] inserted into the same table, for conveniently tracking down the latitudes among the fixed stars, we have obtained these too by the following method. From the equinoctial point A let two arcs run out to the completion of a quadrant, AB and AC, forming the angle BAC. From the demonstration above, the alteration of the ecliptic is determined as $1' 46''$ [1 min. 46 sec.]. For this, having now resolved 1° into 60', you may set the maximum alteration of the ecliptic at 21' [21 min.; no seconds in source], and having laid out the angle at A, you will readily hunt down with us, in the right-angled spherical triangle ADE, the side ED itself and its double EF, which indicates the proportional minutes under any supposition of the quadrant.

These are the things that constitute our hypothesis of the equinoxes and solstices upon the motion of the earth and of the pole of the ecliptic. But if anyone should wish to exhibit the same phenomena with the earth either at rest from all motion, or content with the daily revolution alone, he will necessarily separate the simple progression of the stars from the inequality of the equinoxes and from the maximum alteration of the Sun's obliquity — attributing the former to the starry heaven in the order of the signs, and the latter to the movable poles of the solar orbit on the surface of its own sphere. Thus he will proceed by the same path of demonstration as we have, and he will change nothing in the tables we have constructed beforehand.

Diagram. A large schematic of the precession hypothesis, viewed as nested concentric circles seen on the globe's surface. The construction is centered on the small inner figure at A; concentric circles radiate outward to the zodiacal band at the periphery, which is labeled with zodiac signs. A vertical axis runs from L at the top down through the figure to ♑ (Capricorn) at the bottom; a near-vertical narrow lens/spindle figure (the "subjected" pole assembly) runs from L down to B, carrying the stack of small labels at top.

Inner construction (center, around A):

- A (center): the subjected pole of the ecliptic / common center.
- B (upper center, apex of the spindle): vertex of the spherical triangle.
- C (left of center, on a middle circle): point on the construction.
- D (just above A, on the little circle): point of the little circle (max-obliquity side).
- E (right of A, on the little circle): point of the little circle.
- F (just below A): foot on the little circle (the radius $AF = 10' 53''$).
- G (left of A, on the little circle): point opposite E.

Stacked labels along the upper spindle (top to bottom): L (top of axis); then a small rectangular inset enclosing I, P (upper), Q with the ♋ (Cancer) symbol, R, H, S, and K just above B. These mark the prosthaphaeresis construction (H S the equinoctial arc; K the little-circle foot of the spindle).

Peripheral zodiac labels (reading around the band):

- Top-left, on the outer band: ♋ (Cancer).
- Left side, descending: X with ♎ (Libra); M with ♎ (Libra); and a third ♎ (Libra) lower-left.
- Bottom of axis: ♏ (Capricorn).
- Right side, descending: ♏ (Capricorn) at O; ♈ (Aries) at N; $\text{♈}/\text{♉}$ at V (the right-hand sign group $\text{♈} \dots$ with V).

Diagram. A small right-angled spherical-triangle figure set in the lower text column. A quadrilateral-curved region with vertices:

- B (upper left): start of the upper arc.
- F (upper right): end of the upper arc (the doubled side EF).
- D (interior, between B and the lower arc): point on the construction (side ED).
- C (lower left): lower-left vertex.
- E (lower center): foot of the perpendicular.
- A (lower right): the equinoctial point and right-angle vertex.

The upper curve runs B–F; the lower curve runs C–E–A; D lies on the inner line. This is the triangle ADE (with its double EF) used to compute the proportional minutes.

Tabula Prosthaphaereseon Aequinoctiorum [Table of the Prosthaphaereses of the Equinoxes]

obliquitatis eclipticae & latitudinum stellarum fixarum [of the obliquity of the ecliptic and of the latitudes of the fixed stars]

This is a single large table spanning the opening. The argument runs down the left **Grad.** column 0→30 and continues up the right **Grad.** column 30→0, so each printed row carries two arguments. Six figure-blocks appear across the opening, each block having three sub-columns: **Aequator. aequinoct. S** (Equator of the equinox; "S" = subtractive), **Excessus obliquit. A** (Excess of obliquity; "A" = additive), and **Scrupula proportionalia** (Proportional minutes). On p. 222 the top blocks are headed **fig. 3**, **fig. 1**, **fig. 2**; the foot labels read **fig. 11**, **fig. 10**, **fig. 9**. All values are in minutes (M.) and seconds (S.).

Page 222 — left leaf. Columns, left to right: Grad. | fig.3 [Aeq.S | Exc.obl.A | Scrup.] | fig.1 [Aeq.S | Exc.obl.A | Scrup.] | fig.2 [Aeq.S | Exc.obl.A | Scrup.] | Grad.

				3 3						1 1						2 2			
G	3:	3:	3:	3:	:	:	1:	1:	1:	1:	:	:	2:	2:	2:	2:	:	:	G
r	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	r
a	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	a
d	S	S	A	A	r	r	S	S	A	A	r	r	S	S	A	A	r	r	d
.	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	.
0	0	0	21	46	0	0	13	35	20	19	3	1	23	36	16	19	5	4	3
											0	1					1	9	0
1	0	28	21	46	1	3	14	0	20	12	3	5	23	50	16	9	5	1	2
											1						2	9	9
2	0	56	21	45	2	5	14	25	20	6	3	5	24	4	15	59	5	4	2
											1	8					2	8	8
3	1	24	21	45	3	8	14	41	19	59	3	5	24	17	15	49	5	1	2
											2	0					3	6	7
4	1	52	21	44	4	1	15	12	19	53	3	4	24	30	15	39	5	4	2
						0					3	1					3	4	6
5	2	20	21	43	5	1	15	36	19	48	3	3	24	43	15	29	5	1	2
						2					4	1					4	1	5
6	2	48	21	42	6	1	15	59	19	42	3	1	24	57	15	19	5	3	2

Grade	3:3				3:1				1:1				2:2				Grade		
	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex		S	S
	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	q.	q.	c.	c.		c	c
	S	S	A	A	r	r	S	S	A	A	r	r	S	S	A	A		r	r
M																			
7	3	16	21	40	7	1	16	21	19	35	3	1	25	4	15	8	5	2	2
8	3	44	21	38	8	1	16	43	19	29	3	5	25	16	14	58	5	2	2
9	4	12	21	36	9	2	17	5	19	22	3	4	25	30	14	47	5	5	2
10	4	40	21	33	1	2	17	27	19	15	3	3	25	40	14	37	5	1	2
11	5	8	21	31	1	2	17	49	19	7	3	2	25	49	14	26	5	3	1
12	5	36	21	28	1	2	18	10	19	0	4	6	25	57	14	16	5	5	1
13	6	4	21	26	1	2	18	31	18	51	4	5	26	4	14	5	5	1	1
14	6	32	21	24	1	2	18	52	18	43	4	3	26	11	13	54	5	3	1
15	7	0	21	21	1	2	19	13	18	35	4	2	26	17	13	43	5	5	1
16	7	28	21	18	1	2	19	33	18	27	4	6	26	22	13	32	5	1	1
17	7	55	21	15	1	2	19	52	18	19	4	4	26	27	13	21	5	2	1
18	8	22	21	12	1	2	20	11	18	10	4	3	26	31	13	10	5	4	1
19	8	49	21	9	1	3	20	30	18	2	4	1	26	35	12	58	5	5	1
20	9	16	21	5	2	3	20	50	17	53	4	5	26	40	12	47	5	5	1
29	9	43	21	1	2	2	21	9	17	44	4	3	26	44	12	36	5	1	9

					3	3						1	1				2	2	
G	3:	3:	3:	3:	:	:	1:	1:	1:	1:	:	:	2:	2:	2:	2:	:	:	G
r	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	r
a	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	a
d	S	S	A	A	r	r	S	S	A	A	r	r	S	S	A	A	r	r	d
.	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	.
1					1	9					6	9					9	5	
2	10	10	20	57	2	2	21	27	17	35	4	1	26	48	12	24	5	2	8
2					2	8					7	9					9	4	
2	10	36	20	53	2	2	21	45	17	26	4	5	26	52	12	13	5	3	7
3					3	6					7	7					9	2	
2	11	2	20	50	2	2	22	2	17	16	4	3	26	54	12	2	5	3	6
4					4	4					8	4					9	9	
2	11	29	20	45	2	2	22	18	17	7	4	9	26	57	11	50	5	4	5
5					5	2					9						9	5	
2	11	55	20	40	2	2	22	34	16	57	4	4	26	59	11	39	5	5	4
6					6	0					9	3					9	0	
2	12	20	20	35	2	1	22	50	16	48	5	1	27	1	11	27	5	5	3
7					7	8					0	6					9	4	
2	12	45	20	30	2	1	23	5	16	39	5	4	27	3	11	16	5	5	2
8					8	6					0	8					9	7	
2	13	10	20	25	2	1	23	21	16	29	5	1	27	4	11	4	5	5	1
9					9	4					1	9					9	9	
3	13	35	20	19	3	1	23	36	16	19	5	4	27	5	10	53	6	0	0
0					0	1					1	9					0		

Foot labels under the three blocks (left to right): **fig. 11 · fig. 10 · fig. 9**. The "A" marks beneath each Aequator/Excessus sub-column indicate the additive sense read in the up-direction.

Page 223 — right leaf. Continuation of the same table. Top blocks: **fig. 3, fig. 4, fig. 5**; foot labels: **fig. 8, fig. 7, fig. 6**. Columns left to right: Grad. | fig.3 [Aeq.S | Exc.obl.A | Scrup.] | fig.4 [Aeq.S | Exc.obl.A | Scrup.] | fig.5 [Aeq.S | Exc.obl.A | Scrup.] | Grad.

					3	3						4	4				5	5	
G	3:	3:	3:	3:	:	:	4:	4:	4:	4:	:	:	5:	5:	5:	5:	:	:	G
r	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	r
a	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	a
d	S	S	A	A	r	r	S	S	A	A	r	r	S	S	A	A	r	r	d
.	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	.
0	27	5	10	53	6	0	23	46	5	27	5	4	13	46	1	27	3	1	3
					0						1	9					0	1	0
1	27	4	10	42	5	5	23	31	5	17	5	1	13	21	1	21	2	1	2
					9	9					1	9					9	4	9
2	27	3	10	30	5	5	23	16	5	88	5	4	12	55	1	16	2	1	2
					9	7					0	8					8	6	8
3	27	2	10	19	5	5	23	0	4	5	5	1	12	29	1	11	2	1	2
					9	4					0	6					7	8	7
4	27	1	10	7	5	5	22	44	4	48	4	4	12	3	1	5	2	2	2
					9	0					9	3					6	0	6
5	27	0	9	56	5	4	22	29	4	39	4	9	11	37	1	1	2	2	2
					9	5					9						5	2	5
6	26	58	9	44	5	3	22	12	4	29	4	3	11	11	0	57	2	2	2
					9	9					8	4					4	4	4
7	26	56	9	33	5	3	21	55	4	20	4	5	10	44	0	53	2	2	2
					9	2					7	7					3	6	3
8	26	53	9	21	5	2	21	37	4	11	4	1	10	17	0	49	2	2	2
					9	4					7	9					2	8	2
9	26	49	9	10	5	1	21	20	4	2	4	3	9	50	0	45	2	2	2
					9	5					6	9					1	9	1
1	26	45	8	59	5	4	21	2	3	53	4	5	9	22	0	41	2	3	2
0					9						5	8					0	0	0
1	26	40	8	48	5	5	20	43	3	44	4	1	8	55	0	37	1	3	1
1					8	4					5	6					9	0	9
1	26	37	8	36	5	4	20	24	3	35	4	3	8	27	0	34	1	2	1
2					8	1					4	3					8	9	8
1	26	32	8	25	5	2	20	5	3	27	4	4	8	0	0	31	1	2	1
3					8	7					3	9					7	9	7
1	26	27	8	14	5	1	19	45	3	19	4	6	7	33	0	28	1	2	1
4					8	1					3						6	9	6

					3	3						4	4				5	5	
G	3:	3:	3:	3:	:	:	4:	4:	4:	4:	:	:	5:	5:	5:	5:	:	:	G
r	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	r
a	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	a
d	S	S	A	A	r	r	S	S	A	A	r	r	S	S	A	A	r	r	d
.	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	.
1	26	22	8	3	5	5	19	25	3	11	4	2	7	5	0	25	1	2	1
5					7	4					2	2					5	8	5
1	26	15	7	54	5	3	19	5	3	3	4	3	5	41	0	22	1	2	1
6					7	6					1	7					4	8	4
1	26	8	7	43	5	1	18	44	2	55	4	5	5	12	0	19	1	2	1
7					7	7					0	2					3	7	3
1	26	1	7	32	5	5	18	23	2	47	4	6	6	37	0	17	1	2	1
8					6	7					0						2	6	2
1	25	54	7	21	5	3	18	2	2	39	3	2	5	9	0	15	1	2	1
9					6	6					9	0					1	5	1
2	25	47	7	9	5	1	17	40	2	31	3	3	4	43	0	13	1	2	1
0					6	4					8	3					0	3	0
2	25	38	6	59	5	5	17	18	2	24	3	4	4	15	0	11	9	2	9
1					5	1					7	6						1	
2	25	28	6	48	5	2	16	56	2	17	3	5	3	46	0	9	8	1	8
2					5	7					6	8						8	
2	25	17	6	38	5	2	16	33	2	11	3	1	3	18	0	7	7	1	7
3					5						6	0						6	
2	25	5	6	27	5	3	16	11	2	4	3	2	2	49	0	5	6	1	6
4					4	7					5	1						4	
2	24	52	6	17	5	1	15	48	1	57	3	3	2	20	0	3	5	1	5
5					4	1					4	1						2	
2	24	39	6	7	5	4	15	25	1	51	3	4	1	52	0	2	4	1	4
6					3	4					3	1						0	
2	24	26	5	57	5	1	15	1	1	45	3	5	1	24	0	1	3	8	3
7					3	6					2	0							
2	24	13	5	47	5	4	14	36	1	39	3	5	0	56	0	1	2	5	2
8					2	8					1	8							
2	24	0	5	37	5	1	14	11	1	33	3	5	0	28	0	0	1	3	1
9					2	9					1								

					3	3						4	4				5	5	
G	3:	3:	3:	3:	:	:	4:	4:	4:	4:	:	:	5:	5:	5:	5:	:	:	G
r	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	Ae	Ae	Ex	Ex	S	S	r
a	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	q.	q.	c.	c.	c	c	a
d	S	S	A	A	r	r	S	S	A	A	r	r	S	S	A	A	r	r	d
.	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	.
3	23	46	5	27	5	4	13	46	1	27	3	1	0	0	0	0	0	0	0
0					1	9					0	1							

Foot labels under the three blocks (left to right): **fig. 8** · **fig. 7** · **fig. 6**.

§. IV. *Vsus tabularum praecedentium*. [Use of the preceding tables.]

Since the places of the fixed stars and the obliquity of the ecliptic, on which the preceding mean motions are founded, have been verified from the first bissextile year of the world 5564, and likewise from the completed year 1609 of Christ, accordingly let one first extract the simple motion of the equinoxes and of the anomaly for the given time (at least the completed years and months), adding the epochs from the prior table of mean motions. Then from the following table of prosthaphaereses [προσθαφαίρεσεων, Greek: prosthaphaireseon, "of the equations"] let the appropriate equations be drawn — everywhere including, as is done in all astronomical tables, the proportional part for the minutes and the rest that adhere to signs and degrees, concerning which we have given general instruction in the Logistical Arithmetic. Likewise too, the excess of the minimum obliquity of the ecliptic is always to be added, so that the true obliquity agreeing with the time may be known. But the equation of the equinoxes, according to its own marks, is to be subtracted from or added to the mean precession, so that this too may be had as corrected [coaequata].

Further, the true or corrected precession agreeing with the given time, once the true precession of the completed year 1600 after the birth of Christ — which is 2 signs 17° 6' 10" — has been taken away, leaves a difference (if the time is fixed backward) that is to be subtracted from any star in the upper table of fixed stars. But if instead the time has been drawn forward from that epoch of the stars, let the true precession of the prescribed epoch of the stars be subtracted from the true precession agreeing with such a time; and for any proposed moment the true longitude of the star in the ecliptic is given, by adding the remainder to this star's longitude in the canon.

So that the latitude of the star agreeing with the proposed time may likewise be discerned: enter the latter table again with the true longitude of the star from the vernal equinox previously found, and write down the appropriate proportional minutes; then multiply them into the excess of obliquity previously kept — but now, for convenience, diminished by as many seconds as the number of minutes of that same excess (since these stars suppose the obliquity of the ecliptic of Tycho's age, $23^{\circ} 31\frac{1}{2}'$, and not the minimum $23^{\circ} 31' 7''$). Thus emerges the prosthaphaeresis [προσθαφάρσεις, Greek: prosthaphairesis, "equation"] of the star's latitude, which is then to be accommodated to the star in the upper canon of stars in this way: that if the star's latitude was —

If the latitude was, in signs		applied to that same latitude
Northern [Borealis]	for the Septentrional [Septent.]	Subtract
	for the Meridional [Merid.]	Add
Southern [Meridionalibus]	for the Septentrional [Septent.]	Add
	for the Meridional [Merid.]	Subtract

Thus the latitude of the star for the given time comes out; and over a longer stretch of time it too is variable, as we showed above from Tycho's observation.

For the rest: to this practice concerning the apparent progression of the stars (as it appears), we have also included the method of computing the inequality of the precession of the equinoxes, which we shall need below in the case of the Sun and the other planets. Now we shall illustrate this precept by two examples.

Exemplum I. [Example I.]

Let there be sought the precession of the equinoxes, the obliquity of the ecliptic, and likewise the longitude and latitude of the star Arcturus [Arcturi], in the year 3000 from the first bissextile of the world's founding — around which time, indeed far earlier, namely by 82 years, we know from the evening rising of that same star that Hesiod [Hesiodum] flourished. Therefore, according to what we have prescribed, we gather for 3000 years from the founding of the world:

[Margin: Lib. 2. Sph. cap. 4. prob. 2.]

	Sig.	Gr.	Mi.	Sec.
Simple precession of the equinoxes [Simplicem praecessionem aequinoct.]	1	11	31	1
Anomaly of the equinoxes [Anomaliam aequinoctiorum]	10	0	13	5
Hence from the next table the equation is found [Hinc è tabula proxima invenitur aequatio]			23	32 (Add.)
Likewise the excess of obliquity of the ecliptic [Item excessus obliquitatis eclipticae]			16	21
Therefore the maximum obliquity at that time was [Ergo maxima obliquitas eo tempore fuit]		23	47	28
And the true precession of the equinoxes [Et vera praecessio aequinoctiorum]	1	11	54	33

Now, since the longitude of Arcturus is found in the table of stars to be:

	Sig.	G r	M i	Sec.
[longitude of Arcturus, from the star table]	6	1	3	0
		8	8	
And the difference between the true precession of the given time and that of the year 1600 is gathered to be	1	5	1	37
			1	
Arcturus's longitude is therefore established as	♍ [Virgo]	1	2	23
		3	6	
To this longitude there correspond		1	3	
			7	

Subt. [to be
subtracted]

	G	M	
	r	i	
Sig.	.	.	Sec.
proportional minutes [scrupula proportionalia]			
Which, multiplied into the corrected excess of obliquity	1	3	
	6		
produce the proportional part	4	34	Subt. [to be subtracted]
And since the latitude of Arcturus in the table [abacus] is	3	2 30	B [borea, north]
	1		
its true latitude, congruent with the proposed time, will therefore be	3	5 B [north],	
	0	8 approximately	

So the latitude of Arcturus, fitted to the time in question, comes out as 30° 58' north, very nearly.

Example II.

Let us investigate, in the same way, for the completed year of the first leap year from the founding of the world 6000 — that is, in the year 2034 from the birth of Christ — the true precession of the equinoxes, likewise the maximum obliquity of the Sun, and finally the true longitude and latitude of the first star in the head of Aries.

First, then, from the earlier table the simple precession of the equinoxes for the given time stands at 2 signs 23° 0' 13", and the anomaly at 8 signs 0° 13½'. Entering with this anomaly into the next of the tables already set out, I find the equation of the precession of the equinoxes to be 23' 43", to be added, and the excess to be 5' 29". The true anticipation of the equinoxes will therefore be 2 signs 23° 23' 56", and its difference from the Tychonic epoch of the stars 6° 17' 46", to be added; likewise the maximum obliquity of the Sun, 23° 36' 36".

Now since the longitude of the first star of Aries is found in the canon to be 0 signs 27° 36', with a latitude of 7° 8' north, the longitude of this star at the assumed time will be 1 sign 3° 53' 46". Entering the table of prosthaphaeresis [tabula prosthaphaereseōn] with this value, proportional minutes [scrupula proportionalia] of 33' are offered. When these are multiplied into the excess of the obliquity of the ecliptic for Tycho's time — reduced by the very easy method I set out above, namely 5' 22" — the product comes out as 3' 0", to be subtracted. From this, too, the latitude of this star then works out as 7° 5' north. And this same kind of computation holds in all the other examples we have proposed in this chapter.

CHAPTER V.

On the measure of the tropical and sidereal year, likewise on the mean and apparent motions of the Sun, together with the hypothesis, the tables, and their use.

[Margin: See the commentary on the Sun.]

The tropical year is defined as the measure of the periodic motion of the Sun in the ecliptic, reckoned either from the equinoctial or from the solstitial point; for the start of the simple year we adopt the spring mean equinox.

The sidereal year, by contrast, is the time of the Sun's revolution back to the same fixed star, measured by longitude. The difference between the tropical and the sidereal year is therefore the one determined by the simple anticipation of the equinox — that is, the simple apparent shift of the fixed stars, 49" 45'" — when resolved, by the Sun's daily motion, into a time of 20m 18¹/₃s.

Now in order to attain the tropical year, equal to the first one, in its just measure — so far as is possible (for without this no solid foundation exists for restoring not only the motions but also the times) — it must be elicited from the intervals between the equinoxes most anciently observed by Hipparchus and Ptolemy and restored by us above, the comparison being instituted with that of Tycho. For in this way any error there may be, enclosed within the whole longer span of time, becomes utterly imperceptible once it is broken up into the individual years.

But since we everywhere use the measure of the Julian year, 365¹/₄ days, we shall treat the matter compendiously, as before — without resolution — through the spring equinox anticipation alone. And in order first to reduce the apparent equinoxes everywhere to mean ones,

the inequality both of the Sun and of the [Greek: προσθαφαίρεσις (prosthaphairesis, "equation" — the prosthaphaeretic correction)] of the

equinoxes must be recognized and duly applied. Since this is established from what has gone before, it will be laid out clearly in the following demonstration.

[Margin: See the commentary on the Sun.]

Diagram. A circle representing the Sun's eccentric, labeled E C F D. The horizontal diameter runs from F on the left (marked with the ♎ Libra/autumnal-equinox symbol) through the center A to E (♈ Aries, the spring equinox) on the right. The vertical diameter runs from C at the top (the apogee) through A down to D at the bottom (the perigee). The point B lies below center A on the C–D line; B is the center of the deferent/eccentric, and A–B is the Sun's eccentricity. Lines are drawn from F and from E to the points B and C, forming the triangle A B E (and its mirror A B F) used in the demonstration.

- C (top): apogee [apogaeus]
- D (bottom): perigee [perigaeus]
- E (right, ♈): spring equinox [vernum aequinoctium]
- F (left, ♎): autumnal equinox [autumnale aequinoctium]
- A (center): center of the eccentric
- B (below A): the eccentric's center; A–B is the Sun's eccentricity

Let the Sun's eccentric E C F D be described about center A, and let A B be the Sun's eccentricity itself, 1 part [par. 1], of the kind in which the radius A E is 28 parts [p. 28], as we proved above at the end of the solar commentary. Let the apogee be at C, the perigee at D, and finally the spring equinox at E, the autumnal at F.

Now since it is established that in Hipparchus's time the apogee C was removed from the spring equinox by $65\frac{1}{2}^\circ$ [p. $65\frac{1}{2}$], in Ptolemy's by 70° , and finally in Tycho's and our own age — namely at the start of the year of Christ 1588 — by $95\frac{1}{2}^\circ$, in the triangle A B E there are accordingly given, for the angle A E B (to which let A F B always be equal) — that is, the [Greek: διδόμενα (didomena, "the givens")] — A E, the radius of the Sun's eccentric, 28 parts; A B, the eccentricity, 1 part; and finally the angle E B A, measuring the apogee's removal from the true equinox. Hence the angle A E B is also given — the [Greek: προσθαφάσεις (prosthaphairesis, "equation")] at each equinox: in Hipparchus's time, $1^\circ 51' 44''$; in Ptolemy's, $1^\circ 55' 23''$; and finally in our own, $2^\circ 2' 14''$.

Likewise, from the section immediately above, the equation of the equinox is found, for the Hipparchian age, to be $10' 12''$; for the Ptolemaic, $21' 0''$, ablative [to be subtracted]; but for ours, $7' 12''$, adjective [to be added]. This last is therefore to

be cast off at the spring equinoxes among the ancients, but joined on to the same among us; whereas the opposite procedure will be kept for the autumnal equinoxes. And whatever results is to be converted, in proportion to the Sun's true daily motion, into time; and once this is done I find, according to the following synopsis:

Of the apparent equinox, that the mean may be had	Spring, to be added [Verno addenda]	of the age of [Seculo]	Hipparchus [Hipparchi]
			Ptolemy [Ptolemæi] Ours [Nostro]
	Autumnal, to be subtracted [Autumnali subtrahenda]		Hipparchus [Hipparchi]
			Ptolemy [Ptolemæi] Ours [Nostro]

These equations, with the requisite care, having been set out, we will apply to each equinox — both the Hipparchian and Ptolemaic and the Tychonic. For the spring equinox we still select the very first among the most ancient, which Hipparchus reports having observed twice in one day, and we retain the last one in it, as one in which every part of refraction had by then been dispelled. Further, we shall use the fifth Hipparchian autumnal equinox, likewise determined on the eastern horizon at rising, so that on this account 5 hours are first to be subtracted from it; then whatever remains of the discrepancy from the spring equinox is to be brought into a just limitation, by which not only the Sun's refraction but also other inconveniences of the armillaries — whether contracted of themselves, or committed through an error of the Alexandrian pole — may be opportunely corrected. The procedure now follows.

[Margin: See above in the commentary on the Sun.]

Hipparchus's first spring equinox occurred in the year 146 before the birth of Christ, in the month of March, on the 23rd day, at hour 23, Alexandrian time; but here [at our meridian] at hour 21, minute 25, p.m. When to this time the prefixed

inequality is added — namely, 1 day 17 hours 50 minutes — Hipparchus's mean equinox falls, in our reckoning, on the 25th of March, at hour 15, minute 15. Since, in the Tychonic class, the third [year] from the leap year corresponds to this equinox, that spring equinox will be the one after the birth of the Savior in the year 1587, which at Uraniborg — as also here at Hafnia [Copenhagen], for there is almost no difference at all in the longitude of these two — appeared on the 10th of March, at hour 14, minute 56, p.m.

[Margin: Liber 1, Prog., p. 13.]

When to this the inequality assigned above is added — say, 2 days 4 hours 30 minutes — the sum indicates the 12th of March, hour 19, minute 26, after noon, at which time the mean equinox stood. The difference, then, from the Hipparchian spring equinox is 12 days 19 hours 49 minutes, which is to be ascribed to the anticipation of time over very nearly 1732 Julian years, and is to be preserved.

Further, the fifth autumnal equinox of Hipparchus — once we subtract first, as we indicated, 5 hours, and then the difference of meridians — falls at Uraniborg in the same year 146 before Christ, on the 26th day of September, at hour 11, minute 25; and when the equation of 2 days 0 hours 50 minutes has been deducted from it, the time of this mean equinox was the 24th of September, hour 10, minute 35, p.m. But Tycho's autumnal equinox of the year 1587 was found on the 13th of September, hour 9, minute 16; and the mean one, with the equation subtracted, on the 11th of September, hour 10, minute 36 — from which, then, the Hipparchian autumnal equinox is again distant by 12 days 23 hours 59 minutes. But the anticipation of the spring equinoxes was 12 days 19 hours 49 minutes.

The difference, therefore, is 4 hours 10 minutes, of which the half — namely 2 hours 5 minutes — assigned to the limitation and added to the smaller of the spring anticipations, constitutes the limited anticipation, over completed solar years (which in this business may be used indifferently for Julian ones) 1732, between Hipparchus and Tycho, of 12 days 21 hours 54 minutes. Divided into the proposed 1732 years, this yields a quotient of 10m 44s 8"; and this, in this first experiment, between Hipparchus and us, argues an excess of the Julian year — which stands at 365 days 6 hours — over the mean solar year.

Now, about to weigh Ptolemy's equinox in the same way, we will bring under examination that single spring equinox of his, and another autumnal one verified by us above, making the beginning from the autumnal one in order of time.

Ptolemy's following autumnal equinox, freed above from refraction, fell in the year 139 after Christ, in the month of September, on the 25th day, hour 6, p.m., at

Alexandria; but at Uraniborg on the 25th day, hour 4, minute 25. The equinoctial equation subtracts 2 days 6 hours 45 minutes. Therefore the mean equinox was the 22nd day, hour 21, minute 40. But Tycho's mean autumnal equinox, which occurred in the year 1587 and agrees with the Ptolemaic in respect of the leap year, was found above to have fallen on the 11th of September, hour 10, minute 35. The anticipation, then (over the interval of 1448 solar years), is the difference of these, which is 11 days 11 hours 4 minutes.

By the same reasoning it is gathered that Ptolemy's spring equinox occurred in the year 140 from the birth of Christ, at Alexandria, in the month of March, on the 22nd day, hour 1, p.m.; at Uraniborg, therefore, on the 21st day, hour 23, minute 25, p.m. To this the equation adds 1 day 14 hours 45 minutes. Therefore this mean equinox was the 23rd of March, hour 14, minute 10, p.m. To this Ptolemaic equinox, falling in a leap year, there corresponds Tycho's in the year 1588, which was observed at Uraniborg on the 9th of March, hour 20, minute 45; and with the Tyconic equation of 2 days 4 hours 30 minutes added, this mean equinox occurred on the 12th of March, hour 1, minute 15, p.m. So the difference here between the spring equinoxes of Ptolemy [and Tycho] is found to be 11 days 12 hours 55 minutes. Accordingly this interval, in equal years — namely 1448 — exceeds the one above [the autumnal interval] by 1 hour 51 minutes. The limited anticipation from Ptolemy to Tycho is therefore found to be 11 days 12 hours 0 minutes; which, divided into the 1448 years, shows in the quotient an annual anticipation of 11' 26" 11'''.

The limited anticipation, then, of both Hipparchus and Ptolemy down to our own age, taken through the mean, is 11' 5" 10'''; for which we may now take 11' 5'', since nothing perceptible is lost to the Sun's motion on account of the 10''' — by which we adhere more closely, as is fitting, to Hipparchus, the reason for which has been given above in the commentary. The tropical year, therefore — now sought more subtly, and perhaps with greater diligence, than by anyone hitherto — will be 365 days 5 hours 48 minutes 55 seconds, the limited anticipation, namely, having been subtracted from the measure of the Julian year, 365 days 6 hours. And thus, over 130 solid Julian years, the anticipation of one whole day is distributed as nearly as may be.

And this measure of the solar year is perpetual — namely with respect to the mean equinox — a measure by which we find ourselves surpassed by that of the Persians and of al-Gelālī [Jalālī] [Gelalaei] by only some 1½ seconds; the Tyconic measure had to be a little greater even than this on account of the inequality of the equinoxes increasing in our age. And indeed anyone ought to marvel that

Tycho himself — at an interval of only 100 years from Walther of Nuremberg [Gualthero Noribergensi] — should have come so close to the annual quantity, with no account taken of the unequal precession of the equinoxes, when he sets it at 365 days 5 hours 48 minutes 45 seconds, which ours surpasses by only 10 seconds, but the Alphonsine measure by 41 seconds. This, I judge, happened to them [the Alphonsine astronomers] because, while they

[Margin: See among others, Tom. 1, Ephem., D. O[rigani?] merid., Origanus.]

[Margin: Liber 1, Prog., p. 51 et seqq.]

rejected the equinoxes of al-Battānī [Albategnii] — which we too, both formerly and here, were rejecting (the reason added above in the commentary) — they perhaps adopted these uniquely as the standard of comparison. But as to the fact that earlier, when we examined the motion of the Moon and of the fixed stars among the ancients, we supposed the tropical year to be greater by about 5 minutes than the one now found — that too does nothing other than secure for us, in most matters of the greatest moment, a slightly greater equability, always longed for in our prayers, whenever the examination shall stand up to the norm of this tropical year's measure.

For the rest, it is now easy to deduce the mean quantity of the tropical year we have found into the measure of the sidereal year, by adding — according to the requirement of the Sun's motion, as I indicated above — 20m 18½s. For so it too comes out as 365 days 6 hours 9 minutes 13⅓ seconds. We are not at all troubled that this is surpassed by Copernicus by more than 27 seconds, and by Tycho by more than 13 seconds, since both of them used the places of the stars observed by the ancients without any restoration. We, on the other hand, having corrected those fixed stars and carefully examined them against this constitution of ours, find among the ancients — Timocharis, Hipparchus, and Ptolemy — no greater discrepancy than ⅓ of a degree; but in al-Battānī's case ¼ degree, or a little more, in the contrary direction. This last is rightly to be ascribed to the observations of the ancients as far less certain, and perhaps also to the foundation, from the Sun, used by al-Battānī, which we have not touched, being still more faulty.

[Margin: See above in the commentary on the stars.]

If any, in the ancients' favor, hold against us, they will most easily be convinced by the comparison of the most ancient canon of the stars with the exact Tyconic one. Further, with both quantities of the year having been found, since we discover no difference, whichever in the end we use for determining the Sun's motion — contrary to what Copernicus and Tycho judged against Ptolemy: for the

simple precession of the equinox is the same as the forward motion of the fixed stars in consequence, and the equinoctial inequality is likewise common to the stars too. Accordingly, by a reckoning and method more popular and convenient, we shall begin the course of the Sun and of the stars from the spring equinox — first the mean, then the true, once, that is, we have duly fitted its inequality to this mean precession of the equinoxes, in accordance with what has preceded.

[Margin: Copernicus, Liber 3, Cap. 13; Tycho, Prog., Liber 1, Cap. 2, p. 253.]

[Margin: Cap. 4 of this [book].]

But to arrive at the supposition of the solar motions to be continued by means of tables, we must proceed in this way. First, for the common Julian year, which consists of 365 days: when these are reduced to the whole circle, or 360° , and the result is divided by the measure of the tropical year just found, the Sun's motion proper to the said common year emerges as 11 signs $29^\circ 45' 40'' 13''' 38''''$. When this in turn is divided into 365 days, the Sun's daily motion from the spring mean equinox is elicited as $0^\circ 59' 8'' 19''' 48''''$. And since the Julian leap year is greater than the common one by a whole day, accordingly — the Sun's daily motion being aggregated to the annual common one — the Sun's motion in a leap year is made up to be 0 signs $0^\circ 44' 48'' 33''' 26''''$. From these sources the other motions, which will appear in the following table, are easily deduced. So much for the Sun's motion in longitude.

But the motion of its anomaly — which argues the mean distance of the Sun from the mean apogee at any given time — once the annual motion of the apogee (which we found, from what we left above in the commentary, to be $1^\circ 1' 50''' 14''''$) has been subtracted from the motion of the Sun's longitude ascribed above to any common and leap year, will thus be: in the common year, 11 signs $29^\circ 44' 38'' 23''' 24''''$; and in the leap year, 0 signs $0^\circ 43' 46'' 43''' 12''''$. Now since the daily motion of the Sun's apogee is gathered to be at least $10''' 10''''$, it will accordingly be easy, from the subtraction of this from the simple longitude of the Sun, to make up the motion of the anomaly for the months, days, and the remaining species of time.

From these premises we have collected the following tables of the simple motions of the Sun from the spring mean equinox, and have continued them to myriads of years — enough, perhaps, beyond the future mundane period at least previously set down by us — being about to show next the use of these tables themselves. The epochs of these motions, appearing at the head of the table, are likewise directed, as is fitting, to the mean equinox.

Simple Longitude and Anomaly of the Sun, referred to the mean equinox

[This page completes the canon of the Sun's *simple* mean motions in longitude and anomaly. The epoch radices stand at the head; the left table runs the cumulative motion through scores, centuries, and thousands of years (to 6300); the right table gives the motion for each single year up to 20. Every value is in signs (S), degrees (G), minutes (M), seconds (S).]

Epochs and accumulated years — Longitude and Anomaly

Epoch	Longitude S	G	M	S	Anomaly S	G	M	S
Mundi [World]	8	8	3	5	8	8	2	2
			3	4			8	9
Christi [Christ]	9	8	3	1	7	0	3	4
			8	0			0	4
20	0	0	9	6	11	2	4	3
						9	8	0
40	0	0	1	1	11	2	3	1
			8	2		9	7	
60	0	0	2	1	11	2	2	3
			7	9		9	5	1
80	0	0	3	2	11	2	1	2
			6	5		9	4	
100	0	0	4	3	11	2	2	3
			5	1		9		2
200	0	1	3	2	11	2	5	4
			1			8		
300	0	2	1	3	11	2	7	3
			6	3		7		7
400	0	3	2	4	11	2	1	9
						6	0	
500	0	3	4	3	11	2	1	4
			7	5		5	2	1
600	0	4	3	6	11	2	1	1
			3			4	5	3
700	0	5	1	3	11	2	1	4

Epoch	Longitude S	G	M	S	Anomaly S	G	M	S
			8	7		3	7	6
800	0	6	4	8	11	2	2	1
						2	0	8
900	0	6	4	3	11	2	2	5
			9	9		1	2	0
1000	0	7	3	1	11	2	2	2
			5	0		0	5	2
1100	0	8	2	4	11	1	2	5
			0	0		9	7	4
1200	0	9	6	1	11	1	3	2
				1		8	0	7
1300	0	9	5	4	11	1	3	5
			1	2		7	2	9
1400	0	1	3	1	11	1	3	3
		0	7	3		6	5	1
1500	0	1	2	4	11	1	3	3
		1	2	4		5	8	
1600	0	1	8	1	11	1	4	3
		2		5		4	0	6
1700	0	1	5	4	11	1	4	8
		2	3	6		3	3	
1800	0	1	3	1	11	1	4	4
		3	9	7		2	5	0
1900	0	1	2	4	11	1	4	1
		4	4	8		1	8	2
2000	0	1	1	1	11	1	5	4
		5	0	9		0	0	4
2500	0	1	5	5	11	6	3	2
		8	7	4				6
3000	0	2	4	2	11	1	1	7
		2	5	9			6	
3500	0	2	3	3	10	2	2	4
		6	3			6	8	8
4000	1	0	2	3	10	2	4	2

Epoch	Longitude S	G	M	S	Anomaly S	G	M	S
4500	1		0	8		1	1	9
		4	8	1	10	1	5	1
				3		6	4	0
5000	1	7	5	4	10	1	6	5
			5	8		2		1
5500	1	1	4	2	10	7	1	3
			1	3	3		9	2
6000	1	1	3	5	10	2	3	1
			5	0	7		2	3
6300	1	1	4	3	9	2	3	5
			7	7	0		9	9

In single years, up to 20 — Longitude and Anomaly

Year	Longitude S	G	M	S	Anomaly S	G	M	S
1	11	29	45	40	11	29	44	38
2	11	29	31	20	11	29	29	17
3	11	29	17	1	11	29	13	55
4	0	0	1	49	11	29	57	42
5	11	29	47	29	11	29	42	20
6	11	29	33	10	11	29	26	59
7	11	29	18	50	11	29	11	37
8	0	0	3	38	11	29	55	24
9	11	29	49	19	11	29	40	2
10	11	29	34	59	11	29	24	42
11	11	29	20	39	11	29	9	20
12	0	0	5	28	11	29	53	7
13	11	29	51	8	11	29	37	45
14	11	29	36	48	11	29	22	23
15	11	29	22	28	11	29	7	2
16	0	0	7	17	11	29	50	49
17	11	29	52	57	11	29	35	27
18	11	29	38	37	11	29	20	5

Year	Longitude S	G	M	S	Anomaly S	G	M	S
19	11	29	24	18	11	29	4	44
20	0	0	9	6	11	29	48	30

Simple motion of the Sun — in months, days, and hours

In common months / In leap-year (bisextile) months — Longitude and Anomaly

Month	Common Long. S	Common Anom. S	Bisext. Long. S	Bisext. Anom. S
Ianuarius [January]	1	1	1	1
Februarius [February]	1	1	1	1
Martius [March]	2	2	2	2
Aprilis [April]	3	3	3	3
Majus [May]	4	4	4	4
Iunius [June]	5	5	5	5
Iulius [July]	6	6	6	5 [?]
Augustus [August]	7	7	8	8
September	8	8	9	9
October	9	9	10	10
November	10	10	11	11
December	11	11	0	0

Simple motion of the Sun in days — Longitude and Anomaly

Day	Long. G	M	S	Anom. G	M	S
1	0	59	8	0	59	8
2	1	58	17	1	58	17
3	2	57	25	2	57	25
4	3	56	33	3	56	33
5	4	55	42	4	55	42

Day	Long. G	M	S	Anom. G	M	S
6	5	54	50	5	54	49
7	6	53	58	6	53	57
8	7	53	7	7	53	6
9	8	52	15	8	52	14
10	9	51	23	9	51	22
11	10	50	32	10	50	31
12	11	49	40	11	49	38
13	12	48	48	12	48	46
14	13	47	57	13	47	55
15	14	47	5	14	47	3
16	15	46	13	15	46	11
17	16	45	21	16	45	19
18	17	44	30	17	44	27
19	18	43	38	18	43	35
20	19	42	47	19	42	44
21	20	41	55	20	41	52
22	21	41	3	21	41	0
23	22	40	13	22	40	9
24	23	39	20	23	39	16
25	24	38	28	24	38	24
26	25	37	37	25	37	33
27	26	36	45	26	36	41
28	27	35	53	27	35	49
29	28	35	1	28	34	57
30	29	34	10	29	34	6
31	30	33	18	30	33	13

Simple motion in hours and scruples — Longitude-and-Anomaly together

The two halves of this table share one combined entry for longitude and anomaly. Hours 1–30 are read in minutes (M) and seconds (S); hours 31–60, as the foot of the column marks, are read in seconds (Sec.) and thirds (Tert.).

Hor./Min.	M	S	Hor./Min.	Sec.	Tert.
1	2	28	31	1	16
2	4	56	32	1	19
3	7	24	33	1	21
4	9	51	34	1	24
5	12	19	35	1	26
6	14	47	36	1	29
7	17	15	37	1	30
8	19	34	38	1	34
9	22	11	39	1	36
10	24	38	40	1	39
11	27	6	41	1	41
12	29	34	42	1	43
13	32	2	43	1	46
14	34	30	44	1	48
15	36	58	45	1	51
16	39	26	46	1	53
17	41	53	47	1	56
18	44	21	48	1	58
19	46	49	49	2	1
20	49	17	50	2	3
21	51	45	51	2	6
22	54	13	52	2	8
23	56	40	53	2	11
24	59	8	54	2	13
25	1	2	55	2	15
26	1	4	56	2	18
27	1	7	57	2	20
28	1	9	58	2	33 [?]
29	1	11	59	2	25
30	1	14	60	2	28

There follows the solar Hypothesis, together with the Table of προσθαφάσεις [Greek: prosthaphairesis, "the equation" — the additive-subtractive correction], from which the apparent motion is derived

Up to this point we have been occupied in setting out the Sun's mean motions. Now comes the hypothesis, in which it is shown geometrically how those mean motions are to be reduced to the true or apparent positions at each given moment of time — for this is precisely what most concerns the astronomer. Since the Sun's course is handled by a single, simple anomaly, among the several equivalent hypotheses presented in Chapter 2 we select the homocentric-epicycle [homocentrepicyclum], as being best suited to the nature of things, as we noted earlier. And since previously, in the solar commentary, we had obtained the dimensions of the semidiameters of the Sun's orbs by the due comparison of observations, we shall now treat its hypothesis — founded upon those dimensions — here all the more freely and without hindrance. From this the table of prosthaphaeresis will shortly be set out in the following place; and finally, in the last place, the use of this chapter in computing the Sun's true position for any given times will be clearly demonstrated by examples.

THEORY OF THE SUN [THEORIA SOLIS]

Diagram. A geometric construction of the solar theory on the homocentric-epicycle hypothesis. A large circle (the homocentric BDC) is drawn about the Earth's center A; on its circumference, at the right, lies point B (the vernal mean equinox); the diameter runs horizontally from B (right) through center A to C (left); point D sits on the upper-left arc. A small epicycle EFG is drawn about D at the upper part of the figure, with E (apogee) at top, F and G below it. Two lines run from A up to the epicycle (to D and to F).

- A (center of large circle): the Earth's center.
- B (right end of horizontal diameter): the vernal mean equinox, where the Sun's annual period begins.
- C (left end of horizontal diameter): opposite point on the homocentric.
- D (upper-left, center of epicycle): the point reached by the mean motion of the Sun along the ecliptic from B.
- E (top of epicycle): the apogee, where the anomaly's motion begins.
- F (on epicycle, below E): the Sun's actual body, after the anomaly EF has been traversed.

- G (on epicycle): the further point of the epicycle's circuit (motion E→F→G in the order of the signs' antecedence).

From the Earth's center A is described the homocentric BDC, by which the Sun's annual period, defined above, is laid out starting from the vernal mean equinox B, the motion proceeding in the order of the signs from B toward D — the daily motion having been defined as 59 minutes 8 seconds. Now granting, for the sake of the example, that the arc BD has been completed by the mean motion of the Sun through the ecliptic from B (as it were a quasi-trigon of the circle), and placing D as the center, let the epicycle EFG be drawn in the plane of the homocentric (that is, of the ecliptic). Its semidiameter DE bears to the radius of the Sun's orb AD the ratio that 3571 has to 100000 — that is, 1 part to 28 parts, as we showed above in the commentary. In the inscribed epicycle EFG the motion and period of the anomaly, set out above, are understood to be completed in the order of the signs' antecedence, beginning at the apogee E and carried through F, G. And with the Sun's body placed at F, the Sun's anomaly — reckoned according to its mean motions defined above — will be the arc EF, which here, for the example, let us suppose to be 24 degrees. Therefore the angle FDA is the supplement of this to a semicircle, 156 degrees. About this angle, since each of the two sides AD and DF stands to the other in a fixed ratio — the former namely 28 parts, the latter 1 part — there is given the angle of prosthaphaeresis DAF, 0 degrees 48 minutes 20 seconds, which here, as appears, is to be subtracted from the Sun's mean motion so that the true one may be obtained. Next, likewise, in the same triangle DAF, the angles being given, together with two of the sides (as we have laid down), let the third side AF be sought (whenever there is need), measuring the Sun's distance from the Earth. Here this is found to be 10327, of the kind of which the radius AD is 10000; or 28 parts 55 minutes, of the kind of which AD is 28 parts. By a similar inquiry, made for each five degrees in turn, the following table has been composed.

TABLE OF PROSTHAPHAERESIS, taking the semidiameter of the epicycle as 1 part, of the kind of which [the radius is 28]

This table is read with the argument-degrees (G.) in the leftmost column (running 0→30 downward) and again in the rightmost column (running 30→0 upward, marked G. at the foot). Each pair of columns gives the prosthaphaeresis ("Subtr." = subtractive in the upper half) in degrees-minutes-seconds (G. M. S.) and its difference per degree (Dif. A. = difference of the argument) in minutes-seconds (M. S.). The three header bands across the top read "0 Subtr.", "1 Subtr.", "2

Subtr."; the foot bands read "11 Add.", "10 Add.", "9 Add." — i.e., the same columns serve the additive arguments when entered from below.

G. (top)	0	1			2			Dif.A		G. (foot)
	Subtr. G	Dif.A . M	S	Subtr. G	Dif.A . M	S	Subtr. G	. M	S	
0	0	2	5	0	1	4	1	1	6	30
						9				
1	0	2	4	1	1	4	1	1	4	29
						8				
2	0	2	4	1	1	4	1	1	2	28
						6				
3	0	2	4	1	1	4	1	1	0	27
						6				
4	0	2	3	1	1	4	1	0	59	26
						4				
5	0	2	3	1	1	4	1	0	57	25
						3				
6	0	2	3	1	1	4	1	0	55	24
						1				
7	0	2	3	1	1	4	1	0	51	23
						0				
8	0	2	3	1	1	3	1	0	40	22
						9			[?]	
9	0	2	3	1	1	3	1	0	47	21
						8				
10	0	2	2	1	1	3	1	0	48	20
						7				
11	0	2	1	1	1	3	1	0	46	19
						6				
12	0	2	1	1	1	3	1	0	43	18
						4				
13	0	2	1	1	1	3	1	0	39	17
						3				
14	0	2	1	1	1	3	1	0	38	16
						2				

G. (top)	0	Dif.A . M	1		Dif.A . M	2		Dif.A . M	S	G. (foot)
	Subtr. G		Subtr. S	G		Subtr. S	G			
15	0	2	0	1	1	3	1	0	36	15
						1				
16	0	2	0	1	1	2	1	0	33	14
						9				
17	0	1	5	1	1	2	1	0	32	13
			9			7				
18	0	1	5	1	1	2	1	0	30	12
			9			5				
19	0	1	5	1	1	2	1	0	28	11
			8			4				
20	0	1	5	1	1	2	2	0	26	10
			7			3				
21	0	1	5	1	1	2	2	0	24	9
			6			1				
22	0	1	5	1	1	2	2	0	21	8
			5			0				
23	0	1	5	1	1	1	2	0	19	7
			5			8				
24	0	1	5	1	1	1	2	0	16	6
			4			5				
25	0	1	5	1	1	1	2	0	14	5
			3			4				
26	0	1	5	1	1	1	2	0	12	4
			3			2				
27	0	1	5	1	1	1	2	0	10	3
			2			0				
28	0	1	5	1	1	9	2	0	8	2
			0							
29	0	1	4	1	1	8	2	0	6	1
			9							
30	0	—	—	1	—	—	2	—	—	0
(foot	11	Dif.		10	Dif.		9 Add.	Dif.		G.

	0		1		2		
G.	Subtr.	Dif.A	Subtr.	Dif.A	Subtr.	Dif.A	G.
(top)	G	. M	S	. M	S	. M	(foot)
)	Add.	5	Add.	5		5	

ON SOLIS PERPETVA [The Perpetual Equation of the Sun]

The radius of the Sun's orb is 28 parts.

Table of the solar prosthaphaeresis (continued). This is the right-hand block of the solar prosthaphaeresis canon (epicycle semidiameter 1 part, radius 28). The outer **G.** (degrees of argument) column at far left runs 0→30 downward and governs the **Subtr.** reading; the outer **G.** column at far right runs 30→0 upward and governs the **Add.** reading at the foot. Each of the three equation blocks (**3 Subtr. / 8 Add.**, **4 Subtr. / 7 Add.**, **5 Subtr. / 6 Add.**) carries a degrees–minutes–seconds (G. M. S.) value and an adjacent difference column (M. S.). Within each difference column the printed difference value sits between the two table rows it bridges; it is transcribed here on the row it heads (the upper of the two).

G.							G.
(Subtr.	3 Subtr.	Diffe. A. S.	4 Subtr.	Dif. S.	5 Subtr.	Dif. S.	(Add.
↓)	(G. M. S.)	(M. S.)	(G. M. S.)	(M. S.)	(G. M. S.)	(M. S.)	↑)
0	2 2 42	0 1	1 48 12	1 3	1 3 19	1 53	30
1	2 2 46	0 3	1 47 9	1 4	1 1 26	1 55	29
2	2 2 48	0 0	1 46 5	1 7	0 59 31	1 56	28
3	2 2 48	0 1	1 44 58	1 9	0 57 35	1 58	27
4	2 2 47	0 4	1 43 49	1 11	0 55 37	2 0	26
5	2 2 43	0 9	1 42 38	1 13	0 53 37	2 1	25
6	2 2 34	0 12	1 41 25	1 16	0 51 36	2 2	24
7	2 2 22	0 13	1 40 9	1 17	0 49 34	2 3	23
8	2 2 9	0 16	1 38 52	1 18	0 47 31	2 3	22
9	2 1 53	0 17	1 37 34	1 20	0 45 28	2 3	21
10	2 1 36	0 19	1 36 14	1 22	0 43 25	2 4	20
11	2 1 17	0 21	1 34 52	1 24	0 41 21	2 5	19
12	2 0 56	0 24	1 33 28	1 26	0 39 16	2 6	18
13	2 0 32	0 26	1 32 2	1 28	0 37 10	2 7	17
14	2 0 6	0 28	1 30 34	1 31	0 35 3	2 8	16

G. (Subtr. ↓)	3 Subtr. (G. M. S.)	Diffe. A. S. (M. S.)	4 Subtr. (G. M. S.)	Dif. S. (M. S.)	5 Subtr. (G. M. S.)	Dif. S. (M. S.)	G. (Add. ↑)
15	1 59 38	0 30	1 29 3	1 32	0 32 55	2 9	15
16	1 59 8	0 32	1 27 31	1 34	0 30 46	2 10	14
17	1 58 36	0 35	1 25 57	1 36	0 28 36	2 10	13
18	1 58 1	0 37	1 24 21	1 37	0 26 26	2 10	12
19	1 57 24	0 39	1 22 44	1 38	0 24 16	2 11	11
20	1 56 45	0 42	1 21 6	1 39	0 22 5	2 11	10
21	1 56 3	0 43	1 19 27	1 40	0 19 54	2 11	9
22	1 55 20	0 45	1 17 47	1 42	0 17 43	2 12	8
23	1 54 35	0 47	1 16 5	1 45	0 15 31	2 12	7
24	1 53 48	0 50	1 14 20	1 48	0 13 19	2 13	6
25	1 52 58	0 53	1 12 32	1 49	0 11 6	2 13	5
26	1 52 5	0 56	1 10 43	1 49	0 8 53	2 13	4
27	1 51 9	0 57	1 8 54	1 50	0 6 40	2 13	3
28	1 50 12	0 59	1 7 4	1 52	0 4 27	2 14	2
29	1 49 13	0 1	1 5 12	1 53	0 2 13	2 13	1
30	1 48 12	—	1 3 19	—	0 0 0	—	0
G.	8 Add.	Dif. S. A.	7 Add.	Dif. A.	6 Add.	Dif. A.	G.

De supputatione veri Loci Solis [On the Computation of the True Place of the Sun]

The use of the foregoing material of this chapter follows last of all: namely, computing the true place of the Sun for any given moment. But since the mean motions must be obtained first, and the method of the epochs has been explained above, I will now set out the precept itself, to which three examples are appended that will dispatch each of the various cases. [Margin: Cap. 3.]

I. For the current year and the remaining attached equated time — whether reckoned from the calendar leap year used above, or from the birth of Christ — proceed as follows. (First check whether the number of years, including the current one, divided by 4, comes out as a common year or a leap year; and assume all the time-quantities as full, as is customarily done.) Then extract from the earlier tables of the Sun's mean motions its longitude from the mean vernal

equinox and its anomaly from the apogee, with the individual moments of time indicated under their proper headings. Take careful note of this: if the current year turns out to be a leap year, use the leap-year month as well. [Margin: Cap. 3 & 4.]

II. By means of the anomaly, enter the Table of the Sun [Tabulam προσθαφαίρεσεως [Greek: προσθαφαίρεσεως (prosthaphaireseōs, "of the prosthaphaeresis" — the equation)] Solis], and under the signs and degrees take out the appropriate equation of the Sun, not neglecting the proportional part for the minutes etc., if any have attached to the anomaly.

III. Finally, according to the heading, subtract or add the equation of the Sun thus found to its simple longitude, and you have the true place of the Sun in the ecliptic from the mean equinox. When the equation of the equinox — to be found by the preceding chapter — is duly applied to this, the true place of the Sun from the true vernal equinox stands forth.

Exemplum I [Example I]

Let the true place of the Sun from the true vernal equinox be sought for the current year from the birth of Christ 1596, on the 11th day of March at noon — the time at which, observed at [Uraniborg], the Sun departed most of all from the Tychonic Tables, namely by $3\frac{1}{2}'$ [3 m.], as you will find above in the commentary on the Sun. So that both the motions in the Tables and the epochs may be opportunely tested for correctness, I choose to extract the mean motions of the Sun's longitude and anomaly from the Tables in two ways here. For if to the proposed years from the birth of Christ the 3964 years are added, then from the nearest calendar leap year there are gathered 5560 years, the current one included — which year, like that one from the birth of the Savior, is found to be a leap year, since it admits the divisor 4 completely. With the completed time assumed, and the motions to be drawn from the Tables here set out on both sides, I leave the pattern or paradigm for the inexperienced as follows.

Tempus Mundi conditi			Tempus a nato Christo		
[Time from creation of the world]	Longitudo Solis (S. G. M. S.)	Anomalia Solis (S. G. M. S.)	[Time from the birth of Christ]	Longitudo Solis (S. G. M. S.)	Anomalia Solis (S. G. M. S.)
5500	1 11 43 23	10 7 19 32	1500	0 11 12 24	11 15 38 3
40	18 12	11 29 37 1	80	36 25	11 29 14 2

Tempus Mundi conditi			Tempus a nato Christo		
[Time from creation of the world]	Longitudo Solis (S. G. M. S.)	Anomalia Solis (S. G. M. S.)	[Time from the birth of Christ]	Longitudo Solis (S. G. M. S.)	Anomalia Solis (S. G. M. S.)
19	11 29 24 18	11 29 4 44	15	11 29 22 28	11 29 7 2
Feb. bif.	1 29 8 20	1 29 8 10	Feb. bif.	1 29 8 20	1 29 8 10
Dies 10	0 9 51 23	0 9 51 22	Dies 10	0 9 51 23	0 9 51 22
Summa	3 20 25 36	1 15 0 49	Summa	2 20 21 20	1 22 58 39
Epocha	8 8 33 54	8 8 28 29	Epocha	9 8 38 10	7 0 30 40
Med. Mo.	11 28 59 30	8 23 29 18	Med. Mo.	11 28 59 30	8 23 29 19

Now that the mean motions of the Sun's longitude and anomaly have been found, and verified by the two time-reckonings: with the anomaly at 8 signs, 23 degrees, 29 minutes, 18 seconds, I enter the table

of the solar prosthaphaeresis [tabulam προσθαφαιρέσεως Solis], and the equation of the Sun found there, 2 degrees 2 minutes 28 seconds, I add to the simple longitude, 11 signs 28 degrees 59 minutes 30 seconds. Thus from the mean vernal equinox its longitude is composed as 1 degree 1 minute 58 seconds of Aries [Υ]. When the equinoctial equation of 7 minutes 38 seconds — to be ascertained by the preceding chapter — is added to this, the true place of the Sun from the true equinox comes out as 1 degree 9 minutes 36 seconds of Aries [Υ]. The observation has 1 degree 7 minutes 45 seconds of Aries [Υ]. But the Tables of Tycho, from which (as I said) this observation departs most of all, exhibit the same solar longitude as 1 degree 11 minutes 12 seconds of Aries [Υ]. Our calculation, therefore, on account of the slight limitation which we set out above in the commentary on the Sun together with its cause, is as it were the mean. From this place of the Sun, Copernicus falls short by $33\frac{1}{4}$ minutes; the Alphonsine [tables], by contrast, exceed it by $16\frac{3}{4}$ minutes.

Exemplum II [Example II]

Let Hipparchus's third autumnal equinox be examined, which (as appears above in the commentary) fell in the year 158 before the birth of Christ, at the noon of the 27th day of September, at Alexandria — and therefore at Copenhagen [Hafnia] at 10 hours 25 minutes before noon. Now since (as ought to be done in those current years which precede the epoch of Christ) one year is subtracted

from the number 158, and the remainder from the 3964 of the first mundane leap year, the remainder will be 3807. Then the full days, with the attached hours and minutes in the Copenhagen meridian, are 25 days, 22 hours, 25 minutes; and with these, together with the completed years 3806, there agree from the foregoing tables a longitude motion of the Sun of 6 signs 2 degrees 1 minute 7 seconds, and an anomaly of 3 signs 26 degrees 35 minutes 35 seconds — the epoch of the world being added in both cases. By the anomaly thus expressed, since the equation of the Sun is found to be 1 degree 51 minutes 31 seconds, subtractive, and on that account the equinoctial equation from the foregoing is 9 minutes 42 seconds, likewise subtractive, the sum of these, 2 degrees 1 minute 13 seconds, when subtracted from the mean longitude 6 signs 2 degrees 1 minute 7 seconds, makes this equinox (which is remarkable) fall as if by agreement onto the proposed time, according to our restitution. For although the emendation was not begun from it, yet it must be that this equinox is, as it were, a mean among the autumnal ones, and observed at noon. As for the fact that the remaining Hipparchian and Ptolemaic [results] disagree this way and that, the reasons (I think) have been given by us above in the commentary, sufficiently, in their proper place.

Exemplum III [Example III]

This example we shall direct toward testing how far the restitution of our solar course falls short of the autumnal equinox of [al-Battānī] [Albategnii], which is reported to have been observed at Raqqa in Syria [Aratae Syriae] in the year 882 after the birth of our Savior, on the 18th day of September, at 13 hours 10 minutes — but with respect to the Copenhagen meridian, at 11 hours 10 minutes. For this time, by the method shown above, the mean motions of the Sun are found, namely a longitude of 6 signs 1 degree 49 minutes 0 seconds, and an anomaly of 3 signs 8 degrees 32 minutes 30 seconds. From these the prosthaphaeresis of the Sun is found to be 2 degrees 2 minutes 0 seconds, and the equation of the equinox $22\frac{1}{2}$ minutes, both subtractive. Therefore the true longitude of the Sun for the given time, according to our restitution, falls at 29 degrees $24\frac{1}{2}$ minutes of Virgo [♍], falling short of [al-Battānī]'s notation by $3\frac{1}{2}$ minutes — which puts a discrepancy in time of about $14\frac{1}{2}$ hours in the Sun, though in the Basiliscus [Regulus], as we noted above, scarcely 7 hours was found here. No one before us, however, had reconciled so ancient a figure as Hipparchus, and [al-Battānī] here, more closely with one another, although I recall that I conjectured above in the commentary, in its proper place, the cause of the error in each as well. And so much for the motion of the Sun.

CAPVT VI [Chapter VI]

De cursus Lunae restitutione secundum longitudinem, cum gemina hypothesi, tabulis, et ipsarum usu. [On the Restitution of the Moon's Course according to Longitude, with the twofold hypothesis, the tables, and their use.]

The restitution of the lunar course rightly follows the solar one immediately — both because, according to Tycho, the Moon is bound to the same center as the Sun, and because in part

the Moon, in its own motion, is in every revolution directed with respect to the disposition of the Sun, and from this acquires its other anomalies — analogous in a way to its light, which it truly borrows from the Sun. (This will be more fully grasped in the Plinian description and hypothesis of it.) [Margin: Plinius lib. 2. cap. 9 & 14.] And although this same course of the Moon, both in longitude and in latitude, is found to be subject to a manifold variation beyond the opinion of all the ancients, nevertheless — since in the earlier period, before its hypothesis was made conformable to the celestial appearances, it kept us occupied for a full seven years, with keen meditation and labor, in the observations of the most noble [Tycho Brahe], on those which had meanwhile and earlier been acquired by him — for that reason, at this time and place, we shall be excused if, without the tedious repetition of all the troubles which we drank in there, we first extend the mean motions of the Moon, by a comparison of the ancient observers with the more recent ones through the tables (as we did above for the fixed stars and the Sun), so as to suffice for all the ages of the world; and then construct the hypothesis of the Moon, which will display the lunar phenomena at any given time in the heavens, an invariable one founded upon certain observations; and in addition also another, entirely equivalent to the first, from whose resolution a second prosthaphaeresis table will be made, which will exhibit the lunar phenomena more readily and compendiously. When we have asserted this latter according to its due requisites, and resolved it into tables so far as can be done, there will remain (as in the other cases) the use in the calculation of the lunar motions, to be set out through several examples — which also prove the certainty of our suppositions.

First, as regards the mean motions of the Moon — whose longitude is determined by the Sun, the anomaly from the apogee, and finally the argument of latitude from the ascending node, that is, the head of the dragon [caput draconis] — I find, by the general comparison which follows (but most diligently instituted), not merely of the three most ancient Babylonian lunar eclipses, but also of others, namely the Hipparchian and Ptolemaic in part, and likewise of the subsequent

practitioners — [al-Battānī] [Albategnii], Walther [Gualtheri], [Regiomontanus] [Regiomontani], Copernicus, and the rest — almost nothing perceptible to be corrected in the motions of longitude and anomaly beyond the tables of Reinhold [Reinholdi], except for what we adjusted to the prior file [lima], and except for a single minute still to be subtracted from the longitude, as we indicated above in the commentary. Accordingly, with this our emendation applied to both epochs, we use the Prutenic canon [canone Prutenico] in the Moon's longitude and anomaly entirely. As for the argument of latitude, however, or the motion — for my part, from the comparison of the second of the three most ancient eclipses, occurring at Babylon in the year 720 before Christ, as well as with other particular ancient ones, I see that something must be subtracted here from the Prutenic tables, about 7 minutes, so that that most ancient eclipse which I just named may agree most nearly in magnitude with that of Lord Tycho's, made in the year 1578 after Christ, on the 15th day of September — provided the quantity of the Earth's and the Moon's shadow be duly proportioned in what follows. But in Tycho's earlier restitution, which was carried out at Prague, I recall that 12 minutes were added to the Prutenic motion of latitude; nor can it be doubted by anyone that everything concerning the Moon's latitude was then handled quite well, since we explored it very frequently with new observations, even outside eclipses — both for the discovered variation of latitude around the quadratures, and by investigation of parallax from the meridian altitude of the Moon in each tropic, and finally in the sextile and trine aspect with the Sun, where the variation of the motion of the nodes is found greatest on both sides. Therefore, since from the year 720 before Christ (as I said) up to the completed year 1584 after the birth of Christ — which the earlier particular restitution of the Moon regards — I have distributed the said complicated emendation of the mean motion of latitude, that is, 19 minutes, among the 2305 intermediate equal years, there fall to each common year, in correction of the Prutenic tables, 29 thirds 39 fourths. And from this the latitude table too is extended this way and that by us in this proportion, in the customary manner; and to it likewise is prefixed our epoch of the motion of latitude. And it will suffice to have indicated this much about the mean motions of the Moon, which we shall subjoin here presently.

Table of the Moon's mean motions [Tabula mediorum motuum lunae]

This is the master canon of the lunar mean motions, giving for collected years (epochs and multiples of years up to 6300), individual years 1–20, the months of the year (common and bissextile), and individual days, hours, and minutes-of-

hours, the Moon's mean elongation from the Sun [Longitudo a Sole], the mean anomaly [Anomalia], and the argument of latitude [Latitudo]. Each entry is given in signs (S.), degrees (G.), minutes (M.), and seconds (S.) of the sexagesimal system.

Collected years (left half: epochs and hundreds; right half: years 1–20). The left half is keyed by year-value in the first column; the right half is keyed by single years 1 through 20.

Left half — epochs and collected years

Years (Anni)	Long. from Sun S·G·M·S	Anomaly S·G·M·S	Latitude S·G·M·S
Of the World (Mundi)	9 · 0 · 6 · 15	8 · 25 · 3 · 35	8 · 19 · 4 · 30
Of Christ (Christi)	7 · 6 · 19 · 38	7 · 4 · 14 · 22	7 · 16 · 40 · 0
20	4 · 13 · 24 · 43	1 · 9 · 42 · 12	5 · 10 · 24 · 6
40	8 · 26 · 49 · 25	2 · 19 · 24 · 23	10 · 20 · 48 · 11
60	1 · 10 · 14 · 8	3 · 29 · 6 · 35	4 · 1 · 12 · 17
80	5 · 23 · 38 · 50	5 · 8 · 48 · 46	9 · 11 · 36 · 22
100	10 · 7 · 3 · 33	6 · 18 · 30 · 58	2 · 22 · 0 · 28
200	8 · 14 · 7 · 5	1 · 7 · 1 · 56	5 · 14 · 0 · 56
300	6 · 21 · 10 · 38	7 · 25 · 32 · 54	8 · 6 · 1 · 24
400	4 · 28 · 14 · 11	2 · 14 · 3 · 52	10 · 28 · 1 · 52
500	3 · 5 · 17 · 44	9 · 2 · 34 · 51	1 · 20 · 2 · 20
600	1 · 12 · 21 · 17	3 · 21 · 5 · 49	4 · 12 · 2 · 47
700	11 · 19 · 24 · 49	10 · 9 · 36 · 47	7 · 4 · 3 · 15
800	9 · 26 · 28 · 22	4 · 28 · 7 · 45	9 · 26 · 3 · 43
900	8 · 3 · 31 · 55	11 · 16 · 38 · 43	0 · 18 · 4 · 11
1000	6 · 10 · 35 · 28	6 · 5 · 9 · 41	3 · 10 · 4 · 39
1100	4 · 17 · 39 · 0	0 · 23 · 40 · 39	6 · 2 · 5 · 7
1200	2 · 24 · 42 · 33	7 · 12 · 11 · 37	8 · 24 · 5 · 35
1300	1 · 1 · 46 · 6	2 · 0 · 42 · 35	11 · 16 · 6 · 3
1400	11 · 8 · 49 · 39	8 · 19 · 13 · 34	2 · 8 · 6 · 31
1500	9 · 15 · 53 · 11	3 · 7 · 44 · 32	5 · 0 · 6 · 59
1600	7 · 12 · 56 · 44	9 · 26 · 15 · 30	7 · 22 · 7 · 27

Years (Anni)	Long. from Sun S·G·M·S	Anomaly S·G·M·S	Latitude S·G·M·S
1700	6 · 0 · 0 · 17	4 · 14 · 46 · 28	10 · 14 · 7 · 54
1800	4 · 7 · 3 · 50	11 · 3 · 17 · 26	1 · 6 · 8 · 22
1900	2 · 14 · 7 · 22	5 · 21 · 48 · 24	3 · 28 · 8 · 50
2000	0 · 21 · 10 · 55	0 · 10 · 19 · 22	6 · 20 · 9 · 18
2500	3 · 26 · 28 · 39	9 · 12 · 54 · 13	8 · 10 · 11 · 38
3000	7 · 1 · 46 · 23	6 · 15 · 29 · 3	10 · 0 · 13 · 57
3500	10 · 7 · 4 · 7	3 · 18 · 3 · 54	11 · 20 · 16 · 17 [?]
4000	1 · 12 · 21 · 50	0 · 20 · 38 · 44	1 · 10 · 18 · 36
4500	4 · 17 · 39 · 34	9 · 23 · 13 · 35	3 · 0 · 20 · 56
5000	7 · 22 · 57 · 18	6 · 25 · 48 · 25	4 · 20 · 23 · 16
5500	10 · 28 · 15 · 2	3 · 28 · 23 · 16	6 · 10 · 25 · 35
6000	2 · 3 · 32 · 46	1 · 0 · 58 · 7	8 · 0 · 27 · 55
6300	8 · 24 · 43 · 24	8 · 26 · 31 · 1	4 · 6 · 29 · 18

Right half — single years 1–20

Years	Long. from Sun S·G·M·S	Anomaly S·G·M·S	Latitude S·G·M·S
1	4 · 9 · 37 · 22	2 · 28 · 43 · 8	4 · 28 · 42 · 46
2	8 · 19 · 14 · 45	5 · 27 · 26 · 16	9 · 27 · 25 · 32
3	0 · 28 · 52 · 7	8 · 26 · 9 · 24	2 · 26 · 8 · 18
4	5 · 20 · 40 · 57	0 · 7 · 56 · 26	8 · 8 · 4 · 49
5	10 · 0 · 18 · 19	3 · 6 · 39 · 34	1 · 6 · 47 · 35
6	2 · 9 · 55 · 41	6 · 5 · 22 · 43	6 · 5 · 30 · 21
7	6 · 19 · 33 · 4	9 · 4 · 5 · 50	11 · 4 · 13 · 7
8	11 · 11 · 21 · 53	0 · 15 · 52 · 53	4 · 16 · 9 · 38
9	3 · 20 · 59 · 15	3 · 14 · 36 · 1	9 · 14 · 52 · 24
10	8 · 0 · 36 · 38	6 · 13 · 19 · 9	2 · 13 · 35 · 10
11	0 · 10 · 14 · 0	9 · 12 · 2 · 17	7 · 12 · 17 · 56
12	5 · 2 · 2 · 50	0 · 23 · 49 · 19	0 · 24 · 14 · 27
13	9 · 11 · 40 · 12	3 · 22 · 32 · 27	5 · 22 · 57 · 13
14	1 · 21 · 17 · 34	6 · 21 · 15 · 35	10 · 21 · 39 · 59

Years	Long. from Sun S·G·M·S	Anomaly S·G·M·S	Latitude S·G·M·S
15	6 · 0 · 54 · 57	9 · 19 · 58 · 43	3 · 20 · 22 · 45
16	10 · 22 · 43 · 46	1 · 1 · 45 · 45	9 · 2 · 19 · 16
17	3 · 2 · 21 · 8	4 · 0 · 28 · 53	2 · 1 · 2 · 2
18	7 · 11 · 58 · 31	6 · 29 · 12 · 1	6 · 29 · 44 · 48
19	11 · 21 · 35 · 53	9 · 27 · 55 · 10	11 · 28 · 27 · 34
20	4 · 13 · 24 · 43	1 · 9 · 42 · 12	5 · 10 · 24 · 6

Mean motion in the months

In common months [In mensibus communibus] and in bissextile months [In mensibus bisextilibus], the mean motion accumulated from the start of the year to the start of each named month.

Month	Common: Long. from Sun	Common : Anomaly	Common n: Latitude	Bissextile: Long. from Sun	Bissextile e: Anomaly	Bissextile e: Latitude
January (Ianuarii)	0 · 17 · 54 · 47	1 · 15 · 0 · 52	1 · 20 · 6 · 36	0 · 17 · 54 · 47	1 · 15 · 0 · 52	1 · 20 · 6 · 36
February (Februarius)	11 · 29 · 15 · 15	1 · 20 · 50 · 2	2 · 0 · 31 · 55	0 · 11 · 26 · 41	2 · 3 · 53 · 56	2 · 13 · 45 · 40
March (Martius)	0 · 17 · 10 · 2	3 · 5 · 50 · 55	3 · 20 · 38 · 30	0 · 29 · 21 · 29	3 · 18 · 54 · 49	4 · 3 · 52 · 16
April (Aprilis)	0 · 22 · 53 · 23	4 · 7 · 47 · 53	4 · 27 · 31 · 20	1 · 5 · 4 · 50	4 · 20 · 51 · 47	5 · 10 · 45 · 6
May (Majus)	1 · 10 · 48 · 11	5 · 22 · 48 · 45	6 · 17 · 37 · 55	1 · 22 · 59 · 37	6 · 5 · 52 · 39	7 · 0 · 51 · 41
June (Iunius)	1 · 16 · 31 · 32	6 · 24 · 45 · 43	7 · 24 · 30 · 45	1 · 28 · 42 · 58	7 · 7 · 49 · 37	8 · 7 · 44 · 31
July (Iulius)	2 · 4 · 26 · 20	8 · 9 · 46 · 35	9 · 14 · 37 · 21	2 · 16 · 37 · 45	8 · 22 · 50 · 29	9 · 27 · 51 · 6
August (Augustus)	2 · 22 · 21 · 7	9 · 24 · 47 · 27	11 · 4 · 43 · 56	3 · 4 · 32 · 33	10 · 7 · 51 · 22	11 · 17 · 57 · 41

Month	Common: Long. from Sun	Common : Anomaly	Common n: Latitude	Bissextile: Long. from Sun	Bissextile: e: Anomaly	Bissextile: e: Latitude
)						
September (Septemb.)	2 · 28 · 4 · 28	10 · 26 · 44 · 26	0 · 11 · 36 · 46	3 · 10 · 15 · 53	11 · 9 · 48 · 20	0 · 24 · 50 · 31
October (October)	3 · 15 · 59 · 15	0 · 11 · 45 · 18	2 · 1 · 43 · 21	3 · 28 · 10 · 41	0 · 24 · 49 · 12	2 · 14 · 57 · 7
November (Novemb.)	3 · 21 · 42 · 36	1 · 13 · 42 · 16	3 · 8 · 36 · 11	4 · 3 · 54 · 2	1 · 26 · 46 · 10	3 · 21 · 49 · 57
December (Decemb.)	4 · 9 · 37 · 23	2 · 28 · 43 · 8	4 · 28 · 42 · 45	4 · 21 · 48 · 49	3 · 11 · 47 · 2	5 · 11 · 56 · 33

The Moon's simple motion in days [Simplex motus Lunae in diebus]

Day	Long. from Sun S·G·M·S	Anomaly S·G·M·S	Latitude S·G·M·S
1	0 · 12 · 11 · 27	0 · 13 · 3 · 54	0 · 13 · 13 · 46
2	0 · 24 · 22 · 53	0 · 26 · 7 · 48	0 · 26 · 27 · 31
3	1 · 6 · 34 · 20	1 · 9 · 11 · 42	1 · 9 · 41 · 17
4	1 · 18 · 45 · 47	1 · 22 · 15 · 36	1 · 22 · 55 · 3
5	2 · 0 · 57 · 13	2 · 5 · 19 · 30	2 · 6 · 8 · 48
6	2 · 13 · 8 · 40	2 · 18 · 23 · 24	2 · 19 · 22 · 34
7	2 · 25 · 20 · 7	3 · 1 · 27 · 18	3 · 2 · 36 · 20
8	3 · 7 · 31 · 34	3 · 14 · 31 · 12	3 · 15 · 50 · 5
9	3 · 19 · 43 · 0	3 · 27 · 35 · 5	3 · 29 · 3 · 51
10	4 · 1 · 54 · 27	4 · 10 · 38 · 59	4 · 12 · 17 · 37

D a y	Long. from Sun S·G·M·S	Anomaly S·G·M·S	Latitude S·G·M·S
0			
1	4 · 14 · 5 · 54	4 · 23 · 42 · 53	4 · 25 · 31 · 22
1			
1	4 · 26 · 17 · 20	5 · 6 · 46 · 47	5 · 8 · 45 · 8
2			
1	5 · 8 · 28 · 47	5 · 19 · 50 · 41	5 · 21 · 58 · 54
3			
1	5 · 20 · 40 · 14	6 · 2 · 54 · 35	6 · 5 · 12 · 39
4			
1	6 · 2 · 51 · 40	6 · 15 · 58 · 29	6 · 18 · 26 · 25
5			
1	6 · 15 · 3 · 7	6 · 29 · 2 · 23	7 · 1 · 40 · 11
6			
1	6 · 27 · 14 · 34	7 · 12 · 6 · 17	7 · 14 · 53 · 56
7			
1	7 · 9 · 26 · 0	7 · 25 · 10 · 11	7 · 28 · 7 · 42
8			
1	7 · 21 · 37 · 27	8 · 8 · 14 · 5	8 · 11 · 21 · 28
9			
2	8 · 3 · 48 · 54	8 · 21 · 17 · 59	8 · 24 · 35 · 13
0			
2	8 · 16 · 0 · 21	9 · 4 · 21 · 53	9 · 7 · 48 · 59
1			
2	8 · 28 · 11 · 47	9 · 17 · 25 · 47	9 · 21 · 2 · 44
2			
2	9 · 10 · 23 · 14	10 · 0 · 29 · 41	10 · 4 · 16 · 30
3			
2	9 · 22 · 34 · 41	10 · 13 · 33 · 35	10 · 17 · 30 · 16
4			
2	10 · 4 · 46 · 7	13 · 26 · 37 · 28 [recte: 10 · 26 · 37 · 28]	11 · 0 · 44 · 1
5			
2	10 · 16 · 57 · 34	11 · 9 · 41 · 22	11 · 13 · 57 · 47

D a y	Long. from Sun S·G·M·S	Anomaly S·G·M·S	Latitude S·G·M·S
6			
2	10 · 29 · 9 · 1	11 · 22 · 45 · 16	11 · 27 · 11 · 33
7			
2	11 · 11 · 20 · 27	0 · 5 · 49 · 10	0 · 10 · 25 · 18
8			
2	11 · 23 · 31 · 54	0 · 18 · 53 · 4	0 · 23 · 39 · 4
9			
3	0 · 5 · 43 · 21	1 · 1 · 56 · 58	1 · 6 · 52 · 50
0			
3	0 · 17 · 54 · 47	1 · 15 · 0 · 52	1 · 20 · 6 · 35
1			

Mean motion in hours and minutes of hours

In hours and sixtieths [In horis & scrupulis] (left) — keyed by hour (H.) 1–24;
in sixtieths of an hour [In scrupulis horarum] (right) — keyed by minute (M.)
 25–60. Both give Longitude from the Sun, Anomaly, and Latitude.

In hours (H. 1–24) — in G·M·S

Hou rs	Long. from Sun G·M·S	Anomaly G·M·S	Latitude G·M·S
1	0 · 30 · 29	0 · 32 · 40	0 · 33 · 5
2	1 · 0 · 57	1 · 5 · 19	1 · 6 · 10
3	1 · 31 · 26	1 · 37 · 59	1 · 39 · 14
4	2 · 1 · 54	2 · 10 · 39	2 · 12 · 19
5	2 · 32 · 23	2 · 43 · 19	2 · 45 · 23
6	3 · 2 · 52	3 · 15 · 58	3 · 18 · 27
7	3 · 33 · 20	3 · 48 · 38	3 · 51 · 32
8	4 · 3 · 49	4 · 21 · 18	4 · 24 · 36
9	4 · 34 · 18	4 · 53 · 58	4 · 57 · 41
10	5 · 4 · 46	5 · 26 · 37	5 · 30 · 45
11	5 · 35 · 15	5 · 59 · 17	6 · 3 · 49

Hou rs	Long. from Sun G·M·S	Anomaly G·M·S	Latitude G·M·S
12	6 · 5 · 43	6 · 31 · 57	6 · 36 · 54
13	6 · 36 · 12	7 · 4 · 37	7 · 9 · 58
14	7 · 6 · 41	7 · 37 · 16	7 · 43 · 3
15	7 · 37 · 9	8 · 9 · 56	8 · 16 · 7
16	8 · 7 · 38	8 · 42 · 36	8 · 49 · 11
17	8 · 38 · 6	9 · 15 · 16	9 · 22 · 16
18	9 · 8 · 35	9 · 47 · 55	9 · 55 · 20
19	9 · 39 · 4	10 · 20 · 35	10 · 28 · 25
20	10 · 9 · 32	10 · 53 · 15	11 · 1 · 29
21	10 · 40 · 1	11 · 25 · 55	11 · 34 · 33
22	11 · 10 · 29	11 · 58 · 34	12 · 7 · 38
23	11 · 40 · 58	12 · 31 · 14	12 · 40 · 42
24	12 · 11 · 37	13 · 3 · 54	13 · 13 · 48

In sixtieths of an hour (M. 25–60) — in M·S

Mi n.	Long. from Sun M·S	Anomaly M·S	Latitude M·S
25	12 · 42	13 · 37	13 · 47
26	13 · 12	14 · 9	14 · 20
27	13 · 43	14 · 42	14 · 53
28	14 · 13	15 · 15	15 · 26
29	14 · 44	15 · 47	15 · 59
30	15 · 14	16 · 20	16 · 32
31	15 · 45	16 · 53	17 · 5
32	16 · 15	17 · 25	17 · 38
33	16 · 46	17 · 58	18 · 11
34	17 · 16	18 · 31	18 · 44
35	17 · 47	19 · 3	19 · 18
36	18 · 18	19 · 36	19 · 51
37	18 · 48	20 · 8	20 · 24
38	19 · 19	20 · 41	20 · 57

Mi n.	Long. from Sun M·S	Anomaly M·S	Latitude M·S
39	19 · 49	21 · 14	21 · 30
40	20 · 19	21 · 46	22 · 3
41	20 · 49	22 · 19	22 · 36
42	21 · 20	22 · 51	23 · 9
43	21 · 50	23 · 24	23 · 42
44	22 · 21	23 · 57	24 · 15
45	22 · 51	24 · 30	24 · 48
46	23 · 21	25 · 3	25 · 21
47	23 · 52	25 · 36	25 · 54
48	24 · 22	26 · 8	26 · 27
49	24 · 53	26 · 41	27 · 0
50	25 · 24	27 · 13	27 · 34
51	25 · 54	27 · 46	28 · 7
52	26 · 25	28 · 18	28 · 48
53	26 · 55	28 · 51	29 · 13
54	27 · 26	29 · 24	29 · 64 [recte: 29 · 46]
55	27 · 56	29 · 56	30 · 19
56	28 · 26	30 · 29	30 · 52
57	28 · 57	31 · 1	31 · 25
58	29 · 27	31 · 34	31 · 58
59	29 · 58	32 · 7	32 · 31
60	30 · 29	32 · 40	33 · 5

On the hypothesis of the Moon, and how its apparent motions are derived from its mean motions [De hypothesi lunae, & quemadmodum ex aequalibus motibus ejus apparentes fiant]

Restoring the Moon's mean motions was neither especially difficult nor a matter requiring long delay, given how swiftly the Moon revolves among all the planets. Even so, here too I began from the most ancient observations and, for greater certainty, carried their examination forward through the various centuries down

to our own age, reconciling them all with one another as far as could be done. That difficulty, both in eclipses and outside them, had entangled most practitioners in errors on one side or the other. But to draw out the Moon's apparent positions for each given moment from those assumed mean motions is a task of the art demanding perhaps greater labor than for any other heavenly body — and that not only with respect to longitude but to latitude as well. I shall treat the former first, namely longitude, in this place, and the latter, latitude, afterward; though first I shall briefly set out the errors of my predecessors. Reason itself shows that the phenomena ought to be handled separately, under distinct hypotheses for longitude and for latitude — and this against the fancy of Christmann of Heidelberg [Christmann], who vainly presumed to emend the modern lunar hypothesis found in Tycho [Tycho Brahe], for which he was deservedly refuted by the distinguished David Origanus [Origanus]. But more on this below.

The old Ptolemaic supposition for the Moon, as regards the determination of longitude, suffers — besides the magnitude of the eccentricity and of the epicycle's semidiameter, which disagrees with the celestial appearances — from three further manifest faults. **First**, that it lacks the equant that must necessarily be added. **Second**, that Ptolemy [Ptolemy] supposed the other inequalities, between the least (which occurs at the syzygies of the luminaries) and the greatest (which occurs at their quadratures), could be saved by a single epicycle on the eccentric orbit. For in this way not only do the discrepancies of the lunar parallaxes in these extreme positions — for those who follow the Ptolemaic hypothesis — come out disordered, and moreover nearly eight times greater than the truth in the celestial appearances demands; but also a quite noticeable inequality of the Moon in the sky, away from its mean longitude, when in its mean course it is carried from conjunction or opposition with the Sun toward either quadrature at the apogee and perigee (whereas in the Ptolemaic arrangement there would be none at all, the Moon holding exactly the highest and lowest points of the epicycle) — this declares such a Ptolemaic theory of the Moon to be false. **Third and last**, that a small equation insinuating itself everywhere within these same courses of the Moon was omitted by Ptolemy and by all others down to us. Among these three drawbacks clinging to the lunar supposition of the ancients, Copernicus [Copernicus] removed only the middle one, in a manner — the others perhaps less so — having noticed them from sets of three or four observations, on which the ancients founded their hypotheses far too rashly. But so that these faults should not, through like negligence or license, be handed down to posterity — whereby the exact phenomena of this body for any given moment would remain still unknown to mortals — and for many other

reasons besides, and above all because these things would be of the greatest service in investigating the longitudes of places far apart by land and sea: with truly incredible diligence joined to an equal expenditure of time, at last, by the grace of God, I have here arrived at that knowledge, so that I may hope not only to make the lunar theory everywhere satisfy the celestial appearances, but also to render most of the causes of so wandering and manifold an anomaly in this same body. These I shall therefore set out here first, and then establish them by two hypotheses, of which I have added the latter — entirely equivalent to the former founded on the Tychonic observations — for the sake of the lovers of brevity [Greek: φιλοσυντόμων (philosyntomōn, "lovers of conciseness")], so that perhaps a more expeditious calculation in the tables might be had without the resolution of triangles.

[Margin: See below in this book — Vide infra hoc lib.]

The Moon, on account of certain mysteries which I shall touch upon more fully at the end of this book, was created by the most wise and best Craftsman of things [the Creator] not only with a marvelous joining-together of matter...

created and gathered into a globe; yet it is also subject, in marvelous ways, to its own motions toward each of the two — the Earth and the Sun. For first of all, at the conjunction and opposition with the Sun, the center of the Earth looks back, in its revolution, almost upon the Sun itself — except that it rests upon an equant in an eccentric, or rather upon a twin epicycle in a homocentric, in the manner we sketched out in general toward the end of Cap. 2. Here, however, this is well to observe: that it must reckon the account of its motion against the true course of the Sun in the Zodiac, always counting in as well that proper inequality which it receives in respect of the Earth alone. Such, then, is the lunar hypothesis, suited to the conjunctions and oppositions of the Moon with the Sun alone, and therefore conveniently serving the computation of eclipses.

Next, besides these simple inequalities, the Moon also always shares — apart from syzygies of this kind — in others from the Sun. As I long and earnestly sought the cause of these in nature, the offset of the centers of both luminaries' epicycles from the Earth seemed to hint it to me, as it were obliquely. From this I infer that a power flows out constantly from the center of the Sun's epicycle, and that the Moon's motion too varies under the greatest angle of vision, in proportion to its mean distance from us — yet not exactly as in the other planets, which observe either the center of the Sun's epicycle, or the Sun itself, as their common center. Rather, this power applies whatever it is, properly, to that higher inequality of the Moon which we have described here. Accordingly, it is the

general arrangement of the Moon's orbit, together with its epicycles — not a simple or homocentric one — that this virtual influence for varying the motion considers, its mark being certainly equal at the conjunction and opposition of the luminaries, as will appear afterward in the hypothesis.

Now, however, let us see how, perhaps uniquely through this means, one may investigate the mean distance of the Sun from the Earth elegantly and more reliably. From the Earth (or rather from the center of the Moon's orbit), let the Moon's orbit $C D G E$ be described, and let the line be extended from A to B as the Sun's mean course in the zodiac-bearer. Then let the lines $B C$ and $B G$ be drawn, which touch the Moon's orbit as it were at the quadratures with the Sun, under the greatest angle of elongation. Since the angle $A B C$, or $A B G$ — which measures the greatest digression of the Moon from conjunction with the Sun at D , or from its opposition at E , in the region of the quadratures C or also G , by select Tychonic observations — is established to be $2^{\circ} 30'$; and since the mean distance of the Moon from the Earth, $A C$, has also been found most diligently, through very many of Tycho's observations, to be 56 Earth-radii, or a little beyond; therefore in the right triangle $A B C$, for $A B$ [δίδομεν, "given"] there are the side $A C$ together with all three angles (the right angle at A being of course granted). Hence $A B$ too is given as 1283 Earth-radii — so much so that, if you suppose a somewhat greater distance of the Moon from the Earth (even according to Tycho's earlier constitution), or diminish the angle of greatest digression a very little, and indeed scarcely perceptibly, the distance of the Moon from the Sun will be in that proportion which the diameter of a circle has to its circumference multiplied sevenfold — namely the square root of $1516860 \frac{11}{18} \frac{4}{9}$, or $1231 \frac{65}{88}$ very nearly — and consequently the distance of the Sun from the Earth nearly 1288 Earth-radii.

This I do not doubt can be further confirmed below from eclipses as well — namely that it far exceeds the remoteness of the Sun once established by Ptolemy, and therefore also by Tycho [Tycho Brahe] and Copernicus [Nicolaus Copernicus]. This not only tells against the constitution of the Copernican system (which we set aside above), but also brings with it some weight toward verifying the hypotheses especially of Mars and Venus, as we shall show below in their proper places, God being our helper. Concerning the Moon, however, we have here pursued what seemed probable, lest the things I had not long ago meditated upon this matter should be wrapped in perpetual silence. For although in this business we are not bound to discourse at length, outside the hypothesis, on the special application of such an angle:

[Margin: Lib. 2. Theor. cap. 1.]

Diagram. A geometric construction for the Sun-distance argument. An isosceles triangle stands on its apex at the top, with the two long sides descending and just touching a circle at the bottom; a vertical axis runs down the middle from the apex through the circle's center.

- B (apex, top): the Sun's mean place; the two tangent lines B G and B C descend from here.
- D (upper rim of the circle, on the vertical axis): point where the axis meets the top of the Moon's orbit (conjunction).
- A (center of the circle): the Earth / center of the Moon's orbit.
- G (left end of the horizontal diameter): quadrature point where line B G touches the orbit.
- C (right end of the horizontal diameter): quadrature point where line B C touches the orbit.
- E (bottom of the circle, on the vertical axis): opposition point.

we are not bound: yet that the Moon should likewise direct its courses toward the mean and true course of the Sun, no one ought to wonder at, any more than in certain other planets. Now let us come to the description of the lunar hypotheses, inasmuch as they are founded upon the most certain observations and on geometrical reasonings.

The description and demonstration of the Theory of the Moon, once devised in the household of D. Tycho [Tycho Brahe], B[rahe], in Bohemia, found out together by the author's own labor, and inserted in Book 1 of his *Progymnasmata*.

[Margin: Toward the end of Cap. 2 of this book.]

Having laid some sort of foundation from the premises, we will now build up, by an orderly recapitulation here, the lunar hypothesis which we found in the year of our Savior 1600 in the household of that most noble and, of all, most outstanding astronomer Lord Tycho Brahe.

Let the Moon's orbit B F be described about the Earth's center A, and let the apogee line B A F be drawn. With the center B placed — at which the course of mean longitude under the zodiac is determined — let the first epicycle (as also the rest) be drawn in the same plane with the Moon's orbit. Its topmost apex C is understood to be carried toward G, into the leading parts of the signs, by the

simple motion of the Moon's anomaly assigned above. Again, to the same center C let the second epicycle be joined, carrying the Moon around at D — and that toward I, into the following parts of the signs, by a contrary motion and double that of the center C in all respects, just as we earlier pursued all these revolutions in general.

As regards the measure of the radii in these epicycles, from many and most certain observations of Tycho Brahe here in Denmark, that of the first, B C, has been found to be 5800, while that of the latter, C D, is twice smaller — namely 2900 — in units of which the radius A B of the Moon's orbit is 100000. Now these inequalities, unfolding themselves properly through the whole orbit of the Moon, suffice only for the true conjunctions and oppositions of the luminaries, and are therefore to be ascribed primarily to the Moon's revolution about the Earth. But besides these, two other inequalities also insinuate themselves with respect to the Moon's longitude. One is known to the ancients, by which the Moon's digression from the prior inequalities into the quadratures on both sides is determined; this we here account for by means of a certain little circle, set near the middle, whose center is E, while the center A of the Moon's own orbit is understood to be carried perpetually around within the circuit of that same little circle — whose diameter A M is either united to the Moon's apogee line B F (as it is extended through it at the luminaries' syzygies) or, as at other times, [παράλληλως, "in parallel"] — by the double motion of the Moon from the Sun, on this law: that at the true conjunction and opposition of the Moon with the true motion of the Sun, the center of the Moon's orbit lies at A; but at either quadrature, at M. That this anomaly likewise depends, without doubt, upon the Sun, the Earth, and the motion and distance of the Moon from each of them, we taught just above. The radius of this, namely E A, found by us through Tycho's observations, accurately and carefully obtained, and once limited by us, was found to be 2174 — again in units of which the radius A B of the Moon's orbit is taken as 100000.

Diagram. The lunar epicyclic hypothesis. A large orbit-circle is drawn about the Earth's center, with a vertical apogee line running through it from top to bottom. At the top of the large circle sits the compound epicyclic system; at the lower part of the apogee line is the little circle of the variation.

- A (on the apogee line, lower portion): the Earth's center / mean center of the Moon's orbit.
- B (bottom of the upper epicyclic group, on the orbit): center of the first epicycle, where mean longitude is determined.

- F (bottom of the large orbit, on the apogee line): perigee end of the apogee line B F.
- C (center of the small inner epicycle at top): topmost apex of the first epicycle, carried toward G.
- D (within the upper group, just below C): position of the Moon carried by the second epicycle.
- G (right end of the first epicycle): the leading direction of the signs.
- H (left end of the first epicycle): opposite to G.
- I (right point of the inner second epicycle): the following direction of the signs.
- K (left point of the inner second epicycle): opposite to I.
- N (top of the inner second epicycle).
- E (center of the little circle on the apogee line, lower): center of the variation circle.
- M (top of the little variation circle): the position of the Moon's orbit-center at quadrature; A M is its diameter.

[E A] found, accurately and carefully obtained, and once limited by us, was 2174 — again in units of which the radius of the Moon's orbit A B is taken as 100000.

Finally, the last inequality in the lunar motion — yet the very first of all, in the household of Lord Tycho, to be unfolded by us from his most certain observations and brought to light — we likewise judge to proceed primarily from the Sun, according to the Moon's varying disposition in its encounter toward the Sun and toward the Earth. For while, with respect to the Earth, the Moon is conjoined to the Sun or opposed to it, a greater efficacy seems here to be attributed to the Earth, as the primary center of the Moon's orbit, on account of the solar rays falling upon the Earth and reflecting, together with the motive force from it, jointly and directly toward the center of the Moon's epicyclic system — namely B. From this the Moon is then driven forward more swiftly into the following parts; but about the quadratures more slackly, the virtue of the solar rays being there dissipated and dispersed at the greatest obliquity of aspect — that virtue which had earlier, on account of the bond of the united Moon, mingled itself with the motive faculty of the Earth's center. And so, in the middle course from the luminaries' syzygies, it passes directly into the quadratures and onward. For a virtue more united, or aided by some extrinsic cause, operates more strongly. That such a support emanates from the Sun, we showed not only above in the Earth's motion; but the same is also indicated by a careful consideration of the very position of the luminaries, in the increase and decrease of the tide and the

reciprocation of the ocean's waves, and likewise of the humors in sublunar bodies, each according to its own nature and diversity.

In these premises the cause is plain why such a little equation will be explained outside the Moon's hypothesis; yet nonetheless, with regard to that account by which the apparent accidental celestial motion proceeds most slowly about the beginning of the increase and the end of the decrease, but in the middle of all most swiftly. This is conveniently demonstrated here upon a little circle, the measure of whose radius contains within itself the greatest inequality of this anomaly — namely $40\frac{1}{2}'$. In this way too the other equations of this — which will be inscribed in the lunar table — may be extracted by the right sines of the corresponding arcs. So let the little circle B C F E be drawn about the center A, at which, in the luminaries' syzygies, the center B in the upper lunar hypothesis is understood to dwell; but in the sextile and trigonal aspects, at B and at C — on this condition: that from the Moon's coming-together with the Sun up to the first sextile, the center B, in the upper lunar theory here, runs down the line A B into the following parts of the signs; then betakes itself again into A — that same center B — when the Moon is irradiated by the Sun at quadrate aspect. From here, the other radius likewise, with the point B traversing the Moon's mean motion, determines the first trigonal aspect of the luminaries at C; then returns again to the opposition of the Sun at A; and so it proceeds consecutively. Whence, by the double motion of the Moon from the Sun as well, the commutation of such a center of the Moon's mean course — measured out — displays from the heavens its causes, just as we expressed them above. And these little equations are easily elicited by the following example.

Let the equation of this small circle be sought, corresponding to a double longitude of the Moon from the Sun of 30° . Thus, since the right sine of 30° is 50000 — which is shown by the straight line G E or A D — in units of which the radius A B of that little circle is given as 100000, and to this measure $40\frac{1}{2}'$ are made equal: therefore, as the whole sine A B 100000 is to $40\frac{1}{2}'$, so is A D 50000 to $20' 15''$. In the same way one must proceed in the rest of this little equation, as also in the others, according to the requirement of each, by the geometrical reasonings of plane triangles, throughout the whole theory of the Moon — as will be shown by examples afterward.

And upon these things demonstrated beforehand, we once compiled the whole table of equations of lunar longitude, here translated from Book 1 of the *Progymnasmata* of T[ychon] B[rahe], together with the method of computing the

lunar motions, and a new example besides, by which this hypothesis is more fully illustrated and verified.

Diagram. The little circle of the small lunar equation. A circle with a horizontal diameter C—A—D—B and a vertical diameter, with an inscribed right triangle illustrating the sine construction.

- A (center, on the horizontal diameter): center of the little circle.
- B (right end of the horizontal diameter): syzygy position.
- C (left end of the horizontal diameter): position at the trigonal aspect.
- D (on the horizontal diameter, between A and B): foot showing A D, the right sine.
- G (lower left, inside, near the bottom): endpoint of the line G E.
- E (lower, on the rim near the bottom-right): endpoint paired with G; line G E equals the right sine.
- F (bottom of the circle, on the vertical diameter).

TABLE OF PROSTHAPHAERESES [TABVLA PROSTHAPHÆ-]

0 Sig. [0 Signs]

Degrees (Gradus)	Subtr. Prosthaph. Epicycle (G. M. S.)	Add. Diff. (M. S.)	Elongation from center (Elongat. à centro) (Partic.)	Su b. Di ff.	Eccentric ity (Eccentri citas) (Partic.)	Ad d. Di ff.	Add. Variation (Adde Variatio) (M. S.)	Add. Diff. (M. S.)
0	0 · 0 · 0	—	102900	1	00	38	0 · 0	—
1	0 · 5 · 4	5 · 4	102899	1	38	38	0 · 43	0 · 43
2	0 · 10 · 8	5 · 4	102898	2	76	38	1 · 26	0 · 43
3	0 · 15 · 12	5 · 4	102896	2	114	38	2 · 8	0 · 42
4	0 · 20 · 16	5 · 4	102894	3	152	38	2 · 50	0 · 42
5	0 · 25 · 20	5 · 4	102891	3	190	38	3 · 32	0 · 42
6	0 · 30 · 23	5 · 3	102888	4	228	37	4 · 14	0 · 42
7	0 · 35 · 26	5 · 3	102884	5	265	38	4 · 56	0 · 42
8	0 · 40 · 28	5 · 2	102879	6	303	38	5 · 38	0 · 42
9	0 · 45 · 29	5 · 1	102873	6	341	38	6 · 20	0 · 42
10	0 · 50 · 30	5 · 1	102867	7	379	38	7 · 2	0 · 42

Degrees (Gradus)	Subtr. Prosthaph. Epicycle (G. M. S.)	Add. Diff. (M. S.)	Elongation from center (Elongat. à centro) (Partic.)	Sub. Di. ff.	Eccentricity (Eccentricitas) (Partic.)	Add. Variation (Add. Variatio) (M. S.)	Add. Diff. (M. S.)
11	0 · 55 · 30	5 · 0	102860	7	417	37 7 · 44	0 · 42
12	1 · 0 · 28	4 · 58	102853	8	454	38 8 · 26	0 · 42
13	1 · 5 · 25	4 · 57	102845	9	492	38 9 · 7	0 · 41
14	1 · 10 · 21	4 · 56	102836	10	530	38 9 · 48	0 · 41
15	1 · 15 · 16	4 · 55	102826	10	568	37 10 · 29	0 · 41
16	1 · 20 · 10	4 · 54	102816	11	605	38 11 · 10	0 · 41
17	1 · 25 · 3	4 · 53	102805	12	643	37 11 · 51	0 · 41
18	1 · 29 · 55	4 · 52	102793	12	680	38 12 · 31	0 · 40
19	1 · 34 · 44	4 · 49	102781	13	718	37 13 · 11	0 · 40
20	1 · 39 · 32	4 · 48	102768	13	755	37 13 · 51	0 · 40
21	1 · 44 · 18	4 · 46	102755	14	792	38 14 · 31	0 · 40
22	1 · 49 · 30	4 · 45	102741	15	830	37 15 · 10	0 · 39
23	1 · 53 · 47	4 · 44	102726	15	867	37 15 · 49	0 · 39
24	1 · 58 · 29	4 · 42	102711	16	904	37 16 · 28	0 · 39
25	2 · 3 · 8	4 · 39	102695	17	941	37 17 · 7	0 · 39
26	2 · 7 · 44	4 · 36	102678	18	978	37 17 · 45	0 · 38
27	2 · 12 · 18	4 · 34	102660	18	1015	37 18 · 23	0 · 38
28	2 · 16 · 50	4 · 32	102642	19	1052	37 19 · 1	0 · 38
29	2 · 21 · 20	4 · 30	102623	19	1189 [?]	36 19 · 38	0 · 37
30	2 · 25 · 47	4 · 27	102604	—	1125	— 20 · 15	0 · 37
(foot)	Add.	Subtr.		Add. d.		Sub. b.	Subtr.

11 Sig. [11 Signs] (foot-of-page label, indicating the table is read in the reverse direction for the 11th sign of argument)

Pars Altera, Liber Primus Theoricorum — Cap. VI (continued)

Table of the lunar prosthaphaereses [Tabula prosthaphaereseon lunarium], continued

These are the continuation pages of the great lunar table begun earlier in this chapter. Each opening covers one zodiacal sign (*Signum*) of the argument; the heading "1 Sig.", "2 Sig.", etc. names the sign read from the top, while the foot of the same column carries the complementary sign ("10 Sig.", "9 Sig.", and so on) read from the bottom. The column-foot rubrics (*Adde / Subtr. / Add. / Sub.*) reverse the sense of addition and subtraction between the two reading directions, exactly as in the preceding pages.

The seven data columns are, in order: the **prosthaphaeresis of the epicycle** [prosthaphaeresis epicycli] in degrees, minutes, and seconds (G. M. S.); its **difference** [differentia] in minutes and seconds; the **elongation from the centre** [elongatio a centro] in particles (*Partic.*); its difference; the **eccentricity** [eccentricitas] in particles; its difference; the **variation** [variatio] in minutes and seconds; and the difference of the variation. The marginal scale of the argument (0–30) runs down one side and up the other.

1 Sig.

The argument runs 0–30 from the top (reading "1 Sig."); the same column read upward from the foot gives "10 Sig." The argument index at the right runs 30 down to 0.

Subtr. Prosth. Epicycl. (G. M. S.)	Add. Differ. (M. S.)	Elongat. a centro (Partic.)	Su b. Di ff.	Eccentrici tas (Partic.)	Ad . Di ff.	Adde variatio (M. S.)	Adde Diff. (M. S.)	[a r g.]
2 25 47		102604	20	1125	37	20 15		3 0
2 30 12	4 25	102584	21	1162	36	20 51	0 36	2 9
2 34 34	4 22	102563	21	1198	37	21 27	0 36	2 8
2 38 54	4 20	102542	22	1235	36	22 3	0 36	2 7

Subtr. Prosth. Epicycl. (G. M. S.)	Add. Differ. (M. S.)	Elongat. a centro (Partic.)	Su b. Di ff.	Eccentrici tas (Partic.)	Ad . Di ff.	Adde variatio (M. S.)	Adde Diff. (M. S.)	[a r g.]
2 43 11	4 17	102520	23	1271	36	22 38	0 35	2 6
2 47 25	4 14	120497 [recte 102497]	23	1307	37	23 13	0 35	2 5
2 51 37	4 12	102474	24	1344	36	23 48	0 35	2 4
2 55 46	4 9	102450	24	1380	36	24 22	0 35	2 3
2 59 52	4 6	102426	25	1416	35	24 56	0 34	2 2
3 3 54	4 2	102401	26	1451	36	25 29	0 34	2 1
3 7 53	3 59	102375	27	1487	36	26 2	0 33	2 0
3 11 49	3 56	102348	27	1523	35	26 34	0 33	1 9
3 15 42	3 53	102321	28	1558	36	27 6	0 32	1 8
3 19 31	3 49	102293	28	1594	35	27 39	0 32	1 7
3 23 17	3 46	102265	29	1629	35	28 8	0 32	1 6
3 26 59	3 42	102236	30	1664	35	28 38	0 31	1 5
3 30 38	3 39	102206	30	1699	35	29 8	0 30	1 4
3 34 13	3 35	102176	31	1734	35	29 37	0 30	1 3
3 37 44	3 31	102145	31	1769	35	30 6	0 29	1 2

Subtr. Prosth. Epicycl. (G. M. S.)	Add. Differ. (M. S.)	Elongat. a centro (Partic.)	Su b. Di ff.	Eccentrici tas (Partic.)	Ad . Di ff.	Adde variatio (M. S.)	Adde Diff. (M. S.)	[a r g.]
3 41 12	3 28	102114	32	1804	34	30 34	0 29	1 1
3 44 36	3 24	102082	33	1838	34	31 1	0 28	1 0
3 47 56	3 20	102049	33	1872	34	31 28	0 27	9
3 51 12	3 16	102016	34	1906	34	31 55	0 27	8
3 54 24	3 12	101982	34	1940	34	32 21	0 27	7
3 57 32	3 8	101948	35	1974	34	32 46	0 26	6
4 0 36	3 4	101913	36	2008	34	33 11	0 25	5
4 3 35	2 59	101877	36	2042	33	33 35	0 25	4
4 6 30	2 55	101841	37	2075	33	33 58	0 24	3
4 9 21	2 51	101804	37	2108	33	34 21	0 23	2
4 12 8	2 47	101767	38	2141	33	34 43	0 23	1
4 14 51	2 43	101729		2174		35 4	0 21	0
Adde	Subtr.		Ad .		Su b.	Subtr.	Subtr.	1 0 S i g.

2 Sig.

This opening adds an explicit **Gradus** (degree) column at the left. Argument 0–30 from the top ("2 Sig."); read upward it is "9 Sig."

G r a d u s	Subtr. Prosth. Epicycl. (G. M. S.)	Add. Diffe. (M. S.)	Elongat. à centro (Partic.)	Su b. Di ff.	Eccentrici tas (Partic.)	A d. Di f.	Adde Variatio (M. S.)	Adde Diff. (M. S.)
0	4 14 51		101729	38	2174	33	35 4	

G r a d u s	Subtr. Prosth. Epicycl. (G. M. S.)	Add. Diffe. (M. S.)	Elongat. à centro (Partic.)	Su b. Di ff.	Eccentrici tas (Partic.)	A d. Di f.	Adde Variatio (M. S.)	Adde Diff. (M. S.)
1	4 17 29	2 38	101691	39	2207	33	35 25	0 21
2	4 20 2	2 33	101652	39	2240	32	35 45	0 20
3	4 22 31	2 29	101613	40	2272	32	36 5	0 20
4	4 24 55	2 24	101573	41	2304	32	36 24	0 19
5	4 27 14	2 19	101532	41	2336	32	36 42	0 18
6	4 29 29	2 15	101491	42	2368	32	37 0	0 18
7	4 31 39	2 10	101449	42	2400	32	37 17	0 17
8	4 33 44	2 5	101407	42	2432	31	37 33	0 16
9	4 35 44	2 0	101365	43	2463	31	37 48	0 15
10	4 37 39	1 55	101322	43	2494	31	38 3	0 15
11	4 39 30	1 51	101279	44	2525	31	38 17	0 14
12	4 41 17	1 47	101235	44	2556	31	38 30	0 13
13	4 42 59	1 42	101191	45	2587	30	38 42	0 13
14	4 44 35	1 36	101146	45	2617	30	38 55	0 12
15	4 46 5	1 30	101101	46	2647	30	39 7	0 12
16	4 47 30	1 25	101055	46	2677	30	39 18	0 11
17	4 48 50	1 20	101009	46	2707	30	39 28	0 10
18	4 50 6	1 16	100963	47	2737	29	39 37	0 9
19	4 51 16	1 10	100916	47	2766	29	39 45	0 8

[illegible]

3 Sig.

On this opening the inner column heading reads **Add. Differ.** (the epicycle prosthaphaeresis is now decreasing, so its difference is added). Argument 0–30 from the top ("3 Sig."); read upward it is "8 Sig." The argument index runs down the right-hand margin, 30 to 0.

Subtr. Prosth. Epicicl. (G. M. S.)	Add. Differ. (M. S.)	Elongat. à centro (Partic.)	Su b. Di f.	Eccentrici tas (Partic.)	A d. Di f.	Adde variatio (M. S.)	Adde Diff. (M. S.)	[a r g .]
4 58 20		100378	51	3075	27	40 30		3 0
4 58 26	0 6	100327	51	3102	26	40 29	0 1	2 9
4 58 27	0 1	100276	51	3128	26	40 28	0 1	2 8
4 58 22	0 5	100225	51	3154	26	40 27	0 1	2 7
4 58 14	0 8	100174	51	3180	26	40 25	0 2	2 6
4 57 59	0 15	100123	51	3206	25	40 21	0 4	2 5
4 57 37	0 22	100072	51	3231	25	40 17	0 4	2 4
4 57 10	0 27	100021	52	3256	25	40 12	0 5	2 3
4 56 38	0 32	99969	52	3281	25	40 6	0 6	2 2
4 56 1	0 37	99917	52	3306	25	40 0	0 6	2 1
4 55 18	0 43	99865	52	3331	24	39 53	0 7	2 0
4 54 30	0 48	99813	52	3355	24	39 45	0 8	1 9
4 53 36	0 54	99761	52	3379	24	39 37	0 8	1

Subtr. Prosth. Epicicl. (G. M. S.)	Add. Differ. (M. S.)	Elongat. à centro (Partic.)	Su b. Di f.	Eccentrici tas (Partic.)	A d. Di f.	Adde variatio (M. S.)	Adde Diff. (M. S.)	[a r g .]
								8
4 52 37	0 59	99709	52	3403	23	39 28	0 9	1 7
4 51 33	1 4	99657	52	3426	23	39 18	0 10	1 6
4 50 23	1 10	99605	52	3449	23	39 7	0 11	1 5
4 49 7	1 16	99553	52	3472	23	38 55	0 12	1 4
4 47 46	1 21	99501	52	3495	23	38 43	0 12	1 3
4 46 21	1 25	99449	52	3518	22	38 30	0 13	1 2
4 44 50	1 31	99397	52	3540	22	38 17	0 13	1 1
4 43 13	1 37	99343	52	3562	22	38 3	0 14	1 0
4 41 31	1 42	99293	51	3584	21	37 48	0 15	9
4 39 43	1 48	99242	51	3605	21	37 33	0 15	8
4 37 51	1 52	99191	51	3626	21	37 17	0 16	7
4 35 54	1 57	99140	51	3647	20	37 0	0 17	6
4 33 51	2 3	99089	51	3667	20	36 42	0 18	5
4 31 42	2 9	99038	51	3687	20	36 24	0 18	4
4 29 29	2 13	98937 [?]	50	3707	20	36 5	0 19	3
4 27 11	2 18	98987 [?]	50	3727	19	35 45	0 20	2
4 24 48	2 23	98887	49	3746	19	35 25	0 20	1
4 22 20	2 28	98883		3765		35 0	0 21	0
Adde.	Adde.		Su b.		S u bt	Subt.	Add.	8 S i

G r a d u s	Subtr. Prosth. Epicycl. (G. M. S.)	Subtr. Diff. (M. S.)	Elongat. a centro (Partic.)	Sub t. Diff .	Eccentric itas (Parti.)	Ad d. Di ff.	Adde variatio (M. S.)	Subtr. Diff. (M. S.)
1 3	3 42 50	3 30	98228	43	3987	15	29 37	0 29
1 4	3 39 16	3 34	98185	43	4002	15	29 8	0 29
1 5	3 35 38	3 38	98142	42	4017	14	28 38	0 30
1 6	3 31 55	3 43	98100	41	4031	14	28 8	0 30
1 7	3 28 8	3 47	98059	41	4045	14	27 37	0 31
1 8	3 24 18	3 50	98018	40	4059	14	27 6	0 31
1 9	3 20 24	3 54	97978	40	4073	13	26 34	0 32
2 0	3 16 25	3 58	97938	39	4086	13	26 2	0 32
2 1	3 12 22	4 3	97899	38	4099	12	25 29	0 33
2 2	3 8 15	4 7	97861	37	4111	12	24 56	0 33
2 3	3 4 5	4 10	97824	37	4123	12	24 22	0 34
2 4	2 59 52	4 13	97787	36	4135	12	23 48	0 34
2 5	2 55 35	4 17	97751	35	4147	11	23 13	0 35
2 6	2 51 14	4 21	97716	34	4158	11	22 38	0 35
2 7	2 46 49	4 25	97682	33	4169	11	22 3	0 35

G r a d u s	Subtr. Prosth. Epicycl. (G. M. S.)	Subtr. Diff. (M. S.)	Elongat. a centro (Partic.)	Sub t. Diff .	Eccentric itas (Parti.)	Ad d. Di ff.	Adde variatio (M. S.)	Subtr. Diff. (M. S.)
2 8	2 42 21	4 28	97649	33	4180	10	21 27	0 36
2 9	2 37 51	4 30	97616	32	4190	10	20 51	0 36
3 0	2 33 18	4 33	97584		4200		20 15	0 36
A d d e .	Subtr.		Add.		Subt.	Su bt r.	Add.	7 Sig.

TABLE OF LUNAR PROSTHAPHAERESES [continued]

RESIDVORVM LVNARIVM [of the lunar residues]

5 Sig. [Sign 5 — read top-to-bottom; the foot label is 6 Sig., read bottom-to-top]

Nine columns: the argument (the rightmost column, descending 30→0 at the head for sign 5, ascending 0→30 at the foot for sign 6); the subtractive epicyclic prosthaphaeresis (degrees, minutes, seconds) with its difference (minutes, seconds); the elongation from the centre (in units, *particulae*) with its subtractive difference; the eccentricity (in units, *particulae*) with its additive difference; and the additive variation (minutes, seconds) with its difference (minutes, seconds). The head labels read *Subtr. Prosth. Epicycl. | Add Diff. | Elongat. a centro | Sub. Diff. | Eccentricitas | Add. Diff. | Adde variatio. | Add. Diff.*; the foot labels read *Adde | Add. | Ad. | Sub. | Subt. | Add.*

Subtr. Prosth. Epicycl. (G.M.S.)	Diff. (M.S.)	Elongat. a centro (Partic.)	D i ff f . Eccentricita s (Partic.)	D i ff f . Adde variatio (M.S.)	Diff. (M.S.)	A r g .
2 33 18		97584	3 4200 1	1 20 15 0		3 0
2 28 41	4 37	97553	3 4210 0	9 19 38	0 37	2 9
2 24 1	4 40	97523	2 4219 9	9 19 1	0 37	2 8
2 19 18	4 43	97494	2 4228 8	8 18 23	0 38	2 7
2 14 33	4 45	97466	2 4236 7	9 17 45	0 38	2 6
2 9 45	4 48	97439	2 4245 6	8 17 7	0 38	2 5
2 4 55	4 50	97413	2 4253 5	8 16 28	0 39	2 4
2 0 2	4 53	97388	2 4261 4	7 15 49	0 39	2 3
1 55 7	4 55	97364	2 4268 3	7 15 10	0 39	2 2
1 50 9	4 58	97341	2 4275 1	7 14 31	0 39	2 1
1 45 8	5 1	97320	2 4282 1	6 13 51	0 40	2 0
1 40 5	5 3	97299	2 4288 0	6 13 11	0 40	1 9
1 35 1	5 4	97279	1 4294 9	6 12 31	0 40	1 8
1 29 55	5 6	97260	1 4300 8	6 11 51	0 40	1 7
1 24 47	5 8	97242	1 4306 7	5 11 10	0 41	1 6

Subtr. Prosth. Epicycl. (G.M.S.)	Diff. (M.S.)	Elongat. a centro (Partic.)	D i ff f . Eccentricita s (Partic.)	D i ff f . Adde variatio (M.S.)	Diff. (M.S.)	A r g .
1 19 38	5 9	97225	1 4311 6	5 10 29	0 41	1 5
1 14 27	5 11	97209	1 4316 5	4 9 48	0 41	1 4
1 9 14	5 13	97194	1 4320 4	4 9 7	0 41	1 3
1 3 59	5 15	97180	1 4324 3	4 8 26	0 41	1 2
0 58 43	5 16	97167	1 4328 2	4 7 44	0 42	1 1
0 53 27	5 16	97155	1 4332 1	3 7 2	0 42	1 0
0 48 10	5 17	97144	1 4335 0	3 6 20	0 42	9
0 42 52	5 18	97134	8 4338	2 5 38	0 42	8
0 37 33	5 19	97126	6 4340	2 4 56	0 42	7
0 32 13	5 20	97120	5 4342	2 4 14	0 42	6
0 26 52	5 21	97115	4 4344	1 3 32	0 42	5
0 21 30	5 22	97111	3 4345	1 2 50	0 42	4
0 16 8	5 22	97108	3 4346	1 2 8	0 42	3
0 10 46	5 22	97105	3 4347	1 1 26	0 42	2
0 5 23	5 23	97102	2 4348	0 0 43	0 43	1
0 0 0	5 23	97100	4348	0 0	0 43	0

6 Sig. [foot label — the table is shared between sign 5 (head) and sign 6 (foot)]

On computing the Moon's true longitude from the foregoing tables

[De verae longitudinis Lunae ex tabulis praemissis supputatione]

I.

[Margin: In the commentary on this chapter, sect. 3.]

First of all, for the given moment of time — adjusted according to the method I set out above — determine both the mean longitude of the Sun and its true longitude reckoned from the mean equinox. Then, from the tables of the Moon's mean motions, extract the values that correspond to that same adjusted time, drawing them out in the usual way: namely, the Moon's mean longitude from the Sun, the Moon's anomaly, and the motion of latitude, if that should be needed. Finally, add the Sun's mean longitude to the Moon's mean motion from the Sun, so as to establish the Moon's longitude from the mean vernal equinox.

II.

Second, with the Moon's anomaly enter the table of prosthaphaereses, and take out the epicyclic prosthaphaeresis together with the elongation from the centre (applying the correction by the proportional part, if minutes of arc are attached to the degrees of anomaly). Then add that prosthaphaeresis — or subtract it, as the headings of the columns require — both to the Moon's longitude from the equinox and to the anomaly, so that both come out adjusted.

III.

Third, subtract the Sun's true place from the adjusted longitude of the Moon, and with the doubled remainder look up, from the table of equations, the eccentricity together with the variation (not neglecting, however, the due proportional part, should it be needed); these are to be set aside separately.

IV.

Fourth, if that double distance of the luminaries — the one with which you entered the table — is less than a semicircle, halve its complement to the semicircle; or if it is greater than a semicircle, likewise halve the excess above the semicircle. This will yield the angle of the second equation of the anomaly. Before

the semicircle of the double distance of the luminaries, this angle is to be added to the adjusted anomaly; after the semicircle, it is to be subtracted from it, so that the anomaly comes out adjusted a second time. And if this second-adjusted anomaly is less than a semicircle, bisect it; or if it is greater, bisect the complement of it to the whole circle. Then from the table of tangents look up the tangent of this bisected angle, which will be the third quantity found [the third *inventum*].

V.

Fifth, add together the Moon's elongation from the centre and the eccentricity, and halve the product: you will have the first quantity found [the first *inventum*]. From this, subtract the eccentricity, and there remains the second quantity found [the second *inventum*]. Multiply this by the tangent of the bisected arc — that is, by the third quantity found — and divide the resulting number by the first quantity found. (Or, more concisely, obtain it through the fourth term of the prosthaphaeresis of triangles.) The quotient will be a tangent, and the angle corresponding to it, when subtracted from the bisected angle already mentioned, leaves the prosthaphaeresis of the eccentricity. This is to be subtracted before the semicircle of the last-adjusted anomaly, and added to it after the semicircle.

This, however, must be carefully noted: if the angle of the second equation is subtractive and greater than the anomaly itself, then the anomaly is to be subtracted from it, and the tangent of the bisected remainder is to be used as before; and the angle of the prosthaphaeresis of the eccentricity will then be additive. This prosthaphaeresis, and the variation that you set aside, are — each according to its own sign — to be added to, or subtracted from, the Moon's adjusted longitude from the equinox; and so you will obtain the Moon's true place from the mean equinox. When to this you have applied the inequality of the equinox, to be obtained as shown above, you will have the Moon's true place in its own orbit from the true vernal equinox.

[Margin: Cap. 4.]

FIRST EXAMPLE, ILLUSTRATING THE THEORY OF THE MOON ITSELF, BY MEANS OF THE DOCTRINE OF TRIANGLES

[Exemplum primum super ipsa Theoria lunae, mediante triangulorum doctrina, explicatum]

In the year of Christ 1587, on the 17th day of August, at 19 hours 25 minutes after midday [p.m.], the true place of the Moon's centre was observed at Uraniborg

with the Tychonic instruments at Gemini [Π] $26^{\circ} 21\frac{1}{2}'$ — taking account here, of course, of our correction in the fixed stars. The equation of the natural day, with regard also taken to the meridian of Copenhagen [Hafnia], adds 7 minutes. Therefore, with the completed time — namely, the years 1586, the month of July, 16 days, 19 hours, 32 minutes — I entered the appropriate tables of the mean motions, and I transcribe them in order as set out below.

	Sig.	Gr.	Mi.	Sec.
Anomaly of the equinoxes [Anomalia aequinoctiorum]	6	15	17	0
Whence its equation, additive [eius aequatio adjectiva]			7	13
Longitude of the Sun from the mean equinox [Longitudo Solis a medio aequinoct.]	5	5	42	16
Anomaly of the Sun [Anomalia Solis]	2	0	20	53
Longitude of the Moon from the Sun [Longitudo Lunae a Sole]	9	25	12	6
Anomaly of the Moon [Anomalia Lunae]	1	15	37	21
To the Sun's anomaly there corresponds a subtractive prosthaph. [Profthaph. ablat.]	1	44	46	
And therefore the Sun's true place from the mean equinox [verus locus Solis a medio aequin.]	5	3	57	30

Now let me proceed, in order, with the lunar prosthaphaereses, by means of the following hypothesis [diagram].

The Moon's primary and genuine theory, fitted to the time and to the foregoing mean motions.

[Margin: See chapter 2, sub-section below. (*Vide cap. 2, sub finem.*)]

Diagram. A geometric construction illustrating the Moon's two-epicycle theory. A large eccentric circle is drawn; within it, toward the upper left, sits the double-epicycle apparatus (the small circles around D, K, E, H, G, L, N), and toward the lower centre a smaller circle (the *circellus*) carrying points B, F, A, Q, R, I, used for the libration. The Moon's place E is found in the second epicycle. The labelled points, read from the figure:

- N (top, upper epicycle): apex of the upper small circle
- K (upper left of the epicycle group): on the first epicycle

- E (just right of K): the Moon's place in the second epicycle
- G (left): on the orbit / first epicycle circle
- H (centre of the epicycle group): vertex point of the construction
- D (below H): centre of the first epicycle (and a triangle vertex, D K E)
- L (lower left): point on the construction
- R, I, T (upper right region, along the apsidal line): R and I lettered points on the dotted apsidal/diameter line; T at top right
- F (centre, on the small circle): point on the *circellus* B F C marking the motion of the orbit-centre
- A (centre of the small circle): centre of the *circellus*
- Q (left of A on the small circle): lettered point
- B (foot of the small circle): the Earth's position [a tellure B]
- C (centre of the Moon's orbit / eccentric): the common centre referenced as C E, C D, etc.

The geometric quantities developed in the prose: D K = semidiameter of the first epicycle = 5800; K E = semidiameter of the second epicycle = 2900 (where the radius of the Moon's orbit C D = 100000); the radius of the eccentric D C = 100000. The angle D K E is the doubled anomaly.

Now begins the prose of the worked example accompanying the figure:

Let the mean anomaly of the Moon in the adjacent drawing be I D — sign 1, $15^{\circ} 37' 21''$ — equal (as that which, in the first epicycle reckoned from the apogee, was said above in the explanation of the hypothesis) to G K, up to the centre of the second epicycle, K. Let the same anomaly, doubled — sign 3, $1^{\circ} 14' 42''$ — be counted from H up to E, the Moon's place in the second epicycle, and let the lines E D and K E be drawn.

Now in the plane triangle, two sides are given: D K, the semidiameter of the first epicycle, 5800, and K E, the semidiameter of the second epicycle, 2900 (in units of which the radius of the Moon's orbit C D is 100000), together with the included angle D K E, which is the doubled anomaly. Therefore, by what has been demonstrated concerning triangles, there come out: the angle K D E, $26^{\circ} 18' 43''$, and the side D E, 6541. Then, in the second triangle D C E, the side D E just found, 6541, is given, and D C, the radius of the eccentric, 100000, together with

— together with the included angle C D E, $108^{\circ} 3' 56''$ — that is, the complement of the simple anomaly to the angle K D E. By calculation, then, there comes to light

the angle D C E, the epicyclic prosthaphaeresis, $3^{\circ} 29' 16''$, together with the side, the Moon's elongation from the centre, C E, 102217.

Furthermore, let the Sun's mean motion be added to the Moon's mean longitude from the Sun, so that the Moon's simple longitude from the mean vernal equinox is obtained, sign 3, $0^{\circ} 54' 22''$. From this, subtract the epicyclic prosthaphaeresis, $3^{\circ} 29' 16''$, and there remains the Moon's adjusted longitude, sign 2, $27^{\circ} 25' 6''$. Let the same prosthaphaeresis be taken from the anomaly, so that it too comes out adjusted, sign 1, $12^{\circ} 8' 5''$ — which is contained by the angle I C E.

Next, from the Moon's adjusted longitude subtract the Sun's true place from the mean equinox, sign 5, $3^{\circ} 57\frac{1}{2}'$, and there remains the distance of the luminaries, sign 9, $23^{\circ} 27' 36''$. The double of this, sign 7, $16^{\circ} 55' 12''$, measures in the small circle B F C the motion of the centre of the Moon's orbit C away from the Earth B, in the direction of the signs, along F C — just as that too was set out above.

Now, so that in the same small circle the line C B can be obtained, let the semidiameter A C be drawn. And since in the isosceles triangle the sides A B and A C are each given as 2174, together with the angle C A B, $133^{\circ} 4' 48''$, which is the complement of the double of the Moon's longitude from the Sun to the whole circle, the said line B C is thereby given as 3988. And since the angle F B C is half of F A C — by Elements III, proposition 20 [*per 20 p. 3 element.*] — while this excess of the double distance of the luminaries beyond 6 signs is $46^{\circ} 55' 12''$, the half of it, namely $23^{\circ} 27' 36''$, is therefore the measure of the said angle F B C, to which the angle I C R is also equal. By the subtraction of this angle, in the present example, the Moon's anomaly is adjusted a second time, so that it becomes $18^{\circ} 40' 29''$, which the angle R C E measures.

And so, in the last triangle E C B, from two sides likewise given — C E, 102217, and B C, 3988 — with the angle contained by them, E C B, $161^{\circ} 19' 31''$ (that is, the complement of the last-adjusted anomaly of the Moon to the semicircle), there is found the angle B E C, $0^{\circ} 41' 25''$. When this is added to the prior epicyclic prosthaphaeresis, there results $4^{\circ} 16' 41''$, subtractive.

There remains, in the last place, that libration of the Moon, to be unravelled according to what was demonstrated above. The double longitude of the luminaries is sign 7, $16^{\circ} 55' 12''$; accordingly, the right sine of the excess beyond the semicircle — namely of the arc E C — in the adjacent small circle is 7304007, which is represented by the straight line D C. Therefore, as the radius F A, 10000000, is to $40' 30''$, the maximum prosthaphaeresis of this [libration], so is C D

to the variation at present, 29' 35". Since this is of the same denomination as the remaining prosthaphaeresis — namely, subtractive — it also

it too is added to that same quantity. From the first, second, and now this third, there is aggregated the final prosthaphaeresis of the Moon, found to be $4^{\circ} 40' 16''$. When this is subtracted from the Moon's simple longitude reckoned from the mean equinox, collected above as $3s\ 0^{\circ} 54' 22''$, there remain $2s\ 26^{\circ} 14' 6''$. To these, once the equation of the equinox set out above — namely $7' 13''$ — has been added, the Moon's true longitude sought for the given time is here determined, measured from the true vernal equinox or the beginning of Aries, as $26^{\circ} 21\frac{1}{2}'$ of Gemini. This falls only very slightly short of the observed value, even when account is taken here of the Moon's own proper orbit.

Diagram. A circle with two perpendicular diameters meeting at center A. The vertical diameter runs from F (top) through D and A down to K (bottom); the horizontal diameter runs from E and B (left) through A to H (right). In the upper-left quadrant a small rectangle is drawn with corners at B, C (upper-left), D (upper-right, on the vertical diameter) and A — illustrating the libration/variation construction (the geometry behind the third prosthaphaeresis, the variation, derived above).

- F (top of vertical diameter): apex
- C (upper-left corner of inner rectangle): label C
- D (upper-right corner of rectangle, on vertical diameter): label D
- E (far left of horizontal diameter): label E
- B (left, inner): label B
- A (center): label A
- H (right end of horizontal diameter): label H
- K (bottom of vertical diameter): label K

We shall now adapt this same example to be worked out by the table of lunar prosthaphaereses, following the order of the precepts set out above.

I.

Let there be drawn from the synopsis of mean motions expounded above, in the order required, the relevant quantities. For while the rest remain the same, in this comparison the absolute prosthaphaeresis of the Moon alone will suffice; and from this the use of the table is shown, and the agreement of its result with the hypothesis becomes more evident.

II.

	Sig.	Gr.	Mi.	Sec.
For the simple anomaly of the Moon	1	15	37	21
There corresponds, from the table of prosthaphaereses,		3	29	16

— to be subtracted both from the Moon's longitude from the equinox and from the anomaly.

III.

	Sig.	Gr.	Mi.	Sec.
Simple longitude of the Moon from the Sun	9	25	12	6
Simple longitude of the Sun	5	5	42	16
Therefore simple longitude of the Moon from the equinox	3	0	54	22
Longitude of the Moon from the equinox, adjusted	2	27	25	6
Likewise the anomaly of the Moon, adjusted	1	12	8	5
Elongation of the Moon from the center			102217	

IV.

	Sig.	Gr.	Mi.	Sec.
Excess of the double longitude of the Moon from the Sun beyond a semicircle		46	55	12
Half of this, which is the final equation of the anomaly		23	27	36
Therefore the anomaly of the Moon last adjusted	0	18	40	29
Half of this		9	20	15

Its tangent, 1644280, is the third item found.

V.

Elongation of the Moon from the center	102217
Measure of the eccentricity	3988
Sum	106205

The half, which is the first item found (I.), 53102.

The difference between the lesser side and the first item found, which is the second item found (II.), 49114.

Rule.

I.	III.	II.
53102 —————	1644280 —————	49114
	T. 1520793	Gr. Mi. Sec.
There corresponds the arc		8 38 50
Half of the adjusted anomaly		9 20 15
The difference, which is the second prosthaphaeresis, subtractive	0	[?] 41 25
The third prosthaphaeresis is the variation of the Moon		29 35

Since, then, all the prosthaphaereses of the Moon were here subtractive, in this example they are rightly gathered into a single sum, as follows:

		Gr.	Mi.	Sec.
prosthaph.	Epicycle	3	29	16
	second	0	41	25
	variation	0	29	35
Sum		4	40	16

Now this prosthaphaeresis, since it arose just above plainly from the resolution of the lunar hypothesis into triangles, makes it clear that there is no difference between the triangular computation of the Moon's place and the computation by the table of prosthaphaereses — which indeed was what we set out to demonstrate in this example.

Example II

This is added so that the truth of our present lunar theory may be more fully demonstrated against the celestial phenomena themselves.

Among the observations of the Moon that I made here at Copenhagen [Hafniae] in these last years — both with adequate instruments and by frequent attention

to its conjunctions with the more prominent fixed stars in the zodiac, from which I have found that the foregoing hypothesis, once established upon Tycho's observations, holds most exactly — I shall bring forward here as an example the notable conjunction of the upper horn of the Moon with Aldebaran observed visibly among us in the year of Christ 1608, on February 12, at 8 hours 43 minutes in the evening, when the Moon was near quadrature and at its greatest departure from its mean course; from which both of its prosthaphaereses can be examined to the best advantage. This conjunction, however, was seen three time-minutes later at Wittenberg, namely at 8 hours 46 minutes, as the most distinguished and most sorely missed mathematician Dr. Melchior Jöstel [Melchior Jöstelius], of blessed memory, afterward wrote to me. From this it is surely clear that the time we have assigned to this observation, or visible conjunction, is most correct, since the difference of meridians, already known in advance from eclipses, nearly recurs here. And because the Sun at this time was by its true motion in $3^{\circ} \frac{2}{5}$ of Pisces, the equation of the natural day to be added will be $7\frac{1}{3}'$. [Margin: See Cap. 3.] The mean time, therefore, was 8 hours $50\frac{1}{3}$ minutes on February 12 of the Epoch-year of the Savior currently running, 1608, or 5572 of the world-bissextile; and to this time these motions from the preceding tables correspond.

	Sig.	Gr.	Mi.	Sec.
Mean precession of the vernal equinox	2	17	3	20
Anomaly of the equinox	6	17	19	17
Therefore the inequality of the precession of the equinox			8	9 add.
Whence the true precession of the equinox	2	17	11	29

[Margin: See Cap. 4.]

	Sig.	Gr.	Mi.
Likewise Aldebaran, true { Longitude	II (Gemini)	4	$16\frac{1}{3}$
{ Latitude	♏ (Scorpio)	5	31

This same is the apparent longitude, and the latitude, of the upper horn [of the Moon].

	Sig.	Gr.	Mi.	Sec.
Furthermore, the simple longitude of the Sun	11	1	50	42
Prosthaphaeresis of the Sun		1	43	58 add.

	Sig.	Gr.	Mi.	Sec.
Simple longitude of the Moon from the Sun	2	25	19	38
Anomaly of the Moon	8	18	8	20
Therefore the epicyclic prosthaphaeresis of the Moon		4	53	43 add.
Elongation from the center		99768 p.		

[Margin: Cap. 5.]

	Sig.	Gr.	Mi.	Sec.
Double true longitude of the Moon from the Sun	5	26	58	46
To which corresponds the measure of the eccentricity			4346 p.	

	Sig.	Gr.	Mi.	Sec.
Variation of the center of the Moon			2	9 add.
Anomaly of the Moon first adjusted	8	23	2	3
The angle of the complement of the ecliptic longitude of the Moon from the Sun, contended for, which is the equation of the second anomaly		1	30	37 add.
Therefore the anomaly of the Moon last adjusted	8	24	32	40
The complement of the Moon's anomaly to the whole bisected circle		47	43	40
To which there corresponds, as tangent, the second item found (III.)		11000525		

First item found (I.)	52057

Second item found (II.)	47711

Rule.

I.	II.	III.
52057 —————	11000525 —————	47711

	T. 19082140	Gr.	Mi.	Sec.
There corresponds the arc		45	14	4
The difference from the upper bisection, which is the second angle of the prosthaphaeresis	2		29	36 add.

	Sig.	Gr.	Mi .	Sec.
Whence into a single sum are gathered all the prosthaphaereses of the Moon — being additive — and the aggregate is		7	25	28 add.
Distance of the simple Moon from the mean equinox	1	27	10	20
Prosthaphaeresis of the equinox, additive			8	9
Prosthaphaereses of the Moon gathered		7	25	28 add.
Therefore the true place of the Moon	II (Gemini)	4	44	nearly

Now, since the apparent longitude of the Moon, taken from the place of Aldebaran, comes out at $4^{\circ} 16\frac{1}{3}'$ of Gemini, there appears a parallax of its longitude, at an altitude of about 39° , which around the said conjunction was found to have been $27\frac{1}{3}'$. And since account is taken of the reduction of the Moon to the ecliptic — by which we measure a star's place — there must still be added to the longitude of the Moon found on its own proper orbit $2' 38''$, so that the parallax of longitude grows beyond half a degree, that is, beyond $30'$. That this agrees well enough with our supposition can be shown below, where the parallaxes of the Moon are to be treated. [Margin: Cap. 8 of this book.]

Likewise, that the apparent latitude of the Moon's center was about $5^{\circ} 47'$ north — this too agrees, by necessarily assuming not only its distance from the Earth but also the increase of its latitude around quadrature.

And within these limits the difference between the true and the mean longitude of the Moon is everywhere contained; and to track this difference still more easily, by a table to be derived from the following hypothesis, we shall now strive with all our strength, for the benefit of lovers of the stars.

On the second hypothesis of the Moon, afterward devised by us — which displays the equivalence of the first, but, resolved into tables of numbers, presents an easier practice for the computation of longitude

Up to this point we have produced the Moon's genuine theory from its own causes, and have confirmed it against the celestial phenomena by at least two examples here — though it had already been founded earlier upon many and indeed sufficient observations, covering every kind of variation. So no one can doubt that, if one follows the prescribed tracks, it will represent the Moon's longitude for any time precisely to the heavens in numbers. But since the complaints of certain people have struck our ears

again and again — concerning the ψηφοφορίας [Greek: ψηφοφορία (psēphophoria), "reckoning by counters," i.e. numerical calculation], that is, the lengthy difficulty of calculation from the elaborate table above, and also on account of the analysis of the one triangle necessarily annexed to the precepts there. Now although practiced astronomers dispatch this with little trouble — indeed, those who know the doctrine of triangles perfectly, as is fitting, even take pleasure now and then in the compendious resolution of the whole hypothesis — yet beginners perhaps, and others for whom astronomical tables alone are thought sufficient for the investigation of a phenomenon, since they have judged otherwise: we too, being equally ready to serve all by our labors and to point out in advance the most compendious road we know, shall therefore give here another hypothesis of the Moon, equivalent to the first. From it, accordingly, we have constructed a complete table of prosthaphaereses, designed to dispatch the apparent longitude of the Moon with all the ease that can be achieved. [Margin: Cap. 2 of this book.] Now the syntax of this arises from the equivalences of the hypotheses shown above in general, so long as we use the same eccentric-with-epicycle [eccentrepicyclus] in it. And for lovers of the stars who are moderately versed in the science of the heavens, we shall think it enough here if such a secondary hypothesis of the Moon of ours, accommodated to the first after the example of the foregoing, both proves — in one and the same operation, while it

is being explained — the equivalence I have spoken of, and also shows the structure of the following table from the geometrical source of triangles.

The Secondary Theory of the Moon

[Margin: See Cap. 2 of this book.]

On the straight line F M let two points A and B be conceived; and with B placed as the center of the Earth, let a small circle K L C be drawn about A as center. Then, with C placed as center according to the requirement of the prior example set out above, let the primary eccentric orbit of the Moon F D M itself be described. And when I C is drawn parallel to F A, it is clear that the mean apogee of the eccentric orbit is understood at I. Again, with D made the center, let the equant, that is the epicycle, H E N be added, in which the Moon itself is now understood to be carried at E. For by the law of the revolutions, when D is joined with I, the apogee position, the Moon dwells at H, the lowest part of the epicycle. And because by an equal change H and E come to be with I and D, therefore the angles I C D and H D E are always supposed equal; of which the former, while it measures the simple anomaly of the Moon from the apogee I, the latter drives the same measure of the Moon's revolution onward in the order of the signs through the epicycle, the start being taken from the perigee H. Furthermore, with A B the mean eccentricity, since the center of the Moon's orbit is understood to move likewise in the order of the signs along the small circle K L C, in step with the double distance of the Moon from the Sun, therefore (as in the direct syzygies of the luminaries, that is, in true conjunction and opposition) the same center of the Moon's orbit lies very near to B, the center of the Earth, at K; but in the quadratures it is removed from the same

Diagram. A large geometric construction for the secondary lunar theory, occupying the lower portion of the page. A large circle (the primary eccentric orbit of the Moon) is intersected by a long straight vertical line F M running from F at the top down through the figure to M at the bottom. Near the top, points G, I, and F lie in a row (G to the left, then I, then F), with I and F at the heads of two nearly-vertical lines. In the upper-left region a small circle (the epicycle H E N) carries the labels N (upper-left), D (center of the epicycle), E (right of D), and H (just below D-E); dotted radial lines run from this epicycle down toward the lower-right. In the lower-right region a second small circle (the eccentric's variation circle K L C about center A) carries the labels L (upper), C (left, on the eccentric orbit), A (center), and K (lower). Below it lies B (the center of the Earth), and at the very bottom of the vertical line, M.

- F (top, on vertical diameter F M): apex of the line F M
- I (top, near F): mean apogee of the eccentric orbit (where I C is drawn parallel to F A)
- G (top, left of I): label G
- N (upper-left, on epicycle): label N
- D (center of upper-left epicycle): center of the epicycle H E N
- E (upper-left epicycle, right of D): position of the Moon on the epicycle
- H (upper-left epicycle, below D): perigee (lowest part) of the epicycle
- L (lower-right, on small circle): position of the orbit-center at quadrature
- C (lower-right, left of A, on eccentric orbit): center used to describe the eccentric orbit F D M
- A (center of lower-right small circle K L C): label A
- K (lower-right, below A): position of the orbit-center at syzygy (conjunction/opposition)
- B (below the lower-right circle, on vertical line): center of the Earth
- M (bottom of vertical line F M): foot of the line F M

the farthest from that same [center]. Therefore, the line drawn from B through C to G gives, in the present example, the true eccentricity B C; and G is likewise the true place of the apogee, while the angle A B C — that is, the arc I G — is the subtractive prosthaphaeresis of the anomaly. When, then, the Moon is configured to the true conjunction or opposition of the Sun, a single prosthaphaeresis gathered from one of the two angles suffices for its appearances — namely the angle E K D (to which E C D is always equal) and K E B. Such a combined prosthaphaeresis is the one that the first column of the prosthaphaeresis table presents, as will be said more fully later.

But outside the aforementioned syzygies of the luminaries, since the eccentricity B K keeps increasing right up to the quadratures — that is, until the center K of the Moon's orbit has reached L — the change in the Moon's inequality likewise grows correspondingly greater. This increase of it must be assigned, near the apogee and perigee, to the change in the Moon's distance from the Earth, and then gradually, near the sides, to the angle of greater digression — just as all these things will be able to be shown to the eye in the present hypothesis. That this hypothesis does not differ in the least from the preceding one, so far as the outcome of the calculation is concerned, the outcome alone in the proposed example will presently show, without those demonstrations which I do not consider it necessary to set out at length here.

Finally, there remains that small anomaly belonging to the Moon, which insinuates itself in its ordinary way on both sides of the cardinal syzygies, and which becomes greatest as additive — $40\frac{1}{2}$ min. — when the Moon is set midway on its course in the quadratures, and just as much subtractive in its reception midway from the quadrature. Since this, together with its cause, has been explained above, it will suffice merely to insert it into the following prosthaphaeresis table, from which it too can be drawn for its use.

And this has been the brief sketch of our secondary hypothesis, agreeing in every way with the phenomena of the Moon. Now we will examine that same hypothesis by illustrating it geometrically through an example, and afterwards we shall set out the structure of the following prosthaphaeresis table from the same source, together with the precepts and its use.

An example fitted to the secondary hypothesis of the Moon.

In the year of Christ 1587, on the 17th day of August, at 19 h. 15 m. p.m., the true place of the center of the Moon was observed at [Uraniborg] with the Tychonic instruments, as we set down before, in 26 gr. $21\frac{1}{2}$ min. of Gemini. For, at the time equated as before, the mean longitude of the Sun from the mean vernal equinox is found to be 5 sign 5 gr. 42 min. 16 sec., while the anomaly of the Sun is 2 sign 0 gr. 20 min. 53 sec.; and therefore the subtractive prosthaphaeresis of this is 1 gr. 44 min. 46 sec. Further, the simple longitude of the Moon from the Sun is 9 sign 25 gr. 12 min. 6 sec., with anomaly 1 sign 15 gr. 37 min. 21 sec. This anomaly, in the lunar diagram immediately preceding, measures the angle I C D — that is, the arc I D — namely the simple distance or removal of the Moon from the mean apogee I on its orbit. And since the angle H D E is by hypothesis equal to this, and the radius C D of the Moon's orbit is 100000, of which D E is likewise 2900, in the triangle C D E two sides about the given angle D are given — namely 45 gr. 37 min. 21 sec. From this, by what we prescribed in their proper place in the doctrine of triangles, the angle E C D first emerges as 1 gr. 12 min. 44 sec., and also the side C E as 97994, of which C D the radius, as I said, was 100000.

But when, in a true conjunction or opposition of the luminaries — when the center of the Moon's orbit is at K — the radius of the Moon's orbit is likewise shown by the side K D, as it is now by C D, then, because of these two equal sides, all the rest will be equal: that is, the angle E K D equal to the angle E C D, and the side K E equal to the side C E. Therefore now, in the triangle K E B, with K E 97994 known, likewise K B 5800 (the least eccentricity of the Moon, which befits the conjunction and opposition of the luminaries, as was explained above), and furthermore the comprehended angle E K B 135 gr. 35 min. 23 sec. (which is the

complement of the angle I C E to the semicircle, that is, the equated anomaly of the Moon) —

— the angle F K E being subtracted from that simple supposed one, the angle K E B is thereby brought out into the open as 2 gr. 16 m. 32 s. And so this, together with the earlier-found E K D, composes the ecliptic prosthaphaeresis (which we call it below, because it befits the conjunction and opposition of the luminaries alone, as the difference between the mean and true longitude of the Moon, and therefore befits the eclipses too): 3 gr. 29 m. 16 sec., exactly as it was found in the prior hypothesis of the Moon, agreeing with this same anomaly of the Moon.

Now, this prosthaphaeresis of the Moon being compared with that of the Sun found above, and the smaller subtracted from the greater — since both are subtractive — there remains a residue of 1 gr. 44 m. 30 sec. subtractive. When, therefore, this is taken away from the longitude of the Moon set down above, the remainder is the true ecliptic longitude of the Moon from the Sun: 9 sign 23 gr. 27 m. 36 sec. Doubled, this is found to be 7 sign 16 g. 55 m. 12 s., whose complement to the whole circle is the angle C A K, 133 gr. 4 m. 48 sec. This angle, together with the comprehended sides B A 7974 and A C 2174 (always granted in these parts in which the radius of the orbit C D is 100000 p., as was said above) — since it becomes known from the revolution of the double distance of the Moon from the Sun — gives, in this example too, the angle A B C, 9 gr. 31 min. 49 sec., the subtractive prosthaphaeresis of the anomaly, since the angle I C G — that is, the arc I G — is equal to it. From this the said anomaly is found, finally equated, as 34 gr. 53 m. 43 sec., and the side B C, the true eccentricity, as 9590½. Lastly, in the triangle E C B, with the two sides E C 97994 and B C 9590½ given, likewise the comprehended angle E C B 145 gr. 7 min. 12 sec. (which, as appears, is the complement of the equated anomaly — that is, of the angle G C E — to the semicircle), there is also given the angle C E B, 2 g. 57 m. 57 s.

Diagram. A large circle carrying the geometric construction for the secondary lunar hypothesis. At the top, near the circumference, the labels G, I, and F are set close together; at the upper left lies a small orbit-circle (the Moon's orbit/epicycle) bearing the labels N, D, E, and H. Near the center are A, with K just below it, and a small inner construction at the figure's center. B lies at the bottom of the vertical diameter, and M is at the foot of the figure (the bottom point of the great circle). Dotted construction lines run from B and from the central points out to the orbit-circle at the upper left.

- G (top, near circumference): apogee-related point
- I (top center): mean apogee

- F (top, right of I): apogee/orbit point
- N (upper left, on small circle): orbit point
- D (upper left, center of small orbit): center of the Moon's orbit
- E (upper left, on small circle): place of the Moon on its orbit
- H (upper left, on small circle): orbit point
- A (center): center of the deferent construction
- K (just below A): center of the Moon's orbit at syzygy
- B (lower center, on the diameter): center of the eccentric / Earth-related center
- M (bottom of circle): perigee-related point

— which, added to the angle E C D found at the beginning, 1 g. 12 m. 44 s., constitutes the prosthaphaeresis of the Moon, 4 g. 10 m. 41 sec., subtractive. To this the variation of the Moon adds (as above) 29 m. 36 s. The whole equation of the Moon, then — that is, the difference of the true longitude from the mean — is 4 gr. 40 min. 17 sec., to be subtracted.

But so that the mean longitude may be obtained from the true equinox, let the simple longitude of the Moon from the Sun be added, namely:

	Sign	Gr.	Mi.	Sec.
simple longitude of the Moon from the Sun, namely	9	25	12	6
first, the simple longitude of the Sun from the mean equinox	5	5	42	16
Then the equinoctial equation			7	13
And the mean longitude of the Moon from the true vernal equinox is composed as	3	1	1	35
Finally, let the whole prosthaphaeresis be subtracted		4	40	17
And there remains the longitude of the Moon in its proper orbit, Gemini	II	26	21	19

— as, exquisitely enough, from the theory above. Lastly, the side B E, the distance of the Moon from the Earth, if it be required, is easily brought out into the open from that same last triangle E C B. And from this one may recognize that a pure equivalence with the prior hypothesis is found in this secondary hypothesis of ours for the Moon, which we have also proved true by many other examples.

There now follows the complete prosthaphaeresis table, with its explanation and use.

Second Table of Lunar Prosthaphaereses.

0 Sign

Column structure (left to right): Degrees of anomaly | Subtractive Ecciptic Prosthaphaeresis (G. M. S.) | Additive Difference (M. S.) | Subtractive Prosthaphaeresis of the Equant (G. M. S.) | Additive Difference (M. S.) | Subtractive Excess (G. M. S.) | Additive Difference (M. S.) | Additive Equation of the Anomaly (G. M.) | Proportional Minutes [Scrup. Prop.] (M. S.) | Additive Variation (M. S.) | Degrees of anomaly.

Gra.ano		Ecl.Pros. G		M		S		Diff M		S		Equant G		M		S		Diff M		S		Excess	
G	M	S	Diff M	S	Æq.Anom	G	M	Prop. M	S	Variat. M	S	Gra.ano											
	---		---		---		---		---		---		---		---		---		---		---		0
0	0	0		0	0	0		0	0	0		0	0	0	0	0	0	30		1	0	5	4
5	4	0	1	48	1	48	0	2	13	2	13	0	22	0	0	0	0	43	29		2	0	
10	8	5	4	0	3	36	1	48	0	4	26	2	13	0	45	0	1	1	26	28			
	3	0	15	12	5	4	0	5	23	1	47	0	6	38	2	12	1	7	0	3	2		
8	27		4	0	20	16	5	4	0	7	10	1	47	0	8	50	2	12	1	29	0		
	6	2	50	26		5	0	25	20	5	4	0	8	57	1	47	0	11	1	2	11		
1	52	0	9	3	32	25		6	0	30	23	5	3	0	10	44	1	47	0	13			
11	2	10	2	14	0	13	4	14	24		7	0	35	26	5	3	0	12	31	1			
47	0	15	21	2	10	2	36	0	18	4	56	23		8	0	40	28	5	2	0			
14	17	1	46	0	17	31	2	10	2	59	0	24	5	38	22		9	0	45				
29	5	1	0	16	3	1	46	0	19	41	2	10	3	21	0	31	6	20	21				
10	0	50	30	5	1	0	17	49	1	46	0	21	51	2	10	3	43	0	38				
7	2	20		11	0	55	30	5	0	0	19	34	1	45	0	24	2	2	11	4			
4	0	46	7	44	19		12	1	0	28	4	58	0	21	19	1	45	0	26	12			
	2	10	4	25	0	54	8	26	18		13	1	5	25	4	57	0	23	4	1	45		
	0	28	22	2	10	4	46	1	3	9	7	17		14	1	10	21	4	56	0	24		
	48	1	44	0	30	32	2	10	5	7	1	13	9	48	16		15	1	15	16			
4	55	0	26	32	1	44	0	32	43	2	11	5	28	1	24	10	29	15					
16	1	20	10	4	54	0	28	15	1	43	0	34	54	2	11	5	49	1	36				
	11	10	14		17	1	25	3	4	53	0	29	58	1	43	0	37	6	2	12			
6	9	1	49	11	51	13		18	1	29	55	4	52	0	31	40	1	42	0				
39	17	2	11	6	29	2	2	12	31	12		19	1	34	44	4	49	0	33				
22	1	42	0	41	28	2	11	6	49	2	16	13	11	11		20	1	39	32				

4	48	0	35	3	1	41	0	43	39	2	11	7	9	2	30	13	51	10		21	
	1	44	18	4	46	0	36	44	1	41	0	45	50	2	11	7	28	2	44		
14	31	9		22	1	49	3	4	45	0	38	24	1	40	0	48	0	2	10	7	
47	2	59	15	10	8		23	1	53	47	4	44	0	40	4	1	40	0	50		
10	2	10	8	6	3	14	15	49	7		24	1	58	29	4	42	0	41	43	1	
	39	0	52	18	2	8	8	24	3	29	16	28	6		25	2	3	8	4	39	0
	43	21	1	38	0	54	26	2	8	8	42	3	45	17	7	5		26	2	7	44
	4	36	0	44	57	1	36	0	56	34	2	8	9	0	4	2	17	45	4		27
	2	12	18	4	34	0	46	32	1	35	0	58	41	2	7	9	18	4	19	18	
	23	3		28	2	16	50	4	32	0	48	5	1	33	1	0	48	2	7	9	35
	4	37	19	1	2		29	2	21	20	4	30	0	49	37	1	32	1	2	54	2
	6	9	52	4	55	19	38	1		30	2	25	47	4	27	0	51	8	1	31	1
	5	0	2	6	10	8	5	14	20	15	0										

Column-foot direction labels (left to right): Adde | Sub. | Adde | Sub. | Adde | Sub. | Sub. | — | Subt.

11 Sign

Second Table of Lunar Prosthaphaereses.

1 Sign

	Gra.ano	Ecl.Pros.	G	M	S	Diff M	S	Equant G	M	S	Diff M	S	Excess G	M	S	Diff M	S	Æq.Anom G	M	Prop. M	S	Variat. M	S	Gra.ano	
	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	0
2	25	47			0	51	8			1	5	0			10	8	5	14	20	15	30		1	2	
	30	12	4	25	0	52	39	1	31	1	7	4	2	4	10	24	5	33	20	51					
	29		2	2	34	34	4	22	0	54	9	1	30	1	9	6	2	2	10	39	5				
53	21	27	28		3	2	38	54	4	20	0	55	38	1	29	1	11	5	1						
59	10	54	6	14	22	38	27		4	2	43	11	4	17	0	57	6	1	28						
1	13	4	1	59	11	9	6	36	22	33	26		5	2	47	25	4	14	1						
[recte: 0]	58	33	1	27	1	15	2	1	58	11	23	6	59	23	1	25		6							
	2	51	37	4	12	0	59	59	1	26	1	16	59	1	57	11	37	7	23						
23	48	24		7	2	55	46	4	9	1	1	24	1	25	1	18	55	1	56	11					
	50	7	48	24	22	23		8	2	59	52	4	6	1	2	48	1	24	1	20					
50	1	55	11	3	[recte: 12 3]	8	14	24	56	22		9	3	3	54	4	2	1							
4	11	1	23	1	22	45	1	55	12	16	8	41	25	29	21		10	3	7						
53	3	59	1	5	32	1	21	1	24	39	1	54	12	29	9	8	26	2	20						
	11	3	11	40	3	56	1	6	52	1	20	1	26	32	1	53	12	42	9						
35	26	34	19		12	3	15	42	3	53	1	8	11	1	19	1	28	24	1						

52	12	54	10	1	27	6	18		13	3	19	31	3	49	1	9	28	1	17	
1	30	15	1	51	13	6	10	27	27	39	17		14	3	23	17	3	46	1	
10	44	1	16	1	32	5	1	50	13	17	10	53	28	8	16		15	3	26	
59	3	42	1	11	59	1	15	1	33	54	1	49	13	28	11	19	28	38		
15		16	3	30	38	3	39	1	13	12	1	13	1	35	42	1	48	13	38	
11	45	29	8	14		17	3	34	13	3	35	1	14	24	1	12	1	37	30	
1	48	13	47	12	11	29	37	13		18	3	37	44	3	31	1	15	34	1	
	10	1	39	16	1	46	13	56	12	38	30	6	12		19	3	41	12	3	
28	1	16	42	1	8	1	41	52	1	46	14	5	13	6	30	34	11		20	
3	44	36	3	24	1	17	48	1	6	1	42	37	1	45	14	13	13	34	31	
	1	10		21	3	47	56	3	20	1	18	53	1	5	1	44	30	1	43	14
21	14	3	31	28	9		22	3	51	12	3	16	1	19	56	1	3	1	46	
10	1	40	14	29	14	32	31	55	8		23	3	54	24	3	12	1	20	58	
	1	2	1	47	48	1	38	14	36	15	1	32	21	7		24	3	57	32	3
8	1	21	58	1	0	1	49	25	1	37	14	43	15	30	32	46	6		25	
4	0	36	3	4	1	22	57	0	59	1	51	1	1	36	14	50	15	59	33	
11	5		26	4	3	35	2	59	1	23	55	0	58	1	52	35	1	34	14	
56	16	28	33	35	4		27	4	6	30	2	55	1	24	52	0	57	1	54	
8	1	33	15	2	16	57	33	58	3		28	4	9	21	2	51	1	25	48	0
	56	1	55	40	1	32	15	8	17	26	34	21	2		29	4	12	8	2	47
	1	26	42	0	54	1	57	10	1	30	15	13	17	56	34	43	1		30	4
	14	51	2	43	1	27	35	0	53	1	58	38	1	28	15	18	18	27	35	
	4	0																		

Column-foot direction labels (left to right): Adde | Subt | Adde | Subt | Adde | Subt | Subt | — | Subt.

10 Sign

Second Lunar Prosthaphaeresis Tables (continued)

Pars Altera, Liber Primus Theoricorum, Cap. VI. The "Second Table of Lunar Prosthaphaereses" [Tabula secunda Prosthaphaeresium Lunarium] continues. These are double-entry canons: each table is headed by one zodiacal sign across the top (read top-to-bottom with the left-hand argument 0–30) and by a second sign at its foot (read bottom-to-top with the right-hand argument 30–0). The add/subtract sense of each column reverses between the two reading directions,

as marked by the "Adde" / "Subtr." rubrics printed at the head and foot of every column.

The seven data columns in each table are: the argument of anomaly [Grad. anom.]; the **subtractive prosthaphaeresis of the ecliptic** [Prosth. Eclipt.] with its difference [Diff.]; the **subtractive prosthaphaeresis of the equation** [Prosth. aequat.] with its difference; the **excess** [excessus] with its difference; the added **equation of anomaly** [Aequatio anomaliae]; the **proportional minutes** [Scrupula proportionalia]; and the added **variation** [variatio].

Table for 2 Signs (head) / 9 Signs (foot)

Column add/subtract senses (top → bottom, i.e. for **2 Sig.**): Prosth. Eclipt. = *Subtr.*; Diff. = *Adde*; Prosth. aequat. = *Subtr.*; Diff. = *Subtr.*; excessus = *Adde*; Diff. = *Subtr.*; Aequat. anom. = *Adde*; Scrup. prop.; variatio = *Adde*.

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Di ff. (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
0	4 14 51	2 38	1 27 35	0 51	1 58 38	1 26	15 18	18 27	35 4	30
1	4 17 29	2 33	1 28 26	0 49	2 0 4	1 24	15 23	18 57	35 25	29
2	4 20 2	2 29	1 29 15	0 47	2 1 28	1 22	15 27	19 28	35 45	28
3	4 22 31	2 24	1 30 2	0 46	2 2 50	1 20	15 30	19 59	36 5	27
4	4 24 55	2 19	1 30 48	0 43	2 4 10	1 18	15 33	20 30	36 24	26
5	4 27 14	2 15	1 31 31	0 41	2 5 28	1 17	15 36	21 0	36 42	25
6	4 29 29	2 10	1 32 12	0 39	2 6 45	1 15	15 39	21 30	37 0	24
7	4 31 39	2 5	1 32 51	0 37	2 8 0	1 14	15 42	22 0	37 17	23
8	4 33 44	2 0	1 33 28	0	2 9 14	1	15 44	22 30	37	22

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Di ff. (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
				36		13			33	
9	4 35 44	1 55	1 34 4	0 34	2 10 27	1 11	15 46	23 0	37 48	21
10	4 37 39	1 51	1 34 38	0 33	2 11 38	1 8	15 47	23 31	38 3	20
11	4 39 30	1 47	1 35 11	0 30	2 12 46	1 6	15 48	24 2	38 17	19
12	4 41 17	1 42	1 35 41	0 29	2 13 52	1 3	15 49	24 33	38 30	18
13	4 42 59	1 36	1 36 10	0 27	2 14 55	1 1	15 49	25 4	38 42	17
14	4 44 35	1 30	1 36 37	0 25	2 15 56	1 0	15 49	25 35	38 55	16
15	4 46 5	1 25	1 37 2	0 23	2 16 56	0 58	15 49	26 7	39 7	15
16	4 47 30	1 20	1 37 25	0 20	2 17 54	0 56	15 49	26 39	39 18	14
17	4 48 50	1 16	1 37 45	0 19	2 18 50	0 54	15 48	27 11	39 28	13
18	4 50 6	1 10	1 38 4	0 18	2 19 44	0 52	15 47	27 44	39 37	12
19	4 51 16	1 5	1 38 22	0 16	2 20 36	0 50	15 46	28 17	39 45	11
20	4 52 21	1 0	1 38 38	0 14	2 21 26	0 49	15 45	28 50	39 53	10
21	4 53 21	0 55	1 38 52	0 13	2 22 15	0 47	15 43	29 22	40 0	9
22	4 54 16	0 49	1 39 4	0 12	2 23 2	0 45	15 41	29 54	40 6	8
23	4 55 5	0	1 39 16	0	2 23	0	15 39	30 26	40	7

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Di ff. (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
		44		11	47	42			12	
24	4 55 49	0 39	1 39 27	0 8	2 24 29	0 39	15 36	30 57	40 17	6
25	4 56 28	0 33	1 39 35	0 3	2 25 8	0 36	15 33	31 28	40 21	5
26	4 57 1	0 28	1 39 38	0 3	2 25 44	0 34	15 30	31 59	40 25	4
27	4 57 29	0 22	1 39 41	0 1	2 26 18	0 31	15 27	32 30	40 27	3
28	4 57 51	0 17	1 39 43	<i>Su btr.</i> 0 1	2 26 49	0 29	15 23	33 1	40 28	2
29	4 58 8	0 12	1 39 42	0 2	2 27 18	0 26	15 19	33 32	40 29	1
30	4 58 20	—	1 39 40	—	2 27 44	—	15 15	34 2	40 30	0

Column add/subtract senses (foot, for **9 Sig.**, read bottom → top): Prosth. Eclipt. = *Adde*; Diff. = *Sub.*; Prosth. aequat. = *Adde*; Diff. = *Sub.*; excessus = *Adde*; Diff. = *Sub.*; Aequat. anom. = *Subt.*; variatio = *Subt.*

[Note on the Diff. of the prosth. aequat. column: the printed sub-rubric "Subtr." appears at row 28, where the difference column changes sign — the value at row 28 reads 0 1 and at rows 29–30 the column continues 0 2 with subtractive sense, marking the turning point of the equation.]

Table for 3 Signs (head) / 8 Signs (foot)

Column add/subtract senses (top → bottom, for **3 Sig.**): Prosth. Eclipt. [Subtrahe]; Diff. [Subt.]; Prosth. aequat. [Subtrahe]; Diff. [Subt.]; excessus [Subtrahe]; Diff. [Add. Subt.]; Aequat. anom. [Adde]; Scrup. prop.; variatio [Adde].

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Diff . (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
0	4 58 20	—	1 39 40	—	2 27 44	—	15 15	34 2	40 30	30
1	4 58 26	0 6	1 39 38	0 2	2 28 9	0 25	15 11	34 32	40 29	29
2	4 58 27	0 1	1 39 34	0 4	2 28 31	0 22	15 6	35 2	40 28	28
3	4 58 22	0 5	1 39 27	0 7	2 28 51	0 20	15 1	35 21	40 27	27
4	4 58 14	0 8	1 39 18	0 9	2 29 8	0 17	14 56	36 0	40 25	26
5	4 57 59	0 15	1 39 7	0 11	2 29 22	0 14	14 51	36 29	40 21	25
6	4 57 37	0 22	1 38 54	0 13	2 29 35	0 13	14 46	36 58	40 17	24
7	4 57 10	0 27	1 38 39	0 15	2 29 44	0 9	14 41	37 27	40 12	23
8	4 56 38	0 32	1 38 21	0 18	2 29 50	0 6	14 36	37 56	40 6	22
9	4 56 1	0 37	1 38 0	0 21	2 29 54	0 4	14 30	38 25	40 0	21
10	4 55 18	0 43	1 37 37	0 23	2 29 57	0 3	14 24	38 54	39 53	20
11	4 54 30	0 48	1 37 12	0 25	2 30 0	<i>Sub tra h. 0 0</i>	14 17	39 22	39 45	19
12	4 53 36	0 54	1 36 46	0 26	2 29 59	0 1	14 10	39 50	39 37	18
13	4 52 37	0 59	1 36 20	0 26	2 29 58	0 1	14 3	40 18	39 28	17
14	4 51 33	1 4	1 35 52	0	2 29	0 3	13 56	40 46	39	16

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Diff . (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
				28	55				18	
15	4 50 23	1 10	1 35 24	0 28	2 29 48	0 7	13 49	41 14	39 7	15
16	4 49 7	1 16	1 34 54	0 30	2 29 35	0 13	13 42	41 41	38 55	14
17	4 47 46	1 21	1 34 23	0 31	2 29 15	0 20	13 35	42 8	38 43	13
18	4 46 21	1 25	1 33 5	0 31	2 28 48	0 27	13 27	42 35	38 30	12
19	4 44 50	1 31	1 33 20	0 32	2 28 16	0 32	13 19	43 2	38 17	11
20	4 43 13	1 37	1 32 44	0 36	2 27 39	0 37	13 11	43 29	38 3	10
21	4 41 34	1 42	1 32 6	0 38	2 26 57	0 42	13 3	43 55	37 48	9
22	4 39 43	1 48	1 31 28	0 38	2 26 10	0 47	12 55	44 21	37 33	8
23	4 37 55	1 52	1 30 48	0 40	2 25 20	0 50	12 47	44 47	37 17	7
24	4 35 54	1 57	1 30 5	0 43	2 24 28	0 52	12 38	45 12	37 0	6
25	4 33 51	2 3	1 29 20	0 45	2 23 34	0 54	12 29	45 37	36 42	5
26	4 31 42	2 9	1 28 34	0 46	2 22 39	0 55	12 20	46 2	36 24	4
27	4 29 29	2 13	1 27 47	0 47	2 21 43	0 56	12 11	46 27	36 5	3
28	4 27 11	2 18	1 26 48	0 49	2 20 45	0 58	12 2	46 51	35 45	2
29	4 24 48	2	1 26 6	0	2 19	0	11 53	47 15	35	1

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Diff . (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
		23		52	46	59			25	
30	4 22 20	2	1 25 12	0	2 18	1 2	11 44	47 39	35 4	0
		28		54	45					

Column add/subtract senses (foot, for **8 Sig.**, read bottom → top): Prosth. Eclipt. [Adde]; Prosth. aequat. [Adde]; excessus [Adde]; Aequat. anom. [Subt.]; variatio [Subt.].

Table for 4 Signs (head) / 7 Signs (foot)

Column add/subtract senses (top → bottom, for **4 Sig.**): Prosth. Eclipt. [Subtrah.]; Diff. [Sub.]; Prosth. aequat. [Subtr.]; Diff. [Sub.]; excessus [Subtrah.]; Diff. [Subt.]; Aequat. anom. [Adde]; Scrup. prop.; variatio [Adde].

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Di ff. (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
0	4 22 20	—	1 25 12	—	2 18 45	—	11 44	47 39	35 4	30
1	4 19 46	2 34	1 24 16	0 56	2 17 42	1 3	11 35	48 2	34 43	29
2	4 17 7	2 39	1 23 19	0 57	2 16 38	1 4	11 26	48 25	34 21	28
3	4 14 24	2 43	1 22 21	0 58	2 15 30	1 8	11 17	48 48	33 58	27
4	4 11 36	2 48	1 21 22	0 59	2 14 20	1 10	11 7	49 10	33 35	26
5	4 8 43	2 53	1 20 22	1 0	2 13 6	1 14	10 57	49 32	33 11	25
6	4 5 45	2 58	1 19 21	1 1	2 11 48	1 18	10 47	49 54	32 46	24

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Di ff. (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
7	4 2 42	3 3	1 18 17	1 3	2 10 26	1 22	10 36	50 15	32 21	23
8	3 59 35	3 7	1 17 12	1 5	2 9 0	1 26	10 25	50 36	31 55	22
9	3 56 23	3 12	1 16 6	1 6	2 7 32	1 28	10 15	50 56	31 28	21
10	3 53 6	3 17	1 14 58	1 8	2 6 1	1 31	10 5	51 16	31 1	20
11	3 49 45	3 21	1 13 48	1 10	2 4 27	1 34	9 55	51 36	30 34	19
12	3 46 20	3 25	1 12 37	1 11	2 2 51	1 36	9 44	51 56	30 6	18
13	3 42 50	3 30	1 11 25	1 12	2 1 12	1 39	9 33	52 15	29 37	17
14	3 39 16	3 34	1 10 12	1 13	1 59 30	1 42	9 22	52 34	29 8	16
15	3 35 38	3 38	1 8 58	1 14	1 57 45	1 45	9 11	52 53	28 38	15
16	3 31 55	3 43	1 7 44	1 14	1 55 57	1 48	9 0	53 12	28 8	14
17	3 28 8	3 47	1 6 29	1 15	1 54 6	1 51	8 49	53 30	27 37	13
18	3 24 18	3 50	1 5 13	1 16	1 52 11	1 55	8 38	53 48	27 6	12
19	3 20 24	3 54	1 3 57	1 16	1 50 13	1 58	8 27	54 5	26 34	11
20	3 16 25	3 58	1 2 40	1 17	1 48 12	2 1	8 16	54 22	26 2	10
21	3 12 22	4 3	1 1 21	1 19	1 46 8	2 4	8 5	54 38	25 29	9
22	3 8 15	4 7	1 0 0	1	1 44 0	2 8	7 53	54 54	24	8

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Di ff. (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
				21					56	
23	3 4 5	4 10	0 58 38	1 22	1 41 49	2 11	7 42	55 9	24 22	7
24	2 59 52	4 13	0 57 14	1 24	1 39 36	2 13	7 29	55 24	23 48	6
25	2 55 35	4 17	0 55 49	1 25	1 37 21	2 15	7 17	55 39	23 13	5
26	2 51 14	4 21	0 54 24	1 25	1 35 4	2 17	7 5	55 54	22 38	4
27	2 46 49	4 25	0 52 58	1 26	1 32 45	2 19	6 53	56 8	22 3	3
28	24 2 21 [recte: 2 42 21?]	4 28	0 51 31	1 27	1 30 24	2 21	6 41	56 22	21 27	2
29	23 7 51 [recte: 2 37 51?]	4 30	0 50 3	1 28	1 28 1	2 23	6 29	56 35	20 51	1
30	23 3 18 [recte: 2 33 18?]	4 33	0 48 34	1 29	1 25 36	2 25	6 17	56 48	20 15	0

Column add/subtract senses (foot, for **7 Sig.**, read bottom → top): Prosth. Eclipt. [Adde]; Prosth. aequat. [Adde]; excessus [Adde]; Aequat. anom. [Subt.]; variatio [Subt.].

Table for 5 Signs (head) / 6 Signs (foot)

Column add/subtract senses (top → bottom, for **5 Sig.**): Prosth. Eclipt. [Subtr.]; Diff. [Sub.]; Prosth. aequat. [Subtr.]; Diff. [Sub.]; excessus [Subtr.]; Diff. [Sub.]; Aequat. anom. [Adde]; Scrup. prop.; variatio [Adde].

In this table the first column (Prosth. Eclipt.) has dropped below 3°, so its degree figures read 2, 1, and 0.

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Di ff. (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
0	2 33 18	—	0 48 34	—	1 25 36	—	6 17	56 48	20 15	30
1	2 28 41	4 37	0 47 5	1 29	1 23 9	2 27	6 5	57 0	19 38	29
2	2 24 1	4 40	0 45 35	1 30	1 20 40	2 29	5 53	57 12	19 1	28
3	2 19 18	4 43	0 44 5	1 30	1 18 8	2 32	5 41	57 23	18 23	27
4	2 14 33	4 45	0 42 34	1 31	1 15 34	2 34	5 29	57 34	17 45	26
5	2 9 45	4 48	0 41 3	1 31	1 12 57	2 37	5 17	57 45	17 7	25
6	2 4 55	4 50	0 39 31	1 32	1 10 19	2 38	5 5	57 55	16 28	24
7	2 0 2	4 53	0 37 58	1 33	1 7 39	2 40	4 53	58 5	15 49	23
8	1 55 7	4 55	0 36 24	1 34	1 4 56	2 43	4 41	58 14	15 10	22
9	1 50 9	4 58	0 34 49	1 35	1 2 12	2 44	4 28	58 23	14 31	21
10	1 45 8	5 1	0 33 13	1 36	0 59 26	2 46	4 15	58 32	13 51	20
11	1 40 5	5 3	0 31 37	1 36	0 56 38	2 48	4 3	58 40	13 11	19
12	1 35 1	5 4	0 30 0	1 37	0 53 48	2 50	3 51	58 48	12 31	18
13	1 29 55	5 6	0 28 23	1 37	0 50 56	2 52	3 39	58 56	11 51	17
14	1 24 47	5 8	0 26 45	1 38	0 48 3	2 53	3 26	59 4	11 10	16

Gr ad. an om .	Prosth. Eclipt. (G.M.S)	Di ff. (M. S)	Prosth. aequat. (G.M.S)	Di ff. (M. S)	exces sus (G.M. S)	Di ff. (M. S)	Aequat . anom. (G.M)	Scrup. prop. (M.S)	vari atio (M.S)	Gr ad. an om .
15	1 19 38	5 9	0 25 7	1 38	0 45 9	2 54	3 13	59 11	10 29	15
16	1 14 27	5 11	0 23 28	1 39	0 42 14	2 55	3 0	59 18	9 43	14
17	1 9 14	5 13	0 21 49	1 39	0 39 17	2 57	2 47	59 25	9 7	13
18	1 3 59	5 15	0 20 10	1 39	0 36 19	2 58	2 34	59 31	8 26	12
19	0 58 43	5 16	0 18 30	1 40	0 33 21	2 58	2 21	59 36	7 44	11
20	0 53 27	5 16	0 16 50	1 40	0 30 22	2 59	2 8	59 40	7 2	10
21	0 48 10	5 17	0 15 10	1 40	0 27 23	2 59	1 56	59 44	6 20	9
22	0 42 52	5 18	0 13 30	1 40	0 24 23	3 0	1 43	59 47	5 38	8
23	0 37 33	5 19	0 11 49	1 41	0 21 23	3 0	1 30	59 50	4 56	7
24	0 32 13	5 20	0 10 8	1 41	0 18 21	3 2	1 17	59 53	4 14	6
25	0 26 52	5 21	0 8 27	1 41	0 15 19	3 2	1 4	59 55	3 32	5
26	0 21 30	5 22	0 6 46	1 41	0 12 16	3 3	0 51	59 57	2 50	4
27	0 16 8	5 22	0 5 5	1 41	0 9 13	3 3	0 39	59 58	2 8	3
28	0 10 46	5 22	0 3 24	1 41	0 6 9	3 4	0 26	59 59	1 26	2
29	0 5 23	5 23	0 1 42	1 42	0 3 5	3 4	0 13	59 59	0 43	1
30	0 0 0	5	0 0 0	1	0 0 0	3 5	0 0	60 0	0 0	0

Gr										Gr
ad.		Di		Di	exces	Di			vari	ad.
an	Prosth.	ff.	Prosth.	ff.	sus	ff.	Aequat	Scrup.	atio	an
om	Eclipt.	(M.	aequat.	(M.	(G.M.	(M.	. anom.	prop.	(M.S	om
.	(G.M.S)	S)	(G.M.S)	S)	S)	S)	(G.M)	(M.S)	)	.
23			42							

Column add/subtract senses (foot, for **6 Sig.**, read bottom → top): Prosth. Eclipt. [Adde]; Prosth. aequat. [Adde]; excessus [Adde]; Aequat. anom. [Subt.]; variatio [Subt.].

An explanation of the structure of the preceding table of lunar prosthaphaeresis

The first column, the one nearest the field of the preceding table (for the two outermost columns are not counted here — that is, the degrees of commutation across the signs through the semicircle), is made up of two angles added to each other, namely EKD and KEB. Because this single prosthaphaeresis of the Moon suffices for the true conjunction and opposition of the luminaries — that is, for the true new moon and full moon — and is therefore also common to the eclipses of either luminary, I have for that reason given it the more special name of **the ecliptic prosthaphaeresis** [prosthaphaeresis ecliptica].

The second column contains only that prosthaphaeresis which the angle DCE alone exhibits, on account of the epicycle HEN that I added at the place of the equant. For that reason it is called **the prosthaphaeresis of the equant** [prosthaphaeresis aequantis]. And although such an angle, added to the other one KEB, makes up the prosthaphaeresis sufficient for the conjunction and opposition of the Moon with the Sun, as I said, nevertheless it had to be set out here separately as well — because, once the anomaly of the Moon has been varied through the equation by the angle ICG and so forth, it here fits only that simple anomaly ID. Indeed, the angle HDE is always by hypothesis equal to the angle ICD, that is, to the mean anomaly of the Moon — but not equal to GCD, or rather to the angle GCE, the anomaly finally fully equated. So, in the supposition of the simple anomaly taken degree by degree and so on, when the said prosthaphaeresis of the equant — the angle EKD here — is subtracted from the whole ecliptic prosthaphaeresis, the angle KEB remains by itself (which was one of these two before). Knowledge of this serves what follows.

The third column is assigned to the excess [excessus] — that is, to the excess which falls between the shortest and longest eccentricity (between the eclipse-syzygies of the luminaries and the quadratures), whose limits are K, A, L, as I showed above. This too arises for us from the triangle on the supposed simple anomaly: for when the eccentricity BK 5800 has been increased through KL 4348 — that is, the whole diameter of the little circle KLC — that excess will be the amount by which the angle LEB always exceeds the angle KEB at any distance of the Moon from apogee.

The fourth and last column contains the equation of anomaly [aequatio anomaliae], which in the upper supposition is indicated by the angle ABC. Before the quadratures it is additive, but after them subtractive, as that too appears in the hypothesis itself. This equation is here kept to whole degrees and first minutes [minutes of arc], both because of the want of space here and because nearly every perceptible error is avoided. For half of a single first minute, which might perhaps be in surplus or deficient in the anomaly, scarcely ever produces a deviation of more than 3 seconds along the longitude of the Moon. Accordingly, we judge that this extreme precision [ἀκρίβεια [Greek: akribeia, "exactness, precision"]] need not be commended everywhere, but only a moderate one in the course of the calculation.

Furthermore, so that lovers of the stars [Astrophili] might be relieved of the laborious resolution of the triangle here (which was certainly the aim of this secondary theory of our own devising), I have, in imitation of the ancients, bound the proportion of the aforementioned excess — to be discerned along the whole course of the Moon, outside the cardinal syzygies — to the proportional minutes [scrupula proportionalia] attached in the fifth column. By that means their discovery was likewise easy. For at the contact of the eccentric and epicycle, where the whole prosthaphaeresis is greatest — and now however much the angle CEB always exceeds the angle KEB in the angle of apparent place, the start being made at K — that whole amount too is comprised under the name of excess. Since, then, KL is found at the contact, that is, at the maximum excess of the prosthaphaeresis, to be 2 degrees 30 minutes, and the whole excess is supposed equated to one whole degree (or 60 minutes), the rule of arithmetic proportion gives the way to assign their own proportional minutes to all the other excesses found about the contact, and afterwards, reciprocally by the same rule, to estimate the remaining excesses throughout the whole revolution of the lunar phenomenon.

For the rest, however ingenious this invention of the ancients may seem, and however useful for completing the calculation in a compendious way, nevertheless — to say how the matter stands — it rather often produces some discrepancy from the most exact analysis by triangles, and the greater a discrepancy the larger the excess is (as below in the case of Mars, Venus, and Mercury), or the more complicated the equants are, as here in the case of the Moon.

The sixth and last column of the table, finally, is that variation of the Moon [variatio] which I clearly set out above together with its cause, and afterwards in an example as well. Now, therefore, follow the precepts of the lunar [computation] [ψηφοφορία [Greek: psēphophoria, "reckoning, calculation"]], and then their use, repeated in several worked examples.

The method of computing the true longitude of the Moon from the new table of prosthaphaereses

I.

For the given and equated time, let the mean motions of the luminaries be extracted from their proper tables in every respect, as before, and written out in order: namely, the simple [motion] of the Sun from the mean equinox, the anomaly of the Sun, the simple [motion] of the Moon from the Sun, and the anomaly of the Moon — the inequality of the equinoxes being set out first (since that too is needed, though in the last place).

II.

Then, by the simple anomaly of each luminary, let their prosthaphaereses be extracted one by one: that of the Sun from its own table of prosthaphaeresis, but that of the Moon only from the first column of this table, which we call the ecliptic; and likewise, by the same anomaly, from the second column [let there be extracted] the prosthaphaeresis of the equant, or of the epicycle, which will be reserved for its own use. Account must always be taken of the part of the proportion for the minutes adhering to the anomaly — let it suffice to have set this out generally once elsewhere, but to have given the reminder here and now too.

[Margin: See the logistic of arithmetic at the end.]

III.

Third, let the prosthaphaereses of the luminaries be compared with each other together with their signs. If they are of the same denomination, let them be subtracted from one another — the smaller, of course, from the larger; but if of different [denomination], let them be added to one another. Afterwards the remainder or the sum will be applied to the simple longitude of the Moon from the Sun, and that according to the sign of the prosthaphaeresis — considering, at any rate, [the sign] in the remainder: thus, if it is that of the prevailing Sun, and indeed additive, let it be taken from the simple [longitude] of the Moon, and vice versa. For in the others, as also in the sum, the addition and subtraction always follows the sign of the prosthaphaeresis of the Moon itself. This equated longitude of the Moon from the Sun, when doubled, shows that double true distance of the luminaries from each other which the [present] supposition of the Moon requires.

IV.

With this double distance of the luminaries, entry will be made into the said table of the Moon, and from its last three columns — namely the fourth, fifth, and sixth — let there be excerpted in order, together with their signs: first the equation of anomaly of the Moon; second the proportional minutes; third, finally, the variation of the center of the epicycle, which will be reserved together with the proportional minutes. The equation of anomaly will be accommodated to the simple anomaly of the Moon according to the sign of addition or subtraction, and thus the equated anomaly of the Moon arises.

V.

Further, by the equated anomaly of the Moon, the ecliptic prosthaphaeresis from the first column of the table and that of the equant from the next will again be extracted, and the difference between them — which we seek here — will offer itself at once, provided that no minutes of anomaly adhere; otherwise, in a compendious way, when you have sought only the difference of the first and second columns at the sign and degree of the anomaly offered, and likewise the difference of those same differences which are found set down in the table, and so by the remaining difference have hunted out the proportional part for the adhering minutes in the usual manner, and have duly accommodated it, according to its sign, to the remaining ecliptic prosthaphaeresis — as will be clear in the example. To this, presently, the prosthaphaeresis

of the equant, kept before, is to be added or subtracted according to its sign, but for the most part added. Then, in the same way, by the same equated anomaly,

the excess is sought from the third column, and presently it is multiplied logistically into the proportional minutes kept before, and the proportional part emerging from this is always aggregated to the nearest prosthaphaeresis out of both [of the] remaining [terms].

VI.

To this aggregated prosthaphaeresis, when the variation of the Moon — also kept before — has been duly applied according to its sign, there will at last result from this the whole prosthaphaeresis: that is, how much the true motion of the Moon stands apart from the mean. So let this final prosthaphaeresis of the Moon be accommodated, according to its sign, first to the simple longitude of the Moon from the Sun; then let the simple longitude of the Sun from the mean equinox be added; to which, in the last place, when the equinoctial equation has been duly applied, is given the true longitude of the Moon from the true vernal equinox in its own orbit — which was what was proposed. [Margin: On which see Cap. 4.] These precepts are made clearer by the following examples.

THE FIRST OF THE EARLIER EXAMPLES

	Sig.	Gr.	Mi.	Sec.	
Equation of the precession of the equinoxes	0	0	7	13	
Simple [motion] of the Sun	5	5	42	16	
I. Anomaly of the Sun	2	0	20	53	
Longitude of the Moon from the Sun	9	25	12	6	
Anomaly of the Moon	1	15	37	21	
Therefore — of the Sun		1	44	46	S.
II. Prosthaph. — ecliptic of the Moon		3	29	16	S.
— of the equant		1	12	44	S.
Residue of the ecliptic prosthaph. & of the Sun		1	44	30	S.
III. Therefore double [longitude] of the Moon from the Sun, equated	7	16	15	12	
Equation of anomaly		9	32	7	S.
IV. Proportional minutes			52	6	
Variation of the center			29	35	S.
Anomaly of the Moon, equated	1	6	5	14	

The second part of the example (with the *Differentia* sub-columns in minutes and seconds):

	Sig.	G r.	M i.	Sec .	Diff. Mi.	Diff. Sec.	
To the equated anomaly in signs & degrees there corresponds the prosthaph. — Eclipt.		2	5 1	37	4	9	
— Equant.		0	5 9	59	1	25	
Difference		1	5 1	38	2	44	
Adhering minutes			5	14			
Proportional part				14			Ad d.
Prosthaphaeresis — Eccentric		1	5 1	52			
— Equant, kept		1	1 2	44			
— Aggregate		3	4	36			
Excess		1	1 7	9			
Proportional minutes			5 2	16			
Proportional part		1	7	9			Ad d.
Whole prosthaphaeresis of the Moon, first		4	1 1	48			
Variation of the center of the epicycle			2 9	35			Ad d.
Absolute prosthaphaeresis of the Moon		4	4 1	23			Su b.
Above, from both hypothesis & table, it was the same		4	4 0	36			
Difference				47			+

So the difference from the former [computation] is at most 47 [seconds], for the reason I touched on above; nor will it be necessary in this example to carry the calculation further for the true longitude of the Moon, since everything else will agree with what came before.

THE SECOND EXAMPLE

from the latter table of the prosthaphaereses of the Moon as well, repeated at least for the variation of the new table from the upper [example].

	Sig.	Gr.	Mi.	Sec.	
Simple longitude of the Moon from the Sun	2	25	19	38	
Prosthaphaeresis of the Moon, ecliptic		4	53	43	Add.
Prosthaphaeresis of the Moon, of the equant		1	39	50	Add.
Difference of the prosthaphaereses of the Moon, ecliptic, from the Sun		3	9	45	Add.
Prosthaphaeresis of the Sun		1	43	58	a
Longitude of the Moon, ecliptic, from the Sun	2	28	29	23	
And its double	5	26	58	46	
There corresponds the Equation of anomaly			29	0	Add.
Proportional minutes			59	58	
Variation of the center			2	9	Add.
Anomaly — simple	8	18	8	20	
— equated, of the Moon	8	18	47	20	
There corresponds the prosthaph. of the eccentricity of the Moon		3	17	12	Add.
Prosthaphaeresis of the equant, kept		1	36	50	Add.
Excess		2	30	0	
Proportional part		2	29	55	Add.
Therefore, from these last four to be aggregated here, the absolute additive prosthaphaeresis of the Moon is made up		7	26	6	
Above it was the same		7	25	19	
Difference				47	+

So this too settles within a minute. Nor, if the calculation is duly carried through, shall I judge that any greater deviation anywhere from the exact resolution of the hypothesis itself is to come. In the meantime, however, beginners in astronomy, perhaps weary of the labor as far as the computation of the longitude of the Moon from the earlier [methods] is concerned, are conveniently relieved here.

[Margin: Liber 1, Progymnasmata, on the latitude of the Moon.]

CHAPTER VII

On the latitude of the Moon, and its true computation.

Since I have already attended in the preceding chapter to the perpetual restitution of the mean motion, or argument, of the latitude of the Moon, I shall therefore deal here only with the recession of the Moon from the ecliptic into latitude, together with the inequality of the nodes — that is, of the head and tail of the Dragon [caput et cauda Draconis], as they call them (which are the two points of intersection of the path of the Moon with the ecliptic) — and this from the *Progymnasmata* of [Tycho] Brahe, the heedless drowsiness of the ancients on this matter being passed over.

"The Moon" (says Tycho) "does not run its course beneath the ecliptic, the Sun's path" — this the astronomers of every age have long had thoroughly explored, so much so that it has been found to wander away from it, growing on this side and that to very nearly five degrees, while the nodes (or the intersections with the ecliptic) do not remain in the same places, but anticipate themselves backward, so that each day they move 3 minutes 11 seconds, and complete their period within nearly 19 years. But all practitioners have hitherto been persuaded in vain that the limits of the greatest latitude of the Moon remain perpetually alike to themselves, and reach precisely five degrees — and this above all on the authority of [Ptolemy], whom [al-Battānī] [Albategnius] followed, and after him [the followers of] Alfonso, in whom too [Copernicus], as often elsewhere, acquiesced too carelessly. Indeed, it has hitherto been believed in vain that the nodes themselves keep an equal and regular motion. For in neither of these do our observations — made with the greatest diligence over some years past — agree with the traditions of the ancients up to now, however long received. For not only [did they find] the other limits of greatest latitude

we have also found the [other] limits to differ from those which Ptolemy laid down — and not only that, but to differ among themselves as well. At the new and full moons they correspond as closely as possible to Ptolemy's specification:

there we have established by observation that the Moon can depart from the ecliptic by $4^{\circ} 58' 30''$. But at the quadratures this wandering increases by nearly a third of a degree, so that it then amounts to $5^{\circ} 17' 30''$ — as a careful and often-repeated examination has taught us, conducted attentively about each limit, the northern as much as the southern, and in both tropical positions, so that account was taken everywhere not only of parallax but also of refraction in the more oblique situation.

Now if Ptolemy investigated the Moon's greatest latitude near the quadratures — as is likely — while it lay near the tropic of Cancer [☿] and close to the zenith, then that same latitude must have been about 20' smaller than ours is now. For at that time the obliquity of the ecliptic was greater by just about that many minutes, and that excess could account for nearly the third of a degree by which the Moon's greatest latitude at the quadratures, as we find it, exceeds Ptolemy's value — since he confined himself in this practice solely to the northern limit set near the tropic of Cancer.

In the variation of the nodes too we detect a not inconsiderable inequality. It nevertheless alters the mean motion imperceptibly at the new and full moons, and likewise at the quadratures; so it is the less surprising that earlier astronomers — who chiefly observed the Moon about those positions — did not notice this difference. But in the intermediate places it can grow on either side to nearly two degrees less one quarter, so that the Moon's latitude itself may from this cause be increased or diminished by nearly a sixth of a degree near the nodes, whenever the position is otherwise carefully attended to there as the mean.

Having thus set out, on Tycho's authority [Tycho Brahe], these matters concerning the Moon's latitude — matters founded on the most reliable observations, as may be seen in the four sufficient worked examples in his own work, and confirmed besides by our own most lucid experience in the rest [Margin: Lib. 1. Prog. p. 114 et seq.] — we shall now subjoin here the hypothesis of the Moon's latitude by which both the inequality of the nodes and that of the obliquity of the Moon's orbit with respect to the ecliptic are most correctly handled and preserved. It is plainly similar to the variation of the equinoxes and of the ecliptic that was assigned above, since here too the pole of the lunar orbit is taken to be movable.

For it is not possible to attach these inequalities to the hypothesis of the Moon's longitude, or vice versa, as Jacob Christmann [Jacobus Christmannus] dreamed when by chance the calculation came out for him in a single example. For the

complete revolution of the nodes backward beneath the ecliptic is plainly incommensurable with the period of the lunar apogee: the latter occurs within about 9 years, the former within 19. If from these considerations astronomers have still not understood how that "reformer" — while he claims this theory of the Moon for himself under a borrowed right (ἐν τῷ πιθηκίζειν [Greek: "in playing the ape" — i.e. aping, mimicking], as the proverb says) — has labored in vain, let them set up a comparison of his work with the outcome of Christmann's in the lunar motions, especially outside the eclipses, and they will confirm by experience what we are saying. [Margin: Vide tractatum Christmanni de Theoria lunæ, anno Dn. 1611 excusum pag. 25, 29, etc.]

That Christmann had indeed deserved something rather harsh from us, both on account of this affair and because, ignorant of astronomy, he was not ashamed to instruct anew in his own methods those of us who have grown grey in nearly the final treatment of this art — with new instruments of his own, a new method of observing the times, a preposterous trigonometry handed down (forsooth) from heaven, and in calculation, finally, with shortcuts borrowed in multiplying and dividing, namely from others (as is sufficiently plain also from our *Arithmetic*). And yet we now think his madness should rather be spared — old man that he is, otherwise renowned for learning — whose [book], freely running through it for what it is worth, we deigned to answer no more than the barking of a dog, but aptly. For [as Tycho said], "I do not bother with trifles." But of these things enough, since they are found sufficiently refuted elsewhere by the distinguished Mr. David Origanus [David Origanus]; let me now return to setting out the hypothesis of the Moon's latitude. Z 3

The Hypothesis of the Inequality of the Moon's Latitude and of the Nodes

From B, the pole of the ecliptic, let a great circle be described, set to the plane of this M L N. Then, after the arc of a great circle L I has been drawn, let a distance B A of $5^{\circ} 8'$ be laid off in the sphere, which measures the mean latitude or inclination of the Moon to the ecliptic. From A as pole let both the great circle M H N and the smallest circle D G F E be drawn with compasses. Of these, the former represents the Moon's mean path beneath the zodiac, while the latter shows the pole of the Moon's true orbit, according to the magnitude of its track — and this for the revolution of the double distance of the two luminaries from one another.

Diagram. A geometric construction illustrating the hypothesis of the Moon's variable latitude and the inequality of the nodes. The figure consists of two large

nearly-concentric circles. The outer represents the ecliptic and the inner the Moon's mean path beneath the zodiac; on these are marked the ascending/descending node positions near the head and tail of the dragon. A vertical axis runs through the figure from L (top) through a narrow lens-shaped construction down to a small circle near the center (the locus of the movable pole), and on to P at the bottom. Labels read:

- L (top of vertical axis, on the outer circle): endpoint of arc L I / L K — the greatest-latitude markers
- I (just below L, on the axis): inner endpoint, measuring minimum inclination (arc L I = $4^{\circ} 58\frac{1}{2}'$)
- R — H — S (a horizontal row crossing the lens just below I): R (left), H (center, on axis), S (right); angle H B S is equal to angle A B G
- K (on the axis, below the R–H–S row, within the lens): marks the maximum inclination, measured by arc L K = $5^{\circ} 17\frac{1}{2}'$
- B (on the axis, at the lower vertex of the lens / pole of the ecliptic): apex of triangle B A G
- D (just below B, top of the small circle): position of the movable pole at conjunction and opposition (minimum inclination)
- G (left end of the small circle's horizontal diameter): mid-course position of the movable pole (here latitude is "ablative")
- A (center of the small circle): pole from which circles M H N and D G F E are drawn; mean position
- E (right end of the small circle's horizontal diameter): mid-course position (latitude "adjective")
- F (bottom of the small circle): position of the movable pole at the quadratures (maximum inclination)
- P (bottom of the vertical axis, on the outer circle): opposite endpoint of the axis
- O Ƨ (left side, on the outer circle): point O, marked with the looping Ƨ glyph
- Ƨ N (left side, where the inner and outer curves cross): the node N, near the tail of the dragon (cauda draconis), on the ecliptic; N V marks the cauda inequality; marked with the same looping Ƨ glyph as O and V
- Ƨ V (left side, below N): point V, marked with the same looping Ƨ glyph
- X Ƨ (right side, on the inner curve): point X; M X marks the caput inequality; marked with the same looping Ƨ glyph as the left-side group

- M $\propto$ (right side, where the curves cross): the node M, near the head of the dragon (caput draconis), on the ecliptic; marked with the same looping $\propto$ glyph
- $\propto$ (right side, below M): lower marker on the outer circle, marked with the same looping $\propto$ glyph as X and M

Thus when the said pole is at D — which happens at the conjunction and opposition of the luminaries — the Moon's inclination to the ecliptic is least, namely $4^{\circ} 58\frac{1}{2}'$, which is shown by the arc L I. But when the same pole, having rolled through G, comes to lie at F — which happens at the quadratures of those same luminaries — the Moon's inclination is then greatest, measured by the arc L K, $5^{\circ} 17\frac{1}{2}'$. In these positions no inequality of the nodes is felt, because of the same continued arc L B D F. But in the whole remaining revolution of the pole of the Moon's orbit there is always some inequality, greatest in the mid-courses around G and E — there additive, here subtractive — as is sufficiently understood from the hypothesis itself.

The method of extracting each of the two inequalities depends on the doctrine of spherical triangles, in this way. In the triangle B A G, whose two sides are always B A $5^{\circ} 8'$ and A G $9\frac{1}{2}'$, with the angle B A G to be estimated from the revolution of the true pole of the Moon's path to the true double distance of the luminaries (according as it suits the conjunction or the opposition): first, then, the side B G is given, which determines the angle of the Moon's maximum latitude. Then the angle A B G is given, by which the inequality of the nodes is shown — at the head of the dragon by the ecliptic M X, and at the tail of the dragon by N V on the ecliptic; for the angle H B S is equal to the angle A B G. [Margin: Vide cap. 4 hujus. — Vide demonstrationem apud T. B. lib. 1. Prog. p. 115.]

In acquiring the inequality of latitude compactly, however, we use proportional minutes [scrupula proportionalia], in place of the obliquity of the ecliptic that was substituted in the upper hypothesis of the equinoxes. For whereas there the proportional minutes had to be derived, for any fixed longitude from the equinox we had set, into the excess B G &c. beyond B D for multiplying, here the same minutes arise from the excesses (found by Tycho and by us) and are to be multiplied into the difference of the greatest and least latitude previously acquired for each single degree (after the manner of the Sun's declination in the doctrine of the sphere), so that here too the sought latitude of the Moon may emerge. This permutation of the minutes, besides being agreeable for learners to grasp, will also be useful in that these same minutes can serve the common use in working out the lunar parallaxes and the latitudes of the planets (of which later).

These remarks briefly suffice for setting out the theory of the Moon's latitude, and also for the construction of the tables subjoined below; after which the method of computing the latitude and the motion of the nodes will follow in some example drawn from Tycho's *Progymnasmata*.

Table of the Prosthaphaereses of the Nodes (*Tabula Prosthaphaeresium Nodorum*)

ADDE (Add)

The table is headed by three pairs of column-groups. Each group has a top sign-number and (below) a complementary sign-number: **0 / 6**, **1 / 7**, **2 / 8** (read with ADD); the foot of each group gives its complement for SVBTRACT: **11 / 5**, **10 / 4**, **9 / 3**. Each group lists the Prosthaphaeresis of the Nodes (G. M. S. — degrees, minutes, seconds) and the proportional minutes (Scrup. Pro. — M. S.). The rightmost slim column gives the argument read upward for the subtract entry (30 down to 0). [Margin: Libr. 1. Prog. p. 111 et seq.]

Arg. ↓	0 / 6 —		1 / 7 —		2 / 8 —		A r g . ↑
	Prosth. Nod. (G M S)	0 / 6 — Scrup. Pro. (M S)	Prosth. Nod. (G M S)	1 / 7 — Scrup. Pro. (M S)	Prosth. Nod. (G M S)	2 / 8 — Scrup. Pro. (M S)	
0	0 0 0	0 0	1 33 28	15 22	1 30 32	45 20	3 0
1	0 3 50	0 1	1 35 12	16 18	1 28 34	46 13	2 9
2	0 7 39	0 4	1 36 47	17 15	1 26 30	47 5	2 8
3	0 11 27	0 9	1 38 12	18 13	1 24 21	47 54	2 7
4	0 15 14	0 16	1 39 31	19 11	1 22 6	48 43	2 6
5	0 19 0	0 26	1 40 42	20 10	1 19 45	49 31	2 5
6	0 22 46	0 41	1 41 45	21 9	1 17 18	50 19	2 4
7	0 26 29	0 56	1 42 44	22 9	1 14 46	51 4	2

Arg. ↓	0 / 6 — Prosth. Nod. (G M S)	0 / 6 — Scrup. Pro. (M S)	1 / 7 — Prosth. Nod. (G M S)	1 / 7 — Scrup. Pro. (M S)	2 / 8 — Prosth. Nod. (G M S)	2 / 8 — Scrup. Pro. (M S)	A r g . ↑
							3
8	0 30 9	1 13	1 43 38	23 10	1 12 9	51 47	2 2
9	0 33 47	1 32	1 44 29	24 11	1 9 27	52 28	2 1
10	0 37 23	1 53	1 45 8	25 13	1 6 41	53 8	2 0
11	0 40 56	2 16	1 45 34	26 16	1 3 49	53 46	1 9
12	0 44 26	2 41	1 45 50	27 19	1 0 51	54 23	1 8
13	0 47 52	3 8	1 45 56	28 22	0 57 50	54 59	1 7
14	0 51 14	3 38	1 45 59	29 25	0 54 47	55 33	1 6
15	0 54 32	4 10	1 46 0	30 28	0 51 42	56 6	1 5
16	0 57 47	4 43	1 45 53	31 30	0 48 32	56 36	1 4
17	1 0 56	5 18	1 45 36	32 33	0 45 18	57 3	1 3
18	1 4 0	5 54	1 45 13	33 35	0 42 0	57 28	1 2
19	1 6 59	6 32	1 44 41	34 37	0 38 40	57 52	1 1
20	1 9 53	7 12	1 44 0	35 39	0 35 18	58 14	1 0
21	1 12 42	7 54	1 43 10	36 41	0 31 53	58 35	9
22	1 15 25	8 38	1 42 14	37 42	0 28 26	58 53	8
23	1 18 2	9 24	1 41 10	38 43	0 24 57	59 9	7

Arg. ↓	0 / 6 — Prosth. Nod. (G M S)	0 / 6 — Scrup. Pro. (M S)	1 / 7 — Prosth. Nod. (G M S)	1 / 7 — Scrup. Pro. (M S)	2 / 8 — Prosth. Nod. (G M S)	2 / 8 — Scrup. Pro. (M S)	A r g . ↑
24	1 20 33	10 13	1 39 59	39 43	0 21 26	59 22	6
25	1 22 58	11 2	1 38 42	40 41	0 17 54	59 33	5
26	1 25 16	11 51	1 37 18	41 38	0 14 21	59 43	4
27	1 27 28	12 41	1 35 46	42 35	0 10 46	59 50	3
28	1 29 34	13 33	1 34 8	43 31	0 7 11	59 55	2
29	1 31 34	14 27	1 32 23	44 26	0 3 36	59 58	1
30	1 33 28	15 22	1 30 32	45 20	0 0 0	60 0	0
Subtr . sign →	11 / 5		10 / 4		9 / 3		

SVBTR. (Subtract)

Table of the Latitude of the Moon (*Tabula Latitudinis Lunae*)

The table has three column-groups, each headed by a pair of sign-arguments — **0 Bor. / 6 Mer.**, **1 Bor. / 7 Mer.**, **2 Bor. / 8 Mer.** (Bor. = northern/boreal, Mer. = southern/meridional) — and each carrying a Latitude (Latitudo, G. M. S.) and an Excess (Excessus, M. S.) column. The foot of each group gives the complementary signs read upward: **11 Mer. / 5 Bor.**, **10 Mer. / 4 Bor.**, **9 Mer. / 3 Bor.** The rightmost slim column gives the complementary argument (30 down to 0).

Arg. ↓	0 Bor / 6 Mer — Latitudo (G M S)	Exces sus (M S)	1 Bor / 7 Mer — Latitudo (G M S)	Exces sus (M S)	2 Bor / 8 Mer — Latitudo (G M S)	Exces sus (M S)	A r g . ↑
0	0 0 0	0 0	2 29 6	9 28	4 18 26	16 25	3 0
1	0 5 13	0 20	2 33 36	9 45	4 20 59	16 35	2 9
2	0 10 25	0 40	2 38 3	10 2	4 23 28	16 45	2 8
3	0 15 36	0 59	2 42 26	10 18	4 25 53	16 55	2

Arg. ↓	0 Bor / 6 Mer — Latitudo (G M S)	Exces sus (M S)	1 Bor / 7 Mer — Latitudo (G M S)	Exces sus (M S)	2 Bor / 8 Mer — Latitudo (G M S)	Exces sus (M S)	A rg . ↑
							7
4	0 20 47	1 19	2 46 46	10 35	4 28 13	17 4	2 6
5	0 25 58	1 39	2 51 4	10 51	4 30 28	17 12	2 5
6	0 31 9	1 59	2 55 19	11 7	4 32 38	17 20	2 4
7	0 36 19	2 19	2 59 30	11 23	4 34 43	17 28	2 3
8	0 41 29	2 38	3 3 38	11 39	4 36 43	17 36	2 2
9	0 46 38	2 57	3 7 43	11 55	4 38 38	17 43	2 1
10	0 51 46	3 17	3 11 44	12 11	4 40 27	17 51	2 0
11	0 56 53	3 36	3 15 42	12 26	4 42 11	17 58	1 9
12	1 1 59	3 55	3 19 36	12 41	4 43 50	18 5	1 8
13	1 7 4	4 15	3 23 26	12 56	4 45 23	18 12	1 7
14	1 12 8	4 34	3 27 13	13 10	4 46 52	18 18	1 6
15	1 17 10	4 53	3 30 56	13 24	4 48 18	18 23	1 5
16	1 22 11	5 13	3 34 35	13 38	4 49 35	18 27	1 4
17	1 27 10	5 32	3 38 10	13 52	4 50 49	18 31	1 3
18	1 32 8	5 51	3 41 42	14 5	4 51 58	18 34	1 2

Arg. ↓	0 Bor / 6	Exces sus (M S)	1 Bor / 7	Exces sus (M S)	2 Bor / 8	Exces sus (M S)	A rg . ↑
	Mer — Latitudo (G M S)		Mer — Latitudo (G M S)		Mer — Latitudo (G M S)		
19	1 37 4	6 10	3 45 7	14 18	4 53 0	18 38	1 1
20	1 41 58	6 29	3 48 30	14 31	4 53 57	18 42	1 0
21	1 46 51	6 47	3 51 52	14 44	4 54 49	18 45	9
22	1 51 41	7 6	3 55 9	14 57	4 55 36	18 48	8
23	1 56 30	7 24	3 58 19	15 9	4 56 17	18 51	7
24	1 1 17 [recte: 2 1 17]	7 42	4 1 23	15 21	4 56 52	18 53	6
25	1 6 1 [recte: 2 6 1]	8 0	4 4 24	15 32	4 57 22	18 55	5
26	1 10 43 [recte: 2 10 43]	8 18	4 7 21	15 43	4 57 46	18 56	4
27	1 15 23 [recte: 2 15 23]	8 35	4 10 15	15 54	4 58 5	18 57	3
28	1 20 0 [recte: 2 20 0]	8 53	4 13 5	16 5	4 58 18	18 58	2
29	1 24 34 [recte: 2 24 34]	9 11	4 15 47	16 15	4 58 26	18 59	1
30	1 29 6 [recte: 2 29 6]	9 28	4 18 26	16 25	4 58 30	19 0	0
Subtr. sign →	11 Mer / 5 Bor		10 Mer / 4 Bor		9 Mer / 3 Bor		

Cap. VII.

Deriving the true latitude of the Moon from the tables.

For the proposed moment — the same moment for which the Moon's longitude has been computed — have ready the simple motion of latitude, the true distance of the two luminaries, and the absolute prosthaphaeresis of the Moon. According to what the column-headings indicate, add this prosthaphaeresis to the mean motion of latitude, or subtract it, so as to obtain the true value. Next, using the true distance of the Moon and Sun, take from the first table the proportional minutes [scrupula proportionalia], which you should keep on hand, together with the prosthaphaeresis of the nodes; and, as the headings (additive or subtractive) require, add this to the true motion of latitude, or subtract it from it, so that the motion becomes fully equated. With this equated motion, take from the second table the latitude of the Moon together with its excess, as appropriate. The proportional minutes multiplied into the excess then yield the proportional part, which is always to be added to the latitude, so that the true latitude emerges at this inclination of the Moon's orbit to the ecliptic. Whether it is northern or southern, the headings themselves — added at top and bottom — show plainly.

Example I.

In the year 1596, on 4 January, at 11 hours 43 minutes after midday, the observed apparent latitude of the Moon was $4^{\circ} 31' \text{ N}$. Adding to this a parallax of $28' 30''$ gives the true latitude of the Moon as $4^{\circ} 59' 30''$. Now, for the proposed moment, when equated, the mean motion of latitude comes to 3 signs $4^{\circ} 59' 6''$. Subtracting from this the absolute prosthaphaeresis of the Moon, $5^{\circ} 4' 19''$, leaves the true motion of latitude reckoned from the mean ascending node as 2 signs $29^{\circ} 54' 47''$. Next, with the true ecliptic distance of the luminaries, which is 5 signs $23^{\circ} 38'$, I enter the first table and find the proportional minutes to be $0' 46''$, to be kept on hand, and the prosthaphaeresis of the nodes to be $24' 8''$, to be subtracted from the true motion of latitude so that it may be equated; this gives 2 signs 69° [recte: 29°] $30' 39''$. With this value I go into the latitude table, seeking the latitude and the excess. After making the due correction by the proportional part, I find the latitude to be $4^{\circ} 58' 28''$, and the excess to be $19' 0''$. This excess, multiplied logistically into the $0' 46''$ of proportional minutes kept on hand, yields $15''$ to be added to the latitude — as was said, always to be added — so that the absolute latitude comes out as $4^{\circ} 58' 43''$ northern. This differs from the observed value by only three-quarters of a minute [per dodrantem], $1'$ [recte: $\frac{3}{4}'$], which is of no consequence.

On the method of investigating the nodes.

[Margin: *Lib. 1, Prog., pag. 116.*]

Now that the involved theory of the Moon's latitude has been explained, it is worth our while to inquire also into the nodes — the points where the Moon, free of all latitude, crosses the ecliptic. So, for the northern node (to which the other is opposite), have these simple motions ready for the given time: the simple motion of the Sun from the equinox, the simple motion of the Moon from the Sun, and the simple motion of the Moon's latitude. Once these are obtained, add the simple motion of the Sun to the simple motion of the Moon, so as to compose the simple longitude of the Moon from the vernal equinox. From this sum subtract again the simple motion of the Moon's latitude, and in this way you will obtain the mean anticipation of the nodes — the value that is roughly what those who follow Ptolemy, Alfonso, and Copernicus commonly use, having been persuaded, in vain, that there is no inequality underlying it. But to confine this value within its proper bounds, take from the table of the prosthaphaeresis of the nodes the equation corresponding to the true distance of the luminaries, and — in the opposite sense to what the headings themselves suggest — add it to the mean motion of the nodes, or subtract it; and thus you will arrive at the true place of the node that carries the Moon toward the north. This node they call the head of the dragon [caput draconis], to which, as has been said, the other node, named the tail of the dragon [cauda draconis], is diametrically opposite. Both limits of the greatest latitudes, moreover, are removed a quarter-circle on either side from these points, if one wishes to ascertain them together.

Example II.

In the third example, in which Tycho Brahe was engaged in investigating the Moon's latitude, the simple motion of the Sun is 11 signs $1^{\circ} 51' 1''$. Adding to this the mean motion of the Moon from the Sun, 4 signs $2^{\circ} 20' 59''$, gives the simple motion of the Moon from the equinox as 3 signs $4^{\circ} 12' 0''$. Subtracting from this the simple motion of latitude, $54' 23''$, leaves the mean anticipation of the nodes as 3 signs $3^{\circ} 17' 37''$. Then, with the distance of the luminaries 4 signs $4^{\circ} 13' 11''$, the prosthaphaeresis of the nodes is derived as $1^{\circ} 37' 36''$, which in this case is to be added to the mean motion of the nodes, so that the true place is established as 3 signs $4^{\circ} 55' 13''$ [Margin: *Prog. 1, p. 3.*]. Alternatively, if you have already collected both the longitude and the latitude of the Moon, subtract the equated motion of latitude — the one you used in extracting the latitude — from the true place of the Moon, and the true place of the node will remain. So in the same example, the true place of the Moon is 3 signs $7^{\circ} 48' 45''$; subtracting from this the equated

latitude, $2^{\circ} 53' 32''$, leaves the true place of the ascending node $[\Omega]$ as 3 signs $4^{\circ} 55' 13''$. This exceeds the Alphonsine and Copernican calculation by a degree and a half, and — as the Prutenic Tables give it — by 11 minutes still more, a discrepancy which around the very nodes of the Moon is indeed intolerable.

On the reduction of the Moon's place to the ecliptic.

Since the Moon's motion does not proceed beneath the ecliptic but in another circle inclined to it, and since the calculation yields the motion that takes place in the Moon's own orbit — not the motion that the instruments present by observation, with respect to the ecliptic and its poles — therefore some adjustment is needed here, which is most easily carried out by means of an adjoining little table, in the following way.

The structure of this table of reduction is as follows.

Diagram. A small spherical-triangle figure illustrating the reduction. A nearly horizontal arc runs from A (at left) rightward; a short vertical segment near the right connects to the apex of the arc. The labeled points are:

- A (left vertex): vertex of the triangle
- B (upper right, on the arc): top of the vertical, on the Moon's orbit
- C (lower middle, on the baseline): foot of the perpendicular, on the ecliptic
- D (upper right corner): endpoint above
- E (right end of the baseline): endpoint of the ecliptic arc

In the spherical triangle ABC, the angle at A is given — $5^{\circ} 8'$, the mean latitude of the Moon between its maximum and minimum. Likewise the side AB is given — the Moon's true distance from the node — with the angle at C a right angle. Therefore the side AC, on the ecliptic, is given; and so the difference between AB and AC, which is the reduction that appears in the table, results.

Its use, after Tycho Brahe, is this.

When I wish to reduce the Moon's place — computed from our tables in its own orbit — to the ecliptic, I enter the adjoining table with the true and equated motion of latitude, looking up, in the usual way, the sign at the head or foot, and the degree by descending or ascending. The minutes together with the seconds found in the common cell I add to, or subtract from, the place of the Moon found by calculation, so as to obtain its true position with respect to the ecliptic. I do the contrary, however, with these headings A and S, when a place of the Moon observed by instruments is to be reduced from the ecliptic and its poles to its own

proper orbit. By this concise procedure you will nowhere err from the truth by more than half a minute. But if you wish to follow the utmost precision, then in place of the angle $5^{\circ} 8'$ — which in constructing this table we have treated as invariable — use the true latitude found for the given time, and, according to the doctrine of spherical triangles, seek either the base from both sides, or, from the base and the side of latitude, the side of longitude, as the case requires.

The little table of reduction of the Moon from its own orbit to the ecliptic, and vice versa:

Verus motus	0 / 6 — M. S.	1 / 7 (Subtr.) — M. S.	2 / 8 — M. S.	Verus motus
0	0 0	6 6	6 5	30
1	0 15	6 12	5 57	29
2	0 30	6 18	5 48	28
3	0 45	6 24	5 39	27
4	0 59	6 29	5 30	26
5	1 13	6 35	5 21	25
6	1 27	6 40	5 12	24
7	1 42	6 44	5 1	23
8	1 56	6 47	4 51	22
9	2 10	6 51	4 40	21
10	2 24	6 54	4 29	20
11	2 38	6 56	4 18	19
12	2 52	6 57	4 7	18
13	3 6	6 58	3 55	17
14	3 19	6 59	3 42	16
15	3 32	7 0	3 31	15
16	3 43	6 59	3 18	14
17	3 56	6 58	3 5	13
18	4 8	6 57	2 51	12
19	4 19	6 56	2 38	11
20	4 30	6 54	2 23	10
21	4 41	6 51	2 9	9
22	4 52	6 47	1 55	8

Verus motus	0 / 6 — M. S.	1 / 7 (Subtr.) — M. S.	2 / 8 — M. S.	Verus motus
23	5 2	6 44	1 41	7
24	5 13	6 40	1 26	6
25	5 22	6 35	1 12	5
26	5 31	6 28	0 58	4
27	5 40	6 23	0 45	3
28	5 49	6 17	0 30	2
29	5 58	6 11	0 15	1
30	6 6	6 5	0 0	0
Latitud.	11 / 5	10 / 4 (Adde)	9 / 3	Latitud.

CAPVT VIII.

On the parallaxes of the Sun and Moon, both with respect to the circle of altitude and to the zodiac. Likewise on the refraction of the Sun, Moon, and stars.

Up to this point we have treated the true motions of the stars and luminaries from their restored hypotheses — that is, insofar as they regard the center of the Earth. Now we shall briefly append what deviation in these falls upon the eye: partly by reason of the magnitude and bulging of the Earth, from which the parallaxes arise, and partly by reason of the vapor diffused around it, on which the consideration of refractions depends. For although we have taught about both of these generally in the doctrine of the sphere [Margin: *Lib. 2, Sphae., cap. 10.*], insofar as they depend on observations, I nevertheless think they should be treated specifically here as well — and that the variation in the Sun and Moon should be bound down to tables built upon geometrical demonstrations. This is on account of the doctrine of eclipses that follows shortly, now that the fixed distances of each luminary from the Earth, in the semidiameters of the Earth, have been given above. For since the inference of these distances stands in inverse proportion to their parallaxes in each phenomenon — so that a smaller distance of the same body from the Earth causes greater parallaxes in the vertical circle, and vice versa — the first demonstration concerning these, in each luminary, will accordingly be carried out, for a beginning, in this way.

Diagram. A quadrant figure demonstrating parallax. From center A (lower left) two perpendicular lines are drawn: AD vertical (upward) and AF horizontal (to

the right). About A a quadrant arc DF is described — the quadrant of the Sun's circle; within it the arc CG for the Moon; and within that the arc BK for the terrestrial globe. The line A–H–E is drawn from the Earth's center A, locating the Sun at E and the Moon at H according to their true motions. From the Earth's surface at B, lines are drawn to each luminary — BE to the Sun, BH to the Moon. The labeled points are:

- A (lower-left, center): center of the Earth
- B (left, on innermost arc): point on the Earth's surface
- C (upper left, on the Moon's arc): point of the Moon's quadrant CG
- D (top): top of the Sun's quadrant (vertical from A)
- E (upper middle, on the Sun's arc): the Sun, on line AHE
- F (lower right): foot of the Sun's quadrant (horizontal from A)
- G (lower middle, on baseline): foot of the Moon's arc CG
- H (middle, on the Moon's arc): the Moon, on line AHE
- K (lower left, near A on baseline): foot of the terrestrial arc BK

Here the angle BEA is the angle of the Sun's parallax, and BHA that of the Moon. Their proportional magnitudes depend on the proportion of the Earth's semidiameter AB to the lines AH and AE. For when this semidiameter vanishes — as it does in comparison with the semidiameter of the orbit of the fixed stars — all the angle of parallax is exhausted as well.

The method, then, of deriving the parallaxes in the circle of altitude — or rather of constructing a table of the parallaxes of the Sun and Moon from triangles — is grasped by the following demonstration. In the plane triangle AEB, since two sides are given — AB, the Earth's semidiameter, as one part of those of which AE, the mean distance of the Sun from the Earth, defined by us [Margin: *Cap. 6, lib. hujus.*] as 1288, is another — together with the included angle BAE (which is measured by the true distance of the Sun from the zenith, DE, that is, the complement of the distance from the horizon, FE, in the vertical circle): therefore the angle called the parallactic angle, AEB, also emerges. In exactly the same way, in other phenomena — as for the Moon — given from the hypothesis the elongation AH of the Moon in defined semidiameters of the Earth from its center, together with AB and the angle comprehended (by hypothesis) in the vertical circle, the sought parallax-angle AHB is investigated. Moreover, although according to the hypothesis of the Sun the difference from the mean to the maximum and minimum is found to be 46 semidiameters of the Earth, nevertheless — since on account of the Sun's more remote distance from the

Earth this inequality from the mean nowhere amounts to more than 6 seconds — the mean elongation of the Sun, fixed above at 1288 semidiameters,

For the Moon, whose mean distance from the Earth I had set above at 56 whole Earth-semidiameters — a figure perhaps the best established of all to me, drawn from many and most careful observations — we believe we shall proceed most correctly if we follow Tycho Brahe's earlier reckoning of the Moon's parallaxes in the circle of altitude across a range of 4 whole diameters either side: that is, from 52 semidiameters at the nearest distance up to a maximum of 60 semidiameters. [Margin: See Tycho's *Progymnasmata*, p. 123.] From this it is easy to compute the Moon's individual distances from the Earth as the hypothesis requires — first in semidiameters and minutes, and then, once the appropriate parallaxes for these are given, from what follows. This is especially useful for those who find it tedious to hunt down the Moon's distance from the Earth out of the hypothesis itself, by resolving a single triangle and reducing the result to semidiameters.

Now, since it is established from both branches of our lunar hypothesis that the limits at the quadratures lie farther out — both below and above — than those which obtain at conjunction and opposition, the remaining quantities in this synopsis are given on the assumption, confirmed by repeated experience, that the Moon's mean distance from the Earth is 56 Earth-semidiameters. The Moon's distance from the Earth will therefore be:

		G	M	
		r.	i.	
Maximum at	conjunction & opposition [$\odot$ & $\oslash$]	5	3	Earth-semidiameters
		7	8	
	quadrature [$\square$]	6	4	Earth-semidiameters
		0		
Minimum at	conjunction & opposition [$\odot$ & $\oslash$]	5	2	Earth-semidiameters
		4	3	
	quadrature [$\square$]	5	5	Earth-semidiameters
		1	7	

Therefore the difference between the first and second limit — that is, the quadrature [$\square$] versus conjunction and opposition [$\odot$ & $\oslash$] at apogee and perigee — is $2^{\circ} 26'$. Further, between the second and third limit — that is, between the new or full Moon at apogee and at perigee — the difference is $3^{\circ} 15'$ of semidiameters; and the greatest difference of all is $8^{\circ} 7'$ of semidiameters. Although I recall that a slightly smaller value than this can be drawn from the

Tychonic observations — as indeed Tycho himself records in several places — the difference is still not so great as to undermine our hypothesis. On the contrary, it serves to confirm its truth, far beyond the Copernican value and still more beyond the Ptolemaic.

General method of representing the Moon's distance from the Earth in Earth-semidiameters and their parts, for each of its positions under the hypothesis.

Since the proportional minutes [scrupula proportionalia] attached to the earlier table of the Moon's latitude almost never differ appreciably from those needed for the present task on either side, they are to be applied here too. So, first, the Moon's true distance from the Sun is given — what we have throughout called above its *ecliptic* distance. Then, through this same distance, the corresponding proportional minutes are taken, and by them the difference between the first and second limit (or between the third and fourth), which is $2^{\circ} 26'$ of semidiameters, is always multiplied. The resulting proportional part is to be added to the apogee distance at new or full Moon — namely $57^{\circ} 38'$ of semidiameters — but subtracted from the perigee value, $54^{\circ} 23'$. And so the difference between these is sought.

The Moon's anomaly, corrected by the ecliptic prosthaphaeresis [profthaphaeresis], will then be at hand. We first reduce it by a semicircle, then halve the remainder, and enter with that residue into the same earlier table of the Moon, here too seeking the corresponding proportional minutes. These, multiplied into the difference found earlier, yield the proportional part, which is always added to the Moon's least previously corrected distance from the Earth. In this way the Moon's true elongation from the Earth is obtained compactly, in semidiameters and the minutes thereof.

Thirdly, once the Moon's distance from the Earth has been found in these semidiameters, it is very easy to apply the corresponding parallaxes in the circle of altitude to any distance from the horizon. For the table contains them for each whole diameter from the Earth and each degree from the horizon. The due correction is then made simply for the proportional fractions that attach. Everything will be made clear by the following example.

Example.

Let the Moon's parallax in the circle of altitude be sought for the year 1608, on 12 February, at 8 h 43 m — when we reported, from a careful observation, that the

Moon was conjoined by its upper horn with Aldebaran, in the second of the examples set out earlier on longitude.

	Sig.	gr.	Mi.	Sec.
The Sun's distance from the Moon was then	2	28½		
Corrected anomaly [anomalía coaequata]	8	23	2	
True altitude of the Moon above the horizon, about		39¾		
Proportional minutes answering to the distance of the luminaries			59	56
This, multiplied into the difference — namely 2° 26' of semidiameters — gives			2	26 (nearly)

Which, when added to the second limit — namely the semidiameters 57 38 — *that one will be*: 60° 4' of semidiameters.

And let there be subtracted from the third limit, the Moon's distance from the Earth 54 23 — *this one*: 51° 57' of semidiameters.

Therefore the difference: 8 Gr. 7 Mi. of semidiameters.

	Sig.	Gr.	Mi.
Corrected anomaly	8	23	2
Semicircle to be subtracted	6	0	0
Remainder	2	23	2
Half	[?]	11	31
To which answer the proportional minutes		26	50 (Sec.)
Multiplied by the difference		8	7 (Semid.)
There result, of Earth-semidiameters, to be added		3	38
Fourth limit, corrected		51	57
Therefore there are made up, of Earth-semidiameters		55	35

These prove the Moon's true distance from the Earth in these semidiameters, and so on. To this distance of the Moon from the Earth, and to its altitude above the

horizon — given as $39\frac{3}{4}^{\circ}$ — there answers a parallax of the Moon in the circle of altitude of 48 minutes 7 seconds.

By this example all the rest are to be carried out generally, outside a solar eclipse. In an eclipse, however, nothing is needed except the proportional minutes that emerge through the corrected anomaly of the Moon. When these have been multiplied into the difference of the second and third limit, which is $3^{\circ} 15'$, they show the proportional part always to be added to the third limit, $54^{\circ} 23'$. Once this is done, the parallax in the circle of altitude is likewise sought from the following table, by means of the Moon's true distance from the Earth so found. We shall add an example below, in connection with the solar eclipse.

Now we shall append the table of the parallaxes of the Sun and Moon in the circle of altitude. After it, we shall briefly demonstrate the method by which these same parallaxes come to be distinguished, in length and breadth, across the ecliptic.

[Margin: From the table of parallaxes that follows.]

Table of the parallaxes of the Sun and Moon in the circle of altitude.

Column headings: **G.** = degree of altitude; **Parallaxis Solis media in circulo altitudinis** (the Sun's mean parallax in the circle of altitude), given in M(inutes) S(econds) — but here the leftmost data column is the Sun's parallax expressed as 2 M and a seconds value; **Parallaxes Lunae competentes integris semidiametris terrae in circulo verae altitudinis ab horizonte** (the Moon's parallaxes corresponding to whole Earth-semidiameters in the circle of true altitude from the horizon), in columns for 52–60 semidiameters, each in M(inutes) S(econds); **Scrupula longitudinis & latitudinis** (proportional minutes of longitude and latitude), in M S.

This first table covers degrees of altitude 0–30; the *Residuum* (page 280) continues with degrees 30–60. Table continues into the *Residuum* below.

So		5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6		
1	So	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	0	0	Scr	Scr
M	1 S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	. M	. S
2	40	6	6	6	5	6	3	6	3	6	2	6	2	5	1	5	1	5	1	0	0
		6		4	1	3	9	2	0	1	3	0	0	9	7	8	6	7	8		
2	40	6	6	6	5	6	3	6	2	6	2	6	1	5	1	5	1	5	1	1	3
		6		4	1	3	9	2	9	1	3	0	9	9	7	8	6	7	8		

So		5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6		
l	So	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	0	0	Scr	Scr
M	lS	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	. M	. S
2	40	6	5	6	5	6	3	6	2	6	2	6	1	5	1	5	1	5	1	2	5
		6		4	0	3	8	2	8	1	2	0	8	9	6	8	5	7	7		
2	40	6	4	6	5	6	3	6	2	6	2	6	1	5	1	5	1	5	1	3	8
		6		4	0	3	8	2	8	1	1	0	7	9	5	8	4	7	6		
2	39	6	1	6	4	6	3	6	2	6	1	6	1	5	1	5	1	5	1	4	11
		6		4	7	3	5	2	5	1	8	0	5	9	2	8	1	7	3		
2	39	6	5	6	4	6	3	6	2	6	1	6	1	5	8	5	7	5	9	5	13
		5	7	4	3	3	1	2	1	1	4	0	1	9		8		7			
2	39	6	5	6	3	6	2	6	1	6	1	6	7	5	3	5	3	5	5	6	15
		5	2	4	8	3	6	2	6	1	0	0		9		8		7			
2	38	6	4	6	3	6	2	6	1	6	4	6	1	5	5	5	5	5	0	7	18
		5	6	4	2	3	0	2	0	1		0		8	7	7	8	7			
2	38	6	3	6	2	6	1	6	3	6	5	5	5	5	5	5	5	5	5	8	21
		5	8	4	4	3	2	2		0	7	9	4	8	0	7	1	6	3		
2	37	6	2	6	1	6	3	6	5	6	4	5	4	5	4	5	4	5	4	9	23
		5	9	4	5	3		1	4	0	8	9	6	8	2	7	3	6	4		
2	37	6	1	6	5	6	5	6	4	6	3	5	3	5	3	5	3	5	3	10	25
		5	9	4		2	3	1	5	0	9	9	7	8	3	7	4	6	5		
2	36	6	8	6	5	6	4	6	3	6	2	5	2	5	2	5	2	5	2	11	27
		5		3	4	2	2	1	4	0	8	9	7	8	3	7	4	6	5		
2	36	6	5	6	4	6	3	6	2	6	1	5	1	5	1	5	1	5	1	12	28
		4	5	3	1	2	0	1	2	0	6	9	6	8	2	7	3	6	4		
2	35	6	4	6	2	6	1	6	9	6	3	5	3	5	5	5	1	5	2	13	30
		4	1	3	8	2	7	1		0		9		7	9	7		6			
2	35	6	2	6	1	6	3	6	5	5	4	5	4	5	4	5	4	5	4	14	31
		4	6	3	4	2		0	5	9	9	8	8	7	5	6	7	5	9		
2	34	6	1	6	5	6	4	6	3	5	3	5	3	5	3	5	3	5	3	15	32
		4	0	2	7	1	7	0	9	9	4	8	2	7	0	6	2	5	5		
2	34	6	5	6	4	6	3	6	2	5	1	5	1	5	1	5	1	5	2	16	33
		3	3	2	0	1	1	0	2	9	8	8	6	7	4	6	6	5	0		
2	33	6	3	6	2	6	1	6	4	5	1	5	5	5	5	5	0	5	4	17	33
		3	5	2	2	1	3	0		9		7	9	6	7	6		5			

So		5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6		
l	So	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	0	0	Scr	Scr
M	l S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	. M	. S
2	32	6	1	6	3	6	5	5	4	5	4	5	4	5	4	5	4	5	4	18	32
		3	5	2		0	4	9	5	8	2	7	0	6	0	5	3	4	7		
2	31	6	5	6	4	6	3	5	2	5	2	5	2	5	2	5	2	5	2	19	32
		2	4	1	3	0	4	9	7	8	2	7	1	6	1	5	4	4	9		
2	31	6	3	6	2	6	1	5	6	5	1	5	1	5	1	5	4	5	1	20	31
		2	2	1	1	0	2	9		8		7		6		5		4	0		
2	30	6	8	6	5	5	4	5	4	5	4	5	4	5	4	5	4	5	5	21	30
		2		0	8	9	9	8	4	7	0	6	0	5	0	4	4	3	0		
2	29	6	4	6	3	5	2	5	2	5	1	5	1	5	1	5	2	5	2	22	28
		1	4	0	4	9	6	8	1	7	8	6	8	5	8	4	2	3	8		
2	28	6	1	6	9	5	2	5	5	5	5	5	5	5	5	5	5	5	5	23	26
		1	9	0		9		7	7	6	4	5	4	4	4	3	9	3			
2	27	6	5	5	4	5	3	5	3	5	2	5	2	5	2	5	3	5	4	24	23
		0	2	9	2	8	6	7	1	6	9	5	9	4	9	3	4	2	0		
2	26	6	2	5	1	5	9	5	5	5	3	5	4	5	4	5	9	5	1	25	21
		0	4	9	5	8		7		6		5		4		3		2	5		
2	25	5	5	5	4	5	4	5	3	5	3	5	3	5	3	5	4	5	5	26	18
		9	5	8	7	7	1	6	8	5	6	4	8	3	9	2	4	1	0		
2	24	5	2	5	1	5	1	5	1	5	8	5	1	5	1	5	1	5	2	27	15
		9	5	8	7	7	2	6	0	5		4	1	3	3	2	8	1	5		
2	23	5	5	5	4	5	4	5	4	5	3	5	4	5	4	5	5	5	5	28	11
		8	4	7	6	6	2	5	0	4	9	3	2	2	5	1	1	0	9		
2	21	5	2	5	1	5	1	5	9	5	9	5	1	5	1	5	2	5	3	29	7
		8	2	7	4	6	1	5		4		3	2	2	6	1	3	0	2		
2	19	5	4	5	4	5	3	5	3	5	3	5	4	5	1	5	5	5	5	30	0
		7	8	6	2	5	9	4	7	3	8	2	2	1	7	0	4	0			

Residuum of the table of the parallaxes of the Sun and Moon.

This continues the preceding table for degrees of altitude 30–60. Column structure is identical (the row for G. 30 repeats from the foot of the previous table for continuity).

So		5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6		
l	So	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	0	0	Scr	Scr
M	lS	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	. M	. S
2	19	5	4	5	4	5	3	5	3	5	3	5	4	5	4	5	5	5	5	30	0
		7	8	6	2	5	9	4	7	3	8	2	2	1	7	0	4	0			
2	17	5	1	5	9	5	1	5	4	5	6	5	1	5	1	5	2	4	3	30	51
		7	3	6		5	6	4		3		2	0	1	6	0	4	9	5		
2	16	5	3	5	2	5	3	5	3	5	3	5	3	5	4	4	5	4	7	31	46
		6	8	5	4	4	2	3	0	2	3	1	7	0	4	9	2	9			
2	14	5	2	5	5	5	5	5	5	5	5	5	4	5	1	4	1	4	3	32	40
		6		4	8	3	7	2	5	1	9	1		0	0	9	9	8	4		
2	12	5	2	5	2	5	2	5	2	5	2	5	3	4	3	4	4	4	0	33	32
		5	5	4	1	3	1	2	0	1	4	0	0	9	5	8	6	8			
2	11	5	4	5	4	5	4	5	4	5	4	4	5	4	5	4	1	4	2	34	23
		4	6	3	3	2	3	1	4	0	8	9	5	8	9	8	2	7	5		
2	9	5	5	5	4	5	4	5	7	5	1	4	1	4	2	4	3	4	4	35	15
		4		3		2		1		0	1	9	8	8	2	7	7	6	9		
2	7	5	2	5	2	5	2	5	2	4	3	4	4	4	4	4	1	4	1	36	6
		3	4	2	4	1	4	0	9	9	3	8	1	7	6	7		6	4		
2	6	5	4	5	4	5	4	4	4	4	5	4	3	4	9	4	2	4	3	36	56
		2	2	1	3	0	4	9	9	8	4	8		7		6	4	5	8		
2	5	5	0	5	1	5	3	4	8	4	1	4	2	4	3	4	4	4	0	37	45
		2		1		0		9		8	5	7	4	6	1	5	6	5			
2	3	5	1	5	1	4	2	4	2	4	3	4	4	4	5	4	8	4	2	38	34
		1	7	0	8	9	1	8	7	7	5	6	4	5	3	5		4	2		
2	1	5	3	4	3	4	3	4	4	4	5	4	3	4	1	4	2	4	4	39	22
		0	2	9	4	8	8	7	5	6	3	6		5	4	4	8	3	3		
1	59	4	4	4	4	4	5	4	2	4	1	4	2	4	3	4	4	4	4	40	9
		9	6	8	9	7	4	7		6	0	5	1	4	4	3	7	3			
1	57	4	5	4	3	4	9	4	1	4	2	4	3	4	5	4	6	4	2	40	54
		8	9	8		7		6	8	5	7	4	8	3	2	3		2	4		
1	55	4	1	4	1	4	2	4	2	4	4	4	5	4	1	4	2	4	4	41	40
		8	1	7	6	6	3	5	3	4	3	3	4	3	2	2	3	1	1		
1	53	4	2	4	2	4	3	4	4	4	5	4	1	4	2	4	4	4	0	42	25
		7	3	6	9	5	6	4	6	3	8	3	2	2	8	1	4	1			

So		5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	6	6		
l	So	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	0	0	Scr	Scr
M	lS	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	M	S	. M	. S
1	51	4	3	4	4	4	4	4	5	4	1	4	2	4	4	4	5	4	1	43	10
		6	4	5	1	4	8	3	9	3	2	2	8	1	4	0	8	0	8		
1	49	4	4	4	5	4	0	4	1	4	2	4	4	4	5	4	1	3	3	43	53
		5	4	4	1	4		3	1	2	5	1	3	0	8	0	4	9	5		
1	47	4	5	4	1	4	1	4	2	4	3	4	5	4	9	3	2	3	4	44	35
		4	3	4		3	1	2	3	1	8	0	7	0		9	4	8	4		
1	45	4	1	4	1	4	2	4	3	4	5	4	9	3	2	3	3	3	5	45	17
		4		3	0	2	1	1	4	0	0	0		9	4	8	8	7	8		
1	43	4	8	4	1	4	3	4	4	4	1	3	2	3	3	3	5	3	1	45	58
		3		2	8	1	0	0	4	0		9	0	8	7	7	8	7	9		
1	41	4	1	4	2	4	3	3	5	3	1	3	3	3	4	3	1	3	3	46	38
		2	4	1	6	0	9	9	4	9	1	8	0	7	9	7	1	6	2		
1	38	4	2	4	3	3	4	3	3	3	2	3	4	3	0	3	2	3	4	47	17
		1	0	0	3	9	7	9		8	0	7	0	7		6	3	5	5		
1	36	4	2	3	3	3	5	3	1	3	2	3	4	3	1	3	3	3	5	47	55
		0	5	9	9	8	4	8	1	7	8	6	9	6	0	5	4	4	7		
1	34	3	2	3	4	3	0	3	1	3	3	3	5	3	2	3	4	3	9	48	32
		9	9	8	3	8		7	8	6	7	5	8	5	0	4	4	4			
1	32	3	3	3	4	3	5	3	2	3	4	3	7	3	2	3	5	3	2	49	9
		8	2	7	7	7		6	4	5	5	5		4	9	3	4	3	0		
1	29	3	3	3	5	3	1	3	3	3	5	3	1	3	3	3	4	3	3	49	44
		7	4	6	0	6	0	5	0	4	2	4	5	3	8	3		2	0		
1	27	3	3	3	5	3	1	3	3	3	5	3	2	3	4	3	1	3	4	50	19
		6	6	5	2	5	4	4	5	3	7	3	3	3	7	1	3	1	0		
1	25	3	3	3	5	3	1	3	3	3	2	3	2	3	5	3	2	3	4	50	53
		5	7	4	4	4	7	3	9	3		2	9	2	5	1	1	0	9		
1	23	3	3	3	5	3	1	3	4	3	7	3	3	3	1	3	2	2	5	51	26
		4	7	3	6	3	9	2	3	2		1	4	1		0	8	9	7		
1	20	3	3	3	5	3	2	3	4	3	1	3	3	3	6	2	3	2	5	51	58
		3	7	2	8	2	1	1	9	1	1	0	8	0		9	5	9			

LIBER PRIMUS [Pars Altera, Theoricorum, Cap. VIII] — continued

Remainder of the table of parallaxes of the Sun and Moon [Residuum tabulae parallaxium Solis & Lunae]

This continues (and completes) the great parallax table begun on the preceding page. The first column gives the **mean parallax of the Sun in the circle of altitude** [parallaxis Solis media in circulo altitudinis], indexed by the argument in the leftmost sub-column (degrees, minutes, seconds — G.M.S.). The nine wide columns give the **parallaxes of the Moon corresponding to whole Earth-semidiameters** [parallaxes Lunae competentes integris semidiametris terrae], in the circle of altitude reckoned from the horizon, for distances of 52 through 60 Earth-semidiameters; each cell is minutes and seconds (M. S.). The final column gives the **proportional minutes of longitude and latitude** [Scrupula longitudinis & latitudinis], in minutes and seconds.

The argument column runs from 60 down to 90 (the argument of altitude/zenith distance), the row labeled 90 being the zenith, where every parallax vanishes.

Ar											
g. (G)	Sun (M.S.)	52 (M.S.)	53	54	55	56	57	58	59	60	Scrup. long. & lat. (M.S.)
			(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	(M. S.)	
60	1 20	33 37	32 58	32 21	31 46	31 11	30 38	30 6	29 35	29 5	51 58
61	1 18	32 36	31 58	31 22	30 48	30 14	29 42	29 11	28 41	28 12	52 28
62	1 15	31 34	30 58	30 23	29 49	29 17	28 46	28 16	27 47	27 19	52 59
63	1 13	30 31	29 57	29 23	28 50	28 19	27 49	27 20	26 52	26 25	53 28
64	1 10	29 29	28 56	28 23	27 50	27 21	26 52	26 24	25 57	25 31	53 56
65	1 8	28 26	27 54	27 22	26 50	26 22	25 54	25 27	25 1	24 36	54 22
66	1 5	27 22	26 26	26 26	25 25	25 25	24 24	24 24	24 24	23 23	54 49

Ar g. (G)	Sun (M.S.)	52 (M.S.)	53 (M. S.)	54 (M. S.)	55 (M. S.)	56 (M. S.)	57 (M. S.)	58 (M. S.)	59 (M. S.)	60 (M. S.)	Scrup. long. & lat. (M.S.)
			51	21	50	23	56	29	4	40	
67	1 3	26 18	25	25	24	24	23	23	23	22	55 14
			48	19	50	23	57	32	7	44	
68	1 0	25 13	24	24	23	23	22	22	22	21	55 38
			44	16	49	23	58	34	10	48	
69	0 58	24 8	23	23	22	22	21	21	21	20	56 0
			40	13	48	22	59	36	13	52	
70	0 55	23 2	22	22	21	21	20	20	20	19	56 23
			35	10	46	21	59	37	15	55	
71	0 53	21 56	21	21	20	20	19	19	19	18	56 43
			30	6	43	20	59	38	17	58	
72	0 50	20 49	20	20	19	19	18	18	18	18	57 3
			25	2	40	19	59	39	19	0	
73	0 47	19 41	19	18	18	18	17	17	17	17	57 22
			19	58	36	18	58	39	20	2	
74	0 44	18 32	18	17	17	17	16	16	16	16	57 40
			13	54	32	16	56	38	20	4	
75	0 41	17 22	17	16	16	16	15	15	15	15	57 58
			6	49	28	14	53	36	20	5	
76	0 38	16 15	15	15	15	15	14	14	14	14	58 13
			59	43	24	11	50	35	20	7	
77	0 36	15 8	14	14	14	14	13	13	13	13	58 27
			52	36	19	7	47	34	20	8	
78	0 33	14 1	13	13	13	13	12	12	12	12	58 41
			45	29	14	2	43	33	20	9	
79	0 30	12 52	12	12	12	11	11	11	11	11	58 53
			37	23	9	57	41	31	19	9	
80	0 28	11 43	11	11	11	10	10	10	10	10	59 5
			29	16	4	52	39	29	18	8	
81	0 25	10 33	10	10	9	9	9	9	9	9 7	59 16
			21	9	58	47	36	26	17		

Ar g. (G)	Sun (M.S.)	52 (M.S.)	53 (M. S.)	54 (M. S.)	55 (M. S.)	56 (M. S.)	57 (M. S.)	58 (M. S.)	59 (M. S.)	60 (M. S.)	Scrup. long. & lat. (M.S.)
82	0 22	9 23	9 13	9 2	8 53	8 42	8 33	8 24	8 15	8 7	59 25
83	0 19	8 13	8 5	7 55	7 48	7 37	7 29	7 21	7 13	7 7	59 33
84	0 16	7 3	6 57	6 48	6 43	6 32	6 25	6 18	6 10	6 6	59 40
85	0 13	6 52 [recte: 5 52?]	5 47	5 40	5 36	5 27	5 21	5 16	5 8	5 5	59 46
86	0 16 [recte: 0 10?]	4 42	4 37	4 32	4 29	4 22	4 17	4 13	4 7	4 4	59 51
87	0 9	3 32	3 28	3 24	3 21	3 17	3 13	3 10	3 6	3 3	59 55
88	0 6	2 21	2 18	2 16	2 14	2 11	2 8	2 6	2 3	2 2	59 57
89	0 3	1 10 [recte: 1 9?]	1 10	1 8	1 7	1 5	1 4	1 3	1 1	1 1	59 59
90	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	60 0

282 — THEORICORVM Cap. 8.

On the distinction of parallaxes according to longitude and latitude, chiefly with respect to the ecliptic [De discrimine parallaxium secundum longitudinem & latitudinem in primis respectu Eclipticae]

That, then, was the method for working out the parallaxes of the Sun and Moon in the vertical circle, from the given distance of each luminary from the Earth. Now I shall briefly indicate how these same parallaxes are distributed, in the most general and most compact way, along longitude and latitude according to

the ecliptic. I am well aware how much labor [Ptolemy], Copernicus, Reinhold [Erasmus Reinhold], [Tycho Brahe], and — last of all — [Johannes Kepler] expended on this matter. [Margin: *Lib. Revol. c.* — Copernicus, *De Revolutionibus*.] Nevertheless, once the parallax in the vertical circle is given (which is often required in its own right), we can arrive at the resolution of that same parallax by the same ease, but with a demonstration better suited to the young. I therefore prefer, with Copernicus, to attend to two things especially in this matter, which must first be sought out: namely the altitude of the phenomenon, and the angle that lies between the vertical circle and the ecliptic (or any circle of longitude). For once this alone is had, together with the known parallax in the vertical circle, the distribution of that parallax through longitude and latitude is undertaken with almost no trouble, as will be shown below. First, then, I must demonstrate how that same angle is found, for the purpose at hand, in any given revolution.

Diagram. An uncaptioned spherical-triangle construction illustrating the resolution of parallax into longitude and latitude components. A nearly vertical curved spike rises to the apex **A** (the zenith of the place) at top. From **A** descend three vertical arcs to the horizon arc at the bottom: **A C H**, the arc through the ninetieth degree (nonagesimal), where no parallax of longitude is given; **A B F**, the meridian; and **A D E**, the vertical circle in which the phenomenon is here taken, for the example, to lie. The horizon is the bottom arc **E F H**. Near the foot of the vertical circle, the small triangle formed by points **L**, **D**, **K** (with a dotted line running from **L** through **K** toward **G**) marks the phenomenon's position: its parallax in the vertical circle is **D K**, the latitude-component is **K L**, and the longitude-component is **D L**.

Labels, with position:

- **A** (apex, top): vertex of the place [zenith]
- **C** (upper right, on the nonagesimal arc): point on arc **A C H**
- **B** (upper middle, on the meridian): point on the meridian **A B F**
- **G** (upper right, to the right of **C**): endpoint of the small construction line
- **L** (left, mid-height): vertex of the small parallax triangle (longitude end)
- **D** (left, on the vertical circle **A D E**): foot-point; **D K** = parallax in the vertical circle
- **K** (below **L–D**): vertex; **K L** = latitude-component, **D L** = longitude-component
- **E** (lower left, on the horizon): foot of the vertical circle **A D E**
- **F** (lower middle, on the horizon): foot of the meridian **A B F**

- H (lower right, on the horizon): foot of the nonagesimal arc A C H

[Margin (beside the figure): *Vide 6 problem. lib. 2. Sphaericor.* — See problem 6 of book 2 of the Spherics.]

So, in the present figure, let A be the vertex of the place; D B C some circle of longitude, such as the ecliptic or the Moon's orbit; and E F H the horizon. Let three vertical arcs descend: A C H, of the ninetieth degree, where no parallax of longitude is given; A B F, the meridian; and A D E, the vertical circle, in which the phenomenon here, for the example, is fixed. Its parallax in the vertical circle is D K; the latitude-component is therefore K L, and the longitude-component is D L.

In the triangle A B D, then, for finding the angle A D B, the data needed are available [Greek: διδόμενα (didomena, "the given quantities")]. [Margin: *Per conversionem problematis primi c. 5. lib. 2. Sphaer.* — By the conversion of the first problem, chapter 5 of book 2 of the Spherics.] Given, from the doctrine of the sphere, the declination of the phenomenon, and the elevation of the pole of the place assumed, one easily finds both A B — its distance from the vertex along the meridian — and A D, the complement of the altitude, from the same doctrine. The third side, B D, is supplied by the very distance of the star from the meridian. For when the right ascension M C is present (which is always given for the proposed time), together with the degree of the ecliptic culminating at mid-heaven, that same side B D arises of itself from this and from the position of the phenomenon beneath the ecliptic. Thus, in the spherical triangle of given sides, the sought angle at D is given.

Now this has just been demonstrated somewhat more generally, for a Moon that also has latitude. But if the Moon is nearly free of any appreciable latitude — as happens in lunar eclipses — then it is better to bring the angle that the meridian makes with the ecliptic here from the table of the primum mobile [tabula primi mobilis], and afterward to apply duly the declination of the culminating degree; and finally to count off the arc from the said culminating point of the ecliptic to the position of the luminaries. [Margin: *Per probl. 6. lib. 2. Sphaer. cap. 10.* — By problem 6 of book 2 of the Spherics, chapter 10.] In this way the two sides A B and B D emerge, together with the angle A B D. Hence, by the resolution of triangle A B D, both the side A D — the complement of the altitude of the luminaries — and the angle at D become known. Indeed, once this same angle of intersection of the meridian with the ecliptic, and the side B A, have been brought to light, there is at once given, in the right triangle A B C (the right angle being at C), the side A C, the complement of the altitude of the ninetieth degree. From this, in turn, in the other right triangle A C D — since the base A D, namely the complement of the altitude,

and A C, and the right angle at C are given — the sought angle C D A is therefore given.

This is what was to be demonstrated.

Cap. 8. LIBER PRIMUS — 283

Example. [Exemplum.]

On the Moon having latitude [De luna latitudinem habente]

The worked example below sets out the quantities in degrees and minutes (Gr. Mi.); the right-hand symbols (♋, ♊, ♌, ♍, B) mark the relevant zodiac sign or the designation "Borealis" (northern).

In the year 1608, on 12 February, at hour 8, minute 43, at [Hafnia — Copenhagen], the true place of the Sun was:

Quantity	Gr.	Mi.	Sign / note
The true place of the Sun ☿, in ♋ [Pisces]	3	43	
Its right ascension [Ascensio recta ejusdem]	335	39	
Therefore, the hours and minutes being reduced, there results the right ascension of mid-heaven, M.C.	106	24	
whose culminating point in the ecliptic is	15	8	♋ [Cancer]
But the true place of the Moon according to longitude was	4	44	♊ [Gemini]
The difference, therefore, is the side B D	40	24	
Further, the declination of the culminating place, i.e. M.C., is	22	40	B [northern]
With respect to the Node ♌, which was in	23	40	♌ [Leo]
There is subtracted from the declination of M.C.	3	14	
Therefore the true declination of the Moon's orbit at M.C.	19	26	B [northern]
Which, when subtracted from the elevation of the pole of [Hafnia — Copenhagen]	55	43	
leaves the side B A	36	17	

Quantity	Gr.	Mi.	Sign / note
The side A D, the complement of the Moon's altitude, is likewise	50	15	
Therefore, in the triangle set out above, A D B — since all three sides are given — there results the sought angle A D B	50	10	

It will suffice to have carried out all of this — and likewise the sought angle — in whole degrees and first minutes only; for extreme precision contributes nothing toward altering the distribution of the parallax.

Now, with the angle A D B (that is, L D K) found — the angle always opposed to the parallax of latitude, namely L K — its complement will be the angle L K D, which in this example is indeed 39 degrees, 50 minutes. This angle is likewise opposed to the parallax of longitude L D. For in so small a triangle D L K, since the difference is imperceptible between the sides here taken as spherical and as rectilinear (a point that Copernicus too rightly noted [Margin: *Lib. 4. cap. 6.* — Copernicus, *De Revolutionibus* IV.6], and that we have learned by experience in solving this very triangle) — therefore, given one angle besides the one at L, which is always right, the third angle is given too. And when the side opposite the right angle, by which the parallax in the vertical circle is comprehended, is reckoned as one degree (or 60 minutes), it was according to this same standard that we added, in the preceding table of parallaxes, the proportional minutes for the remaining angles — occupying the last column — derived, that is, from the given right sines of the remaining angles, in the proportion in which 60 minutes correspond to the radius (or whole sine).

So whenever you wish to have the parallax of latitude, extract the proportional minutes that correspond to the angle A D B (or its opposite L D K). But when you want the parallax of longitude, enter the table with the complement of that angle — namely the angle at K — and seek out the proportional minutes; and these, each multiplied into the parallax of altitude, yield from the former the parallaxes of latitude and from the latter those of longitude — as in the present example:

	Gr	M		M	
	.	i.		i.	Sec.
Angle — Latitude	50	10	There correspond proportional minutes	46	5
Angle — Longitude	39	50	proportional minutes	38	26

Now, since the parallax of altitude of the Moon in this example was found earlier to be 48 minutes, 7 seconds, therefore, in multiplying the proportional minutes into that same value, [Margin: *Cap. 6. exemplo 2. lib. 1. hujus.* — chapter 6, example 2, book 1 of this work] there results a parallax of latitude of 36 minutes, 57 seconds, and a parallax of longitude of 30 minutes, 49 seconds. All of these, in this observation as in the rest, agree very well with the restitution of the lunar hypothesis [restitutio hypotheseos lunaris]—

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—agree very well [with the restitution of the lunar hypothesis]. I see that this prescribed method of distinguishing parallaxes is to be used most rightly whenever the Moon has a sufficiently appreciable latitude. For eclipses, however, I shall show another method below.

ON THE REFRACTIONS OF THE HEAVENLY BODIES, WITH THEIR TABLES [DE REFRACTIONIBUS SIDERUM CUM SUIS TABULIS]

The first obstacle — the one that, through the entanglement of parallaxes, intrudes itself into observations — having now been traced back to its causes and resolved and cleared away, a second follows: one entirely new, not yet found among observations before [Tycho Brahe], which indeed arises from the refractions of the heavenly bodies. These refractions, in turn, arise from terrestrial vapors, condensed most of all near the horizon; and it is there, accordingly, that the greatest refractions are detected — just as this too has been demonstrated generally in our *Sphere*. [Margin: *Lib. 2. Sphae. cap. 11.* — book 2 of the *Sphere*, chapter 11.]

But indeed, just as the doctrine of parallaxes affords a most beautiful contemplation — so much so that, given (besides the Earth's semidiameter) either the interval [diastema] of the phenomenon from the Earth, or the angle of diversity from its true motion, the other is obtained with geometric certainty — so, by contrast, the treatment of refractions has hitherto produced no fixed demonstrations, because of the highly variable constitution of the air in different places, such as a geometric account joined with experience everywhere demands. We therefore rely here rather on experience, and we shall retain the refractions just as they were found at [Uraniborg] and gathered into tables throughout the first book of the *Progymnasmata* [Tycho Brahe, *Astronomiae Instauratae*

Progymnasmata], leaving the deeper inquiry — should anyone prefer to learn one beyond that inserted into the doctrine of the sphere — quite willingly to the opticians and others. [Margin: *Legatur lib. 1. Prog. T. B. pag. 91. & seqq. de refractionibus.* — Let book 1 of the *Progymnasmata* of [Tycho Brahe], page 91 and following, on refractions, be read.] A synopsis, then, of the tables themselves for the Sun, the Moon, and the fixed stars follows — that is, of how far they insinuate themselves here in the circle of altitude.

These refractions apply separately to the Sun, the Moon, and the stars in the vertical circle of our hemisphere. They might perhaps vary by some small amount according to the unevenness of the air around sunrise and sunset, and likewise at different seasons of the year. But since, here among us, this has not been sufficiently confirmed by experience in these cases, I willingly defer on so doubtful a matter, and at this point at least give this warning: since the phenomena persist, in proportion to the magnitude of their apparent size, more or less proportionally in the increase and duration of the refractions, something is therefore to be added to the refractions of the planets over and above those values that are assigned here for the fixed stars — and especially for Venus, on account of its torch being brighter than those of the other five planets. As for how much is owed to each individual case, I leave that to the judgment of prudent astronomers — namely those whose charge is the observation of the heavens.

Table of refractions in the circle of altitude, under a pole elevation of 56.

G. ab hor.	Solis — M.	Solis — S.	Lunae — M.	Lunae — S.	Stellar. — M.	Stellar. — S.
0	34	0	33	0	30	0
1	26	0	25	0	21	30
2	20	0	20	0	15	30
3	17	0	17	0	12	30
4	15	30	15	20	11	0
5	14	13	14	20	10	0
6	13	30	13	50	9	0
7	12	45	12	45	8	15
8	11	15	12	0	6	45
9	10	30	11	20	6	0
10	10	0	10	45	5	30
11	9	30	10	10	5	0

G. ab hor.	Solis — M.	Solis — S.	Lunae — M.	Lunae — S.	Stellar. — M.	Stellar. — S.
12	9	0	9	35	4	30
13	8	30	9	0	4	0
14	8	0	8	30	3	30
15	7	30	8	0	3	0
16	7	0	7	30	2	30
17	6	30	7	0	2	0
18	5	45	6	30	1	15
19	5	0	6	0	1	0
20	4	30	5	30	0	0
21	4	0	5	0		
22	3	30	4	35		
23	3	10	4	10		
24	2	50	3	45		
25	2	30	3	20		
26	2	15	3	0		
27	2	0	2	40		
28	1	45	2	20		
29	1	35	2	0		
30	1	25	1	45		
31	1	15	1	30		
32	1	5	1	20		
33	0	55	1	10		
34	0	45	1	0		
35	0	35	0	50		
36	0	30	0	45		
37	0	25	0	40		
38	0	20	0	35		
39	0	15	0	30		
40	0	10	0	25		
41	0	9	0	20		

G. ab hor.	Solis — M.	Solis — S.	Lunae — M.	Lunae — S.	Stellar. — M.	Stellar. — S.
42	0	8	0	15		
43	0	7	0	10		
44	0	6	0	5		
45	0	5	0	0		

[Margin: See above, in the commentary on the refraction of the Sun, which we have attributed to Venus, as observed from the Sun.]

CHAPTER IX.

On the complete doctrine of the eclipses of the Sun and Moon.

Preface, in which are set out the usefulness and antiquity of eclipses, together with their general description and demonstration.

Were it not that I judge it the sole aim of my undertaking to pursue the causes — here, as elsewhere — in the motions, the magnitudes, and finally in the nature of light and darkness of the three world-bodies most affected with respect to one another, namely the Sun, the Moon, and the Earth (from which the eclipses of the luminaries, with their definite magnitudes, necessarily occur at fixed times according to the course of nature), I would assuredly, right here at the very threshold, enrich this doctrine of eclipses richly from the writings of [Pliny], [Plutarch], and other philosophers. [Margin: *Pliny, Book 2, ch. 10. Plutarch, On the Opinions of the Philosophers, Book 2, and elsewhere.*] To read these authors, then, I refer the lovers of the stars, who in place of causes will often gather only the astonishments of the crowd and the follies of the ancient sophists. For since these men were not ashamed to philosophize without any knowledge of astronomy — which it is by no means fitting for a philosopher to lack — the result was that their reasoning, not sufficiently supported by experience, lapsed in great measure into inept and counterfeit fabrications of causes.

To us, who hope to demonstrate these matters solidly, the end or usefulness of this doctrine first of all presents itself. It is not only the contemplation of the other causes in this matter, and the certainty — now beyond all error — of astronomy and of the motion of the luminaries that we have verified above, so that it strikes even the perception of the crowd; it is also the certification of dates in the annals, for the verifying of which the eclipses of the Sun and Moon are very rightly employed by chronologists as the best-known documents. Above all, however, there accrues that remarkable advantage of the restored doctrine of

eclipses: that chiefly from it the geographical tables concerning the longitudes of terrestrial places are refined and made whole again — to say nothing here of the foreknowing and most momentous significance drawn from eclipses, according to the astrologers, in matters of harvests, governments, and the other conditions of the lower world.

But since, in every age from the first origin of the world, eclipses of the luminaries have occurred year by year, and indeed in a roughly equal number (as I shall show below in the place of a corollary), we may surely and with good reason wonder why at last — soon after the elapse of the one-and-a-third Olympiad, and before the birth of Romulus and Remus, that is, in the year 3198 after the beginning of the world — they first of all began to be recorded in the writings of historians, and not even to be reported so frequently; or why their causes were not given before the ancient Babylonians in Syria, or even [Thales of Miletus] in Greece. For in the science of the stars the appearance of eclipses is, as [Josephus] reports, the most ancient of all, and both is and has been the best known. [Margin: *F. Josephus, Jewish Antiquities, Book 1, ch. 3.*] For my part I think that, in those first ages, besides the rarity of practitioners, either antiquity itself begrudged us these most useful monuments of antiquity, or the most lamentable fire stole away the most august library of Alexandria. For it is credible that — apart from the few items there which [Ptolemy] was able to copy out for his own use from the records observed by his astronomical predecessors — very many and most ancient records were foully consumed in the flames.

Be that as it may, even in this late age of the world the magnitude and dignity of these matters deservedly rouse me to the Plinian exclamation: [Margin: *Pliny, Book 2, ch. 12.*] "*Great men, who are interpreters of the heavens, who grasp the nature of things, finders of an argument by which they conquered gods and men in turn! For who, beholding these things and the labors of the stars (since it has so pleased us to call them so), would not pardon his own lot of mortal birth?*" Thus far Pliny.

The sum, moreover, of those things which ought to be set out in this doctrine, and which are thereafter to be demonstrated by us with their anomalies, appears also in the Plinian text in these few words: [Margin: *Book 2, ch. 10.*] "*It is manifest that the Sun is hidden by the interposition of the Moon, and the Moon by the interposition of the Earth, and that these are paid back in turn — the Moon by its own interposition taking away those same rays of the Sun, and the Earth taking them from the Moon.*" From these few words it is established what is the cause of the obscuration of both the Sun and the Moon: [Margin: *See near the end of this*

book, where it is briefly established.] namely, of the Sun, whenever by the law of the motions the Moon thrusts itself between the Sun and our gaze.

Since the light that is seen in the lunar body — otherwise opaque and devoid of light, yet suited, like a mirror, to receiving the solar rays — all flows down from the Sun, and indeed in such a way that about half of the Moon is always illumined by these rays (something easily detected in its various aspects with the Sun), it comes about that, in the visible conjunction of the luminaries beneath the ecliptic, the Moon — passing beneath the Sun from below, and turning its own opaque face toward the Earth — takes away from our sight just so much of the Sun as the visible magnitude of the Moon, in the straight visual line, permits.

Furthermore, the eclipse of the Moon occurs "by the interposition of the Earth" (as Pliny says) for this reason: that in the true opposition of the luminaries as well, the Moon runs into the shadow projected by the Earth toward the part turned away from the Sun, and formed into the figure of a whirl-cone. For since the entire celestial sphere is always illumined by the solar rays — except for that space over which the Earth's shadow extends, and since such a shadow, by its spreading and length, far exceeds from the Earth the region of the Moon (though it does not reach to the remaining stars) — it follows that the Moon, holding the point opposite the Sun beneath the ecliptic, passes through the said shadow, and is therefore for that time necessarily deprived of the solar rays and of the light it enjoys from them. So Pliny aptly argues that they "are paid back in turn," when the Earth here takes from the Moon those same solar rays which earlier the Moon, by its own interposition, took from the Earth — namely whenever the Sun, which of itself suffers nothing of the kind, is on that account hidden from mortals.

Concerning the cause of eclipses, this is established. Now the reason seems to need giving why those obscurations do not occur at every new moon in the Sun, nor likewise at every full moon in the Moon. Also why some are partial and some of them more universal — but in the Moon, total, and indeed sometimes with a longer delay, so that lunar eclipses occur more often than solar ones in the same regions. And finally, why these lunar eclipses present themselves to be seen equally, or at the same instant, in all the same hemispheres, whereas the solar ones — rare and wandering — do not befall observers equally, nor at the same instant, on the face of the Earth far separated.

To know these things, the first and foremost factor is the knowledge of the deflection of the lunar orbit from the ecliptic, set out generally before, in the doctrine concerning the latitude of the Moon. For since these direct syzygies of

the luminaries occur rather rarely near the two intersections of the Moon's path with the ecliptic — which we call the nodes, or commonly the Head and Tail of the Dragon — eclipses must therefore follow these same interpolated conditions of the revolutions. Further, from this same source is drawn the knowledge of both partial and universal obscurations. For the nearer to the nodes these syzygies or conjunctions of the luminaries are seen to be celebrated, the greater and more universal the eclipses become, because of a visible coupling closer to the node; and consequently the more partial, the more remote the said visible syzygies have been from the nodes — so much so that, if the quantity of the visible semidiameters of the Sun and Moon is exceeded by the apparent latitude in a solar eclipse, or that of the shadow and Moon by the true latitude of the Moon in its eclipse, no obscuration on either side can be expected.

Moreover, the greater magnitude of the Earth's shadow in the transit of the Moon causes the eclipses of the Moon itself to appear more frequent. In the Sun, by contrast, they are both rarer in the same tracts of the Earth and more wandering — both on account of its smaller apparent size and on account of the involvement of the parallaxes (about which, in the spherical matters likewise, it was treated above, and will be treated further hereafter). [Margin: *Book 2 of the Spherics, ch. 10; likewise ch. preceding, of this book.*] For to these the Moon, on account of its smaller distance from the Earth, is more subject than any other heavenly body. As for the rest — so that all these things may become clearer for the beginners, to whom we leave the matter — they are to be illustrated by the following demonstrations.

A hypotyposis of the eclipses of the Sun and Moon.

So, in the following diagram, which presents a general consideration of the foregoing, let two circles be drawn from A, the center of the Earth, beyond its surface. Of these, the first is understood to be subject perpetually, in the same plane, to the ecliptic, that is, the path of the Sun; the other, I G, representing the orbit of the Moon, [is] inclined on either side to the ecliptic [in latitude on both sides bending back from the ecliptic].

Hypothesis of the eclipses of the Sun and Moon.

Diagram. A large schematic of solar and lunar eclipse geometry. At the upper right is a small circle representing the Sun (*Sol*), with its center marked. From the Sun, two tangent lines run down to the left, converging far below to form a long, narrow shadow cone (the Earth's shadow) that tapers to a point at the bottom. At the center is the Earth, with two concentric circles drawn about its center A: the

inner is the ecliptic (the Sun's path), the outer is the Moon's orbit. The Moon is shown at several positions along its inclined orbit. The cone of the Earth's shadow is hatched/shaded along its length.

- *Sol* (upper right): the Sun, the small circle at top right.
- E (within the Sun's circle): the center of the Sun.
- M (within/below the Sun's circle): point on the solar configuration.
- P (above the Sun's circle): upper tangent point on the Sun.
- N (lower right of the Sun's circle): lower tangent point on the Sun.
- H (right, on a line extending down-right): point along the lower tangent/shadow line.
- *Via Lunae* (left, on the outer circle): "Path of the Moon" — label of the lunar orbit.
- *Ecliptica* (left, on the inner circle): "Ecliptic" — label of the ecliptic circle.
- I (upper left, on the lunar orbit): node/point I of the Moon's orbit.
- A (center): the center of the Earth.
- O (upper, near the Earth, on the orbit): position of the Moon / intersection point.
- K (center, within the shadow cone near the Earth): point K, the shaded body of the Earth's shadow.
- G (lower right of center, on the orbit): position G of the Moon (full moon outside the eclipse limits).
- S (lower left, within the shadow cone): position S of the Moon (within the shadow).
- Q (bottom): the apex (vertex) of the shadow cone, where the tangent lines converge.

[The label letters P, E, M, N, H, I, O, K, G, S, Q and *Sol*, *Via Lunae*, *Ecliptica* are read from the engraving; the inner/center letters near the shaded cone (K, O, S) are small and partly within the hatching — read with care.]

...**inclined**. But in the two places where the circles intersect each other — namely D is ☊ [the ascending node, Head of the Dragon] and C is ☋ [the descending node, Tail of the Dragon] — it is as was set out before concerning the latitude of the Moon. Now, so that in this single figure it can be discerned when eclipses occur in either luminary and when not at all, let the center of the Sun first be placed at E; and, lines being drawn tangent to the globe of the Earth, N Q and P Q, it is manifest that they meet and terminate at the point Q, and that the shadow projected from the Earth is produced as far as this into the shape of a round cone

or pyramid. Accordingly, when the Moon by its course — which is roughly thirteen times faster than that of the Sun and of the shadow — passes through the said shadow at the node C in a full moon, then, as can be seen, it loses its light, because the Earth, by its interposition, blocks the down-flow of the solar rays meant to illumine the Moon.

And since the pyramidal shadow is far thicker or broader in the transit of the Moon than the lunar diameter, therefore, when the mid-obscurations falls upon the node, it is necessary that not only a total eclipse of the Moon occur, but the same also with a delay — for just so long, that is, until the Moon, by the benefit of its faster motion, frees itself from it. Indeed, from this it also follows that the nearer the full moon is to the nodes, within the eclipse limits, the greater the eclipse the Moon must suffer; and that it is at last gradually diminished, until the latitude of the limb of the Moon turned toward the Earth's shadow has grown beyond the magnitude of this latter's semidiameter. For in all the rest of the tract, as at G and so on, no eclipsed full moon is felt — the orbit of the Moon, that is, bending away from the ecliptic so far that the semidiameter of the shadow, which always faces this with its cone, in no way reaches the Moon.

Furthermore, since the projection of the shadow regards the center of the Earth A as its midpoint, on that account the reckoning here is to be taken of the course of each true light, and not of the apparent — for the former regards the center of the Earth alone, the latter only the surface and the place of the observer.

So much for the eclipse of the Moon. The eclipse of the Sun follows, whose cause we defined above, from Pliny, to have been the interposition of the Moon. Therefore the middle of such an eclipse occurs only at an apparent new moon — I say "apparent," for here we measure the ecliptic conjunction of the luminaries not from the center of the Earth, as before, but rather from the surface. From which it comes about that...

that the true conjunction, which would be observed from its center, is often either anticipated by the apparent or visible one, or itself anticipated by that one. I gave a representation of this in the figure already presented, around the ascending node, where the lunar body D, set against the Sun from the center of the Earth A, is so opposed to it that it is understood to be wholly hidden here and there from someone actually observing — that is, the space of the Sun itself between P and N along the solar diameter. This obscuration, of about the semidiameter M N, is defined as referred to the surface of the Earth B, as appears in the solar semidiameter M N, with the lines B M and B H drawn from B likewise tangent to the lunar body. Yet only those who hold the Sun at their zenith would

perceive the same full defect of the Sun that we earlier reckoned from the center of the Earth. Consequently, for different inhabitants of the Earth, the eclipses of new moon are seen and occur variously — because of the lunar parallax, by which the Sun's parallax is exceeded — at different points of the Sun, and they also set in at a different instant of time.

And although, because the Moon's diameter is smaller than that of the Earth's shadow, these obscurations of the Sun do not so frequently take place at one and the same location on Earth, nevertheless the said parallaxes, by which the lunar appearance is drawn apart in different ways, distribute these obscurations among different inhabitants of the Earth in such a way that their number easily exceeds that of the lunar eclipses — as we shall give demonstrated by way of a coda at the end of this doctrine. Solar eclipses, then, occur at least at an apparent new moon, and on either side too, in the vicinity of each of the nodes, with the magnitude and form that the ratio of the semidiameters of the luminaries and the apparent latitude of the Moon require. For this reason a Moon existing beyond these limits — as at G and the like — never undertakes to eclipse the Sun.

Moreover, the duration of eclipses, as also their magnitude in partial eclipses both of the Moon and of the Sun, and likewise in what part these occur, depends entirely — and is to be estimated — from the motion and from the affection of the Moon's latitude. And this is the first and general outline of eclipses. There follows the first and simple variation, which happens both in the Moon and in the Sun.

On the simple variation of the magnitude or quantity of eclipses.

The heavenly bodies, like the Earth itself, even though they are spherical, are nevertheless perceived by our eyes as flat at any considerable distance — so much so that they can be measured not by their surface but only by their diametral quantity. [Margin: See the ἀκριβέστερα Ἀστρονομικά (akribestera Astronomika, "the more exact astronomical [works]").] Optical theory suggests this perceptible shape of them to us through the near-equality of the visual lines, on account of the more remote distance of those bodies. From the same theory we also recognize that these same bodies so vary their visible magnitude according to their differing distance from the eyes of observers, that the more remote appear smaller and the nearer larger. Finally, those bodies that by their own light cast a shadow from smaller opaque bodies onto the opposite side likewise produce that same shadow at an unequal magnitude proportional to their unequal distance — so that at a greater distance it is gradually enlarged from its

root, but at a lesser distance it is diminished, and for the most part it is cut short at its outermost extent, since the margins of the shadow are licked ever more and more by the light of the rays right up to the final point of intrusion. This consideration must be applied above all to the assessment of the magnitude of eclipses, and it is to be laid out for lovers of the stars by means of the following diagram.

In an eclipse, whether lunar or solar (as set out in the following figure): from the center of the Earth A, let the whole terrestrial globe be described upon the semidiameter A R. Then, in accordance with the theories of the luminaries already presented, let the solar body first be represented at two places — those most distant from one another in respect of the Sun relative to the Earth, namely at apogee B and at perigee C — as if at these centers. Similarly, let the lunar distances be supposed at two places equally distant on either side from the center of the Earth: above, indeed, at new moon, at apogee M and perigee L; but below, at full moon, near the Moon's apogee K and perigee H. Now let straight lines first be drawn from the outermost extremities of the Sun's diameter — namely under the higher apogee, D and E — which touch the terrestrial globe around R and Q, and which therefore end in the point

Diagram. A vertical eclipse-geometry construction showing the shadow cone cast by the Sun past the Earth, with the Sun figured at both apogee and perigee, the Moon figured at four positions (new-moon apogee/perigee above the Earth, full-moon apogee/perigee within the shadow below the Earth), and the cone converging to its apex. The figure occupies the left margin of the page beside the running text.

- B (top, center of upper Sun-circle): solar center at apogee
- C (center of lower Sun-circle): solar center at perigee
- S (left of C, on the F–G line): intersection point on the perigee Sun-circle
- T (right of C, on the F–G line): intersection point on the perigee Sun-circle
- D (left end of upper diameter line): outer extremity of the Sun's diameter at apogee
- E (right end of upper diameter line): outer extremity of the Sun's diameter at apogee
- F (left end of lower diameter line): outer extremity of the Sun's diameter at perigee
- G (right end of lower diameter line): outer extremity of the Sun's diameter at perigee
- M (small circle high in cone): Moon at new-moon apogee

- L (small circle below M): Moon at new-moon perigee
- A (center of Earth-globe, mid-figure): center of the Earth
- R (left edge of Earth-globe): tangent point on the terrestrial globe
- Q (right edge of Earth-globe): tangent point on the terrestrial globe
- H (small circle within the shadow, below the Earth): Moon at full-moon perigee
- K (small circle below H): Moon at full-moon apogee
- N (lower apex region of cone): vertex of the shorter shadow cone (perigee Sun)
- P (lowest apex of cone): vertex of the longer shadow cone (apogee Sun)

[Margin: *Ptol[emy]*, [*Almagest*] V.15; *Copern[icus]*, [*De Revolutionibus*] IV.19.]

P. Furthermore, in the same way, when straight lines are drawn from the outermost diameters of the Sun at its lowest position — namely existing at perigee, the lines I mean F R and G Q — these likewise, touching the Earth, end at N, proving the unequal production of the shadow cone here by the distance P N. They moreover leave traces around each margin of the shadow, from which it is understood to be diminished whenever the Sun descends nearer toward the Earth, and the reverse. For the greater apparent size and presence of the Sun constricts the shadow, as happens in other cases. Similarly the Moon at apogee presents on either side a smaller angle of vision under the ecliptic than at perigee — as may be seen in this same adjoined figure, in the lunar body, with the shadow at apogee K and perigee H immersed.

From these things, generally so sketched, there follows that change in the semidiameters of the Sun, Moon, and shadow which we shall afterward employ, according as the luminaries obtain their determined position relative to the Earth as set out in their hypotheses — a change that must be thoroughly learned from the anomaly of each. The ancient astronomers, from this symmetry of Sun, Moon, Earth, and shadow, reasoned geometrically about both the true distance of the luminaries from the Earth and the determined projection of the shadow from it — but in vain, namely on account of the refractions of the solar rays that render this πραγματεία [Greek: pragmateia, "treatise" or "procedure"] futile and introduce a certain anomaly or exception to what had been demonstrated beforehand. Of this, therefore, we shall presently treat in what follows.

ACCOUNT

On the anomaly of the foregoing, which the thickness of the air around the Earth — and hence the refraction of the solar rays — causes in the eclipse both of the Moon and of the Sun.

The simple and uniform ἐπισκέψεις [Greek: episkepseis, "observations" or "examinations"] of the ancients concerning the eclipses of the luminaries have thus been demonstratively set forth by us. We shall now report what they either noticed less — content as they were with having found the prior causes — or what appeared less to them in those regions (Assyria, that is, and Egypt) where the ancient mathematicians once flourished, whose industry let the sublime knowledge of the stars flow down all the way to us, on account of the purity of the air there.

For from observations made earlier and even now in Denmark, in nearer Germany, and in farther Norway — most diligently by me, and with others standing by — I have detected a quite notable alteration in the magnitude of eclipses in each of the two luminaries. For when, from the year of the Lord 1589 down to the year 1597, together with six, seven, or sometimes more other students whom D. Tycho Brahe [Tycho Brahe] then maintained in his astronomical exercises, I observed with the greatest diligence all the eclipses meanwhile visible — not by the one method through a tube (whose insufficiency we detected easily and soon enough), but by various methods through various instruments, especially the visual one, which deceives less in this business — I certainly found here and there that the diameter of the Moon at an ecliptic new moon undergoes, in these regions, a notable diminishing change from that which appears at full moon. So when a table to set forth this matter was being constructed, we did not hesitate to fit the difference of the Moon's diameter at new moon from that which is seen at full moon — and most of all at an ecliptic one — to the ratio which is that of the number four to five;

even where the same still here somewhat exceeds it. With this assessment the remaining observations — which I had also a little later at Rostock by the banks of the Warnow, and near the shore of the Baltic Sea, and here in Denmark thereafter — all agree as closely as possible. In Norway, however, as is fitting, I observed an even greater diversity. For the solar eclipse of the year of the Lord 1601, on 14 December, which Johannes Kepler [Kepler] cited (as observed at Prague) in defense of his own opinion, I myself was observing 18 miles beyond Nidaros [Trondheim], under an elevation of the pole of $64\frac{2}{3}^\circ$, one hour past noon, on the horizon — the Sun, on account of the parallaxes of the Moon and the

notable refraction there (being, that is, in the neighborhood of the horizon), being sufficiently conspicuous as to its upper limb. [Margin: *See the Appendix, Prog[ymnasmata] 1, T[ychon] B[rahe].*]

And behold, a representation of this same eclipse was afterward shown to me, when I had put in at Bergen in Norway (whose pole is elevated $60\frac{1}{2}^{\circ}$), by the bishop of that place, M. Andreas Foss [Anders Foss] — a theologian, mathematician, historian, botanist, and once a most outstanding man in languages and in all the more polished learning — how, according to the report of fishermen on the neighboring shore who had viewed the said eclipse with the greatest wonder, he had himself sketched it. In that representation the Sun had so embraced the lunar body within its compass, in the middle, that on every side it stood out equally toward the margin with its diffused light by about one and a half digits. And although the Moon was here near apogee, it would not on that account, in the reformed hypothesis, have been seen with so diminished a diameter, had not denser air and, as it were, a vapor of the Ocean intervened.

I shall give another piece of evidence of the present matter (if anyone still doubts) in the solar eclipse occurring in the year 1608, on the last of July. Although that too appeared at Wittenberg of about two digits — as the most illustrious and much-missed D. Melchior Jöstel [Melchior Jöstel] wrote to me — yet at Copenhagen, under a most clear sky at the same time, none at all was perceived. For when, a little before this experience of ours, the most illustrious and most friendly Johannes Kepler, Imperial Mathematician, had departed, having published into the light his *Optics* [*Astronomiae Pars Optica*], I was indeed held by a greater longing to show the truth also through this eclipse. For this reason, of the trace of this investigation on the Sun — which plainly appeared as nothing, as I said — I myself was most zealous, with five students of the keenest sight engaged. And in this eclipse, although the parallax of latitude of the Moon (being lower in our vertical) somewhat greater with us would detract something of the obscuration from us, yet it was not so great that it would diminish it, compared with the observation of the Wittenbergers, even by a single digit — unless the intervention of the thicker air of this place had withdrawn the whole of it from our eyes.

These and many similar observations of eclipses were furnished by maritime places under a higher elevation of the pole; but Bohemia furnished others, where, in air far more cleared, greater defects in the Sun reveal themselves, all else being well considered. This is what, in the year 1600 on the last of June, at Benátky in the same region, while staying in that place with D. Tycho, I was

watching in astonishment. For the eclipse that, from our earlier restitution of the Moon, I was expecting to be scarcely 4 digits, we beheld to be larger by more than one and a half digits. Nor yet was I then able (as it seemed to me) to subordinate so many observations of ours, previously made here in Denmark, to a single marvel, nor to charge these with the earlier weakness of my own sight. Rather, from that time I judged that effort should be devoted to making the causes of so great a variation known — once it had become clearly established to me, from so many observations made in diverse places, that they depend on the unequal constitution of the air (which under a sky nearer the equator is purer, but toward the poles, more remote, is more clouded) — and so to render this ecliptic doctrine as universal as possible over the world, so far as experience and reason would allow. *For he denies the first principles who removes experience in Astronomy.* In the same way, in eclipses of the Moon, whether total or partial, I found a smaller thickness of the shadow in the passage of the Moon under a higher elevation of the pole and a thicker air.

But since it is credible — and so far as sight can attain it — namely that the magnitude of each luminary, at their mean elongation from the Earth, is either made equal to one another as nearly as possible to the senses, or that the Moon somewhat exceeds the Sun — of which we have often made trial, and with various instruments — accordingly, how the solar magnitude surpasses the lunar under an impurer sky at an ecliptic new moon, while at full moon both a shorter transit of the shadow is gathered and the Moon from the same cause rises above its accustomed magnitude, I shall now explain in what follows, to the measure of my own talent, saving the judgment of the opticians, should they be able to find better reasons.

In eclipses, at least two more closely kindred explanations present themselves to us; I shall report both briefly — since it will be beyond our purpose to be too much hindered by contemplations of this kind in the course we have undertaken of restoring the motions.

The prior explanation alleges a dilation of the Sun's or Moon's light in denser air, which reveals itself as soon as an opaque body — the lunar one or the shadow — is interposed. It is just as if the Sun, lingering at its rising or setting, were so hidden from view by some little island in the sea directly interposed between it and the eyes of sailors, that nevertheless on either side something of the solar body (which I have sometimes seen) remained visible. In this spectacle, it is wondrous how the Sun spreads itself out on both sides with its rays, so much so that you would say the very σύστημα [Greek: systēma, "system" or "whole"] of the

Sun is all but torn apart and plainly dissipated from itself. But if the said island insinuates itself between the eyes of the viewers and the Sun in no other way than to lick away, by hiding, only the extreme margin of the Sun on one side, here you will see the Sun gather itself back together and as it were withdraw itself wholly into view — admitting, no doubt, a double refraction: one on account of the nearness of the horizon, the other of the island at the side. When I have duly fitted these spectacles to a solar — or even lunar — eclipse under a thicker sky, I detect no dissimilarity, except that the distances of the Moon in the Sun's obscuration, or of the shadow in the transit of the failing Moon, differ from the island by an enormous difference; and again, that eclipses occur no more in the vicinity of the horizon than at any other position of the daily revolution. For the rest, to this explanation there are two appendages which perhaps can scarcely be sufficiently approved by astronomers: the one, the visible dilation of the solar body, etc., in a fuller eclipse; the other, the change of the same to vision from place to place in a more partial obscuration of the Sun or Moon. Of these two, however, neither can induce a diversification beyond 6 or 7 minutes, under the densest sky, to which experience has thus far extended.

The other explanation depends on the illumination of the thick air that closely surrounds the cone of shadow turned aside by the Moon. For it is credible that this shadow there (perhaps even on account of the reflection of the solar rays from the Earth) is rarer at its margins, the dense and therefore more strongly illuminated air radiating and diminishing it on all sides. And because the lunar obstruction, however small it may be, is represented to the eye by the image of the shadow itself in the air, therefore, when this is made more compressed, less of the Sun is taken from us. Experience seems to dwell alongside this explanation: when in the evening, or even in the morning — the sky being especially clearer in winter time, and the Moon as little distant from the Sun as can be detected by sight — the rays of the Sun so illuminate the nearer semi-periphery of the Moon that this illuminated part is seen to stand out greatly above the remaining part of the opaque lunar body (which is discerned, as it were, enclosed together in the form of a globe), namely by a more ample diffusion of the light made in the denser air. And indeed this image of the dilation of light can be either seen or exist only in the air. For if it really inhered in the Moon, on account of certain περιστάσεις [Greek: *peristaseis*, "circumstances"] causing a refraction of the Sun, then it would appear equally in all the places of the Earth to which the Sun's obscurations reach — on account of the refractive lines descending directly from the Moon's extremities into the air and the eye — and hence no variation of solar eclipses in various climates would occur. Between the two recounted causes, therefore, either this one, which seems to me the stronger, or the prior one, must

be acknowledged as the true one; nor is a third given. So much for the ecliptic anomaly of the Sun. That of the Moon follows.

Since above we showed the cause of the Moon's obscuration to be the terrestrial shadow projected from the opposing rays of the Sun, accordingly, when these [rays pass] into air nearer the Earth and therefore

because the rays strike air that is more vaporous and dense, they are refracted into the shadow in such a way as to draw it together on every side into a thinner, shorter point — and this varies with the differing quality of the air at the several surrounding places of the Earth. Yet I do not believe that this refracted incidence of the rays into the shadow can ever be as great as the refraction of the visible solar body at the horizon that strikes the eye. The strength of such a refraction depends chiefly on the present visibility of the luminous body, as experience in optics confirms. The Sun, by contrast, in the dead of night — when the Moon is most deeply eclipsed — binds the Earth round with rays sent out only obliquely, and casts the Earth's shadow toward the opposite side. So it is one thing for the visible solar body itself to be refracted at the boundary of the horizon, and another for rays cast from the luminous visible source to illuminate the dense air and, so refracted, to attenuate the shadow by leaping into it.

Indeed, experience under a clear sky often reminds me of this here, in a pleasant kind of spectacle: just as the Sun, on the very point of setting, hangs trembling for some little while in a kind of balancing above the horizon (for the same reason), and then, after this struggle, as the wheel of the primary revolution presses on without pause, hides itself below the horizon in a momentary, sinking-like motion — and rises again with equal haste. This happens to our sight because the Sun is refracted, on account of its own visible presence; and I would believe the same occurs in the air with respect to the Moon, which is barely seen at the same instant that the Sun lies hidden. So too the Moon's delay in entering the Earth's shadow, and its haste in leaving it, are observed to follow from the same cause.

Now since optical reasoning, supported by experience, sets out clearly enough before the eyes how visible things present themselves in the same shape, refracted, with which they are in truth marked out and determined, the cause therefore — of Pliny's observation, of Maestlin's [Maestlinianae — Michael Mästlin], indeed of my own as well, on several occasions when a defect was seen in the Moon while both luminaries were visibly above the horizon — must be sought nowhere else than in the Earth's shadow, as I have discussed more fully in the disputation on the diminution of eclipses. But the things of which experience

had reminded me here I shall briefly bind all together in the present figure, as something to be set out before the eyes more clearly there.

Diagram. A vertical geometric construction of a solar eclipse with the Earth's shadow cone. At the top is a large circle representing the Sun, its rim drawn with a hatched (rayed) circumference; its center is **B**, with **C** and **D** marking the ends of the solar diameter (left and right respectively), and **F** flanking on both outer sides at the level of the solar rim, marking the lucid circumference **E F**. Below the Sun, the small circle **H** (with **G** and **K** at its sides) represents the Moon in the solar eclipse. The straight lines from the extremities of the Sun's diameter (**C** and **D**) run down past the Moon to the Earth's surface at **V**. Below, a small circle marked **A** (the center of the terrestrial globe) sits within a region flanked by **R** (left) and **Q** (right); from it the shadow cone tapers downward through **L** (the Moon in its own place, lower down) to the point **M**, and finally to the apex **P** at the bottom.

- **B** (top center): center of the Sun
- **C** (top, left of **B**): one end of the solar diameter
- **D** (top, right of **B**): the other end of the solar diameter
- **E F** (solar rim, both flanks): the lucid circumference remaining visible
- **F** (far left and far right at rim level): endpoints of the lucid circumference
- **H** (upper-middle): the Moon in the solar eclipse (the deliquium of the Sun)
- **G** (left of **H**), **K** (right of **H**): points on the lunar body / sides of the shadow-cone axis **G V K**
- **V** (below the Moon): the point on the Earth's surface where the rays descend into the eye
- **A** (center): center of the terrestrial globe
- **R** (left of **A**), **Q** (right of **A**): points on the Earth flanking center **A**
- **L** (lower): the Moon in its own place
- **M** (low): the gathering of the attenuated shadow into a cone
- **P** (bottom): the apex of the true shadow point

Let **A** be the center of the terrestrial globe; **B** the Sun; **H** the Moon in the eclipse of the Sun; **L** the Moon in its own place. But so that we may first deal with those things that belong to the ecliptic anomaly of the Sun — which do not present themselves equally in all places — let straight lines be drawn from the extremities of the Sun's diameter down to the Earth's surface, **C V** and **D V**, grazing the lunar body at **H** on either side. And although the greatest obscuration of the Sun is brought on here, nevertheless the same total obscuration under a higher pole and in denser air can never be so great that something visible of the

Sun's body does not remain — here represented by the lucid circumference **E F** — which would appear while from those points too straight lines descend into the sight at **V**, passing by the lunar body and so also by the occultation it would induce. So whether the solar rays in the denser air enlarge and dilate the visible circumference of the Sun, so as to thrust it into the view that experience requires; or whether the neighboring air about the axis of the Moon's shadow-cone **G V K**, on account of its density — by which it admits a notable illumination from the solar rays — acts upon the shadow in such a way as to feed upon it on every side with its own stronger illumination,

and as it were eats it away and thins it; and so the image of the lunar body, mimicking with the said air to the eye the residual shadow, takes away from the just obscuration of the Sun just as much as has been consumed from the shadow by the nearer, illuminated air. (For any object whatever is perceived in the shape and magnitude in which the rays flow from it into the eye through the transparent medium.) Whatever the case, I have gathered this through various places, and now affirm it as the most certain experience: namely, that a solar eclipse always turns out smaller the nearer you approach our pole, and consequently the thicker the air through which you view it — provided that everything else required for the changes of parallax or of meridians has been most correctly handled. For this reason, below, the ratio and proportion of this anomaly must be applied by us to each particular climate. But whoever does not hesitate to refer this alteration to the discernment of vision and its defect in certain people — especially in the same or in these places — will sooner convict of a general defect of vision all those who live under a denser sky than he will press such an opinion of his upon us in place of the truth itself.

Furthermore, the denser air around the Earth supplies the cause — as I said above — of carrying off some portion of the terrestrial shadow into which the full and ecliptic Moon is plunged. For although the straight lines **C R P** and **F Q P**, sent from the Sun through the Earth, would really carry the shadow from **A** down to **P**, the air surrounding the Earth, denser and more illuminated beyond **R** and **Q**, causes the solar rays to leap from it into the shadow and to thin it on every side, gathering it again into the cone **M**, in the form seen here. Thus **M P** is the difference of this prolongation; and the space on either side, in the Moon's transit at **L**, is plainly apparent — by which this shadow is shorter than the one that would have ended in the true point **P**. I have also found some variation of this under a denser sky (as reason demands), and that the lucid little part of the Moon in particular eclipses has a greater apparent size under the same sky — although it is not so great as to rival the apparent prominence of the Sun.

Nor am I unaware that the Sun, on account of its true magnitude — which surpasses these bodies by many parasangs [parafangis] — illuminates somewhat more than half the system of the globe of the Moon and the Earth. But both this, and that small quantity which arises from the unequal motion of the nodes about the Moon's latitude, and which does not strike the senses well, I judge should be omitted from the present consideration. For it belongs to the astronomer to be willing to ignore certain imperceptible minutiae. Meanwhile, however, we shall test this geometrically concerning the diminution of the terrestrial shadow, having set the Sun's mean distance from the center of the Earth at 1288 semidiameters, and its visible diameter at 31 minutes, together with the magnitude of the shadow in the Moon's transit observed by us here in Denmark, whose diameter extends to about $1^{\circ} 29 \frac{1}{2}'$. For in vain, as was said above, did the ancients suppose that they could investigate the distance and mutual proportion of these three bodies — the Sun, the Moon, and the Earth — by means of eclipses, even with the Moon's distance from the Earth given in addition; since the anomaly which we have now set forth in the attenuation of the shadow everywhere disturbs the reckoning of the measurement, though unequally.

Demonstration of the true semidiameter of the Sun and the Moon, likewise of the true length and thickness of the terrestrial shadow in the Moon's transit — with, were it possible, the purest air surrounding the horizon — and how far they differ from what is seen among us.

In the present adjoining figure, let **A** be the center of the terrestrial globe, **B** the Sun set in its mean distance from the Earth, which we posited above as 1288 semidiameters of the Earth, of which **A C** is one; this distance is measured by the line **A B**. Likewise let the Moon's mean elongation from the center of the Earth, on the opposite side, be the line **A R**. Now let lines descend from the contacts at the extremities of the diameter **G** and **I**, tangent to the Earth's globe on either side near its diameter at **C** and **D** (for, as I said, these points are found scarcely perceptibly within the contacts); let them therefore end at **H**, as far as they produce the cone of the terrestrial shadow, to which point the line **B A H** is also continued.

Diagram. A tall, narrow geometric construction of the terrestrial shadow cone, drawn vertically. At the top is a circle representing the Sun, with **K** and **B** marked across its upper interior and **G** (left) and **I** (right) at its flanking diameter-ends. Tangent lines descend from **G** and **I**, narrowing downward. Partway down, the

line segment **O — V** is marked (a small horizontal). Below it, a small circle (the Earth, centered at **A**) carries **C** (left) and **D** (right) at its sides and **P** and **A** within. Lower still, the segment **E — R — T** is marked horizontally. The converging tangent lines run down to the apex **H** at the very bottom. Two marginal citations stand to the right: "Caput 8. hujus" ("Chapter 8 of this work") beside the Earth-circle, and "Lib. 5. Almagest. cap. 15." (*Almagest* V.15) beside the apex.

- **K, B** (top, within Sun): points across the solar body
- **G** (top left), **I** (top right): the ends of the solar diameter, points of tangency
- **O — V** (upper-middle): the segment giving the Moon's apparent semidiameter (**O V**)
- **C** (left), **D** (right): tangent points near the Earth's diameter
- **P, A** (within Earth-circle): **P** a point on the radius; **A** the Earth's center
- **E — R — T** (lower-middle): the segment at the Moon's mean distance (**R** the Moon's center, **E** and **T** the shadow edges)
- **H** (bottom): the apex of the terrestrial shadow cone
- [Margin: Caput 8. hujus] (Chapter 8 of this work)
- [Margin: Lib. 5. Almagest. cap. 15.] (*Almagest* V.15)

Since the proposed task is first the true semidiameters of the Sun and the Moon, then the length of the shadow **A H**, let line **A I** be drawn first; then in the right triangle **A B I**, since the givens [$\delta\epsilon\delta\acute{o}\mu\epsilon\nu\alpha$ — *dedomena*, "the given quantities"] are **A B** = 1288 semidiameters of the Earth (the Sun's distance from the Earth), **B A I** = 15' 30" (the apparent semidiameter of the Sun), and finally the angle **A B I** a right angle, the side **B I** is found to be 5 807/1000 semidiameters of the Earth — which measures the true semidiameter of the Sun, to which **B G** is equal. In the same way, for the Moon's true semidiameter, since in the right triangle **A O V** there are given **A O** = 57° 38' semidiameters of the Earth — namely of the full Moon in apogee elongation, when also **O V**, the apparent semidiameter of this Moon, is 16' — there is therefore given the Moon's true semidiameter = 16 1/10', of the kind of which **A D** is 60'.

Further, to **B G** let the line **C K** be drawn parallel to **A B**. Then, the semidiameter **A C** of one Earth being subtracted from **B G**, there remains **K G** = 4 807/1000 semidiameters. Now as this is to **C K** = 1288 semidiameters, so is **B G** to **B H**; and for that reason **B H** = 1556 semidiameters of the Earth, from which, when **A B** = 1288 semidiameters is subtracted, there remains the length of the shadow-cone sought, **A H** = 268 semidiameters of the Earth, very nearly — exactly so with Ptolemy, when the Sun is in its own apogee. Finally, for the apparent semidiameter of the shadow in the Moon's transit according to its mean

elongation from the Earth — namely 56 semidiameters of the Earth — let line **E F** be drawn through **E** parallel to **C D**. And because **R H** is given as 212 semidiameters, through the difference of **A R** from **A H**: therefore again, as **A H** = 268 semidiameters is to **A C** one semidiameter, so is **R H** = 212 semidiameters to **R E** = 212/268 semidiameters, or in resolved minutes about 47' 28" — of the kind of which **A C** is 60'. Then, line **E A** being drawn, since in the right triangle **E R A** the givens [διδόμενα — *didomena*, "the givens"] are **A R** = 56 semidiameters of the Earth and **R E** = 47' 28" semidiameters of one, together with the right angle enclosed at **R**, there is therefore given the angle of vision **R A E** = 48' 34". But among us, under a pole-elevation of about 56°, the same angle in very many and most diligent observations of a lunar eclipse has been found no greater than 44' 10" — to which measure, below, we shall teach the Copernican [reckoning] to come nearer, but the Ptolemaic somewhat to exceed it — so that you may see the quality of the air at the boundaries of the horizon of various tracts of the Earth, upon which the solar rays falling and attenuating the Earth's shadow (as was demonstrated above) vary little; unless perhaps

— unless perhaps in remotest Norway, Iceland, and Greenland, and the lands nearest the pole, where the air, thickest of all, may still subtract something further, namely above $4\frac{3}{5}'$, by which the angle **R A E** just found exceeds the same observed among us; the double of which, with respect to the whole diameter of the shadow, that is, the angle **E A F**, is $8\frac{3}{5}'$.

From the careful consideration of these things it becomes known that what we demonstrated above concerning the Sun's elongation from the Earth, on the hypothesis of the Moon, agrees beautifully with the truth — short of the mean of which the ancients could never arrive at the true distance of the Sun from the Earth, and at the proportion of these three bodies (the Sun, the Moon, and the Earth) which follows from it, to be investigated; the true ratio of **E R** to **C A** always lying hidden from them. For the rest, before we proceed further, we shall presently acquire this same proportion, the opportunity being offered.

On the mutual proportion of the three world-bodies — the Earth, the Sun, and the Moon.

Since the diameter of the Earth, like its whole capacity, is sufficiently known in any proposed measure in geometry, the heavenly bodies will accordingly be made commensurable, correctly, to its proportion — which we are now about to do in the case of the Sun and the Moon, both by way of their symmetry with the Earth and as established among themselves. For astronomy promises not only

knowledge of the motions, but also, so far as it can be done, the magnitude of the heavenly bodies. Now since it is established from geometry that spheres are in the tripled ratio of their diameters, and the semidiameters of the luminaries have been most nearly acquired — that of the Sun indeed $5\frac{807}{1000}$, that of the Moon $16\frac{1}{10}$ minutes, of the kind of which the Earth's is one — therefore, when these numbers are taken cubically by themselves and compared with the cube of one Earth-semidiameter, the Sun is found to be greater than the Earth 196 times, that is, taken twenty-eight times seven; and the Moon smaller than the same Earth about $51\frac{1}{49}$ times, or somewhat further; but, keeping to $51\frac{1}{49}$, the Sun will be greater than the Moon ten thousand times exactly. This is the admirable symmetry of these bodies to one another, just as it is now recognized, drawn from the heavens without any forcing.

DEMONSTRATION

Of the variation of the semidiameters of the Sun, the Moon, and the terrestrial shadow, together with the anomalies subjoined proportionally according to experience.

Having premised those things which concern the properties of the three world-bodies — namely the Sun, the Moon, and the Earth — with respect to their magnitudes and their uniform mutual distances in eclipses, we shall now further pursue, by means of geometrical demonstrations and also experience, the determination of what variations there are in them, according to those things which we gave to be contemplated in the preceding antepenultimate and penultimate figures — so that what the apparent semidiameters in the eclipses have been may here be recognized.

Cap. IX. LIBER PRIMUS [continued]

I.

For the simple variations of the semidiameters that were indicated in the second-to-last figure, let the last one now be taken up again. From it we may first examine the apparent magnitudes of the luminaries — the Sun and the Moon — at apogee and perigee, and afterward the terrestrial shadow, given its variation arising from the distance of both the Sun and the Moon from the Earth. According to the hypotheses for the luminaries set out above, and likewise their determined mean distances from the Earth, we find:

			Degrees (in Earth-radii)	
Sun	altitude above the Earth at	apogee	1334	semidiameters of the Earth
		perigee	1242	

			Deg. Min.	
Moon, new or full	distance from the Earth at	apogee	57 38	semidiameters of the Earth
		perigee	54 23	

From this, in the figure set alongside, with the data [Greek: διδομένοις (didomenois, "given quantities")] $BI = 5 \frac{7}{100} \frac{7}{100}$ [recte: $5 \frac{7}{1000}$?][?] as the true semidiameter of the Sun in Earth-radii, $BA = 1334$ as the distance of the Sun at apogee, and finally with angle ABI a right angle, the angle BAI is obtained as 15 min. 0 sec., very nearly.

When one proceeds by the same method with the Sun at perigee, with only the line AB changed — here put at 1242 — the visible semidiameter of the Sun comes out as $16 \frac{1}{11}$ min. Therefore an imperceptible error is found in the variation of this semidiameter in [Tycho Brahe]: for from experience he puts this semidiameter of the Sun at apogee at 15 min., and at perigee at 16 min.

Likewise for the Moon: since its true semidiameter was found above to be nearly $16 \frac{1}{10}$ of one Earth-radius, then in a similar right triangle AOV , from the given quantities $OV = 16 \frac{1}{10}$ min., the semidiameter of one Earth-radius throughout,

			Deg. Min.	
likewise A	57	Moon at	apogee	with a right angle at O
O	38		perigee	
	semidiameter	in	perigee	
	54 23			

the visible semidiameter of the Moon is given

at	apogee	16 0
in	perigee	17 0

very nearly.

Therefore here in [Tycho Brahe] — I do not know through what error, arising from my departure from Bohemia — a whole minute is in excess in the Moon's semidiameter at perigee; although the same full Moon, set against the terrestrial shadow, still by this discovery of ours always presents a semidiameter somewhat larger, doubtless from one of those causes which I noted above.

II.

Now, as to the terrestrial shadow and its first variation in this same transit of the Moon — the variation it undergoes on account of the Sun's distance from the Earth — we assessed it in this way. Since the portion of the Sun's true semidiameter KG is established above as $4 \frac{1}{100} \frac{0}{1000} \frac{7}{10000}$ [$4 \frac{7}{10000}$][?] in units of which AC is one part, therefore, since CK (which is equal to AB) is given as 1334 with the Sun at apogee, it will be: as KG is to KC , so AC is to AH . By this proportion AH is found to be $277 \frac{1}{2}$ semidiameters of the Earth. Subtracting from these the mean distance of the Moon from the Earth, 56 semidiameters, there remains $RH = 221 \frac{1}{2}$ semidiameters of the Earth. Therefore, that RE may be obtained, it will be: as HA is to AC , so HR is to $RE = 7982$ — namely, in the case where AC , one semidiameter of the Earth, is taken as 10000 parts. We adopt this ratio for the sake of brevity and greater precision, since [...]. And since ARE are given in the same ratio, $AR = 560000$ is the mean distance of the Moon from the Earth, and likewise $RE = 7982$. With a right angle at R , the angle of vision of the semidiameter of the shadow, RAE , with the Sun at apogee, is given as 49 min. 0 sec. of one degree, very nearly; but with the Sun at perigee the angle of vision RAE is found, by the same line of reasoning, to be $48 \frac{1}{15}$ min. There is therefore a difference of $\frac{14}{15}$ min., or 56 sec., entirely consistent with the calculation of earlier astronomers.

III.

Diagram. A geometric construction of the terrestrial shadow-cone and the Moon's transit through it. At the top is a circle representing the Sun, with a horizontal diameter line running $G — K — B — I$ across it (G at left, I at right; K and B near the center). From the two ends of this diameter, the bounding tangent-lines of the shadow-cone descend, converging downward. Lower on the page, a

smaller circle represents the Moon, crossed by a horizontal diameter $C — P — A — D$ (C at left, A near center, D at right; P just left of A). Above and to the right of this Moon-circle, a short segment $O — V$ marks the Moon at one position with a small right-angle bracket. Below the Moon-circle, a segment $E — R — \Gamma$ marks the body at another position, also with a right-angle bracket (E at left, R in the middle, Γ at right). The two tangent lines continue downward to converge at the apex of the cone, labeled H at the bottom.

- G (left end of Sun's diameter): vertex of the shadow-cone's left tangent
- K, B (center of Sun's diameter): interior reference points
- I (right end of Sun's diameter): vertex of the shadow-cone's right tangent
- C (left end of Moon's diameter): left edge of Moon
- P (left of center): interior reference point
- A (center of Moon): center of Moon
- D (right end of Moon's diameter): right edge of Moon
- O, V (above-right of Moon, with right-angle bracket): Moon position / semidiameter reference
- E, R, Γ (below Moon, with right-angle bracket): Moon position / shadow-semidiameter reference
- H (apex at bottom): vertex of the shadow-cone

It remains to treat the variation of the shadow due to the unequal approach of the Moon to the Earth according to our hypothesis. For when the Moon is at apogee, it makes a shorter transit through the shadow than when at perigee, because of the form of the shadow itself, which extends into a cone with its root, as it were, at the Earth — as was shown above. The difference here, however, turns out to be far smaller than the requirement of the ancients' lunar hypotheses; this is because of the equant of the Moon rightly introduced into our hypothesis. We track this down at apogee and perigee as follows.

First, since it is established that the new or full Moon at apogee is distant from the Earth by 57 deg. 38 min. in Earth-radii, it will be: as $H A = 268$ — the found mean length of the whole shadow in Earth-radii — is to $A C$, one semidiameter of the Earth (which here, for the sake of pressing precision, is put at 60000 parts), so $H R = 210$ deg. 22 min. in Earth-radii is to $R E = 47097$ — that is, by resolving the first and third terms to the assumption of the mean. Hence, in the right triangle $A R E$, for the angle of vision $R A E$, since the two sides about the right angle at R are given [Greek: δεδόμενα (dedomena, "given")], the said $R A E$ is given as 46 min. 49

sec. — namely the semidiameter of the true shadow as it appears in the transit of the Moon when at apogee.

By the very same line of derivation the semidiameter of the shadow which the Moon traverses at perigee is found to be 50 min. 24 sec., with only the side H R changed, to suit the distance of the Moon at perigee from the Earth (when it is new or full), 54 deg. 23 min. in Earth-radii. For H R is the difference between A R and A H at any assumed distance of the Moon from the Earth, and the semidiameter of the shadow at the mean distance of the Sun and Moon from the Earth is 48 3/4 min., from the preceding inquiry.

We mean all of this to be computed for the true magnitude of the shadow, so that it may afterward be reduced to the apparent magnitude according to experience, [Greek: καθ' ὅλα (kath' hola, "as a whole" / "on the whole")]. While I was thoroughly anxious about determining this precisely — intending, as it seemed to me, to estimate the attenuation of the shadow from the unequal density of the thicker air around the horizons of various places — I find almost nothing at all that is firm and that undergoes an ordinary, regular change. For when I inquire into the angle of vision of the shadow's semidiameter, R A E, from the mean distance of the Sun and of the Moon from the Earth as given by various astronomers, I find it to be the same:

	[Ptolemy]	44 5/6
	the Alphonsines	41
According to	[Copernicus]	43 1/2
	[Reinhold]	43 3/4
	[Tycho Brahe]	44 1/6

[Margin: See Copernicus, Book IV, Cap. 19.]

To [Tycho Brahe] and to us, then, mere experience assists in this matter; to the Alphonsines, the not-sufficiently-accurate finding of a certain Arab philosopher. As to the ancients' measurement of the shadow's diameter, too: if they measured it — as they doubtless did — by the passage of the Moon through the deep shadow, it must be pointed out that they always fell short of its true magnitude, on account of the swifter motion of the new and full Moon, first of all noticed by us and reckoned by me from the body's own delay in the shadow.

On reducing the Moon's semidiameter at the eclipse new-moon and enlarging it at the full-moon, and that according to the temper of the air as it is successively thickened in different places.

How these phenomena in different places depend on the unequal temperament of the air we have set out abundantly above on optical grounds, but relying chiefly on experience. In accordance with that experience, we now assign here the decrease of the Moon's semidiameter at the eclipse new-moon, and its increase at the full-moon — below and above the magnitude of its mean apparent value, which the table above comprehends — and indeed each of these for the various elevations of the world's pole, always distributing, as it were, two-thirds of the semidiameter at the eclipse new-moon to the same quantity to be augmented at the full-moon. And this at least for the semidiameter of the Moon projecting outside the shadow, which the phenomena of particular lunar eclipses, observed here too by our own diligent observation, themselves demand. No one ought to be surprised at that sudden alteration which sets in at an elevation of 50 and 56 degrees, beyond the usual succession, since experience itself (to which we always appeal) teaches us the same. And why not? For in these maritime regions a greater thickness of air — which is the cause of this appearance — suddenly presses in. Although the same thickness is packed more and more closely the nearer you go toward the pole, it does not, however, answer in a proportion corresponding to the successive change that could occasion an equal change in the Moon's semidiameter; otherwise the same thickness would altogether exhaust every solar eclipse to be expected under an elevation of the pole of 80 parts. Since this would be incredible — and since experience in farther Norway and Iceland, which has come to our knowledge so far, does not demand so great a successive change either — I have accordingly judged that the individual cases should here be adjusted, each one as closely as I could. And because the unequal apparent size of the luminaries at apogee and perigee induces almost no perceptible change through these decrements and increments of the Moon, these are therefore fitted to the Moon's semidiameter just as they come out for the corrected elevations of the pole, by the following little table.

Table of semidiameters in eclipses

[Tabula semidiametrorum in eclipsibus]

The table is read in paired half-leaves: the left half (Anomaly of Sun & Moon, with the Sun's and Moon's semidiameters) is read downward; the right half (semidiameter of the shadow, variation of the shadow, and the mirrored Anomaly of Sun & Moon) corresponds to the same rows. The Anomaly column on

the far left runs from 0s 0° downward; the mirrored Anomaly column on the far right runs from 12s 0° upward (complement). Both anomaly entries are in signs (S.) and degrees (G.); all semidiameter values are in minutes (M.) and seconds (S.); the shadow-variation is in seconds (S.).

		S								
		u	S	M	M	Sha	Sha	Variatio		
Anomal	Anomal	—	n	n	n	w	w	n of	Anomal	Anomal
y ☉ & ☽	y ☉ & ☽	M	—	—	—	—	—	shadow	y ☉ & ☽	y ☉ & ☽
— S.	— G.	.	S.	M.	S.	M.	S.	— S.	— S.	— G.
0	0	15	0	16	0	43	0	0	12	0
0	6	15	0	16	0	43	1	0	11	24
0	12	15	1	16	1	43	2	1	11	18
0	18	15	2	16	2	43	6	2	11	12
0	24	15	3	16	3	43	9	3	11	6
1	0	15	4	16	4	43	14	4	11	0
1	6	15	6	16	6	43	19	6	10	24
1	12	15	8	16	8	43	25	9	10	18
1	18	15	10	16	10	43	33	12	10	12
1	24	15	12	16	12	43	41	15	10	6
2	0	15	15	16	15	43	49	18	10	0
2	6	15	18	16	18	43	58	21	9	24
2	12	15	21	16	21	44	9	24	9	18
2	18	15	24	16	24	44	20	27	9	12
2	24	15	26	16	27	44	31	30	9	6
3	0	15	29	16	30	44	43	34	9	0
3	6	15	32	16	33	44	55	37	8	24
3	12	15	35	16	36	45	7	40	8	18
3	18	15	38	16	39	45	19	43	8	12
3	24	15	41	16	42	45	31	46	8	6
4	0	15	43	16	44	45	44	49	8	0
4	6	15	46	16	46	45	56	51	7	24
4	12	15	49	16	49	46	7	52	7	18

		S u n								
Anomal y ☉ & ☾ — S.	Anomal y ☉ & ☾ — G.	—	n	n	n	Sha do w	Sha do w	Variatio n of shadow — S.	Anomal y ☉ & ☾ — S.	Anomal y ☉ & ☾ — G.
		M .	S. S.	M. M.	S. S.	M. M.	S. S.			
4	18	15	51	16	51	46	17	53	7	12
4	24	15	53	16	53	46	27	54	7	6
5	0	15	55	16	55	46	36	55	7	0
5	6	15	57	16	57	46	44	55	6	24
5	12	15	58	16	58	46	50	55	6	18
5	18	15	59	16	59	46	55	56	6	12
5	24	15	59	16	59	46	58	56	6	6
6	0	16	0	17	0	47	0	56	6	0

Table of the change in the Moon's semidiameter in eclipses

(Tabella mutationis semidiametri lunæ in eclipsibus.)

The top row gives the elevation of the pole, from 20 to 74 degrees (G). For each pole-elevation the table gives three quantities in the Moon's semidiameter at eclipse: the amount to be **subtracted for the Sun** (labeled "S," presumably seconds), the amount to be **added for the Moon**, and the value **outside eclipse** — the latter two given in minutes ("M") once the columns pass into the higher elevations.

Elevation of pole (G)	2	3	4	4	5				5	6		6
	0	0	0	6	0	53	56	9	2	65	9	74
(unit)	S	S	S	S	S	M	M	M	M	M	M	M
Subtract for the Sun / Add for the Moon	0	1	3	4	5	2	3½	4	5	5¾	6	6⅓
		0	0	0	0			½	¼			
Outside eclipse	0	6	2	3	3	1⅓	2⅓	3	3	3⅝	4	4⅓
			0	0	4				½			

Table of the synodic revolution of the Moon with respect to the Sun

(Tabula synodicæ revolutionis lunæ ad solem.)

Mon ths	Co.	B i s .	Time (C) — D.	Anom. of Sun — S.	Anom. of Moon — S.	Lat. of Moon — S.
Janu ary	31	3	29 1	0	0	1
Febr uary	59	6	59 0	1	1	2
Mar ch	90	9	88 1	2	2	3
Apri l	120	1	118 2 1	3	3	4
May	151	1	147 5 2	4	4	5
June	181	1	177 8 2	5	5	6
July	212	2	206 1 3	6	6	7
Aug ust	243	2	236 4 4	7	6	8
Sept emb er	273	2	265 7 4	8	7	9
Octo ber	304	3	295 0 5	9	8	10
Nov emb er	334	3	324 3 5	10	9	11
Dece mbe	365	3	354 6	11	10	0

Mon ths	Co.	B i s . —	Time (C) — D.	Anom. of Sun — S.	Anom. of Moon — S.	Lat. of Moon — S.
r		6				
Half	(M	14		0	9	6
-	ens					
mon	is)					
th						

Table of the hourly motion of the Moon in conjunction and opposition, together with that of the half-day before and after

(*Tabula motus horar. C in ☿ & ♀, & simul dimidii diei ante & post.*)

The argument column on the left, "Ano. 1 gr. coæq." (the coequated anomaly, 1 degree, given as G), runs 0, 3, 6, ... 30. The right-hand column repeats this argument complementarily, running 30, 27, ... 0. The six header-groups across the top are numbered 0, 1, 2, 3, 4, 5 (top) and continue as 11, 10, 9, 8, 7, 6 (bottom of the page, read upward from the complementary argument). Each group carries two subcolumns: "Horar. in ☿ & ♀" (hourly motion in conjunction and opposition) and "Horar. Ultra" (hourly motion beyond). All values are in minutes and seconds (M. S.).

Ar g. (G)	0:		1:		2:		3:		4:		5:		Ar g. (G)
	0: in ☿♀	Ult ra	1: in ☿♀	Ult ra	2: in ☿♀	Ult ra	3: in ☿♀	Ult ra	4: in ☿♀	Ult ra	5: in ☿♀	Ult ra	
0	27 43	27 12	28 4	27 34	29 1	28 46	30 24	30 42	31 51	32 53	32 58	34 45	30
3	27 44	27 12	28 8	27 39	29 9	28 57	30 33	30 55	31 58	33 5	33 3	34 53	27
6	27 45	27 13	28 13	27 45	29 17	29 8	30 41	31 7	32 6	33 18	33 7	35 0	24
9	27 45	27 14	28 18	27 51	29 25	29 19	30 50	31 20	32 14	33 30	33 11	35 8	21
12	27 47	27	28 23	27	29 33	29	30 58	31	32 21	33	33 14	35 18	

Ar g. (G)	0: in 00°	0: Ult ra	1: in 00°	1: Ult ra	2: in 00°	2: Ult ra	3: in 00°	3: Ult ra	4: in 00°	4: Ult ra	5: in 00°	5: Ult ra	Ar g. (G)
		15		58		29		33		41		14	
15	27 48	27	28 29	28	29 41	29	31 7	31	32 28	33	33 17	35	15
		16		5		41		46		54		18	
18	27 51	27	28 35	28	29 49	29	31 17	32	32 35	34	33 20	35	12
		19		12		52		0		5		24	
21	27 53	27	28 41	28	29 58	30	31 26	32	32 41	34	33 21	35	9
		22		20		4		15		17		29	
24	27 57	27	28 47	28	30 6	30	31 34	32	32 47	34	33 23	35	6
		26		29		17		28		26		32	
27	28 0	27	28 54	28	30 15	30	31 43	32	32 53	34	33 24	35	3
		30		38		29		41		36		34	
30	28 4	27	29 1	28	30 24	30	31 51	32	32 58	34	33 24	35	0
		34		46		42		53		45		37	

Precepts [Greek: ψηφοφορίας (psēphophorias, "of reckoning/computation")], or a very brief method for computing eclipses of the Moon and the Sun

Having now set out everything that can in any way cause the eclipses of the two luminaries — and their variations as well — we will add the long-awaited procedure for computing the failing of the light of each. We will make it as light a labor as it can be made, so that the difficulty an astronomer is otherwise forced to endure in this part may not utterly drive him away from an astronomical pursuit that is, of all pursuits in philosophy, the most worthy and the most divine.

The calculation of eclipses turns chiefly on two things, both arising from the motion of each luminary: namely, the **eclipse-bearing syzygy** itself, which from the longitude of the luminaries under the ecliptic furnishes the indication of the time; and the **representation** — that is, of the semidiameters of the Sun, Moon, and shadow, and also of the true latitude of the Moon in a lunar eclipse, but of the

visible latitude in a solar obscuration — which concerns the magnitude and the duration of the eclipse.

I.

[Margin: The detection of a future eclipse. (*Iudicium futuri deliquii.*)]

As to the first point: since the motions, restored from these theories and tables of the two luminaries, are continually arranged into ephemerides — especially for future times — I do not think it any longer necessary to labor over mean eclipse-syzygies. Rather, as soon as it becomes known from the motion of the Moon's latitude (which, in truth, will be contained within $12\frac{1}{2}$ degrees from one of the nodes, in the case of a lunar eclipse, and within $5\frac{1}{2}$ degrees visibly, in the case of any coming solar obscuration) that an eclipse-bearing full moon or new moon is at hand, the calculation should be directed toward the **true syzygy** in each kind of eclipse. Then, with the reduction from the ephemerides made here to the meridian of the place by means of the proper table, the time of the true eclipse-syzygy may come to light.

[Margin: On this reduction see Cap. 3, at the end. (*De hac reductione vide c. 3, in fine.*)]

II.

[Margin: The investigation of the eclipse-syzygy. (*Exploratio syzygiæ eclipticæ.*)]

Next, for this proposed time — not yet equated — the true place of the Sun from the equinox, and of the Moon from the Sun, must each be sought out from their own restored tables. And this is for the sake of greater certainty, so that the motions in eclipses may be weighed out most exactly. Here it must be noted that in the eclipse-syzygy one should use the **prosthaphaeresis of the Moon** alone — the one belonging to the first column, owed to eclipses — taking it according to the simple proposed anomaly of the Moon.

But if anyone, without the continuous computation of the motions that the ephemerides comprehend, should wish to reckon eclipses for ages long past, let him consult the tables of the synodic revolution of the Sun and Moon — those of Reinhold [Erasmus Reinhold] and others; or rather, let him himself, for the beginning of the year in which he desires to learn that such an eclipse occurred, investigate the motions of the luminaries and reduce them to the synodic revolutions. Then let him augment these, for the time being, by the monthly and half-revolutions — by the benefit, that is, of the table I have transferred here

from Tycho's *Progymnasmata* and appended to the rest for this use — until he finds them near the nodes, which are, as it were, the indicators of an eclipse. Afterward, let him correct the calculation from these restored motions of the luminaries.

III.

[Margin: Finding the Moon's hourly motion before and after an eclipse. (*Inventio motus horarii lunæ ante & post eclipsin.*)]

With the time retained as previously supposed for the meridian of the place — which cannot be far from the mean eclipse-syzygy, especially the lunar one, according to our restoration — the Moon's hourly motion from the Sun must be taken, either according to the table just given, or else certainly and compendiously caught in another way, thus: Since in the reduction of the time and of the incidents in each kind of eclipse it is best to have the true motion of the Moon known for one hour both before and after the eclipse-syzygy. The Moon's mean hourly motion is therefore 30 min. 29 sec., and doubled it is 1 deg. 0 min. 58 sec.; to this corresponds, first, an equation of the anomaly (since the proportion of the equating may be neglected here, in so small an interval) of [continued on next page]

[continued:...neglected here, in so small an interval) of] 16 min. 30 sec.; likewise the variation of the Moon, 44 sec. From this the mean anomaly of the Moon corresponding to one hour is 32 min. 40 sec. When the equation is added to this, the hourly motion of the equated anomaly before and after the true syzygy always remains 49 min. 10 sec.

If, therefore, you always multiply the difference of the Moon's prosthaphaeresis of the first column — the one which, for the sign and degree of the anomaly of the mean syzygy, presents itself next below in the hourly motion before, and above in the hourly motion after the syzygy — by 49 min. 10 sec., there emerges the proportional part. This is to be accommodated to the mean hourly motion of the Moon for either of the two hours, complicated with the variation, namely 31 min. 13 sec., according as the Moon shall have been set above or below its mean elongation from the Earth in the hypothesis — which is sufficiently understood from its own anomaly.

[Margin: The reduction of the lunar motions for the conjunction or the opposition. (*Reductio motuum lunarium ad conjunctionem aut oppositionem.*)]

For above [the mean], the proportional part is to be subtracted; below, it is to be added. And this way seems the easiest and safest, nor is there any reason for anyone to be anxious about deriving the proportional part from the excess and the proportional scruples, since it can scarcely ever rise here beyond 1[?] min. By these hourly motions, immediately surrounding the middle of the eclipse, the reduction of the Moon to the mean ecliptic syzygy is also done exactly, as will be evident in the examples.

IV.

[Margin: The estimation of the eclipse's magnitude. (*Æstimatio quantitatis deliquii.*)]

With the hourly motions of the Moon, and the true eclipse-syzygy, found according to what precedes, one must proceed to the magnitude of the eclipse and the time of its duration. Here that other thing comes into the practice which we said is required: concerning the latitude of the Moon, and likewise the magnitude of the semidiameters of the luminaries and of the shadow — which can be sought from the tables above.

The magnitude of a lunar eclipse, therefore, is to be judged from its own true latitude, and likewise from the apparent semidiameters of the Moon and of the shadow; but the magnitude of a solar eclipse from the visible latitude of the Moon, and likewise from the diameters of this and of the Sun. All these are to be sought from the tables above: the semidiameters by the equated anomaly of the luminaries, the latitude of the Moon from the first column of its own table by the true motion of latitude. Now when the latitude of the Moon is subtracted from the two semidiameters added together, there remains the greatest magnitude of the eclipse. If you should wish to have the same in digits — of which the whole luminary is reckoned at 12 digits — it will be: as the whole diameter of the obscured luminary is to 12, so the remaining magnitude is to the fourth term, that is, the digits of obscuration with their scruples.

It must be noted, however, that in a partial lunar eclipse that little portion which we added above for the various elevations of the pole is always to be subtracted, from the deficient part or from the latitude of the Moon, at the middle of the eclipse. But in a solar defect, the Moon's semidiameter is always to be diminished by that little portion which we there left for the corresponding elevation of the pole.

V.

[Margin: Foreknowledge of the eclipse's duration. (*Cognitio durationis eclipseos.*)]

Furthermore, to foreknow how long the defect will last, and likewise how much time of the Moon's incidence into and tarrying within the shadow is required in a total eclipse of this kind: for this, then, let both the aggregate of the semidiameters and the latitude of the Moon at the mean eclipse-syzygy be resolved into seconds; and from the square of the aggregate of the semidiameters let the square of the latitude of the Moon be subtracted, each separately squared. There is left a square, whose root shows the course of the Moon in its proper orbit from the beginning to the middle, or from the middle to the end — to be resolved into time by the Moon's hourly motion.

But in a total lunar eclipse — which is recognized when what is left over from the difference of the semidiameters of the Moon and the shadow exceeds the latitude of the Moon, after the Moon's semidiameter has first been augmented by that little portion which we added above for the various elevations of the pole (which, however, is not added beyond this when, namely, a total eclipse of the Moon is foreknown to occur) — therefore, since the beginning of the whole obscuration, or again the beginning of the first reappearance of the Moon from the shadow at the middle (the time we call the half-tarrying in the shadow) is to be known: the said resolved difference of the semidiameters will be multiplied into itself, and from this number the squared [continued on next page]

[Margin: *Lib. 4 revolut., cap. 32. (De Revolutionibus IV.32)* — Copernicus]

[continued:...and from this number the squared] number of the latitude will be subtracted, for the root from the remainder will show the half-tarrying in the Moon's motion, likewise to be converted into time by the Moon's hourly motion. The demonstration of these things must be sought from Copernicus [Copernicus]; but the practice is helped compendiously by figurate arithmetic concerning square numbers, and so on. The same may also be accomplished more briefly by the tables of Reinhold, rightly accommodated.

[Margin: *Arith. figurata cap. 3. Præcept. tabul. Rhenb. 52. & 63.* (Reinhold's tables, precepts 52 and 63)]

VI.

[Margin: Knowledge of the true and visible latitude of the Moon at the beginning and end of the eclipse. (*Notitia veræ aut visæ latitudinis Lunæ ad initium & finem eclips.*)]

Finally, that the true and visible latitude of the Moon at the beginning and end of the eclipse may be established: let the motion of longitude of the Moon, found between the middle and the beginning, be subtracted from the true or visible motion of latitude at the middle; and through the remainder let the latitude of the Moon at the beginning be sought, by the first column only of the table of the Moon's latitude above — the column which is destined for the conjunction and opposition of the Moon with the Sun.

And although, in the meantime, the prorogation of the nodes' antecedence varies the latitude almost imperceptibly, it can nevertheless be taken into account, by adding to the Moon's motion of latitude $2\frac{1}{7}$ min. for each hour by which it exceeds the hourly motion of the Moon's magnitude from the Sun — whether the motion of latitude is to be reduced backward or forward from the mid-eclipse.

[Margin: This precept pertains chiefly to a lunar eclipse; on another matter, see below, on the solar eclipse. (*Hoc quidem præceptum ad eclipsin lunæ maxime pertinet. Aliud vide infra de eclipsi solis.*)]

For in the same way the motion of the Moon's latitude is recognized for the end of the eclipse, and therefore also the latitude itself, when the same motion of the Moon found from the middle to the beginning is added to the true motion of latitude that agrees with the middle of the eclipse.

Thus far we have delivered, in general, the precepts concerning the defect of both the Sun and the Moon. These, sufficient as they are for the failing Moon, are in no way sufficient for the Sun — on account of the entanglement of the lunar parallaxes — unless these have first both been found and applied to the ecliptic path according to longitude and latitude: since it is not the true but the visible conjunction of the luminaries that is required here.

And although above we treated of the parallaxes of the luminaries in general — both as to their finding in the circle of altitude and their resolution according to longitude and latitude — nevertheless a more compendious method offers itself in eclipses: of seeking not only the distance of the Moon from the Earth in semidiameters (and hence the parallaxes in the vertical circle), but also the angle at the ecliptic, on which the resolution of the parallax chiefly depends, when,

namely, the visible Moon is destitute of almost any sensible latitude, as happens in solar eclipses.

[Margin: The investigation of the Moon's distance in a solar eclipse. (*Investigatio distantiae Lunæ in eclipsi solari.*)]

First, then, for the distance of the Moon from the Earth, let one simply take the distance between the second and third limit, which is 3 deg. 15 min. of the Earth's semidiameters. Then, with the coequated anomaly of the Moon — and, as before, diminished by a semicircle, and the residue halved — let the proportional scruples be sought from the table of the nodes (as above). When these have been multiplied into the difference of 3 deg. 15 min., they leave a proportional part always to be added to the third limit — for example, 54 deg. 23 min. [recte: 0 deg. 54 min.?] of semidiameters [?]. From this, with the corrected semidiameters of the Moon's distance from the Earth, as above, its parallax in the circle of altitude is sought, from which the Sun's parallax at the same altitude is also subtracted.

But since the table of the luminaries' parallaxes is computed for altitudes above the horizon, and here only the Sun's altitude is supposed: if therefore anyone should wish to work out the Moon's eclipse-parallaxes in the circle of altitude more scrupulously than the ancients, let him add to its altitude the parallaxes belonging to it at the supposed solar altitude — or rather, let him take account of its latitude [Greek: ἐν πλάτει (*en platei*, "in latitude")] in the same vertical; thus he will obtain the Moon's true altitude above the horizon as well, and then the parallaxes precisely agreeing with it in the table.

[Margin: The method of investigating the Sun's altitude in a solar eclipse. (*Ratio investigandi altitudinem solis in deliquio solari.*)]

Furthermore, the Sun's altitude above the horizon, and likewise the angle which the vertical circle makes with the ecliptic — on which, too, the remaining parallax of the Moon was above distributed into longitude and latitude — you will catch most compendiously of all, if first, from the right ascension of the mid-heaven (which is given by the benefit of the supposed time), you seek the culminating point of the ecliptic together with its declination. Then, as before, with the elevation of the pole of the place given, and likewise the conjoined place of the luminaries, the two sides of the spherical triangle to be resolved at once present themselves, with the contained angle which the meridian makes with the ecliptic, in the table of the doctrine [continued on next page]

[Margin: *Doctrinæ triang. altitudinis. Lib. 2 Sphæ. probl. 4, c. 1.* (Doctrine of triangles; Book 2 of the *Sphere*, problem 4, chapter 1)]

computed for each individual degree of the ecliptic. Therefore, in resolving the said spherical triangle—whose sides it will here suffice everywhere to be given and worked out in whole degrees and arc-minutes—the complement of the luminaries' altitude above the horizon is what stands opposite this given angle, while the angle that looks toward the side lying on the meridian is the one which the vertical circle marks out with the ecliptic. From this, afterward, the Moon's parallax from the Sun along the circle of altitude is distributed in longitude and latitude, as often as needed, and is fitted appropriately to the Moon's true motion, so that the visible [position] may be obtained.

It should be noted, moreover, that although a general judgment about the ninetieth degree of the ecliptic is drawn from this—namely, that the signs of the Zodiac from ♍ to ♎ require the same [degree] before their passage across the meridian, but afterward the remaining half of the Zodiac, namely from ♎ to ♏—yet from the character of this discovered angle it is reliably known whether the luminaries were in the eastern part of the ecliptic, where the parallax of longitude always increases the true motion of the phenomenon by its own amount; or in the part west of the ninetieth degree, where the same parallax always diminishes the true motion by its own amount. For if that angle has been found to be:

Found	Angle	Conclusion
Before noon	acute	The luminaries are in the eastern part of the ecliptic
After noon	obtuse	The luminaries are in the eastern part of the ecliptic
Before noon	obtuse	The luminaries are in the western part of the ecliptic
After noon	acute	The luminaries are in the western part of the ecliptic

[Margin: A judgment as to in which part or quadrant of the ecliptic the luminaries were, with respect to the ninetieth degree from the horizon.]

This method, for the unraveling and distribution of parallaxes, seems to me the easy one. If certain people would rather proceed by other routes, Reinhold [Erasmus Reinhold], Kepler [Johannes Kepler], and others are at hand. And whom will the variety not occasionally please, since many paths exist within the doctrine of the sphere for reaching those two things we desire? The

demonstration of this, as well as the practice, will be accompanied by the example below on the solar eclipse.

So much in general about the unraveling and distribution of the Moon's parallax from the Sun at any given moment in a solar eclipse. Now we shall briefly and in order append the particular steps that must be worked through in this same matter.

1.

At the time of the true conjunction [synod] of the Sun and Moon, let the Moon's parallax of longitude from the Sun be sought. Converted into time by means of the Moon's hourly motion, this will be applied to the time of true conjunction—to be subtracted, provided the luminaries are found in the part of the ecliptic east of the ninetieth degree; to be added otherwise, in the western part. In this way the estimated apparent time of mid-eclipse emerges, which generally departs from the true time the less, the smaller the parallax of longitude proves to be on either side.

[Margin: Reduction of the true synod to the apparent.]

2.

But so that the Moon's apparent motion away from the Sun over the interval between the true and apparent conjunction may be obtained, let the Moon's parallax of longitude from the Sun again be investigated, this time for the estimated time of apparent mid-eclipse. If this proves smaller than the earlier one, as in the eastern quadrant of the ecliptic, let it be added to the Moon's parallax of longitude found before; but if larger, as in the western part of the ecliptic, let it be subtracted from that same earlier value—and there remains the apparent motion of longitude corresponding to the interval assumed.

[Margin: Inquiry into the apparent motion of the Moon away from the Sun between the true and apparent synod.]

Let the estimated time of the visible conjunction therefore be corrected and precisely reduced to its mid-point, by multiplying the previously found time-interval into the earlier parallax of the Moon's longitude, and dividing the product by the Moon's apparent motion found thereafter. For the amount by which the time can in the meantime be varied on account of a further small change in parallax is tiny. Now it must be that the Moon's parallax of longitude at the time of the apparent synod equals the true distance of the luminaries from

one another, and vice versa—which ought to serve as the test [δοκιμασία (dokimasia, "proof, testing")] and rule of a correctly established time of the visible synod.

[Margin: Exact reduction.]

3.

[Margin: Inquiry into the latitude.]

Once the time of the visible synod has been most correctly established and (as seems convenient, before we proceed further) equated, there is sought both the Moon's latitude in arc-minutes and seconds—by means of its true motion of latitude corresponding to the time of apparent synod—and the Moon's parallax of latitude from the Sun, which is always to be subtracted from the Moon's northern latitude and added to the southern (for us, that is, and for all who look at the ecliptic below their zenith toward the south; for the rest, in the contrary manner). Thus there results the Moon's apparent latitude at the apparent mid-eclipse.

[Margin: Inquiry into the Moon's latitude at the mid-point of a solar eclipse, by the inquiry of the chapter above, without the excess.]

4.

[Margin: Knowing the magnitude of the solar deficiency.]

This apparent latitude of the Moon is compared with the corrected and aggregated semidiameters of the luminaries; from this, by the fourth precept above, the magnitude of the solar deficiency too becomes known. By the fifth, on the other hand, the Moon's equal motion away from the Sun to the beginning and end is derived (here the same as the scruples of incidence and of mid-duration); resolved into time by the Moon's true hourly motion, this gives the estimated time of the beginning and end of the eclipse, once it has been duly applied to this apparent time at the mid-point.

[Margin: The Moon's equal motion from the mid-point to the beginning and end.]

[Margin: The estimated time of the beginning and end of the eclipse.]

5.

[Margin: Knowing the time from the mid-point to the beginning and end; likewise the whole duration of the solar eclipse.]

Furthermore, for both points—the beginning, that is, and the end—each of the Moon's parallaxes from the Sun is sought out, both of longitude and of latitude. The latitude-parallax being kept, and the longitude-parallax converted into time by the hourly motion and duly fitted to that estimated time (in accordance, that is, with what we set before your eyes above in the general application of the parallax of longitude—finding it from the character of the single angle by whose aid the parallax is distributed in length and breadth)—there becomes known the time of incidence from the beginning to the mid-point, and of emersion from the mid-point to the end, and so of the whole duration; and this quite precisely, in proportion as the difference of the longitude-parallax between the estimated and the apparent beginning and end will bring about by thus varying the time.

6.

[Margin: The Moon's apparent latitude at the beginning and end in a solar deficiency, how it is disclosed.]

Finally, so that the Moon's apparent latitude at the beginning and end of the solar deficiency may be disclosed concisely, one must enter the table of the Moon's latitude above, from the beginning, with the mean motion of the Moon's latitude corresponding to the times—namely from the mid-point to the beginning, and from the mid-point to the end (which, as it seems to me, will more correctly be used here than either the motion of longitude or the scruples of incidence, since even the inequality occurring here in the measure of time is nearly recompensed). The scruples of latitude, without excess and so forth, are then to be extracted from it and duly applied to the Moon's apparent latitude at mid-eclipse, according to the disposition of this latitude relative to one or the other of the nodes—account being taken likewise of the difference of the Moon's parallaxes of latitude from the Sun, both at the beginning and at the end, from that same apparent mean latitude. These we accordingly directed above to be kept, once found, for this use; and so too, by the proper application of each difference from the mean, there emerges the Moon's apparent latitude at the beginning and end.

Nothing further now remains except the ordinary delineation of both eclipses, of the Moon and of the Sun. Here, however, once the corrected semidiameters and likewise the Moon's latitude—with its northern or southern denomination at the beginning and end of the eclipse—have been learned from the foregoing, by a common measure laid out in arc-minutes and their customary subdivisions along a straight line up to one degree (or as far as the aggregate of the semidiameters extends), the several items are easily transferred onto paper with the help of

compass and ruler; and so the eclipse is set before the eyes in, as it were, a single figure. Since this matter is plainly mechanical, it will be conveniently shown in its place below, where the individual examples are to be appended.

[Margin: A brief instruction on how eclipses are to be drawn typically on paper.]

An example in the eclipse of the Moon of the Year of the Savior 1612, on the 4th day of the month of May, in the evening.

The foregoing precepts for computing eclipses must now be illustrated by examples, and any deficiency in them, if any there be, supplied. For from examples—which above all instruct beginners in this discipline—each of these may rightly be both expected and provided. We draw our example of the obscuration of each luminary from those eclipses which befell, in each luminary, in the year of the Lord 1612, in the month of May, and which here were observed by us [at Uraniborg] with what diligence we could. The first, of the Moon, whose mid-point—according to the Ephemerides of that most distinguished man Master David Origanus [David Origanus], and the motions of the luminaries restored therein—would have occurred at our meridian on the 4th of May, in the evening at 10 h. 26 m., mean time. That this full Moon was going to be eclipsed is sufficiently shown by the Moon's true motion of latitude at that point, approaching the Tail of the Dragon [Caput/Cauda Draconis, the descending node ☾], and not distant from it by eight whole degrees.

[Margin: I.]

For this time, then, these motions were found from the tables above:

[Margin: II.]

	Sig.	Gr.	Mi.	Sec.
Anomaly of the equinoxes	6	17	45	6
Equation of the equinoxes, to be added			8	20
Mean motion of the Sun	1	22	45	59
Anomaly of the Sun	10	16	59	0
Prosthaphaeresis of the Sun, to be added		1	21	35
Longitude of the Moon from the Sun	5	26	27	30
Anomaly of the Moon	8	18	16	14

	Sig.	Gr.	Mi.	Sec.
Motion of the Moon's latitude	5	17	16	51
Ecliptic prosthaphaeresis of the Moon, to be added		4	53	51
Difference of the prosthaphaereses of Sun and Moon		3	32	16
Which, when it has been added to the longitude of the Moon from the Sun, makes that [longitude]	5	29	59	46
The Moon therefore stands distant from the true full Moon, according to our calculation and the assumed time, at least		0		14

[Margin: III.]

For the Moon's hourly motion before and after this eclipsed full Moon. At the Moon's anomaly 8 [signs] 18 [degrees] &c.:

Difference of the prosthaphaeresis per one degree	Mi.	Sec.
below	0	59
above	0	48
this when it is multiplied separately into the anomaly	49	10

And when the proportional parts have been added to the Moon's hourly motion as if mean, 31 min. 13 sec., there results the true hourly motion of the Moon from the Sun, just before mid-eclipse, 32 min. 1 sec.; but just after, 31 min. 52 sec., with a tiny difference intervening here.

Now since we found the difference in the Moon's motion corresponding to the assumed time, from the true opposition of the luminaries, to be 0 m. 14 sec., this accordingly, divided by the hourly motion 32 m. 1 sec., shows that 26 sec. were lacking in time. The true opposition of the luminaries, therefore, according to our restitution, occurred at 10 h. 26 m. on the 4th of May, to which there corresponds—from the foregoing, reckoned from the equinox:

	Gr.	Mi.	Sec.
true motion of the Sun	24	16	♄ nearly
true motion of the Moon	14	16	♅

	Sig.	Gr.	Mi.	Sec.
Equated anomaly of the Moon	8	23	12	
True motion of latitude	5	22	12	

[Margin: IV.]

Corrected semidiameters	Mi.	Sec.
of the Moon	16	34
of the Shadow [umbra], the variation removed	44	48
Likewise the true latitude of the Moon (northern)	40	56
Therefore the aggregate of the semidiameters of the Moon and the shadow	61	22
Difference from the latitude	20	26

But if 2 min. 16 sec. are added to the Moon's latitude on account of the denser air in this place (which here is most convenient before drawing the eclipse that follows), the difference from the aggregate of the semidiameters will be 18 mi. 10 sec.—which is the part actually deficient here, and to which there correspond eclipsed digits [digiti ecliptici] of 6 gr. 35 mi.

[Margin: V.]

Furthermore, since this partial eclipse of the Moon is determined only for the knowledge of the half-duration, care will be needed: accordingly the Moon's true latitude resolved, namely 40 mi. 56 sec., into seconds of arc, comes to those [values]:

			Mi.	Sec.
The resolved aggregate of the semidiameters of the Moon and shadow, namely 61 min. 22 sec., is	2456 (or) 3822	to which there correspond as squares	6031936 (or) 13557124	
Difference			7525188	
The root of this in seconds of arc is			2743	

			S e c.
		Mi.	
Therefore these reduced are	45	43	

[Margin: See the Prutenic table, fol. 125 and following.]

The same the Prutenic Table [Prutenic Tables] exhibits more concisely, with the things to be corrected corrected, for the appended scruples; but while account is taken of the Moon's increased latitude above, they come to about 45½ min.

	Mi.	Sec.
Now this motion of the Moon from the mid-point to the beginning, or to the end, is	45	30

Converted into time by the hourly motion, it gives the time of incidence from the mid-point to the:

	h.	m.	s.
beginning	1	25	16
end	1	25	40

	h.	m.	s.
The whole duration of the eclipse, therefore, is	2	50	56

Now from the time of the true opposition given above, with the equation of time 9½ min. to be added, it is learned that the apparent time of the mid-deficiency of the Moon in our horizon was 10 h. 38⅓ m. From this the beginning is gathered as 9 h. 13⅓ min., and the end as 12 h. 4⅓ min.

[Margin: VI.]

Finally, when 49½ min.—namely the motion of incidence found—are subtracted, together with the aggregate motion of the nodes corresponding to the true motion of latitude at the true opposition, for the beginning, and the same amount added for the end, there results:

The Moon's true motion of latitude at the	Sig.	Gr.	Mi.	Sec.
Beginning	5	21	23	4

The Moon's true motion of latitude at the	Sig.	Gr.	Mi.	Sec.
End	5	23	1	28

To which corresponds the Moon's latitude itself at the	Mi.	Sec.	
Beginning	44	39	(northern) Sept.
End	36	12	

LIBER PRIMVS [Liber Primus], Cap. 9 (continued)

Figure of the lunar eclipse [Typus eclipseos Lunæ]

Diagram. A circular figure depicting the lunar eclipse. The large outer circle represents the Earth's shadow [terrestrial shadow] at the Moon's distance; its lower half is shaded with horizontal hatching (the dark shadow), and the upper half carries scattered dots (representing stars / the surrounding sky). A horizontal diameter runs across the figure, marking the ecliptic. Three Moon-disks, each drawn with a face, are shown crossing the shadow from left (east) to right (west): the leftmost at the beginning of the eclipse, the middle one at mid-eclipse (deepest immersion in the shaded shadow), and the rightmost at the end. The labels are:

- Left, at the ecliptic line: Oriens ☉ — "East / rising point," with the ascending-node symbol (Head of the Dragon).
- Right, at the ecliptic line: Occidens ☿ — "West / setting point," with the descending-node symbol (Tail of the Dragon).

Of this lunar eclipse, it was permitted to us here to observe only the beginning and the moment of greatest intensity, since the sky was rather overcast at the time. At the beginning, Regulus [Basiliscus] had an altitude above the horizon, in the western quarter of the sky, of 35° 52'. From this the time is derived as 9h 14m. But if we reason — as is likely — that this obscuration had set in a little earlier, no difference will arise between the observation and the calculation.

Furthermore, when we found the altitude of Spica Virginis [spica ♍] in the same part of the sky to be 23° 57', and the time here ἐν πλάτει [Greek: en platei, "broadly / approximately"] 10²/₃h (which falls only very slightly short of the computed mid-eclipse), there were eclipsed of the Moon 6½ digits, and no more.

These were the observations I, the fourth observer, made on this eclipse from our observatory; from which it is clear that it agreed very closely with our own restitution.

Worked example for the solar eclipse which appeared on the 20th day of the same year and month, before noon [Exemplum in eclipsi solis quæ ejusdem anni & mensis die 20 ante meridiem apparuit]

The middle of this true ecliptic conjunction [synodus], according to the ephemerides of Mr. David Origanus [David Origanus], reduced to our meridian, fell on 19 May at 23h 15m. For this instant, then, these motions are collected. To which δειγµασίᾱς [Greek: deigmatias, "by way of demonstration / as a sample"] these motions are gathered:

	Sig.	Gr.	Mi.	Sec.
Anomaly of the equinoxes [anomalía æquinoc-tiorum]	6	17	45	21
Corresponding equation, to be added [æquatio respond. add.]			8	22
Mean motion of the Sun from the mean equinox [medius motus solis a medio æquin.]	2	8	4	39
Anomaly of the Sun [anomalía solis]	11	2	17	37
Prosthaphæresis of the Sun, to be added [prosthaphæresis solis add.]			55	29
Mean motion of the Moon from the Sun [medius motus lunæ a sole]	0	5	49	47
Anomaly of the Moon [anomalía lunæ]	3	11	13	31
Motion of the Moon's latitude [motus latitudinis lunæ]	0	12	47	10
Ecliptic prosthaphæresis of the Moon, to be subtracted [prosthaph. lunæ ecliptica subt.]		4	54	18
Aggregate of the prosthaphæreses [aggregatum prosthaphæresium]		5	49	47
Excess of the Moon's motion over the mean conjunction with the Sun [excessus lunæ motus supra mediam junctionem cum sole]		5	49	47

Therefore the assumed time agrees here precisely with the true conjunction.

The true place of the luminaries, reckoned from the true equinox, is gathered from these as $9^{\circ} 8\frac{1}{2}'$ in Π [Gemini].

THEORICORVM [Theoricorum], Cap. 9 (continued)

I. For the parallax of the Moon's longitude from the Sun under the true conjunction [Pro parallaxi longitudinis lunæ a sole sub vera synodo]

	Gr	Min	
	.	.	
Right ascension of the Sun [ascensio recta solis]	67	27	
Time before noon, 45m [tempus ante merid. 45 m.]	11	15	to be subtracted [Subt.]

	Gr.	Mi.	
Therefore the right ascension of the M.C. [mid-heaven] [ergo ascensio recta M. C.]	56	12	
The culminating point corresponds to [respondet punctum culminans]	28	27	γ [Taurus]
Its declination is [hujus declinatio est]	19	53	N. [B. = Borealis]

The elevation of the pole is $55^{\circ} 43'$; subtracting the declination from it leaves the side in the meridian of $34^{\circ} 16'$. Likewise the distance of the luminaries from the M.C. on the ecliptic is $2^{\circ} 48'$. Finally, the angle of intersection of the meridian and the ecliptic is $80^{\circ} 7'$.

Thus, in the spherical triangle ABC as set out — ex δεδομένοις [Greek: ek dedomenōn, "from the given quantities"]:

Diagram. A thin, narrow spherical triangle labeled at its vertices: A at the top, B at the lower left, C at the lower right (close together at the base). It accompanies the table of sides and angles below.

	Gr.	Mi.	
AB	35	50	from the zenith [a Zenit.], in the meridian [in Merid.]
BC	10	42	distance of the luminaries in the M.C. [distantia luminarium in M. C.]

	Gr.	Mi.	
ABC	77	10	angle of intersection [angulus intersect.]

	Gr.	Mi.	
AC	34	50	complement of the Sun's altitude [compl. altitud. Solis]
ACB	87	45	angle of parallax [angulus parallaxeos]

These are found [inveniuntur]. Since the angle of parallax turns out to be acute, the luminaries have not yet crossed the 90th degree of the ecliptic; the angle of longitude is therefore $2^{\circ} 15'$, to which there correspond from the table proportional minutes [scrupula proportionalia] of longitude of 2m 21s.

Now the Moon's distance from the Earth, in semidiameters, is gathered as $55^{\circ} 34'$ [i.e. 55.34 terrestrial semidiameters]. Therefore the Moon's parallax in the circle of altitude of 56° is taken from the table as 35m 8s — for we reason that the true altitude of the Moon was very nearly this, its latitude having to be added to the Sun's altitude. Likewise the Sun's parallax at that altitude is 1m 29s. And the difference, which is the parallax of the Moon from the Sun in the circle of altitude, is 33m 39s. From these, by means of the proportional minutes, the parallax of the Moon's longitude from the Sun is elicited as 1m 19s.

The Moon's true hourly motion immediately before and after the conjunction is 31m 55s. Dividing the parallax of longitude, 1m 19s, by the Moon's hourly motion yields the time between the true and the apparent conjunction as 2m 8s, to be subtracted; and so the estimated apparent conjunction comes to 11h 13m 52s.

Now since the parallax of the Moon's longitude from the Sun varies imperceptibly over so brief an interval of time, if we then add the equation of time [aequatio temporis] of 6m 50s appropriate to the Sun's place, the apparent time of the apparent conjunction is gathered as 20 May, 11h $20\frac{2}{3}$ m before noon, very nearly.

The angle of latitude at the true conjunction, found earlier from the complement of the longitude, was $87^{\circ} 45'$; and this itself, on account of the brevity of the interval between the true and the visible conjunction, remained virtually unchanged in this example. To it there correspond proportional minutes of 59m 56s, and therefore the parallax of the Moon's latitude from the Sun at the visible conjunction is found to be 33m 36s.

This, subtracted from the Moon's true latitude — namely 40m 50s — leaves the Moon's apparent latitude sought at the visible conjunction: 7m 14s N. [B.]

LIBER PRIMVS [Liber Primus], Cap. 9 (continued)

	Mi.	Sec.
Semidiameter of the Sun [semidiameter solis]	15	4
Corrected semidiameter of the Moon at this place [correcta semidiameter lunæ hoc in loco]	13	13
Therefore the aggregate of the semidiameters [ergo aggregatum semidiametrorum]	28	17
Apparent latitude of the Moon [latitudo lunæ visa]	7	14
The difference, which is the deficient part of the Sun toward the North [differentia quæ est pars deficiens solis ad Septentrionem]	21	3
And in eclipsed digits [et in digitis eclipticis]	8 Part. 23 Min.	

Now from the sum of the semidiameters of the luminaries and the apparent latitude of the Moon there are given proportional minutes of incidence [scrupula incidentiæ] of 27m 10s; and so the time estimated from the visible mid-conjunction to the beginning or to the end is 51m 0s. [Margin: Per tabulam Prut. pag. 114 & seqq. — "By the table, Prutenic Tables, p. 114 and following."]

			Min.		
Therefore the estimated [ergo æstimatum]	beginning [initium]	H. 10	39⅔		
	end [finis]	H. 12	11⅔		
			Mi	Sec	
			n.	.	
Now there is given, by the prescribed method, the parallax of the Moon from the Sun at [datur autem juxta modum præscriptum parallaxis lunæ a sole ad]	beginning [initium]	long.	9	17	Add.

		Mi n.	Sec .	
	latit.	37	22	
end [finem]	long.	5	6	Subt
				.
	latit.	32	36	

Since the parallaxes of longitude at the beginning and the end in this eclipse subtract from the Moon's true motion: from the beginning to mid-eclipse, 7m 58s — which arise by subtracting the parallax of mid-eclipse from the parallax of the beginning (since both happened while the luminaries were in the eastern part of the ecliptic); and from mid-eclipse to the end, 6m 25s, by adding the parallax of mid-eclipse to the parallax of the end. Therefore, in this way, the times from mid-eclipse to the beginning and from mid-eclipse to the end are rightly found, when the parallax of longitude of the angular intervals has been divided by the Moon's true hourly motion — namely 31m 55s — and the time emerging on each occasion is applied to the time of incidence, that is 51m 0s, by addition. For thus I find from mid-eclipse to the beginning 1h 5m 58s, which is called the true time of incidence; and likewise from mid-eclipse to the end 1h 3m 4s, which is called the time of emersion. These times, applied rightly to the mid-eclipse instant (namely 11h 20m 40s), leave the beginning of this eclipse at 10h 14³/₄m and the end at 12h 23¹/₄m.

Finally, in order to obtain the Moon's apparent latitude at the beginning of the eclipse, the mean motion of the Moon's latitude corresponding to the apparent time, from mid-eclipse to:

	Min.	Sec.
beginning [initium]	36	22
end [finem]	34	44

From this, by the table of the Moon's latitude — and indeed at its outset, from one of the nodes — the proper latitude of the Moon in minutes is sought for each terminus, which is done by multiplying the adjusted motion of latitude by 5m 13s, which is the difference of one degree in the table, etc.

		Mi n.	Sec.	
And so there emerges a change	beginning [initium]	3	10	Subt.

	Mi n.	Sec.	
of its latitude from the apparent mean to the [sicque emergit ipsius latit. mutatio a media apparente ad]			
end [finem]	3	3	Add.

since the visible conjunction took place once the Moon had been carried beyond the Head of the Dragon [caput draconis] in the order of the signs [in consequentia signorum], so that it may be perceived from the true motion of latitude up to mid-eclipse.

Finally, in order that the Moon's visible latitude at the beginning and end be obtained precisely, the change too of the parallax of the Moon's latitude of the beginning and end from the visible mean must be taken; this adds to the latitude-minutes above, at the beginning, 3m 46s. Therefore the change of apparent latitude from mid-eclipse to the beginning is 6m 56s, to be subtracted; and the Moon's visible latitude at the beginning is only 0m 18s N. [B.] Similarly, by reasoning about the visible latitude of the Moon at the end, that comes out as 11m 15s N. [B.] Thus far the calculation; there follows the delineation of this solar eclipse.

THEORICORVM [Theoricorum], Cap. 9 (continued)

Figure of the solar eclipse [Typus eclipseos solis]

Diagram. A figure depicting the solar eclipse. At the top is a short graduated scale bar marked 8 16 24 32 (the scale of digits). The central solar disk is drawn with a face and is hatched; its lower portion is occulted, shown by a jagged, serrated (saw-toothed) crescent arc cutting into the disk from below — the Moon's encroaching limb, rendered as the deeply "sickle-shaped" (falcate) bright remnant the author describes. Two Moon-disks flank the Sun, drawn larger and lightly hatched, at left (beginning) and right (end). A vertical line and a horizontal line cross at the Sun, marking the cardinal directions. Labels:

- Top: Septent. — North [Septentrio].
- Bottom: Merid. — South [Meridies].
- Left: Oriens ☉ — East / rising point, with the ascending-node symbol.
- Right: Occid. ☿ — West / setting point, with the descending-node symbol.

The principal moments and phases of this solar eclipse I, the eighth observer, observed most carefully from our observatory, under a sky fairly serene throughout the whole duration.

We detected the visible beginning when the Sun was elevated above the horizon a little beyond 51°, by the large geometric quadrant [quadratum geometricum magnum]. From this the deduced time agrees well enough with the beginning by our calculation above. But the end was, by observation, a little more drawn out. This must be attributed to the uncertainty of determining times near noon from the Sun's altitude — the cause of which I have left to the doctrine of the sphere. [Margin: 2. Sph. cap. 5. — "Book 2 of the Sphere, chapter 5."]

Now the greatest obscuration of the Sun, which set in here at about half past eleven, all of us, contemplating it most carefully and with unanimous agreement — both by direct view through paper [per chartam intuitu] and by a new optical instrument [specillum novum] which projects the rays of the Sun's body distinctly enough onto a little tablet — found by sight to be at most 8 digits; whereas the Copernican calculation supposes it at this place to be about 9½ digits. Moreover — and this is worth recounting — the shining part of the Sun at mid-eclipse was seen by all of us here as strongly sickle-shaped [falcata] on both sides, in the very manner in which you see it in the figure. So much so that, from this eclipse as well as from other solar eclipses observed in these regions, it is readily clear that the visible lunar body at an ecliptic new moon is, on account of the denser constitution of the air among us, perceptibly attenuated or diminished, in the manner I have prescribed above. It will suffice to have reported this here too in good faith.

On the quaternary number of eclipses in each luminary occurring yearly [De quaternario eclipsium in utroque luminari numero quotannis]

On the quaternary number of eclipses in each luminary occurring yearly — or thereabouts — from the beginning of the world, throughout the various tracts and regions of the whole earth: παράδοξον [Greek: paradoxon, "a paradox / contrary to expectation"], against the opinion of the common crowd, ἀποδεικτικόν [Greek: apodeiktikon, "demonstrative / by way of proof"].

Now that the courses of the first revolution and of the luminaries have at last, by God's grace, been restored in this first Book of the Theorics, before I bring it to a close, I have decided to append two things by way of a coda [coronidis loco]: the one

concerning the nearly annual number of eclipses, continued thus from the very beginning of the world; the other concerning the spots in the lunar body that obtrude themselves upon our sight, and the use which these will serve.

I. On the annual number of eclipses [De anniversario numero eclipsium]

The former, even though those who readily understand the foregoing doctrine of eclipses both deduce and recognize it: nevertheless you will persuade the common crowd of the same only with difficulty — especially certain forerunners from the sacred precinct [praecones e sacro loco], who...

These same eclipses also explain a certain human industry that, in these last ages of the world, twists the recurring annual occurrence of eclipses — which are inserted into the ephemerides of certain authors — into miracles or signs portending the world's final day, as if these had been foretold by the prophets and by our Savior himself. [Margin: Joel 3 [recte: 2], v. 15 [recte: 10/31]; Matth. 24, v. 29; Luke 21, v. 25.]

[Margin: Plin. lib. 2, cap. 12.] I am of course not unaware that the failing of the luminaries — like the manifold change, or even perishing, of so many other things in heaven and on earth — does at all times present a general token of the world's consummation. This is something I drew earlier from Pliny [the Elder], elegantly adapted to the mortal condition of the microcosm, where he says: "Who, when he beholds the fixed labors of the stars, would not, being born mortal, pardon his own necessity?" Yet that the eclipses of the Sun and Moon — which occur necessarily according to the course of nature — are those obscurations of the luminaries of which Holy Scripture speaks: scarcely anyone with understanding of these matters would readily believe it. But that belongs to a discussion for another place. What must be shown here is that, in these last ages of the world (contrary to what those people maintain), no more eclipses of the luminaries appear over the whole earth than began at the very beginning of the world, continued through nearly every single revolution of the Sun. For that such annual eclipses in each luminary truly amount, slightly above the count mentioned, to the number four, I shall easily demonstrate by the following proof.

Diagram. A circle representing the celestial sphere. With pole A placed in the sphere, the ecliptic C D E F is drawn; about pole B the lunar circle C G E H is described. At C is the head of the dragon [ascending node Ω], and at E the tail of the dragon, at which intersections the lunar orbit, together with the ecliptic, alone produces (as shown above) the natural eclipses of the luminaries. Along the

right-hand arc, near node C (Ω), the small lettered points R, P, I, Q mark the eclipse-limit arcs (C R for the Sun's limit, R S for the added parallax in latitude); along the left-hand arc the points N, O, M, V (with the node symbols Ω at the upper left and Ξ /cauda) mark the corresponding limits on the opposite side; L is marked at the far left.

- A (interior, upper): pole of the ecliptic
- B (interior, lower): pole of the lunar circle
- C (right, on circle): head of the dragon [ascending node Ω]
- E (left, on circle): tail of the dragon [descending node]
- D (upper left, on ecliptic): ecliptic point
- F (bottom, on ecliptic): ecliptic point
- G (top, on lunar circle): lunar-circle point
- H (bottom, on lunar circle): lunar-circle point
- R, P, I, Q (right arc, near C): eclipse-limit markers (arcs C R, R S)
- S (right arc): endpoint of the parallax-in-latitude arc R S
- N, O, M, V, L (left arc): corresponding eclipse-limit markers on the opposite side

Now, since we everywhere here suppose the mean elongation of the luminaries from the Earth, the greatest eclipse-limit [terminus eclipticus] of the Moon extends no farther than 12° from either of the nodes, and that of the Sun no farther than $6\frac{1}{2}^\circ$, as we have sufficiently set out in the preceding diagram, where all this concerning the nodes is expressed.

In the Moon, then, since its limit is quadrupled (on account of the twofold intersection of the ecliptic by the lunar orbit), the result is 48° , which are to be kept in reserve. Next, since the tropical year, according to our restoration, exhibits a mean course of the Moon away from the Sun, over the year, beyond 12 synodic revolutions, of 4 signs $12^\circ 35'$ — to which are also added the annual regress of the nodes in the order of preceding signs, $19^\circ 20'$ — therefore the whole annual surplus in the Moon's course from the Sun comes to 5 signs $1^\circ 55'$ beyond the 12 synodic returns, that is, beyond the 12 conjunctions of the luminaries under the zodiac. So when, using these $\delta\iota\delta\omicron\mu\acute{\epsilon}\nu\omicron\iota\varsigma$ [Greek: didomenois, dative pl., "the givens" — the data granted], we first compute logistically and in compendious fashion this matter for the Moon to suit our purpose, it will be: as the 12 semi-revolutions, that is —

Sexag. 1	Sex. 2	Sex. 1	gr.	Mi.	gr.	o	Mi.	Sec.
36 ad	1.	14.	31.	55.	Sic 48.	ad (1.	39.	23.

Hence the annual eclipses in the Moon, according to its mean course and that of the nodes, scarcely extend — as you see — to $1\frac{2}{3}$ obscuration-occasions.

Likewise in the Sun, where the limit of this defect was established above as $6\frac{1}{2}^\circ$ — as appears in the diagram alongside Ω , by the arc C R, to which the parallax of the Moon from the Sun in latitude near the horizon adds nearly $58' 40''$, by the arc R S, $11^\circ 20''$ — since always somewhere on Earth, in the first revolution, one or other pole of the ecliptic also grazes the zenith of the place, and so produces such a parallax in latitude of the Moon from the Sun, at least at the mean distance of each from the Earth —

— leaves [it]. The limit, therefore, of a solar eclipse is reckoned over the whole earth as $17\frac{1}{2}^\circ$, which itself, likewise quadrupled (since the solar eclipses also occur at both nodes), yields 1 sexagena [sex.] 10° . For the annual number of solar eclipses, then, the proportion will stand thus:

Sex 1	Sex. 2.	Sex. 1.	gr.	mi	Sex. 1.	gr.	o.	Mi	Sec.
36. ad	1.	14.	31	55	fic 1.	10 ad (2.	24	55	
			.	.			.	.	

Therefore this number of solar eclipses too is left a little below $2\frac{5}{12}$. When compounded in each luminary, it corresponds to the annual revolution and exceeds the number four — which is a trifling amount, and (on account of the thinning of eclipses through the thickness of the air in certain places, discussed above) rightly to be disregarded.

By this reasoning at least — from the mean courses of the luminaries — the matters concerning the annual number of eclipses have been demonstrated. But when we turn to their true motions and the remaining circumstances of time, it may well happen that, with this number posited, the eclipses in certain years are either more or fewer — in alternate years, that is, or thereabouts — and indeed that in some of these years there is no obscuration at all, especially of the Moon. Yet it nonetheless remains true that, around the eight- or nine-year revolution (within which, namely, the apogee of the Moon completes its period under the zodiac), this quaternary number of eclipses, which we elicit from the mean motions, necessarily returns in its cycle. This is sufficiently clear from the ephemerides, above all those of the most distinguished D. Origanus [David

Origanus], who with most studious and laborious industry had computed and described nearly all the eclipses of both luminaries appearing yearly around the whole globe of the Earth, for as long as his ephemerides run — something that lies open to be known by anyone who loves the truth.

[Margin: On the miraculous solar eclipse at the time of the Lord's Passion I here say nothing.] We conclude, therefore, briefly, from all this that the equality of the number of eclipses in every age has flourished in remarkable fashion — like the other natural things — by the ordination and conservation of the most wise Creator, although this is unknown to the common crowd. And no wonder, since the very antiquity of the ancients (and the recent industry set against it here) drew them, near the world's last age, into error about a plurality of eclipses. And whoever imitates this will know that he does not here truthfully interpret the oracles of Christ in the Gospel, his understanding not being inwardly accurate enough — circumstances and inner sense alike not rightly weighed. This is something which D. Tycho Brahe also sets out in fuller words in his progymnasma. [Margin: Lib. 1, Prog. pag. 328.] But as to his saying there that he sees six eclipses in the Sun and Moon occurring yearly, I take it that he posits this according to the ancient Ptolemaic supposition, and indeed at the extreme of the eclipse-limits' extension — which is neither true nor, as ours is, universal. This, however, detracts nothing at all in that place from Tycho's intention, since he asserts, along with us, an equal continuation of the natural eclipses from the first beginning of things to the end. And this indeed had to be demonstrated, at least against those whom ignorance of these matters also in some measure excuses.

I I.

De maculis in Luna, & ipsarum usu.

[On the spots in the Moon, and their use.]

In treating this subject we shall proceed as follows. First we shall briefly set out, by way of induction, the causes that show the spots in the Moon. To this we shall append an epilogism by which — given the pole's elevation and the place's longitude moderately well known — there is shown the time, or moment, in each daily revolution at which the Moon will be situated near the ninetieth degree of the ecliptic from its rising or setting; and from this a judgment can be made about the visible disposition of the horns and of the spots, or of the whole face of the Moon, relative to the place's horizon.

Lastly we shall add the use that this matter will afford to mortals — and above all to those traversing the vast Ocean — namely, that by its help they may duly obtain the longitude of their place, just as they do by means of eclipses, now that the canons of the Moon and of the stars have been so restored that they are found to conform with sufficient exactness to every celestial course.

That the Moon is a body by its own nature and essence not at all translucent, but opaque, several things together prove, among which these chiefly present themselves.

First, the disappearance of the Moon at conjunction with the Sun, so that no bright traces of it remain in the sky. On this likeness a Christian poet of our age, Stigelius [Johann Stigel], elegantly and sweetly played, treating of the immortality of the pious departed, when he sang thus:

The Moon, as though joined more closely to her brother, seems to fools to have utterly perished; Yet she, who turns toward the lights of the Sun once near them, shines forth far brighter. Just so, the crowd thinks those have perished who seem to die; But I would say: he who has departed has not perished — Rather, those who have gained Christ the nearer, obtain what they sought.

Second, a solar eclipse, of whose visible form the Moon is the cause, exhibits the same thing to the eye, when from the Moon a shadow in the shape of a cone is cast downward — which shadow indeed touches no body whatever in its whole substance or surface, but only one that is luminous. As for the Moon's appearing to be endowed with a certain faint glow in its total eclipse, this is to be attributed rather to that glow than to the solar one. (In the latter we have at times even seen the light, as in the solar eclipse of the year 1605, on the 23rd day of September, observed here with a blackish appearance — [Margin: See Joh. Kepler in his Optics (*Astronomiae Pars Optica*), and likewise M. Maestlin's judgment, *ibid.* p. 254.] the Moon, penetrating the earthly shadow, in some way offering itself to be seen by the eyes.) So too in the position closest to the Sun — when in the evening it first appears after the conjunction, and in the morning, nearest again, by visible approach to the Sun, it shows us its whole face, contained in one globe, under a sky now lit beneath the Moon — this is by no means to be attributed to its own native light, but partly to the Sun's rays still lasting to some degree in that upper region above the horizon, and partly also to the light shed by the remaining stars, for receiving which the Moon is likewise fashioned like a mirror.

[Margin: Gen. 1, v. 16; 1 Cor. 15, v. 41.] Nor will it offend anyone that in Scripture the Moon is called "the lesser light," and elsewhere in the same passage is compared with the Sun and stars as to its differing splendor; since Scripture, as is admitted, accommodates itself above all to appearance, as it were to our grasp — which is plain from a hundred other places. The Moon, then, which Scripture names the lesser among the two luminaries, is such not in itself and its own nature, but only by reason and proportion of appearance — being, namely, the nearest of all to the Earth — though it is greater than any of the stars among those called fixed.

[Margin: John 1, cap. 1.] And what mystery's interpretation could be found more fitting — where the Sun [stands for] God of justice, or Christ, and the Moon [for] the Church subject to Him — than that, just as this Moon obtains the whole force and power of its light from the Sun, so the Church, illumined by Him who lights every man coming into the world, obtains it [from Him]? But those who assert that a certain little glow has been left in man, by which even apart from Christ's spirit and total illumination he might be able to do good works unto life, fall — by resisting the testimonies of the sacred codex — most unhappily into the heresy of the Synergists and the like.

Accordingly, since from what has been adduced we gather that the Moon, lacking its own light, borrows it all from the Sun — (to use Pliny's words) showing it back with varied torch and face according to a fixed disposition relative both to the Sun and to the Earth; for in it we discern the boundary-line of its waxing and waning, as —

It is born while dying, dies just now born, and wanders new; Twelve times in the year it takes to itself its faces, six by six.

What, then, prevents the Moon itself from being a body in itself opaque, yet on its surface illuminated by the Sun's rays — when perhaps we rightly determine, with [Galileo] the Florentine, by reasoning from that quality with which the Earth is surrounded [its peristasis], that something similar or thicker exists about the Moon — thicker, I say, in the degree that the lunar body is perceived to abound more, by its richer moisture, than the globe of the Earth, and is, as it were, discerned to do so?

[Margin: Cael. Rhodig. lib. 1, c. 3.] To assert with the ancients, then, that the Moon is an aethereal earth — as nothing forbids — and likewise to attribute to it a peristasis evaporated from enclosed humors, on the part where the Sun's rays fall, which they remarkably illuminate. Without doubt it would, just like the globe

of our habitation, report a history of its surface — and would communicate it to us through a faithful Menippus of the lunar globe, taken from the apparent face — were a reliable inter-messenger not lacking between the worlds. You see, then, by Galileo's demonstration, that the lunar body has vapors about it, and a free incidence of light, by no means impeded — and hence why its visible rim is on every side freed of spots, with which nevertheless the rest of its face (especially the eastern and the upper part) so teems in the manner of a globe, because there are scattered over it, after the fashion of a globe, rough, earthy, and similar fields, which reflect the solar light cast higher [more steeply], and do not absorb it. To these parts of the Moon, rather than to watery ones, I judge that the spots seen in it are to be attributed — with the most excellent Kepler, against Plutarch — and I so reckon them. Accordingly —

[Margin: What the spots in the Moon are.] Accordingly, these scattered spots in the Moon — which are discerned chiefly in it around the full Moon, and indeed most conveniently with the aid of the common spyglass [tube of vision] — are nothing other than the showing-through of the more solid and drier parts of the lunar fabric, through the surrounding compass of vapors about it (which we earlier named the peristasis), illuminated by the solar rays. For such spots cannot, as some hold, result from the form of the Earth, which the Moon, like a mirror, would receive into itself. For if they were formed by such an adventitious impression, they would surely thrust themselves on our gaze diversely in the varied disposition of the Moon at rising or setting, and moreover under various horizons — whereas faithful observations have taught us hitherto the contrary, so that the said spots keep a constant disposition relative to the line drawn from the pole of the ecliptic through the center of the Moon. By this too it is shown that the Moon is not rotatable about its own center, as may be seen in the present diagram. Here, above pole A, the horizon is drawn; but above pole B, the ecliptic, or the orbit of the Moon, is described. To this we attach the same lunar face threefold, as it appears in its uniform spots — and as we have ourselves several times, and indeed at various seasons of the year, diligently observed with the common spyglass, which is believed to have been invented by the Belgians eight years ago.

Diagram. Three full circles in a row, each depicting the same face of the full Moon with identical maculae (the "man-in-the-moon" pattern of shaded fields). Straight lines run from each lunar disk downward, converging first at point A (labeled "Polus Verticis," the pole of the zenith) and then continuing to point B (labeled "Polus Eclipticae," the pole of the ecliptic) at the bottom. The leftmost

disk is lettered D, the central disk C, the rightmost disk E. "Oriens" (East) is printed at the far left, "Occidens" (West) at the far right.

- Oriens (far left): East
- D (left disk): leftmost lunar face, with a diameter-line through it
- C (center disk): central lunar face, with a vertical line through its center
- E (right disk): rightmost lunar face, with a diameter-line through it
- Occidens (far right): West
- A (center, where lines meet): Polus Verticis — pole of the zenith
- B (bottom, where lines converge): Polus Eclipticae — pole of the ecliptic

In such observations of ours we have always noticed that, when an arc is drawn from pole B of the ecliptic through the center of the Moon, the same disposition of the spots remains perpetually; but from vertex A there is seen an inclination of each [face], greatest near the rising and setting of the ecliptic, until B, passing through A into one straight line, halts the Moon at the ninetieth degree by its indication — that is, when the straight line let down from vertex A through the center of the Moon C onto the horizon either touches each of the individual horns, or divides the spots straight across in the manner here shown, so that for this reason the angle in the triangle [in the sphere] A B D, or A B E, which is discerned in the Moon's declination, measures the difference of this from the said ninetieth degree.

Cap. 9.

[continuation of Cap. IX]

Moreover, so that beginners in astronomy may further have it firmly settled at what time the Moon, by calculation under a given elevation of the pole — and also taking the longitude of the place into account — reaches the said ninetieth degree of the ecliptic from its rising, in each of the daily revolutions, I will here append (as I promised) one method, among the various ones that can be drawn from the doctrine of the sphere, in the briefest set of instructions.

For the noon of the day on which one afterward wishes to know the time of the Moon's transit across the ninetieth degree of the ecliptic, first of all the restored motions of the Sun and Moon are to be copied out — either from the Ephemerides of David Origanus [David Origanus], or according to the restitution presented in this book — and reduced in some fashion to the longitude of the place.

Next, from the table of houses, under the elevation of the place in question, the time after noon agreeing with the place of the Sun is to be copied out by looking it

up under the heading of the tenth house. But for the Moon's place, add 90 degrees, or 3 signs, beforehand; then in the same way, under the heading of the first house, set out the time agreeing with the accumulated motion of the Moon.

Afterward these two times are to be subtracted from one another — namely the Sun's from the Moon's — and for the remainder one must seek the proportional daily motion of the Moon from the Sun, by means of the daily motion of each as it presents itself in the ephemerides. This is added to the Moon's residual motion already set down, and the time agreeing with it is again drawn out. From this, when the time previously assigned to the Sun's motion is finally subtracted, there remains in the residue the sought time of the Moon's transit across the ninetieth degree of the ecliptic. This way is the briefest and most expeditious, and we will now illustrate it with an example.

In the current year 1617, on 4 October — the day on which the full Moon occurs — I wish to know at what hour and minute the Moon climbs to the ninetieth degree of the ecliptic from its rising in the daily revolution, and this on our horizon of Copenhagen [Hafniensi], where the pole is elevated 55 degrees 43 minutes.

At noon of that same day, the motions of the luminaries, reduced from the Ephemerides of D. Origanus, are found to be:

	Sign	Gr.	Min.
Of the Sun	♌	22	6
Of the Moon	♍	20	20

To which, when 3 signs are added, the ascending point comes out in ♍ 20°20'.

To these there corresponds the time, from the tenth house and from the first house, as follows:

	H.	M.
From the 10th house [10 Domicilio]	13	18
From the first house [Prima Domo]	23	1
Difference	9	43

To this difference, with respect to the daily motion of the Moon from the Sun — namely 14 degrees 14 minutes — there correspond 5 degrees 46 seconds. When this motion is added to the Moon's residual position previously set down, that figure is made up to 3 signs 26 degrees 6 minutes, that is, 26 degrees 6 minutes of

§. To this, finally, from the first house under the elevation of 56 degrees, there corresponds a time of 23 h. 34 m. From this, again, with the Sun's time removed, there comes out the residue 10 h. 16 min. And so on that day, 10 h. 16 min. after noon, the Moon reaches the ninetieth degree of the ecliptic from its rising on our horizon — which was to be demonstrated.

It must be noted, however, that whenever the time agreeing with the Sun's motion appears at more hours than the Moon's, then 24 hours, or one whole revolution of the heaven, must be added to the time agreeing with the latter's motion, so that the subtraction can be made — as is commonly known.

Once the time has been found in this way at which the Moon ascends to the ninetieth degree on any given day, I have repeatedly applied to its face — and to the arrangement of the spots upon it — that aforementioned Optical instrument of ours [the spyglass]. This I did both at different seasons of the year and with the Moon variously situated, otherwise, above the horizon; and I have always found the Moon clothed with the same aspect of spots, and have moreover found that the previously shown raising-up and disposition of those spots around the ninetieth degree always recurs. And that this likewise happens with the Moon's horns at that place, the angle of its illumination by the Sun, drawn through the same point of the ecliptic — that is, demonstratively — proves. For when the arc of the great circle, drawn from the pole of the ecliptic through the vertex of the place, descends into the horizon and touches the ecliptic at the point of the ninetieth degree from the rising or setting, it always comes about — as has been said — that the said arc, or line, grazes the extremities of the Moon's horns, whenever the Moon happens to be at that place. Nor is there anything

[continuation of Cap. IX]

appreciable by which the Moon's horns deflect from this said line, when its recession into latitude occurs, especially when the Sun and Moon are at a greater distance from one another. Meanwhile, however, this greatest variation occurs when the Moon, having advanced from one node or the other through a semiquadrant, or a little beyond, where its latitude will have become about 4 degrees.

And thus far concerning the indications by which the Moon's lingering around the ninetieth degree of the ecliptic, in every place, is recognized from the upright disposition of the horns and spots. Now let us proceed further to the usefulness which this matter will have in the investigation of an unknown longitude of a place. That use is the determination of the unknown longitude of a place by

means of the motion of the Moon in longitude observed there. For since the Moon, among all the planets, has the swiftest periodic motion — so much so that in single days it completes about $13\frac{1}{2}$ degrees, more or less — it follows that, by the proportion of the Moon's longitude observed at separate places of the Earth (especially those inclined chiefly toward east or west from one another — although this practical method agrees with all places, provided only they lie under different meridians), the difference of longitude of those same places is elicited. But since, through the most accurate restitution of the Moon's course, its longitude is granted relative to the meridian of Hven [Huenen-fem] — or rather to this our meridian of Copenhagen — therefore, this being always presupposed, there is gathered, from the difference of the longitude given here but observed there, that which was proposed: namely the difference in longitude, measured along the equator, of the unknown place from this known one. This holds whenever the time, too, suitable for the place and for the observation to be made, has been known with sufficient precision. For from the difference of this place's time from our own — which difference the proportional difference in the Moon's motion shows by means of its daily motion — there results the sought difference of longitude, once that difference of time has been resolved into equatorial parts and minutes. This is no different from what is customarily done in the observation and comparison of a single eclipse of the Sun or Moon at various places. Therefore this matter, which for those who understand it — that is, for those somewhat versed in astronomy — is all too clear, will be explained to beginners by an example.

Let the example of this be taken for the 6th day of the month of October, of the current year 1617, when the Moon, somewhere toward the rising of our horizon, was found by observation to be distant 10 degrees from Aldebaran [Aldeboran], since it was then near the ninetieth degree. Let that time, moreover, be 11 h. in the afternoon — which is known through the altitude of some fixed star under the given elevation of the pole, or by some other method, [Margin: Lib. 2. Spher. cap. 5.] according to what we have handed down in the doctrine of the sphere concerning the investigation of time — when the Moon there was near the ninetieth degree of the ecliptic, and the said distance of it from Aldebaran was being taken. Further, since on this supposition the longitude of Aldebaran in this year is at Π 4 degrees 26 minutes, while the Moon at 11 h. of the supposed day is at γ 27 degrees 20 minutes on our horizon — wherefore there is given, under this meridian of Copenhagen, a difference in longitude, suitable for 11 h., of 7 degrees 6 minutes. But at the same hour, for the place of unknown longitude, it was by observation 10 degrees 0 minutes; the difference, therefore, is 2 degrees 54 minutes. This, measured according to the Moon's daily motion — namely 14

degrees 41 minutes — in proportional time, gives 4 h. 45 min. By this it is signified that the place where the observation was made is, with respect to the meridians, removed from this our own toward the east by just so many hours and minutes. And that same moment of time — namely 4¾ h. — resolved into equatorial parts, gives 71¼ degrees. Now since the longitude of the city of Copenhagen was 36⅓ degrees, therefore, these 71¼ degrees being added to it, the longitude of the place that was sought is made up to 107 11/12 degrees.

To this example all other cases will be carried out, taking into account besides toward which cardinal point, from us, the unknown place inclines — of which thing there is an easy recognition, whether through the differing motion of the Moon or by some other conjecture. Now we will first transcribe here from their own canon the more notable stars that shine near the ecliptic and are most suited to this practical purpose, reduced to the end of the year 1620. Then we will show the construction of an instrument by which the distances of the Moon from the said fixed stars can be obtained most conveniently; and finally, in what manner these same distances are to be taken, so that they may represent the bare differences of longitude along the ecliptic, according to the hypothesis premised.

[continuation of Cap. IX]

Designation of fifteen of the more notable stars near the ecliptic, according to longitude and latitude, for the year 1620 completed, by which the Moon in its revolution will be most conveniently measured for the knowing of the longitudes of terrestrial places.

N u m .	NAMES OF THE STARS [NOMINA STELLARVM]	Long. Sign	Long. G.	Long. M.	Lat. G.	Lat. M.	Di r.	M ag .
1	Bright star of Aries [Lucida Arietis]	♈	2	22	9	57	B or .	3
2	Aldebaran [Aldeboran]	♌	4	29	5	31	A us .	1
3	Lower horn ♊ [Inferior cornu]	♊	19	28	2	14	A us	3

N u m .	NAMES OF THE STARS [NOMINA STELLARVM]	Long. Sign	Long. G.	Long. M.	Lat. G.	Lat. M.	Di r.	M ag .
4	Propus Π [Propus]	Π	25	38	0	13	A us	3
5	Lower head Π [Caput inferior]	$\mathfrak{S}$	18	0	6	38	B or	2
6	Southern Asellus $\mathfrak{S}$ [Asellus australis]	ϱ	3	24	0	4	A us	4
7	Heart of ϱ [Cor Leonis]	ϱ	24	33	0	26	B or	1
8	First southern of the wing $\mathfrak{M}$ [Prima australis alae]	$\mathfrak{M}$	21	48	0	43	B or	3
9	Spica $\mathfrak{M}$ [Spica]	$\underline{\mathfrak{M}}$	18	32	1	59	A us	1
10	Southern scale $\underline{\mathfrak{M}}$ [Meridional. lanx]	$\mathfrak{M}$	9	47	0	26	B or	2
11	Bright star of the northern forehead $\mathfrak{M}$ [Septent. clara frontis]	$\mathfrak{M}$	27	52	1	5	B or	2
12	Preceding of the three in the head $\mathfrak{M}$ [Trium praecedens in capite]	$\mathfrak{M}$	8	13	1	45	B or	4
13	Lower horn $\mathfrak{M}$ [Inferior cornu]	$\mathfrak{M}$	28	47	4	41	B or	3
14	Following of the tail $\mathfrak{M}$	$\mathfrak{M}$	18	16	2	29	A	3

N u m .	NAMES OF THE STARS [NOMINA STELLARVM]	Long. Sign	Long. G.	Long. M.	Lat. G.	Lat. M.	Di r.	M ag .
	[Sequens cauda]						us	
15	Succeeding in the pouring-out of water ≈ [Succedens in effusione aquae]	✕	6	20	0	20	A us	4

I judged it right to transcribe these fixed stars here separately from their own canon, so that they might be ready at hand whenever the Moon's zodiacal distances from these are to be taken. Now we are about to describe the instrument by whose aid distances of this sort — namely when the Moon lingers near the ninetieth degree of the ecliptic — most fitly come to be worked out. After choosing, among the various kinds that could be devised, I see that the well-known radius [the cross-staff] — which sailors call the Staff of Jacob [Baculum Jacobi], and with which they are also accustomed to investigate the altitude of the pole on the open sea — is most accommodated to this practical purpose. And although we have at one time disapproved of the use of this radius — when, on a given occasion, we treated of the sufficiency and reliability of instruments — [Margin: Lib. 2. Spher. cap. 7.] nevertheless, because the aberration of the radius is smaller in such smaller distances (for measuring which its transversary alone suffices), and because nothing else could perhaps be employed by sailors as conveniently in a turbid and surging Ocean, therefore let a radius of this sort be prepared, whose use besides may serve sailors. Let its transversary stand to the index, or staff, in roughly a sesquialterate-double proportion [in subsesquidupla quasi ratione], so that this transversary, on the radius — that is, on the whole given sinus — takes in only up to 22 degrees. Beyond which measure there is no need of any other in distances of this sort; and so the transversary need not be shifted from its fixed place on the radius here, nor moved nearer to the eye. In such a proportion it suffices to have the radius's length prepared at 5 feet and the transversary's at 2 feet. But since I presented the construction of the radius in the doctrine of the sphere, I therefore do not repeat it here, [Margin: Cap. 7. Lib. 2. Sphaericor.] except that, so far as concerns the completion of this particular matter, I will touch on it briefly.

Since the bare distance of the Moon from a fixed star is to be taken along the path of the zodiac, an arrangement of the transversary uniform to the ecliptic is therefore required, so that it may be done equally by the latitude of the fixed star and of the Moon. Two transverse runners, then, are to be joined to the transversary of this instrument at right angles: one, with its own sight, by which the latitude of the star is taken at the same time; the other, by which the Moon will be indicated, can be without a vane [pinnacidium], and this from the hypothesis — since here directly

[continuation of Cap. IX]

it falls into the horizon of the place, while it duly indicates the point of the ninetieth degree, in which the Moon will be situated — whether the horns, with the Moon thirsting [i.e. waning], are taken so that its center may be had, or whether the limb nearest the Sun, so that nonetheless, by the parallax of the Moon's semidiameter of 16 minutes at the most, the place of the center of the bodies to be applied at the distance may arise — let the transversal graze it. The length of these runners, moreover, will be made nearly equal on both sides to the length of half the transversary, while the degrees and minutes of the star's latitude are to be probed above and below. With these few remarks we will be content for the explanation of this matter; and as for the example used above which remains, we will represent it merely figuratively — in what manner it would have been worked out by means of a radius of this sort.

Diagram. A woodcut showing the practical use of the cross-staff (Jacob's Staff) for measuring the lunar–stellar distance. An arc representing the ecliptic sweeps across the upper part of the figure. The half-illuminated Moon hangs at upper right above the ecliptic arc; the star Aldebaran is shown at left, just below the ecliptic, as a radiant star-burst. A kneeling man at the lower right holds the cross-staff (a long graduated index with a transverse cross-piece bearing upright vanes) up to his eye, sighting along it. Dotted sight-lines run from the observer's eye to the Moon and to Aldebaran, illustrating the angular distance being measured.

- (upper right): **Luna** — the Moon (drawn half-lit)
- (upper arc): **Ecliptica** — the Ecliptic
- (right end of arc): **Occidens** — West
- (left, below arc): **Aldeboran** — Aldebaran (drawn as a star-burst)
- (lower left, at arc's foot): **Oriens** — East

Finis libri primi Theoricorum. [The End of the First Book of the Theorics.]

LIBER SECVNDVS THEORICORVM

De motibus reliquorum quinque planetarum restitutis.

[The Second Book of the Theorics: On the restored motions of the remaining five planets.]

CAPVT I.

De appellationibus quinque minorum planetarum, ὑποθέσεων in motibus horum disquisitione; et quo modo, quibusve observationibus restitutio, ipsorum a nobis suscipienda et pertractanda est.

[On the names of the five lesser planets, on the hypotheses [Greek: ὑποθέσεων (hypotheseōn, "hypotheses" — the underlying geometrical models)] to be investigated in their motions; and in what manner, and from which observations, their restoration is to be undertaken and carried through by us.]

In the previous book of the Theorics, we set out an account of the threefold world-system [Greek: ὑποτυπώσεως (hypotypōseōs, "outline" or "sketch") of the threefold world-system]; we sketched the planetary hypotheses in general terms; and beyond that we restored the motions of the fixed stars, and of the Sun and Moon, fitting them to every age. In doing so we not only laid the foundation for the restoration of astronomy, but also — relying on divine aid — completed the more important part of it. Now, with that same heavenly power favorable to our celestial labor, we undertake with equal effort, and so far as we are able, to restore the motions of the five wandering stars.

Now, since the nature and effects of these five planets — as of the Sun and Moon — belong to the treatment of natural philosophy [physiologia], it will perhaps be necessary here, for the sake of distinction, to say something only about their names, as they were called by the ancient Chaldeans, Greeks, and by Plato, before we come to their motions, the doctrine of which properly belongs to astronomy.

Saturn is called in Hebrew *Schabthai*, "satiety" or "rest"; in Greek χρόνος [Greek: χρόνος (chronos, "time")], but by Plato φαίνων [Greek: φαίνων (phainōn, "the shining one" or "the apparent one")], that is, "the apparent." For because of its motion, the slowest of all, it does not lie hidden beneath the Sun's rays for as long as the others do.

Next, Jupiter is called in Hebrew *Tzedek*, that is, "justice," because it is believed to rule over what is just; in Greek Ζεύς [Greek: Ζεύς (Zeus)]; in Plato φαέθων [Greek: φαέθων (phaethōn, "the blazing one")], from its splendor.

Mars is called in Hebrew *Maadim*, that is, "redness like blood," both from its appearance and from its effect; by the Greeks and by Ptolemy it is called Ἄρης [Greek: Ἄρης (Arēs)], and by Plato πυρόεις [Greek: πυρόεις (pyroëis, "the fiery one")], from fire.

Venus is called by the Hebrews *Nogah*, that is, "light"; by Plato φώσφορος [Greek: φώσφορος (phōsphoros, "light-bringer")] as the morning star, and ἑσπερος [Greek: ἑσπερος (hesperos, "evening star")] as the evening star; but by Ptolemy and the poets Ἀφροδίτη [Greek: Ἀφροδίτη (Aphroditē)], from the foam [spuma] of the sea and of seed.

Mercury, finally, is called by the Hebrews *Choteb*, that is, "scribe," because it is believed to preside over scribes, the lettered, and the ingenious; in Greek Ἑρμῆς [Greek: Ἑρμῆς (Hermēs)], as if "interpreter," and also Στίλβων [Greek: Στίλβων (Stilbōn, "the glittering one")], from its flashing and quivering light. But enough about these names.

As for the construction of the hypotheses by which the phenomena [Greek: φαινόμενα (phainomena, "appearances")] of these planets are to be saved, the occasion would indeed advise us to choose, from the threefold variety of them set out in the first book and laid out in a general sketch, the one most consonant with nature and truth. Nevertheless, we prefer to follow the plan we adopted from the outset — which was also Tycho's [Tycho Brahe] — and to apply to their motions exactly the same devices invented by all three of the most celebrated practitioners; in such a way, indeed, that the restoration of the superior planets, no less than that of the Moon, is everywhere adapted to the true motion of the Sun, and is thus first completed separately as regards longitude and latitude. For the inferior planets likewise, the estimate and judgment is to be made in its proper place from what the observations, compared with one another, have concluded — namely how these too are subordinate, in their own way, to that same mean motion of the Sun. For before all else we hold that the astronomer must do this: that the celestial appearances [Greek: φαινόμενα (phainomena, "appearances")] —

[Margin: Lib. 1. cap. Theor.]

— [that the celestial appearances] be represented and accounted for as correctly and as concisely as possible, at any time whatever, by means of suitable hypotheses constructed on geometrical principles.

For this task we possess observations. Some result from the resolution of that synopsis of restoration which we once labored to carry out at Uraniborg, bringing it to the mean motion of the Sun during our apprenticeship under Tycho — relying then on the tracks of the ancients, before the final lunar restoration, which was carried out in Bohemia. Others we afterward acquired by our own industry in this most celebrated Danish harbor and royal academy, and likewise separated out into longitude and latitude by faithful calculation of triangles. In all these matters I think it should be pointed out in advance that we do not, except where there is need, press that extreme precision — overly meticulous in its attention to seconds — which the observations themselves also reject; nor do we apply the little equation of time we have demonstrated [aequatuncula temporis]; nor, again, that little correction which would arise from reducing the planets' positions from the ecliptic to their proper orbs, since it is of no great importance and could only be applied to the restoration with very great labor. This was the reason why the ancients too neglected it.

CAPVT II.

De restitutione primae inaequalitatis Saturni secundum longitudinem nostro seculo congrua.

[On the restoration of Saturn's first inequality with respect to longitude, suited to our age.]

Besides the simple revolution of the three superior planets in their orbs — by which they repeatedly look back toward their centers — we showed generally in the previous book that they have a twofold anomaly: the first essential, being attached to the centers of the orbs; the other accidental, arising from the course of the Sun alone and from its configuration with the planet's position. Both of these adhere to the motions of these three planets, just as to the lunar motions, though in a different manner. But since the true motions are tracked down from the mean or simple motions by means of certain hypotheses or tables built up from them — and yet the procedure for establishing these true motions depends on the true motions themselves, which in turn come into perception and knowledge through certain celestial observations of the phenomena [Greek: φαινομένων (phainomenōn, "appearances")] — these true motions must

accordingly be unraveled back into the mean ones, which will afterward be subjected to the perpetual laws of the hypotheses.

Now, since the three superiors — Saturn, Jupiter, and Mars, of which we treat first in order — are, in their outermost nocturnal shinings (in which, with the Earth precisely interposed, they are opposed chiefly to the Sun according to its true course, as we now more correctly perceive), and are thus situated in the middle of their retrogradations, conspicuous by their essential anomaly alone, and therefore entirely free of the other, accidental or solar one — our first concern in each case will accordingly be with deducing these acronychal [acronychus, opposition] positions of theirs rightly from observations, and with correcting them here and there chiefly according to their mean motions and eccentricities. For this undertaking we judged the Prutenic Tables [tabulae Prutenicae] to be most suitable, since they were once geometrically and precisely recalled to computation and perfected anew from the Copernican foundation by that most distinguished man Erasmus Reinhold [Erasmus Reinhold]. Therefore, while with these tables the comparison of observations is faithfully carried out by us over repeated trials, and everything is attended to with sufficient diligence — both what pertains to the appearance of the fixed stars, from which the positions of the planets are ascertained, and the course of the Sun, restored by Lord Tycho for the nearest ages but by us in the previous book for perpetuity — we find that the said Prutenic Tables, in the acronychal positions and the first anomaly of Saturn for the completed year of our Savior CICIIC [1600], taken as the middle of the age of our restoration, are to be corrected in the following motions and anomalies in this way, even when the reduction of these to our meridian is omitted. Namely: that from the mean longitude there be sub-

— tracted 20 minutes from the apogee, 3 whole degrees; but that to the prosthaphaeresis [Greek: προσθαφαίρεσις (prosthaphairesis, "the additive-subtractive correction")] of the eccentric there be everywhere added a fiftieth part, the same measure of the semidiameter of the equant being retained here which is assigned to it from a third part of the whole eccentricity, according to Ptolemy and Copernicus.

When this restoration of the said tables has been made, the acronychal positions of Saturn come out, deduced from the calculation and extended over an entire revolution of this star in the eccentric together with the observed positions, in the manner in which they appear in the following synopsis. Into this synopsis we have, for the sake of brevity, contracted the requisite data of times and motions, which others are accustomed to explain at length.

Synopsis of fifteen positions of Saturn, both from observation in opposition with the true motion of the Sun, and from calculation for the same moments, resulting after the prior emendation of the tables.

N u m b e r									
M Y o H									
Long. Longit. Anom. Praec. Long. Diff									
e a t . M obs. (S. G. med. (S. G. ecce. (S. G. aeq. (G. M. sup. (S. G. e.									
r r h . M.) M. S.) M. S.) S.) M.) (M.)									
I	1 A	2	♄ 7 26	10 9 23 40	2 17 34 4	27 58 50	♄ 7 22 $\frac{2}{3}$	3 $\frac{1}{3}$	—
	5 u	0							
	8 g								
	2 .								
I	1 S	1	♄ 19 50	10 28 26 21	3 0 17 15	27 59 46	♄ 19 48 $\frac{1}{3}$	1 $\frac{1}{3}$	—
I	5 e	0							
	8 p								
	3 t								
	.								
I	1 S	6	♄ 2 34	11 11 7 41	3 12 57 58	28 0 40	♄ 2 35 $\frac{3}{4}$	1 $\frac{3}{4}$	+
I	5 e	$\frac{1}{2}$							
I	8 p								
	4 t								
	.								
I	1 S	1	♄ 15 39 $\frac{1}{2}$	11 23 45 38	3 25 35 10	28 1 34	♄ 15 39 $\frac{1}{4}$	0 $\frac{1}{4}$	—
V	5 e	9							
	8 p	$\frac{1}{2}$							
	5 t								
	.								
V	1 O	1	♄ 29 2	0 6 25 47	4 8 14 47	28 2 27	♄ 29 4 0	2 +	
	5 c	1							
	8 t	0							
	6 .								
V	1 O	9	♄ 12 46	0 19 6 29	4 20 44 51	28 3 20	♄ 12 46 0	0	
I	5 c	0							
	8 t								
	7 .								
V	1 N	1	♄ 26 44	1 1 47 28	5 3 35 11	28 4 14	♄ 26 44 $\frac{2}{3}$	$\frac{2}{3}$ +	

N u M m Y o H			Long.	Longit.	Anom.	Praec.	Long.	Diff
b e n .	Long.	Longit.	Anom.	Praec.	Long.	Diff		
e a t M	obs. (S. G.	med. (S. G.	ecce. (S. G.	aeq. (G. M.	sup. (S. G.	e.		
r r h .	M.)	M. S.)	M. S.)	S.)	M.)	(M.)		
I 5 o 0								
I 8 v ⅙								
8 .								
V 1 N 1	Ⅱ 10 53	1 14 28 50	5 16 15 55	28 5 8	Ⅱ 10 53⅔	⅔ +		
I 5 o 4								
I 8 v ½								
I 9 .								
I 1 D 2	Ⅱ 25 10	1 27 10 14	5 28 56 30	28 6 20	Ⅱ 25 8⅓	1⅓		
X 5 e 0						—		
9 c ½								
0 .								
X 1 D 1	⊖ 9 24½	2 9 51 20	6 11 37 3	28 6 58	⊖ 9 23 0	1½		
5 e 0						—		
9 c								
1 .								
X 1 I 2	♂ 21 15½	3 17 53 36	7 19 37 30	28 9 40	♂ 21 18⅓	2⅝		
I 5 a 1						+		
9 n 0								
5 .								
X 1 I 3	♂ 26 53	9 2 7 21	1 3 43 2	28 21 40	♂ 26 57½	4½		
I 6 u 0						+		
I 0 l								
8 i								
.								
X 1 I 1	≈ 8 31	9 14 45 2	1 16 20 55	28 22 33	≈ 8 31⅓	⅓ +		
I 6 u 3								
I 0 l 0								
I 9 i								
.								
X 1 A 2	≈ 20 10	9 27 22 37	1 28 51 57	28 23 30	≈ 20 14½	4½		
I 6 u 2						+		

N									
u	M								
m	Y o	H							
b	e n	.	Long.	Longit.	Anom.	Praec.	Long.	Diff	
e	a t	M	obs. (S. G.	med. (S. G.	ecce. (S. G.	aeq. (G. M.	sup. (S. G.	e.	
r	r h	.	M.)	M. S.)	M. S.)	S.)	M.)	(M.)	
V	1 g	½							
	0 .								
X	1 A	1	✕ 2 12	10 10 1 0	2 11 35 38	28 24 30	✕ 2 13½	1½	
V	6 u	6						+	
	1 g	0							
	1 .								

From this comparison, then, since it is clear that the calculated positions of Saturn nowhere stand apart from the observed ones — the difference rising to 5 minutes one way or the other — I trust that the restoration of Saturn's first anomaly has been so satisfied by this method and industry of ours for our age that I judge it will scarcely be of any moment to wish to repeat it for the sake of greater precision, or even to arrive at greater [precision] here or in what follows by another route, namely by the calculation of triangles. For that route — by the supposition of three observed positions, after the manner of the ancients [Greek: *πραγματεία* (*pragmateia*, "method" or "treatise")] — is far more specialized, and therefore also more uncertain, entangled in a labyrinth and in very many windings, on account of the insinuation of the equant.

[Margin: Vide praefationem in libros Theoricorum ad lectorem. (See the preface to the books of the Theorics, addressed to the reader.)]

Now, before we come to assessing Saturn's second inequality from observations in like manner, the restoration of the first, set out above, must be entrusted to the threefold hypothesis premised earlier — but in such a way that both the old Ptolemaic geometry around the equant, and the remaining eccentricity of Copernicus, are conformed to the two little circles on the circumference of the orb, according to Tycho's restoration, as we showed in the previous book.

[Margin: Lib. 1. cap. 2.]

For in this way the phenomena, in all three suppositions and for these superior planets, will agree with one another to a nicety in calculation and in the tables.

Hypothesis Ptolemaica in Saturno reformata ac emendata, super qua forma motus reliquorum superiorum explicari potest.

[The Ptolemaic hypothesis reformed and emended for Saturn, upon which model the motion of the remaining superior planets can be explained.]

Diagram. A geometric construction of the reformed Ptolemaic model for Saturn. The figure shows a large eccentric orb at the right and a nested cluster of small circles (deferent epicycle, equant circle, and annual epicycle) at the upper left, with sight-lines converging on the Earth.

- A (right of center): the Earth — the vertex toward which the long sight-lines from the epicycle cluster converge.
- B (top of the large circle): top point of the orb BCP on which Saturn's simple revolution is carried through the whole zodiac.
- C (center of the small-circle cluster, lower portion): the center carried on the orb; about C the larger (deferent) epicycle E–D is described.
- P (bottom of the large circle): bottom point of the orb BCP; the line B–A–P is the vertical diameter through the Earth.
- D (upper-middle of the cluster): the point on the larger epicycle E–D; about D the minor epicycle (equant) F–H is described.
- E (left of the cluster): the opposite point of the larger epicycle from D.
- F (lower point of the minor epicycle/equant circle, below D): one end of the equant-circle diameter F–H.
- G (upper point of the minor epicycle/equant circle, above D): point on the minor epicycle.
- H (right point of the minor epicycle/equant circle): the other end; semidiameter D H is a third part of C H. About H the annual Ptolemaic epicycle I K M L is described.
- I (top of the annual epicycle, above K): point on the annual epicycle.
- K (upper-left of the cluster): point on the annual epicycle I K M L.
- L (right side of the cluster, near M): point on the annual epicycle.
- M (between C and L): the point on the annual epicycle at which the planet stands, always opposite the Sun according to its true motion.

[Margin: Vide cap. 2. lib. 1. sub fin. (See chapter 2 of book 1, near the end.)]

Let A be the Earth; let B C P be the orb upon which Saturn's simple revolution is carried out through the whole zodiac. Again, about center C let the larger epicycle E D be described, whose semidiameter C D is gathered, according to our

restoration premised above, as 8721 parts of the kind in which the semidiameter of the orb A B (that is, A C) is 100000. Finally, about center D let the minor epicycle, or equant, F H be drawn, whose semidiameter D H — which is a third part of C H — comes out, from this same emendation of ours, as 2907 parts.

Now the proportion of the revolutions of these circles is this: that the arc E D be commensurable with the arc B C under the same time. Hence it follows that when C has been at B, D will be at E; but the equant F H, with respect to the same first epicycle E D, since it keeps a twofold measure of revolution, therefore — when C has been at B — H will here be in E F D H, the angle being always double the angle B A C, or D C E throughout, as was set out in the previous book.

Now upon these little circles and epicycles, although Saturn's first anomaly, and that of the superior planets, could be explained, nevertheless — so that the acronychal position of the planet may at the same time be exhibited in this Ptolemaic form — let the annual Ptolemaic epicycle I K M L be added, described about center H; and the planet will be at M, always opposed to the Sun according to its true motion. The semidiameter H M of this epicycle will afterward also be emended from our observations, and everything that can pertain to determining the true position of Saturn at any time will be explained. For now it suffices to teach how we may obtain, through the calculation of triangles from the premised supposition, the prosthaphaeresis of the first anomaly which befalls the planet when in opposition to the Sun.

And so, in the plane triangle C D H: since these two sides, namely C D and D H, are always given from what was premised — the former being 8721, the latter 2907 — together with the contained angle C D H, which by hypothesis is always double the angle B A C, or E C D — therefore from these the angle D C H is first given. This is here to be added to the angle E C D, together with the side C H. Then in the triangle A C H, since likewise two sides with the contained angle are given — namely A C, 100000, which is the radius of the planet's orb; and the side C H just now found, together with the included angle A C H, which is the complement of the angle E C H to a semicircle — therefore here too, by analysis of the present triangle A C H, the angle of the entire prosthaphaeresis of the first inequality, C A H, is investigated. This is to be subtracted here from the mean motion of Saturn at C, so that the true [motion] at H, or M, may be had; for the line A I

running across from A grazes the planet stationed at M in the lowest part of the annual epicycle. Together with this prosthaphaeresis angle one usually also obtains the side A H, whose use will appear later in connection with the prosthaphaeresis of the annual orb, to be sought out everywhere outside the

acronychal position. This Ptolemaic supposition is exactly alike for all three superior planets, except that a variation in the semidiameters arises within the individual epicycles. Now let us compare the hypothesis of the moderns — and earlier that of Copernicus — with this Ptolemaic one in a similar configuration.

The hypothesis of the same configuration of Saturn and the rest of the superiors in the Copernican form.

From the Sun, as the center of the universe according to Copernicus, let the orb of the primary revolution of Saturn and of the three superior planets B C be described. With the center placed at C, let the first epicycle E D be drawn, and then the second one from D, namely F H, where the planet is understood to be carried around at H by a motion contrary to E D, as was set out above. Finally, from the same A, that is, the Sun, let the annual orb of the Earth I K M L be described with compasses.

Now whenever the Earth is at M, exactly interposed between H, the place of the planet, and the apparent place of the Sun at A, the prosthaphaeresis of the first inequality is produced by the same computation of triangles from the very *διδόμενοις* [Greek: *didomena*, "the givens" — the given data] (the Sun, that is, being placed in the position of the Earth) which were demonstrated above in the Ptolemaic form. Nothing further is added on account of the second anomaly, except where the Earth M has migrated to another place through its aforesaid annual orb. And since the semidiameter of the annual orb here, A M, is equal to the semidiameter of the superior Ptolemaic epicycle H M, with respect to appearance the same phenomena are elicited here in every way. Moreover, once the distance of the Sun from the Earth, A M, is given in a definite measure, the distance A C of Saturn's orb from the Sun, and so on, can likewise be obtained in the same measure — which the earlier Ptolemaic method denied us the means of investigating.

Diagram. The Copernican-form schematic of Saturn's configuration. A large circle centered near the lower middle of the page is the Sun-centered orb of the primary revolution of Saturn, B at its top and P at its bottom, with the vertical diameter B–P. Inside, near the upper left, sits a small cluster of circles representing the eccentric and the two epicycles E D and F H, with the planet at H; the point G is at the top of this small cluster and C, F at its center. From this cluster a line runs down toward the lower-center small circle, the annual orb of the Earth I K M L, centered on A (the Sun), with M at its upper edge, I and K at its lower right.

- B (top of large outer orb): terminus of the vertical diameter
- P (bottom of large outer orb): opposite terminus of the vertical diameter
- G (top of upper-left epicycle cluster): apex of the first epicycle E D
- E (left of cluster): point on the first epicycle
- D (within cluster): center of the second epicycle
- H (within cluster, near C): planet's position on the second epicycle F H
- F (within cluster): point on the second epicycle
- C (center of cluster): center carrying the epicycles
- A (center of lower annual orb): the Sun
- M (upper edge of annual orb): the Earth's position interposed between H and A
- I (lower edge of annual orb): point on the Earth's annual orb
- K (lower right of annual orb): point on the Earth's annual orb
- L (left, on annual orb): point on the Earth's annual orb

The hypothesis of the same configuration of Saturn, etc., in the Tychonic form, and at the acronychal position.

From the center of the true place of the Sun let the orb of Saturn, etc., be described as before, B C. Then let the epicycles be made from C and D, and let the angle C A H of the entire prosthaphaeresis of Saturn's first inequality, etc., be elicited in every respect as above.

[Margin: Book 1.] [Margin: this chapter's use.]

Afterwards, on the straight line A H, about the center M and with the radius M A, let the orb of the Sun A L be described with compasses; in this orb the center of the planet's orb, A, is understood to be carried likewise in the order of the signs [in signorum consequentialia]. Therefore, when the Earth M is directly interposed between the Sun A — taken according to its true motion — and the planet H, as in this case, the whole second anomaly is entirely exhausted, because our line of sight from the Earth M coincides with the line drawn from A and extended to the position of the planet at H on the second epicycle. But in the other positions, outside the acronychal ones, the planet exhibits everywhere from the annual orb the same anomalies as the Ptolemaic and Copernican do, as will also be shown further below.

[Margin: Cap. 9 of book 2.]

Now this Tychonic supposition, although it is plainly an inversion of the Copernican, nevertheless likewise introduces epicycles in these three planets after the manner of Ptolemy — given the revolution of the Sun as the center of those same orbs — which are wider the smaller the distances of the orbs from the Sun or the Earth, and conversely; as we indicated in its proper place in the preceding book. And so far concerning the first inequality of Saturn, etc.; the other one follows.

[Margin: Book 1 of the Theory, ch. 1, near the end.]

Diagram. The Tychonic-form schematic. A large circle, the orb of Saturn B C, has B at its top and P at its bottom, with the vertical diameter B–P; the center is A, low in the figure. In the upper left corner sits the epicycle cluster: a small circle topped by G, with H at its right, F and C at its center, and F also marked at the bottom left of the cluster. From the cluster at C a long line runs down to A. Between the cluster and A, a smaller circle (the Sun's orb A L) is drawn about the center M, with L at its upper left and M at its center.

- B (top of large orb): terminus of the vertical diameter
- P (bottom of large orb): opposite terminus of the vertical diameter
- A (low center of large orb): center, the planet's place on the second epicycle / the point carried in the Sun's orb
- G (top of upper-left epicycle cluster): apex of the first epicycle
- H (right of cluster): planet's position on the second epicycle
- F (within cluster, and lower left): points on the epicycle
- C (center of cluster): center carrying the epicycles
- M (center of the intermediate small orb): the Earth
- L (upper left of the intermediate small orb): terminus of the Sun's orb A L

CHAPTER III.

ON

The restitution of the second, or annual, inequality in Saturn.

Since for the second inequality — to which these three superior planets are subject on account of the annual revolution of the Sun (or of the Earth in Copernicus) — we have already, in the preceding chapter, prepared the hypotheses together, taking the occasion left to us in the old supposition, it now seems to remain only to investigate, in the present chapter, the measure of an epicycle of the kind by which we said this annual inequality is excused, likewise

from several observations of Saturn — after the semidiameters of the small circles in this same planet have, by the weighing of the acronychal positions, been set forth above in their own numbers.

Since, then, for this reason the entire theory of Saturn is brought back to the computation of triangles — a likeness that is nowhere abandoned in the two that follow — I therefore promise that, if lovers of the stars will once attend with due care to its application and execution, which governs and perfects this art, all the rest that follows will be easier for them.

In the year of Christ our Savior 1590, on February 8, at the 8th hour in the evening, the longitude of Saturn was found by us from observation at Hven [Huena, i.e. Hven/Uraniborg] to be $7^{\circ} 32' \text{ II}$ [Gemini], near the maximum annual digression in the order preceding the signs [in signorum antecedentia]. For again, four days later, namely on February 12 about the same 8th hour, it was observed at $7^{\circ} 38\frac{1}{2}' \text{ II}$ [Gemini] with a southern latitude of $1^{\circ} 30'$.

Now on February 8 at the 8th hour the Sun was, by its own true motion from the true equinox, at $0^{\circ} 0' 40'' \text{ X}$ [Pisces]; but Saturn's mean longitude, corrected from the above likewise from the true equinox, was $2s 15^{\circ} 10' 5''$; and moreover the eccentric anomaly of the same Saturn — that is, its simple distance from the apogee — derived from one acronychal correction up to this time, comes out as $5s 18^{\circ} 51' 40''$. The arrangement of these motions according to the Ptolemaic theory of Saturn set forth above is as follows.

The theory of Saturn exhibiting both of its inequalities from the data above, in the Ptolemaic form.

First, in proportion to the given eccentric anomaly, C is moved away from B through the orb of Saturn's revolution in the order of the signs, and the first epicycle E D is described as before above the point C, with an arc of equal measure in the preceding direction [in antecedentia] as B C has in the order of the signs. Next, from the center D the second epicycle F G H is added, whose revolution from F through G in the forward direction therefore ends here at H, because it completes by its own motion in this small circle a double of the proposed anomaly's arc, H. Finally, about the center H we attach the annual orb of Saturn in this Ptolemaic form, K M L. And since our purpose is to inquire into the semidiameter of this annual orb, H L, in Saturn, we shall accomplish this about the same point in this form by means of celestial observations employed — leaving the rest in order for the following [demonstrations], for the sake of

greater understanding and pleasure, since indeed no difference at all intervenes in that threefold supposition.

First, then, in the triangle C D H, since by what was prescribed the sides are given — namely C D, 8721, and likewise D H, 2907 — together with the angle C D H, which here is the complement to the whole circle of the double anomaly of Saturn given, that is, $22^{\circ} 16' 40''$: there is found both the angle D C H, $10^{\circ} 22' 24''$, and the side C H, 6120 parts. As for the rest,

Diagram. The Ptolemaic-form theory of Saturn showing both inequalities. (The same figure spans the foot of p. 343 and the head of p. 344.) A large outer circle is the orb of Saturn's revolution, with B at top, A on the right (Earth/observer), and P at the lower right. In the lower-left interior sits the epicycle cluster: the first epicycle and the second epicycle F G H with the planet at H, surrounded by the annual orb K M L. Labels in the cluster, reading roughly top to bottom: M, G, D, H, F, C, with K at the left and F repeated at the bottom; L is marked at the left as the upper terminus of the annual orb. Sight-lines run from A down to the cluster.

- B (top of outer orb): apogee terminus of the diameter
- A (right of outer orb): the Earth / observer's vertex
- P (lower right of outer orb): perigee terminus of the diameter
- C (center of the lower-left cluster): center carrying the epicycles
- D (within cluster): center of the second epicycle
- H (within cluster): planet's position on the second epicycle F G H
- G (within cluster): point on the second epicycle (revolution F→G→H)
- F (within cluster, and at bottom): the start-point of the second epicycle's revolution
- M (top of annual orb): point on the annual orb K M L
- K (left of annual orb): point on the annual orb
- L (left, terminus): the semidiameter H L of the annual orb sought

subtracting the found angle D C H from the simple anomaly, there remain $5^{\circ} 8' 29' 16''$, whose complement to the semicircle — namely $21^{\circ} 30' 44''$ — measures the angle A C H. And so in the same triangle A C H, when from there the two sides A C 100000 and C H 6120 are given, together with the said angle A C H contained between them, there is sought likewise the angle of Saturn's entire first prosthaphaeresis, C A H, and it is found to be $1^{\circ} 21' 48''$, together with the side A H 94330, of the kind whose radius of the orb A C is 100000. Now, subtracting the found eccentric prosthaphaeresis from the proposed mean longitude of Saturn, there remain $2^{\circ} 13' 48' 17''$. If this longitude is again subtracted from the true

place of the Sun, 11s 0° 0' 40", the remainder is 8s 16° 12' 23", the anomaly of the annual orb, or the arc K M L. Removing then from each side 6s for the semicircle K M, there remain 2s 16° 12' 23", that is, in resolved [degrees] 76° 12' 13" [recte: 76° 12' 23"?], by which the angle A H L is measured.

Lastly, then, in the triangle A H L, for the sought H L the data are: first, the side A H 94330; second, the angle A H L 76° 12' 13"; third, the angle H A L 6° 16' 17", which results from the difference of comparison of the true place of Saturn on the eccentric H from the true equinox — which was 2s 13° 48' 17" — and of the place observed in the heavens, 2s 7° 32' 0". Having premised here too the sufficient δεδομένα [Greek: dedomena, "the givens"], there comes out, by the analysis of the said triangle A H L, that which we set ourselves to seek from the start, namely the side H L 10398½, which is the measure of the semidiameter of the annual orb in Saturn — arisen for us from observation through all three hypotheses. When this is compared with what likewise comes out from the place of Saturn observed in the year 1591, in the month of March, on the 17th day of that month at the 7½th hour in the evening, at 22° 24½' ♊ [Gemini], an exceedingly small and almost no difference is detected.

For the rest, so that the magnitude of the semidiameter of this annual orb may be compared also from the other side, we have taken up another observation of Saturn, made in the same year as the former — namely on September 7 in the year 1590, at the 12th hour of the following night, where Saturn was found at 28° 6' ♊ [Gemini] (with a southern latitude of 1° 11'), near the maximum eastern digression, that is, the earlier station. For the true place of the Sun at this time was at 24° 28' 35" ♍ [Virgo]; but Saturn's mean longitude from the true equinox was 2s 22° 14' 40", with an eccentric anomaly of 5s 25° 55' 17". These being supposed, the following hypothesis of Saturn in the Ptolemaic form is fitted, as above.

Carrying out the calculation from the given quantities [Greek: δεδομένοις (didomenois, "the things given")] by the method above, through the doctrine of triangles, the angle DCH first comes out as 4° 2' 16"; next the side CH as 5852; third, the angle of the eccentric's prosthaphaeresis CAH as 0° 30' 3", to be subtracted; and fourth, finally, the side AH as 94234, in the units where the radius AC of the orb is 100000. Then, subtracting the true longitude of Saturn's eccentric with respect to the true equinox — namely 2s 21° 44' 37" — from the true place of the Sun, 5s 24° 28' 35", there remain about 3s 2° 44' for the anomaly of the annual orb, KHL, whose complement to a semicircle is the angle AHL.

But likewise: when this same true eccentric place of Saturn, $2s\ 21^\circ 44' 37''$, is subtracted from the observed place at $28^\circ 6' 0'' \times$, there remain $6^\circ 21' 23''$, which measure the angle LAH. This shows Saturn's digression at L — in the annual orb, that is — here in the order of the signs. And so in the triangle AHL, whose angles are given together with the side AH (94234, obtained above), the sought side HL comes out on this side as $10453\frac{3}{4}$. But the same side was found earlier from the western side as $10398\frac{3}{4}$. The difference is therefore 55 parts, half of which, $27\frac{1}{2}$, when added to the smaller value found, or subtracted from the larger value found, yields HL — the semidiameter of Saturn's annual orb — fixed at 10426, in the units where the radius AC of the orb is 100000. And it is fixed so precisely that it did not yet depart by a single minute from the observation or apparent position of Saturn in the heavens.

Now if, with this diameter found — and hence with the complete restitution of Saturn for our age — other observations are compared, or those that appear in Book I of the *Letters* of Tycho Brahe, p. 56, they will fit this emendation of ours nicely and will confirm that we have accomplished the task. For into the apparent position of this star — so far as we can yet determine it with certainty — the unequal distance of the Sun from the Earth introduces no variation. Indeed, if it were so, a smaller distance of the Sun from the Earth would everywhere leave a smaller diameter for the annual orb HL as well, and the reverse. For although this effect — which the eccentricity of the Sun would introduce, given the great extent of Saturn's orb — might be the very smallest of all, nevertheless, if this reasoning here carried any weight, a perceptible apparent effect would surely result from it on one side and the other, and one that would at times extend beyond a fifth part of a degree.

Diagram. A geometric construction for Saturn's annual orb. A large circle (the eccentric orb of Saturn) is drawn, with a nearly vertical line running from B at the top through the interior to E at the bottom. Within the lower part of the large circle sits a smaller circle (the annual orb), and the construction-points cluster in that region.

- B (top of the large circle): endpoint of the central line
- A (interior, upper-middle): center / apex of the triangles
- M (interior, just below A): construction point on the line toward the small circle
- D (inside the small circle): construction point
- L (right of the small circle): position of Saturn in the annual orb (the digression point)

- H (inside the small circle, lower): center of the annual orb
- C (just below the small circle's center): construction point
- K (lower part of the small circle, left): construction point
- E (bottom of the large circle): endpoint of the central line

CAPVT IV.

DE

Mediorum motuum Saturni constitutione atque extensione.

[On the constitution and extension of Saturn's mean motions.]

Since the establishment of mean motions requires the longest possible interval of time between observations made in the heavens, we shall scarcely achieve anything worthwhile here from any later assumption of Ptolemy — even if his observations had been of full reliability (which is not the case). Moreover, it is to be lamented that neither in the time of this most celebrated craftsman among the foremost, nor in that of the others before our Tycho, was anything so certain left behind that one could not doubt it in many ways. If I were to say that the Ptolemaic astrolabe — with which he indicates that he observed these planets — was inaccurate, I would perhaps be saying so with no less right than I showed above in the cases of the Sun and Moon.

But if Ptolemy had indicated his method of observing, and had named the fixed stars one by one to which his sightings were made for the distances, then the remedy for correcting their erroneous places would already have been so well prepared for us, in the restitution of those same stars through Tycho Brahe — indeed, through our own eyes — that we would gladly and deservedly have devoted this care to examining and correcting Ptolemy's own acronychal places. Be that as it may, we assume the apogee places of these planets as his — though they must first be reduced everywhere to the true place of the Sun at the acronychal positions. Then, retaining the semidiameters of the epicycles (which, as I taught above in the case of the Sun, are not altered by time), we shall have recourse to the ancients' observations outside the acronychal positions, for the correction of the mean longitude in individual cases — the same observations from which Ptolemy too once derived the periodic revolutions of these bodies. For at whatever moment these conjunctions of some planet with a fixed star are reckoned — at which both the place of the star and the planet's whole inequality are found from what precedes — it will surely be granted to arrive at what we

intend regarding the correction of this simple longitude far more certainly and securely than from the Ptolemaic data.

Now concerning the advance of the apogee places in the order of the signs — which we shall first of all work out for the individual planets from the ancient source — I judge that I must not pass over the fact that there was, in Ptolemy, among the Alphonsines, and among later astronomers down to N. Copernicus [Nicolaus Copernicus], a unanimous opinion that this advance was the very same as the precession of the equinoxes itself, that is, the progress of the fixed stars in the order of the signs. But as for the motion of the apogee place in Saturn: since it is established, from Ptolemy's acronychal observation — which was made in the 20th year of the Emperor Hadrian, in the year of Christ 136 etc. — to have been at 23° of Scorpio reckoned from the true equinox (to which scarcely half a degree is added), for the reduction of the same to the opposite of the Sun's true place (which addition is here easily gathered from the Sun's prosthaphaeresis, $21'$ additive, and from Saturn's anticipation of motion in the meantime; likewise from the addition of about $21'$ for the inequality of the precession of the equinoxes according to our constitution) — therefore, at the said time of Ptolemy, the apogee's position from the mean equinox, from which we measure the equality of the motions in longitude, is gathered to have been at $23^{\circ} 50'$ of Scorpio, provided that Ptolemy's acronychal observation was otherwise correctly carried out. But in our age — that is, in the year of Christ 1600 completed — through the acronychal restoration and the subtraction of the inequality of the equinox, the same apogee place in Saturn is found at $26^{\circ} 27'$ of Sagittarius. Accordingly, over an interval of 1464 Julian years, Saturn's apogee position has advanced uniformly by $32^{\circ} 37'$.

When this difference is first divided into the interval of years from the first bissextile of the world — likewise from the birth of Christ, and from some one of these epochs — then multiplied into the difference of the years of the proposed age, distributed, and duly adjusted to the others that follow by multiplication of the same, the position of this star's apogee comes out: first, at the first bissextile of the world, $4s\ 22^{\circ} 29' 20''$; then, at the birth of the Saviour of the world, $7s\ 20^{\circ} 48' 12''$.

Then, from the same proportion, the corresponding motion of the same [apogee] is:

		Gr.	Mi.	Sec.	Ter.
1000	annis [years]	22	16	45	0

		Gr.	Mi.	Sec.	Ter.
100	est [is]	2	13	40	30
20		0	26	44	6
1		0	1	20	12

This motion is reserved for future use.

Furthermore, for the restitution of the mean longitude in Saturn, let us take up again the observation of it made by the ancient Assyrians or Egyptians in the year 519 of Nabonassar, in the Egyptian month Tybi, on the 14th day, at the 6th hour in the evening. [Margin: Ptol. l. 11. c. 7. Almag.] This time fell in the year 229 before the birth of Christ; but, reckoned after the first bissextile of the world, in the year 3736, on the first day of March, at the 4th hour, 25 minutes after midday, in our horizon of Copenhagen [Hafniensi]. For on that evening was observed (as Ptolemy reports) Saturn's conjunction with the star following beneath the southern shoulder of Virgo, than which it was more southerly by two digits.

Therefore, since the longitude of the said star from the true vernal equinox at this time is found, by our restitution, to be $9^{\circ} 3'$, with a northern latitude of about $2^{\circ} 43'$, the longitude of Saturn at the same moment was likewise $9^{\circ} 3' \text{ } \mathfrak{M}$, and its latitude $2^{\circ} 38' \text{ N}$. Moreover, the true motion of the Sun, by our restitution at the same time, is found at $7^{\circ} 31\frac{1}{2}' \text{ } \times$ from the mean equinox — from which the simple longitude of Saturn is also given, after a fashion, by the Prutenic tables [tabulis Prutenicis] as $5s 2^{\circ} 24'$. Likewise the apogee place of Saturn from the same equinox, from the corrected values above, is $7s 15^{\circ} 22'$. When this is subtracted from the longitude, there remains the anomaly of Saturn's eccentric, $9s 17^{\circ} 2'$. Since this does not much exceed the mean eccentric elongation in climbing, an error of even one degree, had it been committed here, would introduce only $1\frac{1}{3}$ minutes into the apparent position of Saturn and into its longitude, which we are seeking. With these things thus established, then, for the investigation of Saturn's first inequality, let there be described, as before:

with center A, the orb of Saturn BPC; again, from C, the first epicycle, on whose circumference — given by hypothesis the angle ECD equal to the angle BAC — D is placed; from which, finally, the equant circle FGH is formed. And because the angle in this same circle, FDH, ought (also by hypothesis, in this revolution) to be double the angle BAC, or DCE, H is supposed to be the center of the annual orb in the Ptolemaic form, in all respects as above — but of that, later. For now, since our intention is first to track down the angle of the first inequality, CAH:

and so, as before, in the triangle CDH, since two sides are given — CD 8721, DH 2907 (namely, the semidiameters of the epicycles, from the Tychonic observations [Margin: Lib. priore Theor. in comment. de Sole et Luna.]

Diagram. A geometric construction for Saturn's first inequality. A large circle (the orb of Saturn, BPC) is drawn with center A at the left-center. A nearly vertical diameter runs from B at the top to P at the bottom. From A, two lines run out to the upper-right, where a cluster of small circles (the epicycles and the equant/annual orb) sits on the circumference. The construction-points cluster in that upper-right region.

- B (top of the large circle): endpoint of the vertical diameter
- A (left-center): center of the orb of Saturn
- P (bottom of the large circle): endpoint of the vertical diameter
- K (upper right, outer): construction point
- H (in the cluster): center of the annual orb / equant
- G (in the cluster, near H): point on the equant circle
- D (in the cluster): point on the first epicycle's circumference
- L (left of the cluster): construction point
- M (below the cluster): construction point
- E (right of the cluster): construction point on the epicycle
- C (lower in the cluster): point on the orb / epicycle center
- F (lower in the cluster): point on the equant circle

— from the Tychonic and our own observations, corrected above, which we have supported earlier with reasons that they are in no way to be altered here) — together with the included angle CDH, $45^{\circ} 56'$ (which presents itself when the eccentric's anomaly BPC, $9s\ 17^{\circ} 2'$, is subtracted from the whole circle and what remains is doubled): there is found, therefore, first the angle DCH, $8^{\circ} 19' 28''$, and the side CH, 11246, in the units where AC is 100000. [Margin: Lib. priore Theor. in comment. de Sole et Luna etc.]

Then let the angle DCH be added to the angle ECD, and there accumulate $81^{\circ} 17' 28''$ for the measure of the angle ECH. Accordingly, its complement to a semicircle — namely HCA — is $98^{\circ} 42' 32''$. In this triangle HAC, then, since two sides about the named angle at C are given — namely HC 11246, and CA, the radius of the orb, that is, 100000 — there is obtained, besides the side HA 102261, also the angle CAH of the prosthaphaeresis of Saturn's first inequality, $6^{\circ} 14' 18''$. When this is added to Saturn's estimated simple longitude, $5s\ 2^{\circ} 24'$, there results the true longitude of its eccentric from the mean equinox, $5s\ 8^{\circ} 38' 18''$. When this is

subtracted from the assumed place of the Sun, there remain, for the anomaly of the annual orb, $5^{\circ} 28' 53''$.

This we correct in such a way that an imperceptible error creeps into the disentangling of the annual inequality's given quantities [Greek: διδόμενα (didomena, "things given")]: because the inequality of the precession of the equinoxes this century, according to our constitution, is found to be $6' 25''$ subtractive, therefore, these being added to the place of the fixed star, its longitude from the mean equinox is composed as $5^{\circ} 9' 9' 25''$. And when about $8'$ are subtracted from these — because Saturn was very nearly acronychal — we conclude that, according to the first or eccentric inequality, it was elongated from the mean vernal equinox by $5^{\circ} 9' 1' 25''$. When the place of the Sun, $11^{\circ} 7' 31' 20''$, is subtracted from these, there remains the anomaly of Saturn's annual orb called above — more accurately — $5^{\circ} 28' 30''$ nearly.

Now let there be described, from H, Saturn's annual orb according to Ptolemy, KLM; and with the planet given at the point L on it, it is evident — from the given complement of the anomaly to a semicircle — that the angle LHM is $1^{\circ} 30'$. With this given, in the triangle LAH, together with the two included sides AH (found earlier, 102261) and HL (10426 , the corrected semidiameter of the annual orb), there emerges the angle HAL, $10' 14''$. Earlier, because I supposed only $8'$, the angle LHA was made the annual anomaly, too small by precisely two minutes; and if account is taken of these here, and for them the adhering $14''$ are subtracted, the second inequality — or the angle LAH — comes out as precisely $10'$. When the first inequality, $6^{\circ} 14' 18''$, is joined to these, the whole of Saturn here is aggregated from the mean motion as $6^{\circ} 24\frac{1}{4}'$ nearly, with the place of the fixed star from the mean [continued on following page]

Diagram. A geometric construction for Saturn's first and second inequalities (the same construction as on the facing page, p. 347). A large circle (the orb of Saturn, BPC) is drawn with center A at the left-center. A nearly vertical diameter runs from B at the top to P at the bottom. From A, two lines run out to the upper-right, where a cluster of small circles (the epicycles, equant, and annual orb KLM) sits on the circumference.

- B (top of the large circle): endpoint of the vertical diameter
- A (left-center): center of the orb of Saturn
- P (bottom of the large circle): endpoint of the vertical diameter
- K (upper right, outer): point on the annual orb
- H (in the cluster): center of the annual orb

- G (in the cluster, near H): point on the equant circle
- D (in the cluster): point on the first epicycle
- L (left of the cluster): position of the planet on the annual orb
- M (below the cluster): point on the annual orb
- C (lower in the cluster): epicycle center / point on the orb
- F (lower in the cluster): point on the equant circle

to be subtracted from the mean equinox above, fixed at 9°9'25". Once this is done, there finally emerges from that same equinox a mean longitude of Saturn of 5 signs 2°45' — that is, in the year 229 before the birth of our Savior, on 1 March, at 4h 25m past noon. Now this longitude of Saturn, found and restored in this way, is adjusted to noon on 1 January of the completed year 3736 from the first (namely, the first mundane bissextile) by adding 10°14'40" — this being the motion of Saturn's longitude that corresponds to the interval of time from 1 March and so on to the end of the said mundane year 3736. The simple longitude of Saturn from the mean equinox at the end of the year just mentioned will then be 5 signs 12°59'40". But for our own century — namely the bissextile mundane year 5564, with the year of Christ 1600 completed — we obtain, by the emendation we made above, this same mean longitudinal motion of Saturn as 6 signs 28°21'26", likewise reckoned from the mean equinox. Accordingly, to the proposed interval of 1828 Julian years, or 667677 days, there corresponds a longitudinal motion of Saturn from the mean equinox of 1 sign 15°21¼', over and above sixty-two whole revolutions completed in the meantime.

From the proportion thus granted between the time and Saturn's motion in longitude, once the whole circles are discarded, the epoch of that motion is gathered: first, at the beginning of the first mundane bissextile that we adopted above — specifically at the following noon of 1 January — it is found to be 5 signs 23°28'44"; but the same epoch reckoned to the beginning of the year of our Savior is found to be 2 signs 12°32'51".

Furthermore, from this same proportion there emerges the matching longitudinal motion:

		Sig.	Gr .	M i.	Sec.	Ter.
1000		11	24	52	51	40
100	for [these many] years	4	23	29	17	10
20		8	4	41	51	26

	Sig.	Gr	M	Sec.	Ter.
		.	i.		
Common [year] 1	0	12	13	35	24

All these results — as well as the foregoing ones extricated concerning the motion of the apogee — are to be reserved for the canon of mean motions to be constructed hereafter. And so much for the restitution of Saturn's longitude.

CHAPTER V.

On the restoration of the acronychal longitude — that is, of Jupiter's orb — as it agrees with our own age.

Since, in the foregoing restitution of Saturn, we also touched on the matters that commonly concern the motions of the three superior planets — and, in addition, supplied the description of three hypotheses to which those motions can indifferently be fitted — we are therefore excused for brevity, and shall now set forth only what serves the restoration of what follows. In the case of Jupiter this is the more difficult for us at present, the rarer the observations we have of it. Once its restitution had been worked out long ago on the island of Hven [Huena] with respect to the Sun's mean course, we sought nothing further here until the acronychal appearances of subsequent years showed us a restoration with regard to apogee and perigee difficult enough — the longitude in nearly the same positions of the eccentric gradually increasing beyond the earlier emendation. This was confirmed by the observations we obtained here afterward, indeed carried still further: so much so that we hesitated in doubt whether to assign to the apogee of this star a certain anomaly beyond the custom of the rest (a thing that once seemed so to Tycho Brahe), before we began to extend the first inequality — that is, the inequality of the eccentric of Jupiter — to the true motion of

the Sun. Here that said anomaly about the apogee and perigee became indeed milder — so much so that, although it is not entirely removed, it nonetheless does not appear so great that we should think anything on this account must be ascribed to the hypothesis of the Jovial star as alien to the others; but rather that one must hold to the mean of the restitution.

Having examined most carefully the acronychal positions of Jupiter that we have so far obtained from observations, against the Prutenic Tables [*Tabulae Prutenicae*] — and having repeated the examination quite often, as suits this practical work of restitution — I found that to its simple longitude, for this our

century, $4\frac{1}{2}'$ must be added, and to its apogee $40'$; and likewise that $24'$ must everywhere be added to the prosthaphaeresis of the eccentric. Once this is done, the observed and computed acronychal positions come out, with their requisites and distances, as follows.

A presentation of eight positions of Jupiter, both from acronychal observation at the true opposition of the Sun, and from calculation issuing for the same times according to the aforementioned correction of the Prutenic Tables.

A M n e n n i s.	Log. obs.						
	Long. (S. G.						
	Long. Med.						
	Anomalial						
	Long. (S. G.	Long. Med.	Anomalial	Praec. aeq.	Long.	Diffe	
	M.)	(Sex. ° ' ")	(Sex. ° ' ")	(° ' ")	sup. (° ')	r. (° ')	
I 1 S 5 e 8 p 3 t.	☿ 23 33	5 26 52 42	2 47 31 5	27 59 46	23 38 $\frac{2}{5}$	5 $\frac{2}{5}$ +	
I 1 O I 5 ct 8 o 4 b.	♄ 22 0	0 0 19 12	3 20 57 34	28 0 44	22 0 $\frac{1}{4}$	0 $\frac{1}{4}$ +	
I 1 A I 5 p I 9 ri 1 li s	♂ 13 10	3 18 21 55	0 38 58 30	28 6 30	13 9 0	1 —	
I 1 S V 5 e 9 p 5 t.	☿ 28 56	5 31 30 55	2 52 11 7	28 10 30	28 54 $\frac{3}{4}$	1 $\frac{1}{4}$ —	
V 1 O 5 ct 9 o 6 b.	♄ 5 40	0 4 56 23	3 25 32 23	28 11 25	5 35 0	5 —	
V 1 S I 6 e 0 p 7 t.	♄ 4 10	5 36 8 3	2 56 32 7	28 20 50	4 8 $\frac{1}{6}$	1 $\frac{5}{6}$ —	
V 1 D	♄ 19 36	1 15 51 18	4 36 24 33	28 23 45	19 38 $\frac{1}{12}$	2 $\frac{1}{12}$	

A M n e n n i s.	Log. obs. Long. (S. G. M.)	Long. Med. (Sex. ° ' ")	Anomalial (Sex. ° ' ")	Praec. aeq. (° ' ")	Long. sup. (° ')	Diffe r. (')
I 6 e						+
I 1 c						
0 e.						
V 1 M	♂ 21 45	2 21 41 18	5 42 14 22	28 25 40	21 42¾	2¼
I 6 a						—
I 1 rt						
I 3 .						

And yet, with a restitution of this diligence, the difference here too in Jupiter — the greatest, in either direction, between the observations we obtained in the acronychal positions and the corrected tables — rarely exceeds 5'; nor indeed will anyone suppose that a greater precision can be reached here, once he has compared the first with the fourth and the last.

I hope, however — indeed I am firmly convinced — that the remaining acronychal observations of Jupiter, which are still preserved in the Tyconic treasury, will agree so well with this restitution that nowhere will a greater difference from the calculation appear. The earlier restitution, carried out with respect to the Sun's mean motion, could in a way testify to this, were it too to be brought to light. It now remains for us to fit the semidiameters of the epicycles of Jupiter's orb to this restitution.

[The diagram for this page (the Jupiter orb with its epicyclic cluster) is described below.]

Diagram. A geometric construction for Jupiter's first inequality on the Copernican supposition. A large circle (the orb of Jupiter, BPC) is drawn with diameter B–P vertical; its center is A (near the bottom, the Earth/Sun position in the construction). At the upper left of the figure sits the small epicyclic cluster: a small circle with center C, on which are marked the points E, D (with G just above D), F, and H, the eccentric epicycles C and D from which the planet is carried. Long straight lines run from A up through the cluster to the circumference. Inside the large circle, lower down, sits a second small circle (the annual orb) with center near A, carrying points L and M, with K nearby. Labels read :

- B (top of large circle): apse line / upper apsis
- P (bottom of large circle): lower apsis

- A (center, bottom): center of the deferent (Earth/Sun)
- C (center of the upper small epicyclic circle): first epicycle center
- D (above C, within the cluster): second epicycle center
- E (left of cluster): point on the epicyclic construction
- F (below D): point on the epicyclic construction
- G (above D): point on the epicyclic construction
- H (right of D): position of the planet in this revolution
- K, L, M (lower small circle, the annual orb): construction points on the homocentric annual orb

[Right-column text continues alongside the diagram, transcribed above and below.]

tion which we intimated to be the same as Copernicus's; upon which it is likewise agreeable to explain the remaining phenomena of this Jovial star — although we intimated above that the same can be applied indifferently to any of the three superior planets. Therefore, since the semidiameters of the epicycles on the circumference of the orb — namely C D and D H — take their measure in that proportion in which A C is 100000 parts, drawn from our restitution (which earlier was founded by Reinhold in his tables, which we follow, upon the Copernican foundation): so that C D is reckoned at 6870 parts, and D H a third of the former, namely 2290 — it now suits, by the addition of 1357 (which equal the 24' part of the maximum prosthaphaeresis of the eccentric), to enlarge the said semidiameters above the Copernican supposition so far that C D becomes 7155, and D H, its third part, namely 2385. For this augmentation expresses the said emendation in the supposition of the triangles as nearly as possible, as we shall show afterward in the construction of the table and immediately thereafter, when we are engaged in acquiring the measure of the semidiameter of the orb I K M L to be sought — a task which immediately takes over from the one we have here imposed on emending Jupiter's first inequality.

CHAPTER VI.

On the annual inequality that appears in Jupiter — to be examined and restored.

Among the observations by which we have explored, here and there, the measure of the semidiameter of Jupiter's annual orb, we bring forward two — as we did above in the case of Saturn — to whose standard we measure this inequality in Jupiter that we now treat. The first of these was made by us on Hven [Huenä], in

the year of the Lord 1593, on 28 September, at 8h in the evening, when Jupiter, at an altitude of about 9°, was 33°59' distant from the extreme tail of Capricorn [φ], but 32°33' from the right knee of Ophiuchus, while at the same time having an observed southern declination of 23°7'. From this — by means of the verified positions of the fixed stars and the reasoning of spherical triangles — the longitude of that same Jupiter, delimited and freed from refraction, was found to be 13°56' φ , with a true southern latitude of 0°25'.

For this time, Jupiter's mean longitude from the true equinox was computed at 10 signs 0°20½', and the apogee position at 6 signs 7°32½' — according to our previously mentioned acronychal emendation. Hence the anomaly of the eccentric is likewise found from these to be 3 signs 22°48½', and the prosthaphaeresis, from the same restitution, 5°6½' to be subtracted. From this the true or equated longitude of the eccentric turns out to be 9 signs 25°13'50". But the true position of the Sun at the same time was for us 6 signs 15°17'; and with the equated longitude of the eccentric subtracted from it, there remain 8 signs 20°3'10" for the annual anomaly, of which afterward.

These things, with the hypothesis of Jupiter presupposed in the Copernican form, are applied as in the following figure. From the center of the Sun of the universe, according to Copernicus, let there first be described the orb of Jupiter B C; then, with the eccentric epicycles delineated from C and D, the planet will, in this revolution, be at H. But the side A H is given as 985462, which in triangle C A H — its angles being given — after H C D is found to be 10°55" [recte: 10°55'] and is subtracted from angle D C E, makes angle A C H equal to 78°7'10". And the prosthaphaeresis C A H is 5°6'40"; whence A H C turns out to be 96°45'40"; and A C, the radius of the orb B C, is reckoned at 1000000.

Further, let the annual orb M L K, homocentric to B C, be described

[homocentric to B C], according to Copernicus, through its circumference at M; let M A be sought. In triangle A H M, then — since the angle H A M, the anomaly of the annual orb, was earlier granted to be 80°3'10", but the angle of commutation A H M is found to be 11°17'20", with the observed position of Jupiter on the eccentric subtracted from the true [Sun] — the angle A M H is therefore also given as 88°39'30". And since the side H A in this same triangle was earlier granted to be 985462, from these the semidiameter of the annual orb A M, sought, comes out as 192948 — of the kind of which A B, the radius, is supposed to be 1000000.

A second, quite recent observation was made by us here at Copenhagen [Hafniae], on Jupiter, in the year 1616, on 10 March, at 4h in the morning, just at the vernal equinox. At that time, by means of our sextant, Jupiter was $23^{\circ}46'$ distant from the Heart [of the Scorpion, Antares, $\mathfrak{M}$] but $39^{\circ}44\frac{1}{3}'$ from the Bright [star] of the Vulture [Vega], and this at an altitude of Jupiter of about 10° .

From these data the true longitude of Jupiter was found to be $27^{\circ}41' \nearrow$, with a northern latitude of $0^{\circ}24'$, refraction and parallax being eliminated in both cases. To this time these restored motions correspond:

	Sig.	Gr.	Mi.	Sec.	
Mean longitude of Jupiter from the true equinox	8	21	52	0	
Anomaly of Jupiter's eccentric	2	13	56	30	
To which corresponds the restored prosthaphaeresis		5	10	10	Subt.
Therefore the true longitude of the eccentric is	8	16	41	50	
But the true longitude of the Sun	0	0	1	15	
From which, the true longitude of Jupiter's eccentric being subtracted, there remains the anomaly of annual commutation, in the way we adopt it for these planets	3	13	19	25	

With these granted, let the hypothesis of Jupiter be set up similarly in the Copernican form (as in the following figure), in which everything is ordered as in the preceding, only the present variety of revolutions being considered. For — the other matters being omitted as sufficiently and more than sufficiently inculcated thus far, and since the anomaly of the annual orb somewhat exceeds a quadrant — the Earth will be located at M, and from it the planet is seen at H, namely along the line M H. But since our aim in the inquiry into A M, the semidiameter of the annual orb in Jupiter, is fixed here so that it may be compared with the one found earlier from the other side: therefore let the first angle found, D C H, of $7^{\circ}52'0''$, be added here to E D, the anomaly of the eccentric, and in the complement of the sum to a semi-

Diagram. A geometric construction for Jupiter's annual inequality on the Copernican supposition, the companion to the previous figure. A large circle (the orb of Jupiter, B C) is drawn with diameter B–P vertical, center A near the bottom.

At the upper left is the epicyclic cluster: a small circle with center C, carrying points G, D, H (clustered at top), E (left), F (below D), with H marked by an asterisk as the planet's position. Long lines run from the lower part of the figure (near A) up through the cluster. Lower in the large circle, a small circle (the annual orb) carries points L and M, with K and I nearby; the Earth is located at M, and the line M H runs to the planet. Labels read :

- B (top of large circle): apse line / upper apsis
- P (bottom of large circle): lower apsis
- A (center, near bottom): center of the deferent (Sun)
- C (center of upper small circle): first epicycle center
- D (within the cluster, above C): second epicycle center
- E (left of cluster): construction point
- F (below D): construction point
- G (above D): construction point
- H (top right of cluster, marked with asterisk): position of the planet
- I, K (right of center): construction points on the annual orb
- L, M (small lower circle): the annual orb; M is the Earth's position

Diagram. The Jupiter hypothesis in Copernican form, continued from the previous figure. A large circle (Jupiter's orb) is centered on a point on the horizontal axis; the small annual orb sits to the right of center, and an epicyclic cluster sits at the upper left of the great circle.

- B (top of great circle): cardinal point
- P (bottom of great circle): cardinal point
- A (center, on the axis below M): the Sun / center of the eccentric
- M (small circle right of center): the Earth on the annual orb
- L (just below the small circle): point on the annual orb
- K (right edge of the small circle): point on the annual orb
- C (within the epicyclic cluster, upper left): center of the epicyclic construction
- D, E, F, G, H (epicyclic cluster, upper left): successive epicycle centers and the planet, clustered tightly; individual letters partly overlapping

[Cap. 6 continued]

The angle A C H comes to $98^{\circ} 11' 38''$. But the angle C A H of the prosthaphaeresis is $5^{\circ} 10' 10''$; therefore A H C is $76^{\circ} 38' 20''$. And so, in the triangle A C H, with its

angles given together with the orb's radius $AC = 1000000$, the side $AH = 1017321$ is given.

With these things thus set out and found, the triangle AHM likewise will have its angles given, together with the side AH just now also found. For when the equated longitude of Jupiter's eccentric, $8\text{ signs } 16^{\circ} 41' 50''$, is subtracted from its true observed longitude, $8\text{ signs } 27^{\circ} 41' 0''$, there remains, for the annual commutation, that is, the angle AHM , $10^{\circ} 59' 10''$. The angle HAM , in turn, is known from the complement of LM to a semicircle — the arc by which the annual anomaly is determined — which we earlier set at $3\text{ signs } 13^{\circ} 19' 25''$. The said angle HAM therefore comes out as $76^{\circ} 40' 35''$, and so, by computation, the semidiameter of the annual orb $AM = 194033$, which is what we sought. But above, the same had been found as 192948 . The difference, then, is 1085 ; and when half of this, 542 , is applied to the smaller value, the limited semidiameter of Jupiter's annual orb — the measure of the side AM — comes out as 193490 . Our other observations, made before and after, fall very nearly on either side of this same measurement; these we made in the morning hours of the 18th of February and the 8th of March of this year.

Nor indeed do I see how the Sun's varying distance from the Earth — which is greatest at the opposite and fixed seasons of the year — should introduce a corresponding inequality into the annual orb of this planet. For if even half the ratio of the Sun's eccentricity were taken into account here in this respect alone, the greatest annual commutation in this Jovial star would appear smaller — with the Sun at perigee rather than at apogee — by nearly $\frac{1}{2}^{\circ}$, which certainly does not agree with observation. For when, among others, we consult the observation we made on 1610, January 26, in the evening and on the immediately following days — at least as far as visual and filament-based measurement allowed — when Jupiter stood motionless, that is, stationary, in nearly a straight line and almost midway between Aldebaran and the northern horn of Taurus (except that it had deflected a little eastward from that line, and had been somewhat nearer to Aldebaran), we gather the longitude of this planet to be scarcely up to that which the Prutenic Tables [Prutenic Tables] give for that time, namely $10\frac{2}{3}^{\circ}$ of Gemini Π . And that longitude, even near the Sun's perigee, requires the semidiameter of Jupiter's annual orb, as found, to be still somewhat larger. But so far concerning the restoration of this star's longitude as suited to our own age. There follows a comparison with the ancients.

CAPVT VII.

DE

Restitutione & extensione periodica apogæi & longitudinis Iovis.

(On the periodic restoration and extension of Jupiter's apogee and longitude.)

Since we set forth above, in the case of Saturn, the reasons why the correction of the mean motions in these planets can be derived from Ptolemy [Ptolemy] with less risk — chiefly that of the longitudes — we shall therefore for the present put forward only a few points that bear on the matter concerning this star.

First, for the correction of the apogee's location in Ptolemy, we proceed briefly as follows.

In the first year of Antoninus Pius — that is, in the year 136 completed after the birth of Christ — Ptolemy found the point of Jupiter's apogee, or greatest elongation, in the eccentric to be 5 signs 11° 0', removed from the true vernal equinox; and this by means of three acronychal positions of Jupiter directed against the mean course of the Sun. So even though we are necessarily compelled to retain this same measure of the apogee's location — because the ancients' observations are insufficient, those through which it would be possible to discuss the matter further — nevertheless, when we have regard to the true motion of the Sun and the mean equinox, we find that about 3° should be added to it. For since the Sun's anomaly around the opposite of Jupiter's apogee in Ptolemy's time is found to be 9 signs 15° from the mean equinox, and therefore the Sun's prosthaphaeresis at that place is 2° 2¾' to be added; but the combined daily motion of the Sun and Jupiter ἐν πλάττει [Greek: en platei, "broadly" — i.e. taken roughly], for precision is not required here, around that opposition in their joint revolution was 1° 8'. Therefore, according to the said prosthaphaeresis of the Sun, there are obtained 1 day 19¾ hours, by which Jupiter was exposed to the opposition of the Sun's true place before it was to its mean place. And to this interval there correspond, in Jupiter's own motion in the meantime, 14¾'. For these, the apogee of this planet is to be advanced by 3 whole degrees, together with the distance between the true and mean equinox in Ptolemy's time. Accordingly, the corrected location of the apogee in Jupiter, in the year 136 completed after Christ, in Ptolemy is 5 signs 14° 0'. Nor indeed do we undertake — or even wish — to extract it from Ptolemy more precisely, since the positions of the fixed stars were uncertain to him. But in our own age, that is, in the year 1600 completed after Christ, we have this same apogee — the motion of Jupiter

restored above — at 6 signs 7° 39¾' from the true equinox, but 6 signs 7° 32' from the mean. Over the interval, then, of 1464 Julian years, the location of Jupiter's apogee has been advanced 23° 32' in the order of the signs. These figures, drawn out proportionally over the intervals of years, yield the following:

over years	Gr.	Mi.	Sec.	Ter.
1000	16	4	28	51
100	1	36	26	50
20	0	19	17	23
1	0	0	57	52

By continuing the same proportion, we find the epoch of the apogee's motion in Jupiter, at the beginning of the first bissextile year of the world, to be 3 signs 8° 5' 38"; likewise, at the beginning of [the era of] Christ our Savior, 5 signs 11° 48' 50". All of these will be preserved in the tables of mean motions.

With Jupiter's apogee restored, we come to its longitude — which is indeed of greater weight, since any error it may contain is exposed more evidently with the passage of time. This is the reason why here too we by no means remain within Ptolemy's observations, on account of the uncertainty noted concerning them, but rather, by way of restoration,

we draw the restoration of this [longitude] to a certain conjunction with a fixed star, from Timocharis [Timocharis] or some other observer, about 400 years earlier than Ptolemy — provided that the location of such a coupling can still be precisely known in the sky, with the star's place verified to a nicety through Tyconic [Tycho Brahe] diligence, and with our anticipation of the equinoxes restored to its ancient state.

[Margin: *Lib. 9, c. 3.*]

Ptolemy reckons that a conjunction of this kind of Jupiter with the southern Ass in the asterism of Cancer occurred in the year 45 according to the era of Dionysius, on the 10th day of the month Virginion; but, reckoning from the death of Alexander, in the year 83, on the 18th day of Epiphi according to the Egyptians, in the morning. For at that time he writes that the star of Jupiter covered the southern Ass. And this time, when we count back the Julian years, falls in the year 241 before the birth of Christ, and in the morning hours of the 4th of September, here with us about the 4th completed hour of that same day before noon. For which time I find the longitude of the southern Ass, according to our restoration, to be 3 signs 7° 27' from the true equinox, with a southern latitude of

0° 25'; but the longitude from the mean equinox, 3 signs 7° 33' — exactly as Ptolemy presupposes the same (which is most rare). Therefore, at that time, Jupiter's place must necessarily have been the same. Furthermore, I find the Sun's true place at the same moment — also according to our restoration — to be 5 signs 7° 9½' from the mean equinox. Moreover, since its apogee, previously restored and here reduced, falls at 7° 50' of Virgo ♍, and the mean longitude, derived in whatever fashion from Ptolemy's reasoning, is given from the mean equinox as 22° 59' of Gemini ♊ — therefore, with the former subtracted from the latter, there remains the anomaly of Jupiter's eccentric, 9 signs 15° 3'; to which corresponds the restored prosthaphaeresis, 5° 12' 0". And when this is added to Jupiter's mean longitude, and the sum is taken away from the Sun's true place, there remains Jupiter's annual anomaly, 2 signs 8° 58½'.

With these things supposed, let the Jupiter hypothesis be laid out in Copernican form, agreeing with the earlier demonstrations in the same. In it, since the angle H D C is given as 149° 54' — namely, from the residue of the doubled anomaly to a whole circle — together with the sides comprehending that same angle, C D and D H, which are the semidiameters of the epicycles restored above, there emerges from these the angle D C H of 7° 23' 35". When this is subtracted from the anomaly of the eccentric, there remain 9 signs 7° 33' 25", and therefore the angle H C A is 97° 33' 25". But the angle C A H of the eccentric's prosthaphaeresis was 5° 12' 0"; wherefore the angle C H A will be 77° 14' 35". Also given in the same triangle C H A is the radius A C = 1000000, so that the side A H is found to be 1016410.

Diagram. The Jupiter hypothesis in Copernican form. A large circle (Jupiter's orb) is bisected by a vertical axis B–P and centered on point A; a small inner circle sits at center on the axis. To the right, on the great circle, is a tightly clustered epicyclic group.

- B (top of great circle): cardinal point
- P (bottom of great circle): cardinal point
- A (center of great circle): the Sun / center
- L (left, on the horizontal radius from A): point on the construction
- M (below A): point on the annual orb
- K (between center and the epicyclic cluster, right): point on the construction
- C (lower part of epicyclic cluster, right): epicycle center
- F (within the cluster): epicycle center
- D (within the cluster): epicycle center

- H, G (top of the cluster, lettered together): epicycle center and planet
- E (rightmost edge of the cluster): point of the construction

Furthermore, in the triangle H A M, since two sides are granted — H A = 1016410 and A M = 193490, from our restoration above — together with the angle contained between them, H A M = $111^{\circ} 1' 50''$ (namely the angle that lies in the complement of the arc L M to a semicircle, measuring the anomaly of the annual orb), there is finally acquired from these the angle of the annual commutation, A H M = $9^{\circ} 26' 0''$. When this is added to the equated longitude of Jupiter's eccentric — namely 2 signs $28^{\circ} 11'$ — there is composed the apparent longitude of Jupiter from the mean equinox, $7^{\circ} 37'$ [?] of Gemini Π . But it was observed to be $7^{\circ} 33'$ [?] of Gemini Π . The difference, therefore, is found here to be at least $4'$ [?: on the available scan the apparent-longitude line ends at the bare degree "7"; the $37'/33'/4'$ sub-values could not be located and need checking against a better copy]; and when this has been taken from the Ptolemaic mean longitude, that [longitude] corrected to this time remains 2 signs $22^{\circ} 55'$. Nor indeed here, on account of so small a difference between the assumed mean and the sought longitude of Jupiter — which redounds into both anomalies — does any appreciable error befall the mean longitude to be derived.

Now, since for the completion of the current year from the time mentioned, when this observation was made, there are to be added $9^{\circ} 55' 30''$ which Jupiter would have meanwhile completed in mean motion, there is composed its mean longitude, from the mean equinox to the noon next following the Calends of January of the year 240 before Christ completed, as 3 signs $2^{\circ} 50\frac{1}{2}'$. But in our own age — that is, in the year 1600 after the birth of Christ likewise completed — we find this same corrected mean longitude of Jupiter from the mean equinox to be 5 signs $10^{\circ} 3\frac{1}{2}'$. Over the interval, then, of 1840 Julian years, or 672060 days, Jupiter had completed 155 whole revolutions, and beyond these 2 signs $7^{\circ} 48'$. From this proportion of time and motion given between them, the epoch of Jupiter's mean longitude is found — at the beginning of the first bissextile year of the world (with whole circuits everywhere rejected) — to be 2 signs $1^{\circ} 16' 22''$; likewise [the epoch] of Christ in the flesh, 5 signs $29^{\circ} 56' 46''$. Moreover, from the same proportion, the following agree:

over years	Sig.	Gr.	Min.	Sec.	Tert.
1000	4	2	56	5	13
100	5	6	17	36	31
20	8	7	15	31	18

over years	Sig.	Gr.	Min.	Sec.	Tert.
Comm. 1	1	0	20	31	44

These will be preserved for the motion of the longitude by extending them through the Canon.

And thus far concerning the restoration of Jupiter's longitude.

CHAPTER VIII.

On the restitution of the course of Mars in respect of the first inequality, which appears, around the whole of its orb, in opposition to the true place of the Sun.

In examining and restoring the acronychal phenomena of Mars, we shall not adapt them to the Reinhold tables by quite the same method as we used for the other superior planets. For although the prosthaphaeresis, or first inequality, of this planet — which is large — comes out more contracted, to our satisfaction, when the extreme nocturnal risings of the same planet are also reduced to the true opposition of the Sun: nevertheless those same prosthaphaereses are to be fitted together throughout by means of the eccentric orb — or, what is the same thing for us, the homocentric orb with two small circles (since we prefer to follow the geometrical method that Copernicus introduced) — and they depart but little from the earlier Tyconic restitution, as regards the distribution of the prosthaphaereses around the orbit of Mars. For here too it is not a third part of the whole eccentric inequality that is assigned to the semidiameter of the second, or equant, little circle, as in the two superior planets, but only a fourth part.

[Margin: See the restitution of the acronychal positions in the *Commentary* of Johannes Kepler, *On the motion of Mars*, pp. 54 and 55.] For since the combined magnitude of both epicycles is found by observation to be 18550 (taking the radius of the orb as 100000), in the proportion $5\frac{1}{2}$ to 1 most nearly, as has been shown — a proportion which in Ptolemy was exactly fivefold — the semidiameter of the greater epicycle CD in the adjoined figure, and in the Tyconic hypothesis (to which we promised above to apply the appearances of Mars), will be 14840, and that of the smaller, DH, 3710. As for the rest, the explanation of the last figure of the second chapter is admitted throughout, with this one consideration: that here the orb of the Sun is cut by the orbit of Mars in two places from the Earth M. From this it follows, on both this hypothesis and the Copernican, that the acronychal Mars comes to be far nearer to the Earth than to the Sun.

The hypothesis of Mars in the Tychonic form, representing its acronychal position.

Diagram. A schematic of the Tychonic hypothesis for Mars at acronychal position. A large outer circle is the orbit of Mars, with vertical diameter running from B (upper region) down through A (lower) to P (bottom). Within it sits the smaller orb of the Sun. At upper left, clustered near the top, are the construction points of the double-epicycle arrangement (G, H, D, E, C, F), with M (the Earth's centre) toward the middle-left of the Sun's orb. Lines radiate from A to the epicycle cluster and to B.

- B (upper interior): point on the orbit of Mars
- G, H (top of epicycle cluster, upper left): epicycle points
- D (within cluster): centre of the smaller epicycle
- E (left of cluster): epicycle point
- C (within cluster): centre of the greater epicycle
- F (within cluster): epicycle point
- M (mid-left, within Sun's orb): the Earth's centre
- A (lower centre): point on the orbit / observer's sightline vertex
- P (bottom): lower terminus of the vertical diameter (perigee region)

A representation of the fourteen extreme risings of Mars adjusted to the true opposition of the Sun, together with the moments of time and the mean motions corresponding to these according to the foregoing restitution and converted into true motions, with the differences appended.

N							
u A M							
m n e	n n	Long.		Anomalia	Praec. aeq.	Long.	Di
		obs. ♂	simpl. ♂				
r. . s.		(Sig.)	(Sex.° ' ")	♂ (Sex.° ' ")	(° ' ")	sup. ♂	ffe
						(Sig.)	.
I 1 N	5 o	II 6 27½	0 · 27 54 35	4 · 27 11 10	0 · 1 11	II 6 27½	0
	8 v						
	0 .						
I 1 D	5 e	⊗ 16 54½	1 · 11 28 40	5 · 10 44 20	27 57 50	⊗ 16 55½	1
I 5 e	8 c						+
	2 .						

N u A M m n e e n n r. . s.	Long. obs. ♂ (Sig.)	Long. simpl. ♂ (Sex.°'")	Anomalia ♂ (Sex.°'")	Praec. aeq. (°'")	Long. sup. ♂ (Sig.)	Di ffe .
I 1 I I 5 a I 8 n 5 .	♌ 21 35½	1 · 52 10 10	5 · 51 25 30	27 59 10	♌ 21 37	0 ⅕ +
I 1 M V 5 a 8 r 7 t.	♍ 25 42	2 · 32 47 35	0 · 32 2 40	28 1 5	♍ 25 42⅘	0 ⅙ +
V 1 A 5 p 8 r 9 il .	♍ 4 23	3 · 16 16 20	1 · 15 31 10	28 3 0	♍ 4 26½	3 ½ +
V 1 I I 5 u 9 n 1 ii	♊ 26 42	4 · 7 40 5	2 · 6 54 23	28 5 0	♊ 26 45⅓	3 ⅓ +
V 1 A I 5 u I 9 g 3 .	♋ 12 15	5 · 11 49 20	3 · 11 3 20	28 6 45	♋ 12 14¾	0 ¼ –
V 1 O I 5 c I 9 t I 5 o .	♌ 17 30⅔	0 · 9 6 40	4 · 8 20 0	28 8 40	♌ 17 29½	1 ¼ –
I 1 D X 5 e 9 c 7 .	♍ 2 27	0 · 55 2 30	4 · 54 15 25	28 10 25	♍ 2 26	1 –
X 1 I 6 u	♌ 8 37	1 · 36 24 45	5 · 35 37 10	28 12 20	♌ 8 37⅙	0 ⅙

N u A M m n e e n n r. . s.		Long. obs. ♂ (Sig.)	Long. simpl. ♂ (Sex.° ' ")	Anomalia ♂ (Sex.° ' ")	Praec. aeq. (° ' ")	Long. sup. ♂ (Sig.)	Di ffe .
0 n 0 ii							+
X 1 F I 6 e 0 b 2 .		♊ 12 26	2 · 16 46 35	0 · 15 58 40	28 14 5	♊ 12 24	2 —
X 1 M I 6 a I 0 r 4 t.		♊ 18 36½	2 · 58 45 20	0 · 57 57 0	28 15 55	♊ 18 36½	⅓ +
X 1 I I 6 u I 0 li I 8 i		♊ 11 10	4 · 45 53 15	2 · 45 14 30	28 17 50	♊ 11 13⅓	3 ⅓ +
X 1 O I 6 c V 1 t 0 o .		♊ 25 30	5 · 48 35 0	3 · 47 55 20	28 21 45	♊ 25 26⅔	3 ⅓ —

Among these fourteen acronychal positions of Mars, drawn from the heavens with the utmost diligence and collated with the corrected mean motions, the greatest difference stands almost everywhere within 3 minutes either way — especially after I had subjected the previously demonstrated measure of the epicycles' semidiameters to the calculation. [Margin: *Five Books on the Revolutions* [Copernicus, *De Revolutionibus* V], chapters 16 and 19.] And surely I cannot well understand how Copernicus, by the path of demonstration, did not hesitate to ascribe to an alteration of the Sun's eccentricity those 40 parts (out of the Ptolemaic 1500, where the radius of the orb was reckoned at 10000) which he took away from the distance of the centres. For if any change had ever come about over the course of centuries — which in fact there was none — then it would, by right and with justice, have had to be attributed rather to the annual

inequality of Mars, which proceeds from the Sun; and it is to the examining and restoring of this that we devote the following chapter.

CHAPTER IX.

On the annual inequality of Mars, to be explored and restored.

Just as the first inequality of Mars, which arose from its revolution in its own orb, showed itself easy for us to handle — whether in respect of the mean opposition of the Sun, as formerly, or of the true opposition, as now, when we have taken it up — so in the case of the annual inequality of this planet several anomalies present themselves. Two departures from the pattern of the superior planets chiefly arise here in Mars. The first is one that is manifested in this planet alone (so it seems): namely, that it is distant from the Earth, through its annual revolution, in equally unequal degrees on both sides. From this it comes about that the semidiameter of the annual orb is here contracted and enlarged according to the Sun's approach to and recession from the Earth — and indeed so consistently in this star, once its phenomena have been directed to the true motion of the Sun (for earlier, while we arranged the same phenomena with respect to the mean place of the Sun, we ran the hazard of a contrariety here, the cause of which a careful examiner will explain, by comparison, from that notable change not only of the apogee but also of the prosthaphaeresis) — that Johannes Kepler did not hesitate to assert, indeed to demonstrate by observations as well, that half of the Sun's eccentricity would thus appear, in due proportion, in the annual carrying-round of Mars through the orb of the Earth, that is, of the Sun. From this too he reckons it comes about that the Sun and every one of the wandering stars leave only half of their whole digression in the eccentric of the distance between the centres of the world and of the planet's orb, and assign the remaining half to the equation of apogee and perigee, and so forth. And so from the appearances of Mars alone he judges that it has been shown that the Sun itself is set up as the very centre of all the world's phenomena. This we shall embrace as most true only when Kepler shall have proved the same, without strain, in the rest as well. And indeed we have not been able to be silent about this hitherto, neither in the superior planets Saturn and Jupiter, nor in the Moon, by their appearances; nor shall we expect to be so in what follows, with Venus and Mercury, however much we may be carried that way with every effort. So I shall rather think we must believe that most of these wandering stars suffer their

own peculiar second inequalities — as certain symptoms, as it were — both from the solar rays and from other causes; the Moon especially, and Mars, as more akin by nature to the Sun than the rest.

Concerning Mars, however, it could be a peculiar indication that it imitates the alteration of the Sun from the Earth in its eccentric as well, since in the Sun's opposition — which by a kind of contrary law coincides with the Sun's apogee, Mars's perigee, and vice versa, where the orbs of the revolutions meet — Mars changes its own prosthaphaereses. Hence perhaps it comes about that around the perigee of Mars's eccentric somewhat greater prosthaphaereses are required than in the other planets — the Sun, namely, then dwelling nearer to its apogee and drawing the acronychal Mars into a greater approach toward the Earth — and conversely smaller ones around the apogee, the Sun being lower and farther from the Earth. Yet this conjecture will then at last answer the truth in every part if (which, however, is not the case here) the planet's apogee were conjoined with the Sun's.

The other inequality concerning the annual orb of Mars appears in this: that the semidiameter of the annual orb is always elicited as greater around the apogee than the perigee of the eccentric, from the observations, by the reasoning of triangles, and the distance from the Earth in the eccentric would otherwise allow it. This indeed proclaims itself plainly enough not only in our former restoration based on the Sun's mean position, but also in this new one, although with a far smaller difference of Mars's eccentric here, with both apsides set between. Nor is there any reason why anyone should infer this inequality to be wholly out of place, since (as it also seems) we leave to Ptolemy's manner of equation, which is [Greek: ἀγεωμέτρητος (ageōmetrētos, "ungeometrical")], declining a little more than half of the whole eccentricity, to the difference of the apsides in Mars; for not even this reduction of his to Ptolemy will suffice, but that we should require here a still greater equation of the distances. Hence it had come about that Johannes Kepler, ascribing this discrepancy to the eccentric (as also seems right) in solid, tried to think up hypotheses of revolution other than the customary eccentrics: namely, falling away from a circle and emulating oval or elliptical figures, in which too the arcs of the revolutions do not answer altogether to equal times — the Sun, namely, the director of all motions, driving the planets disposed about it now more intensely, now more remissly. [Margin: Copernicus, *De Revolutionibus* I, c. 4; Tycho Brahe calls it an *axiom of astronomy*, *Progymnasmata* p. 11.] But we, moved by the weightiest reasons, set the greatest store in astronomy by that maxim of Copernicus — that the motion of the heavenly bodies is uniform and circular, perpetual, or compounded of circular motions —

and we do not, and ought not, depart from it on account of such Martian or certain other anomalies before necessity has imposed it, so that all motions of this kind may be described as circular, that is, simple, or, for the most part, complicated and compounded — and through these, indeed, even those curved figurations and likewise plainly straight lines may be described, performing their office around every kind of heavenly phenomenon. Rather, granting that we do not set up these material things as real, we do not hesitate to affirm that things of this kind do exist, things armed with the force and power of the centres, which accomplish their conversions in times that are commensurable with equal times — just as these matters are found more fully discussed by us elsewhere. [Margin: In the *Disputation on the publication of the hypotheses*, etc.]

If, then, we attribute to the libration — whether of the eccentric or of the annual orb of Mars — that further inequality which besides befalls it on the side of its apsides, we commit no offense against the received practice of astronomers; since it is agreed that Copernicus employed it both for the primum mobile and for saving the Mercurial phenomena. And this we ourselves did above in the case of the Moon.

With these premises laid down, we must approach the observations, so that the refutation of what we have just now touched on, concerning the annual inequality, may emerge more clearly. In the examination of the observations we shall observe this method, the same as with the superior planets: first we resolve the hypothesis of Mars to a single observation for the investigation of the semidiameter of the annual orb — and that one which can be had as nearly as possible at the mean place of the eccentric. Then we shall explore the rest, above and below, both at the apogee and perigee of Mars's eccentric and of the Sun's, by the very same procedure and method of calculation.

In the year 1594, January, day 3, hour 7 in the evening, the longitude of Mars was observed by us at Uraniborg [Uraniburgi] as 18 gr. 38 m. ♀; to which time these corrected motions correspond:

	Sex.	Gr.	Mi.	Sec.
Mean longitude of Mars's eccentric	0	48	23	30
Anomaly of the eccentric	4	19	42	20
Therefore the additive prosthaphaeresis		10	33	30
Hence the true longitude of Mars's eccentric from the equinox		58	57	0
But the true place of the Sun ♄ [recte: ♀]		23	30	40

	Sex.	Gr.	Mi.	Sec.
Therefore the anomaly of the annual orb	3	54	33	40

With these found, the hypothesis of Mars in the Tychonian form is applied in this way.

Diagram. A geometric construction of the Tychonic hypothesis for Mars. Let A L N be the orb of the Sun described about the centre of the Earth, M; and about A — distant from L by the arc L A, which the Sun by its true motion has completed from the acronychal position at L — let the orb of Mars B P C be described; then the first epicycle upon C and the second upon D, according to the pattern of revolutions often demonstrated above. The figure shows a large outer circle (orb of Mars) with vertical diameter R...P, and the Sun's orb L–M–N as an interior circle through M (the Earth's centre). At lower right is the double-epicycle cluster with points G, H, D, F around centre C, and the equant/sight points K, E. Lines run A–H, A–C, A–E, and A–K, forming the triangle A M H used to find the semidiameter of the Sun's orb M A.

- R (top): upper terminus of the vertical diameter of Mars's orb (with the dotted line R...Q...P)
- B (upper left interior): point on the orb of Mars
- L (left, on Sun's orb): the Sun's acronychal position
- M (centre): the Earth's centre / centre of the Sun's orb
- N (right, on Sun's orb): point on the Sun's orb opposite L
- A (lower left, on Sun's orb): the Sun's true place, centre of Mars's orb
- Q (lower middle, on dotted vertical): point on the diameter line
- K (lower right): point on the epicycle / sightline
- C (lower right, epicycle centre): centre of the first epicycle
- G, H (top of epicycle cluster): epicycle points
- D (within cluster): centre of the second epicycle
- F (within cluster): epicycle point
- E (far lower right): outer epicycle point
- P (bottom): lower terminus of the vertical diameter

Since, then, for finding the semidiameter of the Sun's orb M A in the annual appearances of Mars, the work had to be done on the triangle A M H — so, after the angle C A H has been sufficiently established from what precedes, for the restitution of the inequality of Mars's eccentric, to be 10 gr. 33 min. 30 sec., we first seek A H in this way.

The anomaly of the eccentric beyond the semicircle — namely the arc P C, to which K D is equal — is 79 gr. 42 m. 20 sec.; therefore its double, 159 gr. 24 m. 40 sec., gives the angle

the angle H D C by the law of the revolutions. But since the sides enclosing this same angle H D C are also given by the preceding chapter — namely, the semidiameters of the epicycle-orbs, and given not merely as such but in a fixed ratio to one another, namely a fourfold proportion — the angle D C H is revealed most concisely (following what we demonstrated for triangles in the first volume) as $4^{\circ} 5' 20''$. [Margin: Prop. 29, *Elements* I.] Adding this excess of the anomaly, the angle A C H comes to $83^{\circ} 47' 40''$. Combining this with the angle of the prosthaphaeresis C A H, and subtracting the sum from a semicircle, leaves the angle C H A at $85^{\circ} 38' 50''$. In the triangle A C H, then, with the angles given and in addition the radius A C = 100000000, the sought side A H is found to be 9969763.

With this side obtained, all the angles besides the said side A H in the triangle A M H are known. For when the line L M N is drawn, as required, parallel to A H, the angle M A H is equal to the angle L M A. But the anomaly of the annual orb M A H is given as $54^{\circ} 33' 40''$, while the angle of commutation at H — the angle of difference between the true place of Mars on the eccentric and its observed place — is found to be $40^{\circ} 19' 0''$. Adding these two and subtracting the sum from a semicircle yields the angle H M A = $85^{\circ} 7' 20''$. From these, then, the sought semidiameter of the orb M A is recovered as 64738, in the units in which the radius A C is 100000 — namely, for the case where the Sun is near perigee but Mars is at about its mean elongation on the eccentric.

For this example, which serves as a kind of paradigm for the remaining observations of Mars that follow — taken near its apogee and perigee, and likewise the Sun's — in order to determine the difference in the semidiameter of Mars's orb: since we have already worked the procedure through, it will accordingly be enough here to lay out for inspection the Tychonic observations themselves, drawn from Kepler's commentary, together with the principal data needed for investigating the said semidiameter of the orb.

The observations are these:

Near the apogee of Mars's eccentric.

In the year 1600, on 6 March at hour $8\frac{1}{2}$, Mars was at $29^{\circ} 18' \text{ } \textcircled{\scriptsize S}$ [Cancer], at about its mean distance from the Earth. [Margin: *Comment. Joh. Kepl. cap. 24.*]

In the year 1587, on 1 January at hour 7, minute 8, in the morning, Mars was at $1^{\circ} 4\frac{3}{5}' \text{ ♎}$ [Libra], near the Sun's perigee.

Near the perigee of Mars's eccentric.

In the year 1591, on 26 September at hour $7\frac{1}{6}$, Mars was at $18^{\circ} 36' \text{ ♎}$ [Capricorn], at the Sun's mean distance from the Earth.

In the year 1589, on 1 November at hour $6\frac{1}{6}$, Mars was at $20^{\circ} 59\frac{1}{4}' \text{ ♎}$ [Capricorn], at a place midway between the perigee and the mean elongation of the Sun from the Earth.

From these, the remaining principal quantities derived by computation are gathered in the following little table.

Near the apogee of Mars.

Verus locus eccentrici ♂ (S. G. M. S.)	Distan. ♂ a centro (Particula)	Verus locus Solis (S. G. M.)	Angulus commuta. ♂ (G. M. S.)	Anomalia orbis (S. G. M.)	Semidiame ter orbis proveniens
4 · 29 · 32 · 5	1112970	11 · 26 · 37 · 0	30 · 14 · 5	6 · 27 · 4 $\frac{5}{6}$	66586
4 · 27 · 17 · 20	1112950	9 · 20 · 39 $\frac{2}{3}$	33 · 47 · 15	4 · 23 · 22 · 0	65691

Near the perigee of Mars.

Verus locus eccentrici ♂ (S. G. M. S.)	Distan. ♂ a centro (Particula)	Verus locus Solis (S. G. M.)	Angulus commuta. ♂ (G. M. S.)	Anomalia orbis (S. G. M.)	Semidiame ter orbis proveniens
11 · 4 · 24 · 0	889149	6 · 12 · 45 $\frac{2}{3}$	45 · 48 · 0	7 · 8 · 21 $\frac{2}{3}$	64077
10 · 29 · 55 · 0	888800	7 · 19 · 14 · 0	38 · 55 · 40	8 · 19 · 19 · 0	63394

(Column headers, in full: *Verus locus eccentrici* ♂ — true place of Mars on the eccentric [signs, degrees, minutes, seconds]; *Distantia ♂ a centro* — distance of Mars from the center, in particles; *Verus locus Solis* — true place of the Sun [signs, degrees, minutes]; *Angulus commutationis* ♂ — angle of commutation [degrees,

minutes, seconds]; *Anomalia orbis* — anomaly of the orb [signs, degrees, minutes]; *Semidiameter orbis proveniens* — resulting semidiameter of the orb.)

From these the difference between the two semidiameters of the annual orb found near the apogee of Mars's eccentric is gathered as 895, but in its perigee 683 — a difference to be assigned directly, in both cases, to the unequal distance of the Sun from the Earth. For as this distance decreases, the compass of the annual orb is contracted, and conversely, as it increases, the orb is enlarged. Moreover, since the Sun is here found to fall short in both cases — both from its mean elongation and from its lowest position — therefore, by bringing in the Sun's true motion together with the place of its apogee, we can certainly, from the data, arrive most conveniently and concisely at a knowledge of the full inequality that the Sun introduces into Mars's annual orb between its own apogee and perigee. We do it this way:

From A let a circle B C D be described, representing the Sun's annual epicycle, and showing the Sun's apogee at B, its perigee at D, and likewise the midpoint of the elongation at C and H, after it has been bisected diametrically at right angles to the center — by the lines B D and C H, that is. Then the Sun's equated anomaly is applied (which we adopt here not only more correctly but also more concisely than the mean anomaly, since both the Sun's true place and the place of its apogee are for the most part known beforehand), as we require it, adapted to the proportion of the sines; and we shall easily obtain it. Thus in the first example, near Mars's apogee, the Sun's place is at 11s 26° 37' and its apogee at 3° 5'. Therefore the anomaly is 8s 20° 58'. From this, having cast away six signs, the Sun is gathered to be raised above its perigee on its own eccentric by 80° 58'. This is represented by the line F G, parallel to and below the semidiameter A H, in which the arc D G is shown as the right sine [sinus rectus]; but the sine of the complement of this arc — namely 9° 2' — is 1570091, indicated by the line F A, by which line the Sun's defect from the mean elongation is also measured.

Diagram. A circle representing the Sun's annual epicycle, centered at A. A vertical diameter runs from B (top) through A (center) and F down to K and D (bottom); a horizontal diameter runs from H (left, with G just below it) through M and A (center) to C (right). The chord-construction lines bisect the figure at right angles. The construction points L (lower left) and K (lower right) lie on an inner rectangle M-F-K-L below the horizontal radius.

- A (center): center of the epicycle (the place of Earth / Mars's-orb center)
- B (top): the Sun's apogee
- D (bottom): the Sun's perigee

- C (right): midpoint of the elongation
- H (left): midpoint of the elongation (opposite C)
- M (on left radius, between H and A): foot of perpendicular / point on line L M N
- F (just below A, on vertical diameter): foot indicating the Sun's defect from mean elongation (line F A)
- G (just below H, on left): endpoint of line F G (parallel to A H)
- L (lower left): corner of the inner construction rectangle
- K (lower right, on vertical diameter above D): corner of the inner construction rectangle

This being given, one proceeds in the same way for the other observation near the apogee of Mars's eccentric, by means of the Sun's equated anomaly, which is found to be $6^{\circ} 15' 11''$. For here too, after removing 6 signs, the Sun's distance from its perigee on its own circle D L is given as $15^{\circ} 11'$; and the sine L M of its complement, $74^{\circ} 49'$, is 9650927. Subtracting from this the earlier value 1570091 leaves the sought line K F = 8080836. As this is to the difference 895 arising from observation, so the whole diameter B D = 20000000 is to the quotient 2215. This number indeed exhibits the whole inequality that the Sun, occupying its apogee and perigee, brings to Mars's annual orb, taking the radius of Mars's eccentric as 100000.

In exactly the same way, in the perigee of Mars's eccentric, by means of the Sun's true anomalies and the difference there found between the two observations and impressed above — 683 — we find the said inequality to be 2415. The difference from the former is therefore 200; half of this, subtracted from the greater or added to the lesser, here yields the same value, limited, as 2315.

Furthermore, to fix the measure of this inequality by other tests as well, relying on Kepler's remarkable industry, we shall produce the following two observations from among those that he himself had carefully worked out from Tycho's — observations made on near-adjacent days at either end at Hven [Huenä], and fitted as closely as possible to one and the same place of Mars on the eccentric. Of these, the former, in the year 1585, on 10 May, at hour 6, minute 11, in the morning, places Mars at $26^{\circ} 54\frac{1}{2}'$ ♌ [Leo]. The latter, in the year 1590, on 31 December, at hour 4, minute 44, likewise in the morning, puts the same Mars at $9^{\circ} 46\frac{1}{2}'$ ♍ [Virgo]. Now at the former time the Sun's true place was at $28^{\circ} 55\frac{1}{2}'$ ♉ [Taurus], and Mars on the eccentric was at $5^{\circ} 22'$ ♎ [Libra]. At the latter time the Sun was at $19^{\circ} 6\frac{1}{4}'$ ♐ [Capricorn], but then Mars was at $6^{\circ} 2'$ ♎ [Libra] — to such a degree, indeed, that, owing to the two distances of the Sun from each other,

almost the greatest inequality arising from them would be implied, while a very small and almost imperceptible error would be left through the eccentric of Mars. From these data the same Kepler, for the earlier time, with the Sun approaching its apogee,

found by his calculation the semidiameter of Mars's annual orb to be 62227; but for the later time — with the Sun, that is, near perigee — he found the same to be 60291, supposing here too, with deliberate care, in such an anomaly of the eccentric, the distance of Mars from the center of the orb to be 100000. But the sure measure of this — given, that is, our restitution of the eccentric, since it stays very nearly around 109000 — therefore if you multiply by 109000 just the difference he left us here, 1936, and divide the sum by the radius, the true value comes out as 2110. In this we shall certainly err least of all, so long as the difference alone is subjected here to the computation for the semidiameter of Mars's annual orb. Furthermore, to extend this found difference to the full inequality that arises from the various solar distances at Mars, by the proportion of sines set out above — in the same manner instituted earlier (except that here the complements of the arcs of the distances are added to each other) — the greatest inequality B D is brought out as 2375. But this previously, by the limitation near the apogee and perigee of the eccentric, was 2315. The difference is therefore 60, and its half 30; from which, finally, this limited inequality comes out as 2345. For this we (so as to be brought nearer to Kepler, even if it slightly exceeds half the measure of the Sun's eccentricity, found above by us) shall set down 2350 as the greatest, in the units in which the radius of Mars's eccentric orb is 100000. [Margin: *Lib. priore Theor. cap. 5.*]

So much, then, concerning the first proposed inequality of Mars's annual orb. The second follows, which concerns the eccentric of the same planet. This stands as follows, from those observations and differences which we set out earlier, worked through near the apogee and perigee of Mars's eccentric: when everywhere the semidiameter of the annual orb at Mars is considered with respect to the Sun's mean distance from the Earth — the distance that resulted for the observations themselves by our customary reduction — it appears as 667875 at the apogee of its eccentric, but 642025 at the perigee, and therefore the mean 654950, in the units in which the radius of the eccentric is reckoned as 1000000. This difference between the greatest and the least, 25850, stands in proportion to the other which flowed from the Sun — namely 23500 — (for that one too must be increased by one digit to this measure of the orb's radius) as 11 to 10. Hence, given one, the other is acquired without difficulty. For this reason, concerning the former — the solar one, that is — we have at least drawn up the following table,

derived by the method just above, for those, namely, who from the hypothesis itself, by the reckoning of triangles, are minded to scrutinize the remaining phenomena of Mars together with us more thoroughly; as we shall then show by example. It is done this way: from each anomaly, the semidiameter of Mars's annual orb that is least of all — with both Mars and the Sun staying at the perigees of their eccentrics — is 630275; but the greatest, at the apogees of both, is 679625, the difference of which is 49350, as is clear from the comparison of the observations and of what we just set out. Half of this — to which, for the rest, it would have been possible to arrive by limitation — would introduce into the phenomena of Mars in those places an aberration of nearly two degrees.

This twofold inequality, residing about Mars's annual orb, although it proceeds from a disparate cause, nevertheless — because it happens only to the annual revolution — we accordingly append both to the annual orb to be saved, as it were, conjointly, by reason of the libration through the diameter of the lunar theory shown above, rather than excusing the other (which flows from the eccentric of Mars) by a libration of the axis on the same.

Besides these two inequalities of Mars's annual orb, a third is also found, which the inclination of this eccentric orb — like that of the rest — to the ecliptic or annual orb (apart from the one that happens by reduction) causes, reaching its maximum chiefly in the places of greatest inclination. But because this is of no great moment, and not even here, where it is found greatest, does it extend as far as 4 minutes — so long as we set and adopt Mars's inclination at $1^{\circ} 50'$ in latitude — for this reason, knowing it, we pass it over both in the higher planets and in the Martian star, lest we should seem to want to burden the theory of this planet with a further and difficult labor.

And so much, thus far, for the recent restitution of Mars according to longitude. The

Table showing the analogy of the distance of the Sun from apogee, in the annual orb of Mars, at each two degrees.

(Tabula analogiam distantiae Solis ab apogaeo, in annuo orbe Martis, ad singulos binos gradus ostendens.)

This is a two-way (re-entrant) table. The degree argument runs **top-down on the left margin** (Gr. $\downarrow$: 0, 2, 4 ... 60) under the sexagene-headings printed above each column-pair (0, 1, 2 Sexagen.), and **bottom-up on the right margin** (Gr. $\uparrow$: 0, 2 ... 60) under the sexagene-headings printed below each column-pair (5, 4, 3

Sexagen.). Each column-pair gives the tabulated value and its first difference (*Diffe.*).

Gr. ↓	0 Sexagen. (value)	Diff e.	1 Sexagen. (value)	Diff e.	2 Sexagen. (value)	Diff e.	Gr. ↑
0	23500	—	17625	—	5875	—	60
2	23493	7	17266	359	5523	352	58
4	23471	22	16900	366	5179	344	56
6	23436	35	16529	371	4843	336	54
8	23386	50	16152	377	4515	328	52
10	23321	65	15770	382	4197	318	50
12	23244	77	15381	389	3888	309	48
14	23151	93	14989	392	3588	300	46
16	23045	106	14593	396	3298	290	44
18	22925	120	14199	400	3018	280	42
20	22791	134	13790	403	3748 [?]	270	40
22	22643	148	13386	404	2490	258	38
24	22484	159	12979	407	2244	246	36
26	22311	173	12570	409	2009	235	34
28	22125	186	12160	410	1784	225	32
30	21927	198	11750	410	1573	211	30
32	21716	211	11340	410	1375	198	28
34	21491	225	10930	410	1189	186	26
36	21256	235	10521	409	1016	173	24
38	21010	246	10114	407	857	159	22
40	20752	258	9710	404	709	148	20
42	20482	270	9307	403	575	134	18
44	20202	280	8907	400	455	120	16
46	19912	290	8511	396	349	106	14
48	19612	300	8119	392	256	93	12
50	19303	309	7730	389	179	77	10
52	18985	318	7348	382	114	65	8
54	18657	328	6971	377	64	50	6

Gr. ↓	0 Sexagen. (value)	Diff e.	1 Sexagen. (value)	Diff e.	2 Sexagen. (value)	Diff e.	Gr. ↑
56	18321	336	6600	371	29	35	4
58	17977	344	6234	366	7	22	2
60	27625 [?]	352	5875	359	0	7	0
	5 Sexagen.		4 Sexagen.		3 Sexagen.		Gr. ↑

The use of the preceding table.

(*Vsus tabulae praecedentis.*)

Given the equated anomaly of both the Sun and Mars: first, the Sun's anomaly is to be used to enter the affixed table, namely with the sexagenes written above and below, and with the whole degrees at either margin that are nearest and next smaller; according to which

—in the adjacent column, where the ratio of the Sun's distance from the perigee is to be taken—with the correction made for the added minutes of anomaly and so on, by the differences inserted between every two degrees, as the need arises. The corrected distance of the Sun is then to be written out and kept separately. Next, in the same way, the entry into the said table is made by means of the eccentric anomaly of Mars, and the distance found in the column is written out and corrected; but this corrected value is at once multiplied by eleven, and the product, divided by ten, is added to the Sun's distance kept earlier above. This sum of the two, when it is added to the measure of the semidiameter of Mars's orb at perigee, 630275, assessed above (as is always to be done), constitutes the said semidiameter of the orb, corrected for this adjustment of each new anomaly, just as has been demonstrated and laid out above; and it will shortly be taught by example.

[Margin: Agreement of the supposed minimal semidiameter of Mars's orb, 631000. See below, near the end of chapter 12.]

An example showing the use of the preceding table, and thereby the full analysis of the theory of Mars in triangles.

So that the truth of the hypothesis of Mars, now restored by us, may become more fully known—and at the same time so that one may see by what reasoning the individual elements are assigned to the computation of the triangles for obtaining its true motion—we have selected a certain observation of our own to

set down in place of an example: namely, when Mars was in conjunction with the star in the northern knee and the topmost of the Twins [Gemini]. This was in the year 1613, on 25 March, at 4 a.m. For in the evening immediately preceding, about 8 o'clock, Mars was distant from that same star toward the leading [westward] direction of the signs by 10'; while in the evening immediately following, near the same hour, Mars had passed the said star toward the trailing [eastward] direction by 20'—so that its daily motion was 30', just as the Copernican calculation also gives the same value very nearly. From this we conclude that the conjunction of Mars and this star at the said time was very nearly exact in degree; and because the reduced longitude of that same star was found to be $4^{\circ}31\frac{1}{2}'$ ♄, with a latitude of $2^{\circ}11'$ north, this was therefore likewise the place of Mars and of the star.

For this time these corrected motions are found:

	Sig.	Gr.	Mi.	Sec.
Mean longitude of Mars from the true equinox	4	7	34	0
Anomaly of Mars	11	8	27	40
True place of the Sun	0	14	31	20
True anomaly of the Sun	9	8	42	0

With these data, the following hypothesis of Mars is fitted to the Tychonic form in this way.

First, let the annual orb of the Sun, LAN, be described upon M; and let the line MK be erected, showing the apogee of the Sun at K, which now stands near 6° ♄. Then, with the center A placed near 14° ♈ in the leading direction, let the orb of Mars BP be described; and let the line BP of the apogee be drawn near the end of ♌ and ♍, and with C removed from B in the leading direction by—

Diagram. A geometric construction of the Tychonic Mars theory. A large circle (the orb of Mars BP) is centered on A; within and to the upper left a smaller circle (the Sun's annual orb LAN) is centered on M, with a small epicycle cluster near the top. Lines radiate from A. The figure continues across the page-opening (the same construction is repeated on the facing page, p. 366).

- A (center, right of middle): center of Mars's orb
- B (upper left, on the inner circle): apogee point of Mars's orb
- C (above B, near the epicycle cluster): displaced eccentric center
- D, E, F, G (small cluster, top): points of the epicycle construction

- H (top cluster): position of Mars on the periphery of the second epicycle
- K (left, below B): apogee of the Sun, on line MK
- L (lower left, on the small circle): point of the annual orb
- M (left-center): center of the Sun's annual orb / position of the Earth
- N (upper left, on the small circle): point of the annual orb
- P (lower right, on the large circle): point on Mars's orb near the perigee line

—2°, namely at 8° &c. $\varnothing$, let the prior epicycle of the eccentric be drawn about that same C, but the posterior one about D. And so, according to the law of the revolutions and the points repeatedly stressed above, the place of Mars will be at H on the periphery of the second epicycle. Then let M be joined to A, and from each of them let a line be drawn to the place of the planet at H; and let another line also be drawn, namely the diameter of the annual orb LN, parallel to AH. All things will then be well expressed in this theory, capable of showing the true motion of Mars even from the Earth at M through the annual orb LAK.

Diagram. The same Tychonic construction repeated at the head of the facing page: the large orb of Mars centered on A, the smaller annual orb of the Sun centered on M to the left, with the epicycle cluster (G, E, F, D, C, H) at the top, and points B, K, N, L, P arranged as on the preceding page.

- A (center): center of Mars's orb
- B (upper left): apogee point of Mars's orb
- C, D (epicycle cluster, top): the two epicycle centers
- E, F, G (top cluster): epicycle construction points
- H (top cluster): position of Mars on the second epicycle's periphery
- K (left, below B): apogee of the Sun
- L (lower left): point of the annual orb
- M (left-center): center of the Sun's annual orb / the Earth
- N (upper left): point of the annual orb
- P (lower right): point on Mars's orb

The procedure for working out this same motion from the foregoing by the application of triangles is this.

First, in triangle CDH, the two sides—which are the semidiameters of the epicycles, CD and DH—are given from what precedes, together with the included angle CDH, 43°4'40" (since by the law of the revolution that angle is reckoned double the angle ECD). Thus first the angle DCH becomes known, 11°48'; then the side CH, 123880 of the units of which the radius AC is 1000000. But when the

angle DCH is added to the arc ED, the angle ECH is constituted, $33^{\circ}20'20''$, whose complement to a semicircle is the angle ACH, $146^{\circ}39'40''$. And so in this same triangle ACH, since likewise the two sides enclosing the given angle at C are granted—namely AC, the radius of Mars's orb, 1000000, and CH just obtained, 123880—there is found from these, first, the angle of the prosthaphaeresis of the eccentric, CAH, $3^{\circ}31\frac{1}{2}'$, additive, so that the place of Mars in the eccentric becomes true or equated, 4 sig. $11^{\circ}5\frac{1}{2}'$; then the side AH, 110530.

With these completed, we come to extricating the prosthaphaeresis, or commutation, of the annual orb. In triangle AHM, the side AH has just been found; likewise also the angle HAM, which—because AH and LM are parallel—is equal to LMA, that is, to the excess of the commutation-anomaly of the annual orb beyond a semicircle, $63^{\circ}25'50''$. For since the true eccentric motion of Mars is subtracted from the true place of the Sun, there remains here the said anomaly, 8 sig. $3^{\circ}25'50''$, which indeed measures the arc NLA. But NL is 6 sig.; therefore what remains, 2 sig. $3^{\circ}25'50''$, is taken for the angle LMA, or HAM. Now with these two data granted in the aforesaid triangle AHM—namely the side AH and the angle HAM—there remains the semidiameter of the annual orb MA, to be equated from the foregoing and at the same time to be supposed, so that finally the angle MHA, or NMH, may be obtained. But to find the said MA in advance, one proceeds as follows.

First, with the equated anomaly of the Sun, 4 sex. $38^{\circ}42'$, the preceding—

—table is entered. Then an entry is made into the same table with the equated anomaly of Mars, which in this example is given as 5 sex. $41^{\circ}59'$ —the simple prosthaphaeresis of the eccentric having been added to the prefixed anomaly of Mars. But the distance at 5 sex. 40° is found to be 22791, with the added difference 134. Therefore, for the $1^{\circ}59'$, the correction is made by the rule of proportion in this way: as 120' is to 134', so is 119' to (about 133); and when these intercepted distances are likewise added here, they make 22924. This number, when it is multiplied by eleven and the sum distributed into ten, yields the inequality of the semidiameter of Mars's annual orb proceeding from the eccentric, 25216.

Combined with the other—that of the Sun, which was found and kept earlier—it produces the sum 38743, to be added to the minimal quantity of the annual orb's semidiameter, 630275; and that semidiameter, equated in this revolution of the eccentric of Mars as well as of the Sun, will be 669018, which in this example the side MA supplies. Therefore, from the three data just found and brought together separately, the angle MHA is obtained, $36^{\circ}35\frac{1}{4}'$, to which HMN is equal, likewise on account of the parallels AH and MN. But the place H in the eccentric of Mars,

reckoned from A—or N in the ecliptic and the orb of the Sun, where from the Earth at M we apprehended the phenomenon of Mars in its acronychal position—was shown to have been 4 sig. $11^{\circ}5\frac{1}{2}'$. Hence the angle MHA just found, subtracted from the eccentric place of Mars in this example, leaves its longitude in the zodiac $4^{\circ}30\frac{1}{4}'$ ☿, falling short of the observed value—according to our diligent and careful observation—by scarcely more than one minute, since that observed value was $4^{\circ}31\frac{1}{2}'$ ☿.

CAPVT X.

On the restitution and extension of the periodic motions of the star Mars.

As for extending the periodic revolutions of Mars to all ages—from which both its apogee place and its mean longitude can be had at any time—we follow the method established in what precedes.

For after we possessed the motion of both the apogee and the longitude, together with the semidiameters of the epicycles and of the annual orb, restored for our own age in Mars as well in the two preceding chapters, we now have recourse to the observations of the ancients. First we borrow from Ptolemy the determination of the apogee place and correct it; then we measure the emendation of the longitude by means of the ancient observation—of Timocharis, as it seems—using the same fixed star that Ptolemy employed here. With both of these acquired, together with the interval of time, we shall thereafter render the task of extending the mean motions easy.

Ptolemy, at the beginning of the reign of the Emperor Antoninus Pius (this was 136 complete years after the birth of Christ), from three acronychal observations directed to the mean opposition of the Sun, found the apogee of Mars at $25^{\circ}30'$ ♄ from the true equinox. We correct this with respect to the true motion of the Sun as follows. Since the apogee place of the Sun in Ptolemy's age was in fact at 10° ♀ [Margin: See book 1, in the Commentary on the Sun.], the prosthaphaeresis of the Sun at the place opposite the apogee of Mars was $1^{\circ}30'$, additive. Moreover, the daily motion of the Sun and Mars combined, when acronychal Mars occupied the Ptolemaic apogee and the Sun the opposite point, comes to $1^{\circ}25'$. From this, the interval of time between the mean place of the Sun [as] Severus and the true opposition of Mars, from the given data, comes out as 25 hours 24 minutes—the prosthaphaeresis of the Sun, namely $1^{\circ}30'$, having been multiplied into 24 hours, and the product divided by the proposed daily motion of both the Sun and Mars together. From this we gather that the earlier arrival of Mars at the true

opposition of the Sun was by an interval of 25 hours 24 minutes; and in this interval of time we reckon Mars's motion, from a daily 24'20", to have been 25³/₄'—which was to be added, namely, to the acronychal, or, what is the same, to its mean motion at the apogee. To preserve this, the apogee place of Mars in Ptolemy's time had to be advanced 2°24' in the trailing direction, so that the acronychal position of Mars might regard the true opposition of the Sun, as is proper.

[Margin: Book 10, chapter 9, on the combined motion.]

There is added besides the difference between the true and the mean equinox, 21', according to our restitution for the time around Ptolemy, so that there are 2³/₄° to be added to the Ptolemaic apogee of Mars. It will therefore stand, for the said year 136 complete after Christ, at 3 sig. 28°15' from the mean equinox. But in our own time, in the year 1600 complete from the Savior of the world, we have found the apogee of Mars at 4 sig. 28°42', likewise prolonged in the trailing direction from the mean equinox. The difference, therefore, of the motion of Mars's apogee from the same equinox over the interval of 1464 Julian years is found to be 30°27'. According to this, in the manner just prescribed above, we have acquired the epoch of the apogee place of Mars at the first bissextile [leap year] of the world-era we customarily use: 1 sig. 2°58'21" from the mean equinox, but from the incarnation of Christ 3 sig. 25°25'20".

Moreover, from the same proportion, the same motion of the apogee in—

annis (years)	Gr.	Mi.	Sec.	Ter.
1000	20	47	56	30
100	2	4	47	39
20	0	24	57	32
1	0	1	14	52

—corresponds; and this indeed will be kept for the tables of the mean motions of Mars to be constructed below.

Further, so that we may also obtain the longitude corrected for the ancient age in Mars, we take up again an ancient observation, which Ptolemy reports was made in the year 13 according to Dionysius, on Capricorn 25 [the 25th day of the Egyptian month], when [Margin: Book 10, chapter 9.] ἕως ὃ τοῦ ἀρέως ἐδόκει προστεθεικέναι τῷ βορέῳ μετώπῳ τοῦ Σκορπίου [Greek: "the morning star of Mars seemed to have been added to the northern forehead of Scorpius"]—that is, the morning star of Mars appeared to have been set against the northern forehead of

Scorpius. For I insert here the very Ptolemaic text, because Johannes Kepler [Johannes Kepler] called into doubt [Margin: In the end of the Commentary on the motion of ♂.] the bright star in the forehead of Scorpius—which is first in number in that asterism, and which Ptolemy had used—and, departing from Ptolemy, chose in its place the most northern star in the forehead of Scorpius, namely the fifth in number, which is of the fourth magnitude and exceeds the bright one of the second rank by $1^{\circ}26\frac{1}{2}'$ in longitude and by $37'$ in latitude. I conjecture that Kepler did this among other reasons because he noticed that his own calculation, previously derived through Ptolemy, agreed more correctly with that fifth star to be supposed—at least in the longitude of Mars, though not likewise in latitude.

For the rest, I do not myself judge it safe enough to depart from Ptolemy in the case of this star—both because the latitude of Mars (as Kepler himself admits, and as is later more evidently argued concerning the emendation of latitude) is better accommodated to the bright star, and because the restitution of Mars, as of the other planets, when derived through Ptolemy, drags more error with it for the reasons set out above; so much so that I have decided these ancient conjunctions of the planets with the fixed stars are the more safely to be chosen. And certainly, when the observer names the northern forehead of Scorpius, it is credible that he was pointing to this bright star par excellence [Greek: κατ' ἐξοχήν (kat' exochēn, "preeminently")], since of all—indeed not of all, but of the three most brilliant in the forehead of Scorpius—this one alone is the most northern and most brilliant, also holding among them the northernmost place in latitude.

This star, then, we shall use with Ptolemy; and we shall further attend to the time of the prosthesis [Greek: προσθέσεως (prostheseōs, "addition, application")], that is, the conjunction of Mars with that same star—which Ptolemy still more clearly sets out, when he reports in explicit words there that this same conjunction took place 52 years from the death of Alexander, and 476 years from Nabonassar, in the month Athyr among the Egyptians, on the 21st day, in the morning. This time we have indeed determined to fall in the current Julian year 272 before the birth of Christ, on 18 January, in the morning about the 5th hour, at Copenhagen [Hafnia]. At this moment the longitude of the said first star of Scorpius, by the Tychonic constitution and by our precession of the equinox, is gathered to have stood at $1^{\circ}29'40''$ Scorpius from the mean equinox, with a latitude of $1^{\circ}16'$ north. From this the Ptolemaic text implies that Mars then very nearly occupied the same place—differing from us in this, that it sets the longitude of the said star at $2^{\circ}15'$ Scorpius with a latitude of $1^{\circ}20'$ north, and consequently assigns the same place to Mars. But at this same time the true place of the Sun from the mean

equinox, according to our computation, was 9 sig. $24^{\circ}38\frac{1}{2}'$, with its equated anomaly 7 sig. $21^{\circ}11'$. Next, the apogee of Mars from the same mean equinox, according—

we find, according to the foregoing correction by means of [Ptolemy], to be 3s $20^{\circ}15'$. Finally, since we still need the mean longitude of Mars from the mean equinox provisionally known in some fashion — so that from it, as from a base, we may first reason out both anomalies, that of the eccentric and that of the annual revolution, before we arrive at the precise correction of that same longitude restored through the hypothesis — for that reason we here place more trust in the account of [Ptolemy] than in the Prutenic Tables [Tabulae Prutenicae], and so set the mean longitude of Mars at this time, reckoned from the true equinox along with [Ptolemy], at $4^{\circ}12' \underline{\alpha}$. When $4\frac{1}{2}'$ of the inequality of the equinox is added to this, that longitude from the mean equinox becomes 6s $4^{\circ}16\frac{1}{2}'$.

Now when the apogeal motion 3s $20^{\circ}15'$ is subtracted from this longitude, there remains the anomaly of the eccentric of Mars, 2s $14^{\circ}1\frac{1}{2}'$. This, diminished by $26\frac{1}{2}'$ in accordance with the curtailed prosthaphaeresis of Mars's eccentric below the Ptolemaic value, which followed from our own correction when we reduced its acronychal positions to the point opposite the true course of the Sun, becomes 2s $13^{\circ}35'$ — although for [Ptolemy] it is only 2s $12^{\circ}47'$, since among other things he sets the apogeal motion here at $21^{\circ}25'$. But since at this point a deviation of one degree in the anomaly of the eccentric scarcely yet produces an error of $4'$ in the prosthaphaeresis of the eccentric, we shall safely use this anomaly as corrected by us — namely 2s $13^{\circ}35'$ — as the value to be investigated in our hypothesis of Mars; and if anything in it should still prove excessive or deficient, we shall amend it later.

Having established these points, then, we shall next describe the following hypothesis of Mars in the Tychonic form, in our usual manner, and shall explain it still more fully for the benefit of students — namely the prosthaphaereses and the discrepancies from the mean motion of Mars that fall to the Sun and to Mars respectively along their orbits, in those places where they are to be drawn out on both sides by calculation.

Diagram. A geometric construction of the Tychonic hypothesis of Mars. From M, the center of the Earth, the orbit of the Sun A L O W is first described, with the line O W drawn to mark the place of the Sun's apogee at O and its perigee at W (which in that ancient time stood at about $31^{\circ} \text{II} - \nearrow$). Taking the beginning of the sign $\var�$ at the top vertex of this circle and $\var�$ at the opposite point, the true place of

the Sun is supposed at A — that is, removed from the sign ♊ by $24^{\circ} 38'$ in the order of the signs (as found above). The Sun's distance from its perigee W is measured by the arc W A, which the excess of the Sun's anomaly written above beyond the semicircle measures, namely $1s 21^{\circ} 11'$. From A a perpendicular is dropped to the axis O W, falling at S, and W S measures the Sun's distance from perigee. Upon A, the true place of the Sun, the orbit of Mars B C P is then described; to it the mark of the sign ♉ is erected upward from A, parallel to M ♉, so that an image of the ecliptic may also exist in the planet's orbit, and here the supposed anomaly of Mars, like every other particular, is arranged in its proper place according to the index of that same apogee — which in this ancient age stood at $20^{\circ} 15' \text{ ♉}$, as established above.

- M (center): the Earth, center of the Sun's orbit
- A (lower right): the true place of the Sun; center of Mars's orbit B C P
- L (right): point on the Sun's orbit
- O (upper right): apogee of the Sun
- W (center-left, below M): perigee of the Sun
- S (mid): foot of the perpendicular from A to axis O W
- B (upper, on Mars's orbit): Mars's apogee position
- C (lower left): mean longitude of Mars on its orbit
- P (bottom): perigee point of Mars's orbit
- N (left): point on the annual orb
- ♉ (top), ♊ (lower center): zodiac-sign markers on the Sun's orbit
- ♉ (small, upper center): sign marker erected parallel from A
- H, G, D, E, F, K (left cluster): epicycle points of Mars's construction

as was proved above, it was situated; let that point be placed at B, occupying so many degrees and minutes in the order of the signs from the sign ♉ on Mars's orbit. Then, with the diameter B P drawn through the line of apsides of Mars's orbit, let its mean longitude be supposed at C, removed from the apogee B in the order of the signs by the measure of the anomaly given above, $2s 13^{\circ} 35'$.

Finally, upon C let the first epicycle of Mars's orbit be drawn, and upon D the second, the distance of whose center from the apogee E of the prior epicycle is equal to the arc B C, while the displacement of the planet at H through the arc F H of the upper epicycle E D is double that — in accordance with what has often been impressed regarding the Tychonic hypothesis of the superior planets, derived from the law of the revolutions. But I pursue all these things by demonstrating them here still more distinctly, so that lovers of learning may both

recognize the excellence of this hypothesis and at last learn to imitate it correctly and aptly in any other supposition of revolution — since it elegantly and καθ' ὑποτύπωσιν [Greek: kath' hypotypōsin, "in outline / by sketch-illustration"] sets before the eye the angles by which the true motions differ from the mean ones in each case.

For the line from A to C is here the indicator of Mars's mean motion in its proper orbit, while A H is the indicator of the true motion. Since the diameter L N of the annual orb is set parallel to A H, the same true place of the eccentric of Mars is shown by it from M to N on the ecliptic. Finally, the line of sight running from M, the Earth, to the planet at H forms with A H the angle of annual commutation, namely M H A, to which the arc N Z on the ecliptic is always equal. Moreover, the angle H A M measures the Sun's distance from the point opposite Mars, of its own accord, from the supposed true place of the Sun, being equal to the arc A L (or the angle L M A) on that same side, because A H and L N are parallel. Finally, with the perpendicular noted from H, the seen place of Mars, extended to Q on the axis B P of Mars's orbit, the distance of Mars itself from its perigee P Q is also indicated — through which, as also through W S, the semidiameter of the annual orb M A is to be corrected in this planet, according to the prescription. But let us return to working out the prosthaphaereses.

The anomaly of Mars being supposed, then — that is, the arc B C, $2s\ 13^{\circ}\ 35'$ — for which the prosthaphaeresis of the eccentric C A H and the side C H are to be sought: first, in the triangle C D H, since from the given semidiameters of the epicycles C D 14840 and D H 3710, together with the contained angle C D H of $147^{\circ}\ 10'$ (double the presupposed anomaly), there is first shown compendiously the angle D C H, $6^{\circ}\ 23\frac{1}{2}'$, to be added to the anomaly E D; and the angle E C H becomes $79^{\circ}\ 58\frac{1}{2}'$. Then in the same triangle C D H the side C H is also given by triangular calculation, 18069.

Descending next into the triangle C A H, we again take there the two sides enclosing the given angle, also granted: of these, A C, the radius of Mars's orbit, is 1000000, and C H, just now found, is 18069, while the angle A C H is $100^{\circ}\ 1\frac{1}{2}'$ —

— namely the supplement of the angle H C A. Wherefore there is found first the angle of Mars's prosthaphaeresis C A H, $9^{\circ}\ 47\frac{1}{4}'$, and then the side A H, 1046688.

But in order to possess here the longitude of Mars's eccentric, corrected to a fair degree of accuracy, before we proceed to draw out the prosthaphaeresis of the annual orb (which is comprehended by the angle M H A), let us at the same time correct M A, the semidiameter of that orb — since in the former the measure of

the angle H A M, to be foreknown together with the true place of the Sun, consists, while in the latter consist the two inequalities of Mars recently found by us regarding its annual orb. Accordingly, first, with the apogee of Mars presupposed at 3s 20° 15', let the anomaly of that same be added, 2s 13° 35', and the mean longitude from the mean equinox is likewise composed, 6s 3° 50'. When the prosthaphaeresis of the eccentric previously sought, 9° 47¼', is taken from this, there remain 5s 24° 2' 45" for the true or equated longitude of Mars's eccentric. This in turn, subtracted from the true place of the Sun set above, 9s 24° 38' 10", leaves 4s 0° 36'. The complement of this arc to the semicircle — namely 1s 29° 24' nearly — measures the angle A M L, that is, the Sun's distance from the point opposite Mars, and consequently also the angle H A M.

Furthermore, in the correction of M A, the semidiameter of the orb: when, by means of the equated anomaly of the Sun, 3 sexagenes 51° 11', and likewise of Mars's eccentric, 1 sexagene 3° 48' (allowing also here that the prosthaphaeresis of the eccentric be subtracted from its simple anomaly, and on both sides a reduction of the signs into sexagenes be made), the several quantities have been found by the benefit of the table set forth — there is found the equation for W S, that is, the distance of the Sun from the Earth, 4465, but for P Q of Mars in the eccentric, 18630. When these, as was prescribed, are added together, and the sum aggregated to the least semidiameter of the orb, 630275, there comes out the corrected radius of the annual orb M A, 653370 — of which units the radius of the eccentric A C is likewise 1000000.

Accordingly, in this last triangle A H M, of the two found and given sides — namely A H 1046688 and A M 653370 — also comprehending the angle M A H given from the foregoing as 59° 24', we at last find the angle of the annual inequality M H A here, 38° 13¼'. From this, with the prosthaphaeresis of the eccentric — say, 9° 47¼' — subtracted, there remains the whole difference between the mean and true longitude of Mars from the mean equinox, 28° 26', with an addition. Therefore, when this is subtracted from the seen longitude of the star — that is, Mars's longitude established above likewise from the mean equinox at 7s 1° 29' 40" — there results the mean longitude of Mars from that same mean vernal equinox, 6s 3° 3⅔', which alone we here sought. This we used just now as 6s 3° 50' while we were hunting the prosthaphaeresis of the eccentric and the anomaly of the annual orb. According to the difference, then, which is about 46', when we carefully examine each particular, we find the prosthaphaeresis of Mars's eccentric to be 9° 44¼', and therefore its true longitude 5s 23° 19' 30", and consequently the anomaly of the annual orb 4s 1° 19⅓', to which corresponds a prosthaphaeresis of 38° 17⅓'. From this, in turn,

with that of the eccentric, $9^{\circ} 44\frac{1}{4}'$, taken away, there remain $28^{\circ} 13\frac{1}{6}'$ to be subtracted from the seen longitude of Mars.

When this is done, the mean longitude of Mars finally sought emerges as $6s\ 2^{\circ} 56\frac{1}{2}'$ from the mean equinox, for the aforementioned year 272 before Christ, the month of January, the 18th day, the 17th hour before noon — worked out, in this undertaking, with sufficient precision. We judged that this path, rather than the Ptolemaic one, should be followed, so as to investigate directly the mean longitude sought. But in order that this same be reduced to the completed year 272, let there be subtracted from the longitude of Mars established at $6s\ 2^{\circ} 56' 30''$ the mean motion from the equinox corresponding to 16 whole days and 17 hours — by which the time of the observation was later than the noon of the next following Calends of January with us — namely $8^{\circ} 45' 22''$; and the mean longitude of Mars is given at the end of the year 272 before Christ, indeed from the mean equinox, $5s\ 24^{\circ} 11' 8''$.

[Margin: Lib. super. ca. 2. & cap. 5.]

But in our own age — that is, in the same year 1600 of our Savior just past — the same longitude of Mars, through the restoration in the acronychal positions described in the higher book, is established by us at $10s\ 7^{\circ} 6' 32''$. Over an interval, therefore, of 1872 Julian years, or 683748 whole days, there falls to Mars a longitude from the mean equinox of $4s\ 12^{\circ} 55\frac{1}{2}'$ beyond 995 complete revolutions.

By this proportion is found the epoch of Mars's longitude from the mean equinox, the complete revolutions being rejected: first, at the beginning of the first bissextile year of the world in our usual reckoning, $4s\ 22^{\circ} 1' 56''$; then, at the epoch of Christ incarnate, $1s\ 9^{\circ} 39' 14''$. Finally, from the same proportion, the same mean longitude of Mars suiting [each interval] is found:

annis	Sig.	Gr.	Mi.	Sec.	Tert.
1000	8	17	9	35	38
100	2	1	42	57	34
20	7	18	20	35	30
com. 1	6	11	17	10	6

All of which serve for arranging the mean longitude by means of the canon — the canon which we shall now at last, the restitution according to longitude being above completed by the grace of God, subjoin in order in each case.

CAPVT XI.

On the canonical arrangement of the mean motions in the three superior planets.

The restitution of the three superior planets in longitude through the ecliptic is followed by the canonical arrangement of their motions, from which indeed their true motions at any time, for all the ages of the world, may readily be sought and computed.

[Margin: Lib. super. ca. 2. & cap. 5.]

Now these motions are called either mean (that is, simple) or prosthaphaeretic — all of which have been sufficiently shown in the foregoing hypotheses. But since the mean ones, because they are simple, regard whole revolutions, we shall briefly treat in this chapter of arranging them — the ones from which the true motions flow. Of this kind are the motions of longitude and of anomaly in each planet. "Longitude" is the name given to that which occurs continually in the planet's primary orbit, ever in the order of the signs, taking its start from the mean vernal equinox. "Anomaly," by contrast, is assigned to the primary epicycle, in which the center of the second epicycle (to which the planet adheres) is carried in the order opposite to the signs, taking its start from the apogee, or the greatest elongation from the center of the orbit, reckoned in that same epicycle. Because this always arises from the motion of the planet's apogee, accordingly, when the anomaly is in turn subtracted from the longitude, the apogee is recovered.

We, however — both for the sake of the clarity of the demonstrations through eccentric epicycles in the hypotheses themselves, and for greater certainty and an easier method in extracting the mean motions — have preferred to use the anomalies of the eccentrics. Nor are any other quantities reckoned by us as simple, since we measure the annual ones from the true courses of the Sun, and the eccentric of the planet, which must always here be subtracted from those of the Sun.

These things considered — which proceed in the same way as the Sun and the Moon did above, and likewise, as there, in the prefixed epochs — the orders of the mean motions, namely of longitude and of anomaly, follow in each case: first, by the Julian years complicated together; then the single ones up to 20; afterward through the remaining classes of completed times — namely the common and bissextile months; and finally days and hours, and the minutes of these. And since enough has been prescribed on these matters by others, especially by Erasmus

Reinhold [Erasmus Reinhold] in his Astronomical Tables [Prutenicae] — whose footsteps we too have followed above in the Sun and the Moon — we consider it superfluous to deliver more on these matters at present. But the use of them we defer to the examples that will follow here and there throughout this work.

Mean motions of the superior planets ♄, ♃, and ♀ in collected Julian years.

The following canon gives, for each completed epoch of collected Julian years (reckoned from the radix at the year of Christ), the mean longitude and the mean anomaly of the eccentric for Saturn, Jupiter, and Mars. Each entry is in signs (S.), degrees (G.), minutes (M.), and seconds (S.).

Years (from Christ)	♄ Long. S	♄ Ano m. S	♃ Long. S	♃ Ano m. S	♀ Long. S	♀ Ano m. S
Christ	5	1	2	10	4	3
(continued)	2	6	5	0	1	9
20	8	8	8	8	7	7
40	4	4	4	4	15	3
60	0	0	0	0	10	11
80	8	8	8	8	6	6
100	4	4	5	5	2	1
200	9	9	10	10	4	3
300	2	2	3	3	6	5
400	7	6	8	8	8	7
500	11	11	2	1	10	9
600	4	4	7	6	0	11
700	9	8	0	0	2	1
800	2	1	5	5	4	3
900	7	6	10	10	6	5
1000	11	11	4	3	8	7
1100	4	3	9	8	10	9
1200	9	8	2	1	0	11

Years (from Christ)	♄ Long. S	♄ Ano m. S	♅ Long. S	♅ Ano m. S	♄ Long. S	♄ Ano m. S
1300	2	1	7	7	2	1
1400	6	5	0	0	4	3
1500	11	10	6	5	6	5
1600	4	3	11	10	8	7
1700	9	1	4	3	10	9
1800	2	0	9	8	1	11
1900	6	5	2	1	3	1
2000	11	10	8	7	5	3
2500	11	9	10	8	3	1
3000	11	9	0	10	1	11
3500	11	8	2	0	0	9
4000	11	8	4	2	10	7
4500	11	7	6	4	8	5
5000	11	7	8	5	6	3
5500	11	6	10	7	5	1
6000	10	6	0	9	3	11
6300	1	8	4	0	9	5

Mean motions of the three superior planets ♄, ♅, and ♄ in expanded Julian years.

This canon resolves the single years 1–20. Each entry is in signs (S.), degrees (G.), minutes (M.), seconds (S.).

A ♄ n Long. . S	♄ Anom. S	♅ Long. S	♅ Anom. S	♄ Long. S	♄ Anom. S
1 0	0	1	1	6	6
2 0	0	2	2	0	0
3 1	1	3	3	7	7
4 1	1	4	4	1	1
5 2	2	5	5	7	7

A ḥ n Long. . S	ḥ Anom. S	𐤁 Long. S	𐤁 Anom. S	♂ Long. S	♂ Anom. S
6 2	2	6	6	2	2
7 2	2	7	7	8	8
8 3	3	8	8	3	3
9 3	3	9	9	9	9
1 4	4	10	10	3	3
0					
1 4	4	11	11	10	10
1					
1 4	4	0	0	4	4
2					
1 5	5	1	1	10	10
3					
1 5	5	2	2	5	5
4					
1 6	6	3	3	11	11
5					
1 6	6	4	4	6	6
6					
1 6	6	5	5	0	0
7					
1 7	7	6	6	6	6
8					
1 7	7	7	7	1	1
9					
2 8	8	8	8	7	7
0					

In common months.

For each completed common month (reckoned from the beginning of the year), the table gives the accumulated mean motion. Entries are in degrees (G.), minutes (M.), seconds (S.) — no signs column, the values remaining within a single sign.

Month	ᵢ Long. G	ᵢ Anom . G	ᵠ Long. G	ᵠ Anom . G	♂ Long. G	♂ Anom . G
Ianuarius (January)	1	1	0	37	34	16
Februarius (February)	1	1	0	16	54	0
Martius (March)	3	3	0	53	28	17
Aprilis (April)	4	4	0	32	58	2
Majus (May)	5	5	0	9	32	19
Iunius (June)	6	6	0	47	2	4
Iulius (July)	7	7	0	24	36	21
Augustus (August)	8	8	0	1	11	7
September	9	9	0	39	40	23
Ocober [October]	10	10	0	17	15	9
November	11	11	0	55	45	25
December	12	12	1	32	19	11

In bisextile [leap] months.

Month	ᵢ Long. G	ᵢ Anom . G	ᵠ Long. G	ᵠ Anom . G	♂ Long. G	♂ Anom . G
Ianua. (January)	1	1	0	37	34	16
Febru. (February)	2	2	0	16	59	1
Mart. (March)	3	3	0	52	33	17
Aprilis (April)	4	4	0	31	3	3
Majus	5	5	0	8	37	19

Month	♄ Long. G	♄ Anom . G	♅ Long. G	♅ Anom . G	♆ Long. G	♆ Anom . G
(May)						
Iunius	6	6	0	46	7	5
(June)						
Iulius (July)	7	7	0	23	41	21
Augu.	8	8	0	0	16	7
(August)						
Septe.	9	9	0	38	45	23
(September)						
Octo.	10	10	0	16	20	9
(October)						
Nove.	11	11	0	54	50	25
(November)						
Dece.	12	12	1	31	24	11
(December)						

Mean motions of the three superior planets ♄, ♅, and ♆ by days.

For each completed day (1–31), the table gives the accumulated mean motion in longitude and anomaly. Entries are in degrees (G.), minutes (M.), seconds (S.).

Dies (Day)	♄ Long. G	♄ Anom. G	♅ Long. G	♅ Anom. G	♆ Long. G	♆ Anom. G
1	0	0	0	0	0	0
2	0	0	0	0	1	1
3	0	0	0	0	1	1
4	0	0	0	0	2	2
5	0	0	0	0	2	2
6	0	0	0	0	3	3
7	0	0	0	0	3	3
8	0	0	0	0	4	4
9	0	0	0	0	4	4

Dies (Day)	ḥ Long. G	ḥ Anom. G	𐤅 Long. G	𐤅 Anom. G	♂ Long. G	♂ Anom. G
10	0	0	0	0	5	5
11	0	0	0	0	5	5
12	0	0	0	0	6	6
13	0	0	1	1	6	6
14	0	0	1	1	7	7
15	0	0	1	1	7	7
16	0	0	1	1	8	8
17	0	0	1	1	8	8
18	0	0	1	1	9	9
19	0	0	1	1	9	9
20	0	0	1	1	10	10
21	0	0	1	1	11	11
22	0	0	1	1	11	11
23	0	0	1	1	12	12
24	0	0	1	1	12	12
25	0	0	2	2	13	13
26	0	0	2	2	13	13
27	0	0	2	2	14	14
28	0	0	2	2	14	14
29	0	0	2	2	15	15
30	1	1	2	2	15	15
31	1	1	2	2	16	16

Mean motions, longitude and anomaly together, of ḥ, 𐤅, and ♂ for hours and for fractions [scruples] of hours.

This canon gives the combined increment (longitude and anomaly move together at this scale) for each planet, in minutes (M.) and seconds (S.). The left half is keyed by hours (H.M., the hours 1–30); the right half by minutes of an hour (M., 31–60).

Left half — by hours (H.):

H.	♂ M	S	♀ M	S	♂ M	S
1	0	5	0	12	1	19
2	0	10	0	25	2	37
3	0	15	0	37	3	56
4	0	20	0	50	5	14
5	0	25	1	2	6	33
6	0	30	1	15	7	52
7	0	35	1	27	9	10
8	0	40	1	40	10	29
9	0	45	1	52	11	47
10	0	50	2	5	13	6
11	0	55	2	17	14	25
12	1	0	2	30	15	43
13	1	5	2	42	17	2
14	1	10	2	54	18	20
15	1	15	3	7	19	39
16	1	20	3	19	20	58
17	1	25	3	32	22	16
18	1	30	3	44	23	35
19	1	35	3	57	24	53
20	1	40	4	9	26	12
21	1	45	4	22	27	31
22	1	50	4	34	28	49
23	1	55	4	47	30	8
24	2	0	4	59	31	27
25	2	5	5	12	32	45
26	2	10	5	24	34	4
27	2	15	5	37	35	22
28	2	20	5	49	36	41
29	2	26	6	1	38	0
30	2	31	6	14	39	18

Right half — by minutes (M.) of an hour:

M.	h M	S	¼ M	S	♂ M	S
31	0	3	0	6	0	40
32	0	3	0	6	0	42
33	0	3	0	6	0	43
34	0	3	0	7	0	44
35	0	3	0	7	0	46
36	0	3	0	7	0	47
37	0	3	0	7	0	48
38	0	3	0	8	0	50
39	0	3	0	8	0	51
40	0	3	0	8	0	52
41	0	4	0	9	0	54
42	0	4	0	9	0	55
43	0	4	0	9	0	56
44	0	4	0	9	0	58
45	0	4	0	9	0	59
46	0	4	0	10	1	0
47	0	4	0	10	1	1
48	0	4	0	10	1	3
49	0	4	0	10	1	4
50	0	4	0	10	1	5
51	0	4	0	11	1	7
52	0	4	0	11	1	8
53	0	4	0	11	1	9
54	0	5	0	11	1	11
55	0	5	0	11	1	12
56	0	5	0	12	1	13
57	0	5	0	12	1	15
58	0	5	0	12	1	16
59	0	5	0	12	1	17
60	0	5	0	12	1	19

CHAPTER XII

On the Construction of the Prosthaphaeresis Tables of the Three Superior Planets

For the two upper planets, Saturn and Jupiter, we present nothing different from Copernicus and the Prutenic Tables [Erasmus Reinhold] except the prosthaphaereses themselves — both of the eccentrics and of the annual commutation — which we have restored above, uniformly, each in its own place. We will therefore first correct the inequalities of the eccentric of these planets according to the indicated ratios, point by point. We will do the same for Mars as well, except that here the ratio is taken only of the fourth part of the whole eccentricity, as a measure of the semidiameter of the second eccentric epicycle into which it enters. This is the reason why we will entrust the work of disentangling the prosthaphaeresis of the eccentric of Mars not to a bare proportion, but rather to the resolution of triangles. The method of calculation has been set out sufficiently in the earlier examples, and will not be repeated here.

Furthermore, in those two superior planets, Saturn and Jupiter, the annual commutation — once it has been computed for the two elongations that differ most from the center of the eccentric, that is, at the apogee and perigee of those eccentrics, through the semicircle of the annual orb, with the semidiameter of this orb also corrected at each point — the difference that results is everywhere to be taken in the table as the excess at the given place. [Margin: *Liber I Theor., Cap. 6, in the second Theory of the Moon, and the table here computed.*] Finally, to this must be acquired and applied the proportional minutes, just as the appearances through the eccentric require, according to the procedure demonstrated above for the Moon; and these here, point by point, differ imperceptibly from the Prutenic values. In this way the prosthaphaeresis table, as they call it, is completed for the two highest planets.

For Mars, however, we compactly include the divergence of the semidiameter of its annual orb, which proceeds from the eccentric, in the excess to be investigated here, in the following way: we suppose that at the apogee of the eccentric the said semidiameter of the orb of Mars is 667875, while at the perigee it is 642025, of those units of which the radius of the eccentric is 1000000. From this, too, the greater equation is derived by computation as somewhat smaller in the excess. For its appearances the proportional minutes are found and arranged in the manner customary for the upper planets, through the revolution of the eccentric.

Moreover, for that part of the annual commutation's inequality which flows from the unequal distance of the Sun from the Earth, the table given above, which we assigned to it, is here resolved and distributed into appearances, to be applied in such a way that they are extracted by the true anomaly of the Sun at any given time, and afterwards applied to the remaining sums of the commutation.

But this inequality, which proceeds from the Sun, since — when it is applied to the annual orb of Mars by means of the latter's eccentric — it produces in itself a fairly notable difference of appearances, I will here set out demonstratively, in the form in which it arises as a maximum at the apogee and perigee of the eccentric of Mars, and indeed at the moment when the planet, the Earth, or the Sun is near the contact of the annual orb. [Margin: *This must be understood according to the triangular hypothesis explained above.*]

So let A in the following figure be the center of the eccentric of Mars, from which a straight line BAE is projected both above and below, serving as the index of the eccentric's apogee at B and of its perigee at E.

Diagram. A geometric construction in the left margin illustrating the eccentric of Mars and the nested annual orbs at apogee and perigee. A central vertical line carries the points B (apogee, upper) and E (perigee, lower), with A as the center of the eccentric between them. Around B (the upper region) two concentric circles are drawn through the points C, H, D, K; around E (the lower region) two concentric circles are drawn through the points F, L, M, G. Lines are drawn from A to the various circle-contact points (C, D at the top; F, G at the bottom), forming the right triangles used in the text's resolution.

- B (top of central axis): apogee of the eccentric
- A (middle of central axis): center of the eccentric of Mars
- E (bottom of central axis): perigee of the eccentric
- C, H, D, K (upper circles, around B): contact points of the annual and circumscribed orbs at apogee
- F, L, M, G (lower circles, around E): contact points of the annual and circumscribed orbs at perigee

[Margin: *See above, Cap. 9 of this book.*] [Margin: *Proposition 18, Book 3, Euclid's Elements.*]

It [the line BAE] exists, then, where the radius of the eccentric is 1000000; and of such units we reckon, from the hypothesis of Mars restored above, AB as 1111300 and AE as 888700. Now, taking the center B near the apogee, let the annual orb of

Mars CH be described, just as it is found there; and at the perigee, with center E, the orb FL — in both cases first without the accession of the anomaly from the Sun, namely in the increment around the apogee and the decrement near the perigee of this orb; then with that same anomaly added; and afterwards with the outer orbs circumscribed, DK at the apogee and MG at the perigee, in like manner, from the given centers B and E. And this with the maximum increment of the Sun above the mean, which is 11750 thousandths — at which value we first operate, for the sake of the demonstration — and afterwards with the like subtraction for the decrement, so that whatever difference there is may be brought into the open.

Around the apogee, of both Mars and the Sun, these things are given:

I. In triangle BAC: AB 1111300, BC 667875, and the angle BCA is right (since it is at the contact of the circle at C). Therefore BAC is 36 degrees, 56 minutes, 26 seconds — which is the maximum commutation at the apogee.

II. In triangle BAD: BA 1111300 and BD 679625 are given, with BDA right. Therefore BAD is 37 degrees, 42 minutes, 9 seconds. The difference of this from the previous BAC is thus 45 minutes, 43 seconds, which is assigned to the maximum inequality of the Sun determined at the apogee of Mars, in terms of the angle CAD.

Furthermore, at the perigee of Mars and the apogee of the Sun, in the right triangle AEF, the given quantities from before are AE 888700, EF 642025, and likewise the angle at F right. Therefore EAF is 46 degrees, 15 minutes, 19 seconds, which includes the maximum commutation of Mars taken separately, together with the maximum excess. Finally, the given quantities of triangle AGE are AE 888700, EG 653775, and likewise the angle at G right; accordingly the angle EAG is given as 47 degrees, 21 minutes, 45 seconds.

From these it is gathered: first, that the maximum excess of Mars here amounts to at least 9 degrees, 18 minutes, 53 seconds, which the difference between the angles BAC and EAF exhibits; and second, that the maximum excess in the Sun is 20 minutes, 43 seconds, which the difference of CAD and FAG demonstrates.

Now such a maximum prosthaphaeresis, together with its excess, comes about from the addition of 11750 particles to the terminus of the annual orb of Mars at apogee and perigee — that is, with the Sun being at apogee — while at perigee, by the subtraction of those same particles, so that there BD becomes 667875 while BC is 656125, and here EG becomes 642025 while EF is 630275. Thus from the resolution of the aforesaid triangles there emerges a smaller difference at the

apogee of Mars than before, by 27 seconds, and at the perigee by 1 minute, 21 seconds. By this slight discrepancy, restricted to the mean by limitation, in place of 11750 particles, 11640 may be used; and however large this terminus turns out to be in the rest, it may always be relaxed by subtracting the hundredth part. Then whatever remains is to be added perpetually to the prescribed termini of the orb's semidiameter at apogee and perigee, and afterwards the computation for this prosthaphaeresis, from the Sun together with its excess, is to be carried out. In aggregating the excess into the appearances proportionally, I do not think that any proportional minutes other than those appropriate to that greater value are required.

For the rest, in applying this appearance — which arises here from the differing distance of the Sun from the Earth —

it must be noted as follows: while the Sun traverses the upper part of its epicycle, its addition is made to the otherwise absolute annual commutation; but in the lower part, the subtraction is made — just as the notes about this matter, inserted in their proper places above and below in the prosthaphaeresis tables, are found.

These instructions, however, I leave to those who abound in the leisure that I, working alone in the restoration of astronomy, am deprived of — if it should please them more to digest all the appearances of this planet too into a single prosthaphaeresis table, and from there to draw out (once the prosthaphaeresis of the eccentric alone and the distance of the planet's side from the center of the orb have been obtained, and finally corrected by the semidiameter of the annual orb in the prescribed manner) the whole annual commutation as well, which Copernicus calls the parallax, in the resolution of a single triangle. This resolution is the most usual of the Theorics, being required for nearly every individual case — namely from two given sides about a likewise given angle. We for the present rest content with this method [Greek: *πραγματεία* (*pragmateia*, "treatment" or "working-out")], as the more reliable one, and also the better suited for more successfully investigating the latitude of this planet, and finally more fitting for the knowledge of the Theory by students of this art. Thus, along with the prosthaphaeresis of the eccentric, we insert only the side of the distance — which in the hypothesis above is everywhere expressed by the note AH — in place of the prosthaphaeresis table of Mars alone, seeking the rest in their proper places, just as the examples above and below will teach.

It must also be noted, concerning the measure of the semidiameter of the annual orb of Mars, that since other observations — both those of Tycho Brahe and those I myself afterwards obtained — everywhere require the same value to be for the

most part somewhat larger (as the example given below will also advise), therefore, in place of the supposed minimum semidiameter 630275, the value 631000 will always be placed in the calculation of Mars, as more correct and better suited to the phenomena, so that in this way satisfaction may be given to the appearances of this planet too with every possible exactness. For I judged it better to neglect the difference of a very few minutes, which may arise in certain places from the inclination of the orb during observation, especially in this planet, and which is treated only with great labor.

Prosthaphaeresis Table of Saturn

0 Sig. [Sign 0]

G r a d u s	Eccentri prosth. (G. M. S.)	Add. Diff. (M.S.)	Scrup. propo. (M. S.)	Ad. Dif (S.)	Orbis An. prosthap. (G. M. S.)	Add. Diff. (M. S.)	Excef. in Per. (M. S.)	Ad. Dif. (S.)	G r a d u s
0	0 0 0	6 35	0 0	0	0 0 0	5 22	0 0	36	3 0
1	0 6 36	6 35	0 0	1	0 5 22	5 22	0 36	36	2 9
2	0 13 11	6 35	0 1	1	0 10 44	5 21	1 12	35	2 8
3	0 19 46	6 35	0 2	2	0 16 5	5 20	1 47	35	2 7
4	0 26 22	6 35	0 4	2	0 21 25	5 20	2 22	35	2 6
5	0 32 56	6 35	0 6	2	0 26 45	5 20	2 57	35	2 5
6	0 39 30	6 34	0 8	2	0 32 5	5 19	3 32	35	2 4
7	0 46 4	6 34	0 10	2	0 37 24	5 19	4 7	35	2 3
8	0 52 37	6 33	0 13	3	0 42 43	5 18	4 42	35	2 2
9	0 59 9	6 32	0 16	3	0 48 1	5 18	5 18	36	2

G r a d u s	Eccentri prosth. (G. M. S.)	Add. Diff. (M.S.)	Scrup. propo. (M. S.)	Ad. Dif (S.)	Orbis An. prosthap. (G. M. S.)	Add. Diff. (M. S.)	Excef. in Per. (M. S.)	Ad. Dif. (S.)	G r a d u s
									1
1 0	1 5 40	6 31	0 20	4	0 53 19	5 18	5 53	35	2 0
1 1	1 12 10	6 30	0 24	4	0 58 36	5 17	6 28	35	1 9
1 2	1 18 39	6 29	0 29	5	1 3 53	5 17	7 3	35	1 8
1 3	1 25 7	6 28	0 34	5	1 9 9	5 16	7 38	35	1 7
1 4	1 31 34	6 27	0 39	5	1 14 25	5 16	8 13	35	1 6
1 5	1 37 59	6 25	0 44	5	1 19 40	5 15	8 48	35	1 5
1 6	1 44 21	6 22	0 50	6	1 24 54	5 14	9 22	34	1 4
1 7	1 50 42	6 21	0 56	6	1 30 6	5 12	9 57	35	1 3
1 8	1 57 2	6 20	1 3	7	1 35 16	5 10	10 32	35	1 2
1 9	2 3 20	6 18	1 10	7	1 40 25	5 9	11 7	35	1 1
2 0	2 9 36	6 16	1 17	7	1 45 33	5 8	11 41	34	1 0
2 1	2 15 50	6 15	1 25	8	1 50 40	5 7	12 16	35	9
2 2	2 22 3	6 12	1 33	8	1 55 45	5 5	12 50	34	8
2 3	2 28 12	6 9	1 42	9	2 0 48	5 3	13 24	34	7
2	2 34 19	6 7	1 51	9	2 5 48	5 0	13 58	34	6

G r a d u s	Eccentri prosth. (G. M. S.)	Add. Diff. (M.S.)	Scrup. propo. (M. S.)	Ad. Dif (S.)	Orbis An. prosthap. (G. M. S.)	Add. Diff. (M. S.)	Excef. in Per. (M. S.)	Ad. Dif. (S.)	G r a d u s
4									
2 5	2 40 24	6 5	2 0	9	2 10 47	4 59	14 32	34	5
2 6	2 46 26	6 2	2 9	10	2 15 45	4 58	15 6	34	4
2 7	2 52 26	6 0	2 19	11	2 20 41	4 56	15 39	33	3
2 8	2 58 23	5 57	2 30	11	2 25 36	4 55	16 12	33	2
2 9	3 4 17	5 54	2 41	11	2 30 29	4 53	16 45	33	1
3 0	3 10 8	5 51	2 52	—	2 35 19	4 50	17 18	33	0

Foot of left columns: **Adde** ... **S.** | Foot of right columns: **Subtr.** ... **S.**

11 Sig. [Sign 11]

Table of the Prosthaphaereses of Saturn [Tabula prosthaphaereseon Saturni]

1 Sig. [the column heading marking the zodiac-sign argument: 1 sign, read with the degree argument in the leftmost "Gradus" column, top sense]

This page continues the double-entry canon of Saturn's prosthaphaeresis (equation). The left block carries the eccentric prosthaphaeresis [Eccentri Prosth.] under the heading **Subtrah.** (subtract), with its tabular difference [Add. Diff.] and the proportional minutes [Scrup. propo] with their difference [Ad. Dif.]. The right block carries the prosthaphaeresis of the annual orb [Orbis An. Prosthap.] under the heading **Adde** (add), with its difference [Add. Diff.] and the excess at perigee [Excess in Per.] with its difference [Ad. Dif.]. The leftmost and

rightmost "Gradus" columns give the degree argument (0–30 read downward at left; 30–0 read upward at right). The foot heading is **10 Sig.**

G r a d .	Eccentr. Prostha. (G.M.S.)	Add. Diff. (M.S.)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
0	3 10 8	5 47	2 52	—	2 35 19	4 48	17 18	—	3 0
1	3 15 55	5 44	3 4	12	2 40 7	4 46	17 51	33	2 9
2	3 21 39	5 41	3 16	12	2 44 53	4 44	18 24	33	2 8
3	3 27 20	5 38	3 28	12	2 49 37	4 42	18 57	33	2 7
4	3 32 58	5 36	3 41	13	2 54 18	4 38	19 29	32	2 6
5	3 38 34	5 33	3 54	13	2 58 56	4 36	20 1	32	2 5
6	3 44 7	5 29	4 8	14	3 3 32	4 33	20 33	32	2 4
7	3 49 36	5 24	4 22	14	3 8 5	4 30	21 5	32	2 3
8	3 55 8	5 20	4 36	14	3 12 35	4 27	21 37	32	2 2
9	4 0 20	5 16	4 51	15	3 17 2	4 24	22 8	31	2 1
10	4 5 36	5 12	5 6	15	3 21 26	4 23	22 39	31	2 0
11	4 10 48	5 9	5 21	15	3 25 49	4 21	23 10	31	1 9
12	4 15 57	5 4	5 37	16	3 30 10	4 17	23 40	30	1 8
13	4 21 1	5 0	5 53	16	3 34 27	4 14	24 11	31	1 7

G r a d .	Eccentr. Prostha. (G.M.S.)	Add. Diff. (M.S.)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
1 4	4 26 1	4 57	6 10	17	3 38 31	4 12	24 42	31	1 6
1 5	4 30 58	4 52	6 27	17	3 42 43	4 9	25 12	30	1 5
1 6	4 35 50	4 46	6 45	18	3 46 56	4 5	25 42	30	1 4
1 7	4 40 36	4 41	7 3	18	3 50 57	4 1	26 12	30	1 3
1 8	4 45 18	4 37	7 21	18	3 54 58	3 57	26 41	29	1 2
1 9	4 49 55	4 32	7 39	18	3 58 55	3 54	27 10	29	1 1
2 0	4 54 27	4 28	7 58	19	4 2 49	3 51	27 39	29	1 0
2 1	4 58 55	4 23	8 17	19	4 6 40	3 47	28 7	28	9
2 2	5 3 18	4 19	8 27	20	4 10 27	3 42	28 35	28	8
2 3	5 7 37	4 13	8 57	20	4 14 9	3 37	29 2	27	7
2 4	5 11 50	4 8	9 17	20	4 17 46	3 34	29 29	27	6
2 5	5 15 58	4 2	9 38	21	4 21 20	3 30	29 55	27	5
2 6	5 20 0	3 57	9 59	21	4 24 56	3 27	30 22	27	4
2 7	5 23 57	3 51	10 20	21	4 28 17	3 23	30 48	26	3
2 8	5 27 48	3 47	10 41	21	4 31 40	3 19	31 14	26	2
2	5 31 36	3 42	11 3	22	4 34 59	3 15	31 40	26	1

G r a d .	Eccentr. Prostha. (G.M.S.)	Add. Diff. (M.S.)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
9 3 5 35 17 0		—	11 25	22	4 38 14	—	32 6	26	0
	Adde	S.		S.	Adde	Sub.		S.	

Foot heading: **10 Sig.**

Table of the Prosthaphaereses of Saturn [Tabula prosthaphaereseon Saturni]

2 Sig. [argument: 2 signs, top sense] — foot heading **9 Sig.**

G r a d .	Eccentr. Prostha. (G.M.S.)	Add. Diff. (M.S.)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
0 5 35 17		3 35	11 25	23	4 38 14	3 10	32 6	25	3 0
1 5 38 52		3 51	11 48	23	4 41 24	3 5	32 31	25	2 9
2 5 42 23		3 25	12 11	23	4 44 29	3 1	32 56	24	2 8
3 5 45 48		3 17	12 34	24	4 47 30	2 56	33 20	24	2 7
4 5 49 5		3 12	12 58	24	4 50 26	2 52	33 44	23	2 6
5 5 52 17		3 6	13 22	24	4 53 18	2 47	34 7	22	2 5
6 5 55 23		3 0	13 46	25	4 56 5	2 43	34 29	22	2 4

G r a d .	Eccentr. Prostha. (G.M.S.)	Add. Diff. (M.S.)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
7 5 58 23		2 55	14 11	25	4 58 48	2 39	34 51	22	2 3
8 6 1 18		2 48	14 36	25	5 1 27	2 34	35 13	22	2 2
9 6 4 6		2 41	15 1	26	5 4 1	2 29	35 35	22	2 1
1 6 6 47 0		2 35	15 27	26	5 6 30	2 23	35 57	21	2 0
1 6 9 22 1		2 29	15 53	26	5 8 53	2 18	36 18	21	1 9
1 6 11 51 2		2 24	16 19	26	5 11 11	2 13	36 39	20	1 8
1 6 14 15 3		2 17	16 45	27	5 13 24	2 8	36 59	19	1 7
1 6 16 32 4		2 10	17 12	27	5 15 32	2 3	37 18	18	1 6
1 6 18 42 5		2 4	17 39	27	5 17 35	1 59	37 36	17	1 5
1 6 20 46 6		1 56	18 6	27	5 19 34	1 54	37 53	17	1 4
1 6 22 42 7		1 50	18 33	28	5 21 28	1 48	38 10	17	1 3
1 6 24 32 8		1 43	19 1	28	5 23 16	1 42	38 27	16	1 2
1 6 26 15 9		1 37	19 29	29	5 24 58	1 35	38 43	16	1 1
2 6 27 52 0		1 30	19 58	29	5 26 33	1 30	38 59	16	1 0
2 6 29 22 1		1 24	20 27	29	5 28 3	1 25	39 15	15	9
2 6 30 46		1 16	20 56	29	5 29 28	1 19	39 30	15	8

G r a d .	Eccentr. Prostha. (G.M.S.)	Add. Diff. (M.S.)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
2									
2	6 32 2	1 9	21 25	29	5 30 47	1 15	39 45	14	7
3									
2	6 33 11	1 3	21 54	30	5 32 2	1 9	39 59	13	6
4									
2	6 34 14	0 56	22 24	30	5 33 11	1 3	40 12	12	5
5									
2	6 35 10	0 48	22 54	30	5 34 14	0 57	40 24	11	4
6									
2	6 35 58	0 41	23 24	30	5 35 11	0 51	40 35	11	3
7									
2	6 36 39	0 34	23 53	30	5 36 2	0 45	40 46	10	2
8									
2	6 37 13	0 28	24 23	31	5 36 47	0 40	40 56	9	1
9									
3	6 37 41	—	24 54	—	5 37 27	—	41 5	—	0
0									
	Adde	S.		S.	Subtr.	S.		S.	

Foot heading: **9 Sig.**

Table of the Prosthaphaereses of Saturn [Tabula prosthaphaereseon Saturni]

3 Sig. [argument: 3 signs, top sense] — foot heading **8 Sig.**

On this opening the left and right blocks swap sense relative to the earlier pages: the **left** block now carries the prosthaphaeresis of the annual orb [Orbis An. Prosthap.] under **Adde** (the upper-left header reads "Adde / Orbis An. Prosthap." with its "Add. Diff." and the "Excess in Per."), and the **right**-hand outer columns carry the eccentric prosthaphaeresis [eccentr. prostha.] under **Subtrah.** Both A.S.Diff (i.e. "Add./Subtr. Diff.") columns and their proportional-minute

companions are given. I transcribe the page in its printed left-to-right column order.

G r a d .	Subtrah. eccentr. prosth. (G.M.S.)	A.S. Diff. (M.S.)	Scrup. prop. (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
0	6 37 41	0 19	24 54	—	5 37 27	0 33	41 5	—	3 0
1	6 38 0	0 13	25 25	31	5 38 0	0 27	41 14	9	2 9
2	6 38 13	0 4	25 56	31	5 38 27	0 21	41 22	8	2 8
3	6 38 17	0 1	26 27	31	5 38 48	0 15	41 29	7	2 7
4	6 38 16	0 8	26 59	32	5 39 3	0 11	41 35	6	2 6
5	6 38 8	0 16	27 30	31	5 39 12	0 9	41 42	7	2 5
6	6 37 52	0 23	28 2	32	5 39 15	0 4	41 48	6	2 4
7	6 37 29	0 32	28 34	32	5 39 11	0 8	41 53	5	2 3
8	6 36 57	0 38	29 6	32	5 39 3	0 17	41 57	4	2 2
9	6 36 19	0 46	29 38	32	5 38 46	0 23	42 0	3	2 1
10	6 35 33	0 52	30 10	32	5 38 23	0 29	42 2	2	2 0
11	6 34 41	1 0	30 42	32	5 37 54	0 35	42 4	2	1 9
12	6 33 41	1 8	31 14	32	5 37 19	0 43	42 5	1	1 8
13	6 32 33	1 15	31 46	32	5 36 36	0 50	42 4	1	1 7

G r a d .	Subtrah. eccentr. prosth. (G.M.S.)	A.S. Diff. (M.S.)	Scrup. prop. (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
1 4	6 31 18	1 22	32 19	33	5 35 46	0 57	42 2	2	1 6
1 5	6 29 56	1 30	32 51	32	5 34 49	1 5	42 0	1	1 5
1 6	6 28 26	1 37	33 23	32	5 33 44	1 8	41 58	2	1 4
1 7	6 26 49	1 44	33 56	33	5 32 36	1 12	41 56	2	1 3
1 8	6 25 5	1 52	34 28	32	5 31 24	1 17	41 53	3	1 2
1 9	6 23 13	1 59	35 1	33	5 30 7	1 22	41 49	4	1 1
2 0	6 21 14	2 6	35 33	32	5 28 45	1 30	41 44	5	1 0
2 1	6 19 8	2 13	36 6	33	5 27 15	1 38	41 38	6	9
2 2	6 16 55	2 21	36 38	32	5 25 37	1 47	40 30	8	8
2 3	6 14 34	2 28	37 10	32	5 23 50	1 57	40 21	9	7
2 4	6 12 6	2 35	37 43	33	5 21 53	2 2	40 19	10	6
2 5	6 9 31	2 41	38 15	32	5 19 51	2 7	40 0	11	5
2 6	6 6 50	2 49	38 47	32	5 17 44	2 11	39 48	12	4
2 7	6 4 1	2 58	39 19	32	5 15 33	2 16	39 35	13	3
2 8	6 1 3	3 5	39 51	32	5 13 17	2 26	39 21	14	2
2	5 57 58	3 11	40 23	32	5 10 51	2 37	39 6	15	1

G r a d .	Subtrah. eccentr. prosth. (G.M.S.)	A.S. Diff. (M.S.)	Scrup. prop. (M.S.)	Ad. Dif. (S.)	Orbis An. Prosthap. (G.M.S.)	Add. Diff. (M.S.)	Excess in Per. (M.S.)	Ad. Dif. (S.)	G r a d .
9 3 5 54 47 0	—	40 55	32	5 8 14	—	38 50	16	0	
	Adde	S. A.		S.	Subtr.	S. A.		S.A.	

Foot heading: **8 Sig.**

Table of the Prosthaphaereses of Saturn [Tabula prosthaphaereseon Saturni]

4 Sig. [argument: 4 signs, top sense] — foot heading **7 Sig.**

G r a d .	Eccent. Prosth. (G.M.S.)	Diff. (M.S .)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis an. Profth. (G.M.S.)	Diff. (M.S .)	Excess Perig. (M.S.)	Ad. Dif. (S.)	G r a d .
0 5 54 47	3 17	40 55	—	5 8 14	2 41	39 50	—	3 0	
1 5 51 30	3 24	41 26	31	5 5 33	2 45	39 35	15	2 9	
2 5 48 6	3 32	41 58	32	5 2 48	2 48	39 19	16	2 8	
3 5 44 34	3 39	42 29	31	5 0 0	2 51	39 2	17	2 7	
4 5 40 55	3 46	43 0	31	4 57 9	2 57	38 45	17	2 6	
5 5 37 9	3 52	43 31	31	4 54 12	3 7	38 27	18	2 5	
6 5 33 17	3 59	44 1	30	4 51 5	3 18	38 7	20	2 4	

G r a d .	Eccent. Prostha. (G.M.S.)	Diff. (M.S .)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis an. Profth. (G.M.S.)	Diff. (M.S .)	Excess Perig. (M.S.)	Ad. Dif. (S.)	G r a d .
7	5 29 18	4 5	44 32	31	4 47 47	3 31	37 45	22	2 3
8	5 25 13	4 12	45 2	30	4 44 16	3 35	37 20	25	2 2
9	5 21 1	4 19	45 32	30	4 40 41	3 38	36 55	25	2 1
1 0	5 16 43	4 25	46 1	29	4 37 3	3 41	36 30	25	2 0
1 1	5 12 18	4 32	46 31	30	4 33 22	3 43	36 4	26	1 9
1 2	5 7 46	4 38	47 0	29	4 29 39	3 48	35 38	26	1 8
1 3	5 3 8	4 44	47 28	28	4 25 51	3 55	35 12	26	1 7
1 4	4 58 24	4 50	47 57	29	4 21 56	4 2	34 45	27	1 6
1 5	4 53 34	4 57	48 15	28	4 17 54	4 9	34 16	29	1 5
1 6	4 48 37	5 2	48 52	27	4 13 45	4 16	33 45	31	1 4
1 7	4 43 35	5 7	49 19	27	4 9 29	4 23	33 14	31	1 3
1 8	4 38 28	5 14	49 46	27	4 5 6	4 30	32 42	32	1 2
1 9	4 33 14	5 20	50 13	26	4 0 36	4 37	32 10	32	1 1
2 0	4 27 54	5 25	50 39	25	3 55 59	4 42	31 37	33	1 0
2 1	4 22 29	5 31	51 4	25	3 51 17	4 47	31 3	34	9
2	4 16 58	5 36	51 29	25	3 46 30	4 50	30 28	35	8

G r a d .	Eccent. Prostha. (G.M.S.)	Diff. (M.S .)	Scrup. propo (M.S.)	Ad. Dif. (S.)	Orbis an. Profth. (G.M.S.)	Diff. (M.S .)	Excess Perig. (M.S.)	Ad. Dif. (S.)	G r a d .
2									
2	4 11 22	5 41	51 54	24	3 41 40	4 53	29 52	36	7
3									
2	4 5 41	5 47	52 18	24	3 36 47	4 57	29 16	36	6
4									
2	3 59 54	5 52	52 42	23	3 31 50	5 3	28 39	37	5
5									
2	3 54 2	5 57	53 5	23	3 26 47	5 8	28 1	38	4
6									
2	3 48 5	6 1	53 28	22	3 21 39	5 14	27 21	40	3
7									
2	3 42 4	6 6	53 50	21	3 16 25	5 18	26 40	41	2
8									
2	3 35 58	6 10	54 11	21	3 11 7	5 23	25 59	41	1
9									
3	3 29 48	—	54 32	—	3 6 44	—	25 17	42	0
0									
	Adde	A.		S.	Subtr.	A.		A.	

Foot heading: **7 Sig.**

Table of the Prosthaphaereses of Saturn **h**

5 Sig.

The argument runs 0–30 down the left and (mirrored) 30–0 down the right. The eccentric prosthaphaeresis [eccentri prosthap.] is to be subtracted (head: *Subtrahe*; foot: *Adde*); the annual-orb prosthaphaeresis [orbis annui prosthap.] is to be added (head: *Adde*; foot: *Subtr.*). Degrees·minutes·seconds (G·M·S) as printed.

G r a d u s	Eccentric prosthap. (G·M·S)	Diff. (G·M)	Scrup. propo. (M·S)	Ad. Dif. (S)	Orbis An. prosthap. (G·M·S)	Diff. (M·S)	Excef. Perig. (M·S)	Diff. (M·S)	G r a d u s
0	3 29 48	6 15	54 32	21	3 5 44	5 27	25 17	43	3 0
1	3 23 33	6 20	54 53	20	3 0 17	5 32	24 34	44	2 9
2	3 17 13	6 24	55 13	19	2 54 45	5 36	23 50	44	2 8
3	3 10 49	6 28	55 32	19	2 49 9	5 40	23 6	45	2 7
4	3 4 21	6 32	55 51	18	2 43 29	5 44	22 21	45	2 6
5	2 57 49	6 36	56 9	17	2 37 45	5 48	21 36	46	2 5
6	2 51 13	6 40	56 26	17	2 31 57	5 51	20 50	47	2 4
7	2 44 33	6 43	56 43	16	2 26 6	5 54	20 3	48	2 3
8	2 37 50	6 46	56 59	16	2 20 12	5 58	19 15	48	2 2
9	2 31 4	6 50	57 15	15	2 14 14	6 2	18 27	49	2 1
10	2 24 14	6 53	57 30	15	2 8 12	6 5	17 38	49	2 0
11	2 17 21	6 56	57 45	14	2 2 7	6 8	16 49	50	1 9
12	2 10 25	6 59	57 59	13	1 55 59	6 11	15 59	50	1 8
13	2 3 26	7 1	58 12	12	1 49 48	6 13	15 9	51	1 7
14	1 56 25	7 3	58 24	12	1 43 35	6 16	14 18	51	1 6

G r a d u s	Eccentric prosthap. (G·M·S)	Diff. (G·M)	Scrup. propo. (M·S)	Ad. Dif. (S)	Orbis An. prosthap. (G·M·S)	Diff. (M·S)	Excef. Perig. (M·S)	Diff. (M·S)	G r a d u s
1 5	1 49 22	7 5	58 36	11	1 37 19	6 18	13 27	52	1 5
1 6	1 42 17	7 8	58 47	10	1 31 1	6 20	12 35	52	1 4
1 7	1 35 9	7 11	58 57	9	1 24 41	6 22	11 43	53	1 3
1 8	1 27 58	7 14	59 6	9	1 18 19	6 25	10 50	53	1 2
1 9	1 20 44	7 16	59 15	8	1 11 54	6 27	9 57	53	1 1
2 0	1 13 28	7 17	59 23	7	1 5 27	6 28	9 4	54	1 0
2 1	1 6 11	7 18	59 30	6	0 58 59	6 30	8 10	54	9
2 2	0 58 53	7 19	59 36	6	0 52 29	6 31	7 16	54	8
2 3	0 51 34	7 19	59 42	5	0 45 58	6 32	6 22	54	7
2 4	0 44 15	7 20	59 47	4	0 39 26	6 33	5 28	54	6
2 5	0 36 55	7 21	59 51	4	0 32 53	6 33	4 34	54	5
2 6	0 29 34	7 23	59 55	3	0 26 20	6 34	3 40	55	4
2 7	0 22 11	7 24	59 58	3	0 19 46	6 35	2 45	55	3
2 8	0 14 47	7 24	59 59	1	0 13 11	6 35	1 50	55	2
2 9	0 7 23	7 23	60 0	0	0 6 36	6 36	0 55	55	1

G r a d u s	Eccentric prosthap. (G·M·S)	Diff. (G·M)	Scrup. propo. (M·S)	Ad. Dif. (S)	Orbis An. prosthap. (G·M·S)	Diff. (M·S)	Excef. Perig. (M·S)	Diff. (M·S)	G r a d u s
3 0	0 0 0	—	60 0	—	0 0 0	—	0 0	—	0

Foot rubrics: *Adde* | A (under the eccentric/diff columns); S. (under proportional minutes); *Subtr.* | A (under the annual-orb columns); A (under the excess column).

6 Sig. (the next sign-page begins; its table is on the following opening)

Table of the Prosthaphaereses of Jupiter ♃

0 Sig.

The argument runs 0–30 down the left and (mirrored) 30–0 down the right. The eccentric prosthaphaeresis is to be subtracted (head: *Subtrah.*; foot: *Adde*); the annual-orb prosthaphaeresis is to be added (head: *Adde*; foot: *Subtr.*). The "A. Diff." and "Add. Diff." columns here are given in minutes:seconds (M·S).

G r a d u s	Eccentr. prostha. (G·M·S)	A. Diff. (M·S)	Scru. prop. (M·S)	Ad. Dif. (S)	Orbis An. prosthap. (G·M·S)	Diff. (M·S)	Excef. Perig. (M·S)	Di ff. (S)	G r a d u s
0	0 0 0	5 28	0 0	0	0 0 0	9 22	0 0	46	3 0
1	0 5 28	5 28	0 1	1	0 9 22	9 22	0 46	46	2 9
2	0 10 56	5 28	0 2	1	0 18 44	9 22	1 32	45	2 8
3	0 16 24	5 27	0 3	1	0 28 6	9 22	2 17	45	2 7
4	0 21 51	5 27	0 4	2	0 37 28	9 21	3 2	45	2 6

G r a d u s	Eccentr. prosth. (G·M·S)	A. Diff. (M·S)	Scru. prop. (M·S)	Ad. Dif. (S)	Orbis An. prosthap. (G·M·S)	Diff. (M·S)	Excef. Perig. (M·S)	Di ff. (S)	G r a d u s
5	0 27 18	5 26	0 6	2	0 46 49	9 21	3 48	45	2 5
6	0 32 44	5 26	0 8	3	0 56 10	9 21	4 34	46	2 4
7	0 38 10	5 26	0 11	3	1 5 31	9 20	5 20	46	2 3
8	0 43 36	5 25	0 14	4	1 14 51	9 19	6 6	46	2 2
9	0 49 1	5 24	0 18	4	1 24 10	9 18	6 52	46	2 1
1 0	0 54 26	5 23	0 22	4	1 33 28	9 17	7 38	46	2 0
1 1	0 59 49	5 22	0 26	4	1 42 45	9 15	8 24	46	1 9
1 2	1 5 12	5 21	0 30	5	1 52 0	9 14	9 10	46	1 8
1 3	1 10 33	5 20	0 35	6	2 1 14	9 13	9 56	46	1 7
1 4	1 15 53	5 19	0 41	6	2 10 27	9 12	10 41	45	1 6
1 5	1 21 12	5 18	0 47	6	2 19 39	9 11	11 26	45	1 5
1 6	1 26 30	5 16	0 53	7	2 28 50	9 9	12 12	46	1 4
1 7	1 31 46	5 15	1 0	7	2 37 59	9 7	12 58	46	1 3
1 8	1 37 1	5 13	1 7	8	2 47 6	9 5	13 43	45	1 2
1 9	1 42 14	5 11	1 15	8	3 56 11 [recte 2 56	9 3	14 28	45	1 1

G r a d u s	Eccentr. prosth. (G·M·S)	A. Diff. (M·S)	Scru. prop. (M·S)	Ad. Dif. (S)	Orbis An. prosthap. (G·M·S)	Diff. (M·S)	Excef. Perig. (M·S)	Di ff. (S)	G r a d u s
					11; see anomaly a]				
2 0	1 47 25	5 10	1 23	9	3 5 14	9 1	15 13	45	1 0
2 1	1 52 35	5 8	1 32	9	3 14 15	8 59	15 58	45	9
2 2	1 57 43	5 6	1 41	9	3 23 14	8 57	16 43	45	8
2 3	2 2 49	5 3	1 50	10	3 32 11	8 55	17 28	45	7
2 4	2 7 52	5 1	2 0	10	3 41 6	8 52	18 13	45	6
2 5	2 12 53	4 59	2 10	10	3 49 58	8 50	18 58	45	5
2 6	2 17 52	4 57	2 20	11	3 58 48	8 47	19 43	45	4
2 7	2 22 49	4 55	2 31	11	4 7 35	8 44	20 28	45	3
2 8	2 27 44	4 53	2 42	12	4 16 19	8 41	21 13	45	2
2 9	2 32 37	4 52	2 54	12	4 25 0	8 38	21 58	44	1
3 0	2 37 29	—	3 6	—	4 33 38	—	22 42	—	0

Foot rubrics: *Adde* (under the eccentric column); *S.* (under the A.Diff column); *S.* (under the Ad.Dif column); *Subtr.* (under the annual-orb column); *S.* (under the excess column).

11 Sig. (the next sign-page; its table is on the following opening)

Table of the Prosthaphaereses of Jupiter ♃

1 Sig.

Argument 0–30 down the left, mirrored 30–0 down the right. Eccentric prosthaphaeresis: head *Subtrah.*, foot *Adde.* Annual-orb prosthaphaeresis: head *Adde.*, foot *Subtr.* The differences are in minutes·seconds (M·S) for the eccentric and annual-orb columns; the proportional-minutes difference and the excess difference are in seconds (S).

G r a d u s	Eccentri prosthapha. (G·M·S)	Add. Diff. (M·S)	Scrup. propo. (M·S)	Ad. Dif. (S)	Orbis An. prosthapha. (G·M·S)	Add. Diff. (M·S)	Excef. in Per. (M·S)	Ad. Dif. (S)	G r a d u s
0	2 37 29	4 48	3 6	12	4 33 38	8 35	22 42	44	30
1	2 42 17	4 45	3 18	13	4 42 13	8 33	23 26	45	29
2	2 47 2	4 42	3 31	13	4 50 46	8 30	24 11	44	28
3	2 51 44	4 38	3 44	14	4 59 16	8 28	24 55	44	27
4	2 56 22	4 36	3 58	14	5 7 44	8 25	25 39	44	26
5	3 0 58	4 33	4 12	15	5 16 9	8 21	26 23	44	25
6	3 5 31	4 31	4 27	15	5 24 30	8 16	27 7	44	24
7	3 10 2	4 28	4 42	15	5 32 46	8 11	27 51	44	23
8	3 14 30	4 25	4 57	15	5 40 57	8 6	28 35	44	22
9	3 18 55	4 21	5 12	16	5 49 3	8 1	29 19	43	21

G r a d u s	Eccentri prosth. (G·M·S)	Add. Diff. (M·S)	Scrup. propo. (M·S)	Ad. Dif. (S)	Orbis An. prosthap. (G·M·S)	Add. Diff. (M·S)	Excef. in Per. (M·S)	Ad. Dif. (S)	G r a d u s
1 0	3 23 16	4 17	5 28	17	5 57 4	7 57	30 2	43	2 0
1 1	3 27 33	4 13	5 45	16	6 5 1	7 52	30 45	43	1 9
1 2	3 31 46	4 10	6 1	17	6 12 53	7 47	31 28	43	1 8
1 3	3 35 56	4 7	6 18	18	6 20 40	7 41	32 11	43	1 7
1 4	3 40 3	4 3	6 36	18	6 28 21	7 38	32 54	42	1 6
1 5	3 44 6	4 0	6 54	18	6 35 59	7 35	33 36	42	1 5
1 6	3 48 6	3 56	7 12	19	6 43 37	7 32	34 18	42	1 4
1 7	3 52 2	3 52	7 31	19	6 51 9	7 28	35 0	42	1 3
1 8	3 55 54	3 48	7 50	19	6 58 37	7 22	35 42	42	1 2
1 9	3 59 42	3 44	8 9	19	7 5 59	7 15	36 24	41	1 1
2 0	4 3 26	3 40	8 28	20	7 13 14	7 9	37 5	41	1 0
2 1	4 7 6	3 36	8 48	21	7 20 23	7 2	37 46	41	9
2 2	4 10 42	3 31	9 9	21	7 27 25	6 58	38 27	41	8
2 3	4 14 13	3 27	9 30	21	7 34 23	6 53	39 8	41	7
2 4	4 17 40	3 22	9 51	22	7 41 16	6 48	39 49	40	6

G r a d u s	Eccentri prosth. (G·M·S)	Add. Diff. (M·S)	Scrup. propo. (M·S)	Ad. Dif. (S)	Orbis An. prosthap. (G·M·S)	Add. Diff. (M·S)	Excef. in Per. (M·S)	Ad. Dif. (S)	G r a d u s
2 5	4 21 2	3 18	10 13	22	7 48 4	6 42	40 29	40	5
2 6	4 24 20	3 14	10 35	22	7 54 46	6 35	41 9	40	4
2 7	4 27 34	3 10	10 57	22	8 1 21	6 29	41 49	40	3
2 8	4 30 44	3 6	11 19	23	8 7 50	6 23	42 29	40	2
2 9	4 33 50	3 1	11 42	23	8 14 13	6 17	43 9	39	1
3 0	4 36 51	—	12 5	—	8 20 30	—	43 48	—	0

Foot rubrics: *Adde* (under the eccentric column); *S.* (under the Ad.Dif column); *Subtr.* (under the annual-orb column); *S.* (under the excess column).

10 Sig. (the next sign-page; its table is on the following opening)

Table of the Prosthaphaereses of Jupiter ♃

2 Sig.

Argument 0–30 down the left, mirrored 30–0 down the right. Eccentric prosthaphaeresis: head *Subtrah.*, foot *Adde*. Annual-orb prosthaphaeresis: head *Adde*, foot *Subtr.* Eccentric and annual-orb differences in minutes·seconds (M·S); proportional-minutes and excess differences in seconds (S).

G r a d u s	Eccentri prosth. (G·M·S)	A. Diff. (M·S)	Scrup. propo. (M·S)	D if. (S)	Orbis An. prosthap. (G·M·S)	Add. Diff. (M·S)	Excef. in Per. (G·M·S)	Dif. (M· S)	G r a d u s
0	4 36 51	2 56	12 5	2	8 20 30	6 11	43 48	39	3

G r a d u s	Eccentri prosth. (G·M·S)	A. Diff. (M·S)	Scrup. propo. (M·S)	D if. (S)	Orbis An. prosthap. (G·M·S)	Add. Diff. (M·S)	Excef. in Per. (G·M·S)	Dif. (M· S)	G r a d u s
				3					0
1	4 39 47	2 50	12 28	2 4	8 26 43	6 3	44 27	39	2 9
2	4 42 37	2 46	12 52	2 4	8 32 44	5 56	45 6	38	2 8
3	4 45 23	2 40	13 16	2 5	8 38 40	5 49	45 44	38	2 7
4	4 48 3	2 36	13 41	2 5	8 44 29	5 43	46 22	37	2 6
5	4 50 39	2 32	14 6	2 5	8 50 12	5 35	46 59	37	2 5
6	4 53 11	2 26	14 31	2 6	8 55 47	5 28	47 36	37	2 4
7	4 55 37	2 21	14 57	2 6	9 1 15	5 21	48 13	36	2 3
8	4 57 58	2 16	15 23	2 6	9 6 36	5 13	48 49	35	2 2
9	5 0 14	2 11	15 49	2 7	9 11 49	5 5	49 24	35	2 1
10	5 2 25	2 6	16 16	2 6	9 16 54	4 56	49 59	35	2 0
11	5 4 31	1 0 [recte 2 0; see anomaly b]	16 42	2 7	9 21 50	4 47	50 34	34	1 9
12	5 6 31	1 55	17 9	2 7	9 26 37	4 40	51 8	34	1 8
13	5 8 26	1 50	17 36	2 7	9 31 17	4 33	51 42	34	1 7
1	5 10 16	1 44	18 3	2	9 35 50	4 25	52 16	34	1

G r a d u s	Eccentri prosth. (G·M·S)	A. Diff. (M·S)	Scrup. propo. (M·S)	D if. (S)	Orbis An. prosthap. (G·M·S)	Add. Diff. (M·S)	Excef. in Per. (G·M·S)	Dif. (M· S)	G r a d u s
4				8					6
1 5	5 12 0	1 38	18 31	2 8	9 40 15	4 18	52 49	33	1 5
1 6	5 13 38	1 33	18 59	2 8	9 44 33	4 8	53 22	33	1 4
1 7	5 15 11	1 27	19 27	2 9	9 48 41	3 58	53 54	32	1 3
1 8	5 16 38	1 22	19 56	2 9	9 52 39	3 47	54 26	32	1 2
1 9	5 18 0	1 16	20 25	2 9	9 56 26	3 35	54 57	32	1 1
2 0	5 19 16	1 11	20 54	2 9	10 0 1	3 28	55 27	31	1 0
2 1	5 20 27	1 5	21 23	2 9	10 3 29	3 22	55 57	30	9
2 2	5 21 32	0 59	21 52	3 0	10 6 51	3 16	56 26	29	8
2 3	5 22 31	0 53	22 22	3 0	10 10 7	3 10	56 54	28	7
2 4	5 23 24	0 48	22 52	3 0	10 13 17	2 58	57 22	28	6
2 5	5 24 12	0 42	23 22	3 0	10 16 15	2 46	57 49	27	5
2 6	5 24 54	0 36	23 52	3 0	10 19 1	2 32	58 15	26	4
2 7	5 25 30	0 30	24 22	3 1	10 21 33	2 17	58 41	26	3
2 8	5 26 0	0 25	24 53	3 1	10 23 50	2 8	59 6	25	2
2	5 26 25	0 19	25 24	3	10 25 58	2 2	59 30	24	1

G r a d u s	Eccentri prosth. (G·M·S)	A. Diff. (M·S)	Scrup. propo. (M·S)	D if. (S)	Orbis An. prosthap. (G·M·S)	Add. Diff. (M·S)	Excef. in Per. (G·M·S)	Dif. (M· S)	G r a d u s
9				1					
3 5 26 44 0	—	—	25 55	—	10 28 0	—	59 54	24	0

Foot rubrics: *Adde* (under the eccentric column); *S.* (under the proportional-minutes column); *S.* (under the Dif. column); *Subtr.* (under the annual-orb column); *S.* (under the excess column).

9 Sig. (the next sign-page; its table is on the following opening)

Liber II Theoricorum, Cap. XII (continued)

Table of the prosthaphaereses of Jupiter [Tabula Prosthaphaereseon Iovis] — concluded; then Mars

These are double-entry equation canons. Each page heads one zodiac sign (read top-down, "Adde / Subtrahe" as labelled at head) and foots the mirror sign (read bottom-up). The column groups are: argument in degrees [Gradus]; the eccentric prosthaphaeresis [Eccentri Prosthap.] with its first difference [A. Differ.]; the proportional minutes [Scrup. propo.] with its difference [Ad. Dif.]; the prosthaphaeresis of the annual orb [Orbis An. Prosthap.] with its difference [A. Diff.]; the excess at perigee [Excessus in perig.] with its difference [Diff.]; and the mirrored argument column.

3 Sig. (head) / 8 Sig. (foot) — Source: PDF p. 389 (body p. 373)

[Margin/head: column-group rubrics — "Subtrahe" over the eccentric prosthaphaeresis; "Adde" over the annual-orb prosthaphaeresis. At the foot these reverse: "Adde" under the eccentric, "Subtr." under the annual orb.]

Eccentri Prosthap. (G M S)	A. Differ. (M S)	Scrup. propo. (M S)	Ad. Dif. (S)	OrbisAn. Prosthap. (G M S)	A. Diff. (M S)	Excessus in perig. (G M S)	Diff . (M S)
5 26 44	0 11	25 55	31	10 28 0	1 26	0 59 54	23

Eccentri Prosthap. (G M S)	A. Differ. (M S)	Scrup. propo. (M S)	Ad. Dif. (S)	OrbisAn. Prosthap. (G M S)	A. Diff. (M S)	Excessus in perig. (G M S)	Diff . (M S)
5 26 55	0 3	26 26	31	10 29 56	1 42	1 0 17	22
5 26 58	0 1	26 57	31	10 31 38	1 30	1 0 39	21
5 26 59	0 0	27 28	31	10 33 8	1 19	1 1 0	20
5 26 59	0 3	27 59	32	10 34 27	1 8	1 1 20	19
5 26 56	0 16	28 31	31	10 35 35	0 58	1 1 39	18
5 26 40	0 26	29 2	32	10 36 33	0 48	1 1 57	17
5 26 14	0 36	29 34	32	10 37 21	0 36	1 2 14	16
5 25 38	0 39	30 6	32	10 37 57	0 25	1 2 30	15
5 24 59	0 43	30 38	32	10 37 22	0 11	1 2 45	15
5 24 16	0 46	31 10	32	10 38 33	0 0	1 3 0	14
5 23 30	0 50	31 42	32	10 38 33	0 12	1 3 14	13
5 22 40	0 56	32 14	32	10 38 21	0 24	1 3 27	12
5 21 44	1 4	32 46	32	10 37 57	0 36	1 3 39	10
5 20 40	1 13	33 19	32	10 37 21	0 48	1 3 49	9
5 19 27	1 20	33 51	32	10 36 33	1 1	1 3 58	9
5 18 7	1 25	34 23	32	10 35 32	1 12	1 4 7	8
5 16 42	1 30	34 55	32	10 34 20	1 25	1 4 15	7
5 15 12	1 34	35 27	32	10 32 55	1 38	1 4 22	5
5 13 38	1 39	35 59	32	10 31 17	1 50	1 4 27	2
5 11 59	1 45	36 31	32	10 29 27	2 4	1 4 29	0
5 10 14	1 53	37 3	32	10 27 23	2 16	1 4 29	1
5 8 21	2 0	37 35	32	10 25 7	2 29	1 4 28	2
5 6 21	2 6	38 7	32	10 22 38	2 42	1 4 26	3
5 4 15	2 11	38 39	32	10 19 56	2 55	1 4 23	4
5 2 4	2 16	39 10	31	10 17 1	3 9	1 4 19	5
4 59 48	2 22	39 42	32	10 13 52	3 21	1 4 14	7
4 57 26	2 28	40 13	31	10 10 31	3 35	1 4 7	9
4 54 58	2 34	40 44	31	10 6 56	3 48	1 3 58	11
4 52 24	2 39	41 15	31	10 3 8	4 3	1 3 47	12

Eccentri Prosthap. (G M S)	A. Differ. (M S)	Scrup. propo. (M S)	Ad. Dif. (S)	OrbisAn. Prosthap. (G M S)	A. Diff. (M S)	Excessus in perig. (G M S)	Diff . (M S)
4 49 45		41 46		10 59 5 [recte 9 59 5]		1 3 45	
(Adde)	S.		S.	(Subtr.)	S.		S.

4 Sig. (head) / 7 Sig. (foot) — Source: PDF p. 390 (body p. 374)

Eccentr. prosth. (G M S)	A. Diff. (M S)	Scrup. prop. (M S)	Ad. Dif. (S)	Orbis An. Prosthap. (G M S)	Add. Diff. (M S)	Excess. in perig. (G M S)	A. Diff. (M S)
4 49 45	2 44	41 46	31	9 59 5	4 15	1 3 35	13
4 47 1	2 50	42 17	31	9 54 50	4 28	1 3 22	15
4 44 11	2 57	42 48	30	9 50 22	4 42	1 3 7	17
4 41 14	3 2	43 18	30	9 45 40	4 56	1 2 50	19
4 38 12	3 7	43 48	30	9 40 44	5 9	1 2 31	20
4 35 5	3 13	44 18	30	9 35 35	5 22	1 2 11	22
4 31 52	3 18	44 48	29	9 30 13	5 36	1 1 49	23
4 28 34	3 24	45 17	30	9 14 37 [recte ~9 24 37?]	5 50	1 1 26	24
4 25 10	3 29	45 47	29	9 18 47	6 3	1 1 2	26
4 21 41	3 34	46 16	29	9 12 44	6 17	1 0 36	28
4 18 7	3 38	46 44	28	9 6 27	6 30	1 0 8	30
4 14 29	3 43	47 12	28	8 59 57	6 43	0 59 38	33
4 10 46	3 48	47 40	28	8 53 14	6 57	0 59 5	34
4 6 58	3 54	48 8	28	8 46 17	7 10	0 58 31	36
4 3 4	3 59	48 35	27	8 39 7	7 24	0 57 55	38
3 59 5	4 5	49 2	27	8 31 43	7 36	0 57 17	40
3 55 0	4 9	49 28	26	8 24 7	7 49	0 56 37	42
3 50 51	4 14	49 55	27	8 16 18	8 3	0 55 55	43
3 46 37	4 17	50 21	26	8 8 15	8 16	0 55 12	45
3 42 20	4 21	50 46	25	7 59 59	8 29	0 54 27	47

Eccentr. prostha. (G M S)	A. Diff. (M S)	Scrup. prop. (M S)	Ad. Dif. (S)	Orbis An. Prosthap. (G M S)	Add. Diff. (M S)	Excess. in perig. (G M S)	A. Diff. (M S)
3 37 59	4 26	51 11	25	7 51 30	8 40	0 53 40	49
3 33 33	4 31	51 35	24	7 42 50	8 54	0 52 51	52
3 29 2	4 36	51 59	24	7 33 56	9 6	0 51 59	54
3 24 26	4 40	52 22	23	7 24 50	9 19	0 51 5	56
3 19 46	4 44	52 45	23	7 15 31	9 30	0 50 9	57
3 15 2	4 48	53 8	23	7 6 1	9 43	0 49 12	59
3 10 14	4 52	53 30	22	6 56 18	9 54	0 48 13	1 1
3 5 22	4 55	53 52	21	6 46 24	10 6	0 47 12	1 3
3 0 27	4 58	54 13	21	6 36 18	10 17	0 46 9	1 4
2 55 29	5 2	54 34	20	6 26 1	10 28	0 45 5	1 5
2 50 27		54 54		6 15 33		0 44 0	
(Adde)	S.		S.	(Subtr.)	S.		S.

5 Sig. (head) / 6 Sig. (foot) — Source: PDF p. 391 (body p. 375)

Eccentri prostha. (G M S)	A. Diff. (M S)	Scrup. prop. (M S)	Ad. Dif. (S)	Orbis An. Prostha. (G M S)	A. Diff. (M S)	Excessus in Perig. (G M S)	A. Diff. (M S)
2 50 27	5 6	54 54	19	6 15 33	10 39	0 44 0	1 8
2 45 21	5 10	55 13	19	6 4 54	10 50	0 42 52	1 10
2 40 11	5 13	55 32	18	5 54 4	10 59	0 41 42	1 12
2 34 58	5 16	55 50	18	5 43 5	11 10	0 40 30	1 14
2 39 43	5 17	56 8	17	5 31 55	11 21	0 39 16	1 15
[recte ~2 29 43]							
2 24 26	5 19	56 25	17	5 20 34	11 30	0 38 1	1 17
2 19 7	5 24	56 42	16	5 9 4	11 38	0 37 44	1 18
2 13 43	5 29	56 58	15	4 57 26	11 47	0 16 26	1 20
						[recte ~0 36 26]	
2 8 14	5 34	57 13	15	4 45 39	11 57	0 35 6	1 21
2 2 40	5 37	57 28	14	4 33 42	12 8	0 33 45	1 23

Eccentri prosth. (G M S)	A. Diff. (M S)	Scrup. prop. (M S)	Ad. Dif. (S)	Orbis An. Prosth. (G M S)	A. Diff. (M S)	Excessus in Perig. (G M S)	A. Diff. (M S)
1 57 3	5 38	57 42	14	4 21 34	12 14	0 31 22	1 24
1 51 25	5 39	57 56	13	4 9 20	12 20	0 29 58	1 26
1 45 46	5 40	58 9	12	3 57 0	12 27	0 28 32	1 28
1 40 6	5 40	58 21	11	3 44 33	12 34	0 27 4	1 29
1 34 26	5 42	58 32	10	3 31 59	12 41	0 25 35	1 30
1 28 44	5 44	58 42	10	3 19 18	12 48	0 24 5	1 31
1 23 0	5 46	58 52	9	3 6 30	12 55	0 22 34	1 32
1 17 14	5 48	59 1	9	2 53 35	13 1	0 21 2	1 33
1 11 26	5 51	59 10	8	2 40 34	13 5	0 19 29	1 34
1 5 35	5 54	59 18	8	2 27 29	13 10	0 18 55	1 35
						[recte ~0 17 55]	
0 59 41	5 56	59 26	7	2 14 19	13 14	0 16 20	1 36
0 53 45	5 56	59 33	7	2 1 5	13 17	0 14 44	1 36
0 47 49	5 57	59 40	6	1 47 48	13 20	0 13 8	1 37
0 41 52	5 57	59 46	5	1 34 28	13 23	0 11 31	1 37
0 35 55	5 58	59 51	4	1 21 5	13 27	0 9 54	1 38
0 29 57	5 58	59 55	3	1 7 38	13 30	0 8 16	1 38
0 22 59	5 58	59 58	2	0 54 8	13 31	0 6 38	1 39
0 18 0	5 59	60 0	0	0 40 37	13 32	0 4 59	1 39
0 12 1	5 59	60 0	0	0 27 5	13 32	0 3 20	1 40
0 6 1	6 0	60 0	0	0 13 33	13 33	0 1 40	1 40
0 0 0		60 0		0 0 0		0 0 0	
(Adde)	S.		S.		S.		S.

Table of the eccentric prosthaphaeresis of Mars, with its distance from the centre [Tabula prosthaph. eccentric. Martis cum distantia à centro]

A new equation canon begins here (still within Cap. XII). Its column structure differs from the planet-prosthaphaeresis tables above: for each argument it gives

the eccentric prosthaphaeresis [Eccentr. Prostha.] with its difference [Ad. Diff.], and the planet's distance from the centre [Distantia à centro] expressed in particles [Particulae] with its difference [Ad. Dif.]. The page heads two signs side by side (0 Sig. and 1 Sig.), each footing its mirror (11 Sig. and 10 Sig.).

0 Sig. (head) / 11 Sig. (foot) — Source: PDF p. 392 (body p. 376)

G Eccentr. Prostha. (G M . S)	Ad. Diff. (M S)	Distat. à centr. (Partic.)	Ad. Dif. (S)	G .
0 0 0 0	10 1	111130	1	3 0
1 0 10 1	10 1	111129	3	2 9
2 0 20 2	10 0	111126	7	2 8
3 0 30 2	10 0	111119	11	2 7
4 0 40 2	9 59	111108	17	2 6
5 0 50 1	9 59	111091	19	2 5
6 1 0 0	9 59	111672 [recte ~110672]	20	2 4
7 1 9 59	9 59	111652	21	2 3
8 1 19 58	9 58	111631	22	2 2
9 1 29 56	9 56	111009 [recte ~110909?]	23	2 1
1 1 39 52 0	9 54	110986	23	2 0
1 1 49 46 1	9 51	110963	24	1 9
1 1 59 37 2	9 48	110939	25	1 8
1 2 9 25 3	9 46	110914	26	1 7

G Eccentr. Prosth. (G M . S)	Ad. Diff. (M S)	Distat. à centr. (Partic.)	Ad. Dif. (S)	G .
1 2 19 11 4	9 44	110888	28	1 6
1 2 28 55 5	9 42	110860	36	1 5
1 2 38 37 6	9 39	110824	43	1 4
1 2 48 16 7	9 36	110781	49	1 3
1 2 57 52 8	9 33	110732	54	1 2
1 3 7 26 9	9 30	110678	57	1 1
2 3 16 55 0	9 27	110621	58	1 0
2 3 26 22 1	9 26	110563	59	9
2 3 35 49 2	9 24	110504	59	8
2 3 45 13 3	9 22	110445	60	7
2 3 54 35 4	9 21	110385	61	6
2 4 3 56 5	9 19	110324	61	5
2 4 13 15 6	9 16	110263	62	4
2 4 22 31 7	9 14	110201	62	3
2 4 31 45 8	9 12	110139	63	2
2 4 40 57 9	9 10	110076	63	1
3 4 50 7		110013		0

G Eccentr. Prosth. (G M . S)	Ad. Diff. (M S)	Distat. à centr. (Partic.)	Ad. Dif. (S)	G .
0 (Adde)	S.		S.	

1 Sig. (head) / 10 Sig. (foot) — Source: PDF p. 392 (body p. 376)

G Eccentr. proftha. (G M . S)	Ad. Diff. (M S)	Distat. à centr. (Partic.)	Ad. Dif. (S)	G .
0 4 50 7	9 6	110013	65	3 0
1 4 59 11	9 0	109948	70	2 9
2 5 8 14	8 54	109878	74	2 8
3 5 17 5	8 48	109804	77	2 7
4 5 25 53	8 42	109727	81	2 6
5 5 34 35	8 35	109646	85	2 5
6 5 43 11	8 30	109561	89	2 4
7 5 51 41	8 24	109472	92	2 3
8 6 0 5	8 18	109380	94	2 2
9 6 8 23	8 13	109286	96	2 1
1 6 16 36 0	8 7	109190	97	2 0
1 6 24 43 1	8 1	109093	100	1 9
1 6 32 44 2	7 55	108993	103	1 8
1 6 40 39	7 48	108890	105	1

G Eccentr. proftha. (G M . S)	Ad. Diff. (M S)	Diftat. à cent. (Partic.)	Ad. Dif. (S)	G .
3				7
1 6 48 27	7 43	108785	107	1
4				6
1 6 56 10	7 37	108678	111	1
5				5
1 7 3 47	7 30	108567	114	1
6				4
1 7 11 17	7 23	108453	117	1
7				3
1 7 18 40	7 17	108336	119	1
8				2
1 7 25 57	7 11	108267 [recte ~108217]	121	1
9				1
2 7 33 18	7 5	108096	123	1
0				0
2 7 40 13	6 59	107973	125	9
1				
2 7 47 12	6 53	107848	128	8
2				
2 7 54 5	6 47	107720	130	7
3				
2 8 0 52	6 41	107590	131	6
4				
2 8 7 33	6 35	107459	133	5
5				
2 8 14 8	6 29	107326	135	4
6				
2 8 20 37	6 23	107191	137	3
7				
2 8 27 0	6 15	107054	139	2
8				
2 8 33 15	6 6	106915	140	1
9				

G Eccentr. proftha. (G M . S)	Ad. Diff. (M S)	Diftat. à cent. (Partic.)	Ad. Dif. (S)	G .
3 8 39 21 0		106775		0
	S.		S.	

Liber Secundus — Cap. XII (continued)

Table of the Eccentric Prosthaphaeresis of Mars, with the Distance from the Centre

[Tabula Prosth. Eccentr. Martis cum distantia à Centro]

The table continues with signs 2 and 3. Each half carries five columns: the argument in degrees (Gradus), the eccentric prosthaphaeresis [Eccentri Prosthap.] in degrees·minutes·seconds together with its difference column (Diff.), the distance from the centre [Distantia a Centro] in particles [Particulae] together with its difference column (Diff.), and the mirrored argument read upward from the foot. For sign 2 the prosthaphaeresis is marked *Subtrahe* (subtract) reading down / *Adde* (add) reading up, and the distance bears the heading *Ad. Dif.* For sign 3 the prosthaphaeresis is marked S. (subtract) with A. difference, and at the foot S. / S. A.

2 Sig. — 3 Sig.

The leftmost and rightmost columns are the shared argument (Gradus): the left runs 0–30 downward for sign 2; the right runs 30→0 upward for sign 3.

Arg. (2 Sig.)	Eccentr. Prosthap. G·M·S	D i ff Distance a Centro (Particulae)	D i ff Eccentr. Prosthap. (3 Sig.) G·M·S	S. Di ff.	Distance a Centro (Particulae)	Diff. (M·S)	Arg. (3 Sig.)
0	8 39 22	— 106775	— 10 30 32	—	101706	192	30
1	8 45 18	5 108633 [recte: 5 106633] 7	1 10 31 35 4 2	1 3	101514	192	29
2	8 51 5	5 106489	1 10 32 31	0	101322	193	28

Arg. (2 Sig.)	Eccentr. Prosthap. G·M·S	D i Distance a ff Centro f (Particulae .)	D i Eccentr. ff Prosthap. f (3 Sig.) . G·M·S	S. Di ff.	Distance a Centro (Particulae)	Diff. (M·S)	Arg. (3 Sig.)
			4	56			
		4	4				
		7					
3	8 56 43	5 106343	1 10 33 19	0 48	101129	194	27
		3	4				
		8	6				
4	9 2 11	5 106194	1 10 33 58	0 39	100935	194	26
		2	4				
		8	9				
5	9 7 29	5 106043	1 10 34 20	0 22	100741	195	25
		1	5				
		8	1				
6	9 12 36	5 105890	1 10 34 20	0 0	100546	195	24
		7	5				
			3				
7	9 17 35	4 105735	1 10 34 15	0 5	100351	195	23
		5	5				
		9	5				
8	9 22 26	4 105578	1 10 34 4	0 11	100156	195	22
		5	5				
		1	7				
9	9 27 10	4 105419	1 10 33 45	0 19	99961	195	21
		4	5				
		4	9				
10	9 31 45	4 105258	1 10 33 18	0 27	99766	195	20
			6				

Arg. (2 Sig.)	Eccentr. Prosthap. G·M·S	D i Distance a ff Centro f (Particulae .)	D i Eccentr. ff Prosthap. f (3 Sig.) . G·M·S	S. Di ff.	Distance a Centro (Particulae)	Diff. (M·S)	Arg. (3 Sig.)
		3 5	1				
11	9 36 12	4 105096	1 10 32 47 6	0 31	99571	195	19
		2 7	2				
12	9 40 32	4 104933	1 10 32 5 6	0 42	99376	195	18
		2 0	3				
13	9 44 44	4 104968 [?]	1 10 31 9 6	0 56	99181	195	17
		1 2	6				
14	9 48 48	4 104601	1 10 29 55 6	1 14	98986	195	16
		4	7				
15	9 52 45	3 104433	1 10 28 22 6	1 33	98791	195	15
		5 7	8				
16	9 56 33	3 104263	1 10 26 31 7	1 51	98596	195	14
		4 8	0				
17	10 0 10	3 104091	1 10 24 22 7	2 9	98901 [recte: 98401]	195	13
		3 7	2				
18	10 3 36	3 103918	1 10 21 55 7	2 27	98206	195	12
		2	3				

Arg. (2 Sig.)	Eccentr. Prosthap. G·M·S	D i Distance a ff Centro f (Particulae .)	D i Eccentr. ff Prosthap. f (3 Sig.) . G·M·S	S. Di ff.	Distance a Centro (Particulae)	Diff. (M·S)	Arg. (3 Sig.)
		6					
19	10 6 50	3 103743	1 10 19 10 7	2 45	98012	195	11
		1 4	5				
20	10 9 52	3 103567	1 10 16 6 7	3 4	97816	195	10
		2	6				
21	10 12 44	2 103389	1 10 12 55 7	3 11	97620	196	9
		5 2	8				
22	10 15 26	2 103209	1 10 9 37 8	3 18	97423	197	8
		4 3	0				
23	10 17 58	2 103027	1 10 6 11 8	3 26	97225	198	7
		3 2	2				
24	10 20 20	2 102843	1 10 2 36 8	3 35	97027	198	6
		2 2	4				
25	10 22 31	2 102657	1 9 58 52 8	3 44	96828	199	5
		1 1	6				
26	10 24 31	2 102469	1 9 55 0 8	3 52	96627	201	4
		0	8				
27	10 26 20	1 102279	1 9 51 0	4	96425	202	3

Arg. (2 Sig.)	Eccentr. Prosthaph. G·M·S	D i Distance a ff Centro (Particulae)	D i Eccentr. ff Prosthaph. (3 Sig.) G·M·S	S. Di ff.	Distance a Centro (Particulae)	Diff. (M·S)	Arg. (3 Sig.)
		4 9	9 0		0		
28	10 27 57	1 102089	1 9 46 52 9	4 8	96222	203	2
		3 7	0				
29	10 29 21	1 101898	1 9 42 35 9	4 17	96018	204	1
		2 4	1				
30	10 30 32	1 101706	1 9 38 9 9	4 26	95814	204	0
		1 1	2				
	<i>Adde</i>	S.	S S. .	S. A.		A.	

Foot signs: below sign 2 reads **9 Sig.**; below sign 3 reads **8 Sig.**

Table of the Eccentric Prosthaphaeresis of Mars, with the Distance from the Centre (continued)

[Tabula prosthaph. eccentr. Martis cum distantia à centro]

4 Sig. — 5 Sig.

For sign 4 the prosthaphaeresis is marked *Subtrah.* with *A.* difference and *Adde* at the foot; the distance bears *Ad. Dif.* For sign 5 the same pattern, *Subtrah.* / *Adde.* The left argument runs 0–30 downward for sign 4; the right argument runs 30→0 upward for sign 5.

Arg. (4 Sig.)	Eccentr. Prosthap. G·M·S	A. Di ff.	Distance a cent. (Particula e)	A d. Di f.	Eccentr. Prosthap. (5 Sig.) G·M·S	A. Di ff.	Distance a cent. (Particula e)	A. Di ff.	Arg. (5 Sig.)
0	9 38 9	—	95814	—	5 51 30	—	90837	—	30
1	9 33 32	4 37	95612	20 2	5 41 18	10 12	90714	12 3	29
2	9 28 42	4 50	95412	20 0	5 30 57	10 21	90594	12 0	28
3	9 23 38	5 4	95214	19 8	5 20 27	10 30	90478	11 6	27
4	9 18 21	5 17	95019	19 5	5 9 48	10 39	90366	11 2	26
5	9 12 49	5 32	94827	19 2	4 59 0	10 48	90258	10 8	25
6	9 7 3	5 46	94640	18 7	4 48 5	10 55	90152	10 6	24
7	9 1 5	5 58	94457	18 3	4 37 3	11 2	90050	10 2	23
8	8 54 54	6 11	94278	17 9	4 25 54	11 9	89950	10 0	22
9	8 48 32	6 22	94103	17 5	4 14 38	11 16	89854	96	21
10	8 42 0	6 32	93932	17 1	4 3 16	11 22	89763	91	20
11	8 35 15	6 45	93762	17 0	3 51 48	11 28	89677	86	19
12	8 28 18	6 57	93593	16 9	3 40 14	11 34	89595	82	18
13	8 21 9	7 9	93424	16 9	3 28 34	11 40	89517	78	17
14	8 13 47	7 22	93256	16 8	3 16 48	11 46	89443	74	16
15	8 6 12	7 35	93088	16 8	3 4 55	11 53	89374	69	15

Arg. (4 Sig.)	Eccentr. Prosthap. G·M·S	A. Di ff.	Distance a cent. (Particula e)	A d. Di f.	Eccentr. Prosthap. (5 Sig.) G·M·S	A. Di ff.	Distance a cent. (Particula e)	A. Di ff.	Arg. (5 Sig.)
16	7 58 25	7 47	92921	16 7	2 52 56	11 59	89310	64	14
17	7 50 27	7 58	92755	16 6	2 40 51	12 9	89251	59	13
18	7 42 17	8 16	92590	16 5	2 28 40	12 11	89197	54	12
19	7 33 57	8 20	92426	16 4	2 16 27	12 13	89146	51	11
20	7 25 27	8 30	92263	16 3	2 4 12	12 15	89099	47	10
21	7 16 47	8 40	92103	16 0	1 51 55	12 17	89055	44	9
22	7 7 57	8 50	91947	15 6	1 39 36	12 19	89015	40	8
23	6 58 57	9 0	91795	15 2	1 27 14	12 22	88979	36	7
24	6 49 47	9 10	91647	14 8	1 14 50	12 24	88948	31	6
25	6 40 27	9 20	91503	14 4	1 2 25	12 25	88922	26	5
26	6 30 58	9 29	91363	14 0	0 59 58 [?]	12 27	88901	21	4
27	6 21 20	9 38	91226	13 7	0 47 30	12 28	88886	15	3
28	6 11 32	9 48	91093	13 3	0 25 1 [recte: 0 35 1]	12 29	88877	9	2
29	6 1 35	9 57	90963	13 0	0 12 31	12 30	88872	5	1
30	5 51 30	10 5	90837	12 6	0 0 0	12 31	88870	2	0

Arg.	Eccentr.	A.	Distance a	A	Eccentr.	A.	Distance a	A.	Arg.
(4	Prosthap.	Di	cent.	d.	Prosthap.	Di	cent.	Di	(5
Sig.)	G·M·S	ff.	(Particula	f.	G·M·S	ff.	(Particula	ff.	Sig.)
	<i>Adde</i>	<i>S.</i>		<i>S.</i>	<i>Adde</i>	<i>S.</i>		<i>S.</i>	

Foot signs: below sign 4 reads **7 Sig.**; below sign 5 reads **6 Sig.**

CAPVT XIII.

On the method of obtaining the longitude of the three superior planets from the preceding tables.

[De modo acquirendi longitudinem trium superiorum planetarum e tabulis praecedentibus.]

The use of the foregoing theories of this book — in deriving the longitudes of the three superior planets that match the heavens for any assumed time — is handled by the worked examples scattered here and there throughout. It remains for us to explain in this chapter that same procedure, carried out by the canons of prosthaphaeresis (which certainly aim at the same target more compendiously), again by means of several examples. But first we must gather together, in a few short precepts, the prerequisites that bear on the matter.

I. The rule prescribed for all celestial appearances to be computed astronomically must be observed here too: namely, that the mean motions of longitude and of anomaly corresponding to the given time must first be drawn out from the tables of mean motions set forth earlier, and entered separately — whether the time falls between the foundation of the world and the birth of Christ, or after our Saviour was shown forth in the flesh, or in any last age of the world. On this matter the model has been given above. [Margin: *Lib. 1. cap. 4, 5, 6.*]

II. Since the anomaly in signs and degrees is the entry-value for the prosthaphaeresis table appropriate to each, the essential or eccentric prosthaphaeresis (as they call it) is extracted, together with the proportional scruples [scrupula proportionalia], which are kept — applying here and everywhere the correction for the minutes and seconds attached to the anomaly, according to what we have appended in our *Logistic Arithmetic* concerning the proportional part, taught there in general terms, and therefore not to be repeated again or in every place. This prosthaphaeresis, applied to the mean longitude of

the planet according to the mark of addition or subtraction expressed in the tables, yields the true longitude of the planet's orb reckoned from the mean equinox.

III. The said equated longitude of the planet's orb is then subtracted from the longitude of the Sun, likewise to be reckoned from the mean equinox — unless the said longitude of the planet too has been established from the true equinox, with the equation of the equinox (which is more convenient) intervening — and there remains the equated anomaly of the annual orb.

IV. With this anomaly, sought again in the prosthaphaeretic canon under the appropriate heading, one obtains the prosthaphaeresis of the annual orb and its excess [excessus]. From that excess is taken the proportional part which the proportional scruples — kept earlier and now multiplied into that same excess — produce. This part is always added to the prosthaphaeresis of the annual orb previously obtained, so that it may suit the particular position of the planet in its proper orb. Finally, this prosthaphaeresis, once added to the previously equated longitude (if the anomaly of the annual orb falls within the semicircle) or subtracted (if that anomaly has exceeded the semicircle), yields the sought true longitude of the planet from the true equinox — provided that the equinoctial inequality (discussed above) has first been applied.

These short precepts suffice for computing the longitudes of the two superior planets, Saturn and Jupiter; but not so for Mars. For since the prosthaphaeresis of the annual orb is in its case also varied by reason of the varying distance of the Sun from the Earth, therefore one must first, with the mean anomaly of this orb, investigate both the eccentric prosthaphaeresis of this planet and its distance from the centre of that same orb — which is just what the prosthaphaeresis table of Mars displays for these purposes only. Then, with the distance kept, one proceeds exactly as in the others, until the equated anomaly of the annual orb of Mars likewise emerges. After that the correction of the semidiameter of that same annual orb is made, according to the tenor of the little table prearranged for this use in the hypothesis of Mars. Hence, given the two sides enclosing the angle that arises from the rightly applied anomaly of the annual orb, the ἀνάλυσις [Greek: analysis — the solving/resolution] of a single plane triangle is sought, just as in the case of the Moon — the prosthaphaeresis of the annual orb of Mars to be left to be applied to its own equated longitude —

— that is, to be added, before the anomaly of the annual orb has completed the measure of the semicircle (i.e., 6 signs), but afterward to be subtracted. This, then,

is the sum of what we have undertaken to set forth in the individual examples, so that they may appear more fully in those examples themselves.

I. Example in Saturn. [Exemplum in Saturno.]

To make our procedure here surer and safer, we shall fit the examples to none but the times of observations actually made, so that afterward the difference can be discerned by comparison. For this reason too — from the Alphonsine [Alphonsinae] as well as the Prutenic [Prutenicae] tables, both built upon the Copernican foundation — we set down the positions of these planets for the same times, so that the necessity of the undertaken restitution may be further recognized.

[Margin: *Lib. 2. Sphaer. cap. 9. probl. 4.*] In the year after the birth of Christ 1591, March day 17, hour 7½, p.m., at Uraniborg [Uraniburgi], the star of Saturn was observed at a distance from Aldebaran [Aldeboran] of 19° 11½'. But at the same time it was 25° 54¼' from Pollux [Polluce], with a declination of 22° 23' north. From this, by the problems set forth in the doctrine of the sphere, the longitude of Saturn is found to be 22° 43' 31" ♊ [Gemini], with a latitude of 0° 56¼' south.

For the same time, these motions agree from our restituted tables:

	Sig.	Gr.	Mi.	Sec.	
Anomaly of the equinox	6	15	37	50	
There corresponds the inequality of the equinox				7 22	<i>Add.</i>
Simple longitude of the Sun	0	4	26	11	
Anomaly of the Sun	8	29	1	18	
Prosthaphaeresis of the Sun		2	2	46	
Therefore the true place of the Sun	0	6	28	57	
Simple longitude of Saturn	2	28	30	50	
Anomaly of Saturn	6	2	16	56	
There corresponds the eccentric prosthaphaeresis			16	52	<i>Add.</i>
Proportional scruples			59	59	
Therefore the eccentric longitude of Saturn	2	28	47	42	
Likewise the anomaly of the annual orb	9	7	41	15	
There corresponds the first		5	29	55	

	Sig.	Gr.	Mi.	Sec.
prosthaphaeresis of the annual orb				
Excess			39	35
Proportional part			39	34
Absolute prosthaphaeresis of the annual orb	0	6	9	29
Therefore the true longitude of Saturn from the true equinox	2	22	45	35

The comparison of the older tables is this, that they give for the same time:

	Alphonsine — Gr.	Mi.	Copernican — Gr.	Mi.
Longitude	II 25	7	II 22	23
Latitude	0	56 south	1	35 south

From this it is clear that the calculation from our restituted tables, in the longitude of Saturn, exceeds the celestial observation by only two minutes and a half; whereas the Alphonsine exceeds that same observation by 2° 24', and the Copernican falls short of it by 20'. As for whatever small difference of latitude has occurred on our side too, it will be indicated in its place for this planet, as for the following ones; for here the excessive deviation of Copernicus lies exposed to the eyes.

II. Example in Jupiter. [Exemplum in Iove.]

In the year of Christ likewise 1591, day 14 of April, at the 11th hour of the following night, Jupiter was observed by us at Uraniborg at 14° 16' ♍ [Scorpio], with a latitude of 1° 23½' N. To the same moment these motions agree from our tables:

(Continuing Example II — Jupiter, 1591 April 14, Uraniborg. The computation from our tables proceeds:)

	Sig.	Gr.	Mi.	Sec.
Anomaly of the equinox [Anomalia aequinocitii]	6	15	38	20
The corresponding equation of the equinox responds			7	23 Add.
Simple longitude of the Sun	1	2	10	42
Anomaly of the Sun	9	27	5	34

	Sig.	Gr.	Mi.	Sec.	
Prosthaphaeresis of the Sun		1	47	29	Add.
Therefore the true place of the Sun from the mean equinox	1	3	58	11	
Mean longitude of Jupiter	7	15	34	15	
Anomaly of Jupiter	1	8	11	38	
The eccentric prosthaphaeresis responds		3	15	21	Subt.
Proportional scruples			5	0	
Therefore the true longitude of Jupiter's eccentric from the mean equinox	7	12	18	54	
Likewise the anomaly of the annual orb, the longitude of the eccentric of Jupiter being subtracted from the true longitude of the Sun	5	21	39	17	
The first, or apogeal, prosthaphaeresis of the annual orb responds		1	52	18	
Excess between this and the perigeal value			13	41	
Proportional part, according to the proportional scruples			1	8	
Therefore the absolute prosthaphaeresis of the orb		1	53	26	Add.
And accordingly the longitude of Jupiter from the mean equinox	7	14	12	20	
But from the true equinox	7	14	19	43	

This therefore differs from the observation obtained from the heavens by about 3½ minutes, exceeding it by roughly that same amount. From the older tables, Jupiter's place here comes out as follows:

from the Alphonsine [tables]		from the Copernican [tables]	
		G M r i.	G M r i.
		.	.
Longitude	ℳ	1 3 ℳ	1 4
		2 2	4 0

	from the Alphonsine [tables]		from the Copernican [tables]	
Latitude,	1	5	North	1 5
North		1		8

III. Example, in Mars.

In the year of our Lord 1587, January 15, at 15½ hours, Mars — observed at [Uraniborg] with the Tyconic instruments — is reported in longitude at 4°1' $\triangle$, and in latitude 3°13' N. But when about 1 minute more is subtracted from the longitude for the parallax of longitude, his true place is found to have been observed at 4°0' $\triangle$. [Margin: See *Progymnasmata* I.2 of [Tycho Brahe], p. 56.]

To the same moment these motions agree:

	Sig.	Gr.	Mi.	Sec.	
Anomaly of the equinox	6	15	13	20	
Therefore the inequality of the same			7	14	
Simple motion of the Sun	10	4	37	27	
Anomaly of the Sun	6	29	16	18	
The corresponding prosthaphaeresis		1	1	57	Add.
Therefore the true place of the Sun from the mean equinox	10	5	39	24	
Mean longitude of Mars	5	4	41	22	
Anomaly of the eccentric of Mars	0	6	24	20	
The eccentric prosthaphaeresis responds		1	4	3	Subt.
And the side, the distance from the center of the eccentric		1110680			
Therefore the true longitude of the eccentric	5	3	37	19	
Likewise the anomaly of the annual orb	5	2	2	5	
Semidiameter of the orb of Mars, at the perigees of the Sun and of Mars		630275			
The addition to the aggregate, from consideration of both Mars and the Sun, from the apogees of the eccentric		27263			

	Sig.	Gr.	Mi.	Sec.
orbs				
Therefore the corrected semidiameter of the annual orb		657538		

Since it is fitting here to work out the prosthaphaeresis of the annual orb by triangular calculation, let the annual orb of Mars alone be drawn from the hypothesis of Mars, together with an index of the planet's elongation from the perigee of the orb. (And in whatever form this is done, it will have the same value and the same demonstration.) Following Copernicus here, let us describe, about the center A, the annual orb of Mars D C E.

Diagram. A circle (the annual orb of Mars), with center marked A near the lower middle and labeled points around the circumference. The construction shows the triangle of distances used to derive the prosthaphaeresis of the annual orb from the planet's elongation.

- A (center of the circle): the center A
- B (upper left, outside/on the circle): vertex B, where the angle A B E is formed
- D (left, on the circle): point D
- E (right of D, on the circle, marked with the elongation index): point E (where the Earth is placed)
- C (bottom, on the circle): point C

Now, because the anomaly of this orb was found above to be $5s2^{\circ}2'5''$ — which is measured by the arc C E — its complement to the semicircle, E D, is therefore $27^{\circ}57'55''$. This measures the angle E A D, with the Earth situated at E. The sides enclosing the given angle are also given: namely A B, the distance of Mars from the center of the eccentric, 1110680 (since one cipher is added here, so that it can be compared with the other value, which has regard to the radius 1000000); and A E, the corrected semidiameter of the annual orb, 657538. Therefore in this example the angle A B E is found to be $30^{\circ}11'$, which measures the whole prosthaphaeresis of the annual orb to be added to the true longitude of the eccentric. This done, the true longitude of Mars from the mean equinox emerges as $6s3^{\circ}48'59''$. And with the inequality of the equinoxes ($7'14''$) supervening, the true longitude of Mars sought by calculation is found at $3^{\circ}56'13''$ $\sphericalangle$, differing from the observed value within the 4 scruples obtained — a difference that is entirely removed once the truer least semidiameter of the annual orb, 631000, is posited, as was noted above. [Margin: See the end of the preceding chapter.]

The older tables, however, have:

Alphonsine		Copernican			
		Gr.	Mi.		Gr. Mi.
Longitude	$\varnothing$	1	32	$\varnothing$	5 5
Latitude	N.	2	58	N.	3 1

CHAPTER XIV.

On the latitudes of the three superior planets — Saturn, Jupiter, and Mars — verified for our own age, and likewise accommodated to the threefold hypothesis.

Now that the motions in longitude of the three superior planets have been verified in the preceding treatise, with the three hypotheses of the three most distinguished practitioners set forth, and finally with everything refined for the present age, it remains for us to pursue, with equal restitution, the doctrine of their latitudes — that is, of their deviations on either side from the ecliptic. For only then are the true places of the planets said to be known, when their latitudes are established together with their longitudes, as Copernicus rightly advises. [Margin: *Preface, Revolutions* book 6.]

The principle of the latitudes in these planets, and their various appearances, follow the hypotheses of the motions in longitude; and this powerfully confirms the truth or certainty of those hypotheses, drawn from the various elevations of the planets above the Earth in their revolutions — being bound to those hypotheses, as it were, by a marvelous connection. For these diversities of latitude do not arise upon the planets' own poles in their sphere, in the manner we displayed for the luminaries; rather they follow the planet's revolution in the eccentric, and above all in the annual orb — that is, the distances of that same orb from the Earth — and from this they vary the angle of latitude to the eye, as the optical principle likewise suggests.

Let the simple inclination, or latitude, of any of these planets lie within the spherical angle B A C, with the quadrant of the ecliptic A B standing fixed, where A C is the path of the planet from the intersection A of the ecliptic and the eccentric orb.

Then, about the center D, let the annual orb of the planet E F be described (in the following figure), after Copernicus — to which Ptolemy and Tycho can here easily be reduced,

as we shall do below. Now let the planet be placed at C, here at its greatest northern latitude and in its acronychal position, while the Earth is at E. Then, with the same planet remaining at C, but the Earth revolved from E to F — so that the planet travels in conjunction with the Sun at D — it is evident, when lines are drawn on both sides (first from the Earth at E to C and to B; then to the same termini from the Earth when it is at F), that by however much the angle C E B exceeds the angle C F B, by just so much the variation of latitude likewise occurs.

Diagram. An arc sweeping from upper left to right (the planet's apparent path), with labeled points and a circle below representing the annual orb of the Earth. Lines are drawn from the Earth's two positions (E and F) up to the planet's positions to display the difference of angles that produces the latitude variation.

- C (top): the planet at C, at its greatest northern latitude
- B (upper middle, on the path): point B
- A (right, end of the arc): node/intersection A
- E (lower middle, on the circle): the Earth's first position, at E
- D (bottom of the circle): point D (Sun's position)
- F (bottom, on the circle): the Earth's second position, at F

The triangular proportion of the planet from the node A (where indeed no latitude is ever found) is preserved everywhere — together with the variation of the planet's distance from the center of its own orb, which, however, is almost interchanged with the uniform change of apogee and the limits of latitude, once the greatest latitude of each — both northern and southern — has been established.

From this it is sufficiently clear that, for discerning the latitude in these three planets, the following are required: first, the point of intersection of the planet's path with the ecliptic; then the quantity of the mean inclination; afterward the planet's distance from the Earth — which, by reason of the annual orb, chiefly alters the latitude (if the other seats of the apsides and the limits of the latitudes were to follow one another in turn).

But indeed, to ascertain first of all the simple inclination of the orbs of these planets from the ecliptic — which is here determined by the angle C D B — we adduce the following from their proper places, as things granted [Greek: διδόμενα (didomena, "things given")]: namely the intersections of the individual orbs with the zodiac, and the greatest latitude of all, both northern and southern, for our age — drawn indeed from acronychal observations once made at [Uraniborg], as they appear in the following synopsis.

PLANE TARV M	Intersection (Intersectio) — Borea	Aust rina			Latitude (Latitudo) — Borea	Aust rina		
	Gr.	M	Gr.	M	Gr.	M	Gr.	M
		i		i		i		i
		.		.		.		.
SATVR NI	20	3	♄ 20	3	♅ 2	4	2	4
		0		0		8		9
IOVIS	7	0	♄ 7	0	♅ 1	3	1	4
						8		0
MARTI S	18	0	♄ 18	0	♅ 4	3	6	4
						3		2

Moreover, concerning the limits of the greatest latitudes — whether northern or southern — there are given, from the hypothesis of each planet's longitude: first, the distance of the planet from the center of the eccentric orb; then the corrected semidiameter of the annual orb, which in the case of Mars alone — because it is variable — must here too be corrected by us first, by the methods prescribed above. These [values], concerning the southern limits as exhibiting the larger latitudes, compiled from the foregoing, are as follows (the finer precision, which does not belong here, being set aside):

PLANETA RVM	Southern Limit (Limes Austr.)	Side of the eccentric (Latus eccen.)	Semidiameter of the orb (Lat. sem. orbis)
SATVRNI	20	Υ 97593	10426
IOVIS	7	Υ 95230	19349
MARTIS emend.	18	≈ 89090	65095

With these things thus acquired and assumed, for demonstrating the said mean inclination of each upon the Copernican hypothesis (to which the suppositions of Ptolemy and Tycho also entirely conform here, as we have said),

we shall reason in the following manner. About the center E let the annual orb of the three superiors, F, be described. Then let the line A B be drawn in the same plane of the ecliptic, but another, K D, to the plane of the eccentric of these planets, intersecting A B at the center E; and let the northern limit of latitude be K, the southern D.

Diagram. A horizontal line (the ecliptic plane A B) crossed at its midpoint E by an inclined line (K D, the plane of the eccentric), with a small circle (the annual orb) at the center; lines are drawn from the ends of the orb's diameter up toward the northern limit and down toward the southern limit to mark the maximum northern and southern latitudes.

- K (upper right): the northern limit of latitude
- A (right end of the horizontal line): point A
- B (left end of the horizontal line): point B
- E (center, intersection of the two lines): the center E
- C (just left of center, on the orb): point C (one end of the annual-orb diameter)
- F (center/right of orb): point F (the other end of the orb's diameter)
- D (lower left): the southern limit of latitude

Furthermore, with lines drawn from the extremities of the diameter of the annual orb — to the north, F K and C K; but to the south, C D and F D — it is evident that the angle K F A designates the greatest northern latitude in the planet's acronychal position, while B C D designates the southern. These latitudes are mobile, according as F and C are changed around the circuit of the annual orb, so that the least northern latitude is seen near the angle K C A, the planet traveling in conjunction with the Sun; but the least southern latitude, at the same time, lies within the angle B F D.

The angle B E D, however — to which K E A is equal — indicates the inclination we are seeking, perpetually conformable to itself, of the ecliptic and of the planet's path. To investigate this inclination in each [planet], the things given [Greek: διδόμενα (didomena, "the givens")] in the triangle C D E — in which the angle of inclination C E D will be sought — are these: E D, the distance of the planet from the center of its orb at the southern limit of latitude; C E, the corrected radius of the annual orb; and besides, the obtuse angle E C D, which is composed of the complement of the greatest southern latitude given in each case to the semicircle.

Having carried out the resolution of the said triangle in the manner taught in our trigonometry, we found the angle D E C — that is, that simple inclination which we sought — to be, in Saturn, $2^{\circ}31'$; in Jupiter, $1^{\circ}19\frac{3}{4}'$; in Mars, $1^{\circ}48\frac{2}{3}'$. [Margin: The angles of inclination of the orbs of the superior planets from the ecliptic.] And although, when set up by way of the northern limit of the superiors, it stands well enough in the rest, in Mars nevertheless — because the inclination there is

elicited at $1^{\circ}50\frac{1}{2}'$ — when the investigation is made on this side in the triangle F E K, the side E K being granted as 110910, and E F (according to our correction, in that place) 895, together with the complement of the angle E F K, $4^{\circ}33'$, as the greatest northern latitude observed was estimated: I suspected that the southern latitude, which we earlier assumed by observation, must come out larger, on account of refractions not sufficiently eliminated here.

Therefore, supposing the perpetual fixed inclination in Mars to be $1^{\circ}50'$ — which is also the finding of the ancients — I found from this the greatest southern latitude to be $6^{\circ}47'$, but the northern $4^{\circ}31'$, just as all these things will appear below by the canon.

And hence, since we find the angle of greatest inclination in the three planets to be fixed — contrary to the opinions of the ancients — we are at last able, by triangular reasoning and induction, to hunt out the latitude of a planet for any of its positions, given of course both its distance from the node in the eccentric, its removal from the center in the same, and the semidiameter of the annual orb.

Our example will be the investigation of the latitude in Mars in the year 1593, on the 10th day of August, and likewise on the 24th day of the same month, when — contrary to the estimate according to the canon of the ancients — its latitude in the sky had decreased, as the planet itself drew near to its acronychal position.

The givens [Greek: *διδόμενα* (*didomena*, "the givens")] for the 10th day of August, at $13\frac{3}{4}$ hours of the aforesaid year, are: the eccentric longitude of Mars from the true equinox, $2^{\circ}42'$ ♄; the place of the Sun, $27^{\circ}38'$ ♌; the anomaly of the Sun, 52° ; the anomaly of the eccentric of Mars, $3s3^{\circ}20'$; the distance of Mars from the center of the eccentric, 88900; the semidiameter of the annual orb, 64928; the anomaly of the annual orb, $5s24^{\circ}56'$; the distance of Mars from the northern node, in the order of preceding signs, $75^{\circ}18'$. With these thus granted, let A — in the following hypothesis of the latitude of Mars — be the center of the Earth's annual orb after Copernicus, upon which the same annual

the annual orb FE, and let the Earth be at F, near its acronychal position at E. Let the lines AD be drawn down to the ecliptic, and AC to the position of the planet on the eccentric at C. Then, the Earth having been placed in the annual orb at F (as I said), draw AF — the semidiameter of the orb — and FD, the distance to the ecliptic, and FC to the planet. It is certain that the latitude of Mars at this time is determined by the angle CFD. But in order to have what is needed within this same triangle CFD, the following are the *διδόμενα* [Greek: *didomena*, "the givens / data"].

First, for the inclination of the latitude of Mars, let two arcs be described on the sphere: BD representing the part of the ecliptic, and BC the part of the planet's orbit, both reckoned from their intersection at B, which inclines toward the preceding signs [in antecendentia signorum]. Now since the angle CBD of the simple inclination was found earlier for Mars to be $1^{\circ} 50'$, while the side BC — the distance of Mars from the northern node on the eccentric — is $75^{\circ} 18'$, and moreover the angle CDB is right (since it stands at the ecliptic in D), CD is therefore given as 30945 (of the kind in which the radius of the eccentric is 1000000), and its arc is $1^{\circ} 46' 23''$.

But since DC is here scaled down [restringitur] to the proportion of CA, 889000, this same line DC is found to be 27510, and likewise DA 888573 — these being the givens, that is, in the right-angled plane triangle CDA, with all its angles together with the side AC. The angle DAC is indeed equal to the arc DC, $1^{\circ} 46' 23''$, and the other angle, the one at D, is always right at the ecliptic.

Diagram. A latitude-hypothesis figure for Mars after Copernicus. A large circle (the annual orb of the Earth) has the point A on its right-hand side (the center of the Earth's annual orb). To the upper left, outside the circle, the cluster of points D, F, E lies near the circle's left edge, with C below and to the left of them, and B at the lower-left terminus of a curved arc descending from C. Straight lines connect A to D, F, E, and C, and the curved arcs BC and BD sweep down to B.

- A (right, on the orb): center of the Earth's annual orb (after Copernicus)
- F (upper left, on the orb): position of the Earth near acronychal point
- E (just right of F): the acronychal position
- D (left of F): foot of the perpendicular on the ecliptic
- C (lower left): position of Mars on the eccentric
- B (lower-left terminus): the intersection (node) of the planet's orbit with the ecliptic

Now in the triangle FAD, from the two given sides — AD, just found, and AF, the corrected semidiameter of the annual orb, 649280 — together with the contained angle at A (which is the complement of the anomaly of the annual orb to a semicircle, namely $5^{\circ} 4'$), FD is yielded as 248517, in the same proportion in which AC was given earlier. Finally, in the triangle CFD, from the givens FD and DC, besides the right angle at D, the angle we sought, CFD, comes to light as $6^{\circ} 19' 0''$. This determines the southern latitude of Mars itself, differing from the observed value only in the smallest amounts.

Now we shall investigate the latitude that Mars yields by computation for 25 August, hour 17½, of that same year 1593 — that is, when it stood at its acronychal position, or was opposite the true place of the Sun. For since from 10 August to the 24th the latitude of Mars decreased by a quarter of a degree or more — as our observations made at that time at Hven [Huena, Uraniborg] confirm — it is likely that over the span of nearly two days, up to the opposition of the Sun's true place, it had continued to decrease still further in the heavens. The reason we reckon the acronychal latitude here is the brevity [compendium] of the ψηφοφορίας [Greek: *psēphophorias*, "of the calculation / reckoning"] at that point.

For given the inclination DBC in the adjoining diagram, and also the right sine CD, together with the side AC, there is first given the true measure of DC in proportion to AC. Then, in the right-angled triangle CDA — which always has its right angle at D, at the ecliptic — there is given the side AD, from which, once AE has been subtracted (AE being the corrected semidiameter of Mars), ED remains. And so in the right-angled triangle CDE, since the two sides CD and ED about the right angle are thus granted, the angle of the latitude as seen from the Earth comes out, namely DEC. For its investigation the διδόμενα and ζητούμενα [Greek: *zētoumena*, "the things sought"] follow in this order. For the given time it is found that:

Diagram. A second latitude-hypothesis figure for Mars. A large circle (the annual orb) has point A on its right side. At upper left, points D and E lie close together near the circle's edge, with F just below E and C to the left of D. A curved arc descends from C to B at the lower left. Lines connect A to D, E, F, and C.

- A (right, on the orb): center of the Earth's annual orb
- D (upper left): foot of the perpendicular on the ecliptic
- E (right of D): the corrected position (after subtraction of AE)
- F (below E): the Earth's position in the annual orb
- C (left): position of Mars on the eccentric
- B (lower-left terminus): the node, intersection of orbit and ecliptic

Longitude of Mars on the eccentric from the true equinox: 11 signs [fig.] 12° 15'

Anomaly { of the Sun: sign 1, 6° 41' corrected { of Mars: sign 3, 13° 20'

To these corresponds the corrected semidiameter of the annual orb EA, 646980, and likewise AC, the distance of Mars from the center of the eccentric, 891310. Now since the intersection B — that is, the northern node — was found at 18° 0' of

Taurus [γ], the eccentric place of Mars (C) is therefore distant from it, toward the preceding signs, by $65^\circ 45'$. Hence, in the given spherical triangle CDB — with BC $65^\circ 45'$, the angle DBC $1^\circ 50'$ of the maximum inclination, and besides the angle CDB right — the arc CD of the inclination is found at this place to be $1^\circ 40' 18''$, whose sine [S R — *sinus rectus*] is 29174, of the kind in which the radius AC is taken as 1000000.

And since CAD, by hypothesis, is equal to CBD, therefore, of the kind in which AC is 891310, CD will be 26003, and DA, in the same parts, 890926. Then, the semidiameter of the annual orb AE having been subtracted from AD, ED remains as 243946. Hence, in the right-angled triangle EDC, from the givens CD 26003 and ED 243946 about the right angle at D, there finally comes out the sought angle LEC [recte: DEC], $6^\circ 5'$, very nearly. Or, far more compendiously, namely in the single triangle CAE: for from the two sides given there about the angle of inclination at A, the angle at E comes to light, whose complement to a semicircle is DEC, $6^\circ 5'$. And this angle exhibits the very latitude of Mars as seen at the appointed time of its opposition with the Sun.

The latitude of this same Mars had therefore decreased over 15 days, or somewhat more, by $14'$ — perhaps three scruples [seconds] less than the observation demanded. These discrepancies, as I see, can easily be remedied by taking one degree at least away from the elongation of the northern intersection [node], so that it falls at 17° of Taurus [γ] and the southern at 17° of Scorpio [$\mathfrak{M}$] — which also agrees more correctly with other observations in this planet.

And so you see a scruple removed for our Tycho [Brahe], who had been stuck on the diminution of Mars's latitude as it approached nearer its acronychal position in the year 1593. I recall that his very words in the book of observations assigned to that same year were these — as they are written in his own hand, and also somewhere repeated by Johannes Kepler [Iohanne Keplero]:

[Margin: In fine Com. de motibus Martis. — "At the end of the *Commentaries on the motions of Mars*."]

It is worthy of consideration that Mars, around 10 August, had its greatest southern latitude, and afterward decreased, so that on the 24th, at opposition, it was rendered nearer the ecliptic by about a quarter of a degree — which nevertheless the canons, even with the correction of the maximum latitude placed at 18 degrees of Aquarius, by no means exhibit, however the maximum latitude be assumed there; the cause of which matter comes to be diligently inquired into.

Now, as for what I promised concerning the uniform representation of latitudes upon the hypotheses of Ptolemy and of Tycho in these three superior planets, I shall show it by a brief recollection of the demonstrations.

For this purpose we shall bring back the second diagram of this chapter, in which — the foundation of the latitudes to be demonstrated here resting upon Copernicus — we derived the angle of maximum inclination in each from observations.

Diagram. The latitude-hypothesis figure for Mars, repeated. A large circle (the annual orb) with point A at its right. At upper left, D and E sit near the circle's edge, F just below E, C to the left of D, and a curved arc descends to B at lower left. Lines join A to D, E, F, and C.

- A (right, on the orb): center of the Earth's annual orb
- D (upper left): foot of perpendicular on the ecliptic
- E (right of D): corrected position
- F (below E): Earth's position
- C (left): position of Mars on the eccentric
- B (lower-left terminus): the node

Moreover, since we showed earlier that Tycho's hypotyposis [scheme] is a mere inversion of Copernicus, here too — the Earth being placed at the middle of the universe at E, and the Sun revolved through the annual orb CF, in such a way that the planet is at C in its acronychal position and at its maximum northern latitude, but likewise toward F in opposition and at its maximum southern latitude — the result is that for Tycho as well, all the things that serve to account for these latitude phenomena agree with Copernicus, while the Sun, at...

its own place, has perpetually held those opposite parts which, in Copernicus's otherwise admirable speculation, the Earth would have occupied — just as the theories of the longitudes proved the identities of their own appearances as well in this inversion.

This, which we have taught in a general way for the reduction of Copernicus to Tycho, may perhaps be a little harder to grasp in the case of Mars than in the other superior planets, Saturn and Jupiter, which encircle the orb of the Sun. Therefore, for the benefit of beginners in astronomy, we shall illustrate the Tychonic supposition by one last example, in the very same data that were transacted there [ibidem] over the Copernican supposition for the unfolding of

the latitude for 10 August of the year 1593 — using in both cases, for the sake of clarity, very nearly the same notation of the demonstrations.

About the point E of the Earth — which dwells perpetually in the same place — let the annual orb of the Sun KFA be described, and let the diameter KA be drawn. Then, with the Sun located at F and removed from A, the acronychal position, by the arc AF — which in the earlier Copernican supposition was FE — let the twin arc BC and BD be drawn on the spherical surface, of which the former (BC) is the part of the eccentric orb of Mars, and the latter (BD) corresponds to the ecliptic, each existing from the common intersection B. Finally, with the planet placed at C, let CD be connected; then let each be joined with the Sun at F. Now from F let the line FD be drawn to the ecliptic at D, so that — always outside the opposition of the planet with the Sun — it will be so accommodated as to be made parallel to the diameter AK. Lastly, let the Earth E also be linked, with the Sun at F by the line EF, and then — with the planet at C and the ecliptic at D — by the emissary lines EC and ED.

Diagram. The Tychonic latitude figure for Mars. A large circle (the annual orb of the Sun about the Earth at E). E is at the upper center of the circle, A at the lower right of the circle, K at the upper left, and F at the lower right near A. To the left, outside the circle, D sits above C, and a curved arc descends from C to B at the lower left. Lines radiate from E and F to C, D, A, and K.

- E (upper center, on the orb): the Earth, fixed at the center of the universe
- A (lower right, on the orb): the acronychal point
- K (upper left, on the orb): opposite end of the diameter KA
- F (lower right, near A, on the orb): position of the Sun
- D (upper left, outside): foot of perpendicular on the ecliptic
- C (left, below D): position of Mars on the eccentric
- B (lower-left terminus): the node, common intersection of orbit and ecliptic

These things being constructed beforehand, since it is manifest that the latitude to be acquired in this example resides in the angle DEC, accordingly: when in the spherical triangle CBD both the right sine CD and the corresponding arc are given, DFC will, by hypothesis, be equal to that same arc CD. But since DC is defined on the sphere, where the radius of the sphere is 1000000, the said sine is therefore restricted to a definite proportion with FC as radius — which line is given from the center of Mars's own orbit (the eccentric). In that same proportion FD is also known, since in the triangle CFD the angle at D — namely at the ecliptic — is perpetually right. Furthermore, in the triangle DFE, since BF [recte: EF — the

semidiameter of the annual orb] is established, namely the corrected semidiameter of the annual orb in Mars, and likewise ED with the contained angle EFD — which, because AE and FD are parallel, is equal to AEF, the distance of the Sun from the opposition of the planet — ED is therefore given here from these. Finally, in the right-angled triangle CDE, from the givens CD and ED, with the angle at D right (as we said), the angle DEC is found, showing the latitude of Mars visible from the Earth at E. And thus neither the demonstration nor the practice in this Tychonic supposition deviates in any way from the Copernican, even in tracking down the latitudes in these highest planets at any time whatever.

Nor, even according to Ptolemy — where an epicycle is supposed in place of the annual orb — does any great difficulty in demonstrating the latitudes present itself in these three superior planets, since the said epicycle has perpetually retained an inclination to the ecliptic that is uniform — which indeed, contrary to Ptolemy's own mind, agrees more correctly with the phenomena in these cases, as we have also intimated before. The corrected Ptolemaic supposition is as follows.

Let there be (in the following figure) in the plane the diameter of the ecliptic LAM, cut by the diameter of the eccentric orbit of one of these three superior planets, BAC, at the center of the world, that is, the Earth A. With the centers B and C placed, let the annual epicycles HF be described toward the...

Diagram. The corrected Ptolemaic latitude figure for the superior planets, a wide horizontal construction. On the left is a circle (the northern epicycle) and on the right a circle (the southern epicycle), joined by the long horizontal diameter of the ecliptic LAM passing through the central point A. Within the left circle the points K, C, G are across the top and M, P, O across the bottom; D lies above A and E below A near the center; on the right circle Q, N, L run across the top and F, B, H across the bottom. Lines radiate from A to the epicycle points; perpendiculars drop to the ecliptic diameter.

- A (center): the Earth, center of the world
- L (right end), M (left end): the two ends of the ecliptic diameter LAM
- B, C (on diameter BAC): centers of the eccentric orbit / epicycles
- K, C, G (top of left circle); M, P, O (bottom of left circle): epicycle points (northern)
- D (above A near center), E (below A near center): points on the construction

- Q, N, L (top of right circle); F, B, H (bottom of right circle): epicycle points (southern)

northern maximum latitude, but toward the southern, KG — whose planes, because they are perpetually conformable to the plane of the ecliptic, accordingly: first the diameters of the epicycles HF and KG having been drawn parallel to the diameter of the ecliptic LM, then the point A connected with F by the line AF, and likewise with H by AH; hence in the same manner, in the south, with AG and AK. From all of these — just as also from B and C — let perpendiculars fall onto the diameter of the ecliptic: toward the north HL, BN, FQ; but toward the south KM, CP, GO. It is manifest that the same latitude phenomena arise here too as were derived above from the Copernican and Tychonic suppositions.

For when, by the law of revolution, the planet has been at F or G, and has held some one part of the epicycle anywhere with respect to the ecliptic — which, as was shown above, happens at the acronychal position — the maximum latitude lies in the angle QAF to the north, but in OAG to the south. These indeed vary and take their decrease according as the sides AB and AH increase toward the north, and likewise AC and AK toward the south — the distance of the planet from the node on the eccentric being always preserved, and its arc toward the ecliptic equal to some one of these angles of inclination NAB and PAC, established by hypothesis (as in the cases above); but afterward being reduced proportionally, in its right sine, to the remaining distances of the planet on the eccentric, AB or AC. To which proportion AN and AP also conform themselves, just as all these things were declared above by examples.

Moreover, the angle of maximum inclination likewise comes into the open in the same way as above, given the maximum latitude, or some acronychal one not too far removed from the maximum, whether northern or southern. For when there [in the Copernican figure] the parallels LA and MA have been drawn from F to E, but here from G to D, it will be fitting that the northern acronychal latitude be defined by the angle AFE, and the southern by AGD — there, because the two sides AB and BF are given; here, AC and HC, together with the angles at F and G, given by the very latitudes seen. For thus the angle of inclination of the planet's orbit to the north, ABF, is made manifest, to which BAN is equal; but to the south ACG, to which CAP is likewise par. This angle must be equal on both sides, so long as the plane of the ecliptic LM stands apart from the plane of the planet's orbit BC by a single, perpetual section A — which the observations prove.

This agreement, through the restored Ptolemaic supposition, it was also necessary to demonstrate here, so that the uniformity of the tables concerning the latitudes of these three planets might likewise be made known from the three [ternis] hypotheses of those same tables.

CAPVT XV.

On deriving the limits of the latitude of the three superior planets for every age of the world; where the canons both of the mean motions of the latitudes and of the prosthaphaereses [prosthaphaereseon] — differing indeed from the ancients — are set out, with their construction and use.

Thus far we have set out the latitudes in the three superior planets, congruent with the heaven itself down to our own age. Now it remains to be explained how...

The nodes — that is, the points of intersection where the eccentric orbs of these planets cut the ecliptic — move, in the order of the signs (contrary to the nature of this motion about the Moon), with an extremely slow motion. Yet their advances are certainly not such as [Ptolemy] and [Copernicus] supposed, with the apogees of these same planets remaining perpetually equidistant; rather, they are far from equal. That [Copernicus], following Ptolemy and the ancients across so great an interval of time, did not notice this, will astonish only someone who fails to understand that in this matter [Copernicus] was content almost entirely with one thing: that, having undertaken his new inversion of the world-system — in which he so greatly applauded himself — the phenomena received from Ptolemy could be allowed to be set out, while the heavens, as regards the doctrine of latitudes (in which the matter is plain enough in his own work), were meanwhile left unconsulted. As for Copernicus's predecessors — namely [Georg Peurbach], [Regiomontanus], and others — why should we not say the same of them? For they never raised their eyes from Ptolemy's footsteps up into the heavens themselves so far as to detect this great discrepancy by proper means, or at length to correct it.

Moreover, not only the nodes but also the greatest departures from the ecliptic, which occur around the quadrants, turned out to be in many respects unlike both the heavens and one another. This is because these were shaped by the aforesaid practitioners — Ptolemy and Copernicus — chiefly to that measure of the epicycle, or annual orb, which each of them had drawn from his own observations (assuming they were observations). But we are not here disputing the reliability of the observations; why, then, may we not still complain of the

means employed, and inquire into them? Certainly the places of the fixed stars, from which Ptolemy had derived his own measurements, were here too least consonant with the constitution of the heavens — some differing from the truth more, others less. [Copernicus] had achieved here only this much: that he established the nodes by an advance equal to that of the apogees (an advance certainly far too rapid and erroneous), and that he constructed the latitudes themselves upon the angles of inclination of Ptolemy — angles too carelessly retained — using his own hypotheses of longitude. Could it not be from this that the motions of latitude observed in these planets are so unequal to those which are today computed from the Reinhold tables [Erasmus Reinhold] and the Copernican foundations? But since scarcely any remedy can be applied to the Ptolemaic observations in such a way as to draw out the precise truth here — nor is there, in the ancients' notes (by which they signified to us the conjunctions of the wandering stars with the fixed) that mark of certainty concerning the latitudes that there was concerning the longitudes (from which we, with Ptolemy, measured out the mean longitudes in these matters, the places of the fixed stars having first been adjusted to our restitution) — and even if there were such a mark in the fullest degree, those notes would still avail us little regarding the constitution of the nodes in that ancient time, being too far removed from them; so that on this account we are compelled to rest in the Ptolemaic principles regarding the epochs of the nodes.

[Margin: See chapter 4 of this book.]

But the latitudes we build up upon the angles of inclination that we have found in these three planets, in accordance with our own restitution — both of the eccentric and of the annual orb — in each individual case; nor do we, in these matters, attribute anything to a change of the semidiameters (which surely would no more occur in the planets than in the Sun) across the various ages. But since the first concern will be the mean motions of the nodes, we therefore measure them from [Ptolemy], following his own words, which — accommodated and rendered for the points of greatest northern latitude in each of these three — run thus: "In Mars, indeed, we shall take the equated longitude itself; in Jupiter, however, we shall subtract 20 degrees from it; but in Saturn we shall add 50 degrees." By these words, since the greatest agreement of the northern latitude in Mars with its own apogee, according to [Ptolemy], is signified for the year 136 after the birth of Christ at 2 degrees 5½' of Cancer [♋], that greatest northern latitude itself was accordingly in that same year at 2 degrees of Cancer. In Jupiter, however, whose apogee Ptolemy fixed at 11 degrees of Virgo [♍] in the aforesaid

year, the northern limit of latitude is gathered, by the addition of 20 degrees, to have been at 1 degree 0' of Libra [♎].

[Margin: See the commentary on the Sun, in the upper book, book 13, chapter 6.]

Finally, in Saturn, since at the same time [Ptolemy] found its apogee at 23 degrees of Scorpio [♏], by the subtraction of 50 degrees the northern limit of latitude of that same planet is likewise found at 3 degrees of Libra [♎] — this with respect to the true equinox. And for our own age, that is, at about the year 1600 completed, these northern limits—

—were found by Tyconic observations to be: in Mars, more correctly, at 17 degrees of Libra [♎]; in Jupiter at 7 degrees of Libra [♎]; and finally in Saturn at 20 degrees of Libra [♎] — likewise with consideration of the true equinox. Now since here too we rightly measure the elongation of the nodes and of the limits in the latitudes of the planets from the mean vernal equinox, and since the inequality of that equinox was found to be, in Ptolemy's time, 21' subtractive, but in our own time 7' additive, therefore by combining these there arise 28' to be subtracted from the interval of the advance of the nodes between the two epochs. Accordingly there are gathered the motions corresponding to each, over the interval of 1464 years — both as to the apogees and as to the northern nodes, reckoned in the order of preceding signs through a quadrant of the circle from the greatest limits — together with the differences, which the following synopsis displays.

Years after Christ	Saturn Apo. Cor. (Gr. Mi.)	Saturn Nod. Bor. (Gr. Mi.)	Jupiter Apo. Cor. (Gr. Mi.)	Jupiter Nod. Bor. (Gr. Mi.)	Mars Apo. Cor. (Gr. Mi.)	Mars Nod. Bor. (Gr. Mi.)
136	23 50 ♏	3 21 ♎	14 0 ♐	1 21 ♎	28 15 ♎	25 51 ♐
1600	26 27 ✎	20 23 ♎	7 32 ♎	6 53 ♎	28 42 ♏	16 53 ✎
Differ.	32 37	17 2	23 32	5 32	30 27	21 2

But these differences, which intervene between the motions of the apogees and of the nodes in these three superior planets, sufficiently indicate that no commensurate agreement holds among them; rather, contrary to the view of both [Ptolemy] and [Copernicus] and of the intermediate astronomers, they are most widely separated from one another. This can be judged from the difference which lies between the apogee of Jupiter and the node — corresponding to the proposed interval of 1464 years, and extending to 18 degrees — as it may also be

judged of the rest in general; and from this one may convict [Copernicus] of an intolerable error in this matter.

Therefore, since we see that the mean motions of latitude of these three planets can be appended, in their own kind, neither to the advance of the apogees, nor to the anticipation of the equinoxes, nor to any other mean motions whatever, we shall — rightly, as before — now set out the motions of latitude of these planets computed separately from the mean equinox, just as we did the longitudes with their epochs. And, as there, the difference of these same motions, which is proportioned to the interval of time between [Ptolemy] and ourselves (that is, the span of 1464 years), will serve for all the ages. For from that interval we gather the motions corresponding proportionally to the nodes in each case, through Julian years, as follows in the synopsis.

Pla net	1000 (gr. min. sec. tert.)	100 (gr. mi. sec. ter.)	20 (gr. mi. sec. ter.)	1 (gr. mi. sec. tert.)
Sat urn	11 38 5 15	1 9 48 31	0 13 57 42	0 0 41 53
Jup iter	3 46 46 33	0 22 40 39	0 4 32 8	0 0 13 36
Ma rs	14 22 1 18	1 26 12 8	0 17 14 26	0 0 51 43

Furthermore, from the same proportion of the given interval and of its motion, no less here than in the longitudes themselves, we forthwith prefix to the following canon of mean motions the epoch of the advance of the northern node, found for each case. Here it should be noted that we accommodate these motions only to whole years and months, on account of their excessive slowness — which is also the reason why we judged that no account whatever should here be taken of leap years among the expanded values; and indeed we have likewise refrained from adding the motions of the months in their own seconds and minute fractions, everywhere apparent, except insofar as we thought that only thus, and not otherwise, the excessive curiosity of certain persons could be satisfied.

Table of the mean motion of the northern node of the three superior planets.

Epoch	Saturn S. G. M. S.	Jupiter S. G. M. S.	Mars S. G. M. S.
Mundi (of the World)	1 15 38 52	2 15 51 13	10 26 54 46

Epoch	Saturn S. G. M. S.	Jupiter S. G. M. S.	Mars S. G. M. S.
Christi (of Christ)	3 1 46 4	3 0 50 10	0 23 53 46
20	0 0 13 58	0 0 4 32	0 0 17 14
40	0 0 27 55	0 0 9 4	0 0 34 29
60	0 0 41 53	0 0 13 36	0 0 51 43
80	0 0 55 51	0 0 13 9 [?]	0 1 8 58
100	0 1 9 48	0 0 22 41	0 1 26 12
200	0 2 19 37	0 0 45 22	0 2 52 24
300	0 3 29 25	0 1 8 2	0 4 18 36
400	0 4 39 14	0 1 30 43	0 5 44 49
500	0 5 49 2	0 1 53 23	0 7 11 1
600	0 6 58 52	0 2 16 4	0 8 37 13
700	0 8 8 39	0 2 38 45	0 10 3 25
800	0 9 18 28	0 3 1 25	0 11 29 37
900	0 10 28 16	0 3 24 6	0 12 55 49
1000	0 11 38 5	0 3 46 46	0 14 22 1
1100	0 12 47 53	0 4 9 27	0 15 48 13
1200	0 13 57 42	0 4 32 8	0 17 14 26
1300	0 15 7 30	0 4 54 48	0 18 40 38
1400	0 16 17 19	0 5 17 29	0 20 6 50
1500	0 17 27 7	0 5 40 10	0 21 33 2
1600	0 18 36 56	0 6 2 50	0 22 59 14
1700	0 19 46 44	0 6 25 31	0 24 25 26
1800	0 20 56 33	0 6 48 12	0 25 51 38
1900	0 22 6 21	0 7 10 52	0 27 17 50
2000	0 23 16 10	0 7 33 33	0 28 44 3
2500	0 29 5 12	0 9 26 56	1 5 55 3
3000	1 4 54 15	0 11 20 20	1 13 6 4
3500	1 10 43 17	0 13 13 43	1 20 17 4
4000	1 16 32 20	0 15 7 6	1 27 28 5
4500	1 22 1 22 [?]	0 17 0 30	2 4 39 6
5000	1 28 0 25	0 18 53 54	2 11 50 7

Epoch	Saturn S. G. M. S.	Jupiter S. G. M. S.	Mars S. G. M. S.
5500	2 3 59 27	0 20 47 17	2 19 1 7
6000	2 9 48 30	0 22 40 40	2 26 12 8
6300	2 13 17 55	0 27 48 42	3 0 30 44

Year	Saturn M. S.	Jupiter M. S.	Mars M. S.
1	0 42	0 14	0 52
2	1 24	0 27	1 43
3	2 6	0 41	2 35
4	2 48	0 54	3 27
5	3 29	1 8	4 19
6	4 11	1 22	5 10
7	4 53	1 35	6 2
8	5 35	1 49	6 54
9	6 17	2 2	7 43
10	6 59	2 16	8 37
11	7 41	2 30	9 29
12	8 23	2 43	10 21
13	9 4	2 57	11 12
14	9 46	3 10	12 4
15	10 28	3 24	12 56
16	11 10	3 38	13 47
17	11 52	3 51	14 39
18	12 34	4 5	15 31
19	13 16	4 18	16 23
20	13 58	4 32	17 14

In mensibus (In the months)

Month	Saturn M. S.	Jupiter M. S.	Mars M. S.
Ianuar. (January)	0 3	0 1	0 4
Februar. (February)	0 7	0 2	0 9
Martius (March)	0 10	0 3	0 13

Month	Saturn M. S.	Jupiter M. S.	Mars M. S.
Aprilis (April)	0 14	0 4	0 18
Majus (May)	0 17	0 5	0 22
Iunius (June)	0 21	0 7	0 26
Iulius (July)	0 24	0 8	0 31
August. (August)	0 28	0 9	0 35
Septemb. (September)	0 31	0 10	0 40
Octob. (October)	0 35	0 11	0 44
Novemb. (November)	0 38	0 12	0 49
Decemb. (December)	0 42	0 14	0 52

The advance of the northern nodes, according to latitude, having thus been separated from the motion of the apogees in the three superior planets, and having been reduced into the foregoing canon — from which it can be sought for each individual age in each planet — it follows that we should set out the method of constructing the tables of the latitudes themselves. This is so even though these tables cannot remain exact in perpetuity, since the motions of the nodes and of the apogees are disjoined and torn apart from one another. For this reason, whenever the latitudes are to be computed most accurately for the higher ages in these planets, it is best to complete the calculation by the triangular procedure, as was demonstrated earlier — although for some of the nearest ages, four or five backward or forward, no perceptible error can creep into our tables, which are constructed upon the demonstration that follows.

Since, then, it will be most convenient and fitting to compute the remaining latitudes (which are imported from every quarter by the annual orb) upon the greatest latitudes, both northern and southern, and afterward to measure these same by proportional minutes to be deduced from the eccentrics:

Diagram. A geometric construction illustrating the latitude of a superior planet. The ecliptic diameter A B runs roughly horizontally, with B at the left and A at the right. The diameter of the planet's orbit, K D, is cut by A B at the point E (the center). About E a circle is described — the annual orb, with points F (lower, near E), L (lower right), C (interior), and I (upper) marked on or near it. From the upper region, K is at the far upper right and G at the far upper left, with dotted lines running to them; D is at the lower left and H at the lower far right (a dotted line descending to H). Lines are drawn from points L and I to A and K (for the northern construction) and to B and D (for the southern construction).

- A (right end of horizontal diameter): terminus of the ecliptic diameter; the right angle of the construction is perpetually at A
- B (left end of horizontal diameter): the opposite terminus of the ecliptic diameter
- K (upper right): terminus of the planet-orbit diameter K D; northern apex
- D (lower left): opposite terminus of the planet-orbit diameter
- E (center, on A B): point where the orbit diameter K D cuts the ecliptic diameter A B; center of the annual orb
- F (interior, below E): point on the annual orb (acronychal-region marker)
- C (interior): point on/within the annual orb
- I (upper, on the orb): position of the Earth in the Copernican form (northern latitude about angle A L K)
- L (lower right, on the orb): position of the Earth (southern latitude residing about B I D)
- G (far upper left): endpoint of a dotted construction line
- H (far lower right): endpoint of a dotted perpendicular construction line

Let it be, then, as before: A B the diameter of the ecliptic, cut by the diameter of the planet's orbit K D at E. About E as center let the annual orb F L C I be described. Now since it is clear from what precedes that the greatest angle of inclination toward the north is A E K — to which the equal angle toward the south is B E D — the rest remaining as above; but if there be taken in the annual orb, outside the acronychal positions, the points F and C — namely L and I — and from these points lines are drawn on the one side to A and K, and on the other to B and D, it is clear that, when in this Copernican form L and I denote the Earth, the northern latitude resides about the angle A L K, while the southern resides at B I D.

Now since, from what precedes, the distance E D of the planet on the eccentric from the center E is given in each case — whose complement to the doubled radius, namely 200000, is E K — therefore, since in the right triangle E A K, which perpetually has its angle at A (the ecliptic) right, the angle of inclination at E is likewise given in each case, first from these E A is found; then it is reduced to the usual radius 100000, of which A K is likewise the tangent; and indeed in the same proportion the radius of the orb E L is given, etc., which has likewise become known above in each case.

But so that the side L A may be obtained more compendiously, let there go out from the radius of the orb E L a line drawn continuously, and another

perpendicular to this, A H. Now since A H is the right sine of the assumed angle A E H, the right sine of the complement of this will be E H; and from this, in this case, the orb-diameter E L being subtracted, the side L H likewise remains given. Hence, easily, in the right triangle A L H, from the two sides thus acquired about the right angle at H, the sought side L A is given. Finally, in the right triangle A L K, which has its right angle at the ecliptic A, since two sides about the right angle are now likewise granted, the latitude of the planet itself — to be measured in the angle A L K — is extracted. In exactly the same way the demonstration proceeds toward the south, after the remaining sides E B and B D, as well as the semidiameter of the annual orb E I, have here too been proportioned to the distance of the planet from the center of the eccentric E.

We will give a worked example for the sake of illustration in the case of Saturn ♄, whose distance from the center of the eccentric — at its maximum southern latitude, as I found it earlier above — is 97593 parts, with the semidiameter of the orb being 10426 in the same proportion; and finally with the maximum angle of inclination $2^{\circ} 31'$. So, first subtracting E D 97593 from 200000, E K 102407 remains. Then in the right triangle K A E — A being the right angle, that is, the whole sine — as the whole sine is to E K, so the right sine of K, $87^{\circ} 29'$, is to E A 102308. But where E A is 100000, the semidiameter of the annual orb E F will be 10191, and A K of those same units will be 4395 — this being the tangent of the given angle A E K.

With these values granted as nearly fixed for the maximum northern latitude of Saturn, let a point L be taken on the circumference of the annual orb, distant for the present example here 30° from the planet's (or the earth's) acronychal position at F. Drawing E L through to H, and dropping a perpendicular from A, A H will be the right sine of the assumed angle A E H, that is, of 30° — namely 50000; while E H is the right sine of its complement, that is, of 60° , which is 86602. But subtracting here E L 10191 from E H, L H 76411 remains. Therefore in the right triangle L H A, since both sides about the right angle at H are given, the sought quantity L A is finally found to be 91315. Now with L A 91315 found, and with A K 4395 given earlier, the last step — in the right triangle L A K, which always holds the right angle to the ecliptic at A (since from the earth at L a radius L A runs out, as it were, perpendicularly into the ecliptic at A) — yields the angle A L K of $2^{\circ} 45\frac{1}{3}'$, the angle that holds Saturn's maximum northern latitude even when it is 30° distant from opposition with the true place of the Sun in the ecliptic.

And it is for this example that I found the numbers in the area of the latitude tables that follow. The annual orbs introduce their own variations into the individual entries, and they recognize no others to be considered — not in the manner of a libration, as Ptolemy and Copernicus wished. For Mars, however, whose annual-orb semidiameter, unlike the others', is not fixed, this can scarcely be provided for in any other way, so long as the values are reduced into a canon. Yet although these matters will be set out more precisely on the basis of the hypotheses presented above — with the semidiameter of each orb first corrected to the prescribed standard — nevertheless, since not much error can result here, I therefore include in the same reckoning with the rest these same values too, computed from what was presented, even though [Greek: διδομένοις (didomenois, "as given quantities")] reckoned only approximately.

Finally, as for what concerns the proportional minutes [scrupula proportionalia]: the table inserted in the first place I have transferred here from the canon of the prosthaphaereses of the equinoctials, computed earlier by me on the spherical principle which is set out in the same place. It will serve indifferently for working out all these latitudes, since a single sexagesimal proportion is commonly included for all of them, applying to the angle of inclination or of latitude.

Table of the latitude of the three superior planets

0 Sig. [0 Signs]

The leftmost column "Gradus" counts the degrees 0→30 top-to-bottom; the rightmost "Gradus" column counts them 30→0. "Scrup. Prop." gives the proportional minutes (M. = minutes, S. = seconds). Each planet has a Northern (Borea) and Southern (Auftr. = Australis) sub-column, each in degrees (G.) and minutes (M.).

G	P	P													G
r	r	r													r
a	o	o	♂	♂			♂	♂			♂	♂	♂	♂	a
d	p	p	Bor	Bor	♂	Auf	Bor	Bor	♂	Auf	Bor	Bor	♂	Auf	d
u	.	.	ea	ea	Auf	tr	ea	ea	Auf	tr	ea	ea	Auf	tr	u
s	M	S	G	M	tr G	M	G	M	tr G	M	G	M	tr G	M	s
0	0	0	2	17	2	16	1	7	1	6	1	9	1	4	30
1	1	3	2	17	2	16	1	7	1	6	1	9	1	4	29

G r a d u s	P r o p . M	P r o p . S	h Bor ea G	h Bor ea M	h Auf tr G	h Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	G r a d u s
2	2	5	2	17	2	16	1	7	1	6	1	9	1	4	28
3	3	8	2	17	2	16	1	7	1	6	1	9	1	4	27
4	4	10	2	17	2	16	1	7	1	6	1	9	1	4	26
5	5	12	2	17	2	16	1	7	1	6	1	9	1	4	25
6	6	14	2	17	2	16	1	7	1	6	1	9	1	5	24
7	7	16	2	17	2	16	1	7	1	6	1	9	1	5	23
8	8	18	2	17	2	16	1	7	1	6	1	9	1	5	22
9	9	21	2	17	2	16	1	7	1	7	1	9	1	5	21
10	10	23	2	17	2	16	1	7	1	7	1	9	1	5	20
11	11	25	2	17	2	16	1	7	1	7	1	9	1	5	19
12	12	26	2	17	2	16	1	7	1	7	1	9	1	6	18
13	13	27	2	17	2	16	1	7	1	7	1	10	1	6	17
14	14	28	2	17	2	16	1	7	1	7	1	10	1	6	16
15	15	28	2	17	2	16	1	8	1	7	1	10	1	6	15
16	16	29	2	17	2	16	1	8	1	7	1	10	1	6	14

G r a d u s	P r o p . M	P r o p . S	ħ Bor ea G	ħ Bor ea M	ħ Auf tr G	ħ Auf tr M	⌞ Bor ea G	⌞ Bor ea M	⌞ Auf tr G	⌞ Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	G r a d u s
17	17	29	2	17	2	16	1	8	1	7	1	10	1	7	13
18	18	29	2	18	2	17	1	8	1	7	1	10	1	7	12
19	19	30	2	18	2	17	1	8	1	7	1	10	1	7	11
20	20	30	2	18	2	17	1	8	1	7	1	10	1	7	10
21	22	29	2	18	2	17	1	8	1	7	1	10	1	7	9
22	22	28	2	18	2	17	1	8	1	7	1	11	1	8	8
23	22	26	2	18	2	17	1	8	1	7	1	11	1	8	7
24	22	24	2	18	2	17	1	8	1	7	1	11	1	8	6
25	22	22	2	18	2	17	1	8	1	7	1	11	1	9	5
26	22	20	2	18	2	17	1	8	1	7	1	11	1	9	4
27	22	18	2	18	2	17	1	8	1	7	1	11	1	9	3
28	22	16	2	18	2	18	1	9	1	7	1	12	1	9	2
29	22	14	2	18	2	18	1	9	1	8	1	12	1	10	1
30	30	11	2	18	2	18	1	9	1	8	1	12	1	10	0

Foot of table: **11 Sig.** [11 Signs — the column read from the bottom serves the eleventh sign]

Table of the latitude of the three superior planets

1 Sig. [1 Sign]

G r a d u s	P r o p . M	P r o p . S	h Bor ea G	h Bor ea M	h Auf tr G	h Auf tr M	ㄷ Bor ea G	ㄷ Bor ea M	ㄷ Auf tr G	ㄷ Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	G r a d u s
0	30	11	2	18	2	18	1	9	1	8	1	12	1	10	30
1	31	5	2	18	2	18	1	9	1	8	1	12	1	10	29
2	31	58	2	18	2	18	1	9	1	8	1	12	1	10	28
3	32	50	2	18	2	18	1	9	1	8	1	12	1	10	27
4	33	51	2	19	2	18	1	9	1	8	1	12	1	11	26
5	34	31	2	19	2	19	1	9	1	8	1	12	1	11	25
6	35	21	2	19	2	19	1	9	1	8	1	13	1	11	24
7	36	10	2	19	2	19	1	10	1	8	1	13	1	11	23
8	36	58	2	19	2	19	1	10	1	8	1	13	1	12	22
9	37	46	2	19	2	19	1	10	1	9	1	13	1	12	21
10	38	3	2	19	2	19	1	10	1	9	1	13	1	12	20
11	39	20	2	20	2	19	1	10	1	9	1	14	1	12	19

G r a d u s	P r o p . M	P r o p . S	ᵢ Bor ea G	ᵢ Bor ea M	ᵢ Auf tr G	ᵢ Auf tr M	ᵢ Bor ea G	ᵢ Bor ea M	ᵢ Auf tr G	ᵢ Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	G r a d u s
1 2	4 0	6	2	20	2	20	1	10	1	9	1	14	1	12	1 8
1 3	4 0	5 2	2	20	2	20	1	10	1	9	1	14	1	13	1 7
1 4	4 1	3 7	2	20	2	20	1	10	1	9	1	14	1	13	1 6
1 5	4 2	2 2	2	21	2	20	1	10	1	9	1	15	1	13	1 5
1 6	4 3	6	2	21	2	20	1	11	1	9	1	15	1	13	1 4
1 7	4 3	4 9	2	21	2	21	1	11	1	9	1	15	1	14	1 3
1 8	4 4	3	2	21	2	21	1	11	1	9	1	15	1	14	1 2
1 9	4 5	1 6	2	21	2	21	1	11	1	10	1	16	1	14	1 1
2 0	4 5	5 8	2	21	2	21	1	11	1	10	1	16	1	14	1 0
2 1	4 6	3 9	2	21	2	21	1	11	1	10	1	16	1	15	9
2 2	4 7	1 9	2	21	2	22	1	11	1	10	1	16	1	15	8
2 3	4 7	5 7	2	21	2	22	1	11	1	10	1	17	1	15	7
2 4	4 8	3 4	2	22	2	22	1	11	1	10	1	17	1	15	6
2 5	4 9	9	2	22	2	22	1	12	1	10	1	17	1	16	5
2 6	4 9	4 3	2	22	2	22	1	12	1	10	1	17	1	16	4

G r a d u s	P r o p . M	P r o p . S	h Bor ea G	h Bor ea M	h Auf tr G	h Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	G r a d u s
2 7	5 0	1 6	2	22	2	23	1	12	1	10	1	18	1	16	3
2 8	5 0	4 8	2	22	2	23	1	12	1	11	1	18	1	16	2
2 9	5 1	1 9	2	23	2	23	1	12	1	11	1	18	1	17	1
3 0	5 1	4 9	2	23	2	23	1	12	1	11	1	19	1	17	0

Foot of table: **10 Sig.** [10 Signs]

Table of the latitude of the three superior planets

2 Sig. [2 Signs]

G r a d u s	P r o p . M	P r o p . S	h Bor ea G	h Bor ea M	h Auf tr G	h Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	G r a d u s
0 1	5 1	4 9	2	23	2	23	1	12	1	11	1	19	1	17	3 0
1 2	5 2	1 9	2	23	2	23	1	13	1	11	1	19	1	17	2 9
2 2	5 2	4 8	2	23	2	23	1	13	1	11	1	19	1	18	2 8
3 3	5 3	1 6	2	23	2	24	1	13	1	12	1	20	1	18	2 7
4 3	5 3	4 4	2	23	2	24	1	13	1	12	1	20	1	18	2 6
5 5	5 5	1 1	2	23	2	24	1	13	1	12	1	20	1	19	2

G r a d u s	P r o p . M	P r o p . S	h Bor ea G	h Bor ea M	h Auf tr G	h Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	G r a d u s
	4	1													5
6	5	3	2	24	2	24	1	14	1	12	1	21	1	19	2
	4	7													4
7	5	2	2	24	2	25	1	14	1	12	1	21	1	19	2
	5														3
8	5	2	2	24	2	25	1	14	1	13	1	21	1	20	2
	5	7													2
9	5	5	2	24	2	25	1	14	1	13	1	22	1	20	2
	5	1													1
1	5	1	2	24	2	25	1	14	1	13	1	22	1	20	2
0	6	4													0
1	5	3	2	24	2	25	1	14	1	13	1	23	1	21	1
1	6	6													9
1	5	5	2	25	2	26	1	15	1	14	1	23	1	21	1
2	6	7													8
1	5	1	2	25	2	26	1	15	1	14	1	24	1	21	1
3	7	7													7
1	5	3	2	25	2	26	1	15	1	14	1	24	1	22	1
4	7	6													6
1	5	5	2	25	2	26	1	15	1	14	1	25	1	22	1
5	7	4													5
1	5	1	2	26	2	27	1	15	1	15	1	25	1	23	1
6	8	1													4
1	5	2	2	26	2	27	1	16	1	15	1	26	1	23	1
7	8	7													3
1	5	4	2	26	2	27	1	16	1	15	1	26	1	23	1
8	8	1													2
1	5	5	2	26	2	27	1	16	1	15	1	27	1	24	1
9	8	4													1
2	5	5	2	27	2	27	1	16	1	16	1	27	1	24	1

G r a d u s	P r o p . M	P r o p . S	♄ Bor ea G	♄ Bor ea M	♄ Auf tr G	♄ Auf tr M	♄ Bor ea G	♄ Bor ea M	♄ Auf tr G	♄ Auf tr M	♂ Bor ea G	♂ Bor ea M	♂ Auf tr G	♂ Auf tr M	G r a d u s
0	9														0
2	5	1	2	27	2	28	1	16	1	16	1	28	1	25	9
1	9	5													
2	5	2	2	27	2	28	1	17	1	16	1	28	1	25	8
2	9	4													
2	5	3	2	27	2	28	1	17	1	16	1	29	1	26	7
3	9	2													
2	5	3	2	28	2	29	1	17	1	17	1	29	1	26	6
4	9	9													
2	5	4	2	28	2	29	1	17	1	17	1	30	1	27	5
5	9	5													
2	5	5	2	28	2	29	1	17	1	17	1	31	1	27	4
6	9	0													
2	5	5	2	29	2	30	1	18	1	17	1	32	1	28	3
7	9	4													
2	5	5	2	29	2	30	1	18	1	17	1	33	1	28	2
8	9	7													
2	5	5	2	29	2	30	1	18	1	18	1	34	1	29	1
9	9	9													
3	6	0	2	30	2	31	1	18	1	18	1	34	1	29	0
0	0														

Foot of table: **9 Sig.** [9 Signs]

Liber II Theoricorum — Cap. XV (concl.) / Cap. XVI

Table of latitude of the three superior planets.

3 Sig.

In each table the first column is the argument; the **Proportional Minutes** [scrupula proportionalia] are given in minutes (M.) and seconds (S.); for each of Saturn, Jupiter, and Mars the **Northern** [Borea] and **Southern** [Auster] latitudes are given in degrees (G.) and minutes (M.). The rightmost column gives the complementary argument, read with the foot-label sign ("8 Sig.").

A	Pro	Pro	h	h	h	h	ꝛ	ꝛ	ꝛ	ꝛ	♂	♂	♂	♂	A
g	p.	p. S.	N	N	h	S	N	N	ꝛ	S	N	N	♂	S	g
.	M.	G	M	S G	M	G	M	S G	M	G	M	S G	M	.	
0	60	0	2	30	2	31	1	18	1	18	1	34	1	29	30
1	59	59	2	30	2	31	1	19	1	18	1	35	1	29	29
2	59	57	2	30	2	31	1	19	1	18	1	36	1	30	28
3	59	54	2	30	2	31	1	19	1	19	1	36	1	30	27
4	59	50	2	31	2	32	1	19	1	19	1	37	1	31	26
5	59	45	2	31	2	32	1	20	1	19	1	38	1	32	25
6	59	39	2	31	2	32	1	20	1	20	1	39	1	33	24
7	59	32	2	32	2	32	1	20	1	20	1	40	1	34	23
8	59	29	2	32	2	33	1	20	1	20	1	40	1	35	22
9	59	15	2	32	2	33	1	21	1	21	1	41	1	36	21
10	59	4	2	32	2	33	1	21	1	21	1	42	1	37	20
11	58	54	2	33	2	34	1	21	1	21	1	43	1	38	19
12	58	41	2	33	2	34	1	21	1	22	1	44	1	39	18

A r g . 1 3	Pro p. M.	Pro p. S.	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	♂ N G	♂ N M	♂ S G	♂ S M	A r g . 1 7
1 3	58	27	2	33	2	34	1	22	1	22	1	45	1	40	1 7
1 4	58	11	2	34	2	34	1	22	1	22	1	46	1	41	1 6
1 5	57	54	2	34	2	35	1	22	1	23	1	47	1	42	1 5
1 6	57	36	2	34	2	35	1	22	1	23	1	48	1	43	1 4
1 7	57	17	2	35	2	35	1	23	1	23	1	49	1	44	1 3
1 8	56	57	2	35	2	36	1	23	1	23	1	50	1	45	1 2
1 9	56	36	2	35	2	36	1	23	1	24	1	52	1	46	1 1
2 0	56	14	2	36	2	36	1	23	1	24	1	53	1	47	1 0
2 1	55	51	2	36	2	37	1	24	1	24	1	54	1	48	9
2 2	55	27	2	36	2	37	1	24	1	25	1	56	1	50	8
2 3	55	2	2	36	2	37	1	24	1	25	1	57	1	51	7
2 4	54	32	2	36	2	37	1	24	1	25	1	59	1	52	6
2 5	54	11	2	37	2	38	1	25	1	26	2	0	1	56	5
2 6	53	44	2	37	2	38	1	25	1	26	2	1	1	56	4
2 7	53	16	2	37	2	38	1	25	1	26	2	3	1	57	3

A r g .	Pro p. M.	Pro p. S.	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	♂ N G	♂ N M	♂ S G	♂ S M	A r g .
28	52	48	2	38	2	39	1	26	1	27	2	4	1	59	2
29	52	19	2	38	2	39	1	26	1	27	2	5	2	1	1
30	51	49	2	38	2	39	1	26	1	27	2	7	2	3	0

8 Sig. (foot-label for the complementary argument column)

Table of latitude of the three superior planets.

4 Sig.

A r g .	Pro p. M.	Pro p. S.	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	♂ N G	♂ N M	♂ S G	♂ S M	A r g .
0	51	49	2	38	2	39	1	26	1	27	2	7	2	3	30
1	51	19	2	38	2	39	1	26	1	28	2	8	2	5	29
2	50	48	2	39	2	39	1	27	1	28	2	10	2	7	28
3	50	16	2	39	2	40	1	27	1	28	2	11	2	9	27
4	49	43	2	39	2	40	1	27	1	28	2	12	2	11	26
5	49	9	2	39	2	40	1	28	1	29	2	14	2	13	25
6	48	34	2	40	2	40	1	28	1	29	2	15	2	15	24
7	47	57	2	40	2	41	1	28	1	29	2	17	2	17	23
8	47	19	2	40	2	41	1	28	1	29	2	19	2	19	2

A r g . M.	Pro p. M.	Pro p. S.	h N G	h N M	h S G	h S M	q N G	q N M	q S G	q S M	♂ N G	♂ N M	♂ S G	♂ S M	A r g .
9	46	39	2	40	2	41	1	29	1	30	2	21	2	21	2
10	45	58	2	40	2	41	1	29	1	30	2	23	2	23	2
11	45	16	2	41	2	42	1	29	1	30	2	25	2	25	1
12	44	33	2	41	2	42	1	30	1	31	2	27	2	28	1
13	43	49	2	41	2	42	1	30	1	31	2	29	2	30	1
14	43	6	2	41	2	42	1	30	1	31	2	31	2	32	1
15	42	22	2	42	2	43	1	30	1	31	2	33	2	35	1
16	41	37	2	42	2	43	1	31	1	32	2	35	2	37	1
17	40	52	2	42	2	43	1	31	1	32	2	37	2	40	1
18	40	6	2	42	2	43	1	31	1	32	2	40	2	43	1
19	39	20	2	43	2	44	1	31	1	32	2	42	2	45	1
20	38	33	2	43	2	44	1	32	1	33	2	45	2	48	1
21	37	46	2	43	2	44	1	32	1	33	2	48	2	52	9
22	36	58	2	43	2	44	1	32	1	33	2	50	2	56	8
23	36	10	2	43	2	44	1	32	1	34	2	52	2	0	7

A r g .	Pro p. M.	Pro p. S.	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	♂ N G	♂ N M	♂ S G	♂ S M	A r g .
2 4	34	21	2	44	2	45	1	33	1	34	2	55	2	4	6
2 5	33	31	2	44	2	45	1	33	1	34	2	57	2	8	5
2 6	32	41	2	44	2	45	1	33	1	34	3	0	2	12	4
2 7	31	50	2	44	2	45	1	33	1	35	3	3	2	16	3
2 8	31	58	2	45	2	46	1	34	1	35	3	7	2	21	2
2 9	31	5	2	45	2	46	1	34	1	35	3	10	2	26	1
3 0	30	11	2	45	2	46	1	34	1	36	3	13	2	32	0

7 Sig. (foot-label for the complementary argument column)

Table of latitude of the three superior planets.

5 Sig.

A r g .	Pro p. M.	Pro p. S.	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	ᵢ N G	ᵢ N M	ᵢ S G	ᵢ S M	♂ N G	♂ N M	♂ S G	♂ S M	A r g .
0	30	11	2	45	2	46	1	34	1	36	3	13	3	32	3 0
1	29	14	2	45	2	46	1	34	1	36	3	15	3	38	2 9
2	28	16	2	45	2	46	1	35	1	36	3	18	3	44	2 8
3	27	18	2	45	2	46	1	35	1	36	3	20	3	50	2 7
4	26	20	2	46	2	47	1	35	1	36	3	22	3	56	2

A r g .	Pro p. M.	Pro p. S.	h N G	h N M	h S G	h S M	ꣳ N G	ꣳ N M	ꣳ S G	ꣳ S M	♂ N G	♂ N M	♂ S G	♂ S M	A r g .
															6
5	25	22	2	46	2	47	1	35	1	37	3	25	4	2	2 5
6	24	24	2	46	2	47	1	35	1	37	3	28	4	8	2 4
7	23	26	2	46	2	47	1	35	1	37	3	32	4	14	2 3
8	22	28	2	46	2	47	1	36	1	37	3	36	4	21	2 2
9	21	29	2	46	2	47	1	36	1	37	3	40	4	28	2 1
1 0	20	30	2	47	2	47	1	36	1	38	3	44	4	35	2 0
1 1	19	30	2	47	2	48	1	36	1	38	3	48	4	42	1 9
1 2	18	29	2	47	2	48	1	36	1	38	3	52	4	50	1 8
1 3	17	29	2	47	2	48	1	37	1	38	3	56	4	58	1 7
1 4	16	29	2	47	2	48	1	37	1	38	4	0	5	6	1 6
1 5	15	28	2	47	2	48	1	37	1	38	4	4	5	14	1 5
1 6	14	28	2	47	2	48	1	37	1	39	4	7	5	22	1 4
1 7	13	27	2	47	2	48	1	37	1	39	4	10	5	31	1 3
1 8	12	26	2	47	2	48	1	37	1	39	4	13	5	39	1 2
1 9	11	25	2	47	2	49	1	37	1	39	4	15	5	47	1 1

A r g . 2 0	Pro p. M.	Pro p. S.	ḥ N G	ḥ N M	ḥ S G	ḥ S M	ꠔ N G	ꠔ N M	ꠔ S G	ꠔ S M	♂ N G	♂ N M	♂ S G	♂ S M	A r g . 1 0
2 1	10	23	2	48	2	49	1	37	1	39	4	17	5	55	1 0
2 2	9	21	2	48	2	49	1	37	1	39	4	19	6	3	9
2 3	8	18	2	48	2	49	1	37	1	39	4	21	6	11	8
2 4	7	16	2	48	2	49	1	37	1	39	4	23	6	18	7
2 5	6	14	2	48	2	49	1	38	1	39	4	25	6	24	6
2 6	5	12	2	48	2	49	1	38	1	40	4	26	6	29	5
2 7	4	10	2	48	2	49	1	38	1	40	4	27	6	34	4
2 8	3	8	2	48	2	49	1	38	1	40	4	28	6	38	3
2 9	2	5	2	48	2	49	1	38	1	40	4	29	6	42	2
3 0	1	3	2	48	2	49	1	38	1	40	4	30	6	45	1
3 0	0	0	2	48	2	49	1	38	1	40	4	31	6	47	0

6 Sig. (foot-label for the complementary argument column)

CHAPTER XVI.

On the computation of the latitudes of the three superior planets.

Two things above all are required for finding the latitudes: the true distance of the planet on its eccentric orb from the northern node of latitude — which yields

the proportional minutes — and the true anomaly of the annual orb, by means of which the latitudes are worked out according to the inclinations, that is, the greatest departures from the ecliptic. So, for a given time reckoned from the Incarnation of Christ, whether backward or forward, you first find from the tables of the mean motions of latitude in these three planets the motion (or position) of the northern node — that is, the point where the planet's path crosses the ecliptic as it climbs toward the north. Once that is found, subtract it from the true longitude of the planet's eccentric (reckoned from the mean vernal equinox), and what remains is the distance from the said northern node. With that distance you look up and copy out the matching proportional minutes from the adjacent table. Then, by the true anomaly of the annual orb, you likewise obtain the planet's latitude itself: this comes out northern so long as the planet's distance on the eccentric from the northern node has not yet completed a semicircle, and afterward southern. This latitude corresponds to the greatest inclination in each position of the planet relative to the Earth or the Sun; once it has been multiplied by (literally, "excerpted with") the proportional minutes previously preserved, the planet's true latitude is finally produced.

In the case of Mars, however, when we set about determining its latitude, we must use the [Greek: ἀκριβέστατος (akribestatos, "most exact")] triangular method set out earlier, once the distance of this planet has been granted not only from the center of the eccentric but also from the Earth, as was shown in the earlier example.

Example in the case of Saturn.

We bring forward two kinds of examples of latitudes in these planets: the first domestic — that is, of those observed by us at [Hven] — which are found appended to the longitude examples already given; the other from the most ancient records of observations, in which, so far as can be done, we shall test for traces of a change in the ecliptic. And since the materials needed for all of this — apart from the mean progressions of the nodes — are at hand, we shall extract those first, and then add the rest.

In the year 1595, in the month of March, on the 17th day, at 7½ hours — the time at which we also noted the latitude of Saturn found above, 56½' in the south — there is found from the tables given above the northern node of Saturn's latitude at 3 signs 20° 16'. But the true longitude of the eccentric was 2 signs 28° 48'; when the node is subtracted from this, there remain 11 signs 8° 32', to which in the table correspond proportional minutes of 21° 56'. (From these it can also be recognized that the latitude is going to be southern, because that number exceeds

a semicircle.) Further, the equated anomaly of the annual orb is 9 signs 7° 41', to which in the table corresponds a southern latitude of 2° 28'; this, multiplied by the proportional minutes preserved, 22 m. 56 s., leaves a computed latitude of 0° 53⅔' — the observed difference extending to only 2⅕'.

Example in the case of Jupiter.

In the same year 1591, on April the 14th, at hour 11, the northern latitude of Jupiter was found from observation to be 1° 23½'.

	Sig.	Gr.	Mi.
At the same time the northern node is found to be	3	6	51
Likewise the true longitude of the eccentric	7	12	19
Hence the anomaly for the proportional minutes	4	5	28
And the proportional minutes themselves, corresponding		48	53
True anomaly of the annual orb	5	21	39
To which corresponds the northern latitude		1	37
Which, multiplied by the proportional minutes, leaves the true — that is, the sought — latitude		1	19 B +

The difference, therefore, of 4½' will easily be excused on account of the refraction of the Jovian radiance, which is set as it were in the midst of [the orb] [in con-].

Example for Mars.

In the year of our Lord 1587, in the month of January, on the 15th day, at 15 hours, 50 minutes, the observed latitude of Mars at Hven [Huæna] was found (as I noted above) to be 3° 13' northern.

	Sig.	Gr.	Mi.
The motion of the node N. matching this time	1	16	41
Likewise the true longitude of the eccentric	5	3	37
Therefore the anomaly for the proportional minutes	3	16	56
And the proportional minutes themselves		57	18
Furthermore the true anomaly of the annual orb	5	2	2
To which corresponds the implicit latitude N.		3	18

	Sig.	Gr.	Mi.
Once this is taken with respect to the proportional minutes, the true latitude sought emerges		3	8⅓ N.

The difference, then, is 4⅓', by which the observed latitude exceeds the computed one — but only just. [Margin: See at the end of ch. 13 of this book.] I see this happening here most of all because the semidiameter of the orb to be assumed in this example, namely 657598, is greater than the mean 654950, and it is to that mean that the latitudes in the table were computed.

So far the examples have been from our own century, for these superior planets, and they conformed well enough to our restitution in the heavens. It remains for us also to subject to examination — for the sake of their latitudes — those ancient observations of these three planets at their conjunctions with the fixed stars, which we reviewed in the restitution of the longitudes. We shall try, as far as can be done, to see whether that shift of the ecliptic (that is, of the latitudes) which was demonstrated above for the fixed stars was perceptible in these planets as well. For without those observations the labor undertaken by Ptolemy for investigating this matter is otherwise useless. Indeed, by the same token, regarding the true latitudes of the wandering stars as of the fixed stars in his work, we remain in doubt, faulting the ancient armillary instruments. I reckon that the result was due partly to their defect, and partly also to the Ptolemaic assumptions themselves, which do not agree well enough — so that the limits of the greatest digressions of these planets from the ecliptic come out larger than ours. For the shift of the ecliptic that would have occurred in the meantime in no way reached, perceptibly, the equinoctial points to which those planets were nearest.

Example of an ancient observation for Saturn.

In the 519th year from Nabonassar [Nabonnassar], in the month of Tybi among the Egyptians, on the 14th day in the evening — this fell in the year 3735 completed from the secular sextile [bissextile] of the world, plus two months — the star of Saturn was in conjunction with the following star beneath the southern shoulder of Virgo [♍], so that it was distant from it two digits toward the south. [Margin: See ch. 4 of this book.] Now since that fixed star, according to the Tychonic rectification — our precession of the equinox and our correction of latitude — was at that time in 9° 3' of Virgo [♍] with a northern latitude of 2° 43', the latitude of Saturn was 2° 37½' N. At this time the northern node of Saturn was

2 signs $29^{\circ} 6'$ distant from the mean equinox, and the equated longitude of the eccentric was 5 signs $9^{\circ} 1'$. Therefore the argument for the proportional minutes was 2 signs $9^{\circ} 55'$, and so the proportional minutes were $56' 12''$; finally the equated anomaly of the annual orb was 5 signs $28^{\circ} 30'$. From this the true latitude follows as $2^{\circ} 37\frac{1}{2}'$ N. But from the Ptolemaic canon of latitudes, assuming the greatest northern latitude in Saturn to be $3^{\circ} 2'$ (which exceeds ours by $14'$), the said latitude comes out as $2^{\circ} 50\frac{1}{2}'$ northern. From this observation of the ancients — since it agrees with our assumption — no trace of a shift of the ecliptic can be discerned distinctly enough; in that elongation from the autumnal equinox toward the leading signs it is scarcely $7'$. But it is sufficiently clear from this that Ptolemy set the northern limit of latitude in Saturn too large, since he draws that latitude still farther north than the assumption for his own fixed star requires.

Example of an ancient latitude for Jupiter.

In the 83rd year from the death of Alexander, in the month Epiphi according to the Egyptians, on the 18th day in the morning — a time which fell in the year 241 before the birth of Christ, in the morning hour of the 4th of September — Ptolemy records that the star of Jupiter occulted the southern Ass of Cancer [asinum Cancr]. [Margin: See ch. 7 of this book. Book 9, ch. 3.] For that time the motion of the northern node of Jupiter is found to be 2 signs $29^{\circ} 56'$ from the vernal equinox, and the longitude of the eccentric 2 signs $28^{\circ} 7'$. From this the argument of latitude for the proportional minutes is 11 signs $28^{\circ} 11'$, and the corresponding proportional minutes themselves are $1^{\circ} 54''$. Furthermore the anomaly of the annual orb is 2 signs $9^{\circ} 2'$, while the implicit latitude itself is $1^{\circ} 14'$, which for Ptolemy is $1^{\circ} 18'$. As for the rest, from the multiplication of the proportional minutes into our latitude there emerges the latitude of Jupiter itself as $2\frac{1}{6}'$ southern. But the latitude of the fixed star which they call the southern Ass, from Lord Tycho's most accurate observation, is $0^{\circ} 4'$ southern in our century; and by the standard of the changed obliquity of the ecliptic it would have been, in that ancient century, $0^{\circ} 25'$ southern. So if the radiance of Jupiter had on average covered the said star, it would have had the same latitude as the star itself. We see, therefore, that on account of such a shift the intersection of the ecliptic with the planet's path was movable, while the planet's orb — no less than the place of the star — remained fixed. This too ought to have appeared here especially near the beginning [of the sign], unless perhaps the light of Jupiter, being too diffuse, could so cover this star (which is reckoned of the fourth magnitude), standing at a middling distance to the north from it — about a third part of a degree — when the planet passed by, that it removed the star from the gaze of onlookers' eyes.

Example for the latitude of Mars observed in an ancient century.

[Margin: See ch. 10 of this book.] In the 52nd year from the departure of Alexander the Great, on the 21st day in the morning, in the month Athyr among the Egyptians — that is, 272 years before the coming of Christ in the flesh, on the 18th day of January in the morning — the star of Mars appeared to have been placed against the northern brow of Scorpio [♏], whose latitude at that time, by the Tychonic restitution, was $1^{\circ} 16'$ N. The latitude of Mars, then, was at that time the same, very nearly — provided that the star in question on the brow of Scorpio [♏] was the bright one, agreeing with Ptolemy, and not some other star, as Johannes Kepler [Johannes Kepplerus] holds it to have been. This we maintain even now, just as we asserted earlier concerning the restoration of the longitude. [Margin: Ptolemy, book 10, ch. 9. Ch. 10 of this book.] For this indeed appears most of all in this comparison of latitudes.

At the same time, from our table, there is found:

	Sig.	Gr.	Min.	Sec.
the motion of the node of the northern latitude of Mars	0	19	20	
the true longitude of its eccentric	5	23	20	
the argument for the proportional minutes	5	4	0	
the corresponding proportional minutes themselves			26	20
the true anomaly of the annual orb	4	1	20	
the corresponding implicit latitude			2	9
Therefore from these the true latitude of Mars N. is given as			0	57 (nearly)

From such an apposition of Mars to the said bright star on the brow of Scorpio [♏] — beyond about an eighth part of a degree, by which it could still have been more southern (for no greater latitude for it would have been computed and expected by the triangular method; rather, somewhat less, on account of the semidiameter of the annual orb falling a little short of the mean) — the same thing that was affirmed above for Jupiter, concerning the unchanged orbs of these planets, will be seen to hold here too: that what suits the fixed stars regarding the shift of latitude is not to be denied to these wandering stars,

whenever anyone wishes to apply the same to those ancient centuries. [Margin: Book of the prior [book], ch. 4.]

CHAPTER XVII.

On the general description of the threefold hypothesis to which the motions of Venus and Mercury are bound; and on its various interchange.

The labor that was owed to the restitution of the motions of the three superior planets — Saturn, Jupiter, and Mars, namely those that encircle the Earth — having now been overcome, it remains for us to devote, with divine grace likewise assisting, the following effort to Venus and Mercury. Both of these are reckoned by Ptolemy to move below the Sun, by Tycho around the Sun, and by Copernicus [Copernico] within the great orb of the Earth.

These planets, however, differ from the superior ones in this: that their orbs (if indeed they are orbs, as Copernicus and Tycho judged them, which Ptolemy calls epicycles) do not have eccentricities of their own, as the superior ones do, but rather belong to the Sun's orb alone, or to the proper orb that encircles the Earth. Nor (so far as I have been able to discover up to now) do these planets, like the Moon and the rest, regard the true course of the Sun, but only its mean course, in their revolutions — just as they likewise acknowledge that same mean course for their own simple and periodic motions through the zodiac. All of this will in due course be shown clearly from the planets' own appearances.

The hypotheses of the three masters [of astronomy], by which the motions of these planets too can be bound together, indifferently and most conveniently, we shall first set out by a general outline according to longitude; afterward we shall work them out more specifically and laboriously, according to the dimensional parts proper to each, in the restitution itself.

Quite varied and diverse reasons present themselves for constructing the hypotheses equipollently here. But among them, lest we seem to wish to trifle at too great a cost to learners, let us follow the easier and most geometrical ones: first, in Ptolemy and Tycho, those that agree in every respect with the Copernican assumption regarding the working-out of the prosthaphaereses [profthaphaereses]; then those bordering on the superior ones, in which a greater ease and a more compendious method of computing the motion presents itself.

As for Ptolemy, in what concerns these matters, in order that here too equal motions be considered about their proper centers, and that all the canons to be

deduced from the threefold assumption breathe conformity, we do indeed assume the eccentricity in the middle, that is, in the diameter of the orb. But we also describe a little circle upon the part of the equant, as upon a diameter, and we dispatch it to the circumference of the orb — under that condition and law of revolution which will be indicated shortly.

The general Ptolemaic hypothesis of the motions of Venus and Mercury, but conformed to geometry.

Diagram. A geometric construction of the Ptolemaic hypothesis for Venus and Mercury, conformed to geometry. The figure shows a large circle — the planet's orb — whose diameter is the straight line A B C D, drawn vertically with A at the top and D at the bottom. The Earth is placed at point C (below center). Upon a small circle near the upper-left interior, the epicycle KFL is drawn about center E, with the inner epicycle GHMN drawn about center F. Radial lines run from C outward to the labeled points N and M, and the auxiliary points P and I lie along the construction lines between the inner figures and C.

- A (top of large orb, on diameter): apex / apogee end of the diameter A B C D
- B (on diameter, above C, near center): center of the planet's orb A D
- C (on diameter, below B): the place of the Earth (tellus), in "sign C"
- D (bottom of large orb, on diameter): lower end of the diameter A B C D
- E (left-interior, center of small circle KFL): center of the epicycle KFL; semidiameter EL
- F (just right of E): center of the inner epicycle GHMN; carries point F revolving on the little circle K L
- G (upper-left, on inner epicycle): true apogee of the inner epicycle
- H (upper-left, outermost on inner epicycle): mean apogee of the inner epicycle
- K (lower part of small circle, on diameter K L): endpoint of the little-circle diameter K L (parallel to B A)
- L (upper part of small circle, on diameter K L): opposite endpoint of the little-circle diameter K L
- M (lower-left on inner epicycle): point of the morning/evening contact (digression) of the planet
- N (right, on inner epicycle): point of the evening/morning contact (digression) of the planet
- P (interior, between F and C, on line B–C): the prosthaphaeretic point P (angle B P C / F P E)

- I (interior, right of P): auxiliary point on the construction line

Upon the straight line A B C D, as a diameter, and about center B, let the planet's orb A D be described, with the Earth placed in sign C, so that the line B C is three quarters [dodrans] of the whole eccentricity. For the remaining quarter, let the semidiameter E L be taken and drawn out to the circumference of the orb. Now, with the center of the epicycle placed at E, and with the distance from A taken as the semi-quadrantal apogee, let K F L be drawn as a circle, whose diameter K L always remains parallel to B A in every revolution. Upon this circle, then, and upon the eccentricity B C of the orb, the prosthaphaeresis is worked out in the following way.

First, in the triangle B E C, the sides being given — B E the radius and B C the first eccentricity — together with the angle E B C, which is the complement of the eccentric anomaly A E, there is found both the angle B E C and the side C E. Next, since F on the little circle K L has, by hypothesis, a double revolution in the direction of the following signs with respect to A E — so that when E is at A or D, F will be at K, completing in the meantime a whole revolution, and this in Venus; whereas in Mercury in just the contrary way (as Copernicus holds, and as we shall examine in its proper place), and otherwise commensurable by a revolution and a half — therefore in this position, as always, the angle K E F is double the angle A B E, and so B E F is perpetually equal to that same A B E. The triangle C E F thus likewise has two sides known, C E and E F, together with the angle C E F, which here is greater than the angle F E B previously acquired, B E C. Consequently the said angle F C E is given, which together with the prior B E C constitutes, in this place, the whole prosthaphaeretic [quantity] of the eccentric, B P C or F P E, to be subtracted, along with the side C F.

Further, with the center placed at F and the epicycle G H M N described — in which the planet is likewise carried in the direction of the following signs — and with the straight line from C through F produced to G, and another, I H, drawn through center F parallel to B E, it is clear that the mean apogee of the epicycle is at H and the true apogee at G. The angle G F H, to which B P C is equal, is here to be added, on the contrary, so that the distance of the planet — whether at the contacts M or N, or elsewhere in the epicycle — may come out from its own true apogee. Once this is done, and with the semidiameter of the epicycle F M established, there are finally given, in the triangle C F M, again two sides C F and F M, together with the included angle C F M. Therefore the angle F C M will not be hidden, which contains at last the prosthaphaeresis of the epicycle according to

the evening digression of the planet; and in the same way the angle $F C N$ is obtained for the morning digression.

And this is the reformed Ptolemaic assumption for these planets, from which neither the Copernican nor the Tychonic departs, not even slightly, as regards the appearances — as will be gathered from those same hypotheses shortly.

The Copernican Hypothesis for Venus and Mercury.

On the line $A D$, with the center of the Sun at B on it, let the great orb of the Earth $A D$ be described. Then on its diameter — that is, on the same line $A D$ — take $B K$ as the eccentricity, and placing the center K at one third of $B K$, let the little circle $C L$ be drawn; its diameter $C L$ will accordingly be equal to $B C$.

Next, let the Earth be set at E in the revolution of its orb, advancing in the order of the signs, so that its distance here from the apogee A becomes half a quadrant. Now since the motion of F on the little circle is of double measure relative to $A E$ — and likewise in the order of the signs — let the last orb of the planet $C H M N$ be drawn as from a center at F , which lies here a quadrant away from C . From this, draw the lines from the Earth E : first $E B$ and $E K$, then $E F$ continued as far as G , and another through the center of the orb $H I$ parallel to $B E$; finally those seen at the points of contact of the orb, $E M$ and $E N$. It is then clear that everything here agrees with the appearances of the hypothesis set out above. For the two angles $B E K$ and $K E F$ here complete the whole prosthaphaeresis of the eccentricity, and the rest stand in the same way with respect to the appearances of the orb. Therefore, sparing the reader further detail, let us hurry on to set out — briefly as well — the Tychonic hypothesis.

Diagram. The Copernican inner-planet construction. A large circle (the great orb of the Earth, $A D$) is centered near the middle of the figure. The vertical diameter runs from A at the top to D at the bottom, with Q just to the lower left of D .

- A (top): apogee of the line $A D$
- P (top, right of A): endpoint of the dotted line (the planet's mean apogee direction)
- E (upper left, on the great orb): the Earth, from which the sight-lines are drawn
- B (center): the Sun, center of the great orb
- C (just below B): on the little circle, the planet-orb's reference point
- K (center of the little circle, right of C): center of the little circle $C L$
- L (just below K): on the little circle

- F (lower left of the little circle): center of the planet's orb (a quadrant from C)
- M (just above B, near the contact point): point of contact of the planet's orb (one elongation)
- N (left, on the horizontal line): the other point of contact of the planet's orb
- G, H (lower center): G the planet's true place on the orb, H its apogee; H I drawn parallel to B E
- I (left of center): on the line H I parallel to B E
- Q, D (bottom): foot of the figure

The Tychonic Hypothesis for Venus and Mercury.

In the following diagram, let the orb of the Sun A D be described about the center B. Conceiving, as before, in this same center a distance E from A of half a quadrant, let the first Tychonic epicycle K P be drawn, whose diameter P K will always be parallel to the line of the apogee B A — understood, that is, through the revolution of the center E around the orb A D. Then, fixing K as center, let a second little circle, or Tychonic epicycle, L C be described, on which the center of the planet's own orb is carried forward in the order of the signs, with a revolution double that of E on A D, beginning in Venus from C through F, but in Mercury from L to C. Finally, with F as center, let the planet's own orb about the Sun, G H M N, be drawn, as the Tychonic hypothesis carries it. Then, with B G drawn and another line H I parallel to B E, it is plain here too that H is the mean apogee of the planet's orb, while G is the true apogee. From this true apogee the place of the planet will be reckoned, in the order of the signs, to the point of contact at M or N, or to whatever part of this circumference the planet may dwell upon. In all other respects too this Tychonic supposition — which we mark with this name on account of those two little circles by which the prosthaphaeresis of the eccentric is explained — fully agrees with the foregoing ones,

Diagram. The Tychonic inner-planet construction (the right-hand figure on the page). The orb of the Sun A D is the large circle; the two small concentric circles at center carry the epicycles.

- A (top), D (bottom): the vertical diameter of the Sun's orb A D, with B at center
- P (top right): apogee point of the first Tychonic epicycle K P
- E (center): the carried center on the Sun's orb
- K, L (center): the second Tychonic epicycle L C
- C, F (center): C and F on the epicycle; F the center of the planet's own orb

- B (center): center of the Sun's orb
- M (center, near contact): point of contact of the planet's orb
- N (left): the other point of contact
- G, H (lower center): H the mean apogee, G the true apogee of the planet's orb
- I (left of center): on H I parallel to B E
- Q, D (bottom)

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and no less than the first, the Ptolemaic, yields to the Copernican demonstration in the unfolding of the prosthaphaereses, when the true motions of these planets are derived from their mean motions. So let it be, just as in the figure immediately preceding: B is the center of the Earth, about which the orb of the Sun A D is described; and when, in a like revolution, a distance E P has been laid off from the center, let the first Tychonic epicycle P K be drawn, the apogee point of which is P. Next, an arc P K equal to A E having been taken backward (in the order against the signs) along the circumference of this epicycle, let a second epicycle C F L be made. On its small circumference, by a forward motion in the order of the signs and a double commensurable revolution relative to A D, let the center F be taken; and from this same point let the last orb of the planet about the Sun, G H M N, be described, and let I H be drawn parallel to B E.

In every respect this proceeds as the Ptolemaic supposition does in our earlier treatment. I assert that the equivalence of the appearances is plainly displayed here, in all things, with the foregoing models of this chapter that rival the Copernican — and that there is no different procedure, as he supposes, in exhibiting the inequality of the eccentrics in these planets (Venus especially) from the foregoing.

For if the angle K E F has been given more compendiously, together with the side E F, there is afterwards obtained both the angle of the whole prosthaphaeresis E B F and the side B F, the distance of the center of the planet's orb from the Earth — in every respect here as in the foregoing, that is, in the Ptolemaic or Tychonic supposition, or finally in the Copernican E F. Now their equivalence, which anyone (as we ourselves also have done) will be free to test further by calculation, having thus been shown, the rest, which concerns the planet's orb, will be squared to a wholly similar measurement in all things —

Diagram. The reduction of the Tychonic construction to the Copernican (upper figure). A large circle (the planet's orb G H M N about the Sun) fills most of the

figure; two small concentric circles (the two epicycles) sit at upper left, with the Earth B at the bottom.

- G, H (top left): G the true apogee, H the mean apogee of the planet's orb
- L (center of small circles): on the epicycle
- K (left of L): center / apogee region of the first epicycle
- F (right of K): center of the planet's orb on the second epicycle
- P (left): apogee point P of the first Tychonic epicycle
- C (below K): on the second epicycle C F L
- E (below C): the carried center
- A (right, on the orb): apogee direction of A D
- N (right, near contact): point of contact of the orb
- I (lower right): on I H parallel to B E
- M (lower left, marked with a star): point of contact of the orb
- B (bottom): the Earth
- D (bottom): foot of the diameter A D

Diagram. The same equivalence shown in a second figure (lower figure). Again a large circle for the planet's orb, the two small epicycle-circles at center, and the Earth B at lower right with D below it.

- G, H (top): G the true apogee, H the mean apogee
- L (center): on the epicycle
- K (left of F): the first epicycle's reference point
- F (right of K): center of the planet's orb
- C, E (center): on the epicycle and the carried center
- A (right, on the orb): apogee direction
- N (right, marked with a star): point of contact of the orb
- I (lower right of center): on I H parallel to B E
- P (below center): apogee point of the first epicycle
- M (lower left): point of contact of the orb
- B (lower right): the Earth
- D (bottom): foot of the diameter A D

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so much so, indeed, that I should think it idle to dwell on more of these matters at present. Moreover, this supposition can readily be carried over into the Copernican, and can satisfy it in every respect — indeed, it can even be adapted,

in that manner, to the center of the eccentric orb, just as is evident in the later lunar hypothesis.

What more shall I say? So great is the kinship of the hypotheses among the planets that almost every one of them admits all the modes we have set forth through these books — modes drawn from the judgment of those craftsmen who recognize celestial motions as solely either circular or compounded of such circular ones, with only the mean motions of the orbs and their semidiameters verified against the celestial appearances. Since this labor still remains to be carried through in these two, Venus and Mercury, we shall therefore boldly undertake in what follows that same task, which is our chief concern.

CHAPTER XVIII.

ON

The Restitution of the Motions of Venus according to Longitude, in our own age.

The hypotheses for these planets having been prepared in their manifold forms, it follows that we should duly measure the restitution of their motions from the celestial observations. In attempting this, we certainly proceed by a path of unequal ease compared with the foregoing cases. For although here, in each of the two stars, it would be most fitting to have the observed greatest elongations from the Sun on both sides, which occur near the contacts of the orbs — so that, in imitation of our predecessors Ptolemy and Copernicus, we might more easily arrive from these at the knowledge both of the apogee and of the prosthaphaeresis of the eccentric, and of the size of the orb, for each planet on either side — it has happened that at other seasons this was rare, and in summer scarcely ever possible (on account of the obliquity of our sphere and the perpetual twilights) to continue observations of these planets. We have therefore been compelled to gather the scattered observations here and there made by Tycho Brahe, to which we also add our own, such as they are, though by no means doubtful — fully confident that, by an often-repeated effort at restitution, the truth may at last appear more correctly from all these collated together than from a few of them otherwise reduced. This is especially so when, in the reduction of observations of this kind, the path is so slippery — not only on account of some inequality between the heavens and the Ephemerides hitherto computed in the motions, but also because of the entanglement of refractions, which mostly cling to the observations of these stars. Such observations of Venus,

then, both Tycho's and our own, carried out at length and breadth, are included in the following little table.

Nu mb er	Ye ars	Mo nth s	D a ys	H M Longitude . . G.	M.	(sig n)	Latitude G.	M(B/ . M)
1	15 85	Sep t.	1 4	1 1 15 7 5	58	♌	2	8 M
2	15 87	Jan.	1 5	4 4 16 0	55	♋	2	3 B 9
3	15 87	Feb.	2 1	6 5 16	1	♋	8	5 B 6 *
4	15 87	Mar .	2	1 3 10 7 0	7 *	♋	8	2 B 6 *
5	15 88	Dec.	1 4	1 4 17 9 0	10 *	♏	3	1 B 0
6	15 90	Dec.	1 7	2 0 20 0	0 *	♏	0	2 B 7
7	15 94	Dec.	1 5	5 1 21 0	0	♏	1	1 M 6
8	16 10	Dec.	1 2	4 4 17 0	58	♏	1	2 M 9
9	16 16	Mar .	9 7	1 0 15 7	24	♏	— (non est obs. / "not observed")	

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These observations having first been freed, as far as could be done, from parallaxes and refractions (except the third and fourth, of which I shall give notice later), I used the following method to restore the course of Venus according to longitude.

First, for the investigation of the place of the apogee, I brought into comparison with the Prutenic Tables those observations you see acquired on both sides in December, which (among other indications) the equality of the digression on either side from the mean motion of the Sun proves to lie not far from the perigee of Venus — for it is certain that Venus constantly regards this mean motion rather than the true. Submitting these first to a rough and provisional examination, I found that the apogee of Venus is far more advanced than by the

Copernican restitution, and that the Alphonsine reckoning agrees more correctly at this time. Then, for the greatest prosthaphaeresis, together with the greatest size of the orb of Venus, I directed two of them — namely the first and the last in number — into a δοκιμασίαν [Greek: dokimasian, "test" or "proof" (accusative of δοκιμασία, an assaying/examination)]. From this I at once clearly perceived (keeping the equant of Venus at one third of the whole eccentricity, as in most of the rest) that about a twelfth part must be subtracted from the whole prosthaphaeresis, and about a three-hundredth part added to the digression from the Sun — which Copernicus calls the parallax.

But all this, as I said, I attempted with a rough hand, until the mean anomaly of the orb — insofar as it too in Venus exceeds the course assigned by the Prutenic canon — was thoroughly known. To this end I also brought in by number those two observations, namely the third and the fourth, which Tycho had once obtained with Venus nearly at the perigee of its orb — that is, at conjunction with the Sun. These he afterward carefully preserved on that account, because it had seemed marvelous that Venus should ever appear to us in that place. Indeed, on the morning of 24 February she even offered herself to be seen (if this deserves belief), after she had been sufficiently observable on the evening of the 21st of the same month — something I remember seeing among the Tychonic records. Nor would this surely have befallen us Danes had not the obliquity of our sphere here been greater, and had not, in addition, the wandering of Venus's latitude toward the north far surpassed the Ptolemaic and Copernican supposition — indeed, that of the Alphonsines too.

Let certain opticians of our age therefore look to it (that I may insert this in passing) — by what right they assert, rather boldly, that the remaining stars, like the Moon, borrow all their light from the Sun, and produce Venus as a witness for this assertion of theirs, on the grounds that she, for her visible magnitude, appears at her greatest elongations from the Sun half-full, like the Moon at quadrature. What sort of appearance, then, is she in at the near-conjunction with the Sun? But let them answer: that was caused by her own latitude of nearly 9 degrees.

But again it is asked: when the Moon stands so very little distance from the Sun, is she ever seen by us evening or morning? Therefore, from this spectacle, and from what the opticians seem to wish to prove from Venus's greatest revolution from the Sun, I do indeed judge it may be concluded that the Sun adds something to condensing the light of the stars, when they are presented to our gaze more firmly under a definite angle of vision — but that it never imparts that light to the

Moon alone. This too is not without its own mystery, unless perhaps we prefer that the remaining stars draw their light from the Sun not, like the Moon, at the surface, but rather καθόλου [Greek: katholou, "wholly / throughout"] — which, however, the traces of their tails show to belong more properly to comets.

But to return to the motions of Venus that are to be restored: from the two observations mentioned, which in the year 1587, about the spring season, in the space of nine days, were both acquired, it was most opportune to amend the anomaly of that same orb, since a change of one minute in this amounted to nearly three seconds in longitude, on account of the very great approach of Venus to the Earth at that time. These two observations, received as it were from heaven and arranged at length and breadth, I decided to retain for this reason: that I might more correctly demonstrate, by their mutual confirmation, the refractions far greater at the very threshold of the horizon than those that could be drawn out from the parallaxes, and might take heed —

the disposition of the signiferous circle [signiferi] at its setting or rising had been taken into account — from which a roughly equal excess of refractions on both sides resulted, so that we assigned about half again as much to the former as to the latter. And so, once the restitution of the said anomaly had been secured (which was done by adding $1^{\circ} 18'$ to the Prutenic value), and once that same restitution had been made more reliable, among other observations, by the sixth observation by number — in which Venus was nearer to the apogee of its epicycle — I tested all the remaining cases too, again and again, until the restitution had been brought to within a difference of a tenth of a degree, never sensibly exceeded in the rest.

I found that, at the end of the year of our Savior 1585, the apogee of Venus had moved into the preceding signs to 2 signs $29\frac{1}{4}^{\circ}$ from the true vernal equinox, but to 2 signs $29^{\circ} 8'$ from the mean. Further, the whole eccentricity of Venus is no greater than 3208 parts, of which the radius of the eccentric is 100000. Accordingly, the semidiameter of the prior eccentric epicycle is to be set at 2406 parts, and to the latter epicycle a third of this, namely 802, is to be assigned.

This eccentricity, although it comes out more contracted for us in the case of Venus than for all our predecessors — and although, gradually from Ptolemy [Ptolemaeus] down to us, it falls short by more than half a degree — I could nevertheless scarcely affirm to be truly changed. This is both because of the doubtful recording of the Sun and the stars among the ancients, and because of the refraction they perpetually neglected, which (for Venus just as for the Sun) demands a greater delay in the northern signs and therefore also a looser

derivation of the eccentricity, apparently against the truth — since, in respect of their apogees, the Sun and Venus are not far apart from one another. And surely, if anyone should say that we are appealing to refraction here in vain — since all those data are admitted into the annotations of Theon [Theon] and Ptolemy with reference to the fixed stars — yet these are so slippery and conjectural [stochastical] that it would be impossible to know what truth is to be established from them, as will be indicated below more fully and more clearly.

By the same observed positions through which the eccentricity of Venus was restored, and by others near perigee close to the contacts of the orb — after it had been established with sufficient certainty that the amplitude of that perigee was to be augmented by a 300th part, as I reported above — and with the center of the orb accordingly rightly placed at the apogee of the eccentric, the semidiameter of this orb near the contact comes out (in the same units in which the radius of the eccentric is 100000) as $72248\frac{1}{2}$, which for Copernicus [Nicolaus Copernicus] was 71930.

And so from these data we have constructed the whole hypothesis of Venus, and from it we have further proved the restitution of the motions in this star through all its previously given observed positions — with the calculation directed to the analysis of triangles (lest any error perhaps admitted into the Prutenic numbers should inconvenience us). I will now elicit one of these worked examples by way of a paradigm, and indeed according to each of the two hypotheses set out above, so that their equivalence may be shown precisely enough in the numbers as well.

Let us therefore take up for this purpose the observation numbered first, which was made at Hven [Huæna] in the year 1585, on September 15, in the morning at hour $5\frac{1}{4}$. There, with parallaxes and refractions excluded, and the rest carried out according to our supposition of the fixed stars, the star of Venus was found at $15^{\circ} 58'$ in Leo [♌]. At the same time, the mean longitude of the Sun (or of Venus) from the true vernal equinox was 6 signs $3^{\circ} 48\frac{1}{3}'$, but the restored apogee, likewise from the true equinox, was 2 signs $29^{\circ} 14\frac{1}{2}'$. With this longitude subtracted, there remains the anomaly of the eccentric, 3 signs $4^{\circ} 33\frac{1}{2}'$. Moreover, the anomaly of the epicycle (or orb) of Venus, similarly restored to the same time according to the method indicated, is found to be 7 signs $7^{\circ} 10'$.

With these things thus found, two hypotheses are laid beneath them: one Copernican, the other Ptolemaic rather than Tychonic, differing only in the placement of the eccentric's epicycles from those we gave above — so that it may be recognized that there still survives in the planets that hypothesis whose likeness we left behind in the Moon (under a different rotation, however), which

differs in no way at all from the rest as to the exhibition of the phenomenon, since the same thing occurs at the middle of the eccentric, and

behind the circumference it represents the first inequality of these same motions. First, therefore, let the φαινόμενα [Greek: phainomena, "appearances" — the observed data] presupposed for the Copernican supposition be applied as follows, taking the position just described as the true place of Venus for the prescribed time.

Diagram. A geometric construction for the Copernican version of the Venus hypothesis. A large arc (a semicircle) sweeps from the topmost point A down to the bottom point D. To the left lies point E (the observer's eye / Earth in the Copernican arrangement) with several lines drawn from it across the figure. Within the upper-middle region are point B and, nested within a small circle around it, the points C, K, and F (the eccentric-center mechanism and the little circle of doubled revolution). Below and left of this cluster are points M, P, and N; at the right margin lie points H and G where the lines from E meet the far side. The accompanying text-flow caption beside the figure reads:

The Copernican hypothesis having been described in all respects like the one set out above, except as regards the given revolution of the Earth E (that is, its distance from the apogee A), and then the planet N placed in its own orb — the rest is here conformed to that earlier construction. Therefore, since the anomaly of the eccentric A E was granted to be 3 signs $4^{\circ} 33\frac{1}{2}'$, the angle D B E is recognized, in the complement to the semicircle, as $85^{\circ} 26\frac{1}{2}'$. This angle, together with the comprehended sides B E 100000 (that is, the radius of the eccentric) and B K 2406 (the restored eccentricity), constitutes the first of the three given triangles. On account of which there is also given the angle B E K, $1^{\circ} 22' 36''$, together with the side E K 99837. Further, since by the law of the double revolution of the center of the planet's orb F about E, the Earth in its own orb...

- A (top): apex of the semicircle (apogee)
- D (bottom): foot of the diameter (perigee end)
- E (far left): the eye / Earth in the Copernican layout
- B (upper middle): the deferent/eccentric reference point
- C, K, F (nested in the little circle about B): the doubled-revolution mechanism (eccentric center K, F the center of the planet's orb)
- M, P (lower left of the cluster): auxiliary points of the construction
- N (below the cluster): the planet (Venus) in its orb

- H, G (right margin): intersections of the sightlines on the far side

let F now be removed $189^{\circ} 7'$ from C in the consequent direction, and let the angle B K E come to $93^{\circ} 10' 54''$. Therefore, with this subtracted from that, the angle F K E remains $95^{\circ} 56' 6''$. With the surrounding sides known — namely E K 99837 and K F 802 (that is, a third part of B K) — the angle K E F will not stay hidden: $0^{\circ} 27' 26''$. This angle, taken with the earlier B E K, constitutes the whole prosthaphaeresis of the eccentric for Venus at this place, $1^{\circ} 50' 2''$, which is what we first sought here.

Next, in the same latter triangle K E F, the side E F is given as 99925. Third, since the mean anomaly of the orb of Venus, H P N, is established as 7 signs $7^{\circ} 10'$, then by adding the angle H F G of $1^{\circ} 50'$ (the 2 seconds being neglected, since they are of no moment) — equal to B E F, which is the difference between the true and the mean anomaly — the said true anomaly becomes 7 signs $9^{\circ} 0'$, and therefore the angle E F N is $39^{\circ} 0'$. There are also given, in the triangle E F N, the sides E F 99925 and E N, the restored semidiameter of the orb of Venus, $72248\frac{1}{2}$. From these, finally, the angle F E N is found to be $46^{\circ} 5' 0''$: this is the commutation, or parallax of the orb, as Copernicus named it. Joined with B E F of $1^{\circ} 50'$ — since both are subtractive here — there is given the difference of the mean motion of Venus from the true, $47^{\circ} 55' 0''$. And since the mean motion stood before at 6 signs $3^{\circ} 48\frac{1}{2}'$, then with the said difference subtracted from this, Venus is found by calculation at $15^{\circ} 53\frac{1}{3}'$ in Leo [♌], differing from the observed longitude by only $4\frac{2}{3}'$. This is certainly very small, and — on account of the inclination of its orb to the signiferous circle, and the other factors that intervene in observations — could scarcely ever be determined more precisely by any hypothesis.

We will now at some point test these same motions of Venus against the other hypothesis we spoke of, to see whether the same phenomena can be drawn out from it. And if so, there can be no doubt about all the rest, that they will follow precisely the same δοκιμασίαν [Greek: dokimasian, "scrutiny" or "test"].

Let there be, as before (in the following figure), the straight line A D, upon which the Earth is set, according to Ptolemy, at B. With the given eccentricity B K, let there be drawn upon K the little circle C L, whose semidiameter is in subtriple proportion to B K. But in this little circle,

the center of the eccentric is understood to revolve in the consequence of the signs, at F, in a doubled rotation of the center of the planet's epicycle. Therefore, about center F let the half-orb of the planet itself be described according to Ptolemy, A E D — according to Ptolemy, I say; for neither the hypothesis of

Copernicus nor that of Tycho [Tycho Brahe] permits that orb to be foreign to the orb of the Earth or of the Sun, and therefore its center cannot in the least be carried around in accordance with another planet's eccentricity.

Diagram. A geometric construction for the Ptolemaic version of the Venus hypothesis. A large arc runs from the top-right point A down through B to D at the bottom right. The points O and P lie just below A at the upper right. To the left are points G and H at the margin, with E near the center as the focus of several sightlines. Below center is M; above and right of E is N (the planet). At the far right, clustered in a small circle, are F, L, K, and C — the little-circle mechanism near B.

- A (top right): apex (apogee)
- O, P (just left of and below A): points of the construction at the top
- B (right, below the little circle): the Earth's position (Ptolemaic)
- D (bottom right): foot of the diameter
- G, H (left margin): intersections of sightlines on the far side
- E (center): the eye / observation point
- N (above center): the planet (Venus)
- M (below center): auxiliary point
- F, L, K, C (in the little circle near B): the doubled-revolution mechanism (K the eccentric center, F the orb's center)

But let some Ptolemy press the objection: nowhere else is it ever permitted that an eccentricity should unfold itself upon a foreign orb — from which (he might say) it can not unreasonably be concluded that to each planet its own eccentric is more rightly assigned. For an eccentricity could be established within the epicycle at the same time, in the way we have done in several of the preceding suppositions, where the whole circumference of the primary orb is bound up in the Tychonic form; yet the disposition of the revolution will always look toward the apogee in the primary, that is, the ecliptic, circle. This was certainly not the least among the causes why we granted the Ptolemaic typology a place between the Copernican and the Tychonic — though first reduced (as we have done) to the standard of geometry. But let us press on with our undertaking.

Therefore, in this planetary orb A E D, with the center of the epicycle removed from the apogee A by 3 signs $4^{\circ} 33\frac{1}{2}'$, it follows that the center F is distant from C by a doubled measure, namely 6 signs $9^{\circ} 7'$, likewise in the consequence of the signs. Accordingly, the remainder to complete the circle, F C, of 5 signs $20^{\circ} 53'$, measures the angle C K F, about which the sides B K 2406 and K F 802 are granted

— in the units in which the radius of the orb F E is supposed to be 100000. From these, then, is found first the angle K B F, $2^{\circ} 16\frac{1}{2}'$, and then the side B F, nearly 3204. But the angle K B F here (to which the arc P O is equal, when K A and E P are parallel) is taken away from the mean anomaly of the eccentric, and there remains the angle O F E, 3 signs $2^{\circ} 17'$; and therefore the angle B F E is 2 signs $27^{\circ} 43'$. Accordingly, in the triangle B F E, with the two sides about the angle B F E (likewise given, as above) granted, there is offered by calculation, first, the whole prosthaphaeresis F E B, $1^{\circ} 50' 4''$, differing imperceptibly from the earlier one; and then the side B E 99925, plainly agreeing with the earlier F E.

On account of this agreement everywhere, the angle E B N does not differ in the least from the angle F E N in the preceding Copernican supposition, since the two are entirely alike in both cases. For the angle H E G here is equal to the angle H F G there; whence, supposing the same orb-anomaly H N in both cases, the angle N E B here will be equal to the angle N F E there. And because B E and E N in this hypothesis are equal to E F and F N in the prior one, there is therefore given here the angle E B N equal to that angle F E N — namely $46^{\circ} 5'$. And so we have shown the equivalence in these two suppositions. It would not be difficult to demonstrate this in all the rest as well, so that the appearances can be made to vary only by revolutions (which is mostly the case) and by varying the

semidiameters of the epicycles (which happens in the individual planets) — something that, although it would perhaps have been more opportune to have demonstrated from the start, nevertheless the occasion first transmitted us here, so that in luxuriant Venus we too might luxuriate in the affluence and abundance of hypotheses for setting out, and indiscriminately saving, all the phenomena of the wandering stars.

To the same standard that we left behind in one of these suppositions — especially the latter, which is similar, for the sake of brevity, to the Tychonic — the remaining positions of Venus too, computed for the previously given times of the observations, stand as follows in the comparison.

Comparison of the previously given observations of Venus with the restored hypothesis of the same.

[Margin: See a little above the observed positions of Venus with their times, etc.]

Mean longit. of Venus from true equinox (S. G. M. S.)	Anomaly of the eccentric (S. G. M. S.)	Anomaly of the epicycle (S. G. M. S.)	True longit. of Venus computed (S. G. M.)	Observed longit. of Venus (S. G. M.)	Differ. (Min.)
6 · 3 · 48 · 20	3 · 4 · 33 · 30	7 · 7 · 10 · 0	♌ 15 · 53½	♌ 15 · 58	4⅔ —
10 · 4 · 17 · 15	7 · 5 · 0 · 45	5 · 7 · 42 · 17	♋ 16 · 50½	♋ 16 · 55	4½ —
11 · 10 · 48 · 48	8 · 11 · 32 · 15	6 · 0 · 33 · 7	♋* 15 · 49¼	♋□ 16 · 1	□ 11¾ —
11 · 20 · 9 · 10	8 · 20 · 52 · 30	6 · 6 · 23 · 38	♋□ 10 · 14½	♋□ 10 · 7	□ 7½ +
9 · 3 · 52 · 14	6 · 4 · 33 · 15	7 · 19 · 2 · 10	♍ 17 · 6	♍ 17 · 10	4 —
9 · 6 · 21 · 15	6 · 6 · 59 · 25	10 · 20 · 57 · 9	♊ 20 · 4⅓	♊ 20 · 0	4⅓ +
9 · 3 · 48 · 5	6 · 4 · 21 · 45	4 · 20 · 4 · 20	♊ 21 · 6½	♊ 21 · 0	6½ +
9 · 0 · 57 · 48	6 · 1 · 9 · 30	4 · 21 · 8 · 38	♊ 18 · 4½	♊ 17 · 58	6½ +
11 · 28 · 0 · 53	8 · 28 · 6 · 15	8 · 1 · 28 · 40	♊ 15 · 24¼	♊ 15 · 24	0¼ +

CAPVT XIX.

On the derivation of the periodic motions of Venus.

[Margin: Ptolemy [Ptolemaeus], bk. 10, ch. 2; Copernicus [Nicolaus Copernicus], bk. 5, ch. 20.]

This was the restitution of the motions of Venus in longitude for our own age. But since the mean course of the Sun is also established as common to these — namely to Venus and to Mercury — it remains, therefore, that in this chapter we treat of each anomaly of Venus, and how it can be derived for every age, by taking up the recorded monuments of the ancients into the investigation, just as we did above in the rest. Our first concern, then, will be the carrying-forward of the eccentric's apogee in this star.

[Margin: Bk. 10, ch. 1 & 2.]

We found in the foregoing restitution that this carrying-forward, from Ptolemy down to us, had been far swifter than Copernicus established it to be in his own time. For the point of Venus's apsis [Aulis], which Ptolemy wished to be immobile (apart from the portion of the precession of the equinox in the meanwhile), resting at $48\frac{1}{2}^{\circ}$ of the orb of the fixed stars — we found above, at the end of the year 1585, to have been removed $89\frac{1}{4}^{\circ}$ from the true vernal equinox; and with the precession of the equinox subtracted, according to the mind of Copernicus, $61\frac{1}{3}^{\circ}$ — a difference notable enough, which extends to nearly 13° .

But here both Ptolemy and Copernicus were deceived: the former because, apart from the slippery and almost uncertain observations he trusted, he had also formed a judgment about the very slow motion of this point — scarcely even perceptible between Theon and himself; the latter because he had suspended his own judgment, as in other matters, upon Ptolemy's, and had no longer drawn out the truth of this matter through his own experiments directed at the heavens.

But the annotations of Venus with respect to the fixed stars which Ptolemy had left us, from Theon and from himself — from which he had estimated not only the position of the apogee at that time but the whole digression of Venus from its mean motion — when we first examined them, we found that, contrary to his own words, they had been removed too far on this side and that from the greatest digressions, which occur uniquely at the contact of the epicycle. So that this may be made manifest to all, let us include both in the following synopsis, together with the moments of time (very largely restored to the ancient Greek exemplar), and likewise exhibiting our own restitution of the Sun and the fixed stars — with the anomaly of the epicycle also added,

with the mean anomaly everywhere matching times of this kind, which we drew from the Prutenic Tables [tabulae Prutenicae] and corrected by subtracting 3 degrees, just as best suited the restitution of that same anomaly in this present age.

A synopsis of the observations of Venus relative to the fixed stars made long ago by Theon [Theon] and Ptolemy [Ptolemy] at Alexandria [Alexandria], together with the place of the Sun, the place of Venus, and the greatest digression of Venus from the Sun (as Ptolemy holds).

Ep oc h	Mo nt h	Time of day	Med. Solis [Mean Sun] S.	Loc. Ven. [Place of Venus] S.	Digref. Vener. à Sole [Venus's digression from the Sun] G.	
Ha dri an	Ph ar mu thi	Evenin g	♄	♅	47	
An ton inu s	Th oth	Mornin g followi ng	♈	♊	47	
Ha dri an	At hy r	Mornin g followi ng	♌	♍	47	(<i>Alma gest</i> X.1)
Ha dri an	Me ch eir	Evenin g	♎	♎	47	
Ha dri an	Epi phi	Mornin g followi ng	♅	♈	44	(<i>Alma gest</i> X.2)
Ha dri an	Ty bi	Evenin g	♍	♎	47	

[Margin, against rows 3 and 5 respectively: *Lib. 10. cap. 1.* and *Lib. 10. cap. 2.* — i.e. *Almagest* X.1 and X.2]

This synopsis was Ptolemy's record. But since these same times reckoned from the epoch of Christ, with the months and days likewise Julian, and besides with one hour in the morning before sunrise (and as much in the evening after sunset, that is, whenever Venus might at all be observable among the fixed stars that Ptolemy lists); and since to our restitution there has been added — for the Sun

and the stars reckoned from the true equinox — the single anomaly of the orb [orbis] as well: all of this is set before the eye in the following little table, computed for the horizon of Alexandria.

Nu. Ann. post Chr. [Year after Christ] Month D. H.M. Medius Solis motus [Mean motion of the Sun] S. G. M. Longitudo Veneris [Longitude of Venus] S. G. M. Distantia Solis [Distance of the Sun] G. M. Anomalia orbis Veneris [Anomaly of the orb of Venus] S. G. M.																															
--- --- --- --- --- --- --- --- --- --- --- --- --- --- --- ---																								1	132	March	8	7			
15	♈	14	49	♉	1	39	46	50	4	26	22		2	140	July	29	16	12													
♊	6	12	♋	19	1	47	11	7	26	13		3	127	October	11	17	20		♌												
	18	20	♍	2	6	46	14	7	24	2		4	136	December	25	6	0		♎	2											
	36	≈	19	36	47	0	4	27	5		5	129	May	19	16	10		♏	25	53											
♊	10	57	45	4	7	25	16		6	136	November	18	6	10		♏	26	9													
♎	13	4	46	55	4	4	17																								

Such, then, are the determinations of the places of Venus and of her digressions from the Sun, once the restitution of the Sun and stars set out in the prior book is extended back to Ptolemy's era. Any reader of understanding will readily see — once he has examined the anomaly of the orb added to each entry — how far the heaven itself and its own constant revolutions departed from the Ptolemaic program for constructing a theory of Venus on observations of this kind. As for ourselves, even though we can draw nothing very certain out of these uncertain materials, we do take this much away from them: that if the apogee of the eccentric of Venus is placed at $25\frac{1}{2}^\circ$ of Taurus [⚄], and the eccentricity and the greatest digression are then kept as we found them, the present observations generally support us better than they do Ptolemy, at least when they are subjected to a fair test. For as to the third observation, which Ptolemy ascribes to Theon and which disagrees by a full degree both from us and from the truth, I conjecture the cause to be either an anticipation of one day in Ptolemy's citing of Theon, or some other deliberate error that could serve his purpose — something that betrays itself plainly enough under careful examination, when one looks at and weighs that same star, which is the fifth in the asterism of Virgo [♍], in Ptolemy's canon and in the heaven itself. And there is no reason for anyone rashly, on account of this discrepancy from Ptolemy, to set a battering-ram against the foundation of the restored reduction of the Sun and fixed stars.

For this cannot be done without retortion that is dangerous enough, seeing that the arrangement of the Sun and the stars is almost uniform in both ages, ours and Ptolemy's; so that, from our restitution, a redundancy is offered over and above

the Ptolemaic numbers. We shall therefore keep the apogee of Venus as accepted from Ptolemy — and perhaps discovered by him from other, better observations as well, and tested rather often, and moreover increased by our own proportion for the Sun's more advanced position — namely at $25\frac{1}{2}^\circ$ of Taurus [♉] for the completed year 140 after the birth of the world's Savior; which in our own age (that was the completed year 1585) was found, as above, at $29\frac{1}{4}^\circ$ of Gemini [♊] from the true equinox. Accordingly, with the inequality of the precession of the equinox duly applied for the interval (which there was 20 minutes to be added, here 7 minutes to be subtracted), the said apogee point of Venus has been advanced by $33^\circ 18'$, and that over an interval of 1445 Julian years.

By that proportion the following correspond:

	Gr. [Degrees]	Min. [Minutes]	Sec. [Seconds]	Tert. [Thirds]
100 23 0		2	41	56
100 2		18	16	12
20		27	39	14
1		1	22	$57\frac{1}{2}$

[The left-hand brace labels these four rows collectively "Annis" — i.e. for 1000, 100, 20, and 1 years respectively.]

These quantities, suitably subtracted from the mean Sun, leave everywhere the anomaly of the eccentric that is to be entered in the table below.

Now, as concerns the anomaly of the epicycle, or orb, of Venus: for this we first examined an observation of Ptolemy's made in the year — as he reports — the second of Antoninus, on the 29th day of the month Tybi, in the morning at 4 hours 45 minutes, Alexandrian time. That time fell in the year 138 from the birth of Christ, December 15th, at 15 hours 15 minutes, in our own horizon. And since at that same time the Sun, by our restitution, was found according to its mean motion to be at $22^\circ 43'$ from the true equinox, while the star in the forehead of Scorpius [♏] (not to mention the spica of Virgo [♍], from a measurement of which Ptolemy's instrument might have erred) was at $6^\circ 54'$ of Scorpius [♏]: Venus accordingly, by Ptolemy's record, occupied $7\frac{1}{4}^\circ$ of Scorpius [♏]. For although we find the apparent place of the Moon — by his computation, which I think truer than the observation — assigned to $6^\circ 45'$ of Scorpius [♏], so that it would then form a straight line with Venus and the bright star of the forehead of Scorpius, and in that way the planet's place could be more fully verified: still, that

the Moon was in such a line, arranged toward the leading parts of the signs together with the planet and the star, no one will persuade himself who has rightly understood the Moon's course from a legitimate hypothesis that agrees with the celestial phenomena. For when, for the time prescribed by Ptolemy and the Alexandrian horizon, the mean motion of the Sun is found by our restitution at $22^{\circ} 43'$ from the true equinox, and likewise the Moon's distance from the Sun at 10 signs $19^{\circ} 14'$: from these combined it is gathered that the Moon's mean motion was at $11^{\circ} 57'$ — which Ptolemy puts at $11^{\circ} 24'$ of Scorpius [♏]. And the Moon's anomaly too, or its distance from the apogee, was $87^{\circ} 49'$, which Ptolemy also has at $87^{\circ} 30'$. Hence we also see the Moon then placed at nearly the mean elongation between last quarter and conjunction with the Sun, and likewise in very nearly the greatest subtractive prosthaphaeresis of the eccentric. On this point ours is nearly the same as Ptolemy's, namely $4^{\circ} 58'$; but in the rest of the Moon's deviation there is a fairly notable difference between us. For he heaps up only 41 minutes for the former (which is what the distance of $5^{\circ} 39'$ between the mean and true longitude of the Moon implies); we, by contrast, assert most certainly — given the Moon's disposition relative to the Sun and Earth — that it was $1^{\circ} 6'$ greater than Ptolemy's. Hence with us the Moon's own prosthaphaeresis had increased to $6^{\circ} 45'$, to be subtracted from the proposed mean longitude, so that its true longitude comes out at $5^{\circ} 12'$ of Scorpius [♏], which Ptolemy puts at $5^{\circ} 45'$ of the same sign. Then, as regards the Moon's parallax in longitude, we find it at an

altitude of about 20° above the Alexandrian horizon, and at a distance of the Moon from the Earth of 57 semidiameters, to be as many minutes at most — which Ptolemy too wished to be a full degree. It appears, therefore, that the visible longitude of the Moon at that time could not have been greater than $6\frac{1}{2}^{\circ}$ of Scorpius [♏], whereas Ptolemy nevertheless put it at $6^{\circ} 45'$ of Scorpius [♏]. I have worked this out somewhat scrupulously so that the discerning reader might recognize that the universal restitution of the Moon which we have made is not on that account undermined merely because it differs from Ptolemy — since we have sufficiently shown elsewhere that the digression of the Moon from its mean course was no other in his age than in ours. But since we have very often found more confidence wanting in Ptolemy's observations, we judge even here that that straight line cannot otherwise be excused than by supposing that the star in the forehead of Scorpius [♏] looked across at Venus and the Moon along the same line, toward the north, in such a way that the Moon itself would appear more advanced by three quarters of a degree — from which Venus could have been seen at $6^{\circ} 33'$ of Scorpius [♏]. But lest we seem to want to settle the matter here by conjecture, we therefore rather set this Ptolemaic observation aside and choose, for restoring Venus's motion through the epicycle, the ancient

observation of Timocharis [Timocharis], which Ptolemy too describes as having been made in the year 13 of Philadelphus [Ptolemy II Philadelphus], on the 17th day of the month Messori completed — or at the 12th hour, that is, in the morning at the first light of the 18th day. This year indeed falls in the year 3693 after the first leap-year of the world, or 272 before Christ, on the 11th day of October at hour 18 at Alexandria — but at Copenhagen [Hafnia] at hour 16 minutes 25. For at that time the star of Venus was seen to have exactly occulted the star opposite to Vindemiatrix [Vindemiatrix], which fixed star, by our restitution and reduction, was at that time at $3\frac{1}{4}^{\circ}$ of Virgo [♍] with a latitude of $1^{\circ} 16'$ north — and there the observation has Venus too as having lodged. At this moment of time the mean motion of the Sun was elongated from the true equinox by 6 signs $16^{\circ} 6\frac{1}{2}'$; from which the distance of Venus from it is found to have been $42^{\circ} 56\frac{1}{2}'$, so that Venus had passed beyond her greatest morning elongation. Furthermore, since the motion of Venus's apogee, reduced hither proportionally from the principles we set out for it in the same manner, is found at $16\frac{1}{4}^{\circ}$ of Taurus [♉]: therefore, when a subtraction is made of this from the place of the Sun, there is shown in the remainder the anomaly of the eccentric at 4 signs $29^{\circ} 56\frac{1}{2}'$. With these things thus prepared, let the hypothesis of Venus be drawn out in yet another form — indeed a sixth form together with those already presented — one which to Copernicus, with the eccentricity expressed at its mean and the epicycle alongside the circumference of the eccentric, is in the foregoing most familiar; but, on account of the position of the epicycle or orb of Venus, agreeable to Ptolemy or to Tycho [Tycho Brahe], in this way.

Diagram. A geometric construction of the Venus hypothesis in Tychonic/Ptolemaic form, with the eccentric circle, an equant circlet about its center, and Venus's epicycle.

- A (top of figure): apogee of the eccentric; upper end of diameter A D.
- B (upper middle, on diameter A D): center of the eccentric circle of Venus.
- C (below B, on diameter A D): the Earth (tellus); seat of eccentricity B C = 2406 (where the eccentric's radius B A = 100000).
- D (lower end of diameter, on the eccentric's circumference at right): perigee end of diameter A D.
- E (center of the small circlet near figure-center): center of the epicycle; distant from apogee A by the given anomaly of Venus (4 signs $29^{\circ} 56\frac{1}{2}'$); also the center about which the equant circlet is reckoned.

- F (left of E, on the equant circlet): center of the epicycle of Venus carried on the eccentric's circumference; when E is at apogee A, F is at perigee P of the said equant.
- P (perigee of the equant circlet — named in the text, not separately lettered on the figure): point diametrically opposite E's apogee position; the angle P E F is always equal to the angle A B E.
- G (lower left, on the epicycle): one end of the epicycle, where line C F produced meets the epicycle.
- H (lower left, just below G, on the eccentric circumference): point on the eccentric; arc H G equals the prosthaphaeretic arc B R C.
- K (just below F–E, at center): point near the epicycle's lower margin (the equant/epicycle construction).
- L (left of K, at center): point near the epicycle's lower margin.
- M (right of center, on the epicycle): the perigee line M of the epicycle; line from which the morning Venus at N is reckoned.
- N (upper left, on the epicycle): the position of the star Venus on the epicycle; locus of Venus.
- Q (upper middle, on the epicycle, between N and M): point on the epicycle (H Q N measures the mean anomaly of the epicycle).
- R (right of center, on line C F produced toward M): point at which the eccentric prosthaphaeresis is determined by angle C R B.

For the equatoreal circlet, alongside the eccentric's circumference following Copernicus's practice in the foregoing, having been relegated [there], the center of this epicycle will be E, distant from the apogee A by the given anomaly of Venus, 4 signs $29^{\circ} 56\frac{1}{2}'$; and its semidiameter E K = 802, as were found above in our own restitution — and never, as we suppose, distinctly varied in the Theonine and Ptolemaic records adduced. Since, however, the revolution of the epicycle's center F is established as commensurable, on the equant circlet, to E the center of that same [epicycle] through the eccentric — so that when E is at the apogee A, F will be at the perigee P of the said equant — the angle P E F is therefore always equal to the angle A B E.

In the working-out of the present hypothesis, then, first, in the triangle C B E, there are given: B C = 2406; B E the radius, namely 100000; and besides the included angle C B E = $30^{\circ} 3\frac{1}{2}'$ — for this is the complement to a semicircle of the given anomaly of the eccentric. From these is found the angle B E C = $0^{\circ} 42' 20''$, together with the side C E = 97940. Then, in the triangle E C F: since C E just found = 97940 is granted, and likewise E F = 802, and the included angle besides, C E F =

150° 38' 50", exceeding B E F by the acquired angle B E C — there emerges here the angle E C F = 0° 13' 34", and besides the side C F = 98655. But from the aggregate of B E C and E C F there is compounded the whole prosthaphaeresis of the eccentric, 0° 56' 3", determined by the angle C R B.

Furthermore, about the center F let the epicycle of Venus H M Q N G be described. With C F produced straight to G, and another diameter of the epicycle Q H drawn parallel to B E, and finally the star of Venus placed at N, it is plain that H Q N is the measure of the mean anomaly of the epicycle, but G H Q N contains the true anomaly — the difference of which is the arc H G, equal to the prosthaphaeretic [arc] B R C. For the rest, since it is the same anomaly that we aim to track down here, we must see how, in the triangle F C N, the three givens [δίδομενα, dídomena: "things given"] may be had for the sought angle C F N — the angle by which the morning Venus at N is made known from the perigee line M through the epicycle E. Since, then, N itself, or the place of Venus, previously stood (as reckoned from the true equinox) at 3° 10' of Virgo [♍], while the mean motion of the Sun (or of Venus) from the same equinox, at the point E, was at 16° 6½' of Libra [♎]: therefore, when the angle of the eccentric prosthaphaeresis B R C, 56' 3", is subtracted from this, the point F is left at about 15° 10½' of Libra [♎], distant in the leading direction by 42° 0½' from the observed longitude of Venus — from which the measure of the angle F C N is established.

The sides are also given: C F = 98655, and F N = 72248½, from our highest restitution, which we judge to be perpetually ratified. Therefore there comes to light

This produces the required angle C F N, 71° 55', which measures the arc M N on the epicycle. When M Q (56') is subtracted from this, and the semicircle H Q is added to the remainder, the resulting quantity is the mean anomaly of commutation [media anomalia commutationis] of Venus at the moment of this ancient observation: 250° 59'. The Prutenic Tables, when worked out, give this as 252° 6', very nearly in agreement with Ptolemy. No one who understands the matter will be surprised at the difference between the Ptolemaic restoration and ours, which here amounts to a little more than a full degree. For much must be attributed to the correction of the Sun and the fixed stars, and this whole inquiry [πραγματεία [Greek: pragmateia, "investigation"]] into restoring such an anomaly was set up at the point of nearly the greatest digression of Venus, which is hardly opportune and indeed scarcely certain enough. For here a very small change in the appearance entails a very large change in the anomaly. A far more certain test, therefore, would have been carried out nearer the apogee or perigee of the

epicycle — provided that ancient observations had been available, as ours are, at those very places. Since we are deprived of them, we must rest content with what we have found.

For extending the said anomaly (which we commonly call the anomaly of commutation) to all the ages of the world, we use here a quicker and larger shortcut than in the case of the superior planets. For after we had found Reinhold's [Erasmus Reinhold] diligence in the mean motions computed through his Prutenic Tables sufficiently sound, and had here too subjected the interval of Julian years to computation, we found this: in the year 3693 from the first bissextile [leap] year of the world, the said tables run ahead by $1^{\circ} 7'$ in the motion of commutation of Venus. But at the time of our own restoration — which was in the year of the world 5550, falling in the year 1586 from the birth of the Savior — the Prutenic Tables fall short by $1^{\circ} 18'$ in the same motion. Therefore the two combined differences — namely $2^{\circ} 25'$ over the interval of 1857 Julian years, by which our computation exceeds the Prutenic — when duly distributed among the individual divisions of time, work out from this:

	Gr.	Mi.	Sec.	Tert.
For 1000 years	1	18	5	0
100	0	7	48	30
20	0	1	33	42
1	0	0	4	41

When these have been added to the individual divisions of years in the Prutenic Tables, and the customary epochs afterward established from them, the restoration of this motion also emerges, which the table below will show in its proper place.

CHAPTER XX.

ON THE CORRECTION OF THE MOTIONS OF MERCURY BY THE LONGITUDE SUITED TO OUR OWN AGE.

In the foregoing restoration of Venus we offered several different hypotheses by which the same phenomena of Venus are accounted for. There too all the observations we brought to bear on the restoration of this planet plainly testify that its mean longitude through the ecliptic is indistinguishable from the simple longitude of the Sun. For had the eccentricity been established otherwise, the celestial appearances would by no means have agreed in every respect, and at

every distance of the Sun from the Earth — especially given the great size of the orbit, where a slight change of distance would otherwise have introduced a sufficiently perceptible change in the phenomenon. But now, since the divine mercy has so willed, we shall lay our final hand on Mercury. Astronomers down to the present have complained of its slipperiness no less than the alchemists have of terrestrial mercury [quicksilver] — astronomers who clung stubbornly and ignorantly to the traces and surmises of the ancients about this planet rather than to true celestial observations.

For our part, after we had detected the immense discrepancy in this star between the Alphonsine [calculus Alphonsinus] and the Copernican calculation — a discrepancy that sometimes rises beyond 7° — we first (as with the others) began to collect observations, which are rare for us in the case of Mercury, owing to the rather oblique situation of our sphere, since its digressions from the Sun are briefer and swifter. Then we carefully compared these same observations with the Copernican hypothesis, here too keeping the mean motion of the Sun the same, and everywhere taking, for the moments of the observations made, the Tychonic restoration — which agrees well enough with the heavens in this age — as the mean of Mercury supposed on the eccentric. From the comparison between the observed places of Mercury and those that result from Copernican calculation, we found the greatest differences near the apogee and perigee of the eccentric, the celestial phenomena always requiring a greater amplitude of the orbit, beyond two degrees, and demanding (as is proper) a somewhat swifter motion of Mercury at those same points.

Here I at once began to suspect that the Copernican epicycle — which he attaches to the orb of Mercury, and in which he makes this star ascend and descend in a straight line — was illegitimate. For in this way, with a second equant taken up from it as one in the middle, a double diminution of the orbit's phenomenon is created with respect to the apogee of the eccentric: one on account of the longer distance from the Earth, the other on account of the dimension of the planet perpetually occupying the lowest parts of the epicycle, as is [shown] sufficiently in the *Work of the Revolutions* [margin reference]. [Margin: *De Revolutionibus* V.25 — "Lib. 5. Revol. cap. 25."]

For the first trial of the restoration, then, after the position of the apogee of the eccentric — clinging in our age near the beginning of Sagittarius — had been established moderately well, in accord with Ptolemy's investigation [πραγματεία], I assumed a simple eccentricity in Mercury, as before in the Sun, but one that extended to a greatest prosthaphaeresis of 3° , or 524 parts of the kind of which

the radius of the eccentric is 10000. From this, with the operation derived through the whole body of observations according to the principles of triangles, although I found somewhat smaller differences in the expected measure of the same semidiameter of the orbit — which everywhere came out from the observations — than I had earlier found by comparison through Copernicus, yet among other things I perceived that a greater relaxation took place at the apogee of the eccentric, here and there, than the phenomena themselves required. Therefore, after everything had again been weighed generally, as is most fitting, I saw that there was need of a little circle or epicycle, in which the center either of the eccentric (according to Ptolemy) or of the orb of Mercury (according to Copernicus) would turn with a revolution commensurable with a point on the eccentric — yet not into the preceding signs at a double rate, as Copernicus wished, nor at an equal rate, as Ptolemy wished, but rather into the following signs at a triple rate relative to that point of revolution; and in such a way, moreover, that it could nonetheless take on each of the forms of the foregoing hypotheses. The following three figures present these same forms, in imitation of all three masters, generally at a single glance.

The Hypothesis of Mercury in [its three] forms. Tychonic.

Diagram. A composite plate presenting Mercury's hypothesis in three forms side by side — Ptolemaic (left), Copernican (center), and Tychonic (right) — so that the single new construction can be seen to reproduce all three. Each form shows a large eccentric circle with a small circle (the little circle or epicycle carrying centers B, D, E) near its apogee, the deferent/orbit carried on it, and the apsidal line running through the apogee region (D) down to the perigee (K) below.

- "Ptolemaica" (heading, left figure): the Ptolemaic form. Large circle with center A; horizontal diameter terminating at F (left) and H (right); vertical apsidal line through A downward to a small circle (with D above, B inside, E below) and continuing to C, then to K at the bottom arc.
- "Copernicaea" (heading, center figure): the Copernican form. Apex point A at top (drawn as a small circle on the apsidal line); the deferent arc carrying F and H; the large lower circle with C at its top, a small circle inside (E above, B inside, D below), and the apsidal line continuing to K at the bottom.
- "Tychonica" (heading, right figure): the Tychonic form. A large circle at top; on its lower part a small circle with D (top), B (inside), then E and A on the apsidal line; the deferent arc crossing through F (lower left) and H (lower right); the apsidal line descending to C and to K at the bottom.

- A — center of the figure / point on the apsidal line (recurs in each form)
- B — center of the little circle (epicycle of the centers)
- D — apogee position of the carried center
- E — point on the apsidal line within the little circle
- F — terminus of the horizontal/deferent line (left)
- H — terminus of the horizontal/deferent line (right)
- C — point on the apsidal line below the eccentric center
- K — perigee point at the foot of the apsidal line

First, on the line A C K let C be placed for the Earth in the Ptolemaic and Tychonic forms — which point we here also assign to Copernicus as the center of the great orb of the Earth. Next, let the eccentricity B be assumed, as in the two prior forms; to this corresponds A B in the Tychonic. Then, with B taken as center, let there be described a little circle or epicycle E D, of at least a fifth part (as I find) of the eccentricity C B or A B. In this little circle A D, at E, let the center of the epicycle or orb of Mercury H F be revolved in the order of the signs, by this law: that whenever the center of the planet's orb — or also of the Earth, according to Copernicus — is at the apogee of the eccentric, the center of this orb will be at D; but at the perigee K, it will be at E.

Now since the reduction or period of the said E, by a triple revolution, will (by our discovery) measure off one revolution proceeding from A, it happens that not only in these but also in certain other places along the eccentric, D falls at the greatest and least distance from C according to Ptolemy and Copernicus, but from A according to Tycho. And this is what afterward corrects all the errors that Copernicus assigned to Mercury, so that both the celestial phenomena are satisfied for our age, and our hypothesis agrees with the Ptolemaic observation of the wider digression of Mercury — near its sextile position on the eccentric — as nearly as possible on that side from which, according to Ptolemy, its greatest digression from the mean longitude, measured in Aquarius in the morning and in Gemini in the evening, is reported. For as to what this same Ptolemy strives to prove — namely, that Mercury, when in the sextile as if from the perigee, is found on either side nearer to the Earth than when it is in the perigee itself — that he can hardly maintain from the very observations he there adduces, let alone from truer ones. For in his own investigation [πραγματεία] he neither rightly applied the account of the eccentric's prosthaphaeresis in those places (that is, in the alternating evening and morning greatest digression) all at once, nor had he sufficiently exactly determined the limits of the greatest digressions. [Margin: *Ptolemy, Almagest IX.8 — "Ptol. l. 9. c. 8."*]

This will easily be proved by comparison, if anyone binds Ptolemy to his own hypothesis and to the numbers arising on either side — to say nothing of the truer correction of Mercury for the prescribed times, derived from the restored places of the Sun and the fixed stars, which we carry out in what follows. Pressing further upon this, then, within our foregoing hypothesis: after the nine more recent observations of Mercury that follow had been carefully examined by us, and the calculation reiterated several times, we found that no proportion of the eccentricity to the radius of the eccentric could be given closer or more consonant with the phenomena than that we should set, as in the preceding diagram, C B equal to 5685 very nearly, in agreement with Ptolemy, and B D equal to 1137 parts — namely a fifth part of C B — of the kind of which the radius of the eccentric is 100000; and also the radius of the orb 38500 parts of the same. With these so established, and besides with the apogee of the eccentric placed, at the end of the Christian year 1585, at $0^{\circ} 30'$ of Sagittarius, with respect to the true equinox (which is nearly the motion of Copernicus), and finally with the anomaly of the orb at the same term set at $1 \text{ sign } 13^{\circ} 2' 8''$, so that here $2\frac{1}{2}$ Prutenic degrees are added — all things from such a hypothesis of ours agree well enough with what is observed in Mercury, so far as the slippery observations of this star permit. As is seen in the following synopsis, in the customary manner, in which both the observed places of Mercury are faithfully carried over here, and freed (so far as could be done) from refractions and parallaxes, and the rest computed from the added mean motions — to a single example, which we now set forth.

An example of the fifth of the subjected places of Mercury, worked out from our new hypothesis in the Tychonian form.

In the year of Christ 1587, on January 9, at hour 4, minute 50 in the evening, we found Mercury observed at Uraniborg to be in longitude $17^{\circ} 44'$ of Virgo [♍], with a latitude of barely $2'$ North.

For the same time, from the true equinox, there is found, first, the mean motion of the Sun, or of Mercury, on its eccentric: $9 \text{ signs } 28^{\circ} 22' 30''$; with the corrected anomaly of the eccentric $1 \text{ sign } 27^{\circ} 53'$ very nearly, and the anomaly of the orb similarly corrected $4 \text{ signs } 2^{\circ} 28'$.

Now these are the mean motions of Mercury, from which, by means of the hypothesis, we hunt down the true [place] according to longitude in this way:

Diagram. A geometric construction for the worked Tychonic example of Mercury. A large primary circle of the Sun (or of Mercury), B C, is described about center A (the Earth); the vertical diameter B O serves as the index of the apsides

(with B at top, O at bottom). About the cluster of centers near the top of this circle are drawn the eccentric and the little circle: the apogee/center points D, G, E, F, and the orb-circle, with sight-lines drawn out to the labels H/I (upper left), L (right), and K (lower left), and continuing down to A.

- B (top): apex of the vertical apsidal diameter (apogee index of the primary circle)
- O (bottom): foot of the vertical apsidal diameter
- A (center): the Earth, center of the primary circle of the Sun / Mercury
- F (upper center): point near the top of the inner construction
- E (just below F): point on the apsidal line within the little circle
- D (left of center cluster): point on the eccentric construction
- G (center cluster): center point of the little circle
- C (just below the cluster): point on the apsidal line
- L (right): terminus of a sight-line on the eccentric / orb circle
- H and I (upper left, stacked): termini of sight-lines from the construction
- K (lower left): point on the orb circle (the swung-out orbit position)

In the adjoined Tychonian hypothesis — to which on this occasion it pleases [us] to accommodate these mean motions of Mercury — let there be described about A, the center of the Earth, the primary orb of the Sun, or of Mercury, B C. Then, as an index of the apsides, let the diameter B O be drawn in this primary circle. Next, for the measure of the anomaly of this eccentric (for it is permitted to call it so here too, in conformity with the constitution of the rest), which is given as $57^{\circ} 53'$, let there be removed [off]...

in the order of the signs, away from the point B about C, on which in turn the first Tychonic epicycle is described. A line A C D drawn from the Earth A points to D, the apogee of this epicycle. On the circumference of that epicycle the point E is conceived to move backward (against the order of the signs) by an arc equal to B C, namely $57^{\circ} 53'$. About this center E the second Tychonic epicycle F G is then drawn; and when the line C F is carried through E, it also marks out F as the apogee of this epicycle. From F, in the order of the signs, the point G along this epicycle will stand $173^{\circ} 39'$ away, according to the tripled measure of the revolution B C (or D E) — all of which was set out in general terms above.

Finally, then, the orb of Mercury H I K is described about the center G; its diameter I L is drawn parallel to D A; and another line is drawn from A through G to H. By this construction the true apogee H of this orb is distinguished from its mean apogee I.

On the periphery of this orb, in the order of the signs, let the planet be supposed at the sign B, according to the given mean anomaly of $122^{\circ}28'$. To this same sign the semidiameter of the orb G K is carried out from the center G, and finally the line A K from the Earth A. With these preparations made, the calculation must now be set up — by resolving plane triangles, in the manner that follows — to find the true longitude of Mercury from the given mean motions, and it is to be completed according to what we set out earlier in the trigonometry.

First of all, in triangle G C E, from C F = 5685 [Greek: διδομένοις (didomenois, "given quantities")] and E G = 1137 — in the units in which the radius A C is taken as 100000 — together with the included angle C E G, which is the complement of F G, $6^{\circ}21'$, one obtains very compendiously (by the theorem of plane triangles) first the angle E C G = $1^{\circ}35'$, then the side C G = 4551. But the angle H C G is here subtracted from the angle E C D = $57^{\circ}53'$, and there remains D C G, the equated anomaly of the eccentric, = $56^{\circ}18'$.

Next, in triangle A C G, for the angle C A G and the side G A, the [Greek: διδόμενα (didomena, "given quantities")] are A C, the radius = 100000, and C G = 4551, together with the included angle G C A = $123^{\circ}42'$ — for this is the complement of the equated anomaly D C G to a semicircle. Accordingly, from these are derived: first the angle C A G $\square 2^{\circ}7'$; then the side A G $\square 102600$, very nearly. But the angle so found, C A G — which is the prosthaphaeresis of the eccentric — when it has been subtracted from the mean motion of Mercury along the zodiac prescribed as $28^{\circ}22\frac{5}{6}'$, and added to the corrected anomaly of the orb $4s2^{\circ}28'$, yields the following: from the true equinox these appear there as $9s26^{\circ}15\frac{1}{2}'$, and here as $4s4^{\circ}35'$ — this latter motion being reckoned from the true apogee H of the orb to K.

Finally, in triangle K G A, because of G A = 102600 and likewise G K = 38500 (which we judged above to be the restored semidiameter of Mercury's orb), with the angle K G A = $55^{\circ}25'$ exhibiting the excess of H K beyond a semicircle: from these is obtained the second prosthaphaeresis G A K $\square 21^{\circ}29'$. When this is added to the true place of Mercury's eccentric, which was previously $9s26^{\circ}15\frac{1}{2}'$, we at last reach the sought true place of the longitude, $\square 17^{\circ}45' \mathbb{M}$, from the true equinox. But the observed true place was at $17^{\circ}44' \mathbb{M}$; the difference therefore is at most $1'$. From this example — or also from the table of prosthaphaereses that follows (since it is built upon the same foundation) — the remaining subjoined places of Mercury have been computed, as appears in the following synopsis.

Synopsis of the comparison of the observed places of Mercury with the restored hypothesis.

Table. Columns: index | year (Anni) | month (Mens.) | day (D.) | hour (H.) | minute (M.) | observed true longitude (Longit. obser. vera: Sign, °, ') | observed true latitude (Latit. obser. vera: °, ') | longitude from the true equinox (Longitud. à vero aequinoct.: Sign, °, ', ") | anomaly of the eccentric (Anomalia eccentrici: Sign, °, ', ") | anomaly of the orb (Anomalia orbis: Sign, °, ', ") | supposed longitude (Longitud. Supp.: Sign, °, ') | difference (Differentia: ').

M Y o e n a t r h	Obs. long. (S °)	Obs. lat. (°)	Long. from true equinox (S °''')	Anom. eccentric (S °''')	Anom. orb (S °''')	Suppos. long. (S °)	Diff . (')
1 N 5 o 8 v 5 .	♌ 13 4	B 2 18	8 4 0 15	0 3 30 0	8 16 22 20	♌ 12 57	7
1 N 5 o 8 v 5 .	♌ 25 3	B 1 25	8 12 53 20	0 12 23 0	9 14 22 30	♌ 24 58	5
1 O 5 c 8 t 6 .	♊ 22 35	Non obs.	7 13 4 30	11 12 34 0	8 5 6 30	♊ 22 28	7
1 O 5 c 8 t 6 .	♊ 26 33	B 2 17	7 16 49 30	11 16 29 0	8 17 27 0	♊ 26 28	5
1 J 5 a 8 n 7 .	♊ 17 48	B 0 1	9 28 22 50	1 16 52 0	4 2 28 20	♊ 17 49	1
1 M 5 a 9 r	♊ 13 44	B 1 42	11 23 55 20	3 23 20 0	3 11 41 10	♊ 13 36	8

M Y o e n a t r h	Obs. long. (S °)	Obs. lat. (°)	Long. from true equinox (S °′′)	Anom. eccentric (S °′′)	Anom. orb (S °′′)	Suppos. long. (S °)	Diff . (′)
0 t . 1 F 5 e 9 b 2 r . 1 M 5 a 9 j 3 u s 1 A 6 p 0 r 7 i l. 1 D 6 e 1 c 0 e .	✕ 12 20	B 0 47	10 22 50 45	2 22 15 0	3 23 8 30	✕ 12 29	9
	II 23 16	B 2 0	11 29 23 0	5 28 51 0	3 22 4 0	II 23 4	12
	✕ 21 5	B 1 40	1 3 20 50	5 2 18 0	2 15 50 6	✕ 21 12	7
	✕ 2 42	Non obs.	8 24 40 0	0 23 34 0	8 10 11 0	✕ 2 35	7

So in this way the restitution of the Mercurial phenomena has been carried through, such that — when account is taken of the parallaxes and refractions in them — the difference between the appearances obtained from the heavens and those which our now-reformed hypothesis of Mercury yields nowhere exceeds a sixth part of a degree, except in the third-from-last entry, which we borrowed from the observations of the most illustrious William, Landgrave [of Hesse]. Since that one was obtained with a sextant, by a single observer (as was the practice there at the time), and at that season of the year when perpetual twilight continues even at that place, it is no wonder that Mercury, being nearer the horizon by about 12', appeared more advanced in longitude than it should — both

on account of refractions greater than those reckoned for the fixed stars, and because of its own scintillation, by which this volatile star has more than once deceived us near its setting, presenting itself to the eye as higher than it truly is.

[Margin: See Pliny, *Natural History* II.15.]

Moreover, I think it sufficiently established by this restitution that Mercury, like Venus, regards in its revolutions the *simple* course of the Sun rather than its true course. Otherwise these planets would exhibit a far greater anomaly than the superior ones, had they (like the superior planets) adapted their motions through their eccentrics to the *true* path of the Sun. For this reason — among the causes we may conjecture — the Moon and the superior planets, circling the Earth separately, are thrust into the rays of the Sun; whereas these inferior planets, which never circle the same Earth except in company with the Sun (because of the shorter span of their circles), are permitted to be contained, according to the opinion of the ancients about all the planets, within the Sun's simple course.

CAPVT XXI.

On the further extension of the mean motions of Mercury in longitude.

[Margin: Book 9, ch. 7.]

As for the ancient observations of Mercury which Ptolemy [Ptolemy] had collected into a single chapter, we have refrained from examining them further for the restitution of the planet itself, hoping here for a sure outcome similar to the one we obtained earlier with Venus by [Greek: δοκιμασίας (dokimasia), "testing, verification"]]. For these too are found to have been made either by the crude and conjectural [Greek: στοχαστικά (stochastika), "conjectural"] assignment of the ancients relative to the fixed stars, or to have been taken with the Ptolemaic astrolabe — which is likewise not to be trusted here, since its construction, once understood, excludes precision (among other things) to within ten, if not more, minutes of arc, and since other recognized impediments during observation have counseled the same caution. Meanwhile, however, we have found that Ptolemy pronounced more correctly for his own age regarding the apogee of Mercury when in the ancient epoch — namely in the year 3700 of our world-era — it was lingering at 6° [of Libra]; this will become clear both from other comparisons and especially from those which we shall presently bring forward below, from the monuments of the ancients, for the restitution of the anomaly of the orb. Accordingly, since we possess the apogee of this same

Mercury, measured from the mean equinox, in our age — that is, in the year 1586 completed since Christ — at $8s0^{\circ}12'$ (as given above); but in the ancient epoch, in the ancient epoch, that is, 264 years before Christ, the same apogee measured from the same equinox stood at $6s[\text{degree: ?}]^{\circ}5'$ [printed "sig. 6 gr. 5 mi." — only a single "6" appears before "gr.", so the degree digit is not independently inked; possibly a dropped compositor digit or a degree reconstructed from context]. Thus over an interval of 1850 Julian years the motion of the apogee in Mercury, measured from the mean equinox, was carried forward by $54^{\circ}7'$. When this is distributed over the proposed intervals of years, the following amounts correspond to the Julian years:

Table. Apogee motion per number of Julian years — columns: number of years | degrees (Gr.) | minutes (Min.) | seconds (Sec.) | thirds (Tert.).

Years	°	'	"	'''
1000	29	15	8	6
100	2	55	30	49
20	0	35	6	10
1	0	1	45	18

From this same proportion the customary epochs are also established. And when the individual species of these are subtracted from the corresponding values of the mean Sun measured from the mean equinox, there remains the anomaly of Mercury's eccentric, which will appear afterward in the table of its mean motions.

There remains the last motion of this planet — that on the epicycle, or rather on its proper orb — which we commonly call the *anomaly of commutation*. Ptolemy certainly did not handle this successfully enough, leaving it to posterity, since he set it down as far more sluggish and slow than even Copernicus [Nicolaus Copernicus] could later correct — as appears above in our own restitution, where $2\frac{1}{4}$ degrees had to be added besides in our age. And indeed this error is by far more intolerable than any deviation of the apsides, especially in the apsides of the orb, or thereabouts incidentally, where a single degree in the more remote apsis produces about a quarter of its own size, and in the lower apsis two-thirds of itself — although near contact, back and forth, only a slight difference appears. We shall therefore most conveniently use an ancient observation for the examination and derivation of this motion, because it is found at a proper

distance below the contact of the orb. Ptolemy describes an observation of this kind, made as follows.

In the 22nd year according to Dionysius — which time is the 484th year from Nabonassar — on the 22nd day of Scorpio, by the Egyptian reckoning Thot 18, with the following day being the 19th, Mercury at morning rising stood off from the line which passes through the northern forehead of Scorpius and through the middle of the [Greek: τὰ ὑπολειπόμενα (ta hypoleipomena, "the lagging stars, those left behind")] — that is, falling short toward the preceding part by one lunar diameter; and toward the north it stood off from the northern forehead by two lunar diameters. Thus far Ptolemy. [Margin: Ptolemy, *Almagest* IX, ch. 10.]

The time of this observation, when accommodated to the years of Christ (or of the world) and to our Copenhagen horizon [Hafniensi, Copenhagen], falls in the year 265 before Christ — which was the year 3700 from the first bissextile of the world — and moreover on the 14th of November, in the morning hours, 17 hours after the noon of the preceding (stated) 14th. For this very moment these motions are gathered from the preceding tables:

Table. Columns: quantity | Sign (Sig.) | degrees (Gr.) | minutes (Min.).

Quantity	Sign	°	'
Simple [motion] of the Sun from the mean equinox	7	2	1¼ 0
Precession of the equinox	1	2	11⅔ 1
Anomaly of the same	0	1	⅓ 0
Whence the equinoctial inequality			4¾
When this is taken from the mean Sun, there remains the Sun from the true equinox	7	1	56½ 9
Likewise from the precession of the equinox, that becomes	1	2	6 11/12 [?] 1
This again from the epoch of precession in our age makes the remainder	0	2	59¼ 5

This is, of course, the elapsed precession from that ancient epoch to our age. When, therefore, we subtract this amount from the two named stars in the forehead of Scorpio, the longitude of the bright one [in the forehead] is restored to 1°36' ♍ (whose latitude is likewise 1°16' N); and that of the third of the same

Scorpio to $6^{\circ}59'$ $\mathcal{M}$. With these carefully weighed against the standard of the observation, we find the longitude of Mercury at $2^{\circ}35'$ $\mathcal{M}$, with a northern latitude of $2^{\circ}20'$.

But since we apply the prescribed mean longitude of the Sun (or of Mercury) to this hypothesis, so that from it we may reason about the said anomaly of the orb, the arrangement,

the arrangement, in the form of Copernicus, appears as follows — after, here too, the apogee being supposed according to Ptolemy at 6° [of Libra], the anomaly of the eccentric has come out as $43^{\circ}56\frac{1}{2}'$.

First, since AH in the great annual orb of the Earth is given from the anomaly of the eccentric as $43^{\circ}56\frac{1}{2}'$, its triple will be $131^{\circ}49\frac{1}{2}'$, the arc DF . Accordingly, since in triangle CBF two sides CB and BF have been granted by our restitution — the former 5685 parts, the latter 1137, in the units where the radius CH is reckoned as 100000 — there is given besides the angle included by these, $48^{\circ}10'$, being the complement of the external angle DBF . Therefore from these is found first the angle $BCF \approx 9^{\circ}45'$, then the side $CF = 5002$.

Next, in triangle CHF , the sides being likewise given — $CH = 100000$ and $CF = 5002$ — with the included angle $FCH = 145^{\circ}48'$ (for this is established through the complement of the anomaly ACH , together with the angle FCB previously found), there is finally obtained the prosthaphaeresis of the eccentric in the single angle $CHF \approx 1^{\circ}33'$. When at this point this is subtracted from the mean motion of the Earth toward H — which for Copernicus corresponds to the motion of the Sun (or of Mercury) in the eccentric, in the judgment of Tycho and Ptolemy — there remain $7s18^{\circ}24'$, very nearly, for the equated eccentric motion of H from the true vernal equinox. There is found besides, in the same proposed triangle CHF , the side $HF = 103902$. But since the true place of the star Mercury from the true equinox had previously, by the ancient observation, been found at $7s2^{\circ}35'$, in the difference between these — namely the visible digression of Mercury along its proper orb, here described as $LMPN$ about the mobile center F — there stands $15^{\circ}49'$. Therefore, when HIL is drawn, the true apogee of the epicycle (or orb) is shown at L , and likewise the mean apogee at M , MP being drawn parallel to CH . Furthermore, since the morning digression of Mercury from the equated eccentric is established as $15^{\circ}49'$, let a line HN be drawn from the Earth H into the planet's orb, which closes the triangle FHN — in which the angle FHN includes the said digression.

Diagram. A geometric construction for Mercury's orb on the Copernican plan. The large outer circle is the great annual orb; within it a smaller circle (the planet's proper orb L M P N) is drawn about a mobile center F. The labels read, from the figure:

- A (top, just right of center): endpoint of the vertical apsidal line
- C (top center): a point on the vertical apsidal line, below A
- H (upper left): the Earth, from which lines radiate to the orb
- P (left of center): a point on the planet's orb
- N (left): the planet's place on its orb, where line H N meets it
- F (center): the mobile center of the proper orb (with C and B clustered near it)
- B (center, below F): center point near F, enclosed in the small inner circle
- D (center, below B): point on the vertical apsidal line below the inner circle
- M (lower right of center): the mean apogee of the orb
- L (lower center/right): the true apogee of the epicycle/orb
- I (bottom): the lower endpoint of the vertical apsidal diameter

There are besides, in the same triangle F H N, two given sides — namely F H = 103902 and F N = 38500. The angle H F N therefore cannot but be $\approx 31^{\circ}33'$. When to this is first added a semicircle of 180° , and then from the sum is subtracted the arc L M = $1^{\circ}33'$ (which was earlier equal to C H F), there remains at last the sought motion of Mercury, continued from the mean apogee M of the orb to the planet at N: $210^{\circ}0'$ — which Copernicus found $1\frac{3}{4}^{\circ}$ greater, and Ptolemy $2^{\circ}34'$ greater. But since this is the only observation among the ancient ones especially suited to investigating the anomaly of the orb in Mercury...

it had been suitable, on account of Mercury's smaller digression from the Sun; and so we settle on that one. For convenience, when we carry this anomaly forward to the end of the year of our earlier epoch, 3700, we find it to be $356^{\circ}55'20''$ on our horizon at the meridian of the following first of January. And because this year is bissextile, we rightly take into the comparison, for our own age, the year 1588 from the birth of Christ — that is, 5552 of the world, the first bissextile of the era. To the close of that same year we likewise hold the anomaly, corrected from the earlier Tyconic observations and our own, to be very nearly $208^{\circ}1'20''$, the Prutenic value reduced to our meridian, with $2^{\circ}10'40''$ added to the same as the correction up to the assumed year.

Since, therefore, we reckon that over the interval of 1852 Julian years Mercury has completed, on its epicycle (that is, on its proper orb), 5837 whole revolutions

plus a further $211^{\circ} 6'$, it follows from this proportion of motion to time that, for each of the following classes of Julian years, the motions beyond the whole revolutions are assigned as below.

	Sig.	Gr.	Min.	Sec.	Tert.
For years 1000	0	16	0	54	25
100	2	13	36	5	26
20	0	14	43	13	6
Common year, 1	1	23	57	33	35

Moreover, the epochs of this motion — which are set at the head of the following canon of Mercury's mean motions — arise by the same proportion.

TABLE OF THE MEAN MOTIONS [TABVLA MEDIORVM MOTVVM]

Collected years [Anni collecti]

The two left-hand value-pairs give the anomaly of Venus (of the eccentric, and of the orb); the two right-hand pairs give the anomaly of Mercury (of the eccentric, and of the orb). Each value is given in signs, degrees, minutes, seconds (S. G. M. S.).

Years	Venus Ecc. (S G M S)	Venus Orb. (S G M S)	Mercury Ecc. (S G M S)	Mercury Orb. (S G M S)
Epoch of the World [Epoc. Mundi]	9 17 18 31	9 24 13 46	5 20 42 54	5 4 40 0
Of Christ [Christ.]	7 16 1 45	4 6 21 20	2 24 49 49	1 18 13 48
20	11 29 41 27	6 3 41 38	11 29 34 0	0 14 43 13
40	11 29 22 54	0 7 23 15	11 29 8 0	0 29 26 26
60	11 29 4 20	6 11 4 52	11 28 42 0	1 14 9 39
80	11 28 45 47	0 14 46 30	11 28 16 0	1 28 52 52
100	11 28 27 14	6 18 28 10	11 27 50 0	2 13 36 5
200	11 26 54 29	1 6 56 20	11 25 40 0	4 27 12 11
300	11 25 21 44	7 25 24 29	11 23 30 1	7 10 48 16
400	11 23 48 59	2 13 52 39	11 21 20 1	9 24 24 21

Years	Venus Ecc. (S G M S)	Venus Orb. (S G M S)	Mercury Ecc. (S G M S)	Mercury Orb. (S G M S)
500	11 22 16 13	9 2 20 48	11 19 10 1	0 8 0 27
600	11 20 43 28	9 20 48 58	11 17 0 1	2 21 36 33
700	11 19 10 43	10 9 17 7	11 14 50 1	5 5 12 38
800	11 17 37 57	4 27 45 17	11 12 40 1	7 18 48 44
900	11 16 5 12	11 16 13 26	11 10 30 2	10 2 24 49
1000	11 14 32 27	6 4 41 37	11 8 20 2	0 16 0 54
1100	11 12 59 43	0 23 9 46	11 6 10 2	2 29 37 0
1200	11 11 26 58	7 11 37 56	11 4 0 2	5 13 13 5
1300	11 9 54 12	2 0 6 6	11 1 50 2	7 26 49 11
1400	11 8 21 27	8 18 34 16	10 29 40 3	10 10 25 16
1500	11 6 48 42	3 7 2 25	10 27 30 3	0 24 1 22
1600	11 5 15 57	9 25 30 35	10 25 20 3	3 7 37 27
1700	11 3 43 12	4 13 58 44	10 23 10 3	5 21 13 32
1800	11 2 10 26	11 2 26 54	10 21 0 3	8 4 49 38
1900	11 0 37 41	5 20 55 4	10 18 50 4	10 18 25 43
2000	10 29 4 56	0 9 23 14	10 16 40 4	1 2 1 49
2500	10 21 21 10	9 11 44 2	10 5 50 5	1 10 2 16
3000	10 13 37 24	6 14 4 51	9 25 0 6	1 18 2 43
3500	10 5 53 38	3 16 25 39	9 14 10 7	1 26 3 10
4000	9 28 9 52	0 18 46 28	9 3 20 7	2 4 3 48
4500	9 20 26 6	9 21 7 16	8 22 30 8	2 12 4 5
5000	9 12 42 20	6 23 28 5	8 11 40 9	2 20 4 32
5500	9 4 58 34	3 25 48 13	8 0 50 10	2 28 4 59
6000	8 27 14 48	0 28 9 42	7 20 0 11	3 6 5 27
6300	8 22 36 32	8 23 34 11	7 13 30 12	10 17 53 43

OF VENUS AND MERCURY [VENERIS ET MERCVRII]

For single years, up to 20 [In annis singulis ad 20]

Year	Venus Ecc. (S G M S)	Venus Orb. (S G M S)	Mercury Ecc. (S G M S)	Mercury Orb. (S G M S)
1	11 29 44 17	7 15 1 50	11 29 43 55	1 23 57 34
2	11 29 28 34	3 0 3 40	11 29 27 50	3 17 55 7
3	11 29 12 52	10 15 5 30	11 29 11 45	5 11 52 41
4	11 29 56 17	6 0 44 20	11 29 54 48	7 8 56 39
5	11 29 40 34	1 15 46 10	11 29 38 43	9 2 54 12
6	11 29 24 52	9 0 48 0	11 29 22 38	10 26 51 46
7	11 29 9 9	4 15 49 50	11 29 6 33	0 20 49 19
8	11 29 52 34	0 1 28 39	11 29 49 35	2 17 53 17
9	11 29 36 52	7 16 30 29	11 29 33 30	4 11 50 51
10	11 29 21 9	3 1 32 19	11 29 17 25	6 5 48 24
11	11 29 5 26	10 16 34 9	11 29 1 20	7 29 45 58
12	11 29 48 52	6 2 12 59	11 29 44 33	9 26 49 56
13	11 29 33 9	1 17 14 49	11 29 28 18	11 20 47 29
14	11 29 17 26	9 2 16 38	11 29 12 12	1 14 45 3
15	11 29 1 44	4 17 18 29	11 29 56 8	3 8 42 36
16	11 29 45 9	0 2 57 18	11 29 39 11	5 5 46 34
17	11 29 29 26	7 17 59 8	11 29 23 6	6 29 44 8
18	11 29 13 44	3 3 0 58	11 29 7 1	8 23 41 41
19	11 28 58 2	10 18 2 48	11 28 50 56	10 17 39 15

Y
e

r	Venus Ecc. (S G M S)	Venus Orb. (S G M S)	Mercury Ecc. (S G M S)	Mercury Orb. (S G M S)
9				
2	11 29 41 27	6 3 41 28	11 29 34 0	0 14 43
0				

In common months [In mensibus communibus]

The month-name column [Menfis denomin.] runs January through December. The four value-pairs are, as before, Venus (eccentric, orb) and Mercury (eccentric, orb), in S. G. M. S.

Month	Venus Ecc. (S G M S)	Venus Orb. (S G M S)	Mercury Ecc. (S G M S)	Mercury Orb. (S G M S)
January [Ianuarius]	1 0 33 11	0 19 6 43	1 0 33 9	3 6 18 31
February [Februarius]	1 28 8 57	1 6 22 30	1 28 8 53	6 3 17 51
March [Martius]	2 28 42 9	1 25 29 13	2 28 42 3	9 9 36 22
April [Aprilis]	3 28 16 11	2 13 58 58	3 28 16 3	0 12 48 30
May [Majus]	4 28 49 23	3 3 5 41	4 28 49 13	3 19 7 1
June [Iunius]	5 28 23 25	3 21 35 6	5 28 23 14	6 22 19 10
July [Iulius]	6 28 56 37	4 10 42 10	6 28 56 23	9 28 37 41
August [Augustus]	7 29 29 48	4 29 48 53	7 29 29 32	1 4 56 13
September	8 29 3 52	5 18 18 38	8 29 3 33	4 8 8 20
October [October]	9 29 37 3	6 7 25 22	9 29 36 43	7 14 26 52
November	10 29 12 6	6 25 55 6	10 29 10 44	10 17 38 59
December	11 29 44 17	7 15 1 50	11 29 43 55	1 23 57 32

In bissextile months [In mensibus bissextilibus]

Month	Venus Ecc. (S G M S)	Venus Orb. (S G M S)	Mercury Ecc. (S G M S)	Mercury Orb. (S G M S)
January [Ianuarius]	1 0 33 11	0 19 6 43	1 0 33 9	3 6 18 31
February [Februarius]	1 29 8 6	1 6 59 29	1 29 8 1	6 6 24 15
March [Martius]	2 29 41 17	1 26 6 12	2 29 41 11	9 12 42 46
April [Aprilis]	3 29 15 20	2 14 35 57	2 29 15 11	0 15 54 54
May [Majus]	4 29 48 31	3 3 42 41	4 29 48 21	3 22 13 26
June [Iunius]	5 29 22 34	3 22 12 26	5 29 22 22	6 25 25 34
July [Iulius]	6 29 55 45	4 11 19 8	6 29 55 31	10 1 44 5
August [Augustus]	8 0 28 47	5 0 25 54	8 0 28 40	1 8 2 37
September	9 0 3 1	5 18 55 38	9 0 2 41	4 11 14 44
October [October]	10 0 36 12	6 8 2 21	10 0 35 51	7 17 33 17
November	11 0 10 15	6 26 32 6	11 0 9 52	10 20 45 24
December	0 0 43 26	7 15 38 50	0 0 43 3	1 27 3 56

Liber II Theoricorum — Cap. XXI (concl.) / Cap. XXII

Cap. XXI (continued) — Mean motions of Venus and Mercury

Anomaly of Venus and Mercury, in days together

This table (the left half gives the daily increments of anomaly. Columns: the day-count (1–31); the anomaly of the eccentric [anomalía eccentrici] shared by Venus and Mercury; the anomaly of the orb [anomalía orbis] of Venus; and the anomaly of the orb of Mercury. Each value is given in degrees (G.), minutes (M.), and seconds (S.).

D
a

y	Ecc. Ven. & Merc. G. M. S.	Orb. Ven. G. M. S.	Orb. Merc. G. M. S.
1	0 59 8	0 36 59	0 3 6 24
2	1 58 17	1 13 59	0 6 12 48
3	2 57 25	1 50 58	0 9 19 13
4	3 56 32	2 27 58	0 12 25 37
5	4 55 41	3 4 57	0 15 32 1
6	5 54 49	3 41 57	0 18 38 25
7	6 53 57	4 18 56	0 21 44 50
8	7 53 5	4 15 56 [recte: 4 55 56]	0 24 51 14
9	8 52 13	5 32 55	0 27 57 38
10	9 51 21	6 9 55	1 1 4 2
11	10 50 30	6 46 54	1 4 10 27
12	11 49 37	7 23 54	1 7 16 51
13	12 48 45	8 0 53	1 10 23 15
14	13 47 54	8 37 53	1 13 29 39
15	14 47 2	9 14 52	1 16 36 3
16	15 46 9	9 51 51	1 19 42 28
17	16 45 17	10 28 51	1 22 48 52
18	17 44 26	11 5 50	1 25 55 16
19	18 43 34	11 42 49	1 29 1 40
20	19 42 42	12 19 49	2 2 8 5

D a y	Ecc. Ven. & Merc. G. M. S.			Orb. Ven. G. M. S.			Orb. Merc. G. M. S.		
2 1	20	41	50	12	56	48	2	5	14 29
2 2	21	40	58	13	33	48	2	8	20 53
2 3	22	40	7	14	10	47	2	11	27 17
2 4	23	39	14	14	47	47	2	14	33 42
2 5	24	38	22	15	24	47	2	17	40 6
2 6	25	37	31	16	1	46	2	20	46 30
2 7	26	36	39	16	38	45	2	23	52 54
2 8	27	35	46	17	15	45	2	26	59 19
2 9	28	34	54	17	52	44	3	0	5 43
3 0	29	34	3	18	29	44	3	3	12 7
3 1	30	33	10	18	6	43 [recte: 19 6 43]	3	6	18 31

Anomaly of Venus and Mercury, in hours and scruples of hours

The right half carries the increments per hour and per minute (scruple of an hour). It is laid out as two side-by-side blocks. The first block is keyed by **whole hours** (Hor. & Min., column "H."); the second block is keyed by **minutes** (Minut., column "M.").

By hours (1–24): the anomaly of the eccentric of Venus and Mercury together (Eccen. Vener. & Mer. simul, H. M. S.); the anomaly of the orb of Venus (Orbis Vene., M. S.); the anomaly of the orb of Mercury (Orbis Mercurii, G. M. S.). For entries 25–30 the eccentric column shifts to seconds and thirds (the small sub-columns headed "Sec." and "Tert.").

H			
r.	Ecc. Ven. & Mer. simul H. M. S.	Orb. Ven. M. S.	Orb. Merc. G. M. S.
1	2 28	1 32	7 46
2	4 56	3 5	15 32
3	7 24	4 37	23 18
4	9 51	6 10	31 4
5	12 19	7 42	38 50
6	14 47	9 15	46 36
7	17 15	10 47	54 22
8	19 34	12 20	1 2 8
9	22 11	13 52	1 9 54
1	24 38	15 25	1 17 40
0			
1	27 6	16 57	1 25 26
1			
1	29 34	18 30	1 33 12
2			
1	32 2	20 2	1 40 58
3			
1	34 30	21 35	1 48 44
4			
1	36 58	23 7	1 56 30
5			
1	39 26	24 40	2 4 16
6			
1	41 53	26 12	2 12 2
7			
1	44 21	27 45	2 19 48
8			
1	46 49	29 17	2 27 37
9			
2	49 17	30 50	2 35 20
0			
2	51 45	32 22	2 43 6

H

r.	Ecc. Ven. & Mer. simul H. M. S.	Orb. Ven. M. S.	Orb. Merc. G. M. S.
1			
2	45 13 [recte: 54 13]	33 55	2 50 52
2			
2	46 40 [recte: 56 40]	35 27	2 58 38
3			
2	59 8	36 59	3 6 24
4			

For hours 25–30 the eccentric increment is given in seconds (Sec.) and thirds (Tert.), with the orb of Venus and orb of Mercury continuing in M. S.:

Hr

.	Ecc. (Sec. Tert.)	Orb. Ven. M. S.	Orb. Merc. M. S.
25	1 2	0 39	3 14
26	1 4	0 40	3 22
27	1 7	0 42	3 30
28	1 9	0 43	3 37
29	1 11	0 44	3 45
30	1 14	0 46	3 53

By minutes: to the right is the minute-keyed block (Minut., column "M.," 31–60), giving the same three quantities per minute of an hour: anomaly of the eccentric of Venus and Mercury together (Ecce. Ven. & M. simul, M. S.), anomaly of the orb of Venus (Orb. Ven., M. S.), and anomaly of the orb of Mercury (Orb. Mer., M. S.).

Mi

n.	Ecc. Ven. & M. simul M. S.	Orb. Ven. M. S.	Orb. Mer. M. S.
31	1 16	0 48	4 1
32	1 19	0 49	4 9
33	1 21	0 51	4 16
34	1 24	0 52	4 24
35	1 26	0 54	4 32
36	1 29	0 55	4 40
37	1 30	0 57	4 47

Mi n.	Ecc. Ven. & M. simul M. S.	Orb. Ven. M. S.	Orb. Mer. M. S.
38	1 34	0 58	4 55
39	1 36	1 0	5 3
40	1 39	1 2	5 11
41	1 41	1 3	5 18
42	1 43	1 5	5 26
43	1 46	1 6	5 34
44	1 48	1 8	5 42
45	1 51	1 9	5 50
46	1 53	1 10	5 57
47	1 56	1 12	6 5
48	1 58	1 14	6 13
49	2 1	1 15	6 21
50	2 3	1 17	6 28
51	2 6	1 18	6 36
52	2 8	1 20	6 44
53	2 11	1 22	6 52
54	2 13	1 23	6 59
55	2 15	1 25	7 7
56	2 18	1 26	7 15
57	2 20	1 28	7 23
58	2 23	1 29	7 30
59	2 25	1 41 [recte: 1 31]	7 38
60	2 28	1 42 [recte: 1 32]	7 46

CAPVT XXII

On the construction of the prosthaphaeresis canons of the two lower planets, Venus and Mercury

The simple motions of these two planets, which were treated earlier, are identical in their setup and use to those of the superior planets, so I prescribe nothing further about them. The same holds for the texture of the following canon in the case of Venus. For once the semidiameters are assumed in accordance with the

true celestial appearances, the hypothesis of Venus has been resolved by us into numbers and into a table in exactly the way Copernicus [Copernicus] used — so much so that not even here would I suppose the proportional scruples [scrupula proportionalia] to come out any differently from the Prutenic Tables [the *Prutenic Tables*].

The renewal of the Mercurial theory, on the other hand, does not depart much here from the common construction of the prosthaphaeretic canon used for the two superior planets or for Venus — provided one takes into account the threefold analogy of the small equating circle [circellus aequans], commensurable with the revolution, to the eccentric. For when this theory is restored to numbers, the proportional scruples are drawn out, in proportion, from the distance of the center of the planet's orb (that is, the epicycle) from the Earth. The difference between the greatest and least of the distances — which the apsides set down also exhibit in this star, namely 11370, of the kind in which the radius of the eccentric is 100000 — is converted into 60 and placed in the middle position of the rule of three [regula aurea], where the said difference 11370 will occupy the first place; the third place is then any distance you please reckoned from the maximum, namely 106822, by means of the anomaly of the eccentric previously computed and just now taken up. From these the proportional scruples for any given degree, appropriate to the eccentric, emerge easily. We do the same in the remaining cases as well, once we have obtained the distances together with the equations of the eccentric, by the proper operations, from the hypothesis of this star established above.

This method of constructing the proportional scruples — which we also applied above to the lunar appearances — I judge to be the most certain and most correct of all, especially for the purpose of recovering in turn the distances from which these scruples arise, since one sometimes has need of those same distances, namely for a more accurate assessment of the parallaxes and latitudes.

For the rest — and as concerns the appearances of the commutations [commutationes] — nothing new is found in this star, since it will be plain enough to those who understand that the parallax (or commutation) of the orb arises at its greatest in the apogee of the eccentric, at the given distance 106822 and the radius of Mercury's orb 38500; while the excess beyond this arises, similarly, from the least distance in the perigee of the eccentric, 95452, the same radius of the orb being retained — and this at each individual degree of the anomaly taken up.

Let these few remarks on the construction of the following prosthaphaeretic canons in Venus and Mercury suffice; their use will be demonstrated afterward in the several short precepts and examples.

Table of the prosthaphaereses of Venus

0 Sig. (zero signs — the heading at the top of the page; the foot of the page is marked **11 Sig.**, since the column is read upward as well as downward.)

This double-entry canon is read in both directions: the degree-argument runs 0→30 down the leftmost column (Gradus) and 30→0 up the rightmost column. The left half gives the eccentric prosthaphaeresis to be **subtracted** (Subtrah. — Eccentr. Profta., G. M. S.), its difference (Diff., M. S.), and the proportional scruples (Scru. prop., M. S.) with their difference (Dif., S.). The right half gives the orb prosthaphaeresis to be **added** (Adde — Orbis prosthap., G. M. S.) with its difference (Diff., M. S.), and the excess in the perigee (Excef. Perig., G. M. S.) with its difference (Diff., M. S.).

G. ow n)	Eccentr. Prostha. (Subtr.) G. M. S.	Diff. M. S.	Scru. prop. M. S.	D if. S.	Orbis prosthap. (Add) G. M. S.	Diff. M. S.	Excef. Perig. G. M. S.	Diff. M. S.	G. (u p)
0	0 0 0	1 54	0 0	0	0 0 0	24 57	0 0 0	0 28	3 0
1	0 1 54	1 54	0 0	0	0 24 57	24 57	0 0 28	0 28	2 9
2	0 3 48	1 54	0 0	1	0 49 54	24 56	0 0 56	0 28	2 8
3	0 5 42	1 54	0 1	1	1 14 50	24 55	0 1 24	0 28	2 7
4	0 7 36	1 53	0 2	1	1 39 45	24 55	0 1 52	0 28	2 6
5	0 9 29	1 53	0 3	2	2 4 40	24 55	0 2 20	0 28	2 5
6	0 11 22	1 53	0 5	3	2 29 35	24 55	0 2 48	0 28	2 4
7	0 13 15	1 53	0 8	3	2 54 30	24 54	0 3 16	0 28	2 3

G. (down)	Eccentr. Prostha. (Subtr.) G. M. S.	Diff. M. S.	Scru. prop. M. S.	D if. S.	Orbis prosthap. (Add) G. M. S.	Diff. M. S.	Excef. Perig. G. M. S.	Diff. M. S.	G. (up)
8	0 15 8	1 53	0 11	4	3 19 24	24 54	0 3 44	0 28	2 2
9	0 17 1	1 52	0 15	5	3 44 18	24 54	0 4 12	0 28	2 1
10	0 18 53	1 52	0 20	5	4 9 12	24 54	0 4 40	0 28	2 0
11	0 20 45	1 52	0 25	6	4 34 6	24 53	0 5 8	0 28	1 9
12	0 22 37	1 51	0 31	6	4 58 59	24 53	0 5 36	0 28	1 8
13	0 24 28	1 51	0 37	7	5 23 52	24 52	0 6 4	0 28	1 7
14	0 26 19	1 50	0 44	8	5 48 44	24 52	0 6 32	0 28	1 6
15	0 29 9 [recte: 0 28 9]	1 50	0 52	8	6 13 36	24 51	0 7 0	0 28	1 5
16	0 29 59	1 49	1 0	8	6 38 27	24 50	0 7 28	0 28	1 4
17	0 31 48	1 49	1 8	9	7 3 17	24 49	0 7 56	0 28	1 3
18	0 33 37	1 49	1 17	9	7 28 6	24 48	0 8 24	0 28	1 2
19	0 35 26	1 48	1 26	1 0	7 52 54	24 47	0 8 53	0 29	1 1
20	0 37 14	1 47	1 36	1 1	8 17 41	24 47	0 9 22	0 29	1 0
21	0 39 1	1 47	1 47	1 0	8 42 28	24 46	0 9 51	0 29	9
22	0 40 48	1 46	2 57 [recte: 1 57]	1 1	9 7 14	24 45	0 10 20	0 29	8

G. ow n)	Eccentr. Prostha. (Subtr.) G. M. S.	Diff. M. S.	Scru. prop. M. S.	D if. S.	Orbis prosthaph. (Add) G. M. S.	Diff. M. S.	Excef. Perig. G. M. S.	Diff. M. S.	G. (u p)
23	0 42 34	1 45	2 8	1 1	9 31 59	24 44	0 10 49	0 29	7
24	0 44 19	1 45	2 19	1 2	9 56 43	24 43	0 11 19	0 30	6
25	0 46 2	1 44	2 30	1 2	10 21 26	24 41	0 11 49	0 30	5
26	0 47 46	1 43	2 42	1 2	10 46 7	24 39	0 12 19	0 30	4
27	0 49 28	1 42	2 54	1 3	11 10 46	24 36	0 12 49	0 30	3
28	0 51 9	1 41	3 7	1 3	11 35 24	24 36	0 13 19	0 31	2
29	0 52 49	1 40	3 20	—	12 0 0	24 35	0 13 50	0 31	1
30	0 54 28	1 39	3 33	—	12 24 35	—	0 14 21	—	0

Foot labels: under the eccentric column "Adde"; under the difference and scruple columns "S."; under the orb-prosthaphaeresis and excess columns "S." The reversed page-foot heading reads **11 Sig.**

Table of the prosthaphaereses of Venus (continued)

1 Sig. (one sign — top heading; the page-foot reverse heading reads **10 Sig.**)

This is the continuation canon for the first sign of argument. Same column structure as the 0 Sig. table: degree argument down the left (Gradus, 0→30) and up the right (30→0); on the left, the eccentric prosthaphaeresis to be **subtracted** (Subtra. — Eccentr. proftha., G. M. S.) with its difference (Ad. Diff., M. S.) and the proportional scruples (Scru. Prop., M. S.) with their difference (Ad. Dif., S.); on the right, the orb prosthaphaeresis to be **added** (Adde — Orbis Profthap., G. M. S.) with its difference (Add. Diff., M. S.), and the excess in the perigee (Exceffus in perig., G. M. S.) with its difference (Add. Diff., M. S.).

G. (d o w n)	Eccent. prostha. (Subtr.) G. M. S.	Ad. Diff. M. S.	Scru. Prop. M. S.	Ad. Dif. S.	Orbis Prosthap. (Add) G. M. S.	Add. Diff. M. S.	Excessus in perig. G. M. S.	Add. Diff. M. S.	G . (u p)
0	0 54 28	1 39	3 33	14	12 24 35	24 33	0 14 21	0 31	3 0
1	0 56 7	1 38	3 47	14	12 49 8	24 32	0 14 52	0 31	2 9
2	0 57 45	1 37	4 1	15	13 13 40	24 31	0 15 23	0 31	2 8
3	0 59 22	1 36	4 16	16	13 38 11	24 30	0 15 54	0 31	2 7
4	1 0 58	1 35	4 32	16	14 2 41	24 28	0 16 25	0 31	2 6
5	1 2 33	1 33	4 48	16	14 27 9	24 26	0 16 56	0 31	2 5
6	1 4 6	1 32	5 4	16	14 51 35	24 24	0 17 27	0 31	2 4
7	1 5 38	1 31	5 21	17	15 15 59	24 22	0 17 58	0 31	2 3
8	1 7 9	1 30	5 39	18	15 40 21	24 20	0 18 29	0 31	2 2
9	1 8 39	1 29	5 57	18	16 4 41	24 18	0 19 0	0 31	2 1
10	1 10 8	1 27	6 16	19	16 28 59	24 16	0 19 31	0 31	2 0
11	1 11 35	1 26	6 35	19	16 53 15	24 14	0 20 2	0 31	1 9
12	1 13 1	1 25	6 54	19	17 17 29	24 12	0 20 34	0 32	1 8
13	1 14 26	1 24	7 13	19	17 41 41	24 9	0 21 6	0 32	1 7
14	1 15 50	1 22	7 33	20	18 5 50	24 7	0 21 38	0 32	1 6

G. (d o w n)	Eccent. prosth. (Subtr.) G. M. S.	Ad. Diff. M. S.	Scru. Prop. M. S.	Ad. Dif. S.	Orbis Prosthap. (Add) G. M. S.	Add. Diff. M. S.	Excessus in perig. G. M. S.	Add. Diff. M. S.	G . (u p)
15	1 17 12	1 21	7 53	20	18 29 57	24 5	0 22 16 [recte: 0 22 10]	0 32	1 5
16	1 18 33	1 20	8 13	20	18 54 2	24 2	0 22 42	0 32	1 4
17	1 19 53	1 18	8 34	21	19 18 4	23 59	0 23 15	0 33	1 3
18	1 21 11	1 17	8 55	21	19 42 3	23 56	0 23 48	0 33	1 2
19	1 22 28	1 15	9 17	22	20 5 59	23 54	0 24 21	0 33	1 1
20	1 23 43	1 13	9 39	22	20 29 53	23 51	0 24 54	0 33	1 0
21	1 24 56	1 12	10 1	22	20 53 44	23 48	0 25 28	0 34	9
22	1 26 8	1 11	10 24	23	21 17 32	23 45	0 26 2	0 34	8
23	1 27 19	1 10	10 47	23	21 41 17	23 42	0 26 36	0 34	7
24	1 28 29	1 8	11 10	23	22 4 59	23 39	0 27 10	0 34	6
25	1 29 37	1 6	11 33	23	22 28 38	23 35	0 27 44	0 34	5
26	1 30 43	1 4	11 57	24	22 52 13	23 32	0 28 19	0 35	4
27	1 31 47	1 2	12 21	24	23 15 45	23 28	0 28 54	0 35	3
28	1 32 49	1 0	12 45	24	23 39 13	23 23	0 29 29	0 35	2
29	1 33 49	0 59	13 10	25	24 2 36	23 19	0 30 5	0 36	1
30	1 34 48	—	13 35	25	24 25 55	—	0 30 41	0 36	0

Foot labels: under the eccentric column "Adde"; the difference, scruple, orb-prosthaphaeresis, and excess columns are footed "S." The reversed page-foot heading reads **10 Sig.**

Liber II Theoricorum, Cap. 22

Table of the prosthaphaereses of Venus [Tabula prosthaphaereseon Veneris]

The Venus prosthaphaeresis canon continues. Each two-page opening presents one zodiacal sign-block as a double-entry table: the argument [Gradus] runs 0–30 down the left margin (read with the top heading) and 30–0 up the right margin (read with the foot heading). The columns are: the eccentric prosthaphaeresis [Eccentr. Prosthap.] in degrees·minutes·seconds with its difference [Diff.]; the proportional scruples [Scrupula proportionalia] in minutes·seconds with its difference; the orb prosthaphaeresis [Orbis prosthaph.] in degrees·minutes·seconds with its difference; and the excess at perigee [Excessus in perigaeo] in degrees·minutes·seconds with its difference. The Add/Subtract directive for each column is printed at the head (for descending argument) and at the foot (for ascending argument).

2 Sig. (page running header: 433)

For the descending argument (0→30, left column): the eccentric prosthaphaeresis is **Subtract**; the orb prosthaphaeresis is **Add**; the excess at perigee is **Add**. For the ascending argument (30→0, right column) the foot directives apply.

G. (desc.)	Eccentr. Prosthap. G·M·S	Di ff. M·S	Scrup. prop. M·S	D i ff. S	Orbis prosthap. G·M·S	Di ff. M·S	Excess in Peri. G·M·S	Di ff. M·S	G. (as c.)
0	1 34 48	0 58	13 35	2 5	24 25 55	23 16	0 30 41	0 37	30
1	1 35 46	0 56	14 0	2 5	24 49 11	23 12	0 31 18	0 37	29
2	1 36 42	0 55	14 25	2 6	25 12 23	23 8	0 31 55	0 38	28
3	1 37 37	0 52	14 51	2 6	25 35 31	23 4	0 32 33	0 38	27
4	1 38 29	0 51	15 17	2 7	25 58 35	23 0	0 33 11	0 39	26
5	1 39 20	0	15 44	2	26 21 35	22	0 33 50	0	25

G. (desc.)	Eccentr. Prosthap. G·M·S	Di ff. M·S	Scrup. prop. M·S	D i ff. S	Orbis prosthap. G·M·S	Di ff. M·S	Excess in Peri. G·M·S	Di ff. M·S	G. (as c.)
		49		7		56		39	
6	1 40 9	0 47	16 11	2 8	26 44 31	22 51	0 34 29	0 39	24
7	1 40 56	0 45	16 39	2 8	27 7 22	22 46	0 35 8	0 39	23
8	1 41 41	0 43	17 7	2 8	27 30 8	22 41	0 35 47	0 39	22
9	1 42 24	0 42	17 35	2 8	27 52 49	22 36	0 36 26	0 39	21
10	1 43 6	0 40	18 3	2 8	28 15 25	22 31	0 37 6	0 40	20
11	1 43 46	0 38	18 31	2 9	28 37 56	22 25	0 37 46	0 40	19
12	1 44 24	0 36	19 0	2 9	29 0 21	22 29 [?]	0 38 27	0 41	18
13	1 45 0	0 34	19 29	2 9	29 22 41	22 14	0 39 9	0 42	17
14	1 45 34	0 32	19 58	2 9	29 44 55	22 8	0 39 51	0 42	16
15	1 46 6	0 30	20 27	3 0	30 7 3	22 2	0 40 34	0 43	15
16	1 46 36	0 29	20 57	2 9	30 29 5	21 55	0 41 17	0 43	14
17	1 47 5	0 27	21 26	3 0	30 51 0	21 49	0 42 1	0 44	13
18	1 47 32	0 24	21 56	3 0	31 12 49	21 43	0 42 45	0 44	12
19	1 47 56	0 22	22 26	3 0	31 34 32	21 37	0 43 30	0 45	11
20	1 48 18	0	22 55	2	31 56 9	21	0 44 16	0	10

G. (desc.)	Eccentr. Prosthap. G·M·S	Di ff. M·S	Scrup. prop. M·S	D i ff. S	Orbis prosthap. G·M·S	Di ff. M·S	Excess in Peri. G·M·S	Di ff. M·S	G. (as c.)
		21		9		30		46	
21	1 48 39	0	23 25	3	32 17 39	21	0 45 2	0	9
		19		0		22		46	
22	1 48 58	0	23 56	3	32 39 1	21	0 45 48	0	8
		17		1		14		46	
23	1 49 15	0	24 26	3	33 0 15	21	0 46 35	0	7
		14		0		5		47	
24	1 49 29	0	24 57	3	33 21 20	20	0 47 22	0	6
		13		1		57		47	
25	1 49 42	0	25 28	3	33 42 17	20	0 48 10	0	5
		11		1		49		48	
26	1 49 53	0 8	25 59	3	34 3 6	20	0 48 59	0	4
				1		40		49	
27	1 50 1	0 6	26 30	3	34 23 46	20	0 49 48	0	3
				1		32		49	
28	1 50 7	0 5	27 1	3	34 44 18	20	0 50 38	0	2
				1		24		50	
29	1 50 12	0 3	27 32	3	35 4 42	20	0 51 29	0	1
				1		15		51	
30	1 50 15		28 13		35 24 57		0 52 21	0	0
								52	
Foot direct ive	Adde	S.		S.	Subtr.	S.		S.	G. (as c.)

Foot heading of this block: **9 Sig.** [i.e. the right/ascending margin serves sign 9].

3 Sig. (page running header: 434 THEORICORVM Cap.22.)

For the descending argument: eccentric prosthaphaeresis **Subtract**; orb prosthaphaeresis **Add**; excess at perigee **Add**.

G. (desc.)	Eccentr. Prosthap. G·M·S	Di ff. M·S	Scrup. prop. M·S	D i ff . S	Orbis prosthap. G·M·S	Di ff. M· S	Excess in perig. G·M·S	Di ff. M·S	G. (as c.)
0	1 50 15	0 1	28 3	3 2	35 24 57	20 5	0 52 21	0 53	30
1	1 50 16	0 1	28 35	3 1	35 45 2	19 54	0 53 14	0 53	29
2	1 50 15	0 4	29 6	3 2	36 4 59	19 43	0 54 7	0 54	28
3	1 50 11	0 5	29 38	3 1	36 24 39	19 32	0 55 1	0 55	27
4	1 50 6	0 8	30 9	3 2	36 44 11	19 21	0 55 56	0 56	26
5	1 49 58	0 10	30 41	3 1	37 3 32	19 10	0 56 52	0 57	25
6	1 49 48	0 11	31 21	3 2	37 22 42	18 58	0 57 49	0 58	24
7	1 49 37	0 13	31 44	3 1	37 41 40	18 46	0 58 47	0 58	23
8	1 49 24	0 16	32 15	3 2	38 0 26	18 33	0 59 45	0 59	22
9	1 49 8	0 18	32 47	3 1	38 18 59	18 20	1 0 44	1 0	21
10	1 48 50	0 19	33 18	3 2	38 37 19	18 6	1 1 44	1 1	20
11	1 48 31	0 21	33 50	3 1	38 55 25	17 52	1 2 45	1 3	19
12	1 48 10	0 24	34 21	3 1	39 13 17	17 37	1 3 48	1 4	18
13	1 47 46	0 25	34 52	3 2	39 30 54	17 22	1 4 52	1 5	17
14	1 47 21	0 27	35 24	3 2	39 48 16	17 7	1 5 57	1 6	16

G. (desc.)	Eccentr. Prosthap. G·M·S	Di ff. M·S	Scrup. prop. M·S	D i ff . S	Orbis prosthap. G·M·S	Di ff. M· S	Excess in perig. G·M·S	Di ff. M·S	G. (as c.)
15	1 46 54	0 29	35 56	3 1	40 5 23	16 51	1 7 3	1 7	15
16	1 46 25	0 32	36 27	3 2	40 22 14	16 33	1 8 10	1 8	14
17	1 45 53	0 34	36 59	3 1	40 38 47	16 14	1 9 18	1 10	13
18	1 45 19	0 35	37 30	3 1	40 55 1	15 55	1 10 28	1 11	12
19	1 44 44	0 37	38 1	3 1	41 10 56	15 36	1 11 39	1 13	11
20	1 44 7	0 40	38 22 [?]	3 1	41 26 32	15 16	1 12 52	1 14	10
21	1 43 27	0 41	39 3	3 1	41 41 48	14 55	1 14 6	1 15	9
22	1 42 46	0 43	39 44	3 1	41 56 43	14 34	1 15 21	1 17	8
23	1 42 3	0 45	40 5	3 0	42 11 17	14 12	1 16 38	1 18	7
24	1 41 18	0 47	40 35	3 0	42 25 29	13 50	1 17 56	1 19	6
25	1 40 31	0 48	41 5	2 9	42 39 19	13 26	1 19 15	1 22	5
26	1 39 43	0 50	41 34	3 0	42 52 45	13 0	1 20 36	1 23	4
27	1 38 53	0 53	42 4	2 9	43 5 45	12 32	1 21 59	1 24	3
28	1 38 0	0 54	42 33	2 9	43 18 17	12 3	1 23 23	1 26	2
29	1 37 6	0 56	43 1	2 9	43 30 20	11 33	1 24 49	1 28	1

G. (desc.)	Eccentr. Prosthap. G·M·S	Di ff. M·S	Scrup. prop. M·S	D i ff . S	Orbis prosthap. G·M·S	Di ff. M· S	Excess in perig. G·M·S	Di ff. M·S	G. (as c.)
30	1 37 10		43 30		43 41 53		1 26 17		0
Foot direct ive	Addē	S. A.			Subtr.	S.		S.	G. (as c.)

Foot heading of this block: **8 Sig.**

4 Sig. (page running header: Cap. 22. LIBER SECVNDVS. 435)

For the descending argument: eccentric prosthaphaeresis **Subtract**; orb anomaly prosthaphaeresis **Add**; excess at perigee **Add**. (The orb column on this page is headed *Orbis An. Prosthap.* — orb-anomaly prosthaphaeresis.)

G. (desc)	Eccentr. Prosthap. G·M·S	Sub. Diff. M·S	Scru. Prop. M·S	D i f. S	Orbis An. Prosthap. G·M·S	Di ff. M· S	Excess in perig. G·M·S	Di ff. M· S	G. (as c.)
0	1 36 10	0 58	43 30	2 8	43 41 53	11 2	1 26 17	1 30	30
1	1 35 12	0 59	43 58	2 9	43 52 55	10 33	1 27 47	1 32	29
2	1 34 13	1 1	44 27	2 8	44 3 28	10 0	1 29 19	1 35	28
3	1 33 12	1 3	44 55	2 8	44 13 28	9 26	1 30 54	1 37	27
4	1 32 9	1 5	45 23	2 8	44 22 54	8 50	1 32 31	1 39	26
5	1 31 4	1 7	45 51	2 8	44 31 44	8 11	1 34 10	1 40	25
6	1 29 57	1 8	46 19	2 8	44 39 55	7 31	1 35 50	1 43	24
7	1 28 49	1 9	46 47	2 7	44 47 26	6 50	1 37 33	1 45	23

G. (desc)	Eccentr. Prosthap. G·M·S	Sub. Diff. M·S	Scru. Prop. M·S	D i f. S	Orbis An. Prosthap. G·M·S	Di ff. M· S	Excess in perig. G·M·S	Di ff. M· S	G. (as c.)
8	1 27 40	1 11	47 14	2 7	44 54 16	6 8	1 39 18	1 47	22
9	1 26 29	1 13	47 41	2 7	45 0 24	5 15	1 41 5	1 49	21
10	1 25 15	1 15	48 8	2 6	45 5 39	4 30	1 42 54	1 52	20
11	1 24 1	1 17	48 34	2 5	45 10 9	3 42	1 44 46	1 54	19
12	1 22 44	1 19	48 59	2 5	45 13 51	2 49	1 46 40	1 57	18
13	1 21 25	1 19	49 24	2 5	45 16 40	1 52	1 48 37	1 59	17
14	1 20 6	1 20	49 49	2 4	45 18 32	0 48	1 50 36	2 2	16
15	1 18 46	1 22	50 13	2 4	45 19 20	0 13	1 52 38	2 4	15
16	1 17 24	1 24	50 37	2 4	45 19 7	1 27	1 54 42	2 7	14
17	1 16 0	1 26	51 1	2 3	45 17 40	2 17	1 56 49	2 9	13
18	1 14 34	1 27	51 24	2 3	45 15 23	3 40	1 58 58	2 12	12
19	1 13 7	1 28	51 47	2 2	45 11 43	4 50	2 1 10	2 15	11
20	1 11 39	1 29	52 9	2 2	45 6 45	6 20	2 3 25	2 17	10
21	1 10 10	1 30	52 31	2 2	45 0 25	7 46	2 5 42	2 18	9
22	1 8 40	1 32	52 53	2 2	44 52 39	9 19	2 8 0	2 20	8
23	1 7 8	1 33	53 15	2	44 43 20	10	2 10 20	2	7

G. (desc)	Eccentr. Prosthap. G·M·S	Sub. Diff. M·S	Scru. Prop. M·S	D i f. S	Orbis An. Prosthap. G·M·S	Di ff. M· S	Excess in perig. G·M·S	Di ff. M· S	G. (as c.)
				1		55		22	
24	1 5 35	1 35	53 36	2	44 32 25	12	2 12 42	2	6
				0		39		24	
25	1 4 0	1 36	53 56	2	44 19 46	14	2 15 6	2	5
				0		28		26	
26	1 2 24	1 37	54 16	1	44 5 18	16	2 17 32	2	4
				9		23		27	
27	1 0 47	1 38	54 35	1	43 48 55	18	2 19 56	2	3
				9		26		27	
28	0 59 9	1 39	54 54	1	43 30 29	20	2 22 26	2	2
				8		38		26	
29	0 57 30	1 40	55 12	1	43 9 51	22	2 24 52	2	1
				8		55		24	
30	0 55 50		55 30		42 46 56		2 27 16		0
Foot direc tive	Adde	Ad.		S .	Subtr.	S. A.		S. A.	G. (as c.)

Foot heading of this block: **7 Sig.**

5 Sig. (page running header: 436 THEORICORVM Cap.22.)

For the descending argument: eccentric prosthaphaeresis **Subtract**; orb prosthaphaeresis **Add**; excess at perigee **Subtract**. (At the foot, the directives are: eccentric **Add**; orb **Subtract**; excess **Add**.)

G. (desc)	Eccentr. Prosthap. G·M·S	Diff e. M·S	Scrup. propo. M·S	D i f. S	Orbis Prosthap. G·M·S	Differ . G·M·S	Excess in perig. G·M·S	Di f. M· S	G. (as c.)
0	0 55 50	1	55 30	1	42 46 56		2 27 16		30
		41		7					
1	0 54 9	1	55 47	1	42 21 33	0 25	2 29 39	2	29
		43		6		23		23	

G. (desc)	Eccentr. Prosthap. G·M·S	Diff e. M·S	Scrup. propo. M·S	D i Orbis Prosthap. G·M·S	Differ G·M·S	Excess in perig. G·M·S	Di f. M· S	G. (as c.)
2	0 52 26	1 44	56 3	1 41 53 34 6	0 27 59	2 32 0	2 21	28
3	0 50 42	1 45	56 19	1 41 22 49 5	0 30 45	2 34 19	2 19	27
4	0 48 57	1 45	56 34	1 40 49 8 5	0 33 41	2 36 22	2 13	26
5	0 47 13	1 46	56 49	1 40 12 19 4	0 36 49	2 38 38	2 6	25
6	0 45 27	1 47	57 3	1 39 32 12 4	0 40 7	2 40 34	1 56	24
7	0 43 40	1 48	57 17	1 38 48 34 4	0 43 38	2 42 20	1 46	23
8	0 41 52	1 49	57 31	1 38 1 12 3	0 47 22	2 43 54	1 34	22
9	0 40 3	1 50	57 44	1 37 9 54 3	0 51 18	3 45 11 [recte: 2 45 11]	1 17	21
10	0 38 13	1 50	57 57	1 36 14 27 2	0 55 37	2 46 5	0 54	20
11	0 36 23	1 51	58 9	1 35 14 34 2	0 59 53	2 46 42	0 37	19
12	0 34 22	1 51	58 21	1 34 10 4 2	1 4 30	2 46 49	0 7 [?]	18
13	0 32 41	1 52	58 33	1 33 0 42 1	1 9 22	2 46 24	0 25	17
14	0 30 49	1 53	58 44	1 31 46 17 0	1 14 25	2 45 20	1 4	16
15	0 28 56	1 53	58 54	8 30 26 34	1 19 43	2 40 34	1 46 [?]	15
16	0 27 3	1 54	59 2	7 29 1 25	1 25 9	2 40 57 [?]	2 37	14

G. (desc)	Eccentr. Prosthap. G·M·S	Diff e. M·S	Scrup. propo. M·S	D i Orbis Prosthap. S G·M·S	Differ G·M·S	Excess in perig. G·M·S	Di f. M· S	G. (as c.)
17	0 25 9	1 54	59 9	7 27 30 36	1 30 49	2 37 24	3 33	13
18	0 23 15	1 55	59 16	7 25 54 3	1 36 33	2 32 48	4 36	12
19	0 21 20	1 55	59 23	6 24 11 38	1 42 25	2 27 5	5 43	11
20	0 19 25	1 56	59 28	6 22 23 25	1 48 13	2 20 4	7 1	10
21	0 17 30	1 56	59 34	5 20 29 21	1 54 4	2 11 49	8 15	9
22	0 15 34	1 56	59 39	4 18 29 45	1 59 36	2 2 3	9 46	8
23	0 13 38	1 56	59 43	4 16 24 42	2 5 3	1 50 14	11 9	7
24	0 11 42	1 57	59 47	3 14 14 33	2 10 9	1 38 22	12 32	6
25	0 9 45	1 57	59 50	3 11 59 52	2 14 41	1 24 24	13 58	5
26	0 7 48	1 57	59 53	3 9 41 2	2 18 12	1 9 17	15 7	4
27	0 5 51	1 57	59 56	2 7 18 56	2 22 6	0 52 56	16 22	3
28	0 3 54	1 57	59 58	1 4 54 4	2 24 52	0 35 40	17 8	2
29	0 1 57	1 57	59 59	1 2 27 31	2 26 33	0 18 4	17 44	1
30	0 0 0		60 0	0 0 0	2 27 31	0 0 0	18 4	0
Foot direc tive	Adde	Ad.		Subtr.	Add.		Ad d	G. (as c.)

Foot heading of this block: **6 Sig.**

Liber II Theoricorum, Cap. XXII

Table of the prosthaphaereses of Mercury [Tabula prosthaphæreseon Mercurii]

These are the continuation of the double-entry canon for Mercury, giving for each degree of argument (Gradus): the eccentric prosthaphaeresis [Eccentr. prosthap.] with its difference, the proportional scruples [Scru. Prop.] with their difference, the orb prosthaphaeresis [Orbis Prosthap.] with its difference, and the excess at perigee [Excessus in perig.] with its difference. The operating instruction printed in each super-header — "Subtrahe" (subtract), "Adde" (add) — and the foot-of-column note ("Adde", "Sub.", "S.", "S. A.", "Su.Ad") govern whether the tabulated quantity is added or subtracted; these are transcribed verbatim. The right-hand Gradus column gives the complementary (mirrored) argument 30→0 for use with the sign printed at the foot of the page.

0 Sig. (page 437)

Column operating-headers: Eccentr. prosthap. = **Subtrahe**; Scru. Prop. and Orbis Prosthap. and Excessus in perig. = **Adde**.

G r a d u s	Eccentr. prosthap. (G M S)	Diff. (M S)	Scru. Prop. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff. (M S)	Excessus in perig. (G M S)	Diff. (M S)	G r a d u s
0	0 0 0	—	0 0	—	0 0 0	—	0 0 0	—	3 0
1	0 1 54	1 54	0 1	0	0 15 54	15 54	0 1 20	1 20	2 9
2	0 3 47	1 53	0 2	1	0 31 48	15 54	0 2 40	1 20	2 8
3	0 5 41	1 54	0 3	1	0 47 41	15 53	0 4 1	1 21	2 7
4	0 7 35	1 54	0 6	3	1 3 34	15	0 5 22	1 21	2

G r a d u s	Eccentr. prosthap. (G M S)	Diff. (M S)	Scru. Prop. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff. (M S)	Excessus in perig. (G M S)	Diff. (M S)	G r a d u s
						53			6
5	0 9 29	1 54	0 10	4	1 19 26	15	0 6 43	1 21	2
						52			5
6	0 11 23	1 54	0 17	7	1 35 18	15	0 8 4	1 21	2
						52			4
7	0 13 18	1 55	0 24	7	1 51 9	15	0 9 25	1 21	2
						51			3
8	0 15 13	1 55	0 32	8	2 6 59	15	0 10 46	1 21	2
						50			2
9	0 17 9	1 56	0 42	10	2 22 49	15	0 12 8	1 22	2
						50			1
1 0	0 19 6	1 57	0 53	11	2 38 40	15	0 13 30	1 22	2
						50			0
1 1	0 21 4	1 59	1 4	11	2 54 30	15	0 14 51	1 21	1
						50			9
1 2	0 23 3	1 59	1 16	12	3 10 19	15	0 16 12	1 21	1
						49			8
1 3	0 25 2	1 59	1 29	13	3 26 8	15	0 17 32	1 20	1
						49			7
1 4	0 27 1	1 59	1 31	12	3 41 57	15	0 18 53	1 21	1
						49			6
1 5	0 29 0	1 59	1 43	12	3 57 46	15	0 20 14	1 21	1
						49			5
1 6	0 31 0	2 0	1 55	12	4 13 34	15	0 21 34	1 20	1
						48			4
1 7	0 33 0	2 0	2 8	13	4 29 22	15	0 22 55	1 21	1
						48			3
1 8	0 35 0	2 0	2 26	18	4 45 9	15	0 24 16	1 21	1
						47			2
1	0 37 0	2 0	2 47	21	5 0 55	15	0 25 38	1 22	1

G r a d u s	Eccentr. prosthap. (G M S)	Diff. (M S)	Scru. Prop. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff. (M S)	Excessus in perig. (G M S)	Diff. (M S)	G r a d u s
9						47			1
2 0	0 39 0	2 0	3 10	23	5 16 41	15 46	0 27 0	1 22	1 0
2 1	0 41 0	2 0	3 35	25	5 32 25	15 44	0 28 22	1 22	9
2 2	0 43 0	2 0	4 0	25	5 48 3	15 38	0 29 44	1 22	8
2 3	0 45 0	2 0	4 26	26	6 3 35	15 32	0 31 7	1 23	7
2 4	0 47 0	2 0	4 50	24	6 19 1	15 26	0 32 30	1 23	6
2 5	0 49 1	2 1	5 15	25	6 34 21	15 20	0 33 53	1 23	5
2 6	0 51 4	2 3	5 39	24	6 49 34	15 13	0 35 16	1 23	4
2 7	0 53 9	2 5	6 4	25	7 4 42	15 8	0 36 39	1 23	3
2 8	0 55 17	2 8	6 28	24	7 19 45	15 3	0 38 2	1 24	2
2 9	0 57 28	2 11	6 52	24	7 34 44	14 59	0 39 26	1 24	1
3 0	0 59 42	2 14	7 16	24	7 49 40	14 56	0 40 50	1	0

Foot operating-notes: Eccentr. prosthap. → **Adde**; Scru. Prop. → **Sub.**; Dif. → **S.**;
Orbis Prosthap. → **Subtr.**; Diff. → **Sub.**; Excessus in perig. → **A.**

Sign at foot of page: **11 Sig.**

1 Sig. (page 438)

Column operating-headers: Eccentri prosthap. = **Subtrahe**; Scrup. propo. = **Add.**; Orbis Profthaph. and Excessus in perig. = **Adde**.

G r a d u s	Eccentr. prosthap. (G M S)	Diff. (M S)	Scrup. propo. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff e. (M S)	Excessus in perig. (G M S)	Diff . (M S)	G r a d u s
0	0 59 42	2 16	7 16	24	7 49 40	14 54	0 40 50	1 24	3 0
1	1 1 58	2 18	7 40	24	8 4 34	14 51	0 42 14	1 24	2 9
2	1 4 16	2 20	8 4	25	8 19 25	14 49	0 43 38	1 24	2 8
3	1 6 36	2 22	8 29	25	8 34 14	14 47	0 45 2	1 24	2 7
4	1 8 58	2 24	8 54	25	8 49 1	14 45	0 46 26	1 24	2 6
5	1 11 22	2 25	9 19	26	9 3 46	14 43	0 47 50	1 24	2 5
6	1 13 47	2 24	9 45	27	9 18 29	14 41	0 49 14	1 24	2 4
7	1 16 10	2 24	10 12	29	9 33 10	14 39	0 50 58 [recte: 0 50 38?]	1 24	2 3
8	1 18 34	2 24	10 41	31	9 47 49	14 37	0 52 2	1 24	2 2
9	1 20 58	2 24	11 13	33	10 2 26	14 35	0 53 26	1 24	2 1
10	1 23 22	2 24	11 46	34	10 17 1	14 32	0 54 50	1 25	2 0
11	1 25 46	2 24	12 20	35	10 31 33	14 27	0 56 15	1 25	1 9
11	1 28 10	2 25	12 55	34	10 46 0	14	0 57 40	1	1

G r a d u s	Eccentr. prosthap. (G M S)	Diff. (M S)	Scrup. propo. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff e. (M S)	Excessus in perig. (G M S)	Diff . (M S)	G r a d u s
2						22		25	8
1 3	1 30 35	2 25	13 29	34	11 0 22	14 17	0 59 5	1 25	1 7
1 4	1 33 0	2 25	14 3	34	11 14 39	14 11	1 0 30	1 25	1 6
1 5	1 35 25	2 25	14 37	34	11 28 50	14 5	1 1 55	1 26	1 5
1 6	1 37 50	2 25	15 11	34	11 42 55	13 58	1 3 21	1 26	1 4
1 7	1 40 15	2 25	15 45	35	11 56 53	13 51	1 4 47	1 26	1 3
1 8	1 42 40	2 25	16 20	35	12 10 44	13 44	1 6 13	1 26	1 2
1 9	1 45 5	2 26	16 55	35	12 24 28	13 36	1 7 39	1 26	1 1
2 0	1 47 31	2 26	17 31	36	12 38 4	13 30	1 9 5	1 27	1 0
2 1	1 49 57	2 26	18 7	36	12 51 35	13 26	1 10 32	1 27	9
2 2	1 52 23	2 26	18 43	36	13 5 1	13 20	1 11 59	1 27	8
2 3	1 54 49	2 26	19 20	37	13 18 21	13 14	1 13 26	1 27	7
2 4	1 57 15	2 27	19 56	36	13 31 35	13 9	1 14 53	1 27	6
2 5	1 59 32	2 27 [recte: 2 17?]	20 33	37	13 44 44	13 3	1 16 20	1 28	5
2 6	2 1 59	2 27	21 10	37	13 57 47	12 56	1 17 48	1 28	4

G r a d u s	Eccentr. prosthap. (G M S)	Diff. (M S)	Scrup. propo. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff e. (M S)	Excessus in perig. (G M S)	Diff . (M S)	G r a d u s
2 7	2 4 26	2 27	21 47	38	14 10 43	12 49	1 19 16	1 28	3
2 8	2 6 53	2 27	22 25	37	14 23 32	12 41	1 20 44	1 28	2
2 9	2 9 20	2 27	23 2	37	14 36 13	12 34	1 22 12	1 28	1
3 0	2 11 47	—	23 39	—	14 48 47	—	1 23 40	—	0

Foot operating-notes: Eccentr. prosthap. → **Adde**; Scrup. propo. → **Subt.**; Dif. → **S.**; Orbis Prosthap. → **Subtr.**; Excessus in perig. → **S.**

Sign at foot of page: **10 Sig.**

2 Sig. (page 439)

Column operating-headers: Eccentr. prosthap. = **Subtrahe**; Scrup. Propo. = **Add.**; Orbis Prosthap. and Excessus in perig. = **Adde**.

G r a d u s	Eccentr. prosthap. (G M S)	Diff . (M S)	Scrup. Propo. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff. (M S)	Excessus in perig. (G M S)	Di ff. (S)	G r a d u s
0	2 11 47	2 28	23 39	—	14 48 47	12 33	1 23 40	1 29	3 0
1	2 14 15	2 28	24 16	37	15 1 20	12 22	1 25 9	1 29	2 9
2	2 16 43	2 28	24 54	38	15 13 42	12 12	1 26 38	1 29	2 8
3	2 19 11	2 28	25 31	37	15 25 54	12 1	1 28 7	1 29	2 7

G r a d u s	Eccentr. prosthap. (G M S)	Diff . (M S)	Scrup. Propo. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff. (M S)	Excessus in perig. (G M S)	Di ff. (S)	G r a d u s
4	2 21 39	2 28	26 9	38	15 37 55	11 50	1 29 36	1 29	2 6
5	2 24 7	2 28	26 46	37	15 49 45	11 39	1 31 7	1 30	2 5
6	2 26 35	2 28	27 23	37	16 1 24	11 28	1 32 37	1 30	2 4
7	2 29 3	2 28	28 0	37	16 12 52	11 18	1 34 7	1 30	2 3
8	2 31 31	2 28	28 37	37	16 24 10	11 8	1 35 37	1 30	2 2
9	2 33 59	2 28	29 14	37	16 35 18	10 58	1 37 7	1 30	2 1
10	2 36 28	2 29	29 51	37	16 46 16	10 49	1 38 37	1 30	2 0
11	2 38 57	2 29	30 28	37	16 57 5	10 41	1 40 8	1 31	1 9
12	2 41 24	2 27	31 4	36	17 7 46	10 33	1 41 39	1 31	1 8
13	2 43 51	2 27	31 41	37	17 18 19	10 25	1 43 10	1 31	1 7
14	2 46 14	2 23	32 17	36	17 28 44	10 18	1 44 41	1 31	1 6
15	2 48 33	2 19	32 53	36	17 39 2	10 9	1 46 12	1 31	1 5
16	2 50 47	2 14	33 29	36	17 49 11	10 0	1 47 42	1 30	1 4
17	2 53 0	2 13	34 4	35	17 59 11	9 52	1 49 12	1 30	1 3
18	2 55 10	2 10	34 38	34	18 9 3	9 45	1 50 42	1 30	1 2

G r a d u s	Eccentr. prosthap. (G M S)	Diff . (M S)	Scrup. Propo. (M S)	Di f. (S)	Orbis Prosthap. (G M S)	Diff. (M S)	Excessus in perig. (G M S)	Di ff. (S)	G r a d u s
1 9	2 57 15	2 5	35 11	33	18 18 48	9 36	1 52 12	1 30	1 1
2 0	2 59 18	2 3	35 44	33	18 28 24	9 26	1 53 40	1 30	1 0
2 1	3 1 18	2 0	36 17	33	18 37 47	9 6	1 55 11	1 31	9
2 2	3 3 14	1 56	36 50	33	18 46 53	8 49	1 56 42	1 31	8
2 3	3 5 6	1 52	37 24	33	18 55 42	8 32	1 58 12	1 30	7
2 4	3 6 50	1 44	37 57	33	19 4 14	8 15	1 59 13	1 30	6
2 5	3 6 30 [recte: 3 8 30?]	1 40	38 30	33	19 12 29	8 58 [recte: 7 58?]	2 1 14	1 31	5
2 6	3 10 8	1 38	39 3	33	19 20 27	7 42	2 2 45	1 31	4
2 7	3 11 41	1 33	39 37	34	19 28 9	7 25	2 4 16	1 31	3
2 8	3 13 8	1 27	40 11	34	19 35 34	7 8	2 5 46	1 30	2
2 9	3 14 30	1 22	40 44	33	19 42 42	6 50	2 7 17	1 31	1
3 0	3 15 47	1 17	41 16	32	19 49 32	—	2 8 47	1 30	0

Foot operating-notes: Eccentr. prosthap. → **Adde**; Scrup. Propo. → **S.**; Dif. → **S.**;
Orbis Prosthap. → **Subtr.**; Diff. → **Sub.**; Excessus in perig. → **Sub.**

Sign at foot of page: **9 Sig.**

3 Sig. (page 440)

Column operating-headers: Eccentri Prosthaph. = **Subtrahe**; Orbis Prosthaph. = **Adde**.

G r a d u s	Eccentri Prosthaph. (G M S)	Diff e. (M S)	Scrup. Propo. (M S)	Di f. (S)	Orbis Prosthaph. (G M S)	Diffe. (M S)	Excessus in perig. (G M S)	Diff . (M S)	G r a d u s
0	3 15 47	1 12	41 16	—	19 49 32	6 35	2 8 47	1 29	3 0
1	3 16 59	1 10	41 47	30	19 56 7	6 20	2 10 16	1 29	2 9
2	3 18 9	1 9	42 17	30	20 2 27	6 4	2 11 45	1 28	2 8
3	3 19 18	1 7	42 46	29	20 8 31	6 51 [recte: 5 51?]	2 13 13	1 28	2 7
4	3 20 25	1 6	43 15	29	20 14 22	5 37	2 14 41	1 28	2 6
5	3 21 31	1 5	43 43	28	20 19 59	5 24	2 16 9	1 28	2 5
6	3 22 36	1 4	44 11	28	20 25 23	5 10	2 17 37	1 28	2 4
7	3 23 40	1 4	44 38	27	20 30 33	4 56	2 19 5	1 28	2 3
8	3 24 44	1 3	45 6	28	20 35 29	4 39	2 20 33	1 27	2 2
9	3 25 47	1 2	45 33	27	20 40 8	4 20	2 22 0	1 26	2 1
10	3 26 49	1 1	46 0	27	20 44 28	3 50	2 23 26	1 25	2 0
11	3 27 50	1 0	46 26	26	20 48 18	3 30	2 24 51	1 24	1 9
11	3 28 50	0 59	46 51	25	20 51 48	3 10	2 26 15	1	1

G r a d u s	Eccentri Prosthaph. (G M S)	Diff e. (M S)	Scrup. Propo. (M S)	Di f. (S)	Orbis Prosthaph. (G M S)	Diffe. (M S)	Excessus in perig. (G M S)	Diff . (M S)	G r a d u s
2								23	8
1 3	3 29 49	0 58	47 16	25	20 54 58	2 50	2 27 38	1 21	1 7
1 4	3 30 47	0 56	47 40	24	20 57 48	2 30	2 28 59	1 20	1 6
1 5	3 31 43	0 50	48 4	24	21 0 18	2 10	2 30 19	1 19	1 5
1 6	3 32 33	0 46	48 18	24	21 2 38	1 48	2 31 38	1 18	1 4
1 7	3 33 19	0 31	48 52	23	21 4 26	1 25	2 32 56	1 17	1 3
1 8	3 34 0	0 0	49 15	23	21 5 51	0 56	2 34 13	1 16	1 2
1 9	3 34 0	0 0	49 38	22	21 6 47	0 31	2 35 29	1 16	1 1
2 0	3 33 56	0 4	50 0	22	21 7 18	0 15	2 36 45	1 15	1 0
2 1	3 33 50	0 6	50 22	21	21 7 33	0 10	2 38 0	1 13	9
2 2	3 33 42	0 8	50 43	20	21 7 23	0 20	2 39 13	1 10	8
2 3	3 33 31	0 11	51 3	19	21 7 3	1 0	2 40 23	1 8	7
2 4	3 33 13	0 18	51 22	19	21 6 3	1 25	2 41 31	1 5	6
2 5	3 32 51	0 22	51 41	19	21 4 38	1 50	2 42 36	1 2	5
2 6	3 32 25	0 26	52 0	18	21 2 48	2 20	2 43 38	0 59	4
2	3 31 55	0 30	52 18	18	21 0 28	2 50	2 44 37	0	3

G r a d u s	Eccentri Prosthaph. (G M S)	Diff e. (M S)	Scrup. Propo. (M S)	Di f. (S)	Orbis Prosthaph. (G M S)	Diffe. (M S)	Excessus in perig. (G M S)	Diff . (M S)	G r a d u s
7								56	
2 8	3 31 25	0 34	52 36	17	20 57 38	3 17	2 45 33	0	2
2 9	3 30 45	0 40	52 53	17	20 54 21	3 43	2 46 26	0	1
3 0	3 30 0	0 45	53 10	—	20 50 38	—	2 47 16	—	0

Foot operating-notes: Eccentri Prosthaph. → **Adde**; Diffe. → **S. A.**; Scrup. Propo. → **S.**; Orbis Prosthaph. → **Subt**; Diffe. → **Su.Ad**; Excessus in perig. → **S.**

Sign at foot of page: **8 Sig.**

Liber II Theoricorum — Cap. XXII–XXIII

Table of the Prosthaphaereses of Mercury [Tabula prosthaphaereseon Mercurii]

Sign 4 [4 Sig.] (head) / Sign 7 [7 Sig.] (foot) — Source:

Column structure (double-entry): the leftmost **Gradus (G.)** column is read top-to-bottom for signs 4→5; the rightmost **Gradus (G.)** column is read bottom-to-top for the mirrored argument (signs 7→6). For the head (sign 4): the eccentric prosthaphaeresis is *subtractive* [Subtra.], the proportional scruples and orb prosthaphaeresis are *additive* [Adde]. The footer labels reverse these (Adde / Ad. / S. / Subtr. / A.).

Gradus	Eccent. Prostha. (G.M.S.)	Diff. (M.S .)	Scru. Prop. (M.S.)	Di f. (S.)	Orbis Prosthap. (G.M.S.)	Diff. (M.S .)	Exceff. in perig. (G.M.S.)	Diff. (M.S .)	Gr ad u s
0	3 30 0	0 50	53 10	17	20 50 38	4 15	2 47 16	0 48	3 0
1	3 29 10	0 54	53 27	16	20 46 23	4 44	2 48 4	0 45	2 9
2	3 28 16	1 0	53 43	16	20 41 39	5 13	2 48 49	0 41	2 8
3	3 27 16	1 6	53 59	15	20 36 26	5 41	2 49 30	0 38	2 7
4	3 26 10	1 14	54 14	15	20 30 45	6 9	2 50 8	0 34	2 6
5	3 24 56	1 22	54 29	14	20 24 36	6 39	2 50 42	0 30	2 5
6	3 23 34	1 32	54 43	14	20 17 57	7 8	2 51 12	0 25	2 4
7	3 22 2	1 43	54 57	14	20 10 49	7 40	2 51 37	0 22	2 3
8	3 20 19	1 55	55 11	13	20 3 9	8 16	2 51 59	0 19	2 2
9	3 18 24	2 6	55 24	13	19 54 53	8 55	2 52 18	0 18	2 1
10	3 16 18	2 18	55 37	13	19 45 58	9 25	2 52 36	0 16	2 0
11	3 14 0	2 30	55 49	12	19 36 33	9 56	2 52 54	0 6	1 9
12	3 11 30	2 37	56 1	12	19 26 37	10 30	2 53 0	0 7	1 8
13	3 8 53	2 45	56 13	11	19 16 7	11 6	2 52 53	0 12	1 7
14	3 6 8	2 53	56 24	11	19 5 1	11 40	2 52 41	0 19	1 6

									G
									r
									a
									d
									u
									s
Grad	Eccent.	Diff.	Scru.	Di	Orbis	Diff.	Exceff. in	Diff.	
us	Prostha.	(M.S	Prop.	f.	Prosthap.	(M.S	perig.	(M.S	
	(G.M.S.)	.)	(M.S.)	(S.	(G.M.S.)	.)	(G.M.S.)	.)	
15	3 3 15	2 57	56 35	10	18 53 21	12	2 52 22	0 25	1
						22			5
16	3 0 18	3 0	56 45	10	18 40 59	13 8	2 51 57	0 31	1
									4
17	2 57 18	3 4	56 55	10	18 27 51	13	2 51 26	0 39	1
						56			3
18	2 54 14	3 5	57 5	10	18 13 55	14	2 50 47	0 44	1
						18			2
19	2 51 9	3 6	57 15	9	17 59 37	14	2 50 3	0 52	1
						45			1
20	2 48 3	3 7	57 24	8	17 44 52	15	2 49 11	1 3	1
						30			0
21	2 44 56	3 8	57 32	9	17 29 22	16 0	2 48 8	1 12	9
22	2 41 48	3 9	57 41	8	17 13 22	16	2 46 56	1 18	8
						34			
23	2 38 39	3 10	57 49	8	16 56 48	17 6	2 45 38	1 24	7
24	2 35 29	3 12	57 57	8	16 39 42	17	2 44 14	1 31	6
						46			
25	2 32 17	3 14	58 5	8	16 21 36	18	2 42 43	1 38	5
						31			
26	2 29 3	3 16	58 13	8	16 3 25	19	2 41 5	1 50	4
						25			
27	2 25 47	3 19	58 21	8	15 44 0	20	2 39 15	2 6	3
						15			
28	2 22 28	3 22	58 29	7	15 23 45	21 5	2 37 9	2 22	2
29	2 19 6	3 26	58 36	6	15 2 40	21	2 35 47	2 34	1
						56			
30	2 15 40	—	58 42	—	14 40 44	—	2 33 13	—	0
(foot	Adde	Ad.	—	S.	Subtr.	A.	—	S. A.	G

	Eccent.	Diff.	Scru.	Di	Orbis	Diff.	Exceff. in	Diff.	G
Grad	Prostha.	(M.S	Prop.	f.	Prosthap.	(M.S	perig.	(M.S	r
us	(G.M.S.)	.)	(M.S.)	(S.	(G.M.S.)	.)	(G.M.S.)	.)	a
label									d
s)									u
									s

Table of the Prosthaphaereses of Mercury [Tabula prosthaphaereseon Mercurii]

Sign 5 [5 Sig.] (head) / Sign 6 [6 Sig.] (foot) — Source:

Head (sign 5): eccentric prosthaphaeresis *subtractive* [Subtrahe]; proportional scruples and orb prosthaphaeresis *additive* [Adde]. Footer labels: Adde / S. / S. / Subtr. / A.

	Eccentri	Diff	Scrup.	Di	Orbis	Diff	Exceffus	Diff.	G
Gra	prosthap.	(M.	propo.	f.	Prosthap	e.	in perig.	(M.S.)	r
us	(G.M.S.)	S.)	(M.S.)	(S.	h. (G.M.S.)	(M.S	(G.M.S.)		a
0	2 15 40	3 30	58 42	5	14 40 44	22 34	2 33 13	2 36	d
1	2 12 10	3 34	58 47	5	14 18 12	23 2	2 30 37	2 40	u
2	2 8 36	3 38	58 52	4	13 55 10	23 37	2 27 57	2 44	s
3	2 4 58	3 42	58 56	4	13 31 33	24 2	2 25 13	2 48	
4	2 1 16	3 46	59 0	5	13 7 31	24 24	2 22 25	2 5 [recte: 2 51?]	

Gr a d u s	Eccentri prosthap. (G.M.S.)	Diff . (M. S.)	Scrup. propo. (M.S.)	Di f. (S.)	Orbis Prosthap h. (G.M.S.)	Diff e. (M.S)	Exceffus in perig. (G.M.S.)	Diff. (M.S.)	G r a d u s
5	1 57 30	3 50	59 5	4	12 43 7	24 44	2 19 24	2 54	2 5
6	1 53 40	3 56	59 9	5	12 18 23	25 19	2 16 30	3 4	2 4
7	1 49 44	4 1	59 14	4	11 53 4	25 54	2 13 26	3 15	2 3
8	1 45 43	4 7	59 18	5	11 27 10	26 32	2 10 11	3 31	2 2
9	1 41 38	4 13	59 23	4	11 0 38	27 13	2 6 40	3 47	2 1
10	1 37 25	4 20	59 27	4	10 33 25	27 54	2 2 53	4 10	2 0
11	1 33 5	4 27	59 31	4	10 5 31	28 35	1 58 43	4 40	1 9
12	1 28 38	4 35	59 35	4	9 36 56	29 11	1 54 3	5 10	1 8
13	1 24 3	4 43	59 38	3	9 7 45	29 41	1 48 53	5 40	1 7
14	1 19 20	4 49	59 41	3	8 38 4	30 6	1 43 13	6 5	1 6
15	1 14 31	4 53	59 43	2	8 7 58	30 26	1 37 8	6 15	1 5
16	1 9 38	4 55	59 46	3	7 37 32	30 46	1 30 53	6 20	1 4
17	1 4 43	4 56	59 48	2	7 6 46	31 7	1 24 33	6 23	1 3
18	0 59 47	4 57	59 50	2	6 35 39	31 27	1 18 10	6 24	1 2
19	0 54 50	4 58	59 52	2	6 4 12	31 48	1 11 46	6 25	1 1

Gra dus	Eccentri prosthap. (G.M.S.)	Diff		Di f. (S.)	Orbis Prosthap h. (G.M.S.)	Diff		Exceffus in perig. (G.M.S.)	Diff. (M.S.)	G r a d u s
		.	Scrup. propo. (M.S.)			e. (M.S)				
20	0 49 52	4 58	59 54	2	5 32 24	32 17		1 5 21	6 27	1 0
21	0 44 54	4 58	59 56	1	5 0 7	32 36		0 58 54	6 29	9
22	0 39 56	4 59	59 57	1	4 27 31	32 56		0 52 25	6 30	8
23	0 34 57	4 59	59 58	1	3 54 35	33 10		0 45 55	6 31	7
24	0 29 58	4 59	59 59	0	3 21 25	33 20		0 39 24	6 32	6
25	0 24 59	4 59	59 59	0	2 48 5	33 30		0 32 52	6 33	5
26	0 20 0	5 0	59 59	0	2 14 35	33 35		0 26 19	6 34	4
27	0 15 0	5 0	59 59	0	1 41 0	33 35		0 19 45	6 34	3
28	0 10 0	5 0	60 0	0	1 7 21	33 39		0 13 11	6 35	2
29	0 5 0	5 0	60 0	0	0 33 41	33 40		0 6 36	6 36	1
30	0 0 0	—	60 0	—	0 0 0	33 41		0 0 0	—	0
(foo t labe ls)	Adde	S.	—	S.	Subtr.	A.	—	A.		G r a d u s

CAPVT XXIII.

On the method of computing the true longitudes of Venus and Mercury by the preceding tables [De ratione computandi veras longitudes Veneris & Mercurii per canones praecedentes]

The method of computing the phenomena of these planets [Venus and Mercury] in longitude is for that reason somewhat easier than for the rest, and scarcely differs at all from the Prutenic method, since it everywhere assumes the mean motion of the Sun.

First, for the required time the mean motion of the Sun is taken out of the solar table inserted in the preceding book — and that with respect to the mean (as it has it) vernal equinox. When the inequality of this motion is applied to it, the mean elongation of these planets from the true equinox is established within this motion of the Sun.

Second, in the same way, for the same time, the mean motions of the anomaly of each of these planets are worked out and copied off, as contained in the tables just set out above.

Third, by means of the eccentric anomaly of each planet, the prosthaphaeresis of this is sought, together with the proportional scruples attached to it. Now when this prosthaphaeresis is applied to the mean eccentric motion according to the headings inscribed on the table, the true motion of the planet in its eccentric, reckoned from the true equinox, is produced. By the contrary procedure, however, the same prosthaphaeresis is applied to the anomaly of commutation [anomalía commutationis], so that this too is brought to its equated value.

Fourth, by means of the anomaly of commutation thus equated, the equation of the orb [aequatio orbis] of each planet is found in the prosthaphaeresis table — the equation that Copernicus calls the parallax [parallaxin] — together with the excess, of which, according to the scruples kept, a proportional part is taken, always to be added to the equation, so that the latter may be seen for every position of the center of the orb upon the eccentric. Afterwards, according to the headings of subtraction or addition, it is to be applied to the true elongation of the planet in its eccentric. When this has been done, the sought longitude of the planet from the true equinox will be established — which was what was proposed.

An example in both, namely Venus and Mercury together [Exemplum in utroque, Venere scilicet & Mercurio simul]

In the year 1607, in the month of April, on the 14th day, in the evening at hour 8½, when Venus was about 2 degrees above our horizon and Mercury was above it in almost a straight line, tending toward Capella — from which, however, it was declining quite perceptibly toward the north — I observed here, by means of the astronomical radius [radius Astronomicus], at least the distance of these planets from one another to be 2⅔ degrees. Now at that time the latitude of Venus was some southerly amount, about 10 minutes (at most), as will be shown later; but that of Mercury about 1⅓ degrees north, as on the following days I investigated this more precisely through other observations. With these things noted beforehand in the sky, it must now be seen what disposition in longitude the calculation of the reformed hypotheses assigns to these two relative to one another.

So, **first**, to the assumed time there corresponds in common the longitude of the Sun from the mean equinox: 1 sign, 2 degrees, 11 minutes, 41 seconds. When the inequality of the precession is added to this — namely 8 minutes, 9 seconds — the mean elongation of both from the true equinox becomes 1 sign, 2 degrees, 19 minutes, 50 seconds. Then for the same time are found:

		Sig.	Gr .	Mi .	Sec.
anomaly in Venus	Eccentric [Eccentri]	10	1	34	16
anomaly in Venus	Orb [Orbis]	1	6	23	7
anomaly in Mercury	Eccentric [Eccentri]	5	1	24	5
anomaly in Mercury	Orb [Orbis]	2	12	48	11

In addition, from the prosthaphaeresis table of each there is given:

	Gr .	Mi .	Sec.	
the prosthaphaeresis of the eccentric in Venus	1	33	15	additive [add.]
with the proportional scruples	—	12	56	
likewise in Mercury	2	10	46	subtractive [ablat.]
with the proportional scruples	—	58	48	

Now when these prosthaphaereses have been duly applied according to their own headings, there results:

		Sig.	Gr.	Mi.	Sec.
in Venus	True longitude of the eccentric [Longitudo eccentrici vera]	1	3	53	5
in Venus	Equated anomaly [Anomalia coaequata]	1	4	49	52
in Mercury	True longitude of the eccentric [Longitudo eccentrici vera]	1	0	9	4
in Mercury	Equated anomaly of the orb [Anomalia orbis coaequata]	2	14	58	57

Furthermore, with the equated orb-anomaly of each prescribed, the equation of the orb is likewise found in the tables set out above:

	G r .	M i.	Se c.		Mi.	Se c.
In Venus	1 4	2 3	1	additive, with the excess [add. cum excessu]; hence the proportional part [Pars ideo proport.]	16	25
In Venus (proportional part cont.)					3	27
In Mercury	1 7	3 8	53	additive, with the excess [add. cum exceff.]; whence the proportional part [unde pars proportion.]	(see below)	

For Mercury, the proportional-part figures given are:

Gr.	Mi.	Sec.
1	33	18
1	44	10

Now since to the equations of the orbs at apogee their proportional parts are added — and those too, as the headings require, to the eccentric elongations from

the true equinox obtained earlier — there finally emerges the sought longitude of Venus: 18 degrees, 19 minutes, 33 seconds in Taurus [♉]. Likewise, the sought longitude of Mercury: 19 degrees, 32 minutes, 7 seconds in Taurus [♉]. So it appears that Mercury, by our calculation, followed Venus as to longitude alone by about $1\frac{1}{2}$ degrees, the remaining part of their distance — which had been about equal to this — being made up in the recession of the two contrary latitudes.

And so much, then, also for the restitution of these planets, Venus and Mercury, through their longitudes.

CAPVT XXIV.

On the demonstration of the latitudes of Venus and Mercury, agreeing with the celestial appearances, by their canon [De latitudinum Veneris & Mercurii demonstratione, apparentiis caelestibus congruentium cum ipsarum canone]

The latitudes of these planets, when rightly considered, certify the hypotheses of their longitudes no less than those of the three superior planets; indeed there is a great agreement of these with the superior ones, provided we weigh it correctly. For just as there the inclination of the eccentric orbs to the ecliptic is perceptible on both sides — so much so that they are varied only by the common sections of these circles over a fairly long stretch of time — in the same way the latitudes of these planets behave too: their epicycles, or rather their orbs, are found to be inclined to the ecliptic by a perceptible inclination on both sides, in such a way that only the common segments are altered in these over the course of time.

As to segments of circles of this kind, which in the case of the superior planets were by no means equally bound to the motions of the apsides — contrary to the opinion of the ancients, and especially the Ptolemaic view, which Copernicus followed — although in Mercury they are understood to pass through its apsides (which we have shown to lie, in the beginning of our century, in [Pisces ♉] and [Gemini ♊]), nevertheless in Venus these same points of the sections anticipate its apsides — which were found near the end of [Gemini ♊] and [Pisces ♉] — by 16 degrees, so that in this century they fall at about 13 degrees of [Gemini ♊], as the celestial observations have taught us.

To the said common sections of the ecliptic and the planet's orb we refer the stable and unvaried inclination of the latitude, in every [position] through the eccentric or...

— whether by the carrying-round of the center of the planet's orb, as Ptolemy and Tycho [Tycho Brahe] hold, or of the earth itself, as Copernicus [Nicolaus Copernicus] holds. The motion is of one kind from the first departures of the ecliptic's section, namely the orb's: for Venus toward the north, but for Mercury toward the south, and so by turns, in keeping with the reckoning of the section, everywhere reversed into its opposite. And because the visible angles of inclination of these planets too are regularly altered according to their distances from the earth, it cannot be that there should be some eccentric, distinct from the plane of the ecliptic, to which the planet's orb would accommodate itself uniformly here, as the epicycle does in the case of the Moon. For this reason it has by now become more consistent with the truth that these two bodies acknowledge the supposition of Copernicus or even of Tycho, in which the orbs themselves are substituted for the Ptolemaic epicycles.

Furthermore, the observations decisively reject the two variations that Ptolemy devised concerning these planets' latitudes. The first is the one by which he supposed that the angles of the declinations and reflexions (which are in fact one and the same thing) are increased at the contact of the epicycle — that is, at the planet's greatest excursion to the sides. It was chiefly the apparent latitudes of Venus around these said places that led Ptolemy into this error, after he had made the perpetual angle of inclination a full degree smaller than the one that results from the observations taken near the perigee of the epicycle and the quadrants of the eccentric — observations of a kind that could scarcely ever have been obtained by Ptolemy in a less oblique sphere. Accordingly, what was so unwittingly left out he chose to make good by some addition or other, even a fictitious one, but only in the manner used above, with the opposite tendency. Yet in fact the whole latitude of these five planets together, and its variation, depends on the removal of the center of the planet's orb from the said common section (in which resides the angle of perpetual inclination, to be ascertained before everything else in each individual case); and likewise on the planet's motion in the epicycle or its own orb; nor any less on its distance from the earth, as the longitude-hypothesis requires.

The second variation of latitude in these planets, which they call the *deviation*, was supposed to appear at its greatest near the apsides of the eccentric — such that in Venus it would extend toward the north to a sixth of a degree [ad

sextantem gradus], while in Mercury, on the contrary, with its greatest perpetual declination toward the south, it would extend to three-quarters of a degree [ad dodrantem]. This we shall truly call a *deviation*, that is, a Ptolemaic error, since no cause for it is anywhere manifest in the longitude-hypothesis, and since, in our observations or even in those of the ancients once they have been duly examined and restored, we still detect no sure and firm traces of it. After all, even that variation which will be produced near the latitudes of Mercury at apogee and perigee, below what the celestial appearances give, will be excused by the reformed longitude-hypothesis, so that it does not square here — and for that reason it is celestial experience, rather than the common imitations, that has given us the occasion both to clear part of Augeas's stable [Augiae stabuli] and to restore the suppositions of the latitudes here too to their genuine simplicity. Now from observations compared with one another, we find the angle of mean inclination in Venus to be $3\frac{1}{2}^\circ$, fuller by a whole degree, that is, than the Ptolemaic and Copernican supposition gives; but in Mercury $5\frac{2}{3}^\circ$.

With these things supposed, and the rest that we have just pressed home as required for tracking down the latitudes, we shall carry out the computation [epilogismum] of these latitudes — far more in harmony with the heavens — with an almost uniform ease. For each of the two, a general scheme must first be set out by means of the received figures, and briefly explained, so that it may lie open to the reader's eyes, as it were at a single glance, how far in longitude each of these planets passes from the ecliptic at any time throughout the whole eccentric — that is, in the Ptolemaic and the Tychonic form at once, since in both the epicycle or orb of the planet stands obliquely upon the eccentric with respect to the plane of the ecliptic.

Scheme of the latitudes of Venus and Mercury in the Ptolemaic and Tychonic form.

Diagram. A central eccentric circle is drawn about the center A, with two perpendicular diameters crossing at A. Around its circumference, at eight positions, stand eight small circles (the epicycle or orb of the planet, shown in eight successive positions through the eccentric). The eight positions are lettered B (top), C (upper-left), D (left), E (lower-left), F (bottom), G (lower-right), H (right), I (upper-right). The word **Orient**. [East] is printed at the left margin, **Occident**. [West] at the right margin. Each of the eight epicycles bears the same set of internal labels, marking its inclined diameters and the common-section line:

- A (center): center of the eccentric; the two main diameters B–F and D–H cross here at right angles.
- B, C, D, E, F, G, H, I: the eight successive positions of the planet's epicycle/orb around the eccentric.
- Within each epicycle — R (top): one end of the diameter R–S.
- S (bottom): the other end of the diameter R–S (R–S corresponds to the diameter B–F of the eccentric).
- K (upper-right) and L (left): the ends of the diameter K–L (K–L regards H–D of the eccentric).
- N (upper-left) and M (lower-center): the ends of the inclined diameter M–N drawn through the center of the epicycle, where the common intersection is made with L–K. The angle L–B–N (or K–B–M) holds the measure of the mean inclination.

Let an eccentric B D F &c. be described about the center A; and with the diameters B F and D H drawn to right angles, let there be supposed, at the positions B C D E F G H I, the epicycle or orb of Venus — or also of Mercury — the eccentricity (which saves their inequality in longitude) being here omitted for the sake of brevity and ease.

Further, this epicycle, in any position through the eccentric, takes on sections similar to the eccentric's and plainly parallel to them, so that R S answers to B F, and K L regards H D — and this in every carrying-round of the epicycle through the eccentric.

Now since the orb or epicycle of the planet is perceived to be inclined to the ecliptic, let there finally be drawn everywhere a diameter M N through the center of the epicycle, where its common intersection is made with L K, so that the angle L B N (or K B M) holds the measure of the mean inclination in each of these planets. This angle we asserted above to be $3\frac{1}{2}^{\circ}$ in Venus, and $5\frac{2}{3}^{\circ}$ in Mercury — found by us from the heavens — and we shall make it plainer below as well.

When the scheme of the latitudes of these planets is set up in this way, in either the Ptolemaic or the Tychonic form, the denominations and variations of the latitudes are sufficiently in view, and they follow the planet's revolution both through the eccentric and through the epicycle (its own orb). Thus we, for example, beginning our consideration of the latitudes in Venus from the intersection B — which runs 16° ahead of the apogee, as was said before — declare that the whole semicircle R N S, which looks toward the rising, is of northern latitude perpetually and in every revolution; but the remainder, S M R,

of southern. The turns, however, are reversed throughout, so that the hemisphere of the epicycle toward the rising completes the southern latitude, the other, toward the setting, the northern. The crossings, meanwhile, fall at R and S — the line of which is perpetually parallel, in these planets, to the apogee line B F.

So much for the general indication of the latitudes in these two planets according to the Ptolemaic or Tyconic form. There follows the Copernican, which in the same equivalence is still easier, since the planet's orb occupies the middle invariably, in this way.

Diagram of the latitudes of Venus and Mercury in the Copernican form.

Diagram. About the center A a large circle (the eccentric, that is, the great orb of the earth) is drawn, with point B at top and C at bottom marking the ends of the vertical diameter B–C. Within it sits a smaller, inclined circle homocentric with it (the planet's orb), bearing the point K on the upper part of the vertical diameter. Through the center A run the line of common intersection B–D, the diameter F–E (drawn through the center, at right angles to B–D, marking the plane of the ecliptic / great orb of the earth), and the line G–H carried through A. The labels:

- A (center): the common center of the eccentric and the planet's orb.
- B (top) and D (bottom, below): the ends of the line of common intersection B–D, which in Mercury is at the apogee, in Venus anticipates the apogee by 16°.
- K (on the inner circle, upper part) and C (lower): common segments through which the line B–D passes.
- E (right) and F (left): the ends of the diameter E–F, set through the center at right angles to B–D, showing the plane of the ecliptic.
- G (upper-right) and H (left): the ends of the line G–H drawn through A; the angle H A F (or G A E) equals the perpetual inclination of the planet's orb to the plane of the eccentric (the great orb of the earth).

About the center A of the eccentric (as before) let the eccentric itself, that is, the great orb of the earth, B D, be described; and upon this let there be set, for the sake of ease, the planet's orb K C, homocentric with it and inclined — although it would be more correct to fix the center of this orb at the moveable terminus of the eccentricity, since that carries some weight in the latitudes of Mercury, even though in Venus (because of the eccentricity, and the orb's inclination, both

nearly double the smaller ones in Mercury) it would bring in almost nothing perceptible.

Let the line of common intersection B D be drawn — which in Mercury is at the apogee, in Venus anticipates the apogee by 16° . This passes through the common segments K and C; let E F be set at right angles to it, through the center, showing the plane of the ecliptic, that is, of the great orb of the earth. Finally let G H be carried through the center A, which here makes with E F on either side the angle H A F (or G A E) equal to the perpetual inclination of the planet's orb to the plane of the eccentric, that is, to the great orb of the earth.

With these things so set up — and the opposite disposition of the latitudes in Venus and Mercury (as shown above) being taken into account — the latitude of Venus, as it traverses the eastward semicircle K H C of its circle B D, will in any situation of the earth be always northern; but in the other semicircle, C G K, everything is the other way round, so that the latitude of Venus through that one becomes southern, while Mercury's, on the contrary, is northern. Nothing more is required for knowing the state of the latitudes in these planets in a general way. Indeed — and this is what we hoped for from the outcome — they differ in no respect from what went before, since in both cases we are considering a fixed and stable excursion of the orbs into latitude with respect to the ecliptic, and we measure everything that touches the distances from the longitude-hypothesis of each. How these things are now carried out, and how the latitude of each of these planets at any time can be obtained by a uniform application of plane triangles, we shall make plain in the example here appended.

Let us take that example from the first observation of Venus made among us, where we found its true southern latitude, parallax removed, to be $2^\circ 8'$ — namely about the end of the 16th degree of Leo [♌], which happened in the year 1585, September the 15th, in the morning at hour $5\frac{1}{4}$, as was noted above.

[Margin: Cap. 18 hujus — "Chapter 18 of this book."]

	Sig.	Gr.	Mt.
The coequated anomaly of the eccentric was	3	2	43
The orb, that is, of the epicycle, likewise coequated	7	9	0

— although it would have been the same had one applied these simple values on both sides.

Further, the distance of Venus from the earth was 63120, of the kind in which the radius of the eccentric is 100000.

With these given, and granting at the same time that the northern node in Venus anticipates the apogee by 16° , the scheme of the latitude of Venus is supposed in the Ptolemaic or Tychonic form, as before.

Diagram. A geometric construction for the worked latitude example. A large circle (the eccentric) has point B at top and D at bottom, with the vertical diameter B–D and the center A on it (to the right). Inscribed is the planet's orb (drawn partly with a dotted arc), within which the construction triangle is formed. The readable labels:

- A: the eye/earth (the apex of the latitude triangle, on the vertical B–D line at right).
- B (top) and D (bottom): ends of the vertical diameter of the eccentric.
- C: the center of the planet's orb.
- E: the point on the orb (one end of the diameter E–K, parallel to B–D).
- K: the lower end of the diameter E–K (below C).
- L: the planet's position on its orb (apex of the latitude, above M).
- M: the foot below L, on the line of intersection E–K.
- P: the foot of the perpendicular from M (or L) used for the sine M P.
- N: the point on line E–F where N C is perpendicular to E K.
- F: the right-hand end of the line E–F through C.
- H: the left-hand end of the diameter through C (opposite F).

For the arc B C of the eccentric is made up by adding to the anomaly of the eccentric the anticipation of the node, namely 16° , and it comes to 3 signs $18^\circ 43'$. Equal to this, in the orb, is the arc E H, since B D and E K are parallel. But with Venus dwelling at M, H M is 7 signs 9° . So, adding E H and H M together, E H M amounts to 10 signs $27^\circ 43'$.

Now since N C is perpendicular to E K, the angle N C M — that is, the arc N M — is likewise established as 57° , the three-quarters of the circle [dodrans, 270°] being subtracted, of course, from the single given arc E H M. And since Venus here holds the western part of its epicycle, with respect to the line of intersection E K, therefore, by what was shown above, its latitude is pronounced to be southern. This is to be elicited in the following way, and then compared with the observed value.

First, in order to have the side M L in the triangle L M E, that side must be sought beforehand in proportion to the right sine M P — namely the sine of the complement of the angle M C N. For as the radius 100000 is to the said sine, which

in this example is 53411, so the angle of the perpetual inclination of the orb of Venus, demonstrated earlier to be $3^{\circ} 30'$, L E M, is to the said M L, 3261 — which, obtained more quickly κατὰ πρόσθαφάιρεσιν [Greek: kata prosthaphairesin, "by way of addition-and-subtraction" (the prosthaphaeresis method)], and proportioned a second time with the radius of the orb of Venus, 72248, comes out as 2356.

Finally, in the triangle A M L, from the given M A (the distance of Venus from the earth, 63120), together with the recently found side M L, 2356, and the angle at L (always as it were a right angle to the ecliptic), the sought latitude of Venus is found, at the angle M A L, to be $2^{\circ} 8'$ — which agrees fully with the observation. The Alphonsine calculation [calculus Alphonsinus] exhibits this latitude of Venus, both found and observed, quite competently in this place; whereas Copernicus falls short of it by half a degree.

So much for Venus. As for one further slight variation observed near the apogee and perigee of the eccentric of Mercury, the celestial observations inform us — those inserted in the synopsis above. For in the year of the Lord 1585, on the 15th of November, in the morning at hour 7, about the greatest elongation of Mercury from the Sun, say, when the anomaly of the corrected orb was 8 signs $16^{\circ} 22'$, the true observed latitude of Mercury at Uraniborg [Vraniburgi] was $2^{\circ} 18'$ north [B = Borea]; and this near the apogee of the eccentric, or at least $3\frac{1}{2}^{\circ}$ [?] of it [?] in consequence.

Further, in the year of the Lord 1593, I find that Mercury, likewise near its perigee and at its evening greatest elongation, was observed at Kassel [Caffellis] by the Landgrave's men [a Landgravianis] over several continuous days, about the middle of May — and this not without astonishment, since the perpetual twilight, even in that locale, then lasts, though perhaps not so conspicuously as among us.

Among these observations, however, when that one which was taken on the 11th of May in the evening at hour $9\frac{1}{2}$ —

which was taken from the observation of those men, gives the latitude of Mercury [Mercurii] as seen at $2^{\circ} 4\frac{3}{4}'$. If I assume B and free it, as far as I can, from the parallaxes that overcome the refractions in the opposite direction, I do not find that this same latitude was in fact greater than 2° . At that time the anomaly of Mercury's eccentric [anomalía eccentrici] was 5 signs 29° nearly — that is, 1° short of perigee; while the anomaly of commutation [commutationis anomalía] was 3 signs 22° , or thereabouts, as these motions too are seen more precisely once

corrected above. Mercury was therefore also then near its greatest elongation from the Sun.

Now between these observed latitudes there is a difference of 18 minutes, which seems quite contrary to the hypothesis of longitude: for the angle of the planet's orb departing from the ecliptic appears greater at apogee than at perigee, whereas on optical grounds it ought to be smaller. And so this can be saved neither by the Ptolemaic deviation [*deviatio Ptolemaica*], nor by the Sun's being then at a greater distance from the Earth (since the planet had been at the perigee of the eccentric), nor by the reverse. For that account makes those leaps of latitude (which [Nicolaus Copernicus] complains of) suddenly, into opposite directions, and on top of that it here has the appearance near apogee working against it; while this latter account at the same time leaves Mercury's excursions in longitude near perigee shorter within its orb — which likewise contradicts the appearance in longitude.

Therefore: what [Copernicus] supposed was inherent in his hypothesis of Mercury's longitude — namely, the epicyclet he himself devised — for the purpose of representing the Ptolemaic deviation, we with better right assign to our own little circle [*circellus*], the equant being granted. That is, it is set inclined toward the north at apogee, and equally much toward the south at perigee; since, for half the measure of the eccentric, its restitution proceeds at one-and-a-half times the rate, so that on that account it obtains the contrary part toward the opposite apsides of the eccentric. This inclination, on either side, close to the latitude, is felt to amount to roughly half the apparent longitude — namely about 15' near apogee, but 18' at perigee.

Accordingly, with the angle of perpetual inclination of Mercury's orb to the ecliptic set at 5°40', and the rest retained as established by us above in the reformed hypothesis of longitude — namely Mercury's distance at the apogee of the eccentric 106822, at perigee 95452, of which the radius of the eccentric is 100000, and likewise, on the same proportion, the radius of the orb 38500 — there results for this last planet a declination near the quadrants of 1°34' at the apogee of the orb, and 3°33' at perigee; but the greatest obliquation or reflexion [*obliquatio seu reflexio*] is 2°3' at the apogee of the eccentric and 2°19' at perigee. These are limited to the inclination of the little circle, so that there remains 2°18' in the one place and 2°1' in the other — that is, while an eighth part of each of those is reckoned for the said inclination, to be added to the northern obliquation near apogee, and subtracted from the southern near perigee; which holds the contrary turn in the remaining opposite quadrants of the orb and their latitudes.

In this way the phenomena of the Mercurial latitudes are accounted for, and the following table for them is composed — in such a way, indeed, that the inclination of that little circle just named is included in the latitudes of reflexion, which can also be determined more exactly together with the nodes [nodi], and is not thought by us to transmit anything of itself, in any sensible degree, to the latitudes of declination. Lastly, in the case of Venus too, the canon next following sets out the quantities of the latitudes, both as to declination and as to reflexion (as they call it).

Table of the latitude of declination of Venus

Tabula latitud. declinationis Veneris. (p. 450)

This is a double-entry canon. The argument runs down the left in **Grad. def.** (degrees defective, 0→30) and up in **Grad. asc.** (degrees ascending, 30→0). The body holds three groups: **Scrupula proport. / Dodecatemoria** (proportional minutes, columns indexed at the head 0, 1, 2 and at the foot 5, 4, 3, each as M[inutes].S[econds]); **Declinat. Sept. / Dodecatemoria** (northern declination, columns 0, 1, 2 at head / 11, 10, 9 at foot, each G[radius].M[inute]); and **Declinat. Austr. / Anomaliae** (southern declination, columns 3, 4, 5 at head / 8, 7, 6 at foot, each G.M.).

Gr ad . de f.	Gr ad . as c.	Pro p. 0 (M S)	Pro p. 1 (M S)	Pro p. 2 (M S)	Decl.S ept. 0 (G M)	Decl.S ept. 1 (G M)	Decl.S ept. 2 (G M)	Decl.A ustr. 3 (G M)	Decl.A ustr. 4 (G M)	Decl.A ustr. 5 (G M)
0	30	0 0	30 11	51 49	1 28	1 19	0 51	0 0	1 25	4 12
1	29	1 3	31 5	52 19	1 28	1 18	0 50	0 1	1 29	4 21
2	28	2 5	31 58	52 48	1 28	1 18	0 48	0 3	1 33	4 30
3	27	3 8	32 50	53 16	1 28	1 17	0 47	0 6	1 37	4 39
4	26	4 10	33 41	53 44	1 28	1 17	0 45	0 8	1 40	4 48
5	25	5 12	34 31	54 11	1 28	1 16	0 44	0 10	1 44	4 58

Gr ad . de f.	Gr ad . as c.	Pro p. 0 (M S)	Pro p. 1 (M S)	Pro p. 2 (M S)	Decl.S ept. 0 (G M)	Decl.S ept. 1 (G M)	Decl.S ept. 2 (G M)	Decl.A ustr. 3 (G M)	Decl.A ustr. 4 (G M)	Decl.A ustr. 5 (G M)
6	24	6 14	35 21	54 37	1 27	1 15	0 42	0 13	1 48	5 8
7	23	7 16	36 16	55 2	1 27	1 15	0 41	0 15	1 52	5 18
8	22	8 18	36 58	55 27	1 27	1 14	0 40	0 18	1 56	5 28
9	21	9 21	37 46	55 51	1 27	1 13	0 38	0 21	2 1	5 38
10	20	10 23	38 33	56 14	1 27	1 12	0 36	0 24	2 6	5 48
11	19	11 25	39 20	56 36	1 26	1 11	0 35	0 26	2 11	5 59
12	18	12 26	40 6	56 57	1 26	1 10	0 33	0 29	2 16	6 10
13	17	13 27	40 52	57 17	1 26	1 9	0 32	0 31	2 21	6 22
14	16	14 28	41 37	57 36	1 26	1 8	0 30	0 34	2 26	6 34
15	15	15 28	42 22	57 54	1 25	1 7	0 29	0 37	2 31	6 47
16	14	16 29	43 6	58 11	1 25	1 6	0 27	0 40	2 36	6 59
17	13	17 29	43 49	58 27	1 25	1 5	0 25	0 43	2 42	7 11
18	12	18 29	44 33	58 41	1 24	1 4	0 24	0 46	2 48	7 23
19	11	19 30	45 16	58 54	1 24	1 3	0 22	0 48	2 54	7 34
20	10	20 30	45 58	59 5	1 24	1 2	0 20	0 51	3 0	7 44
21	9	21	46	59	1 23	1 1	0 18	0 54	3 6	7 54

Gr ad de f.	Gr ad as c.	Pro p. 0 (M S)	Pro p. 1 (M S)	Pro p. 2 (M S)	Decl.S ept. 0 (G M)	Decl.S ept. 1 (G M)	Decl.S ept. 2 (G M)	Decl.A ustr. 3 (G M)	Decl.A ustr. 4 (G M)	Decl.A ustr. 5 (G M)
		29	39	15						
22	8	22 28	47 19	59 24	1 23	1 0	0 16	0 57	3 12	8 3
23	7	23 26	47 57	59 32	1 22	0 59	0 14	1 0	3 19	8 12
24	6	24 24	48 34	59 39	1 22	0 58	0 12	1 4	3 26	8 21
25	5	25 22	49 9	59 45	1 21	0 57	0 10	1 7	3 33	8 29
26	4	26 20	49 43	59 50	1 21	0 56	0 8	1 10	3 40	8 37
27	3	27 18	50 16	59 54	1 20	0 55	0 6	1 14	3 48	8 44
28	2	28 16	50 48	59 57	1 20	0 54	0 4	1 17	3 55	8 50
29	1	29 14	51 19	59 59	1 19	0 52	0 2	1 21	4 3	8 56
30	0	30 11	51 49	60 0	1 19	0 51	0 0	1 25	4 12	9 2
(fo ot)		(5)	(4)	(3)	(11)	(10)	(9)	(8)	(7)	(6)

Foot rubrics: under the proportional-minutes group, **Anomalia Eccentrici**; under the declination groups, **Orbis seu Epicycli**.

Table of the latitude of declination of Venus (continued)

Tabula latitud. declinationis Veneris. (p. 451)

This opening repeats the same canon body, read from the other direction. Here the proportional-minutes group carries head indices **6, 7, 8** (foot 11, 10, 9), the **Declina. Austr. / Dodecatemoria** group carries 0, 1, 2 (foot 11, 10, 9), and the **Declina. Septen. / Anomalia** group carries 3, 4, 5 (foot 8, 7, 6). The numerical

body is identical to the preceding table; the two halves are mirror-paired argument-wise.

Gr ad . de f.	Gr ad . as c.	Pro p. 6 (M S)	Pro p. 7 (M S)	Prop. 8 (M S)	Decl.A ustr. 0 (G M)	Decl.A ustr. 1 (G M)	Decl.A ustr. 2 (G M)	Decl.S epten. 3 (G M)	Decl.S epten. 4 (G M)	Decl.S epten. 5 (G M)
0	30	0 0	30 11	51 49	1 28	1 19	0 51	0 0	1 25	4 12
1	29	1 3	31 5	52 19	1 28	1 18	0 50	0 1	1 29	4 21
2	28	2 5	31 58	52 48	1 28	1 18	0 48	0 3	1 33	4 30
3	27	3 8	32 50	53 16	1 28	1 17	0 47	0 6	1 37	4 39
4	26	4 10	33 41	53 44	1 28	1 17	0 45	0 8	1 40	4 48
5	25	5 12	34 31	54 11	1 28	1 16	0 44	0 10	1 44	4 58
6	24	6 14	35 21	54 37	1 27	1 15	0 42	0 13	1 48	5 8
7	23	7 16	36 16	55 2	1 27	1 15	0 41	0 15	1 52	5 18
8	22	8 18	36 58	55 27	1 27	1 14	0 40	0 18	1 56	5 28
9	21	9 21	37 46	55 51	1 27	1 13	0 38	0 21	2 1	5 38
10	20	10 23	38 33	56 14	1 27	1 12	0 36	0 24	2 6	5 48
11	19	11 25	39 20	56 36	1 26	1 11	0 35	0 26	2 11	5 59
12	18	12 26	40 6	56 57	1 26	1 10	0 33	0 29	2 16	6 10
13	17	13 27	40 52	57 17	1 26	1 9	0 32	0 31	2 21	6 22

Gr ad f.	Gr ad c.	Pro p. 6 (M S)	Pro p. 7 (M S)	Prop. 8 (M S)	Decl.A ustr. 0 (G M)	Decl.A ustr. 1 (G M)	Decl.A ustr. 2 (G M)	Decl.S epten. 3 (G M)	Decl.S epten. 4 (G M)	Decl.S epten. 5 (G M)
14	16	14 28	41 37	57 36	1 26	1 8	0 30	0 34	2 26	6 34
15	15	15 28	42 22	57 54	1 25	1 7	0 29	0 37	2 31	6 47
16	14	16 29	43 6	58 11	1 25	1 6	0 27	0 40	2 36	6 59
17	13	17 29	43 49	58 27	1 25	1 5	0 25	0 43	2 42	7 11
18	12	18 29	44 33	58 41	1 24	1 4	0 24	0 46	2 48	7 23
19	11	19 30	45 16	58 44 [recte: 58 54]	1 24	1 3	0 22	0 48	2 54	7 34
20	10	20 30	45 56	59 5	1 24	1 2	0 20	0 51	3 0	7 44
21	9	21 29	46 39	59 15	1 23	1 1	0 18	0 54	3 6	7 54
22	8	22 28	47 19	59 24	1 23	1 0	0 16	0 57	3 12	8 3
23	7	23 26	47 57	59 32	1 22	0 59	0 14	1 0	3 19	8 12
24	6	24 24	48 34	59 39	1 22	0 58	0 12	1 4	3 26	8 21
25	5	25 22	49 9	59 45	1 21	0 57	0 10	1 7	3 33	8 29
26	4	26 20	49 43	59 50	1 21	0 56	0 8	1 10	3 40	8 37
27	3	27 18	50 16	59 54	1 20	0 55	0 6	1 14	3 48	8 44
28	2	28 16	50 48	59 57	1 20	0 54	0 4	1 17	3 55	8 50

Gr ad . de f.	Gr ad . as c.	Pro p. 6 (M S)	Pro p. 7 (M S)	Prop. 8 (M S)	Decl.A ustr. 0 (G M)	Decl.A ustr. 1 (G M)	Decl.A ustr. 2 (G M)	Decl.S epten. 3 (G M)	Decl.S epten. 4 (G M)	Decl.S epten. 5 (G M)
29	1	29 14	51 19	59 59	1 19	0 52	0 2	1 21	4 3	8 56
30	0	30 11	51 49	60 0	1 19	0 51	0 0	1 25	4 12	9 2
(f oo t)		(11)	(10)	(9)	(11)	(10)	(9)	(8)	(7)	(6)

Foot rubrics: **Anomalia Eccentri** under the proportional-minutes group; **Orbis seu Epicycli** under the declination groups.

Table of the latitude of reflexion or obliquation of Venus

Tabula latitud. reflexionis seu obliquationis Veneris. (p. 452)

A wide double-entry canon. Argument: **Grad. def.** (0→30) and **Grad. asc.** (30→0). Body groups: **Scrupula propor.** / **Dodecatemoria** (columns 0, 1, 2, each M.S.); **Reflexio Septentrionalis** / **Dodecatemoria** (columns 0, 1, 2, 3, 4, 5, each G.M.); **Reflexio Austrina** / **Anomalia orbis seu epicycli** (columns 6, 7, 8, 9, 10, 11, each G.M.). Foot indices for the proportional-minutes group run 11, 10, 9.

G r. d ef	G r. as c	Pr.0 (M S)	Pr.1 (M S)	Pr.2 (M S)	R S 0	R S 1	RS 2	R S 3	R S 4	RS 5	RA 6	R A 7	R A 8	R A 9	R A 10	R A 11
0	3 0	60 0	51 49	30 11	0 0	0 4	1 28	2 3	2 2	2 25	0 0	2 2	2 27	2 3	1 2	0 45
						5 ½		3 7	2			2 5			8	½
1	2 9	59 59	51 19	29 14	0 1	0 4	1 29	2 4	2 3	2 24	0 10	2 2	2 26	2 2	1 2	0 44
						7		4 7	3			2 6			6	
2	2	59	50	28	0	0	1	2	2	2	0	2	2	2 1	1	0

G r. d ef	G r. as c	Pr.0 (M S)	Pr.1 (M S)	Pr.2 (M S)	R S 0	R S 1	RS 2	R S 3	R S 4	RS 5	RA 6	R A 7	RA 8	RA 9	R A 10	R A 11	
	8	57	48	16	3	4	30			23	19		26		2	42	
						8		5	2			2			5		
								8				7					
3	2	59	50	27	0	0	1	2	2	2	0	2	2	2	0	1	0
	7	54	16	18	4	5	31			22	28		25		2	41	
						0		6	2			2			4		
								8				8					
4	2	59	49	26	0	0	1	2	2	2	0	2	2	1	1	0	
	6	50	43	20	6	5	33			20	37		25	59	2	39	
						1		7	2			2			3		
								8				8					
5	2	59	49 9	25	0	0	1	2	2	2	0	2	2	1	1	0	
	5	45		22	7	5	34			19	46		24	58	2	38	
						3		8	2			2			2		
								9				9					
6	2	59	48	24	0	0	1	2	2	2	0	2	2	1	1	0	
	4	34	34	24	9	5	35			17	54		23	57	2	36	
						4		9	2			3			0		
								9				0					
7	2	59	47	23	0	0	1	2	2	2	1 2	2	2	1	1	0	
	3	32	57	26	1	5	37			15			23	56	1	35	
					0	6		1	3			3			9		
								0	0			0					
8	2	59	47	22	0	0	1	2	2	2	1 9	2	2	1	1	0	
	2	24	19	28	1	5	38			13			22	54	1	33	
					2	8		1	3			3			7		
								1	0			1					
9	2	59	46	22	0	0	1	2	2	2	1	2	2	1	1	0	
	1	15	39	29	1	5	39			10	16		21	53	1	32	
				[?]	4	9		1	3			3			6		
								2	0			1					
1	2	59 4	45	22	0	1	1	2	2	2 7	1	2	2	1	1	0	
0	0		58	30	1	1	40				22		21	52	1	30	

G r. d ef	G r. as c	Pr.0 (M S)	Pr.1 (M S)	Pr.2 (M S)	R S 0	R S 1	RS 2	R S 3	R S 4	RS 5	RA 6	R A 7	RA 8	RA 9	R A 10	R A 11
					5 ½			1 2	3 1			3 2			4	½
1 1	1 9	58 54	45 16	19 30	0 1 7	1 2	1 42	2 1 3	2 2 1	2 4	1 28	2 2 3	2 20	1 51	1 1 3	0 29
1 2	1 8	58 41	44 33	18 29	0 1 9	1 3	1 43	2 1 4	2 2 1	2 0	1 33	2 2 3	2 19	1 50	1 1 2	0 27
1 3	1 7	58 27	43 49	17 29	0 2 0	1 5	1 44	2 1 5	2 2 1	1 46	1 38	2 2 3	2 18	1 49	1 1 1	0 26
1 4	1 6	58 11	43 6	16 29	0 2 2	1 6	1 45	2 1 5	2 2 1	1 12	1 43	2 2 3	2 17	1 48	1 9	0 25
1 5	1 5	57 54	42 22	15 28	0 2 3	1 8	1 46	2 1 6	2 2 1	1 48	1 48	2 2 3	2 16	1 46	1 8	0 23
1 6	1 4	57 36	41 37	14 28	0 2 5	1 9	1 48	2 1 7	2 2 1	1 43	1 52	2 2 3	2 15	1 45	1 6	0 22
1 7	1 3	57 17	40 52	13 27	0 2 6	1 1	1 49	2 1 8	2 2 1	1 9 [?]	1 56	2 2 3	2 15	1 44	1 5	0 20
1 8	1 2	56 57	40 6	12 26	0 2 7	1 1	1 50	2 1 1	2 2 1	1 2 [?]	2 0	2 2 3	2 14	1 43	1 3	0 19

G r. d ef	G r. as c	Pr.0 (M S)	Pr.1 (M S)	Pr.2 (M S)	R S 0	R S 1	RS 2	R S 3	R S 4	RS 5	RA 6	R A 7	RA 8	RA 9	R A 10	R A 11
								9	2			1				
1	1	56	39	11	0	1	1	2	2	1	2 4	2	2	1	1	0
9	1	36	20	25	2	1	51			33			13	42	2	17
					9	3		2	3	[?]		3				
								0	2			0				
2	1	56	38	10	0	1	1	2	2	1	2 7	2	2	1	1	0
0	0	14	33	23	3	1	52			22			12	40	0	15
					0	4		2	3	[?]		2				½
					½			1	2			9				
2	9	55	37	9 21	0	1	1	2	2	1	2	2	2	1	0	0
1		51	46		3	1	53			16	10		11	39	5	14
					2	6		2	3	[?]		2			9	
								1	1			9				
2	8	55	36	8 18	0	1	1	2	2	0 9	2	2	2	1	0	0
2		27	58		3	1	54			[?]	13		11	38	5	12
					3	7		2	3			2			8	
								2	1			9				
2	7	55 2	36	7 16	0	1	1	2	2	0	2	2	2	1	0	0
3			16		3	1	56			37	15		10	37	5	10
					5	9		2	3	[?]		2			6	
								3	0			8				
2	6	54	35	6 14	0	1	1	2	2	0	2	2	2 8	1	0	0
4		37	21		3	2	57			28	17			35	5	9
					6	0		2	3			2			4	
								3	0			8				
2	5	54	34	5 12	0	1	1	2	2	0	2	2	2 7	1	0	0
5		11	31		3	2	58			19	19			34	5	7
					8	1		2	2			2			3	
								4	9			8				
2	4	53	33	4 10	0	1	2 0	2	2	0	2	2	2 6	1	0	0
6		44	41		3	2				10	24			33	5	6
					9	3		2	2		[?]	2			1	
								4	8			7				

G r. d ef	G r. as c	Pr.0 (M S)	Pr.1 (M S)	Pr.2 (M S)	R S 0	R S 1	RS 2	R S 3	R S 4	RS 5	RA 6	R A 7	RA 8	RA 9	R A 10	R A 11
2 7	3	53 16	32 50	3 8	0 4 1	1 2 4	2 2	2 2	0 0	2	25	2 2 5	1	30	0 5 0	0 4
								2 2	5 8			2 7		[?]		
2 8	2	52 48	31 58	2 5	0 4 2	1 2 5	2 6	2 2	0	2	23	2 2 4	1	30	0 4 8	0 3
								2 2	[?]	19		[?]				
								2 2	[?]			2 8				
2 9	1	52 19	31 5	1 3	0 4 4	1 2 6	2 8	2 2	0	2	24	2 2 2	1	29	0 4 7	0 1
								2 2	[?]			2 7				
								6 6								
3 0	0	51 49	30 11	0 0	0 4 5 ½	1 2 8	2 3	2 2	0 0	2	25	2 2 3	1	28	0 4 5 ½	0 0
								2 2				2 7				
								7 5								
(f o ot)		(11)	(10)	(9)												

Foot rubric under the proportional-minutes group: indices 11, 10, 9.

Table of the latitude of reflexion or obliquation of Venus [Tabula latitud. reflexionis seu obliquationis Veneris]

(This is the second, right-hand half of the Venus latitude-of-reflexion table begun on the facing page. The two outer columns give the degrees of descending and ascending motion [Gradus descensionis / Gradus ascensionis]; the next group, "Proportional minutes" [Scrupula proportionalia], gives the dodecatemoria columns 3, 4, 5; the central block, "Southern reflexion" [Reflexio Austrina], gives dodecatemoria 0–5; and the right block, "Northern reflexion" [Reflexio Borea], gives the orbital-anomaly columns 6–11. The foot legends read "Anomaly of the eccentric"

[Anomalía Eccentri] beneath the proportional-minutes block. All paired G.M. entries are degrees and minutes.)

G r a. d e sc .	G r a. s c. .	Pr op. 3 (M S)	Pr op. 4 (M S)	Pr op. 5 (M S)	Re fl. 0 (G M)	Re fl. 1 (G M)	Re fl. 2 (G M)	Re fl. 3 (G M)	Re fl. 4 (G M)	Re fl. 5 (G M)	Bo r. 6 (G M)	Bo r. 7 (G M)	Bo r. 8 (G M)	Bo r. 9 (G M)	Bo r. 10 (G M)	Bo r. 11 (G M)
0	3 0	0 0	30 11	51 49	0 0	0 45 ½	1 28	2 3 27	2 25	2	0 0	2 25	2 27	2 3	1 28	0 45 ½
1	2 9	1 3	31 5	52 19	0 1	0 47	1 29	2 4 27	2 24	2	0 10	2 26	2 26	2 2	1 26	0 44
2	2 8	2 5	31 58	52 48	0 3	0 48	1 30	2 5 28	2 23	2	0 19	2 27	2 26	2 1	1 25	0 42
3	2 7	3 8	32 50	53 16	0 4	0 50	1 31	2 6 28	2 22	2	0 28	2 28	2 25	2 0	1 24	0 41
4	2 6	4 10	33 41	53 44	0 6	0 51	1 33	2 7 28	2 20	2	0 37	2 28	2 25	1 59	1 23	0 39
5	2 5	5 12	34 31	54 11	0 7	0 53	1 34	2 8 29	2 19	2	0 46	2 29	2 24	1 58	1 22	0 38
6	2 4	6 14	35 21	54 37	0 9	0 54	1 35	2 9 29	2 17	2	0 54	2 30	2 23	1 57	1 20	0 36
7	2 3	7 16	36 10	55 2	0 10	0 56	1 37	2 10 30	2 15	2	1 2	2 30	2 23	1 56	1 19	0 35
8	2 2	8 18	36 38	55 27	0 12	0 58	1 38	2 11 30	2 13	2	1 9	2 31	2 22	1 54	1 17	0 33
9	2 1	9 21	37 46	55 51	0 14	0 59	1 39	2 12 30	2 10	2	1 16	2 31	2 21	1 53	1 16	0 32
10	2 0	10 23	38 33	56 14	0 15	1 0	1 40	2 12 31	2 31	2 7	1 22	2 32	2 21	1 52	1 14	0 30 ½
11	1 9	11 25	39 20	56 36	0 17	1 2	1 42	2 13 31	2 31	2 4	1 28	2 32	2 20	1 51	1 13	0 29
12	1 1	12 40	40 56	56 0	0 13	1 3	1 1	2 2	2 20	2 0	1 1	2 2	2 1	1 1	1 1	0 0

G r a. d e sc .	G r a . a s c.	Pr op. 3 (M S)	Pr op. 4 (M S)	Pr op. 5 (M S)	Re fl. 0 (G M)	Re fl. 1 (G M)	Re fl. 2 (G M)	Re fl. 3 (G M)	Re fl. 4 (G M)	Re fl. 5 (G M)	Bo r. 6 (G M)	Bo r. 7 (G M)	Bo r. 8 (G M)	Bo r. 9 (G M)	Bo r. 10 (G M)	Bo r. 11 (G M)
2	8	26	6	57	19		43	14	31		33	32	19	50	12	27
1	1	13	40	57	0	1 5	1	2	2	1	1	2	2	1	1	0
3	7	27	52	17	20		44	15	31	56	38	32	18	49	11	26
1	1	14	41	57	0	1 6	1	2	2	1	1	2	2	1	1 9	0
4	6	28	37	36	22		45	15	32	52	43	33	17	48		25
1	1	15	42	57	0	1 8	1	2	2	1	1	2	2	1	1 8	0
5	5	28	22	54	23		46	16	32	48	48	32	16	46		23
1	1	16	43	58	0	1 9	1	2	2	1	1	2	2	1	1 6	0
6	4	29	6	11	25		48	17	33	43	52	32	15	45		22
1	1	17	43	58	0	1	1	2	2	1	1	2	2	1	1 5	0
7	3	29	49	27	26	11	49	18	32	38	56	31	15	44		20
1	1	18	44	58	0	1	1	2	2	1	2	2	2	1	1 3	0
8	2	29	33	41	27	12	50	19	32	33	0	31	14	43		19
1	1	19	45	58	0	1	1	2	2	1	2	2	2	1	1 2	0
9	1	30	16	54	29	13	51	20	32	28	4	31	13	42		17
2	1	20	45	59	0	1	1	2	2	1	2	2	2	1	1 0	0
0	0	30	58	5	30	14	52	21	32	22	7	31	12	40		15
$\frac{1}{2}$																
2	9	21	46	59	0	1	1	2	2	1	2	2	2	1	0	0
1		29	39	15	32	16	53	21	31	16	9	30	12	39	59	14
2	8	22	47	59	0	1	1	2	2	1 9	2	2	2	1	0	0
2		28	19	24	33	17	54	22	31		13	30	11	38	58	12
2	7	23	47	59	0	1	1	2	2	1 2	2	2	2	1	0	0
3		26	57	32	35	19	56	23	30		15	30	10	37	56	10
2	6	24	48	59	0	1	1	2	2	0	2	2	2	1	0	0 9
4		24	34	39	36	20	57	23	30	54	17	29	9	35	54	
2	5	25	49	59	0	1	1	2	2	0	2	2	2	1	0	0 7
5		22	9	45	38	21	58	24	29	46	19	29	8	34	53	
2	4	26	49	59	0	1	1	2	2	0	2	2	2	1	0	0 6

G r a. d e sc. .	G r a. .	Pr op. 3 (M S)	Pr op. 4 (M S)	Pr op. 5 (M S)	Re fl. 0 (G M)	Re fl. 1 (G M)	Re fl. 2 (G M)	Re fl. 3 (G M)	Re fl. 4 (G M)	Re fl. 5 (G M)	Bo r. 6 (G M)	Bo r. 7 (G M)	Bo r. 8 (G M)	Bo r. 9 (G M)	Bo r. 10 (G M)	Bo r. 11 (G M)
6		20	43	50	39	23	59	25	28	37	20	28	7	33	51	
2	3	27	50	59	0	1	2 0	2	2	0	2	2	2	1	0	0 4
7		18	16	54	41	24		25	28	28	22	28	6	31	50	
2	2	28	50	59	0	1	2 1	2	2	0	2	2	2	1	0	0 3
8		16	48	57	42	25		26	27	19	23	28	5	30	48	
2	1	29	51	59	0	1	2 2	2	2	0	2	2	2	1	0	0 1
9		14	19	59	44	26		26	26	10	24	27	4	29	47	
3	0	30	51	60	0	1	2 3	2	2	0 0	2	2	2	1	0	0 0
0		11	49	0	45	28		27	25		25	27	3	28	45	
					$\frac{1}{2}$										$\frac{1}{2}$	

Foot legends (read bottom-to-top): proportional-minutes columns are also indexed 8, 7, 6; reflexion columns are indexed 6, 7, 8, 9, 10, 11; the right block carries indices 5, 4, 3, 2, 1, 0. **Anomaly of the eccentric** [Anomalía Eccentri].

Table of the latitude of declination of Mercury [Tabula latitud. declinationis Mercurii]

(First, left-hand half. Outer columns: degrees of descending / ascending motion. "Proportional minutes" [Scrupula proport.] gives dodecatemoria 0, 1, 2; "Southern declination" [Declina. Austral.] gives dodecatemoria 0, 1, 2; "Northern declination" [Declinat. Borea] gives orbital-anomaly columns 3, 4, 5. Foot legends: "Anomaly of the eccentric" [Anomalíae eccentrici] beneath the proportional/declination block, "Anomaly of the orb" [Anomalíae Orbis] beneath the northern-declination block.)

Gra. desc.	Gra. asc.	Prop. 0 (M S)	Prop. 1 (M S)	Prop. 2 (M S)	Decl.S 0 (G M)	Decl.S 1 (G M)	Decl.S 2 (G M)	Decl.B 3 (G M)	Decl.B 4 (G M)	Decl.B 5 (G M)		0	30	0 0	30 11	51 49	1 34	1									
24	0 52	0 0	1 16	2 43	1	29	1 3	31 5	52 19	1 34	1 23	0 50	0 2	1 19	2 46	2	28	2 5	31 58	52 48	1 34	1 23	0 49	0 5	1 21	2 49	
3	27	3 8	32 50	53 16	1 34	1 22	0 48	0 7	1 25	2 51	4	26	4 10	33 41	53 44	1 34	1 21	0 47	0 9	1 28	2 53	5	25	5 12	34 31	54 11	

	1	34		1	20		0	45		0	11		1	30		2	56			6		24		6	14		35	21		54	37		1	33		1	19	
	0	44		0	14		1	33		3	59	[?]			7		23		7	16		36	10		55	2		1	33		1	18		0	42		0	
15		1	36		3	0			8		22		8	18		36	58		55	27		1	33		1	17		0	40		0	18		1	38		3	3
		9		21		9	21		37	46		55	51		1	33		1	16		0	39		0	20		1	41		3	6			10		20		
10	23		38	33		56	14		1	33		1	15		0	37		0	23		1	45		3	8			11		19		11	25		39			
20		56	36		1	33		1	15		0	36		0	24		1	48		3	11			12		18		12	26		40	6		56	57			
1	33		1	14		0	34		0	27		1	50		3	13			13		17		13	27		40	52		57	17		1	32		1	13		
	0	32		0	30		1	53		3	15			14		16		14	28		41	37		57	36		1	32		1	12		0	31		0		
32		1	56		3	16			15		15		15	28		42	22		57	54		1	32		1	11		0	29		0	35		1	59			
3	18			16		14		16	29		43	6		58	11		1	32		1	10		0	27		0	37		2	2		3	19			17		
13		17	29		43	49		58	27		1	31		1	9		0	26		0	40		2	5		3	21			18		12		18	29			
44	33		58	41		1	31		1	7		0	24		0	42		2	8		3	23			19		11		19	30		45	16		58	54		
	1	31		1	6		0	22		0	45		2	11		3	24			20		10		20	30		45	58		59	15		1	30		1	5	
	0	20		0	48		2	14		3	25			21		9		21	29		46	39		59	24		1	30		1	4		0	19		0	51	
	2	17		3	26			22		8		22	28		47	19		59	24		1	29		1	3		0	17		0	53		2	20		3	27	
		23		7		23	26		47	57		59	32		1	29		1	2		0	15		0	56		2	23		3	28			24		6		
24	24		48	34		59	39		1	28		1	0		0	14		0	59		2	26		3	29			25		5		25	22		49	9		
	59	45		1	28		0	59		0	11		1	1		2	27		3	30			26		4		26	20		49	43		59	50		1		
27		0	58		0	9		1	5		2	31		3	31			27		3		27	18		50	16		59	54		1	26		0	57		0	7
	1	8		2	34		3	32			28		2		28	16		50	48		59	57		1	26		0	55		0	4		1	10		2	37	
3	32			29		1		29	14		51	19		59	59		1	25		0	53		0	2		1	13		2	40		3	33			30		
0		30	11		51	49		60	0		1	24		0	52		0	0		1	16		2	43		3	33											

Foot legends: proportional/southern-declination columns are also indexed 5, 4, 3 / 11, 10, 9; northern-declination columns indexed 9, 8, 7, 6. **Anomaly of the eccentric** [Anomaliae eccentrici] · **Anomaly of the orb** [Anomaliae Orbis].

Table of the latitude of declination of Mercury [Tabula latitud. Declinationis Mercurii]

(Second, right-hand (mirror) half. The body figures repeat those of the left half; only the column headings are interchanged. "Proportional minutes" [Scrupula proporti.] here gives dodecatemoria 6, 7, 8; "Northern declination" [Declina. Borea] gives dodecatemoria 0, 1, 2; "Southern declination" [Declinat. Austr.] gives orbital-anomaly columns 3, 4, 5. Because the proportional-minutes values are symmetric, columns 6/7/8 carry the same M S figures as columns 0/1/2 of the facing page.)

	Gra. desc.		Gra. asc.		Prop. 6 (M S)		Prop. 7 (M S)		Prop. 8 (M S)		Decl.B 0 (G M)		Decl.B 1 (G M)		Decl.B 2 (G M)		Decl.A 3 (G M)		Decl.A 4 (G M)		Decl.A 5 (G M)			---		---		---		---		---		---		---		---		---		---		0		30		0	0		30	11		51	49		1	34		1

24 | 0 52 | 0 0 | 1 16 | 2 43 | | 1 | 29 | 1 3 | 31 5 | 52 19 | 1 34 | 1 23 | 0 50 | 0 2 |
 1 19 | 2 46 | | 2 | 28 | 2 5 | 31 58 | 52 48 | 1 34 | 1 23 | 0 49 | 0 5 | 1 21 | 2 49 | |
 3 | 27 | 3 8 | 32 50 | 53 16 | 1 34 | 1 22 | 0 48 | 0 7 | 1 25 | 2 51 | | 4 | 26 | 4 10 |
 33 41 | 53 44 | 1 34 | 1 21 | 0 47 | 0 9 | 1 28 | 2 53 | | 5 | 25 | 5 12 | 34 31 | 54 11
 | 1 34 | 1 20 | 0 45 | 0 11 | 1 30 | 2 56 | | 6 | 24 | 6 14 | 35 21 | 54 37 | 1 33 | 1 19
 | 0 44 | 0 14 | 1 33 | 2 59 | | 7 | 23 | 7 16 | 36 10 | 55 2 | 1 33 | 1 18 | 0 42 | 0 15 |
 1 36 | 3 0 | | 8 | 22 | 8 18 | 36 58 | 55 27 | 1 33 | 1 17 | 0 40 | 0 18 | 1 38 | 3 3 | |
 9 | 21 | 9 21 | 37 46 | 55 51 | 1 33 | 1 16 | 0 39 | 0 20 | 1 41 | 3 6 | | 10 | 20 | 10
 23 | 38 33 | 56 14 | 1 33 | 1 15 | 0 37 | 0 23 | 1 45 | 3 8 | | 11 | 19 | 11 25 | 39 20 |
 56 36 | 1 33 | 1 15 | 0 36 | 0 24 | 1 48 | 3 11 | | 12 | 18 | 12 26 | 40 6 | 56 59 | 1 33
 | 1 14 | 0 34 | 0 27 | 1 50 | 3 13 | | 13 | 17 | 13 27 | 40 52 | 57 17 | 1 32 | 1 13 | 0
 32 | 0 30 | 1 53 | 3 15 | | 14 | 16 | 14 28 | 41 37 | 57 36 | 1 32 | 1 12 | 0 31 | 0 32 |
 1 56 | 3 16 | | 15 | 15 | 15 28 | 42 22 | 57 54 | 1 32 | 1 11 | 0 29 | 0 35 | 1 59 | 3 18
 | | 16 | 14 | 16 29 | 43 6 | 58 11 | 1 32 | 1 10 | 0 27 | 0 37 | 2 2 | 3 19 | | 17 | 13 |
 17 29 | 43 49 | 58 27 | 1 31 | 1 9 | 0 26 | 0 40 | 2 5 | 3 21 | | 18 | 12 | 18 29 | 44 33
 | 58 41 | 1 31 | 1 7 | 0 24 | 0 42 | 2 8 | 3 23 | | 19 | 11 | 19 30 | 45 16 | 58 54 | 1
 31 | 1 6 | 0 22 | 0 45 | 2 11 | 3 24 | | 20 | 10 | 20 30 | 45 58 | 59 5 | 1 30 | 1 5 | 0
 20 | 0 48 | 2 14 | 3 25 | | 21 | 9 | 21 29 | 46 39 | 59 15 | 1 30 | 1 4 | 0 19 | 0 51 | 2
 17 | 3 26 | | 22 | 8 | 22 28 | 47 19 | 59 24 | 1 29 | 1 3 | 0 17 | 0 53 | 2 20 | 3 27 | |
 23 | 7 | 23 26 | 47 57 | 59 32 | 1 29 | 1 2 | 0 15 | 0 56 | 2 23 | 3 28 | | 24 | 6 | 24
 24 | 48 34 | 59 39 | 1 28 | 1 0 | 0 13 | 0 59 | 2 26 | 3 29 | | 25 | 5 | 25 22 | 49 9 |
 59 45 | 1 28 | 0 59 | 0 11 | 1 1 | 2 27 | 3 30 | | 26 | 4 | 26 20 | 49 43 | 59 50 | 1 27
 | 0 58 | 0 9 | 1 5 | 2 31 | 3 31 | | 27 | 3 | 27 18 | 50 16 | 59 54 | 1 26 | 0 57 | 0 7 |
 1 8 | 2 34 | 3 32 | | 28 | 2 | 28 16 | 50 48 | 59 57 | 1 26 | 0 55 | 0 4 | 1 10 | 2 37 | 3
 32 | | 29 | 1 | 29 14 | 51 19 | 59 59 | 1 25 | 0 53 | 0 2 | 1 13 | 2 40 | 3 33 | | 30 | 0
 | 30 11 | 51 49 | 60 0 | 1 24 | 0 52 | 0 0 | 1 16 | 2 43 | 3 33 |

Foot legends: proportional columns also indexed 11, 10, 9; northern-declination
 columns 11, 10, 9; southern-declination columns 8, 7, 6. **Anomaly of the
 eccentric** [Anomalia Eccentri] · **Anomaly of the orb** [Anomalia Orbis].

Table of the reflexion of Mercury [Tabula reflexionis Mercurii]

*(Outer columns: degrees of descending / ascending motion. "Proportional minutes"
 [Scrupula propor.] gives dodecatemoria 0, 1, 2; "Southern reflexion" [Reflexio
 Austral.] gives dodecatemoria 0–5; "Northern reflexion" [Reflexio Borea] gives
 orbital-anomaly columns 6–11. Foot legend: "Anomaly of the eccentric" [Anomalia
 Eccentri].)*

G ra d es c.	G r a s c.	Pro p. 0 (M S)	Pro p. 1 (M S)	Pro p. 2 (M S)	R A 0 (G M)	RA 1 (G M)	R A 2 (G M)	R A 3 (G M)	R A 4 (G M)	R A 5 (G M)	R B 6 (G M)	RB 7 (G M)	R B 8 (G M)	R B 9 (G M)	R B 10 (G M)	R B 11 (G M)
0	3 0	60 0	51 49	30 11	0 0	0 41	1 1 6	1 4 2	1 4 6	1 1 5	0 0	1 37	2 1 6	2 1 0	1 38	0 53
1	2 9	59 59	51 19	29 14	0 2	0 43	1 1 7	1 4 3	1 4 6	1 1 3	0 4	1 38	2 1 6	2 9	1 37	0 51
2	2 8	59 57	50 48	28 16	0 3	0 44	1 1 8	1 4 3	1 4 5	1 1 1	0 8	1 40	2 1 7	2 9	1 35	0 49
3	2 7	59 54	50 16	27 18	0 4	0 45	1 1 9	1 4 4	1 4 4	1 9 1	0 1	1 42	2 1 7	2 8	1 34	0 48
4	2 6	59 50	49 43	26 20	0 5	0 47	1 2 0	1 4 4	1 4 4	1 7 5	0 1	1 44	2 1 7	2 7	1 33	0 46
5	2 5	59 45	49 9	25 22	0 7	0 48	1 2 1	1 4 4	1 4 3	1 5 9	0 1	1 46	2 1 7	2 6	1 31	0 45
6	2 4	59 39	48 34	24 24	0 8	0 40 [rect e: 0 49]	1 2 2	1 4 4	1 4 2	1 3 2	0 2 2	1 48	2 1 8	2 5	1 30	0 43
7	2 3	59 32	47 57	23 26	0 1 0	0 50	1 2 3	1 4 5	1 4 1	0 1 6	0 2	1 50	2 1 8	2 4	1 28	0 41
8	2 2	59 24	47 19	22 28	0 1 2	0 52	1 2 4	1 4 5	1 4 0	0 5 9	0 2 9	1 52	2 1 8	2 3	1 27	0 40
9	2 1	59 15	46 39	21 29	0 1 3	0 53	1 2 5	1 4 6	1 3 9	0 5 7	0 3 3	1 54	2 1 8	2 2	1 25	0 38

G ra . d es c.	G r a s c.	Pro p. 0 (M S)	Pro p. 1 (M S)	Pro p. 2 (M S)	R A 0 (G M)	RA 1 (G M)	R A 2 (G M)	R A 3 (G M)	R A 4 (G M)	R A 5 (G M)	R B 6 (G M)	RB 7 (G M)	R B 8 (G M)	R B 9 (G M)	R B 10 (G M)	R B 11 (G M)
1 0	2 0	59 4	45 58	20 30	0 1 5	0 54	1 2 6	1 4 6	1 3 8	0 5 5	0 3 7	1 55	2 1 8	2 1	1 24	0 35
1 1	1 9	58 54	45 16	19 30	0 1 6	0 55	1 2 7	1 4 6	1 3 7	0 5 3	0 4 1	1 57	2 1 8	2 0	1 22	0 34
1 2	1 8	58 41	44 33	18 29	0 1 7	0 56	1 2 8	1 4 7	1 3 6	0 5 0	0 4 5	1 48 [rect e: 1 58]	2 1 8	1 5 9	1 21	0 32
1 3	1 7	58 27	43 49	17 29	0 1 9	0 57	1 2 9	1 4 7	1 3 5	0 4 8	0 4 7	1 59	2 1 8	1 5 8	1 19	0 31
1 4	1 6	58 11	43 6	16 29	0 2 1	0 59	1 3 0	1 4 7	1 3 4	0 4 6	0 5 2	2 0	2 1 8	1 5 7	1 18	0 29
1 5	1 5	57 54	42 22	15 28	0 2 2	1 0	1 3 0	1 4 7	1 3 3	0 4 3	0 5 3	2 1	2 1 8	1 5 6	1 16	0 27
1 6	1 4	57 36	41 37	14 28	0 2 3	1 1	1 3 1	1 4 8	1 3 2	0 4 1	0 5 8	2 3	2 1 7	1 5 5	1 15	0 25
1 7	1 3	57 17	40 52	13 27	0 2 4	1 2	1 3 2	1 4 8	1 3 1	0 3 8	1 3 8	2 4	2 1 7	1 5 4	1 13	0 23
1 8	1 2	56 57	40 6	12 26	0 2 5	1 3	1 3 3	1 4 8	1 3 0	0 3 5	1 4 5	2 6	2 1 7	1 5 3	1 12	0 21
1 9	1 1	56 36	39 20	11 25	0 2 7	1 4	1 3 4	1 4 8	1 2 9	0 3 3	1 7 3	2 7	2 1 6	1 5 2	1 10	0 19

G ra . d es c.	G r a s c.	Pro p. 0 (M S)	Pro p. 1 (M S)	Pro p. 2 (M S)	R A 0 (G M)	RA 1 (G M)	R A 2 (G M)	R A 3 (G M)	R A 4 (G M)	R A 5 (G M)	R B 6 (G M)	RB 7 (G M)	R B 8 (G M)	R B 9 (G M)	R B 10 (G M)	R B 11 (G M)
2 0	1 0	56 14	38 33	10 23	0 2 8	1 5	1 3 5	1 4 8	1 2 8	0 3 0	1 1 0	2 8	2 1 6	1 5 0	1 9	0 18
2 1	9	55 51	37 46	9 21	0 3 0	1 6	1 3 5	1 4 8	1 2 7	0 2 7	1 1 3	2 9	2 1 6	1 4 9	1 7	0 16
2 2	8	55 27	36 58	8 18	0 3 1	1 8	1 3 6	1 4 8	1 2 6	0 2 4	1 1 6	2 10	2 1 5	1 4 8	1 6	0 14
2 3	7	55 2	36 10	7 16	0 3 2	1 9	1 3 7	1 4 8	1 2 5	0 2 1	1 1 9	2 11	2 1 5	1 4 7	1 4	0 12
2 4	6	54 37	35 21	6 14	0 3 3	1 10	1 3 8	1 4 8	1 2 4	0 1 7	1 2 1	2 12	2 1 4	1 4 6	1 3	0 10
2 5	5	54 11	34 31	5 12	0 3 5	1 11	1 3 9	1 4 8	1 2 3	0 1 5	1 2 4	2 13	2 1 4	1 4 5	1 2	0 8
2 6	4	53 44	33 41	4 10	0 3 6	1 12	1 4 0	1 4 7	1 2 2	0 1 2	1 2 6	2 14	2 1 3	1 4 4	1 0	0 6
2 7	3	53 16	32 50	3 8	0 3 7	1 13	1 4 0	1 4 7	1 2 0	0 9 9	1 2 9	2 15	2 1 3	1 4 2	0 58	0 5
2 8	2	52 48	31 58	2 5	0 3 8	1 14	1 4 1	1 4 7	1 1 8	0 6 1	1 3 1	2 15	2 1 2	1 4 1	0 56	0 3
2 9	1	52 19	31 5	1 3	0 3 9	1 15	1 4 1	1 4 6	1 1 7	0 3 4	1 3 4	2 16	2 1 0	1 4 0	0 55	0 2
3	0	51	30	0 0	0	1 16	1	1	1	0	1	2 16	2	1	0	0

G	G	R					R	R	R	R	R	R				R
ra	r	A					A	A	A	A	B	B		B	R	R
.	a.	Pro	Pro	Pro	0		2	3	4	5	6		8	9	B	B
d	a	p. 0	p. 1	p. 2	(G		(G	(G	(G	(G	(G		(G	(G	10	11
es	s	(M	(M	(M	M	RA 1	M	M	M	M	M	RB 7	M	M	(G	(G
c.	c.	S)	S)	S)	)	(G M)	)	)	)	)	)	(G M)	)	)	M)	M)
0		49	11		4		4	4	1	0	3		1	3	53	0
					1		2	6	5		7		0	8		

Foot legends (read bottom-to-top): the proportional columns are indexed 11, 10, 9; the orbital-anomaly columns of the Northern-reflexion block carry the descending series 5, 3, 2, 0 at the foot. **Anomaly of the eccentric** [Anomalia Eccentri].

Table of the Reflexion of Mercury [Tabula Reflexionis Mercurii]

The table is read with two complementary degree-arguments in the left margin: **Gra. desc.** (descending degree, G) running 0–30 downward, and **Gra. asce.** (ascending degree, G) running the reverse. The next block, **Proportional minutes** [Scrupula Proportionalia], gives the *Dodecatemoria of the Eccentric* under column-heads 3, 4, 5 (each in M. = minutes, S. = seconds), with the complementary heads 8, 7, 6 printed at the foot beneath the legend **Anomaly of the Eccentric** [Anomalia Eccentri]. The right block, **Northern Reflexion** [Reflexio Borea], lists the *Dodecatemoria* (twelfth-parts) under heads 0, 1, 2, 3, 4, 5, 6, and **Southern Reflexion** [Reflexio Austrina] lists the *Anomaly of the Orb* [Anomaliae Orbis] under heads 7, 8, 9, 10, 11 — every reflexion entry given as G. (degrees) M. (minutes).

de	as	3	4	5	0	1	2	3	4	5	6	7	8	9	10	11
sc.	c.	(M	(M	(M	(G	(G	(G	(G	(G	(G	(G	(G	(G	(G	(G	(G
G	G	S)	S)	S)	M)	M)	M)	M)	M)	M)	M)	M)	M)	M)	M)	M)
0	3	0	30	51	0	0	1	1	2	1	0	1	2	2	1	0
	0	0	11	49	0	46	24	54	0	23	0	53	35	26	48	56
1	2	1	31	52	0	0	1	1	2	1	0	1	2	2	1	0
	9	3	5	19	2	47	25	55	0	26	5	55	35	25	46	56
2	2	2	31	52	0	0	1	1	1	1	0	1	2	2	1	0
	8	5	58	48	3	49	27	55	59	24	9	57	35	24	45	55
3	2	3	32	53	0	0	1	1	1	1	0	1	2	2	1	0

de	as	3	4	5	0	1	2	3	4	5	6	7	8	9	10	11
sc.	c.	(M	(M	(M	(G	(G	(G	(G	(G	(G	(G	(G	(G	(G	(G	(G
G	G	S)	S)	S)	M)	M)	M)	M)	M)	M)	M)	M)	M)	M)	M)	M)
	7	8	50	16	4	50	28	56	59	21	13	59	35	23	43	52
4	2	4	33	53	0	0	1	1	1	1	0	2	2	2	1	0
	6	10	41	44	5	51	29	56	59	19	17	2	35	22	42	51
5	2	5	34	54	0	0	1	1	1	1	0	2	2	2	1	0
	5	12	31	11	7	53	30	57	58	17	21	4	36	21	40	49
6	2	6	35	54	0	0	1	1	1	1	0	2	2	2	1	0
	4	14	27	37	9	54	31	57	57	15	25	6	36	20	39	47
7	2	7	36	55	0	0	1	1	1	1	0	2	2	2	1	0
	3	16	10	2	11	55	33	57	57	12	30	8	36	19	37	45
8	2	8	36	55	0	0	1	1	1	1	0	2	2	2	1	0
	2	18	58	27	12	57	34	57	56	9	34	10	36	18	35	43
9	2	9	37	55	0	0	1	1	1	1	0	2	2	2	1	0
	1	21	46	51	14	58	35	58	55	6	38	12	36	17	33	41
10	2	10	38	56	0	0	1	1	1	1	0	2	2	2	1	0
	0	23	33	14	15	59	36	58	55	4	42	14	36	16	32	39
11	1	11	39	56	0	1	1	1	1	1	0	2	2	2	1	0
	9	25	20	36	17	0	37	59	54	2	46	16	36	14	30	37
12	1	12	40	56	0	1	1	1	1	1	0	2	2	2	1	0
	8	26	6	57	18	1	30	59	53	0	51	17	36	13	28	35
							[?]									
13	1	13	40	57	0	1	1	2	1	0	0	2	2	2	1	0
	7	27	52	17	20	3	40	0	52	57	55	18	35	12	27	33
14	1	14	41	57	0	1	1	2	1	0	0	2	2	2	1	0
	6	28	37	36	21	5	41	0	51	53	59	19	35	11	25	31
15	1	15	42	57	0	1	1	2	1	0	1	2	2	2	1	0
	5	28	22	54	23	6	42	0	50	50	3	21	35	10	23	29
16	1	16	43	58	0	1	1	2	1	0	1	2	2	2	1	0
	4	29	6	11	24	7	43	0	49	47	7	23	34	9	22	27
17	1	17	43	58	0	1	1	2	1	0	1	2	2	2	1	0
	3	29	49	27	26	9	44	0	47	44	11	24	34	8	20	24
18	1	18	44	58	0	1	1	2	1	0	1	2	2	2	1	0
	2	29	33	41	27	10	45	1	47	40	16	26	33	6	18	22

de sc. G	as c. G	3 (M S)	4 (M S)	5 (M S)	0 (G M)	1 (G M)	2 (G M)	3 (G M)	4 (G M)	5 (G M)	6 (G M)	7 (G M)	8 (G M)	9 (G M)	10 (G M)	11 (G M)
19	1	19	45	58	0	1	1	2	1	0	1	2	2	2	1	0
	1	30	16	54	29	11	45	1	46	37	18	27	33	5	16	20
20	1	20	45	59	0	1	1	2	1	0	1	2	2	2	1	0
	0	30	58	5	31	13	46	1	45	34	22	28	34	3	15	19
21	9	21	46	59	0	1	1	2	1	0	1	2	2	2	1	0
		29	39	15	32	14	47	1	44	31	25	28	32	2	13	17
22	8	22	47	59	0	1	1	2	1	0	1	2	2	2	1	0
		28	19	24	34	15	48	1	43	28	27	29	31	0	12	15
23	7	23	47	59	0	1	1	2	1	0	1	2	2	1	1	0
		26	57	32	36	16	49	1	41	24	32	30	30	59	10	13
24	6	24	48	59	0	1	1	2	1	0	1	2	2	1	18	0
		24	34	39	37	17	50	1	39	20	33	31	30	57		11
25	5	25	49	59	0	1	1	2	1	0	1	2	2	1	17	09
		22	9	45	39	18	51	1	37	16	38	32	29	56		
26	4	26	49	59	0	1	1	2	1	0	1	2	2	1	15	07
		20	43	50	41	19	51	1	36	13	41	33	28	54		
27	3	27	50	59	0	1	1	2	1	0	1	2	2	1	13	05
		18	16	54	42	20	52	0	33	10	44	33	28	52		
28	2	28	50	59	0	1	1	2	1	0	1	2	2	1	12	03
		16	48	57	43	21	53	0	31	7	47	34	27	51		
29	1	29	51	59	0	1	1	2	1	0	1	2	2	1	10	02
		14	19	59	45	22	54	0	20	4	50	35	26	49		
30	0	30	57	60	0	1	1	2	1	0	1	2	2	1	0	00
		11	49	0	46	24	54	0	23	0	53	35	26	48	56	

Foot of the proportional-minutes block (complementary heads): **8 · 7 · 6**, below the legend **Anomaly of the Eccentric** [Anomalia Eccentri].

[Margin: Precept for computing the latitude in Venus and Mercury.]

CHAPTER XXV [CAPVT XXV]

On computing the latitudes of Venus and Mercury from the preceding tables [De latitudinum Veneris ac Mercurii e tabulis praecedentibus computatione]

We constructed these rules for the latitudes of Venus and Mercury throughout on the model of the Prutenic Tables [Tabulae Prutenicae], both for greater convenience and to avoid error here; consequently we scarcely depart at all from the precept of [Erasmus] Reinhold in the same place regarding how these latitudes are represented. Even so, an enormous difference is sometimes found in the actual phenomena — so great, indeed, that it often amounts to a full three degrees, chiefly in Venus, as the reformation of the tables shows when set against the old ones. The latitude, then — although in itself it is uniform, as its computation by triangles demonstrated above — was nonetheless distributed, so that it might be the better caught in tables and made serviceable through the help of proportional minutes, in some imitation of the older astronomers, into a *declination*, computed only at the actual quadrants of the eccentric, and a *reflexion* (or *obliquation*), likewise computed only at each of the two apsides. These two species, mixed together, are what we everywhere hunt for at the intermediate places in these tables — the *deviation* (as they call it) being set aside, for the reason mentioned above.

The substance of the precept, then, is this. By the **equated anomaly of the eccentric** — after, in Venus, 16 degrees have been added to it for our own age — take out the proportional minutes; and by the **equated anomaly of commutation** take out the latitude, with this caution: that from whichever part of the canon supplies the proportional minutes you draw the latitude as well, both the declination and the reflexion. The column-titles indicate which kind of latitude it is.

Then, applying the proportional minutes, take from each of the latitudes the parts that correspond to their respective latitudes. Finally, if both — the declination and the reflexion, thus corrected — belong to the same direction, the latitude sought is their sum; but if not, the smaller species subtracted from the larger leaves the difference, which is to be carried over as the latitude sought, and it will keep the name (north or south) belonging to whichever was the excess, that is, the remainder.

Worked example [Exemplum]

We will illustrate this general precept of the latitude of Venus here only — and chiefly — by comparing it with an ancient observation, so that it may be established in some measure that the motion of its nodes does not perpetually cling to the apsides, or that it keeps in some way to a motion commensurable with these.

[Margin: Cap. 19 of this book.]

For this purpose I think we should use that same observation of Timocharis which we employed for correcting the longitude, since its requisites were ready to hand there. This, as we reported above from [Ptolemy], was made in year 13 of [Ptolemy] Philadelphus, on the day beginning the 18th of the month Messori, in the morning; where, among other things, we found the latitude of Venus, at its conjunction with the fixed star in the southern wing — namely the one opposite the Vintager [Vindemiatrix], in Virgo (♍) — to be $1^{\circ} 16'$ north.

At that same time the equated anomaly of the eccentric was 4 signs $29^{\circ} 0'$, and that of the restored orb — the equated anomaly of commutation — was 8 signs $11^{\circ} 55'$. Accordingly, in the former table of Venus, the one inscribed *of declination*, by the proposed anomaly of the eccentric there are first found proportional minutes $31' 5''$; and by the anomaly of the orb the southern declination itself, $0^{\circ} 46'$, from which the proportional part is deduced as $23' 50''$, to be kept. Then in the same way one enters the following table of Venus, the one that gives the *reflexion*, and likewise, with 4 signs $23^{\circ} 0'$ of the anomaly of the eccentric there are found —

— in the field, proportional minutes $52' 19''$; and with the anomaly of the orb likewise 8 signs $11^{\circ} 55'$, the northern obliquation is taken as $2^{\circ} 19'$, from which too, by the proportional minutes already noted, the proportional part is drawn out as $1^{\circ} 48' 53''$. Since this turns out to be greater than the declination, and moreover of the opposite direction, it is therefore subtracted from that one, and there is left the latitude of Venus sought, $1^{\circ} 25' 3''$ north. This exceeds the observed value by only 9 minutes — provided we judge that the orb of the planet has been unalterable as to any change in the ecliptic; but otherwise it agrees entirely. For the said star in the wing holds a latitude of $1^{\circ} 25'$ north in our own age.

But the ancient observations are not to be trusted so far, nor is such precision to be granted, that anything should here be concluded precisely — contrary to what was done above concerning the latitudes of the three superior planets. Rather, only this much, on account of which we adduced this example earlier as

indicated: namely, that we may understand that the northern section of the latitude of Venus in that age clung as nearly as possible to its own apogee, or had run a little ahead of it in the order of the signs — whereas now in our own age, as we have said, and as all our modern observations prove, it has fallen back at least 16 degrees behind the same point. Around the time of Christ's birth, then, since we gather that the point of the said intersection lingered near the apogee, and since therefore it must be carried back to intermediate times, whether earlier or later, it is established by reasoning that about one degree corresponds to each hundred years or so — by which the motion of the ecliptic section and of the orb of Venus will be reckoned slower than that of this apogee. In Mercury, by contrast, the section of the southern node accompanies its own apogee unceasingly and without dislocation, just as our observations sufficiently support the opinion of [Ptolemy] in this respect.

And here, by the grace of God, we set an end to the **Restoration of Astronomy** [Astronomiae Redintegratio]; in which, according to our plan, we have corrected the hypotheses and the Tables — which are to display the celestial phenomena — against the infallible observations that we were able to obtain. And we have done so on these terms: that we would have these persevering labors of ours — which we hope will profit all honest people and all who love these disciplines — judged by no one, that is, branded or perversely disdained, except by one who takes pains to institute properly their comparison with the celestial appearances, so that from this he may rather pronounce a just verdict upon them; or who, at length setting his hand to the table, may himself feel how difficult this province has been.

As for the *station*, *retrogression*, and likewise the *appearance* and *occultation* of the planets, we think there is no need at all of a separate treatment in imitation of [Ptolemy] or of [Copernicus], since the recognition of these things falls out better in the computation of the true motions of those same planets, as [Copernicus] rightly judged long ago; and the science of these matters is most correctly drawn from our *Doctrine of the Sphere* [Doctrina Sphaerica], Book 2, Chapter 4, which clearly sets out the rising and setting of the stars.

To God alone be glory. [Soli Deo Gloria.]

THE END. [FINIS.]

[The fourth page of this opening is blank.]

Appendix to the *Astronomia Danica*

Here begins the **Appendix** to the *Astronomia Danica**, a self-contained treatise on the new stars (of 1572 and 1604) and the comets (of 1607 and 1618). The Appendix carries its own title page and its own independent pagination, beginning again from page 1.*

Appendix title page

APPENDIX TO THE *ASTRONOMIA DANICA* [*Astromiæ Danicæ Appendix* — so printed, with the title-page misspelling "ASTROMIÆ" for "ASTRONOMIÆ"], ON THE ADVENTITIOUS PHENOMENA OF THE HEAVENS [*De Asscitiis Coeli Phænomenis*], namely THE NEW STARS AND COMETS [*Stellis Novis et Cometis*].

BY THE AUTHOR Christian Severinsen [*Christiano Severini F. Longomontano*], son of Severin, Longomontanus.

AMSTERDAM, at the press of Johannes and Cornelius Blaeu [*Apud Ioannem & Cornelium Blaeu*]. In the year 1640 [M D C X L].

Diagram. Printer's device of the house of Blaeu. At the center stands an armillary sphere on a stand, encircled by graduated rings. To the left stands a winged figure (Mercury, with caduceus and winged cap); to the right stands a standing male figure (a savage or Hercules-type figure holding a staff). Below, a banderole carries the motto.

- Banderole (center, below sphere): *INDEFESSVS AGENDO* ("Tireless in action")

Commendatory poem

To the distinguished and most excellent gentleman, Sir Christian Severinsen Longomontanus [Dn. Christianum S. Longomontanum], Royal Professor of Mathematics in the Academy of Copenhagen [in Academia Hafniensi], on his publishing a new work on comets.

Far beneath your spirit, great man, lies any care for the lowly earth or for whatever this slippery globe contains. Your one concern is to leave mortal cares behind and to climb, in spirit, to the roofs of the lofty heaven. You enter the venerable palaces of the sky; with shrewd mind you mark the hidden causes of things. You declare what the hostile torch of a comet portends, and what God himself means by these sacred fires. For not in vain are new portents seen in the

heavens: they are sure tokens of an impending evil. Those believe it who have learned by experience — those whom, with a harsh lash, the hostile force of Jove's hand has struck down until now. Yet the end of the punishment has not come, because the end of the crimes has not come. Take away the crime, and no star does harm.

Caspar Erasmus Brochmand [*Casp. Erasm. Brochmand*], Professor of Sacred Literature in the Academy of Copenhagen [*in Acad. Hafniensi*].

Dedicatory Epistle

[*Margin: This dedication is dated 1 November 1621 (see its closing), preserved from the 1622 first edition of the appendix; the present volume was printed in 1640.*]

TO THE MOST SERENE AND MOST EXALTED PRINCE, Sir CHRISTIAN IV [*Dn. Christianum IV*], ELECTED KING OF DENMARK AND NORWAY [*Danice et Norvegiæ Regem Electum*], HIS OWN MOST GRACIOUS LORD.

Just as it was my duty, most exalted Prince, to dedicate the *Astronomia Danica* — worked out so far as the strength of my mind and body had thus far allowed — to the most serene and most powerful King of Denmark, the parent of Your Highness, indeed to the renowned father of his country, with all the submission of spirit that is fitting: so now it is almost equally my duty humbly to inscribe and present to Your Most Serene Highness its appendix on the new phenomena of the heavens. For since God, the best and greatest, has in these times shown us such clemency that these ancestral kingdoms not only begin to lift themselves higher from the ground day by day and to flourish under so great a king, but also — and this must be reckoned a matter of the greatest moment — that the sound confession of the triune God and of the mystery of our salvation in Christ Jesus is preserved with one accord, while in the meantime no honorable arts or disciplines are neglected: I, for my part and according to my slender strength, once I had been appointed to this Royal Academy, judged it a sin if either those things which had earlier been accomplished at the greatest expense for the restoration of astronomy, sent down from heaven in our homeland, or those things which God in the meantime willed to shine forth in the heavens as a warning to mortals, should be suppressed and utterly perish — especially since I had learned that whatever industry of mine was directed toward bringing these things to light would be welcome to the gracious will of my superiors. Among other things, I was diligently to direct my efforts to this end: to record carefully that fateful comet which, this last three years past, blazed out in the heavens menacingly enough. And not in vain — since with its rising, and thereafter, the

Roman Empire was so shaken and thrown into confusion that even at present its very safety is imperiled, and any solid restoration is thrown into doubt, unless the merciful God himself shall interpose, as it were directly, and bring timely aid to our exhausted affairs.

But although, as long as it lasted, I omitted here neither opportunity nor any industry for grasping the motion and position of this tailed star too — as my observations, compared with those of others, will, I hope, sufficiently prove — yet very many causes of delay pressed themselves upon me, which too long held in suspense the dutiful service I owed in setting down its record in good time. Among these was that deadly plague, sent by God at the same time as a punishment for our sins, which began to rage at the rising of this comet and lasted a year and a half; on account of which this Academy was for about three-quarters of a year suspended and dispersed. But lest I should bring to birth a piece concerning this one comet alone too late, amid the others which others have lately hurried into the light, I thought it would be worth my while to make the subject — so specialized, concerning a single comet — richer and fuller, and to bring before the public a general treatise on the new celestial phenomena, derived, so far as I could, through their causes as through first principles; and to append to it, by way of examples, the phenomena and such astrological judgments as there may be of the two most recent comets; so that in this way at last I might return to discharging humbly, as I wished, the service I owed.

This treatise, however — though I confess it to be far smaller than befits so great a Prince, who, like a new Sun, pours himself out upon all with the rays and brightness of his ancestral, paternal, and wholly heroic virtue and clemency, so that [a 2 — gathering signature] the hope of these kingdoms

the hope of these kingdoms rests solely upon Your Highness, with good God's help, secured for the future — nevertheless, while among the languages, arts, and sciences in which Your Highness is so excellently skilled you also handle, with singular skill, the exercises of mathematical matters, there can be no doubt that that lofty mind of Your Highness will also, as occasion offers, take much delight in the celestial and more sublime subjects which Astronomy, the queen of disciplines, embraces and shares with many other matters, and especially in the proper investigation of the nature of new phenomena, and of comets above all. For this study, which calls the noble mind away from earthly dregs and sets it, as in its own inspired place, into admiration — indeed into knowledge — of the well-ordered arrangement and motion of the heavenly bodies, has always been sought after by kings and princes and by men in public office, as the examples among

the most serene and most praiseworthy Kings of the Danes remain to the present day.

Go on, then, in virtue, most exalted Prince, and emulate your great-grandfather and your father's grandfather hereafter, not only in piety and clemency, but also in the skill and delight of this sublime science; and emulate your father besides in his effective desire to advance this commonwealth. Their virtues and good deeds toward these kingdoms no memory will ever blot out. For (as our own Saxo [Saxo Grammaticus], no less a poet than the most elegant historian of his time, sang):

Glory follows the dead, and over the ashes of each a surviving fame shall stand; nor does it sink into any age, the virtue that a man has wrought in his own time to perfection.

But lest this glory should prove vain, those Heroes so devoted themselves to piety toward God and clemency toward their subjects that they had Him above all favorable, and their subjects bound to them — Him, I say, the δημιουργόν [Greek: dēmiourgon, "maker, craftsman" — the Creator] of all things, on whom depend all the safeguards and saving counsels for protecting and enlarging the commonwealth. For it is true, as Plato somewhere says: Where not God but some mortal man reigns, there is no escape from calamities and evils. This I think must be religiously observed hereafter, lest — if either other evils, or those which these latest comets threaten the world with (not in vain), should reach us too by the divine will and (alas) by our deserts — we should once more have God on our side, who alone knows how, in his inmost heart, to bend the fates and to pour down from on high a blessing in place of a portentous omen, and who most graciously wills to do so.

To Him we devoutly commend Your Most Serene Highness, together with your most illustrious brothers, that the piety and the rest of the ancestral virtues — in whose footsteps you tread so beautifully, by divine grace — may extend themselves to become the surest supports of these Kingdoms, may continue, and may be happily propagated to distant posterity, if any there is to be. At Copenhagen [*Hafniæ*], on the eve of All Saints', in the year of Christ our Redeemer's redemption 1621 [*clw lwc xxi*].

Your Most Serene Highness's most submissive

Christian Severinsen Longomontanus [*Christianus Severini F. Longomontanus*].

PROOEMIUM [Preface]

Since all the luminous bodies displayed to our eyes in the theater of the heavens are subjects of astronomical consideration — whether, like the fixed stars, they remain motionless, or whether they free themselves from the fixed stars by motions of any kind whatever — it is fitting that something concerning the adventitious phenomena of the sky should likewise be made subject to our astronomy, now that it has been completed in its main part. For although these phenomena have not been bound by the eternal laws of motion as the other bodies coeval with the world are — being, as it were, less durable — nevertheless the investigation of them must by no means be neglected, insofar as the truth can be brought near, both by external aids (namely by duly conducted observations) and by skillful reasoning here and there. Indeed, the purpose and the very fruit, so to speak, of this contemplation — a contemplation so illustrious and most worthy of a person of natural gifts — urges this. And by it, at the same time, an immense longing of souls, and an almost extreme [yearning] of those committed to security, is rightly drawn, in the fear and reverence of the Deity, through the appearance of new portents in the sky. For just as the stars are seen to be present for the general governance of the world, according to God who created them in the beginning — not only when they mark out the seasons, but also when, in those things which happen here, they operate certain effects according to the course of nature, and signify other things to come — so these new phenomena, whenever they blaze forth in the sky, doubtless portend something beyond the accustomed law of nature; and by the singular providence and dispensation of the Deity in this worldly economy, they intimate it before it has come to pass, and they exhort the pious and the impious alike to repentance. For signs of this kind — and above all a comet — are usually ill-omened, a matter to which the history of all ages bears witness, so much so that from it an apotelesma [astrological prognostic] has been constructed:

[Greek: οὐδεις κομήτης ὅστις οὐ κακὰ Φέρει [oudeis komētēs hostis ou kaka pherei, "no comet that does not bring evils"]] Never has a comet been seen to shine without bringing harm.

The example of Hipparchus of Rhodes [Hipparchus], the ancient and distinguished astronomer — but most of all the example of our Danish Atlas, Tycho [Tycho Brahe] — will likewise move posterity not to wrap such new celestial phenomena in silence, but rather, on their occasion, to cultivate the art of astronomy more intensely. For both of them observed two new stars, blazing in the sky in their own centuries, and observed the remaining fixed stars almost

universally, and arranged them into a canonical disposition with respect to the ecliptic. Moreover, our Tycho devoted no less care — if not greater — in his earliest years to observing comets besides, and to assigning them rightly among the celestial bodies, than he did to the planets and the other stars coeval with the heavens. Accordingly, since new stars [Greek: τερατώδεις [teratōdeis, "portentous"]] of this kind do not cease to be seen in the sky in our own times as well, but appear more frequently as the world — otherwise effete — nears its end, which without doubt is shortly at hand (by which, namely, divine providence wishes pious minds to be forewarned of the most disturbed state of the world to come, which is everywhere indicated in Scripture) [Margin: Ezech. 38, item 39; Apoc. 20] — the care of these things has rightly roused me too, who profess the mathematical arts, and especially the astronomical, in this Royal Academy; and it has impelled me, as before to observe, so now to write. And although this comes about later than I had intended, for various reasons — after the last of the comets, which was terrible enough, had been taken up by many even distinguished men almost immediately upon its first beginning, described, and reported by most with a dire and ominous reputation, just as the present state of affairs seemed to demand — nevertheless, because I have neither found observations so settled and precise of that same comet made by others, nor, even amid so great a variety of judgments, has there been such disputation by them concerning its causes or effects, I have confidence that my labor (although a "postliminium" [a belated return]) will find a place among the discerning. And so, that I might more rightly serve the usefulness of the mathematical youth, to whom I had destined my chief effort, it seemed good to me: first to weave together a general treatise on new phenomena of this kind, through their causes and effects — the opinions of the ancients and the moderns having first been collected, recalled to the anvil of reason, and adjudicated; then, from my own observations, to work out the appearances, motions, and positions of the two [comets] which presented themselves in cometic form, with contrary motion, as if struggling against one another — namely those that blazed from the heavens in the year from the birth of Christ 1607 and 1608 — and to append whatever significations of theirs there might be. And this both on account of these perilous times, of which those comets were without doubt the signs — of which times (a thing to be feared) [the worst] had not yet fully arrived — and also lest the youth should [hereafter] abandon the examples concerning the observation of comets, the unraveling of parallaxes, and finally the prediction of effects of whatever kind, rightly and astrologically, that is, in the most rational manner —

— may they [hereafter] abandon [these]. And what, for the small things which I acknowledge, I attempt by the powers of my own ability, and helped here by the divine Deity — so that the material I have undertaken may be directed and decided toward those principles which I too judge to be most likely true in nature — I see that pardon must be obtained for me from those very people who otherwise cherish opinions on new celestial phenomena that are far at variance even among themselves; opinions which nevertheless must most of all be tested by true and competent astronomical observations, those which fall within the senses, and which afterward, by the benefit of geometry and arithmetic, are subjected to a higher reasoning. For these are the only steps — and indeed the most direct, above all the other inventions of many ancients and moderns alike — by which to enter beneath the threshold of so lofty a philosophy.

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Cap. XI. On the effects or significations of these two comets, to be adjudicated separately and briefly according to the common decrees of the astrologers.

Cap. XII. On the common character of the significations of the two aforesaid comets, from the same efficient causes — with [a note on] their mutual contrariety; likewise on the nature of the signs or asterisms through which they passed; and further, on two others, which arose at nearly the same time; from all of which the detestable security [heedlessness] of human beings in these last times is above all things argued.

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CH. S. LONGOMONTANI — DE NOVIS COELI PHAENOMENIS, APPENDIX.

[Ch. S. Longomontanus, On the New Phenomena of the Sky, an Appendix.]

CAP. I.

On the species of new phenomena.

The method of weaving together the treatise I have undertaken departs somewhat from the natural one, which is commonly accustomed to begin from the definition of the thing. For what we chiefly seek here — namely, of what nature at last such new celestial phenomena have been — will first be transmitted through external criteria, and will be accurately adjudicated by them through various circumstances, especially by the sense of the eye and by history; and it can be moderately taught or expounded by reasoning, which will follow afterward, on the nature of the thing to be discerned through its causes. For although a new phenomenon can generally be said to be that which, in the manner of a fixed or cometic star, arises anew above [us], whether in the air or rather in the celestial theater — being, namely, by no means seen beforehand — nevertheless, however imperfect this description may be, the opinions of the ancients and moderns, which will be recited in the following chapter, will plainly show it.

Therefore, beginning from the ocular criterion, the species of those phenomena which thus, beyond what is customary for the sky, are sometimes seen to appear in it, we will here review first of all — where even Plinius [Pliny] alone would suffice for us, provided he had not extended too far the cometic material into the distinction of such species, in imitation of the Greeks, from whom the doctrine of comets was transferred to the Latins (as others hold) and first took its origin. For he enumerates these twelve species of comets (*Natural History* II.25), which it is better to transcribe in Greek characters, so that their etymology may be more rightly perceived, and [to what things] they were conformable.

- Κομήτης [komētēs]: a hairy or curled star.
- Πωγωνίας [pōgōnias]: bearded, from whose lower part a beard, as it were in the form of a long beard, is sent forth as a mane.
- Ακοντίας [akontias]: from the form of a javelin.
- Σιφίας [siphias — so set; the gloss "from the form of a sword" requires Ξιφίας, and the compositor's sigma is transcribed as printed]: from the form of a sword.
- Δισκεὺς [diskeus], or rather δισκοειδής [diskoeidēs, "disc-shaped"]: almost without rays, except for rare ones at the margin, of an amber [electric] color.
- Πιθίτης [pithitēs]: like a cask, in the hollow of a smoky light.
- Κερατοειδής [keratoeidēs]: has the form of a horn.
- λαμπάδες [lampades]: stars which present the appearance of burning torches.
- Ἴππεὺς [hippeus]: presenting equine manes, of very swift motion, and going round about themselves in an orbit.
- Αργυρόκομης [argyrokomēs]: shining with so silvery a hair that it can scarcely be looked upon, showing in itself the appearance of the Deity in human form.
- λογχίτης [lonchitēs]: the form of a spear-head [lance].

There also occur [comets] with the shaggy appearance of goats and surrounded by a certain mane; at the same time the appearance of the mane changed into the form of a spear-head [?] — was, in Olympiad 108. Thus far Pliny.

Moreover, although Pliny establishes these differences of comets, having imitated the Greeks (as I said), yet [I follow] the opinion of Seneca (*Natural Questions* VII.11), where he discourses of comets thus: "Whether there are some differences of those [comets], or whether there are not, they must come to be by the same

principle." To which I gladly subscribe, a comet [?]; and therefore I add that [observation] of Pliny concerning the Plinian differences — namely, that a great part of them, or [these] alone, are expressed by the disposition of our sight alone between the Sun and the comet — so that, as will be shown more rightly below, with the material suited to comets having been given. But concerning the Plinian forms of comets, such as it is, certainly those which were seen by our own eye, or were comate [hairy], like that one of the year 1585, or caudate [tailed], like all the rest — except that one which the distinguished Johannes Keppler [Johannes Kepler], in the year 1618, reports to have been seen in the sky on the day before [the others], among the three which were visible the same year, [and which] he detected in the north: whose tail, like that of the others diametrically averted from the Sun, had appeared in the manner of a wardrobe-broom, so that on this account it might seem able to be referred to the second species, [Greek: πωγωνία] [pōgōnīa, "bearded"]] named above. So that I dispute nothing concerning that form which the same Kepler mentions — first noticed by others, then by himself, near the rising of the last comet, and observed a few days before it itself — having the appearance of a Ceratias, or rather of a beam (concerning which Seneca, *Natural Questions* VII.5), and stretched out lengthwise from the Sun.

But we, treating of new phenomena in general, see that that difference of them can rightly be given which Epigenes attributes to comets alone, and indeed in another sense, who is read (*Seneca, On Natural Matters*, ch. 6, book 7) in these words: "There are," says Epigenes, "two genera of comets: some pour out a blaze on every side, and do not change place; others gather a wandering fire into one part, in the manner of a [coma] beard, and traverse the stars." Of which species, I say, although Epigenes contends that the former is born from earthly matter, and afterward raised on high by the force of spirit — which opinion of his, mentioned by Seneca in several places, he refutes below — nevertheless, while he admits that the same [genus] is, with respect to the stars (indeed, of the fixed stars, without doubt) — not [moved] by the first motion — unmoved; but the other truly traverses the stars: he does not obscurely intimate what otherwise popular observation can attain both in new stars and in comets, and bear witness to; which, without distinct observations and parallaxes — drawn this way and that, the knowledge of which is lacking — sustains nothing certain among the phenomena appearing above by night, [nor] to distinguish the sublimations from the face of the earth. Therefore, what Epigenes there calls unmoved, but low, we too declare to be unmoved among the stars, but the most sublime of all the things which are seen anew in the sky. And so, from this we established the first genus, called a new star. But from the other — that which appears mobile among the stars, and

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— stretches out a wandering fire in the manner of a beard — we make the genus of comets, namely [of those] wandering in the lower or planetary [region], and appearing sometimes in various forms; the causes of which opinion will be given in what follows. And so much for the species of new phenomena — these being, namely, the first of all to fall upon our sense, and therefore also assigned the first place in this treatise.

CAP. II.

On the various opinions of both the ancients and the moderns concerning the origin and definition of new phenomena, and especially of comets; how the causes of these are briefly handed down by them, and reduced under certain genera.

If the new stars of which we speak must be referred to causes in nature, and not rather — as some hold — counted plainly among miracles, then certainly no matter in all natural philosophy seems to me (whether it is found in the lower elements, or above in the sky itself, where the most excellent nature flourishes) to have given rise to more mutually discrepant opinions of philosophers than the generation and origin of comets, to be demonstrated through their causes — to say nothing of the new stars, whose appearances were either never detected by the ancients in those rude ages, or never propagated in writing to posterity. For that one which Pliny (*Natural History* II.26) reports to have been observed by Hipparchus of Rhodes — since it was not without a peculiar motion (provided that credit be given to Pliny elsewhere) — must rather be counted among comets, appearing in the opposition of the Sun and the earth. But, that I may return to comets, it will pertain to my purpose to first propose briefly in this chapter the chief opinions concerning their nature and causes, both of the ancients and of the moderns; then to distinguish them into certain genera. As for the opinions of the ancients on these matters, Plutarch (*On the Opinions of the Philosophers* III.2) collected them with marvelous brevity, and proposes them in this manner:

"Some of the disciples of Pythagoras [hold] a comet to be one of those [stars] which are not perpetually seen, but are revealed at a fixed time, recurring in their orbit. Others [say it is] of our sight reflected back to the Sun, in the manner of a mirror-image. Anaxagoras [and] Democritus [say it is] the conjunction among themselves of two or more stars mutually shining together. Aristotle [says it is] an ignited concretion of dry exhalation. Strabo [says it is] the light of a star

surrounded by a dense cloud, just as happens with torches. Pontius Heraclides [says it is] a lofty cloud illuminated by a lofty light. Epigenes [says it is] an earthly spirit, which seeks the heights, becoming ignited. Boethus [says it is] a species of fiery air. Diogenes [says] comets are stars. Anaxagoras [says they are] sparkling [things] which are said to fall from the aether in the manner of sparks, and therefore are immediately extinguished. Metrodorus [says it is] a violent compression sparkling in a cloud. Xenophanes [says] all such things are clusters of ignited clouds, or motions [of them]." — Thus far Plutarch has reviewed most of the opinions of the ancients concerning the causes and generation of comets, which it is probable he took from others, but especially from Seneca; who (*Natural Questions* VII) disputes more fully about these same matters, and refutes these same [opinions] — yet in such a way that he renders even himself uncertain about the generation of comets, except that he rightly holds them to come to be in the sky.

When we turn ourselves from these ancients to the moderns, we will bring into the open the assessments — not indeed of all, but of the chief mathematicians and natural philosophers — concerning the causes of comets, and, emulating Plutarch, we will bind [them] to a brief unfolding. For one can scarcely easily comprehend the multitude of those opinions which have been produced in public by so many writers concerning the last comet that shone forth. Cardanus [Cardano] and our Tycho hold a comet to be celestial matter, generated anew, gathered into a globe, and suddenly illuminated as a whole [Greek: καθ' ὅλου [kath' holou, "as a whole"]] by the Sun. Julius Scaliger [Scaliger], on the contrary, contends that the same [comet] comes to be from an earthly exhalation elevated into the supreme aerial region by the force of the heavenly bodies, and is not kindled, but illuminated only by the rays of the Sun. To particulars of this opinion Rothmannus [Rothmann] assents, under this condition: that, once he had understood the heaven to be sufficiently pervious, he did not doubt that an earthly exhalation could ascend the aethereal region, and there first the comet be generated, and be illuminated only by the rays of the Sun (as Cardano, Tycho, and Scaliger [hold]).

Keppler [holds] that comets are generated in the vast aether, as fishes in the Ocean; and as these [swim] in the water, or birds in the air, so comets traverse the whole universe by a rectilinear course. From this he supposes it to come about that, falling perchance sometimes upon the earth, they are perforated by it as by something more solid. Nor can anything else, according to Kepler's hypothesis, come about, among other things, by that cometic rectilinear passage. Ambrosius Rhodius [Ambrosius Rhodius] [holds] comets to be celestial bodies,

luminous indeed, but outside the order of nature. Willebrordus Snellius [Willebrord Snell] discourses that comets are [Greek: κατακα[?]ματα [katakaōmata — one accented vowel unresolved on the page] [?]], or exustulations [scorchings], emitted from the Sun, which therefore, like the Sun, will glow with anthracic [coal-like] ignition. D. Habrechtus [Habrecht], finally, defines a comet to be a new star, generated from celestial matter, and sometimes existing in the supreme air, but for the most part in the aether itself — as did L. Regiomontanus [Regiomontanus], P. Apianus [Peter Apian], Andreas Craigius the Scot [Andrew Craig, the Scot], and others, otherwise excellent men. Nor will I touch on those whom I [count] also as natural philosophers, who [hold] no...

held that no opinion about the generation of comets is to be admitted except Aristotle's [Aristotle]—lest perhaps they should seem to wish to bespatter and pollute the heavens with new impressions—but rather that the hypothesis of the one philosopher of Stagira [Aristotle] concerning these matters, as in other things, is to be followed.

Now this collection of views, ancient and modern, about the causes of comets, however varied it has been, nonetheless falls under three heads, considered according to the diversity of *matter*, of *appearance*, and of *place*. First, with respect to matter, a comet is produced as arising either from celestial or from earthly material. As to the form of its appearance—which will not differ from the true one—a comet either burns ablaze like a fire, or at least appears illuminated by the stars, and above all by the Sun. Finally, with respect to place, comets will be situated either in the heavens above the Moon, or below it, that is, in the elementary or sublunary region; or sometimes in the aether, and sometimes also in the sublunary region—for the judgments of Kepler [Johannes Kepler] and Habrecht [Habrecht] extend to this point as well.

CAP. III.

On the assessment of the opinions of the ancients and the moderns about comets, and what in them we judge should be rejected and what retained.

To assess the foregoing opinions of the ancients and the more recent writers about those new phenomena, and especially about comets, two, as it were, κριτήρια [Greek: kritēria, "criteria, means of judgment"] serve us in a certain way. The first consists in the observations made beforehand of such phenomena—not, however, with any instruments whatever, but with suitable ones, and acquired no less prudently with respect to the celestial revolution. From these the astronomer's judgment is made firm concerning the distance of such phenomena

from the face of the earth, likewise their magnitude, and finally a certain natural advance and duration—also from the appearance of the comet's tail, which perpetually points to the part opposite the Sun.

The second means is joined with a still greater skill of reasoning: namely, when we inquire about the matter from which those new phenomena chiefly arise; and likewise about their inner form, which is indeed luminous—so that you might call it nothing but an offspring of light—yet you anxiously ask in the end whether it was inborn and as it were embedded like coal, as fire is in charcoal, or whether it came from elsewhere, as the Sun's light comes to illuminate the Moon.

As to the first, which is formed from infallible observations: certainly all phenomena of this kind can and ought to be judged celestial. From this, by the industry of the one D. Tycho [Tycho Brahe]—in his books, the *Progymnasmata*—the whole Peripatetic doctrine of the sublunary generation of comets easily collapses and is overthrown from its foundation; as also are the opinions of however many others that are in any part akin to the Aristotelian view—namely with respect to the sublunary place—since no one has ever yet come forward who could prove, from his own observations of equal certainty with the Tychonic ones (or with the exquisite observations of others, or with those I have since made myself), that there were such great parallaxes in new stars or in comets that these anywhere exceeded the lunar parallax.

Indeed, against the unenviable matter of parallaxes I shall not touch upon the trifles of Keckermann [Keckermann], nor those of Julius Scaliger [Julius Caesar Scaliger]—least of all the obstinacy of John Craig the Scot [John Craig], and his tangled excuses, or rather rejections, of the parallaxes in defense of Aristotle. These I shall not refute at greater length here than was once done. For since sublunary meteors, as well as the Moon itself and everything observed on high—things to which the diameter of the earth's globe holds a sufficiently perceptible comparison—present and submit themselves, in their own order, to the laws of rightly observed parallaxes and to the analogy of these, and are found subject to them (examples of which matter I had left behind throughout our *Astronomia*, especially in Book 2 of the *Sphere*, chapters 10 and 11): therefore John Kepler [Johannes Kepler], in his cometary hypothesis, seems to me not to have attended sufficiently to this—because by that same hypothesis he grants that comets sometimes become sublunary, indeed even strike against the earthly globe itself, though he is destitute of any prior observations of their descent among sublunary things.

Accordingly, we truly maintain (as we said) that all those new phenomena are celestial, which either do not move from their place like the fixed stars and yet last for several months and whole years; or which take on the nature of comets and continue their motion for a proper duration beneath the greatest circle of the heavens, and finally deflect their tails to the parts opposite the Sun so exactly, in every revolution. For from such signs too the analogy of these things with the celestial bodies is excellently established—as will afterward be more fully shown in the examples, together with those matters that bear on the explication and certainty of their parallaxes.

Now once it is obtained—from the doctrine of parallax, likewise from the constant advance of the comet beneath the greatest circle, and finally from the extension of the tail toward the parts opposite the Sun—that phenomena of this new kind occupy places and motions within the very system of the heavens, the consequence is that we must either pronounce them perpetual stars (which nonetheless appear only at certain times), with the Pythagoreans; or acknowledge that their generation is new, arising namely from matter and form. But from where these principles of generation are taken, let us investigate.

But first we shall answer the Pythagoreans, with whom others in our age also side, and especially the Nolan—

—Brunus the Italian [Giordano Bruno]. Against these, we deny with Seneca [Seneca] that new stars, or even comets, belong to the number of perpetually existing things—because (for the most part) they display their greatest increase and bulk at the very onset of their appearance. This could not happen if they migrated step by step toward the earth out of the inmost recess of the heavens, until, on account of their evident magnitude—by which nearer bodies are enlarged with respect to the angle of vision—they came into our sight. Nor does this occur on account of a fixed disposition with respect to the Sun (as the Nolan [Giordano Bruno] held)—as though stars of this kind, otherwise dark in themselves, should receive that illumination from the Sun and so become visible to us. For a disposition of this sort with respect to the Sun is by no means uniform in them, as it is in the Moon.

I add that the time of their duration could by no means be thus continued—both because of the Sun's motion in the meantime, and because of the comet's own motion—since that disparity of such motions would dissolve it sooner. And what, I ask, is the geometrical reasoning by which a comet, now risen high in the north, now in the south, now before, now after the Sun, would configure such an angle of appearance with the Sun and the eyes of beholders as falls under neither a

fixed ratio nor a fixed measure? To say nothing of the fact that obscure heavenly bodies of this kind—from which they suppose new stars and comets to be formed—while they wander among the other luminous bodies of the heavens, would conceal these no less than the Moon sometimes conceals the Sun; which, however, has not yet been noticed by astronomers. Let the Pythagorean opinion therefore be gone, and moreover that of all those who would have comets be perpetual stars, or be produced by the mutual illumination of the fixed stars or planets, as Anaxagoras [Anaxagoras] held. For this illumination would likewise occur irregularly through the motions of the producing planets, or would be dissolved too immaturely. [Margin: See *Progymn.* 1; *Tych.* p. 652.]

It remains, therefore, that we believe those new phenomena are generated anew. Therefore from some matter and form, which generation presupposes as its principles. But concerning matter first: shall we establish it as earthly, with Epigenes [Epigenes] and Rothmann [Rothmann]—matter which, restrained by no obstacle or barrier, climbs the heavens, and there, gradually collected, compacted into a globe, and at last illuminated by the Sun's rays, is discerned? Or rather as comets being κατακάύματα [Greek: katakaumata, "things burned up," i.e., scorchings], that is, burnings, leaping out from the Sun, or rather from its peristatic matter, as Willebrord Snell [Willebrord Snell] holds? Or as formed from other planets, chiefly Mars and Mercury, as others—and especially Ptolemy [Ptolemy]—judged; yet under this condition, that those planets hold the place not so much of matter as of the efficient cause, since that matter was, with Aristotle, believed to be earthly?

Against these opinions our common reasoning and discussion is this: although it has been quite credible that there are certain peristases [Latin: peristases, "surrounding envelopes"] about the heavenly bodies, which there perform the office of the air spread around our earth—as in the Moon, the Sun, Jupiter and Saturn, we have lately learned by more intimate observations through Galileo's [Galileo Galilei] optical instrument—nonetheless I see that these are so appropriated and bound to the very bodies and globes about which they stand, that in no way do they pass over to other bodies, or mingle themselves with the otherwise vast and empty heaven. For whatever communication of the celestial bodies there may be among themselves and with the earth, that must be referred solely to the light of the stars and its expanded vehicle—namely, lest those world-bodies should perish, with their congenital parts thus excluded and heterogeneous ones introduced in their place, which is something that utterly recoils from the nature of homogeneous things; just as this will be more clearly

understood in the other part of the *Astronomia of Prognostics*, since it is there set out more fully.

Finally, if anything further is to be answered to Kepler [Johannes Kepler], it must certainly be said that even rectilinear trajectories of comets of this kind, such as he constructs, manifestly contradict the Copernican [Copernicus] hypothesis, to which Kepler is otherwise singularly bound. [Margin: *Lib. 1, cap. 8 Revol.*] For since Copernicus acknowledges that circular motion alone is natural, from where then is that straight motion introduced by Kepler, which is attributed to comets? Certainly not from nature. Unless, therefore, Kepler shall have proved that comets are endowed with sense, and recognize the end of such a straight motion of theirs, he will scarcely find anything else to make that motion so regular—to say nothing of the planets, which the form and apparent motion of comets in some way imitates, being really moved by a circular motion. But he who has ascribed knowledge to the planets perhaps does not deny it to the comets. However that may be, we wonder at this matter: that while a comet is believed to be merely natural, and it is not yet established that any Spirit presides over its governance, it should forthwith be held similar to an animal—especially when it does not last long, but quickly either disappears in the heaven itself, or (by the Keplerian hypothesis) is dissipated and vanishes by striking against the earth. When anyone wishes rightly to weigh these arguments, taken together with the foregoing, let him say that Kepler excuses the generation of comets and the motion of their trajectories by his hypothesis more ingeniously—indeed more laboriously—than truly.

And these were the chief authors, distributed by us as if into classes, around whose opinions about the origin of new phenomena this διάσκεψις καὶ διάκρισις [Greek: diaskepsis kai diakrasis, "examination and discernment"] of ours had to be extended. For the rest, whom Plutarch [Plutarch] reviews, do not deserve refutation. Among these, however—as also from those, and likewise from Cardano [Cardano] and our Tycho [Tycho Brahe]—we gather this: that the matter of the new phenomena is certainly celestial, but from

—what part of the heaven it arises, they pronounced little or nothing definite. Therefore in the following chapters, as far as God shall have deigned to grant to my ability, I shall set forth my assessment about this cause as well as the others, so that what I think a new star is, and what a comet is, may afterward be established by the reasons to be drawn out here and there, and be ready at hand—yet in such a way that room is always left for anyone who brings forward better things.

CAP. IV.

On the efficient cause of the origin and generation of the new celestial phenomena.

Since the judgments of the most learned men (even besides those we have reviewed) about the origin and generation of new phenomena in the heavens are varied—for, although that they come to be within the enclosures of the heavens is today agreed among nearly all the more recent and intelligent astronomers, after Tycho [Tycho Brahe], yet most think this generation so prodigious, and so anomalous in comparison with the generation of the other elemental things, that their causes, even if they are contained only within nature, can scarcely, indeed scarcely at all, be thoroughly searched out by any mortal—therefore before the candid reader I see that I have need of excuse here, lest he suppose that I wish, by presumptuous and rash judgment, to discourse fully and without doubt about their causes, or even to assert something that transcends the candid confession of such men, and especially of our Tycho himself.

[Margin: *T. B. Lib. Progymn. P. 513.*] There, in his own words, he subscribes to the doubt of Thaddaeus Hagecius [Tadeáš Hájek], when he explains him, in these words: *At the end of the same chapter (the 4th of the διαλέξεως [Greek: dialexeōs, "dissertation, discourse"])* he [Hagecius] *touches on some things about the celestial generation of comets; and although he candidly confesses that he is ignorant of it, nevertheless he does not deny that on this account it can come to be, but piously ascribes it to the admirable works of God—how it comes about—and asserts that it is removed from our full knowledge, which I too do not deny. Yet what can probably be affirmed about the procreation of these things, beyond the common opinion of the philosophizers, I shall (with the favor of the Author of the heavens) signify, as my opinion, near the end of the whole work, namely at the close of the third volume; meanwhile I gladly hold with those, not unwillingly, who place comets among natures whose secrets are not yet explained, and who assert that they arise from hidden causes (as Regiomontanus [Regiomontanus] proves from Messala [Māshā'allāh]).* Thus Tycho. From this anyone too will recognize that we at least follow what is most probable about the generation of these things, especially when astronomical observations do not permit us to ascend higher into this knowledge than this: that first, through the smallness of the parallaxes, the celestial bodies are certainly detected; then that here they either have a fixed seat—since they appear motionless among the fixed stars, the new stars—or that those which have taken on the form of comets are borne with a motion below the fixed stars, namely in the planetary heaven, beneath the greatest circle of the sphere, with respect to the senses; and finally from the streaming-out by which

they are seen, and which they call the tail or coma, they display a constant extension from the Sun in every revolution.

From these signs, as it were the more manifest ones, we conjecture that there is something natural in them. Yet some of them—namely those concerning their first appearance, since these are usually for the most part the greatest and most powerful, both as to quantity and as to motion—can scarcely be accommodated to the elemental generation of the lower things. For we observe that sublunary things grow gradually and strive toward an ἀκμή [Greek: *akmē*, "peak," i.e., culmination] before their perishing, not so that they diminish gradually like the new celestial phenomena; but rather they last for the most part in the same magnitude, until the predestined time and the age of each works out their resolution and perishing in such a way that they are at last subjected to change, and take on a new form, to which—after Tycho [Tycho Brahe]—the continuation and succession of generation, the seeds of the species, are preserved in nature; as may be observed in plants, in their fruits, likewise in animals and in all things growing beneath the Moon, apart from the meteors, which—because their generation, or mixture, is imperfect, and their duration not so much fixed to a certain time—are also deservedly excluded from that celestial generation.

There is therefore one power in these sublunary and elemental things, another in those superior or celestial things, which in generating them suddenly expands them into the greatest mass, and at last gradually consumes them—not otherwise than as we observe a bubble suddenly formed on water by the wind: encrusted, that is, or expanded into the thinnest little films; then (the one that is not at once abolished) sometimes gradually subsiding into itself, since the water beneath withdraws its matter from it. Or again, such a generation of the superior things could in some way be likened to the sudden coming-forth of earthly mushrooms, which reach their perfection within the space of at most seven days. Yet these likenesses are so set down here that they contain much dissimilarity with the generation and condition of a new star or comet in themselves—so that I report nothing here about Pliny's [Pliny] ephemeral little birds, or even the homunculi of the same name, because of some dissimilarity.

In this doubtful assessment and judgment, therefore, while we are engaged about the generation of the new celestial phenomena—yet we admit that nothing comes to be without causes—accordingly, when we now discourse about these in order, we shall imitate the counsel and consensus of Aristotle [Aristotle] in *Metaphysics* Book 1, chapter 7 [recte: *Metaph.* I.7], where he says: *Those things that are hidden*

from sense we say have been demonstrated sufficiently by reason, if we have recalled that which is possible.

Now in the beginning, concerning the efficient cause of the new celestial phenomena, which in the order of causes occurs first, we shall certainly not err at all when we devolve it primarily upon God himself, the creator and Lord of all things:

—especially since this new and sudden generation of celestial things seems to exceed the laws of nature, inasmuch as it observes and measures no fixed course of nature, nor any period and restitution of time in it; so that here and everywhere that saying of Psalm 115, verse 3, has its place: *Our God is in the heavens; whatever he willed, he did.* Likewise Psalm 136, verse 6: *Whatever pleases Jehovah, he does, in the heavens and on earth, throughout the seas and all the abysses.* But because God, on account of the most wise ordering of nature from the beginning, acts thereafter for the most part through secondary causes, it seems to me that for the generation of new phenomena, as often as they occur, there can be assumed as the efficient cause—together with the divine will itself—the mutual aspect of certain stars among themselves and with the Sun; and for a comet especially (as Ptolemy [Ptolemy] held), the conjunction, opposition, and quadrature of Mars and Mercury, or some other sinister regard of theirs—above all on account of those effects which are wont to follow the inauspicious disposition of these toward one another and toward the others; likewise the powerful irradiations of the other planets also with the Sun, as will be said later.

But here certain questions or doubts arise, which are wont to leave goads in the minds of some men, and so must first be explained, according to our small ability, before we hasten on to other matters. First, then, it can be asked: since God the Best and Greatest has, doubtless for this reason especially, set such new celestial portents and signs before the eyes of mortals—so that they may come to their senses from their crimes and, in due time, return to saving repentance before they are dragged to the just punishments which such new celestial phenomena seem to announce—and yet these same portents are nonetheless affirmed to come about from secondary causes: what impediment, then, can there be to prevent God being bound to the secondary causes, according to the Stoics?

Likewise, if new stars and comets, which among other signs are presented to the eyes of mortals about repentance from sins, will at the same time draw any effecting causes from the heavenly bodies. Or, to extend this question more generally: if from the stars coeval with the world future things are affirmed to be foretold—whether those be general or special, such as the things the astrologers

prophecy from the natal configurations, which likewise also come about from the divine ordination—and these are such as to incline at least toward evils, if they do not (as others would have it) effectively operate the same: it can certainly be asked, how from that same divine ordination crimes and sins can be excluded, since these seem to be comprehended, as effected or signified, in those certain causes or signs?

And here I think it must be answered most rightly that God by no means ordained sins for that reason—sins which certainly arise from the will of the devil and of men, apart from and beyond all divine ordination. Likewise, that these signs in some way emanate from the secondary causes—that is, from the celestial heavenly bodies, which God created from the beginning together with the heaven—this, according to Scripture, indeed according to the philosophers themselves, and especially Plato [Plato], plainly conflicts with the divine nature, which is wholly good. Rather, here it must be established that, since God is without time and place—and indeed in such a way that he governs by the most exquisite harmony the things that come to be in time and place—so much so that in that divine economy all things, from the first time of the world's beginning to its end, that is, to the consummation of the whole globe, are perpetually placed in the sight of the Creator: namely both the crimes and sins of men, and all other things that will come to be at their certain times. It was therefore not impossible for God, in the primeval creation, so to ordain the heavenly bodies that perpetually afterward, among their other ministries, they should either be themselves about to be signs, or should give signs of themselves, of his benevolence toward men—of which, as also of his justice, the avenging of crimes is no small part, which avenging either follows such celestial portents or is joined to their appearances: as war, plague, dearth, the death of great men in the commonwealth, the transmutations of kingdoms, and other such means, which often correct the wicked and draw them back to repentance, but in no way trouble the good. For to these, namely those who love God, all things work together for good. [Margin: *Rom.* 8, v. 28.]

Secondly, since it was affirmed a little before that the aspects of the heavenly bodies—chiefly of Mars and Mercury—contribute to the generation of comets (although certainly the aspects of other planets too are not excluded, as will appear below), it can be asked: why then, when aspects of this kind occur every year, are comets not also produced from them every time, so that the origin of comets could be predicted at a certain time? This, however, has scarcely been done by anyone hitherto—even if certain men who wish to be heard as astrologers boast that they do it by an inept presumption.

But here too it will be answered not inconveniently, by that which I admonished at the beginning: namely, that the action of God is utterly free; and it must be added besides that by what hidden part of nature the Creator uses it, as it were as a secondary cause, for this purpose, is plainly unknown to us. Whence Hippocrates [Hippocrates] aptly says, of those things that come to be while we are ignorant of them: ἡμῖν μὲν αὐτόματα, τῷ δὲ Θεῷ μηδὲν αὐτόματον [Greek: hēmin men automata, tō de Theō mēden automaton, "to us indeed they are spontaneous, but to God nothing is spontaneous"]. Meanwhile it must also be said that the course of nature is not on that account entirely impeded in other respects, because various delays occur, arising from more powerful unknown causes; likewise defects of matter supervening, which either defer or vary the generation of new phenomena, and render the predictions from them—as also from the other stars—uncertain.

often render them outright void — quite apart from the practitioners' own weakness and laziness. Ptolemy [Ptolemy] sets all this out fully and at length at the beginning of his *Quadripartite* [*Tetrabiblos*], in his treatment of the effects of the stars. So much, then, for the efficient cause of new phenomena.

As for what Paracelsus [Paracelsus] says in one passage — that the heavenly stars (because of their [Greek: συμπάθειαν (sympatheian, "sympathy" — a shared affinity with things below)], and above all Venus) conceive, as it were, from the perverse thoughts of men and from their premeditation of evil deeds, and bring forth comets even against their will, even though by their nature the heavenly bodies are otherwise chaste and pure; and that these new offspring, the comets, are afterwards resolved into effects which carry over and bring out such conceptions through wars, thefts, robberies, adulteries, and the like — this surely seems to rest on the reasoning of some fictitious cause rather than to be grounded in nature or in Scripture. For it is agreed that the higher act upon the lower and not the reverse. The stars are so ordained by God that they can serve at once as either the forewarners or the intermediaries of those evils which proceed from free human action — or rather enslaved action, that is, action in bondage to the devil — exactly as was set out just above, and as will be explained more fully below by way of a similar cause.

CHAPTER V.

On the material cause of new celestial phenomena.

The transition from the efficient cause to the matter of new phenomena is now to be made — but a transition of a kind that nature scarcely acknowledges in the

case of its other generations. For once the generation of these phenomena has been rightly denied on Aristotelian principles and hypothesis, and their matter has accordingly been sought in the heavens, this has produced various opinions among the more recent astronomers. Nearly all of these were so thoroughly discussed in the preceding chapters that there is no need to bring them back under the anvil afresh, or to resist them with more arguments than were employed there. Yet although we also proved sufficiently from what came before that this matter of new phenomena is from nowhere other than the heavens, still — so that we do not conclude that its concurrence in generating those new phenomena is merely fortuitous — we shall here examine the matter a little more deeply. First, so far as we can, we shall consider whether the heaven was so disposed, or (as they say) so qualified, that it could produce of itself any new things of this kind, since the ancient philosophers, and the Peripatetics above all, flatly deny it. Once this point has been established, we shall inquire further into the discovery of such matter — namely, from what part of the heaven it is drawn: whether from the stars, or from that part which we perceive to be empty of stars and which we properly call the heaven; or, finally, from both.

For although we here acknowledge such a matter — one that is the receptacle of a form, or of a kind of seed operating after the manner of a form — we do not mean that notorious prime matter which, according to the Platonists, is virtually nowhere and nothing, but a matter really existing in the heaven. For that prime matter, which others call chaos and Scripture an unformed mass, was — as is our view, with some others — absorbed into the original creation of things. For this reason, even though the lower bodies do not differ in their whole genus from the higher, there is nevertheless a great variety of matter in heaven and on earth, from which diverse species are produced throughout the world (see Antonius Flaminius on Book 11 of Aristotle's *Physics*). But let us return to the first question.

The contraries that Aristotle [Aristotle] requires for generation are chiefly of two kinds, and they bear upon matter. For either they consist in contrary qualities — the qualities felt to be present in the elements as they are transmuted into one another — or in the destruction, or privation, of a form in matter while a like or a different form is introduced into it. As for the third contrariety, which arises from upward and downward motion according to the nature of the elements (namely, lightness and heaviness), it contributes little or nothing to generation; and I have shown in the *Prognosis* of the *Astronomia*, in the section on the matter of the heaven, that this belongs not so much to the sublunary elements as to the individual stars, regarded as great cosmic bodies. Those contraries, I say, which Aristotle excludes from the heaven, we assert to be present in it. For as regards

the prime qualities — and above all the active ones, namely cold and heat — we proved earlier in the *Prognosis* of the *Astronomia*, by no doubtful arguments, that they too are included in the heaven: namely, that that heavenly expanse is exceedingly cold, while the stars, and the Sun above all, excel in heat. Therefore this first principle, drawn from contrariety, stands ready for us in support of the new generation of celestial things.

As for securing the second principle, we shall not have to labor much, since trust can safely be placed in the things we make use of with our eyes, that is, the things we see. Such are those phenomena which have lately arisen in the heaven, namely new stars and comets. That these are really generated in the heaven I think no more necessary to prove against the Peripatetics by many arguments than what our most noble Tycho [Tycho Brahe] formerly brought forward against them in his *Astronomical Progymnasmata* [*Astronomiae Instauratae Progymnasmata*]; so that now once more we may touch on this point — which we pressed above and shall also demonstrate below in examples — concisely and, as it were, in a single word. And although neither new stars nor

Neither new stars nor comets are perpetuated in the heaven as individuals — as among us, by the alternating succession of plants or of animals in their generation — so that on this very ground the heavenly bodies differ from the generation of things below. Nevertheless it is necessary that some matter has put off one form, or in some way altered it, the form from which a new star or comet is displayed; as we shall more conveniently explain below by an example, since matter is nowhere found wholly bare of any form. It now remains to explain from what part of the heaven we are to suppose the matter of new phenomena comes.

Is it, then, from the stars themselves? For these new phenomena point that way — and above all the new stars — as though they were the offspring of the stars, or as though begotten by them. For thus the new star of the year 1572 in Cassiopeia [Cassiopeia], and likewise the other of the year 1604 in Sagittarius, equalled Venus in magnitude, and also matched the other fixed stars in visible brightness — and that in the same shape (namely round) in which the perpetual stars appear; which round shape also belongs to comets, apart from the traces of tails or beards. Comets too do not differ much from the other stars in brightness. But since this brightness, or innate light, seems to be either a form, or else adventitious — that is, emitted from the Sun and the stars — we therefore refer it rather to the form, of which we treat in the following chapter, though in such a way that we leave the matter here suited to receiving such a form.

Is such matter, then, to be found in the rest of the heaven, which Scripture calls the expanse [expansum], from which the new stars — and comets above all, like fish in the sea, according to the opinion put forward by D. Johann Kepler [Johannes Kepler] — are generated? But this I can neither plainly affirm, since it has been shown in the *Prognosis* that that Rhachia, or expanse, is [Greek: ὁμοιομερής (homoioimerēs, "of uniform parts" — homogeneous)], and of an equal and most subtle rarity, exquisitely extended everywhere throughout the whole finite universe. Now this expanse, because it is the vehicle of light, or the substrate, through which the lights or rays of the stars unfold themselves swiftly, in the twinkling of an eye, even to the ends of the world (whence perhaps some hold that the heaven consists in a flowing — which, however, seems to be nothing other than the nature of the expanse, or Rhachia, permeating all things) — this expanse, then, of itself scarcely furnishes matter for new phenomena, except insofar as this matter has been interwoven into it, or so determined in potency that it ensnares and contains within itself that new luminous phenomenon as a form, or as something formed.

The matter of new celestial phenomena must therefore be, as it were, of a third nature, since it results from the stars and the expanse. And this is so on account of the qualities of these two — which we have suggested above to be contrary — and from a consideration of the prime creation of the heaven and the stars. For since places and the things placed in them must agree in some part of their nature — just as the elements, namely earth, water, and air, agree with the things generated within each of them — accordingly the heavenly bodies, which were without doubt created and brought forth from the primeval light by the best and wisest Creator, and which are read to have been set and located in that expanse by the powerful hand, or word, of Jehovah [Jehovah], are plainly not contrary to nature: as is set out more clearly and at greater length in the later *Prognosis* of the *Astronomia*, where they are, as it were, displayed before the eye. For these things are not of a contrary nature on account of the contrary qualities we have asserted to be in them, since every corruptible nature includes that contrariety of qualities.

Since, then, such matter reflects in some way the nature both of the stars and of the expanse (as I just said), it must accordingly be presented to the eyes through some communication of the light of the primeval light that shines in the stars. Yet nothing is found in the heaven, apart from the stars, except the belt or circle which the Greeks call the [Greek: γαλαξίαν (galaxian, "galaxy" — the Milky Way)] and the Latins the Milky Way [via lactea]. For which reason all those seem to me to reason consistently who hold that this circle, appearing at night in the heaven,

supplies the matter for new phenomena — a circle which, in the likeness of a great circle (though not perfectly everywhere), seems to encircle the whole starry heaven, equally distant, by about one sign, from each pole, namely the poles of the ecliptic and of the equator. Its eastern edge today, crossing the ecliptic in two places, has crept a little beyond the beginning of Cancer [♋], though with unequal breadth. For toward the north and the beginning of the constellation Cassiopeia — where we make its beginning in this description — it is thicker than in the places where it crosses the ecliptic and the equator. It spreads most broadly toward the Swan, or the constellation Cygnus [Olor/Cygnus], where it is also split into two parts, or paths: one of which descends through the bow of Sagittarius, while the other extends exactly through the neck and beak of the Swan, and likewise grazes the right shoulder of the Serpent-bearer [Serpentarius/Ophiuchus], and there gathers itself, as it were, into a fold near the equator. But the former path persists on its course and does not approach much nearer to the Antarctic pole before it rises again toward the north — namely through the ship Argo and the back of Canis Major, taking up its path there, and grazing the feet of Gemini [♊], and also Eridanus and Perseus, it returns again into Cassiopeia, whence its beginning had been taken by us — as though in the heaven itself, and in this rival globe of it (the celestial, material globe), one may recognize it.

From this description, such as it is, of the Galaxy or Milky Way, and from a comparison with the ancients, we understand that its motion in the order of the signs is the same as the apparent motion of the fixed stars. For even by Aristotle's designation

[Margin: Genesis 1.] [Margin: v. 14.]

(which is found in *Meteorologica* I, chapter 8) it was, by about one sign, earlier in that age, and was bounded on both sides and all around by the same stars as now: which fact, among others, furnishes a great argument for establishing the immobility of the fixed stars.

After this description, such as it is, of the Milky Way, we must inquire further: first, whether (against Aristotle) it is located wholly in the heaven; then, once this has been established by sufficient argument, whether it is a bare phantasm in the heaven, formed from the illumination of the many stars found in it; or whether it is something real and material, from which it can plausibly be held that new phenomena — and new stars above all — are generated.

First, as concerns Aristotle's opinion in the passage cited just above, it falls easily — even by the testimony of the Philosopher himself, set out in the same place, when this is weighed against the doctrine of parallax. For if in the regions bordering on the Milky Way the same fixed stars are seen at every revolution (as they certainly are seen), and if those stars never abandon the western edge of that path, or circle, at their rising, nor its eastern edge at their setting, but are perceived to keep plainly the same place within it — then it must certainly be that whatever reflects the Galaxy described above subsists either in the heaven, and indeed far above the Moon. For this too leaves parallaxes everywhere outside the zenith of the place, the greatest being for the regions nearest the horizon, as is clear in the doctrine of the sphere, and still more evidently in the theory of solar eclipses.

Now, in order to fit to a demonstration what we have said, from the foregoing reasoning, about asserting the Milky Way to be in the heaven, and to set it before the eye, we shall proceed.

Diagram. A geometric construction in the form of a semicircle, used to demonstrate that the Milky Way lies in the heaven (above the Moon) rather than below it, by a parallax argument. From a center, a semicircle of the sphere of the fixed stars is described, together with a vertical D C R; that of the earth is described as P F O. Then from the same center the circular boundaries of the Milky Way are drawn, as though below the Moon, between the semicircles I B K and H G. Its boundary I B lies about the setting (west), to which B K about the rising (east) is opposite. On the outer edge of the galaxy a vertical point B is given, which a fixed star, likewise vertical at C, exactly determines. The construction shows that a line drawn from the earth's surface (F), when extended into the sphere of the fixed stars, falls not at E but at N — the arc N E (equivalently L M) exhibiting the parallactic difference between the point the star would occupy in the Milky Way (L) and the point M into which the star never actually passes.

- C (apex of the vertical, top of figure): vertex / zenith point on the sphere of the fixed stars
- Q (upper left, outer arc): point on the sphere of the fixed stars
- B (upper middle, on inner galaxy edge): vertical point on the outer margin of the galaxy
- M (right of center, inner): point into which the star never passes
- E (right end of an upper chord): point on the sphere of the fixed stars
- F (center area): point on the earth's surface (observer)
- L (below B, inner): point the star would occupy in the Milky Way

- N (right, below E): point where the line from F falls in the sphere of the fixed stars
- R (left end of baseline): terminus of the diameter (west)
- K (baseline): point on inner galaxy boundary B K
- G (baseline): point on galaxy boundary H G
- O (baseline): point on the earth's semicircle P F O
- A (baseline center): center of the construction
- P (baseline): point on the earth's semicircle P F O
- H (baseline): point on galaxy boundary H G
- I (baseline, right): point on galaxy boundary I B
- D (right end of baseline): terminus of the vertical/diameter D C R (east)

A semicircle of the sphere of the fixed stars is described from the center, together with the vertical D C R; the earth's, however, as P F O. Finally, from the same center, the circular boundaries of the Milky Way are drawn, as though below the Moon, within the semicircles I B K and H G; and its boundary is I B, about the setting, to which B K about the rising is the contrary. On the same outer edge of the galaxy is given the vertical point B, which the fixed star, likewise vertical at C, exactly determines. Then, with that same point B revolved to L toward the west above the world's axis A Q, the fixed star is carried from C to E by an equal motion of revolution together with B L — which the line A E, passing from the center of the earth through L, attests. Therefore from F, the earth's surface, when the line is drawn and likewise extended into the sphere of the fixed stars, it certainly falls there not at E but at N.

But if you should send out another from F to E, this by no means passes through the point L (where, however, experience proves it would fall, since a single point — at B and L — would be occupied by the same star at C and E, with the observations preserved), but passes elsewhere, through M; and thus, through the arc L M, or N E, the difference is shown between the point that the star would possess at L in the Milky Way (if one had to abide by the observations) and the other point at M, into which the star nevertheless never passes, nor does it change place (as was said above). Had Aristotle considered this, he would never have pronounced the Galaxy to be a collected vapor, or its sublunary concretion — that vapor by which, in the firmament, he supposed the stars (and especially those tiny ones which are seen in the Milky Way, and are themselves called [Greek: σποράδες (sporades, "scattered ones" — scattered stars)]) to be gathered together and held suspended beneath itself at every revolution. Let Aristotle be

read, *Meteorologica* I, chapter 8, where at the same time the opinions of others about the Milky Way are reviewed and refuted by Aristotle in his own fashion.

The second opinion has, among the ancients, Democritus [Democritus] as its author. For he, as Plutarch [Plutarch] reports, supposed the Milky circle to be nothing other than the shining-together of many tiny contiguous stars (he means, without doubt, those which appear in it) — a shining which they cast forth among themselves because of their being drawn close together.

[Margin: Let Plutarch be read, on comets.]

This opinion of Democritus is at the present time vigorously confirmed, it seems, by Galileo Galilei [Galileo Galilei], the mathematician of Padua, who, with the spyglass recently invented and applied by him, discovered in the traces of the Milky Way nothing but a heap, as it were, of countless stars set close beside one another. Our most illustrious Doctor Caspar Bartholin [Caspar Bartholin] also affirms this. For when, some years ago at Padua, he was present with and a friend of the aforesaid Galileo, he was admonished by the same spectacle to change his earlier opinion (inserted in his treatise on the stars), where he had previously argued that the Milky Way was a celestial substance, and as it were the relics of the primordial light, after the view of Tycho Brahe. Moreover, for the greater credibility of this matter, he takes an argument from other similar little patches of a white color, like whitish

[Margin: Let D. Caspar's *On the Heaven*, chapter 3, be read.]

clouds, which elsewhere shine through the aether; where, by the benefit of the said spyglass, a heap of crowded stars is seen — namely, where the so-called nebulous stars are set close together.

But indeed, although the things that proceed from ocular demonstration seem on that account to deserve trust in physics, the reasons that follow bring it about that I cannot fully acquiesce, nor firmly believe the Milky Way to be solely the shining-together of the tiny stars seen in it, without any other underlying matter.

1. Because the nature of the celestial expanse (which expanse we have shown, in the *Prognosis* of the *Astronomia*, to be the vehicle of the light of the stars) is such that the same light passes freely through the expanse and is transmitted, and is nowhere bounded except at material objects, such as the stars and the earth. But that light, namely of the tiny stars seen in the Milky Way, is bounded by the same: therefore the Milky Way is not entirely

and [Greek: καθ' ὅλου (kath' holou, "in general" — wholly)] of the nature of the rest of the expanse, but contains besides some matter in itself.

2. A reason is given at once from ocular sense and from the authority of D. Tycho Brahe, our most excellent astronomer of all ages, who noted a gap in the Milky Way and in the constellation of Cassiopeia, in the same celestial place where the new star of the year 1572 had lodged — as he testified in plain words in the conclusion of the first volume of the *Progymnasmata*, and as our own ocular experience formerly confirmed independently. Whether this gap is found today I cannot for my part discern by eye quite so exactly. From this, then, one may infer as follows.

If the Milky Way were only the light of the stars set within it, certainly nothing could thence be taken away or perish, so long as those stars remain in their several places, as they do. For while the causes endure in nature, the effects endure — that is, while those stars remain, the light remains. But that great and most brilliant new star of the year 1572, after its disappearance in the following year, left in that place a dark gap — one not unlike the color of the rest of the heaven outside the Milky Way — where it had been found to shine at the margin of the Galaxy; that gap not being at once filled or covered over by the flowing of the diffused light (as the nature of light otherwise is). Therefore it is consistent that there underlay, besides that light of the stars, a certain matter, of which that new star consisted — which star, once dissolved and dissipated, the matter was at the same time absorbed and transported elsewhere, away from the place where it had existed.

3. A reason is taken for us from the usual place of the generation of new phenomena, and of new stars above all; which place is, for the most part, in the Milky Way. For in it arose both the said star of the year 1572 in Cassiopeia, and the one of the year 1600 in Cygnus, which is even now seen there. Finally, the new star of the year 1604 took its origin in the knee of the Serpent-bearer, and so on — not to mention the comets, which from time to time also referred their beginnings to the Milky Way, since they shone with a more brilliant color when they crossed it. From which it is very probable that the matter of the Milky Way is akin to these as well — as Tycho also judged in the conclusion of the *Astronomical Progymnasmata*. From the reason adduced, therefore, we conclude that, just as herbs or other things growing from the earth acquire their matter not from afar but in the places where they spring up, so the new stars draw their chief matter from the Milky Way, in which they are for the most part seen to

arise; and that this matter, as I said, is in some way different from that which otherwise yields to the rest of the heaven — the expanse, or Rhachia, as we have declared.

With these reasons laid down, it must be seen by what means the observations of Galileo can be saved when reduced to them. Among these, we seek first and foremost in the admirable power of Galileo's spyglass — for enlarging and dilating, in wondrous ways, the objects that strike the sight. This power seems to me so great that, while the observer scans the stars set in the Milky Way, even the tiniest, he apprehends them and exhibits them to the sight through that tube, and sets them in view stripped and laid bare of that rare underlying matter.

This conjecture of mine proceeds from the fact that Galileo had exhibited to us, through that same spyglass of his, in his *Sidereal Messenger* [*Sidereus Nuncius*], the spots on the Moon as interspersed and not appearing quite in the manner in which they otherwise appear — where, however, he attained even the smallest particle of the Moon, illuminated in its thirsty and dark part. The other point is that, although that milky color does not appear otherwise than in a manifold heap of stars here and there throughout the milky zone, by the benefit of Galileo's optic, nevertheless I do not lay it down as following from this that a color of this kind is nothing other than the shining-together of those stars among themselves, without any other underlying matter into which that confused light of the stars is determined; for this matter is necessarily required by the opticians, as Johann Kepler demonstrates — on which medium too our first reason rests; and this is also the common opinion of D. Isaac Habrecht [Isaac Habrecht], in his description of the last comet (or the one that shone in the year 1618),

) on page 14, rendered into Latin almost word for word as follows: "Although through the spyglass the Milky Way is perceived to be nothing other than a sheer crowding-together, or heap, of tiny stars set close beside one another — just as Albertus Magnus, Scaliger, Cardanus [Cardano], Goclenius [Goclenius], and others have supposed — I myself nevertheless judge that this path may here and there be composed of smaller and larger stars, even of stars that otherwise escape our ordinary sight; and that nonetheless what remains over is matter of the very kind that Brahe believed it to be."

These considerations vindicate for the Milky Way a certain matter of its own, beyond that diffused starlight within it. For as concerns the little patches (or nebulous whorls) visible at certain places throughout the aether, these introduce, by their own obscurity, a certain dissimilarity from the milky brightness — as near the Manger ☽ [Praesepe in Cancer], the Hair of Berenice [Coma Berenices],

the forefoot [thigh] of the Great Bear [Ursa Major], and so on. Nor are such little patches seen elsewhere wherever stars are set densely beside one another. Examples in Taurus are the Pleiades, and elsewhere, throughout other asterisms, other clusters of stars more tightly packed and gathered together. Indeed, if anyone in winter, when the Moon is hidden, should carefully contemplate a perfectly clear sky, he will see everywhere throughout the aether — even outside the Milky Way — a numerous and as it were crowded chorus and host of stars scattered abroad. It would therefore be a wonder if the whole aether did not, like the rest, shine with brilliance — unless it were that this peculiar matter had been prepared in it by God's foresight; or rather, that it had been left over from the primal light (as Tycho [Tycho Brahe] held), set apart for determining the light of the stars in the Milky Way, and at the same time for the successive generation, in due time (as God pleases), of new stars and comets.

Nor is this same milky matter to be reckoned of slight thickness, or as a mere whiteness with which a wall is overlaid — as D. Habrecht [Isaac Habrecht] rightly says — but rather endowed with a depth great enough, though not so great that it descends to the borders of the Moon, much less, as Aristotle [Aristotle] supposed, that it stands beneath her. For the vanishing of parallax, which I demonstrated above, proves this.

By these premises and arguments we now gather that this matter is above all the matter of the new celestial phenomena. We have a foretaste of this view, agreeing with Tycho's, in several places throughout his *Progymnasmata*. Indeed, had that man — by the judgment of all the greatest of astronomers — survived us longer, and entrusted to me, as he had intended, the task of the new phenomena, to be carried forward thereafter by my own diligence, I do not doubt that he would have handed over to me, together with that task, this very matter of theirs — concerning which he hinted, not obscurely, both elsewhere and in the epilogue of the *Progymnasmata*, that he would conclude his third volume on the new phenomena (which, however, did not appear after his death) directly in favor of this opinion of his, suggested there in a few places.

What, then, that great man among the astronomers once judged about the matter of the new phenomena, I too now judge, having adduced my reasons: namely, that it is celestial, and prepared chiefly in the Milky Way. And why should I not judge so? For if we have laid it down, following the teachings of Pythagoras [Pythagoras] and Plato [Plato], that this world is conformable to the condition of a living creature, we shall surely nowhere locate a more fitting and suitable matter within that vast body than the Milky Way is — inasmuch as it corresponds to the

spinal marrow running down the back of a man or of any other animal. For just as the marrow of the brain and spine supplies matter to the nerves spread everywhere throughout an animal's body — where those luminous spirits called the animal spirits, elaborated out of the vital spirits (which are first generated from the blood in the heart), obtain either their dwelling-place or their nourishment, and contribute not the least to generation — so perhaps it will not be absurd to hold that this milky matter has accordingly been prepared in the heaven, so that through its passages the structures of rays, or rather the invisible spirits of the fixed stars, may first be prepared, before they are communicated to the Sun, to the planets, and indeed to the Earth itself. And from this elaboration, if ever the hidden seeds are at hand, and the divine will — without which nothing comes to be — is at the same time present, either new stars are generated (and they do not sink themselves more deeply), or else comets, whose matter, torn away from the Milky Way by the attraction and force of the planets, is on that account rarer and perpetually subject to the motions that prevail throughout the regions of the planets across the universe. But enough about the matter of the new phenomena.

Cap. VI.

On the formal cause of the new celestial phenomena.

We treated the matter of the new phenomena at greater length in the preceding chapter, because it was set in the greatest controversy among philosophers and astronomers. And although we frankly admit that the opinion we have put forward concerning it — like that of others — has thus far been ἀναπόδεικτον [Greek: anapodeikton, "undemonstrated"], yet that same opinion, established by so many reasons and authorities, scarcely anyone will derive more fully from any other source than from the Milky Way. But now the order of inquiry requires that we advance further to the form and end of the new phenomena — to the form, I say, which chiefly gives a thing its being, and which separates it from all other things by a certain peculiar property of nature that it acquires. Diligently inquiring here into such a form, we find that in the new phenomena it is either general — namely, that which belongs equally to all of them — or special, that is, the form that displays first the differences of the new stars from the comets, and then those of the comets among themselves. Both must be treated, but chiefly the latter, and the more concisely in the present chapter, because in the preceding chapters not a few things were already said about them, as the need for understanding demanded.

First, as concerns the general form of the new phenomena — that is, the form so informing the matter we have spoken of that celestial bodies begin to exist out of it — although they do not last long, they nonetheless rival the other stars in the splendor of their light, in their round shape, and in their motion, to such a degree that these cannot be mere apparitions in the air, as is the opinion of certain people — an opinion held with no greater height, nor with any more reason, than the common crowd, indeed than the other animals that raise their eyes upward. Now this form, which is no ordinary one, ought not to be light derived from the Sun and the stars, but rather seminal (so to speak), or potent to generate, as happens in the rest of nature — the form, namely, which God infuses into the matter when it has seemed good to His providence, for the end which I touched on above to some extent under the efficient cause, but will set out more fully below under the final cause, since the period of a generation of this kind can be determined with certainty by no mortal. Meanwhile, however, it is credible that God — who in nature generally acts through means — even when those things are produced through which He wishes men to be admonished of His mercy, His wrath, their own repentance, and so forth (as we have an example in the rainbow), here too makes use of means: namely, of the arrangement of particular stars in a sort of rational relation to one another, through which their influences operate efficaciously at the appointed time, first seizing the center of the matter, then suddenly its whole destined and as it were torn-off mass, and, among other things, so filling it with the vigor of light that those new stars appear at their rising not much unlike the stars coeval with the world — although afterward, successively, on account of their lesser exaltation or perfection of form, these in turn gradually fail, or by the wearing-away of the solar rays, as Kepler [Johannes Kepler] perhaps rightly thinks. For just as to terrestrial metals their excellence and durability are imparted by their greater or lesser exaltation — whence gold, the most precious among them, is so durable that it can scarcely be dissolved by any art, or be transformed into anything other than gold, whereas rust easily consumes iron — so it is likely to happen in celestial things, namely that the new phenomena are the more quickly consumed and vanish again, precisely because they do not stand in equal exaltation with those that were first created by God, the formal potency in the new ones being feebler.

Furthermore, since mention falls here of light as the primary instrument, as it were, of this form, I do not judge it absurd that that internal [light] too, in the new phenomena, should be an offspring of the primal light which Scripture, first among all created things, so greatly extols. For it is not credible that this whole [primal light] yielded to the creation of the Sun and the stars in the place of matter, so that no traces of it are anywhere any longer to be found — since the

divine, perpetually admirable and most powerful sustaining of created things suggests to us that none of those things which God had made will be abolished, or wholly perish, before the final consummation of the world. Indeed, as far as my own judgment is concerned, I am of this view or opinion: that although that primal light, after the creation of the Sun (which by divine ordinance succeeded into its ministry), is less apparent to us, yet through the Milky Way, and moreover through all the larger bodies of the world, its hidden traces are left behind; and that those bodies, no otherwise than as internal or innate heat warms our own bodies — to which, while adventitious light and heat from the Sun and stars join themselves, this animal which is called μικρόκοσμος [Greek: mikrokosmos, "microcosm"] preserves its vigor. For concerning that which the senses sufficiently apprehend, I do not think there need be much inquiry — namely, whether heat is present in light; but rather whether that heat be of the coal-like kind, that is, whether it is as vigorous in the stars and new phenomena as fire kindled in a coal. This question that point which I just adduced concerning the primal light surely resolves without doubt — so that they are neither mere solar rays which illuminate and besiege the stars, both the perpetual and the new (as some would have it), nor do they possess light and heat in themselves by this or that coal-like faculty: just as not even the Sun itself does, although this divine star is compacted of copious matter, inasmuch as it pours out a light so dense that it dulls and darkens the rest of the luminous stars whenever they are found together with it above the horizon. Finally, among the other causes for which the great bodies of the world

are round — which we proved in the latter parts of the *Prognosis* of Astronomy — that light or internal heat seems to be not the least. For such heat alone brings it about that the new phenomena too contain themselves within a spherical figure by a certain natural force. For since the internal heat — which besieges the matter or body of the star — recoils from the external cold (which we showed, in the *Prognosis*, to prevail chiefly in the expanse), on account of the contrariety of qualities, it happens that, by leaning upon the center, and by compressing and constraining on every side the mass it occupies equally, it so contains it that its surface is everywhere distant from that same center by an equal interval — which rightly suits the definition of a sphere. Moreover, this is further confirmed by the fact that this spherical figure is preserved throughout every diminution of the star, right up to its final disappearance — the internal heat, of course, failing in these gradually, as in other bodies that live. Hence the head of the comet appears more compact and also more luminous than the rest of its outflow, namely the hair or tail. But to return to the primal light: if that is the light out of which the luminous celestial bodies were, according to most, founded by God, it

will perhaps be far less absurd to hold that this same light is diffused throughout all the larger bodies of the world in place of internal fire (which Aristotle [Aristotle] otherwise falsely set up as a sublunary element), than to ascribe it, with the Zwinglians and certain others, to the empyrean heaven; or, with the Rosicrucian brethren [Brothers of the Rose Cross], to reckon that light which was created as the very Son of God Himself. And thus far concerning the general informing form of the new phenomena. The special form follows.

We call this form "special" because it makes a distinction between new stars and comets, and that both in respect of outward form and of motion; but among the comets themselves, only in respect of outward form.

First, that the new stars differ from the comets in that external form and in motion (or rather rest), must be ascribed to the place in which they are situated. For since the supreme starry heaven differs from the lower, the planetary one, chiefly in this — that there all the fixed stars are carried either by one and uniform motion from east to west, or (those being at rest) the little globe, on which we are thought to be at rest, is turned by a contrary motion (that is, from west toward east, conformable, namely, with the planets) — which I had always rather believed, for reasons you will find in the *Astronomia Danica*. For astronomers do not admit, as the natural philosophers do, that the fixed stars are moved separately through the aether, however fluid it be, for reasons which I had likewise left scattered throughout the *Astronomia*. Therefore, since the new stars are found in that supreme heaven — which Tycho [Tycho Brahe] showed most fluidly in the phenomena of the star of the year [15]72 — and moreover in the Milky Way (as most of the other new stars too argue by their positions), it accordingly suits the new stars also to be subject to none but the first revolution along with the fixed stars, and at the same time to be so endowed with an abundance of matter in that supreme heaven that they both display a greater brightness than a comet and lack any perceptible outflow of matter. Although another reason can also remain why we perceive such an exhalation from this matter less than in the comets, especially when assent is given to the Copernican hypothesis [hypotyposis]. For if the fixed stars are at so great an exaltation from the Sun and Earth that the solar rays transmitted thither do not differ from the rays of our own sight — the semidiameter of the eighth sphere [sphaera octava], namely, which the fixed stars occupy, excluding every perceptible comparison with the semidiameter of the Earth's orbit (which to Ptolemy [Ptolemy] and Tycho is the semidiameter of the Sun) — [then we should perceive no such outflow]. But the former reason, which attributes it to the density of the matter — that the solar rays do not pass through the new stars as perceptibly as through the comets — is

preferable to us. Although certainly from that other reason too, which has an attached demonstration, a comet of the higher planetary region would, in the disposition of its tail, vary somewhat with respect to the regard of the Sun and of our sight from those comets which occupy the lower region nearest the Moon — as will be shown below.

Further, concerning the comets, I think the following must be inquired into. First, whether there was an internal form by which they are immediately moved; or rather whether by some external force, within the celestial intervals, they produce those various motions? Then, about which center — Earth or Sun — they completed the portion of their circuit? For I hold, with other astronomers, that their motion is circular rather than — with Kepler [Johannes Kepler] — rectilinear, for the reasons given above. Third, what that outflow was which the comets are seen to hurl from themselves in place of a tail or hair? And why does that same outflow perpetually incline toward the parts opposite the Sun? Finally, whence arises the difference of tails and hair among comets? These four questions, which suit this place — besides those already dispatched — are to be inquired into further concerning comets, since they are weighed only at the bar of reason, insofar, that is, as the origin and generation of these things is founded in the nature of things. Therefore I think no one should be begrudged who shall have discussed these matters more probably, but rather that better reasons are to be received with all applause.

As to the first point, we here certainly judge the same to hold concerning the nature of the motions in comets as we have otherwise left discussed, in the *Prognosis* and elsewhere too, concerning the motion of the planets: namely, that comets are not moved from their place by an altogether internal force — since Aristotle [Aristotle], in

book 1 of *On the Soul*, chapter 3, denies against Plato [Plato] that even our soul has this force within it, so as to move itself. If, then, the soul does not move itself, how will a star, entirely inanimate, be moved without some external impulse? Even so, I do not deny that the stars, as well as the comets, are so constituted by nature, and endowed with such a form, that they are most apt for local motion through the aether. Therefore we acknowledge that the motion of the comets is from the prime mover. For I suppose that, by divine providence, it was brought about from the beginning that that planetary heaven — which stands beneath the supreme starry one — should be wholly in motion, distributed through its kinds (that is, the planets and comets) according to the directories which proceed from the divine ordinance, so that each of these moves beneath a great circle: the

planets indeed along the zodiacal track, the middle of which the course of the Sun holds, for the end which is best in nature — namely, that the Sun and the other planets, by the alternation of the fixed motion toward north and south along that same oblique track, may continually serve to vivify and cherish the things below; as will be said more fully in *Astrology*. But the comets accordingly obtain various courses and motions through the aether, so that they too may fulfill the fixed end for which, like divine messengers, they are sent out at various times — as we touched on under the efficient cause, and shall set out more fully below under the final cause. These, however, although on account of the track of the great circle which by their fixed motions they trace out almost to the very end, indicate constantly, two or three days after their first appearance, into what region of the world, and under which asterisms chiefly, they will afterward direct their fixed courses — yet I would suppose that this can be predicted by no mortal at their very rising, or before, because they have no definite circles (as the planets do) along which they complete their fixed motions.

Furthermore, since the Tychonic hypothesis sets up two centers in the orb most rightly, as it seems, about which the heavenly bodies turn — for according to this hypothesis the two luminaries, and the whole system of the fixed stars, regard the Earth, while the remaining five [planets] regard the Sun itself, as their respective midpoints — these reasons compel us to believe that the comets have the Earth rather than the Sun for their center.

The first: that the comets follow the track of a great circle in a fixed motion. This course could rightly be denied them if, for the sake of some constancy of hypothesis, they were to circle the Sun in such a way as to admit a difference of distances from the Earth equal to, or even greater than, what the planets manifestly display in their annual motions about the Sun. For the orbit of a planet (which cuts the ecliptic in two opposite places, as suits great circles) alters and changes the angle of intersection into inequality as often as the planet [changes] its distance from the Earth. For that angle is greater when the distance from the Earth is smaller, and conversely. This is the cause why even a planet's own path sometimes falls short of the track of a great circle — as is gathered in book 2, part 2, of our *Astronomy*, concerning the latitudes of the planets. But in a comet's motion this is not observed; rather, its course through the aether is such that, whatever great circle may cut it — whether the equator or the ecliptic — it perpetually retains an equality of angle with that same circle.

The second reason seems to be drawn from the end and effect of comets. For the new phenomena are immediately destined, like the Sun and Moon, to the Earth

and its inhabitants, beyond doubt; and they exercise their fixed effects about it before the influences of those bodies obtain in these lower regions their full and fixed disposition with respect to their decrees, which are at the same time infused from those planets. Although I would not deny that the effects of comets are greatly intensified and remitted according to the rational aspects of the planets with these, and among themselves — but chiefly of the Sun and Moon, with which the new stars and comets occupy the same center — as will be said below concerning the effects of the new heavenly bodies.

These things being well considered, as for the fact that Tycho [Tycho Brahe], and likewise Maestlin [Michael Mästlin], referred the comet of the year [15]77 to hypotheses of this kind — in which the Sun is the center, as among the planets — you see in them something more ingenious and affected, namely so that they might somehow show that comets too have fixed hypotheses, such as the planets have, than [something] tending toward the truth: just as Tycho seems somehow to retract this opinion in book 2 of the *Progymnasmata*, page 268 and following. And indeed I do not know how comets, like the planets, should be subjected to the obedience of the Sun as a fixed director, when they by no means follow the track of the zodiac as the planets do, but for the most part rise far from the ecliptic and are swept quite transversely across the heaven with respect to the zodiac. And for that reason they seem to rest on no other hypothesis than that in which the analogy of motion answers to the magnitude of those same comets, gradually diminishing while they last — just as we mentioned above the analogical decreases of both magnitude and motion in comets, to be illustrated below also from examples.

The third question discerns the nature of the hair or tail in comets. Now this outflow, whatever it is, since it perpetually inclines toward the parts opposite the Sun, Tycho [Tycho Brahe] not inelegantly once reasoned out, together with Petrus Apianus [Peter Apian] and Gemma Frisius [Gemma Frisius], to be [produced] by the projection of solar rays

be produced and formed rather than to be flame, of the sort that a comet sends out or shoots up from itself like a burning lamp. This indeed had been Aristotle's account of the nature of the comet's tail; see [Tycho Brahe's] *Progymnasmata* [*Astronomiae Instauratae Progymnasmata*], pp. 714 and 651.

Moreover, the requirement that arises from optical reasoning adds this point as well: namely, that without some material substance, the said rays of the Sun, transmitted through the body of the comet, could not appear in this way — just as was reasoned out above concerning the Milky Way [via lactea], and was

demonstrated soundly enough in optics by the distinguished [Johannes Kepler]. Nor is it an objection that elsewhere, in Book 2 of his *Progymnasmata*, [Tycho Brahe] argues about the orientation of the tail of the comet of '77 as though it were projected toward the opposite parts by Venus rather than by the Sun — a view which he also shows by demonstrations deduced from his own observations. For although Venus's remarkable brilliance, which is so splendid and luminous that it even casts a shadow from itself at night, may have contributed something to the nature of that comet's tail and to the bending of that light, nevertheless this was not so great as to make it less probable that the Sun was the primary producer and director of that comet's tail. And [Tycho Brahe] did not hesitate to affirm this in other comets seen and observed afterward, and to confirm the aforesaid opinion about the tail of comets, as you will find it scattered throughout the first *Progymnasmata* — the second one, namely, having been worked out and written in our own time at Hven [Huaena], so that in it one finds what the proverb says, φροντίδες δευτέραι σοφώτεραι [Greek: phrontides deuterai sophōterai, "second thoughts are wiser"].

Nor in what follows will we lack a way of showing what was needed: namely, that the tail of that same comet stretched somewhat upward with respect to us, away from that straight line which [Tycho Brahe] demonstrates in his hypothesis of comets, and this on account of its greater elevation above the Earth. To produce and form the hair, or tail, of a comet, then, two things combine: the solar rays (and perhaps also those of the other heavenly bodies, which mingle with the solar rays) and the material around the body of the comet, or out of which that body consists. Upon this material these solar rays act powerfully — indeed almost violently — and they lick and graze it at its surface. The result is that the lighter particles of that material, and especially the most yielding ones, are snatched up by these rays and, so far as is possible, are continuously projected into the space outside the rest of the comet's body, into the parts opposite the Sun — yet in such a way that they do not entirely abandon the head of the comet, from which they take their origin, nor vanish into the aether [aether], except as they are gradually worn down and consumed by the solar rays. And this seems to be the reason why comets gradually

diminish, and are not extinguished at the magnitude in which they begin to arise, but rather that central vigor we mentioned above is at last exhausted and emptied out together with the material. For toward the end the internal light grows faint, as it were, in all the new phenomena; and then too, in comets the traces of the tail and the motion are scarcely perceived, since the material is now

found in these to be thinner, less compact and consolidated, and less suited to motion. The formal vigor too, as I said, gradually flows away.

But if anyone should take an argument against us from this, pressing the point that a comet moves by an internal force, without any external impulse, so that the internal force is made the cause of the motion — let him know that this is not sufficient to establish such an opinion. For a body that is also moved by an external cause provides a great occasion for more intense or more remitted motion, either by gradually failing or by measuring out a longer course violently and contrary to nature. This can be observed in fire-darts [pyrobola], which are moved chiefly by an external impulse and meanwhile gradually fail; likewise in arrows, and in the cannon-balls of artillery, which the external moving force gradually abandons. Nor do we imagine that any external impulse of this kind is poured out by the mover of the comet — a mover that merely pushes it along from place to place and does not rather occupy the whole mass and at the same time project its internal powers into motion.

But as for what was earlier affirmed about the material of the comet being torn away and led outward, this will be further confirmed. For if anyone holds a candle or a torch up against the brightly shining Sun, he will notice a certain subtle smoke or kindling matter which the Sun's rays drive toward the part of it turned away from the Sun. Or if this comparison does not fit well enough — since a comet is not kindled fire, but rather the entire cometic material so illuminated that the whole mass appears as if fiery, as was affirmed earlier — let one consider in what way these same solar rays, collected around the stone the common people call crystalline [lapis crystallinus] when it is held up to them, leave a luminous trace through the air in the part turned away from the Sun, and with such heat that even gunpowder [pulvis pyrius] can be ignited from it at a moderate distance. Hence, on account of the presence of the light, that outflow must in some way be hot — which the effects of comets also show, as will be discussed below.

There remains here the fourth and last question concerning the cause, or reason, from which that difference in the outflow which is observed around comets takes its origin. For since the verdict of [Seneca] about comets — which I quoted above as well, and which runs thus: *Whether there are some differences among comets, or none, they necessarily come to be by the same principle* — was quite plausible, we are therefore not asking in vain here what the cause finally will be why comets — whether seen at different times, or the same ones sometimes passing

into other forms — appear in that manifold difference, chiefly with respect to the outflow

which I had recounted from [Pliny] in Chapter 1. For as to what concerns the heads themselves, or the round bodies themselves, those can be known in a way both from the efficient causes — namely the planets, which contribute the greatest power to the generation of comets at different times — and from the density and rarity of the material. From this, indeed, the difference of consistency and of color in comets is rightly derived. But for the discernment of the matter we are presently investigating, the disposition of the comet with respect to the Sun and the Earth is chiefly required — although the quality of the interior form, likewise the material, and the end as well, also enter into this consideration.

For the rest, in treating this question for us the twelve cometary species of [Pliny] are restricted to that small number which we indicated above in Chapter 1 to have been observed in our own age: namely, that the outflows of comets terminate either in the form or figure of a tail (as most frequently), or of hair, as the comet of the year 1585, or finally of a beard — or rather of brooms for cleaning clothes — such as [Johannes Kepler] writes he observed in the north [Septentrio] in the year 1618. For by the observation of such an outflow, and at the same time by the disposition of the comet with respect to the Sun, these species are in some way distinguished by ocular perception and discerned from one another. For the fact that a comet is seen as tailed is brought about chiefly by its lateral disposition with respect to the Sun and the Earth — especially while the comet is within a quadrant of the Sun, and not too close to it. But the hairy, the bearded — or if you prefer, the one more truncated of its tail and surrounded on all sides by traces of the outflow — these require either the Sun more nearly opposite (as those two which I just mentioned), or a sublime elevation above the Sun. For the place of the comet relative to the Earth also varies its appearance somewhat: the more it has been elevated above the Earth, the more it draws the outflow upward, out of our sight — as that great comet of the year '77, of which I made mention above. Besides which, too great a distance of the comet from the Earth, in [Tycho Brahe]'s opinion, greatly diminishes the magnitude of the tail, unless the material has been more abundant, as in that same comet. But in those which we saw later, namely in the years 1607 and 1618, these varieties are shown as follows.

Diagram. A geometric construction illustrating how the apparent projection of comet tails varies with the comet's position relative to the Sun and the Earth. At the center is point A, the Earth, which is the common center of both the Sun's

orbit and the comets. From A three concentric circles are drawn. The Sun is placed at **B**, on the rightmost point of the middle circle (the orbit of the Sun, whose semidiameter is **A B**). Comets are shown at successive positions around the two other circles — an inner circle below the Sun (semidiameter **A D**) and an outer circle above the Sun (semidiameter **A H**) — each drawn as a small star-like head with a tail directed away from the Sun (toward the left, i.e., the part opposite B). Straight lines radiate from A out to the heads and tails of the comets, marking the visual angles by which each tail's apparent length is measured.

Readable labels, with approximate positions:

- A (center): the Earth, common center.
- B (right, on the middle circle): the Sun.
- D (just left of center, inner orbit): comet position on the lower/inner circle, near the head O.
- O (center-upper, inner circle): tail-endpoint of an inner comet; the line C O / the true tail-length referenced in the text.
- C (right of center, inner circle): comet on the inner circle, with the visual angle E A Q / C A O.
- E (left of center, inner circle): comet position on the inner circle (trine aspect with the Sun); the angle E A Q measures its tail.
- Q (left, inner circle): tail-endpoint associated with comet E.
- P (upper-center, inner circle): comet position on the inner circle above center.
- F (lower-center, just left of A): point on the lowest/innermost division.
- R (lower-left): comet head low on the inner region.
- K (lower-left, on the middle circle): comet head with tail at S.
- S (far lower-left, on the middle circle): tail-endpoint of comet K.
- H (upper-center, on the outer circle): comet on the outer/upper circle; the angle A H B referenced in the text.
- M (top, above H, outer): tail-endpoint of the upper comet H.
- I (upper-left, outer circle): comet head; the line I N, visual angle I A N.
- N (upper-left, beyond I): tail-endpoint of comet I.
- L (upper-right, outer circle): tail-endpoint of comet G.
- G (right, outer circle, above B): comet head on the upper circle, with tail G L; visual angle G A L.

- The far-right point on the middle circle, on the horizontal diameter beyond B, is marked with a symbol resembling II [?] (the opposite end of the Sun's orbit diameter from the left side).

Let A in the adjoining figure be the Earth, the center, that is, of both the Sun and the comets, as was shown above. From this center let three circles be described, and let A B be the semidiameter of the Sun's orbit; let A D be the semidiameter of a certain circle below the Sun, and A H that of another above

the same — which latter circles the comets occupy in their various dispositions toward the Sun. For these semicircles being divided over the center A into four equal parts or angles, and the Sun being placed at B, if through each circle, and through their quadrants and half-quadrants, comets are positioned this way and that with respect to the Sun's aspect, and their tails are extended equally and precisely into the part turned away from the Sun, while from the Earth A other straight lines are drawn out both to the heads of the comets and to their equally extended tails — then the variation in the projection of the outflows or tails which falls within our sight is evident from the mere inspection of this figure. For when we have generally concluded all comets to be tailed, the differences reveal themselves from the visual angles which measure the equal tails: that is, the differences of those comets which are placed at the same or at different sections of the circles, or aspects with the Sun, below and above this place. For in those that are above, the tails are perceived by our sight as turned away, or more erected upward, until one arrives plainly at the point opposite the Sun on either side, where, by the interposition of the comet's head, the prolongation of the tail is plainly hidden and concealed. Yet around the body of that same comet the solar rays rebound in the manner of a coma or hair, and are scattered, busied around that superficial material which (as was said above) is gradually worn and abraded — just as is the nature of light leaping back upon a smooth and polished bodily object, when it is gathered again to a central point before the dispersion of the rays.

Now since the cause of the direction of the cometic tails comes from the solar rays — as, besides our own observations, we have [Cardano], [Apian], [Gemma Frisius], and Cornelius Frisius, our own [Tycho Brahe], and many others as witnesses to this matter — and since we further retain in these cases the hypothesis we established above, holding the Earth to be the center of the circular motions

of the comets, it cannot be otherwise than that our sight, raising itself from the Earth up to the comet, measures the length of the tail by that angle two of whose

sides — those, namely, which enclose this same visual angle — extend, the one from the Earth to the head of the comet, the other to the extremity of the tail; while the third side is the true length of the tail projected from the Sun. For example: let a comet be set (as is most often the case) below the Sun, and in its sextile aspect, namely at C, whose tail's true length is C O; then the apparent length of that tail will be the one which the angle C A O measures. The same happens if a comet is placed at G above the Sun and likewise in the sextile of the Sun: for since the true length of this tail is G L, its apparent length is grasped by the angle G A L. By exactly as much as this angle is exceeded by the angle C A O (other things being equal), by so much must the tail of the comet at G exceed in length the tail of the comet at C, since the former stretches more upward — as the comet of the year 1577, which [Tycho Brahe] found, while it was near the sextile of the Sun, had stretched its tail upward with respect to the Earth in just this way, as though it appeared to have been projected by Venus, since Venus was at that time set in its orbit in its morning departure and a little below the Sun.

[Margin: See Book 2 of the *Progymnasmata*, T.B. [Tychonis Brahe], p. 203.]

Had [Tycho Brahe] only placed the comet above the Sun in his own true hypothesis — as his observed parallaxes also demanded, which in truth scarcely exceeded three minutes at the horizon, provided the observations were duly examined — then he would certainly have found that same direction of the tail, which he reports, to come from the Sun itself. For it made a very great difference here whether you set the center of the comet at the Earth or at the Sun. But if at the Sun, as [Tycho Brahe] supposed, then why did the comet, in the lowest position of its orbit, keep the same track under a great circle that it kept when made more elevated in the same orbit? Whereas the planets, leading similar circuits around the Sun, vary this track greatly through approach and recession in latitude, so much so that for the same reason they cannot by any means travel under a circle that is greatest on the sphere. For comets, therefore, the Earth is the center, and the Sun is the director of their tails, in the same way as happens with the rest as well, whose types and tail-figures the present figure sufficiently expresses.

From this it also appears that comets cannot be seen in invisible conjunction with the Sun and Earth — whether they are below or above the Sun — because these, like the rest of the stars, are hidden beneath the Sun's rays. And the more the comets incline toward the point opposite the Sun, the shorter their tails become — as happens with the comets at E and I, near the trine of the Sun, where in the latter, I, a very slender angle I A N measures the apparent tail; while in the

former, which is below the Sun, the measure of the tail is the angle $E A Q$, which, by however much it is found wider than the higher angle $I A N$, by so much in it is the tail more produced.

Finally, the tails of comets which appear plainly opposite the Sun lie hidden behind the heads themselves, and only that which the Sun's rays draw out around the cometic mass itself, and onto which they reflect themselves, appears — from which a hairy comet arises, as a bearded one does when it is not far distant from the point opposite the Sun. For of this kind, by this hypothesis, it bends a short and dense tail to the side and below itself. It appears, moreover, in this figure — the very thing I was suggesting above — that no tail can be detected in those comets or new phenomena whose orbit's semidiameter, like $A H$, exhausts every proportion of the Sun's orbit's semidiameter, namely $A B$, in respect of its own magnitude. For when the angle $A H B$ so completely vanishes that $B H$ and $A H$ are taken, as to sight, for one straight line, the tail tends straight upward no differently than if the phenomenon had been set at the point opposite the Sun.

Accordingly, by such a hypothesis the new stars too could be accommodated — chiefly according to [Copernicus] — if it were not for this obstacle: namely, that the new stars display no trace of an outflow above or around themselves, but distribute their quite clear and brilliant light, like that of the rest of the fixed stars, equally throughout their whole mass. And so we perhaps more correctly judge that they are destitute of tails — or of any visible outflow — just as they are of motions. As for the rest, concerning such a hypothesis, by which the differences of comets are in some way shown: since the causes judged above gather themselves together, so that comets differ from one another in all the kinds of causes mostly by "more and less," and the higher they are the more powers they contain within themselves — and moreover these (as [Tycho Brahe] supposed) withdraw the truer part of the outflow from our sight, on account of their too great distance from the Earth. Accordingly, since from this the true production of the tail can vary much in different comets, so much so that the true length of the tail is not the same in most of them — therefore from the merely formal prominence of the tail it will be difficult to make a judgment about the altitude and magnitude of a comet. For this reason a verdict is more correctly given about these matters from precise observations, from their comparison with one another, and from the subtle working-out of the parallaxes from this.

Lastly in this chapter, in response to the opinion of those who say comets are angels, good or bad — taking these portentous forms, namely, so that they extend

those outflows around comets, of which we made mention, into various significations of punishments, by which they would have the figure of hair or beard represent a rod, but the likeness of a tail a sword or spear, as though instruments of punishments and afflictions, of wars and other evils that are at the doors, are shown forth from heaven by the metonymy of the cause — to this I think the answer must be that it is not necessary to draw at once, from supernal and entirely hidden causes, those things which (in the number of which we showed comets to be

above) tend to denote something singular and are wont to be marked out by God. We have an example in the celestial rainbow [iris], which, although it is a natural thing — namely an appearance [emphasis] in the air from solar rays reflected onto a dewy cloud — nevertheless very significantly expresses that covenant which God made with men after the flood: that this world would no more perish by water. For the rainbow displays this by its blue circle appearing in the clouds. But also by another cloud-circle, which is seen alongside it, this without doubt seems to signify what the holy Apostle [Peter] elsewhere prophesies: namely that the world will one day be consumed by fire. [Margin: 2 Pet. 3.]

Meanwhile, however, it must not be denied that the things which comets signify are very closely akin to those which are foretold elsewhere in Scripture — and most of all in the Apocalypse [Revelation] of John, in the visions, as to come to pass through the ministries of angels — if indeed they are not entirely the same. About the comparison of these we will say a few things below, when we have reviewed the effects of comets — and above all of the last one, in particular — according to our understanding and judgment. But now we will complete, by an interpretation of the end, what remains among the causes.

CAP. VII.

On the final cause of the new phenomena, and their description.

If God and nature, according to the philosopher's [Aristotle's] axiom, do nothing in vain, then surely there will be some end — and indeed a weighty enough one — on account of which both the new stars and the comets are seen in the heavens at different times. And this end, had it but moved [Thomas Erastus] and the other grave despisers of the stars and of their effects, according to its dignity, would surely have left neither to the rest of the stars a quite scanty efficacy, nor to comets, or to new phenomena in general, none at all.

[Margin: Gen. 1, v. 14 and following.] Concerning the Sun, the Moon, and the stars coeval with the world, Scripture defines briefly but most pithily and most truly, as it does all things — namely, that almighty God created them so that, among the other offices assigned to each, they should be for signs, should illuminate the Earth, and should foster by their light what we perceive by sense; for this is the εὐπραγία [Greek: eupragia, "well-doing" — the beneficial functioning] of the stars, on account of which they too, like the rest, were pronounced very good by the Creator. But the word *signs* which Scripture uses is not to be taken entirely in that sense in which the Sun is there declared to be about to profit the day, and the Moon with the stars the night — since these particular phrases, which impose their offices or ministries upon the stars, are sufficiently distinguished from one another in the text — but rather in the sense that the stars are also signs of this kind, by which they will at times, at appointed seasons, indicate the will of God, and above all proclaim

among men His glory, on account of which, as the highest end, all things were created, and continually proclaim His wisdom. [Margin: Psal. 19.] For Christian natural philosophers rightly consider that all things were made by God for the sake and benefit of man, but man for the sake of God — so that this most noble creature on Earth should, from this, acknowledge its Creator above the rest, and duly worship Him — since Scripture by no means departs from this opinion. [Margin: Psal. 8 and elsewhere.] Therefore, so that the best Creator might both consult best for the benefit of men, and might inform each individual man — even the heathen [Ethnici] — more clearly and more manifestly about His omnipotence and will than through those obscure earthly things, it pleased the most wise God to place these most splendid signs, the stars, in the celestial theater and to set them before the eyes of mortals — which Ovid aptly noted in these little verses: [Margin: Ovid, *Metam.*]

*And whereas the other animals look down upon the earth, He gave to man
an uplifted face, and bade him behold the heaven and raise his upright
countenance to the stars.*

From the things just now adduced, anyone understands that the end of the creation of the stars — even the new ones — is in general twofold: natural and supernatural. The natural end is that earthly things, being made up of the elements and perishable, should be fostered and governed by the conversion and influence of the world's light from the stars; that they should stir up seeds in these, and should contribute greatly to the generation of individuals, so that the species in nature may be preserved and may endure to the very consummation of

the age. And since the common material, of which both the stars and these lower things consist, binds the latter very much to the former — and most of all in what concerns the seminal condition and material, which [Aristotle] for that reason not ineptly says is to be compared to the starry heaven — for this reason it should be a wonder to no one why the stars act upon these lower things not only quite evidently, as the Sun and the Moon do, but also by a certain hidden reason, as do these same luminaries, likewise the rest of the planets and the fixed stars: and this most of all, and most diversely, according to their aspects and the different angles of their lines of regard, as is taught in astrology. [Margin: Arist., *De gener.*, c. 2 and 3, *Tab. anim.* (?)]

But this concerns the natural end of the stars insofar as they are coeval with the world, if they alone are understood. What, then, is to be said here about the new phenomena, namely the new stars and the comets? Surely nothing else than that neither do these wholly lack their own end in nature — since they have certainly not been entirely supernatural. For in the first place, since for their generation the particular aspects of the planets concur, as was said above, there can therefore be no doubt that those new phenomena, at the same time, by their own aspects to the rest of the heavenly bodies, so operate upon these lower things that they mingle themselves with the influences of those bodies — but above all with those of the Sun and the Moon, to which they are homocentric. From this it comes about that the effects of comets — at whatever times

times they will occur, we may measure chiefly by the course of the Moon. As for the new stars, whose effects are both more lasting and reach farther into the future, these too adjust themselves to the conjunctions of the superior planets, so that the times of their onset can be discerned either through their direction along the zodiac — as will be set out more correctly here below, and more conveniently in the proper place when their effects are treated — or afterward, more fully and more specifically, through examples.

Now since the effects of the new phenomena — and above all of comets — or at least the signs they give along with other things, are commonly held to be evil omens and portents (so much so that we frequently find true what Claudian, like a prophet, sang):

And never was a comet sighted in the sky without harm. [*comet seen in the sky*]

— yet, by contrast, the whole of nature appears to have been made by God toward the best end, since everything created in it by that supreme creator was

seen to be very good. It is therefore not without reason that the question may be raised: how do those evil and deadly effects, which for the most part emanate from comets — or which at least loom over the world at their appearance — have a place in nature? To this I reply briefly, on the authority of Scripture and of the philosophers, and finally by means of a few examples:

After the curse brought upon nature through the transgression of the first men, nature was never free from evil. Rather, it held evil so closely bound to itself that, before you could remove evil — or anything worse — from nature itself, or from that which is better in nature, you would necessarily destroy nature itself, indeed providence itself (as Plotinus puts it). Yet meanwhile the omnipotent God, by this admirable providence of his, by which he governs all things in nature, so moderates all the evils in it, and so disposes them in time, that they are carried toward a good — indeed the best — end. [Margin: Boethius, *De consolazione philosophiae*, Book 4, prose 6] This is just what Boethius argues, with great skill and force, among others. For since it is the nature of the devil always to strive to turn good into evil, this skill must be ascribed uniquely to the divine — whether to nature or to omnipotence — namely that through it evil is directed to a good end and passes over into it.

Furthermore, that the things which now seem to us worse do little or nothing to hold back nature's course as it now is — this examples joined with experience show. The dearness of grain, of which too we find the comet to be a forerunner: does it not bring it about that, while the land lies fallow and does not yield an abundant crop each year, it is prepared anew, and in the following years becomes more fit to put forth its fruits? The same must be said of the storms that most often follow the appearance of comets. For while these stir up the air and the waters, they render them far purer than they were before. Nor is there sometimes any other reason in those things that befall men through the affliction of diseases and wars, or even of other evils, of which comets too are wont to be the forerunners. Among other things, Palingenius notes the case of wars in these verses: [Margin: Palingenius, in Capricorn]

But although war is foul, deadly, and abominable, still it is not without all usefulness. The cunning, the deceitful men, given to usury and frauds, heap and pile up for themselves in the time of peace. But when wars roar, the soldier seizes their ill-gotten gains. Then deservedly the foxes — the wolves' anger takes its vengeance; then the brave win rewards worthy of their cunning, and pay back like for like: so does time change all things. Moreover, kings, when there is peace, despoil their peoples by

a thousand arts and robberies, and from every side scrape together coin, whence all the gold and silver would be devoured by the tax-collector, had not war torn it loose, which had lain hidden in peace; it distributed to many what one man had amassed. Lastly there is no small crowd of men who have neither property, nor learning, nor craft, and who flee toil, given over to wickedness, prone to crime, impious and bold. Such men, then — nay, shadows — at a fixed time God sends into arms, to be slain by a cruel death. Thus he purges the human race, and those who remain live merry through many years, once this plague is removed; until the evil weeds spring up again, once more to be torn out by fierce war, and cut down by the sword; then Jupiter again skims his foul pots; then again, with furies sent in, he sweeps away the filth.

But you object: the innocent too often pay undeserved penalties, when disaster falls upon the commonwealth and lays it low by plague, war, flood, or some other common calamity; or even when the guiltless are condemned to punishment by the perverse and erroneous verdict of judges; indeed when they suffer various and unexpected adversities and pressures, as if sent from God alone. But I answer with the divine Chrysostom: these are not evils, nor is anyone harmed except by himself. [Margin: Chrysostom, in oration *That no one is harmed except by himself*] And it is certain that evils which were free of fault have never harmed the good and the just. For these things are evil only in appearance, and adverse only to the senses, which are here deceived; whereas Horatius [Horace] bravely encourages the just man in the commonwealth, namely that

If the whole world should collapse upon him, its ruins will strike him
unafraid.

But I have here digressed farther than I intended, in order somehow to satisfy the solution of the proposed question. Elsewhere, both the sacred books and the philosophers — Plato, Plotinus, Boethius, and very many others — bear witness to and discuss this divine economy and providence in nature with full clarity.

And so it is apparent — even from the effects of the new phenomena (which in part we know, and have here accommodated to nature, to be set out more fully in what follows) — that our claim holds: namely that they look to some end in nature, an end which nothing prevents from being manifold as well. For what if, besides those effects which I have pointed out that these share in common with the other stars, the new phenomena — and above all comets — are also appointed for purging the celestial element and freeing it from the impurities with which the sublunary region, by analogy, often has it mixed? For just as the

upper and middle region of the air is wont to be frequently purged by [Greek: φλόγωσις (phlogōsis, "inflammation" or "burning")] — that is, by the inflammation of a falling star, a leaping goat, lightning that accompanies thunder, and other fiery meteors of this kind — so it is likewise probable that the heaven too is purged in its own way through the new phenomena. While these are worn away by the solar rays, as if set ablaze, they wrap up together the superfluous matter into which they strike and which they touch in their course, and expose it to be consumed by those same solar rays. From this, perhaps, the inequalities of the comet's tail on different days in the same comets draw their cause, since the circumfluent cometary matter — that adventitious matter scattered everywhere beneath the heaven — will have been increased. Anyone can test this in the flame of a lamp, if he brings combustible matter up to it from above. Meanwhile, such matter floating upon the celestial sea is, among other things, endowed with such thinness and subtlety that of itself it admits no illumination of itself from the Sun and the stars. And this can be the reason why, while the airy region is purged by the frequent ignition of nearly continual fiery meteors, the ethereal region is freed of its defilements roughly every ten years, by the rising and flaring-up of comets — and this in its own peculiar way, according as the heaven differs from the sublunary region.

But enough of the end of the new phenomena in nature; there remains the supernatural end, of which we shall speak in the last place. We ought to recall what was stated from the outset: namely that the new phenomena, although in a way they are found within the enclosures of nature, nevertheless, as often as they are generated and appear, are by divine foresight appointed chiefly to signify supernatural things — and above all those which seem to be evil punishments in nature. For if even from the Sun and the Moon — which, together with the rest of the stars known to the world, continually serve the elemental nature — signs of supernatural things are taken, as often as they apparently fail in their appointed offices and fall into eclipses (which indeed those luminaries admit in their wholly natural courses, and bring upon the world, so that anyone seeing them readily forgives his own mortality, to speak almost with Pliny) — and certainly signs from the natural eclipses of the luminaries have existed at all times, concerning the world's eventual destruction (against Aristotle's opinion), besides the other things that for the most part usually follow those portentous darkenings — such as wars, plague, famine, diseases, storms, and other similar evils, likewise the deaths of princes in the commonwealth — what wonder is it, then, that as often as divine providence, besides the stars created from the beginning (which continually foster the lower nature with their light and motion), wishes also new and unusual things to be generated, doubtless to this end: that among other

things they may plainly bear witness concerning the overthrows of the commonwealth, and above all to make detestable that complacency in which (at all times indeed, but most of all under this last age of the world's exhaustion) the greatest part of the human race lies buried; and that they may graciously invite mortals to repentance (so far as it can even thus be done) before the evils they foretell? For these new phenomena, and above all comets, are prophets — no less than Jonah once was among the Ninevites — mute, yet, as the Psalm has it, sufficiently vocal. [Margin: Psalm 19] This too the inhabitants of Jerusalem heeded too late and too unhappily before the final and fatal overthrow of their city, since, as Josephus writes, a comet like a drawn sword shone for an entire year in the celestial theater and stood out over their temple. To say nothing of the other flarings-up of comets, which histories record here and there, and which fatal calamities for the most part followed — just as the last one seen by us bears witness this very day.

Since, then, God and nature do nothing in vain, as was said before — while nature often produces monsters beyond its own intention, yet never apart from the divine will, namely so that these too may bear witness clearly, and as it were [Greek: κατ' ὑποτύπωσιν (kat' hypotypōsin, "by way of outline" or "as a sketch")], concerning human wickedness, sloth, and other enormous vices, and the punishments deservedly to follow — whether they arise in the air, the waters, or the earth, among animals or men (as now everywhere) — what then is to be judged of those vast, translucent bodies of stupendous magnitude, which God wills to be newly generated, affixed to the heaven, set in motion as if by rational minds, and finally configured with the rest of the stars by what seem fatal aspects? What else than that these must be true prophets, or preachers, and as it were witnesses before men — both of the immense [Greek: φιλανθρωπία (philanthrōpia, "love of mankind")] of God, and of his mercy toward those who repent, as well as of his most just anger against the impenitent? By the kindled heat of that anger these men will not only be justly punished in this age — and, as the new phenomena are consumed from the heaven, so these will at last be consumed from the face of the earth — but will also in their own time be afflicted with infernal punishment. Not to mention that the new phenomena, and chiefly comets, are signs of the world's coming destruction no smaller than the eclipses of the luminaries — provided only that by their disappearance, and as if by a visible conflagration, they have not been more conspicuous than these.

But since a pious mind, by no means a despiser of divine signs, is wont to ponder and conjecture more things with itself, of these too something has been said here and there above, and more will be said below, more fully and specifically in the

examples. Therefore we here draw in our sails concerning these final indications of the new phenomena, and at last, from the causes investigated and set forth, we shall construct their descriptions and, as it were, definitions — first in general, then in particular. Therefore the new phenomenon in general (under which we have so far comprehended both new stars and comets alike):

It is a heavenly body that is new in the sky — generated, that is, from celestial matter, from the Milky Way and from the concourse of the stars, or from their other aspects. Although by its splendor and its regular motion about the center of the earth it in a way rivals the stars coeval with the world, yet because the excellence of its inner form is far lower and meaner than theirs, it is therefore also found to be less durable than they; whence it comes about that it gradually decreases to its final disappearance. Meanwhile, by singular divine providence, it is also ordained to this end: that, by its outward form, position, motion, and aspect toward the other stars, and by its ominous relation to the earth, it may seriously admonish men — for the most part concerning great public calamities and just punishments for their various offenses — into an expectation of repentance; and it likewise in a way exhibits a sign of the consummation of the age (which now without doubt presses on briefly) by its more frequent flaring-up and disappearance.

From the foregoing, moreover, it is understood that the new phenomenon falls into two distinct kinds, so that it is predicated both of the new star and of the comet — two things which differ between themselves chiefly in form, place, and motion, as also in their special signification, and in the duration of that signification and of its events, as well as of the time. For the new star is a star, or new phenomenon, located in the highest heaven among the fixed stars, and almost always in the Milky Way, from which it draws its matter; it for the most part shows itself from the start with a great, white, and splendid radiance, so much so that it sometimes thereby equals the star of Jupiter or even of Venus; and it is subject to no motion other than that of the fixed stars — that is, to the prime revolution. Moreover, the time of its duration is for the most part beyond a full year; and then, gradually decreasing, it altogether disappears — it continues, and extends the duration of its significations into longer periods, which, although from the start they are wont to be of a better and more brilliant omen, nevertheless for the most part end in a notable calamity, as the casters of nativities [Genethliaci] confess concerning the other fixed stars as well.

The comet, however, is likewise a new celestial phenomenon, generated, that is, from the common celestial matter — which we judge to be the Milky Way — and

furthermore from the aspects of the planets in the lower, or planetary, heaven. It shines with a varied color, and is often conspicuous in a varied and terrible form, since for the most part there is seen attached to it some efflux — which the rays of the Sun drive perpetually toward the part turned away from itself, that is, toward that quarter of the world — and which most often takes the figure either of a tail, or of hair, or of a beard, or of brooms. And although the comet tends toward various quarters of the world beyond the prime motion, yet it carries on its course constantly beneath the greatest circle of the heaven right up to the end: and that too by this rule of proportion, namely that, as its motion from the start becomes for the most part slacker day by day, the cometary body at the same time decreases. At last it denounces dire calamities — according to the quality of the form it presents, and the stars and aspects of the planets to which it applies — to the world, or to certain tracts of it.

And thus far concerning the causes of the new phenomena, and their descriptions drawn from those same causes, so far as they could hitherto be investigated by the reason and criteria of our rather feeble understanding. There follows the discussion of their effects.

CAP. VIII.

On judging the effects of the new celestial phenomena, and above all of comets.

At the beginning of this chapter it must above all be warned that predictions about effects of this kind are by no means definitive decrees, since the effects of the stars in general vary infinitely according to the diverse nature and condition of the subjects that receive them. Besides the other causes, or hindrances, that are taught in astrology — by reason of which, although the stars act naturally, nevertheless their effects in these sublunary things are not felt uniformly, because the natures and conditions of these things are not uniform, but differ in virtually infinite ways — I add this: that things which include free and voluntary causes, such as human minds, can indeed be inclined by the stars, but by no means compelled; because the soul of man — and above all of one who continually cultivates God and the virtues and directs his actions to the divine statutes and the norm of right reason — is both superior to the stars and more excellent than they. Hence the story told of Socrates: he once confessed that his own nature was indeed the very nature which a certain astrologer expounded to his face — namely depraved and distorted — but that he had corrected it according to the institutes and prescriptions of philosophy. And so Socrates, in

the way it is said of the wise man, splendidly mastered the stars, when he rendered a nature otherwise distorted most gentle by the discipline of philosophy — so much so that, by the testimony of his wife Xanthippe, he never changed his expression. These things being premised — since it was asserted above that those new phenomena,

whenever they blaze in the heaven as harbingers of God, beyond doubt portend new things, that is, new changes in the political world, and those more universal than the ones that other sublunary prodigies indicate. Therefore, so far as I can reason about their effects or significations — given the feebleness of my talent, which I know well enough — I shall set them out in order in what follows; where I shall first render the causes of these things in nature that seem to me most to show why those new heavenly phenomena, and above all comets, signify certain things diverse from and surpassing the rest of what is to come, as often as they blaze forth. To render these causes — and likewise to practice the whole of astrology rightly — I find nothing more suitable, for my part, than the universal Tychonic hypotyposis [hypotyposis universa Tychoniana], which, after the Ptolemaic and the Copernican, I had inserted in the frontispiece of the second part of the *Astronomia Danica* [Tychone, Ptolemaic, Copernican] with that eulogy of which it was worthy. For in it, besides the center of the universe — namely the earth, toward which all the lights and operations of the stars are directed, even according to the sacred pages, since it is uniquely appointed to be fostered — there is another, secondary and mobile center, which the Sun itself occupies as its royal seat. [Margin: Genesis 1] The Sun, in the midst, as it were the [Greek: δεσπότης (despotēs, "master" or "lord")] and lord of this celestial economy, draws the other planets — except the Moon — like a household, and with its annual motion carries them along with itself around the earth, while it holds them within the fixed limits of their circles, and meanwhile sends some of them — the three superior ones, Saturn, Jupiter, and Mars — off to such a distance from itself that they even encircle the earth at their opposition to it; whereas it confines the others, Venus and Mercury, within shorter orbits around itself, so that, while the Sun is borne in its chariot to survey the region of the heaven and to render the annual vicissitudes among us in its slanting circle, these may be at hand and, each according to its nature, motion, and ministry, carry out its commands — as the king's chamberlains are wont to do. To the Moon, moreover, as queen of the Sun and the heaven, the lower celestial region — that is, the one nearer the earth — is assigned. This Moon shines and thrives by the perpetual communication and, as it were, impregnation of the Sun's light — both that light innate to it and that acquired by the ministry of the other planets — and from this it shows the earth its various torches and faces; whence these lower things are manifestly

moved and affected, as one may observe in the tide and the reciprocation of the Ocean, likewise in living creatures and other sublunary bodies. Furthermore, around the planetary region (and indeed from the center of the earth itself), in which, as I said, the Sun is lord and king, and the Moon mistress and queen, there stands the outermost celestial region, for the most part filled with what are called the fixed stars; with these — as with foreign peoples and nations, though endowed with the same or a more excellent kind of light — the Sun and its envoys, the planets, as also its very queen, the Moon, carry on a continual commerce [Greek: κοινωνία (koinōnia, "fellowship" or "communion")] of light, and first import it into the planetary region, then, the more abundant it is, the more eagerly they strive after it, and, made more accommodated to lower things through the ministry of the planets, they pour it into the lunar and at last into the sublunary region; and finally they affect the latter variously, according to the disposition of the stars and planets in relation to the Sun, Moon, and earth, and according to the condition of the underlying matter. This very thing the risings and settings of the planets — and above all of the Sun — together with those fixed stars, evidently prove year by year.

And this is the customary governance of the mundane economy in nature, which, prepared long ago together with the heaven, out of the divine word of the Omnipotent in these secondary causes — for the purpose of fostering, changing, multiplying, and so forth — is thereafter continually sustained by the same word right up to the consummation of the age. On this matter the philosophers, and above all Trismegistus [Hermes Trismegistus], once discoursed. Indeed more things, and more specific, will be inferred for astrology — if ever it is rightly to be practiced — from the consideration of this same thing. But now, to our subject concerning the new phenomena. That which is annual, then, and which, on account of its continual periods, is less unusual, this governance of the mundane economy in nature disturbs less — even the antagonism of aspects — than that which happens rarely; as, among other things, the conjunctions of the superior planets, which occur only rarely, convince us by the manifest indication of their effects. And why, I ask, should not those phenomena which are far more unusual — which either, like the new stars, associate themselves with the fixed stars, or, like comets, especially the more powerful ones, pour themselves into the planetary region, and for the most part with their transverse and distorted motions, against the advance of the planets, variously cut across the courses or paths of these — why, I say, should their governance, penetrating on every side to the lowest earth, not disturb these new heavenly prodigies, and incline and indeed compel things toward the unusual? For although comets are the offspring of the Sun and the planets, yet because they are for the most part generated

under the sinister aspects of these very bodies, they are deservedly accounted by our lord Tycho [Tycho Brahe], and by certain other learned men, as nothing but bastards, or spurious, degenerate, and as it were alien in this order of the mundane ministry. Therefore, while most of the planets contribute their work, each according to its nature, to fostering the lower things and to freeing the air from the noxious humors that perpetually exhale from the earth and the seas — but most of all of them the Sun, as a star most gentle to terrestrial things and above all to the human race — by contrast the new heavenly phenomena, and before all comets, by their inauspicious generation, by their no less disparate irradiation and motion, and finally of noxious vapors ...the attraction of noxious matter, and the dispersal through the air of things only half-concocted, violently afflict earthly things, when these operations of theirs mix disastrously with the qualities of the elements. From this arise storms and barren years, and likewise *δυσκρασίαι* [Greek: *dyskrasiai*, "ill-temperings" — disturbances of the elemental balance] spread through the other tempering-qualities. Out of these, in turn, come the morals of men — which usually follow such conditions — driving worldly mortals into various unlawful appetites. From this take their rise wars, the contentions of kingdoms and the overthrow of commonwealths, together with countless other crimes and pollutions, and likewise diseases and other evils of that kind.

So much from purely physical reasoning about the new phenomena, and chiefly comets — apart from what is mystical in them, which bears more on the divine providence in the world that operates even around their generation [Margin: The cause of producing the new phenomena]. Some of this I have treated above; some I will treat more fully below, as occasion offers. For now I will move on to the division of the effects, or significations, that are derived from these new heavenly bodies.

The effects of new phenomena — that is, both of new stars and of comets — whenever by the divine will they appear in the sky, are gauged either by the force and power of their effects (or rather their significations), or by the display of these at certain places and times in a particular way; and they are taken, as it were, as predictions about them. Now the significations of new phenomena are either supernatural or natural. In the supernatural order, divine providence reigns — or, what is the same thing, the divine decree — which is exercised with precision in the prophets' oracles concerning things to come upon the world right up to the consummation of the age. Of these things, and of greater ones too, these new phenomena are without doubt temporal signs; but since they were treated more fully in the chapter immediately preceding, they need not be repeated here.

Among the other significations of new phenomena, the first to be noted in general is the position of the new body in longitude and latitude with respect to the ecliptic, and likewise with respect to the equator — so that one may properly recognize, by reference both to the ecliptic and to the equator, which region of the world, and which point of the spherical surface, the new star or comet has kept to throughout its whole course in its rising. This matters especially for grasping and analyzing the significations of these bodies more thoroughly. For one thing, it is through the ecliptic (or zodiac) that the aspects of the Sun and planets with the new phenomenon are perceived — from which it is reckoned to draw most of its origin. Moreover, the regions and tracts of the earth are judged, from [Ptolemy] and others, by the signs of the zodiac, since they are especially affected when the new body runs through signs of this kind. For another thing, it matters with respect to the equator, when one considers the daily revolution of the new body and, within it, the vertical point that the comet reaches on its individual days. Hence, besides its daily motion in its proper circle and its progress through the ecliptic, its declination is also required, as will appear in the examples later.

Furthermore, the astrologer is not idly bidden to contemplate the celestial houses in the scheme of the new body's rising (provided its time can be had), and likewise at the new moon or full moon next following — these confined to the dimensions of these two circles under the given pole-elevations. So much so that knowledge of all these is required, besides the nativities of leading men of those regions toward which the effects or significations of the new body are observed to incline most — some of which matters will also be recalled more fully below. Again, the nature or quality of that celestial asterism — whether within the zodiac or outside it — to which the new star is attached, or in which the comet either takes its origin or chiefly directs its motion, must be diligently investigated and confirmed by an induction of examples. For besides the fact that the signs of the zodiac are wisely distributed into as many triplicities as there are primary qualities in nature —

Hot	Cold
♈ ♌ ♉	♊ ♋ ♏
Dry	Moist
♈ ♌ ♉	♊ ♋ ♏ [recte: ♈]

— so that an astrological judgment can be woven from these as it were houses and properties assigned to the individual planets, the nature of each individual asterism must also be examined. For their very names, although they appear to

have been imposed on them from the outcome, are nonetheless not idle in producing effects, but at times very much συμπάθῃ [Greek: sympathē, "sympathetic" — in active sympathy], so that of these is verified the common saying: *Names often agree with things* [*Conveniunt rebus nomina saepe suis*]. But the qualities and natures of the signs of the zodiac, of the asterisms, and of the planets, by which the new bodies are affected, need not be treated here at length; rather, on these [Ptolemy] in his fourfold work [the *Tetrabiblos*] is above all to be read and consulted.

Finally, as to the aspects or configurations of the new bodies with the rest of the heavenly bodies — and chiefly the planets — let the judgment likewise be made according to their nature. Hence square aspects with the malefics Mars and Saturn for the most part mark the significations of the new body with an unlucky outcome, while benefics on either side mitigate them. And since these aspects are detected through a rational division of the circle, it is not in vain that through these same...

same divisions a judgment is also taken about the influences of these stars too, fictitious as they are. To these matters, considered in general, is added the observation of the magnitude and color of the new body, and from this too the judgment about the magnitude of the events is confirmed: a new star, indeed, with another star fixed or wandering, to which it is thus likened; a comet, similarly, whose color and the shape of its head most often resembles that planet which had contributed the greatest force to its rising, or generation.

There follows the display of the significations derived from the new phenomena. This is subject to a twofold consideration: namely, either of the places — the earthly regions — to which the effects of the new phenomena are chiefly applied, or of the times by which effects of this kind are determined. To indicate the places, or inhabitants, of these in a general way, one must note those signs to which the most approved astrologers, and above all [Ptolemy], had assigned the individual regions or earthly districts; and these are to be considered carefully according to the nature, qualities, and the morals of the men under them, whenever the new body is found either to rise in those signs, or to be determined by its course in them. Further, for the knowledge of the places, knowledge of the vertical point is required — the point that the new body has shown by its motion in the syzygy of the luminaries immediately following its rising. From this the place on the terrestrial globe, subject to such a meridian or vertical, is elicited quite exactly. For which time, as I said above, it is best to construct a celestial scheme, and to consider carefully the signs of the zodiac and likewise the

situation of the luminaries and of the rest of the planets in the individual celestial houses.

But as for [Ptolemy]'s instruction to observe the direction of the tail in comets — toward which region of the world it points — and from this to derive a judgment about the tracts of the earth that it thus indicates: this without doubt is mistaken, since that whole disposition of the tail proceeds from the Sun, as was shown above. Yet the observation of the tail's magnitude, and of the motion of the whole comet across the regions of the world, is not empty: for since the one argues an abundance of matter, and the gradual dissipation of the comet accompanies the other, there will be no lack of a hidden basis of influence by which the subject earthly places are affected in no small degree with respect to the meridian, or vertical place. And so much for the places assigned in a general way, which are most strongly drawn to the perception of the new body's influences.

Further, the special places to which the influence or significations of the new body most reach are the Ascendants, and likewise the apheta [Aphetica] and hyleg [Hylegialia] of those born under such conditions — but chiefly, as I suggested above, of Emperors, Kings, and rulers in the worldly commonwealth. For since these are public persons, namely those bearing public offices, by no idle reasoning, and not without the authority of [Ptolemy] — indeed by experience itself — such significations of the new phenomena are most of all applied to them. So much so that these, wrapped up in the decrees of the anaereta (the killer planet) that come from the directions, very often cut short the threads of life and hasten on death or peril for these illustrious persons all the sooner, as will be evident in the examples below.

Thus far concerning the earthly places to which the significations of the new bodies are usually applied from indications of this kind. There follow the times by which astrologers measure the future events from such stars. These are determined either in general or in particular. In general, for instance: if new phenomena first appear in the eastern region of the sky, their effects will come upon the world early; if in the west, later. And if they are in the remaining regions, the times are accommodated to such a consideration as well. In new stars, however, the measuring-out of times extends to far more years than in comets — so that, regarding the effects of the new star of the year 1572, it is verified down to this very limit of ours, by the most certain prediction of [Tycho Brahe]. [Margin: T.B. in the conclusion of Book 1 of the Progymnasmata.]

But the special times that determine the effects of the new body are referred to the decrees of the directions, which are excellent documents of astrological

certainty. Here the direction may be made either according to the zodiacal circle — as in new stars, with a conjunction of the superior planets among themselves, either preceding or rather following next, with which a new star has a great kinship: as in the same star of the year 1572, with the conjunction of Saturn and Jupiter made afterward in the year 1583 in Pisces. Likewise in the new star of the year 1604 in Sagittarius, similarly, with the conjunction of the superiors Jupiter and Mars in the year 1600 in ♈, a fiery sign. Concerning this latter, [Johannes Kepler] did write — but not enough, as it seems to me; he did not bring out to the public the operations of such conjunctions, since scarcely from that time has there been any rest in Russia, Sweden, Livonia, Poland, Denmark, and thereafter throughout the whole Roman Empire, from wars and dreadful contentions, chiefly under the pretext of religion.

But such a direction is determined entirely by reference to the Sun's daily motion in the ecliptic at that time, for the individual years. Comets, however, since they generally last a space of two or three months, accordingly have their decrees of events rolled out from these partly along the daily circle, and partly include their measure within their own proper motions — that is, either considered along their daily motion in their proper circle, or rather by comparison of that motion to the zodiac, made as it were the proper circle of men and of all things that live. Here for the most part some...

are usually presented, through which the comets will unfold their decrees at various and fixed times, so long as they are duly weighed — not by empty pretenders, but by skilled astrologers — for an estimate of the celestial harmony with these lower things. And in this manifold path of the directions, back and forth, what experience has most proved in other cases of old is to be followed; and much is to be left to the astrologer who is carried by natural inclination to discern, judge, and bring forth in their proper times the effects of the stars. Yet so that he does not — by some rash, enthusiastic impulse of producing results, as certain men otherwise ignorant of the art are accustomed to do — concoct and press certain singular things as effects of the stars, and pronounce these by his inept and immature judgment; whereas every wise and circumspect man judges nothing before he has surveyed the whole series of gathered causes universally, and so too keeps himself as much as possible within the bounds of a general judgment. *For only those inspired by the divine spirit foretell particulars [Margin: Ptolemy, Centiloquy 1].* Meanwhile, those things that are more special, and those that are supernatural and have a symphony with sacred scripture (as was said above), are to be set forth not so much by prediction from the stars as by timely admonition to the people from the word of God and the law of nature, should it

be violated — and nonetheless to be determined in some way by the circumstances of the directions, regarding the times of those things that fall within the judgments for a new body. With these things I will have sufficiently completed the first part of this treatise, according to my purpose and the powers of my ability.

CAP. IX.

On the delineation of the path of the comet of the year 1607, from my own observations; and on its small parallaxes, from which we conclude that it lay within the celestial region, far above the Moon.

Now that the causes and effects of the new phenomena have been sketched in a general way through the preceding chapters — the first part of this treatise — there follows the second part of the undertaken labor, in which I will append the promised examples among comets: among comets, I say, the two most recent, namely those of the year 1607 and of 1618. For the positions of the new stars that have appeared in our age, hitherto in Cassiopeia, Cygnus, and Serpentarius, are found to be sufficiently delineated by others — chiefly by our [Tycho Brahe], and by [Johannes Kepler], the Imperial Mathematician. But the comets that I will now touch upon were, among those that blazed forth in the sky in this most recent time — and after the death of our Atlas [Tycho Brahe] — not only remarkable and astonishing in magnitude, motion, and duration, but also signified by their almost contrary thrust, and by their encounter through the asterism of Boötes, certain things beyond the rest. Beyond doubt they signify things which it befits anyone skilled in astronomy and astrology, according to the powers of his ability and his gift of interpretation, to treat soberly rather than to conceal — or, in supine negligence, to despise along with the common crowd. For portents of this kind are most of all those things through which God in these last times warns men of their security, and at the same time severely rebukes them, as I had warned above, and as below I will rightly enlarge upon at greater length.

Now then, I will pass to describing, from my observations, the earlier comet — which in the year of our Lord 1607, in the months of September and October, took its rise in the North, and bent its course rather toward the South.

In the year of our Savior Jesus Christ 1607, on the evening of 18 September — scarcely one week after I had completely finished the construction of our sextant of five cubits' radius (the sextant whose mention I had made in the *Astronomia*), and after I had finally made several experiments, for the sake of its infallible certainty, in taking the distances of the fixed stars from one another; and indeed

after I had observed Jupiter near its acronychal situation with the same instrument on 14 September, observations that I had also continued on the following days, had not the clouds and mists here on our Copenhagen [Haunia] horizon interposed themselves for three days — when serenity returned on the evening of 18 September, I first noticed and at the same time observed a tailed comet in the North. In its magnitude it almost rivaled Jupiter itself to the eye, but in its more obscure aspect — that is, in its livid color — it rivaled Saturn itself. It also had a fairly long and dense tail, which the solar rays drove directly into the region turned away from the Sun, and this throughout the comet's whole duration. For on this evening, after the 7th hour, when the comet stood scarcely at the distance of one Moon below the small unformed star behind the Great Bear — which I noted here with no less diligence than [Johannes Kepler] in Bohemia on the same evening — I found also that the comet pointed with its tail at the two unformed stars below the root of the [Bear's] tail in the Great Bear, the Sun of course passing through 5 degrees of Libra. The comet's tail was also dense, as I said, and perhaps more compact than the rest usually are, and of the same color as the head itself — namely livid and somewhat Saturnine. But now I come to our observations.

There follow the observations.

So then, on the evening of the same day, 18 September, at hour $7\frac{3}{4}$, the comet was found in a disposition with a small unformed star — one called "inermis" [unarmed] — in the manner we expressed before, such that it was scarcely distant from it by the diameter of one Moon, that is 34 minutes below, and declining a little toward the east.

That this star's longitude is 12 degrees 16 minutes $\mathfrak{M}$, and its latitude $40\frac{1}{2}$ degrees North, are well enough assumed by [Johannes Kepler] — as our following observations also confirm. For around the same time, namely hour $7\frac{1}{4}$, the comet was distant from the preceding upper star in the square of the Great Bear, by the sextant, 24 degrees 32 minutes, the observation being repeated in the moment; likewise from the lower of the following stars of the same square of the Great Bear, 14 degrees 14 minutes. But since these two stars were arranged too nearly in a straight line with the comet — so much so that the comet's position could scarcely be had without error, as I cautioned in the doctrine of the sphere in our work — therefore, the comet's latitude being supposed 40 degrees 5 minutes (namely by the estimate of sight, in the disposition it had with the said little star, whose latitude was 40 degrees 30 minutes), there comes out, by the reasoning of spherical triangles, the comet's longitude from each of the two distances from the

stars named in the square of the Great Bear, as 11 degrees 57 minutes $\mathbb{M}$ — agreeing closely enough on each computation, when the longitudes and latitudes of these stars too at this time are applied from the corrected canon of fixed stars.

Further, on the following day, morning and evening, the sky was not clear. For with the north wind blowing, the clouds continually rose up and hid the comet.

But on 20 September, in the morning, with serenity somewhat restored — when the altitude of the middle star in the tail of the Great Bear was 34 degrees 36 minutes, observed by our geometric quadrant, to which the time corresponds very nearly hour 5½, while the comet's altitude in this horizon was about 17 degrees — the comet was distant, ἐν πλάτει [Greek: en platei, "in latitude" — i.e. measured along latitude] from the middle star in the tail of the Great Bear, 20 degrees 46 minutes, by a reiterated and limited observation. Moreover, at the same time the comet's tail, projected from the Sun, pointed quite precisely at the last star in the tail of the Great Bear, while the Sun stood in 6 degrees 22 minutes of Libra. But since more could not be had by us this morning, both because of the twilight and especially because of the clouds rising up, accordingly, from the given data, in inquiring into the comet's situation, we proceed by this method — which therefore I leave here in full and pursue, for the students' sake, because it was not set out in our work [on the sphere].

In the following figure, let A be the pole of the ecliptic, B the middle star in the tail of the Great Bear, C the extreme of the tail of the same. From pole A, at the distance of a quadrant, let there be described a portion of the ecliptic H E; and the Sun being placed at point E of the ecliptic, with the line E C given on the sphere, it is manifest that the comet is found on that same line, its place being determined at D — namely by the distance from the middle star of the Great Bear's tail, B D. But since we intend to have D itself, the point or situation on the sphere with respect to the ecliptic...

Diagram. A spherical-triangle construction for fixing the comet's position relative to the ecliptic, accompanying the worked computation of Cap. IX. The figure is a near-vertical curvilinear triangle whose apex is at top, with the ecliptic arc forming the base at bottom. Quadrant arcs descend from the apex A through the four points B, C, D, E down to the corresponding ecliptic points H, G, F, E. A small sun-disk symbol marks the base at E (lower right), and another decorative sun-disk is engraved at the base near E.

- A (apex, top): pole of the ecliptic
- B (upper left, on the left limb): the middle star in the tail of the Great Bear

- I (just below and right of B): intersection point on arc B–H
- O (right of I): point on the right-hand descending arc
- C (below B): the extreme star of the Great Bear's tail
- K (right of C, mid-figure): intersection point
- D (mid-lower left, on the left limb): the comet's place
- O (lower, on the descending arc toward E): intersection point
- H (base, far left): ecliptic point below B
- G (base, left of center): ecliptic point below C
- F (base, right of center): ecliptic point below D
- E (base, right): the Sun's place on the ecliptic (marked with a sun-disk)

accordingly, with quadrants dropped from A through those four points B, C, D, E, and these marked on the ecliptic by the distinct points H, G, F, E, it is manifest that the comet's latitude is found by the arc F D, and its longitude by the difference between H and F — that is, by the angle B A D; and likewise by G F or F E, for the operation here can be set up in various ways.

First, then, in the triangle B A C, from the known B A and C A, and likewise the angle B A C, is found the sought distance of the stars in the tail of the Great Bear, B C, 6 degrees 41 minutes.

Next, in the triangle B C A, with the sides given, the angle B C A is sought, whose complement to two right angles (180 degrees) is the angle B C G. The angle B C A is found to be 68 degrees 29 minutes, and therefore B C G is 111 degrees 31 minutes.

Third, in the right triangle C G E — having a right angle at G — from the given C G, the latitude of the extreme of the Great Bear's tail, 54 degrees 25 minutes, and besides G E, the distance of that same tail and of the Sun along the ecliptic, 15 degrees 4 minutes, there is given by computation the angle G C E, 18 degrees 19 minutes; and adding to this the angle B C G previously acquired, the whole angle B C D is formed, 129 degrees 50 minutes.

Fourth, then, in the triangle B C D we have known two sides, B C and B D, together with the angle at C; therefore the distance from the last [star] of the Bear's tail to the comet, D C, will not be hidden. But the analysis of the proposed triangle, since it is conveniently resolved for the sought quantity by right angles, [proceeds thus]: with the arc D C produced indefinitely, and a perpendicular dropped from B to I, there will be, first, in the right triangle C I B — given B C 6 degrees 41 minutes, the angle B I C (which is the complement of the angle B C D to two right angles, namely 50 degrees 10 minutes), together with the right angle at I.

Therefore first is given the perpendicular B I, 5 degrees 7½ minutes; then the arc C I, 4 degrees 38½ minutes. From this, in the whole triangle B D I — from the given B D 20 degrees 46 minutes, B I 5 degrees 7½ minutes, with the right angle at I — the whole side I D is acquired, 20 degrees 9½ minutes; and when from this I C, 4 degrees 38½ minutes, the side previously found, is subtracted, there remains the sought D C, 15 degrees 31 minutes — namely the distance of the Bear's tail and the comet.

Fifth, in the triangle A C D, since two sides are given — A C 35 degrees 35 minutes, D C 15 degrees 31 minutes — together with the angle A C D, 161 degrees 41 minutes...

is 41 minutes, since it is the complement of angle G. For C E, together with [the angle] at Arcturus, makes two right angles, that is 180 degrees. So from these the side D A finally emerges as 50 degrees 31 minutes — this being the complement of the comet's latitude with respect to the quadrant, that is, 90 degrees. Therefore the comet's latitude itself is 39 degrees 29 minutes north.

Finally, in triangle B A D, since all the sides are now known, the angle B A D works out to 18 degrees 24 minutes. Adding this to the longitude of the middle of the Great Bear's tail at B — which at this time was 10 degrees 2 minutes in Virgo — gives the comet's longitude as 28 degrees 26 minutes in Virgo.

And so, by a double approach to this day's position of the comet, we possess the comet's situation as it was at the start, that is, at 5¼ o'clock (5:10) in the morning of this day, from our observations. Now we move on to the following day.

Furthermore, on 21 September in the evening, at about 7:30 by the clock, with the altitude of the bright star of the Corona [Borealis] set at 34.5 degrees, the comet stood 30 degrees 59 minutes from the middle of the Great Bear's tail; and at about the same time it stood 16 degrees 41 minutes from the bright star of the Corona. These measurements I repeated by means of our cross-staff [sextans]. Here, with the positions of the fixed stars assumed for this time — namely the longitude of the middle of the tail at 10 degrees 2 minutes Virgo, and that of the [bright star of the] Corona at 6 degrees 44 minutes Scorpio, with its latitude 56 degrees 22 minutes north, and the latitude of the former 44 degrees 23 minutes — as the computation set out in the [section on the] sphere is found [to require]: when these data are worked through by this calculation, reduced to the first minutes throughout, the comet's longitude at this time comes out, as written above, at 16 degrees 44 minutes in Libra, with a latitude of 37 degrees 14 minutes north.

For the rest, because on the following days the sky was more overcast, and various other obstructions also arose to keep me from observing, I no longer pursued this comet with the [usual] instruments, but recorded it instead by solar cross-threads [filaribus extensionibus] against straight lines aligned with the fixed stars. Accordingly, I caught these three positions of it in the same way as the three previous days. First I sought it out by means of this rather large globe — about three feet across — onto which it could finally be brought under a great-circle arc; and by working this through mechanically and accurately with an incurved arc and a great-circle quadrant carefully extended on the globe, I found a point on the surface of the globe, which I took as the north pole of this comet's track. Thus, reduced to the ecliptic, it had its longitude at $17\frac{3}{4}$ degrees in Pisces, and its latitude at $49\frac{1}{3}$ degrees north.

To such a position of the comet the following circle-observations conformed as closely as possible. For on the first day of October, at Malmö [Malmogia], in the evening at about $6\frac{1}{2}$ o'clock, the comet stood off from the bright star in the left hand of Ophiuchus — by sight, scarcely 0.5 degree — toward the west and the horizon. At that time it stood in a straight line with Arcturus and with the eastern star in the same hand of Ophiuchus, so that, from the [now] rectified positions of the fixed stars, its position at this time finally came out at longitude 18 degrees 45 minutes Libra, and latitude 31 degrees 2 minutes [north]. Again at the same time it was occidental [west of] and north of the western and northern star in the hand of Ophiuchus, at longitude 26 degrees 50 minutes Scorpio, with a latitude of 17 degrees 19 minutes north. Following this, finally, in the same hand, [it was in line] at longitude 28 degrees 3 minutes Scorpio with a latitude of 16 degrees 30 minutes. From this the comet's longitude comes out at 26 degrees 30 minutes in Scorpio, and its latitude at 17 degrees 40 minutes north — which here too it observed [in keeping] with the ecliptic, in such a way that it scarcely deviated perceptibly from it, either downward or toward setting, by at most 4 minutes.

Furthermore, on 2 October in the evening around 6 o'clock, or a little later, the comet had advanced, so that it appeared as if at the apex of a blunt isosceles triangle formed with the stars named [above] in the hand of Ophiuchus, though it inclined nearer to the preceding one of them; and in apparent size it was larger than any of these, but the trace of its tail was rather faint. As for its thread-extension [filar extension], it made a straight line with the second-to-last star in the [path] toward the Corona, of fourth magnitude and northern, in the said hand of Ophiuchus; likewise a straight line with the lower star in the head [of Scorpio] and the eastern star in the same hand, which lines also argue for that same

isosceles-triangle form, and place it, at the said moment of this day, at longitude 27 degrees 20 minutes Scorpio, with a latitude of nearly 17 degrees north.

On 6 October in the evening, at 6¼ o'clock, with the middle of the sky moderately clear — I being still at Malmö — the comet seemed to me to form an isosceles triangle with the eastern star in the hand of Ophiuchus and the bright star in the nearby knee; for it had now deviated well off the line between these two by about 1 degree, by sight, from which the comet's longitude would be 0½ degree in Sagittarius, and its latitude [Greek: ἐν πλάτει (en platei, "in latitude")] 13¼ degrees north.

On 12 October at 6:30 in the evening, and a little before, after I had returned to Copenhagen [Hafnia], the comet seemed to be below the knee of Ophiuchus, at a distance of about 2.5 degrees, or a little more. It made a straight line with the lower star in the drawing-out of the jaws of the Serpent and the upper star in the hand of Ophiuchus; likewise another line with the lower and following star in the right shoulder of Ophiuchus, and the bright star in his left knee. From this, with the positions of the stars verified and laid down on the globe, the comet's longitude is established at 1½ degrees [in Sagittarius], with a latitude of 9¾ degrees north — a position certainly the same as the one Mr. Kepler [Johannes Kepler] had assigned to it from his own observation, which I regard not without wonder, [as confirming] the smallness, or rather the very nullity (so to speak), of the parallaxes.

On 13 October in the evening, around 6 o'clock, the comet was seen to have advanced very little, so that a straight line from it through the knee of Ophiuchus would hold the midpoint between the two stars in his right shoulder.

would hold [that midpoint]. But the comet was very dim, and scarcely any trace of its tail appeared, from which its longitude is reckoned to have been about 2 degrees in Sagittarius, with a latitude of 9¼ degrees north.

On 14 October in the evening, at 5¾ o'clock, the comet was seen very dimly, in nearly the same place as the preceding evening, so that its longitude might be reckoned at 2 1/12 degrees in Sagittarius, and its latitude at 9 degrees north.

After this the comet was no longer seen by me. As for what Mr. Kepler writes — that toward the end, namely from the observations of 9, 12, and 16 October, it was retrograde with respect to the ecliptic, and that moreover its latitude changed so remarkably from the 12th to the 16th, namely by three degrees — since therefore its motion on the intermediate days seemed to us here far slower, I made a test concerning Kepler's last observation, where he writes that on 16

October at 6.5 hours after noon he snatched a glimpse of the comet between the clouds, as it then stood very low in the vertical, and was about half a Moon's diameter more to the west than the knee of Ophiuchus itself, and that three, or at least more, of the distances of the two [stars] in the hand below that same knee were seen [between them]. This observation indeed argues for a longitude of the comet not less than $2\frac{1}{2}$ degrees in Sagittarius, nor a latitude less than 8 degrees north — all things carefully weighed and verified on the larger globe; but even Kepler himself is doubtful about this last observation of his.

Now we will go back to those earlier days which — with the Keplerian observations of this comet also serving as intermediaries — recall it, traced back, to its origin. For it is certain that before 15 September this comet had not shone out, since we remember that on the 14th of the same month we too, attending to the sky, surveyed it carefully with our eyes from that quarter as well.

Now since it is highly consistent that this comet first began [to appear] in Virgo with the simple [conjunction] of Sun and Jupiter — which indeed, according to the restored motions, is found to have fallen on 15 September at $15\frac{2}{3}$ hours [after midnight], when Jupiter was at 4 degrees 25 minutes Aries, [Jupiter] standing opposite the Sun, which was running through its simple course at 4 degrees 25 minutes [in Libra — the sign is not inked at this spot; inferred from the opposition to Jupiter in Aries]; at which time, too, there was nearly [a conjunction of] Mars with Mercury, and likewise a trine aspect of Saturn and Mars. On which account, when Mr. Kepler's observations made on the evening of 16 September are carefully reviewed and drawn back to this time along a great-circle arc, I find the comet then in

[col. 2:] $10\frac{1}{3}$ degrees Leo, with a latitude of about 34 degrees north.

As for the comet's diurnal motion — which will be considered correctly along its own circle, that is, its track — that motion was indeed at the start the most rapid of all, and then, little by little, day by day, it slackened somewhat from this, together with its magnitude, which agrees with the hypothesis of comets stated above. For on 16 September at 9 p.m., according to Kepler's observation of the comet carefully reviewed, it was at 18 degrees 30 minutes [Leo], with a latitude scarcely less than 36 degrees north — when we likewise consider the comet's track. And on 18 September at $7\frac{1}{4}$ hours in the evening, by the upper [stars], the comet's longitude was 11 degrees 57 minutes Virgo, with a latitude of 40 degrees 5 minutes. From this, the distance, or motion along its own circle, is gathered to have been 18 degrees 20 minutes, and therefore the diurnal motion $9\frac{1}{2}$ degrees in the same circle.

Furthermore, on 20 September in the morning at 5 o'clock, the comet was at longitude 28.5 degrees Virgo, with a latitude of 39.5 degrees. At which place, as also from the foregoing one just premised, the comet's motion along its own track over 34 hours is found to have been 12 degrees 50 minutes; from which the diurnal motion scarcely amounts to 9 degrees.

Third, when the comet's diurnal motion is investigated for the following day, namely 21 September — whose own longitude at 7.5 hours in the evening was found at 16 degrees 44 minutes [Libra], with a latitude of 37 degrees 14 minutes north — then this position, when compared with the preceding one with respect to the comet's own track, [shows that] it had advanced along the same over 38.5 hours by 14.5 degrees, and the diurnal motion comes out from this at very nearly 8 degrees 55 minutes — all this being worked out and carried through in a broad way, applied only to our larger globe. From this one may note that the motion of this comet was greatest and most rapid at the beginning along its own circle, and gradually grew slower; for from this time on it decreased quite perceptibly and proportionally, together with its magnitude, as I have said.

But so that not only the diurnal motion of this comet along its own circle throughout its whole duration may be had from the foregoing observations, but in addition its longitude and declination as well — from which a more correct judgment about it may be formed — behold the following table, which will contain these in order.

THE MOTION OF THE COMET OF THE YEAR 1607 [MOTVS COMETÆ ANNI MDCVII]

diurnal, at the 7th evening hour

WITH RESPECT TO [these circles]

[The table below occupies the left column of p. 29; its three left marginal genitive labels — "Sept." and "Oct." in the Dies column — are transcribed inline. The three paired heading columns are "Proprii circuli" (own circle), "Eclipticæ" (ecliptic), and "Circuli declinationis" (declination circle).]

Mo nt h	Di es		Proprii circuli (G. M.)		Eclipticæ (G. M. S.)		Circuli declinationis (G. M.)	
	Sept.	Oct.						
Se pt.	16	9 40			18 0 2 (Leo)		49 0 B.	

Mo nt h	Di es	Proprii circuli (G. M.)	Eclipticæ (G. M. S.)	Circuli declinationis (G. M.)
	17	9 30	0 0 ♍ (Virgo)	47 0
	18	9 10	11 50 ♍	44 fere
	19	9 0	23 30 ♍	39 15
	20	8 40	5 15 ♎ (Libra)	34 20
	21	7 50	16 30 ♎	28 30
	22	7 0	27 0 ♎	21 20
	23	6 0	4 0 ♏ (Scorpio)	16 40
	24	4 30	9 30 ♏	12 20
	25	3 40	13 30 ♏	9 30
	26	3 0	17 0 ♏	6 20
	27	2 30	20 0 ♏	4 10
	28	2 10	22 30 ♏	1 50
	29	2 0	24 0 ♏	0 25
	30	1 50	25 30 ♏	1 0 Mer.
Oct	1	1 40	26 40 ♏	1 50
.	2	1 30	27 30 ♏	2 40
	3	1 25	28 20 ♏	5 0
	4	1 15	29 10 ♏	6 45
	5	1 0	0 0 ♐ (Sagittarius)	8 15
	6	0 45	0 50 ♐	9 30
	7	0 40	1 20 ♐	10 30
	8	0 30	1 30 ♐	11 fere
	9	0 25	1 40 ♐	11 10
	10	0 20	1 45 ♐	11 18
	11	0 18	1 50 ♐	11 25
	12	0 15	1 55 ♐	11 32
	13	0 12	2 0 ♐	11 40
	14	0 10	2 5 ♐	11 50
	15	0 8	2 8 ♐	12 0

Mo nt h	Di es	Proprii circuli (G. M.)	Eclipticæ (G. M. S.)	Circuli declinationis (G. M.)
	16 0 5		2 10 8	12 10

[col. 2 of p. 29, alongside the table:]

And yet this diurnal motion of the comet — with respect to these circles, [taken] [Greek: ἐν πλάτει (en platei, "in breadth," i.e. loosely)] — had been [as tabulated]. Nor indeed is greater precision needed here to accomplish our purpose.

It is to be noted, moreover, that the said comet crossed the equator on 29 September a little later [than the tabulated hour], at 236 degrees [printed thus; the value is likely a compositor error — editor's conjecture: 23½ degrees in Scorpio, not on the page]. Indeed, that it first clearly deviated from the path of a great circle was detected on 1 October, where it had nearly reached the stars in the left hand of Ophiuchus. Finally, as for the beginning of the comet estimated by us, at 10½ degrees Leo, the declination of the point corresponding to this was 49 degrees 50 minutes north. Now, lastly in this chapter, we shall treat briefly of the parallaxes of this comet; from which it will be established that it was not sublunar, but existed in the heaven itself.

ON THE PARALLAXES of this comet [DE PARALLAXIBVS hujus cometæ]

Although in the case of this comet it had not fallen to us to have such observations arranged in order, from which we might subtly tease out its parallaxes — in whatever measure it had any — precisely according to what was set forth in the doctrine of the sphere: nevertheless, among the rest, even that single observation of the comet's position, made by us on the evening of 18 September at 7.25 hours, jointly as it were with Mr. Johannes Kepler — who was then at Prague in Bohemia — will suffice for what alone we here intend.

Our observation at that time, which was recited above, was this: that the comet at that time was seen by us scarcely half a Moon, or 34 minutes, below and a little to the east — in that revolution — of a small unformed star behind the Bear. But Kepler writes this:

"On 28 September (understand the new style, and the evening time) the comet was seen standing below a small unformed star, companion of the great unformed one, between the tails of the Bear and the Lion; and it was distant by a

Moon's diameter, on the line which runs along the last [star] of the Bear's tail toward the hand of Arcturus." Thus he. From which it is to be understood that there was scarcely a perceptible difference between the position of the comet at this time as observed by each of us. This also appears from the comparison above. For when, from our other observations made on the same evening and hour, we sought out its longitude at 11 degrees 57 minutes Virgo, with a latitude of 40 degrees 5 minutes — Kepler here afterwards subjoins and gathers, from his own observation, the longitude of the comet to have been [continued on next page:]

[col. 1:] ...to have been 12 degrees 0 minutes Virgo, and the latitude 40 degrees 4 minutes north — the difference of these from ours extending scarcely beyond 3 minutes in the vertical circle, at the very most. Moreover, since Kepler sets the comet's longitude smaller than the longitude of that unformed star to which it was joined, by 16 minutes — and the comet likewise, in the straight line which he mentions, had at the same time advanced — there need have been no very perceptible difference between the time at which this observation was made at Prague and the time when the observation was being made at our place. For this the comet's diurnal motion argues.

These things being settled: since Prague is distant scarcely 6 minutes of time from the Copenhagen [Hafnia] meridian toward the east — but the elevation of the pole there is 50 degrees 6 minutes, here 55 degrees 43 minutes, the difference therefore being 5 degrees 37 minutes — there was, at that time, a difference of about 5.5 degrees [recte: per text, 5 gr. 37 min.] in the altitude-circle; and the altitude of the comet then at Prague was about 20 degrees, but here 25.5 degrees, all things considered in a broad way. For although, without the assignment of azimuths — which is especially suited to the precise teasing-out of parallaxes — these things are conceived by me for the present [only thus], they nevertheless have safeguard enough to convince Aristotle [Peripatus], if this comet had existed below the Moon in the highest border of the air (as the Peripatetics maintain): namely when the premises are drawn out to a demonstration and to numbers in this way.

From a center, let a semicircle of the orb of the earth E F be described, whose semidiameter E A is known to mathematicians as 860 German miles [milliarium German.].

Diagram. A geometric construction for the comet's parallax. A semicircle stands on a horizontal base line, with its center A at the foot. The base line carries, from left to right, the points G, E, A (center), F, H. The radius E A is the earth's

semidiameter (860 German miles); the larger arc, of radius A G (= A H), bounds the supreme region of the air (G H), at the additional altitude E G = 13 German miles, so that A G = 873 German miles. Two observing stations sit on the earth's surface: B (Prague) and C (Copenhagen), C lying to the left (west) of B. The apparent horizontal lines drawn through each station are I K (through Prague) and L M (through Copenhagen). The comet sits at D, near the top under the common vertical, marked with a small radiant star-symbol; lines run from center A up through C, B to D, and the slanting horizon-lines L–M and I–K cross near C and B.

- A (foot/center of semicircle): center of the earth
- E (on base, left of A): point where radius E A meets the earth's surface
- F (on base, right of A): the opposite end of the earth's semicircle E F
- G (on base, far left): inner end of the air-region's outer arc; E G = 13 mi.
- H (on base, far right): outer end of the air-region's outer arc (G H = supreme air-region)
- B (on the surface arc, right of center): station of Prague [urbs Pragensis in Bohemia]
- C (on the surface arc, left of B): station of Copenhagen [Hafnia in Dania]
- D (top, with star-symbol): the comet, under the common vertical
- I (near L, upper left): point on the Prague horizontal line I K
- K (right side): the other end of the Prague horizontal line I K
- L (upper left): point on the Copenhagen horizontal line L M
- M (right side): the other end of the Copenhagen horizontal line L M

The semidiameter E A is known to mathematicians as 860 German miles.

[col. 1 continues:] Furthermore, about the same center A, at the distance A G, let a circle G H be described, determining the supreme region of the air, while E A is augmented by the altitude E G of 13 German miles — which [altitude] indeed the opticians, from observation of the twilights, rightly trace as the extreme sublimation of the earthly vapors (out of which Aristotle wished comets to be composed), as we showed this in our doctrine of the sphere, Book 2, chapter 9. The radius A G will therefore be 873 German miles. Then, with the places of the observations made laid out on the surface of the terrestrial globe, let B be the city of Prague in Bohemia, C [the city] of Copenhagen [Hafnia] in Denmark; and likewise, with the apparent horizontal lines drawn — namely I K through Prague, L M through Copenhagen — let the comet finally be placed at [continued in col. 2:]

[col. 2:] D, under the common vertical (from which indeed it was imperceptibly distant). Therefore the arc I D will be 20 degrees, but L D 25.5 degrees, by the comet's altitudes at both places premised above. With these things thus established, let straight lines be drawn from center A to B, C, and D; let D likewise be connected with C and with B; and there arise the plane triangles A D B and A D C. Of these two triangles, the parallax angle at D must be elicited separately from the given premises, so that the difference between [the two] may be had for arguing what we intend.

First, then, in triangle A D B, since the [Greek: δεδομένα (dedomena, "the givens")] are

- A B 860 German miles
- A D 873 German miles
- angle D B A 110 degrees 0 minutes — that is, the right angle I B A augmented by the angle D B I, which is the altitude of the Prague observation of the comet, 20 degrees —

therefore the angle A D B will not be hidden: 67 degrees 46 minutes.

Again in triangle A D C, with only the angle D C A altered, augmented by the 5.5 degrees by which the altitude of the comet here was higher than at Prague, the angle D C A becomes 115.5 degrees. From these data, then, comes the angle A D C, 62 degrees 46 minutes; and therefore the difference at angle B D C is 5 degrees-plus. And certainly by this same difference, nearly — in the altitude-circle, beyond a diameter of one Moon — the comet here would have been torn away from that tiny unformed star, and would have stood below it for Kepler, if the comet had been some meteor. And conversely, for us, by the same difference (with the space of one Moon excepted), the comet would become more exalted in the vertical circle [relative to] that little star. But the difference which the observations made on both sides demanded can be rated at scarcely beyond 3 minutes; so that I think it easily established from this that Aristotle's hypothesis about comets is by no means legitimate — however keenly it has been defended, down to this very day, in the Schools and Academies by certain Peripatetics (that is, by the common run of philosophizers): so much so that not even these has our Tycho [Tycho Brahe], through so many demonstrations drawn from the new stars and comets which were observed in his time, been able to bring back into the way of truth.

Moreover, the same thing we can gather — indeed plainly establish — from the proportional change of the proper motion and latitude of this comet, and likewise

from its steady progress under a great circle almost to the last days of its disappearance: that this comet was celestial. Although, in proportion as it had climbed aloft through the aether, the observations of it, taken less precisely and less timely, scarcely permit us to reason [about it]. Therefore now, with the thread broken off, we shall hasten on to the phenomena of the comet of the year 1618, likewise to be explained from our own observations, so that at last, in the final place, we may set forth our judgments about each, such as they are. For of these comets...

the things signified by comets are, beyond doubt, bound together by some connection — a fact which, among other indications, the oblique passages of these bodies across the upper air argue and prove in remarkable ways.

CAP. X.

On the phenomena, parallaxes, distance from the Earth, and magnitude of the comet of the year 1618, drawn together from my own observations; with which I add an account of this comet as observed also in Persia.

We now turn to that astonishing and fateful comet which, in the year 1618, in November and December, shone upon nearly the whole world. For of this one comet I do not recall that the phenomena were previously observed by so many people at once anywhere else. And certainly, although they were not observed by all of those people with equal or sufficiently exact diligence — by most, indeed, only carelessly — nevertheless their effort deserves to be praised on the whole, since they looked with unblinking eyes upon so evident and conspicuous a messenger of divine mercy and at the same time of divine vengeance, unlike the common crowd that despises such signs. Rather, each one, hunting out the path and traces of this comet, carefully noted for himself its rising, its course, and the way it took through the upper air, and inserted these things into their annals or writings for posterity. It was a matter of all the greater concern to me to do this too, because our most Christian magistrate had diligently seen to it that this task was laid upon me — not, as I judge, out of any superstition, from which the mind of our present most merciful king is removed by countless leagues, but rather with this end in view: that we should by no means despise so distinguished a legate of the most powerful King of all kings — a sign, as it were, of divine wrath and at the same time a kind and fatherly invitation to a timely repentance — nor unhappily consign it to oblivion.

Now as to the time when this comet first began to rise: whether it was at the conjunction with Saturn that preceded our observations by three days, when

Saturn was in sextile with Mars; or even at the opposition of the Sun and Saturn, which had occurred about a day and a half earlier still; or finally — what is still more credible — whether it had had its first rising before the start of the 12th of November, namely at the quadrature of Jupiter and the Sun, which the trine aspect of Mars and Venus soon took up afterward: this is settled chiefly by conjecture. For not only did the rumor of this comet's appearance grow strong some days before the 6th of November — the day on which we first observed it — but we have also received word that the same comet was observed in Persia around the 12th of November, as will be set down later in its proper place. For even if it had begun within the 12th of November, or a little earlier, it would not have been possible for us to observe it before the 20th of that same month, on account of the cloudy, rainy, and windy condition of the air over our hemisphere during that interval. And certainly on the 12th of November, around the quadrature itself of Jupiter and the Sun, and for several days following (about which time four ships, together with a frigate, set out from this Danish harbor for the East Indies), the weather did not let up; yet they did not slip out of our Hellespont before the 29th of November, on account of the adverse wind and the more frequent storms. For so violent a tempest occurred in the meantime that other ships here and there were dashed against the sandbanks and shores of our Baltic where they had been scattered.

Therefore, on the 21st of November, in the morning between the 5th and 6th hours, I first caught sight of this comet — for the sky during the 10 preceding days had been cloudy for us, as we said before, and thick with rain-clouds and showers. In its apparent magnitude the comet was then somewhat larger than Spica Virginis [Spica ♍], and it had a tail reaching, by sight, to the height of a rather tall man's stature; this tail it cast out quite directly into the part away from the Sun, which was then in the 9th degree of Sagittarius [♐]. The color of its head inclined toward red, so that it was roughly midway between the color of Mars (which was seen by us, appearing at the same time) and that of Spica.

That same morning, when the comet was at an altitude of 18° , to which corresponds the time $5\frac{1}{4}$ hours, the distance of the comet from Spica through our sextant was $24^{\circ} 56'$ and so on; and shortly after, from the bright star of the Corona [Lucida Coronae] $32^{\circ} 41\frac{1}{2}'$. And so that you may be assured of the instrument's reliability in exhibiting the distances of the stars, we observed a little later these distances of the fixed stars, repeated and determined several times: between Spica ♍ and Arcturus — with which the comet was then almost in the same vertical — $33^{\circ} 4\frac{3}{4}'$, a distance very close to the truth.

Now, with the positions of the fixed stars supposed — Spica ♀ in longitude $18^{\circ} 31'$ ♀ with latitude $1^{\circ} 59'$ south; the bright star of the Corona in longitude $6^{\circ} 53'$ ♀ with latitude $44^{\circ} 23\frac{1}{2}'$ north — there results from this, by the triangular computation in the manner we set forth in the doctrine of spherics, the longitude of the comet at $9^{\circ} 26\frac{2}{3}'$ ♀, with latitude $11^{\circ} 46\frac{1}{2}'$ north; and this, as we said, for the 21st of November, at $5\frac{1}{4}$ hours in the morning.

On the 22nd of November, in the morning, we did snatch a view of the comet several times among the clouds; but we did not properly observe it, because the said intervening clouds took this opportunity away from us.

On the 23rd of November, in the morning, from the 5th hour up to the 7th, we carefully observed the comet as to its distances and altitudes. It appeared to be in the same magnitude as before; likewise as to its color, except insofar as the Moon's splendor had somewhat affected it. We measured the length of the tail too at roughly 20° or a little more, although we could scarcely follow its outermost flickering with our eyes.

When the comet was at an altitude of 20° through our geometric square, to which the time corresponds very nearly to $6\frac{1}{2}$ hours, these were its distances from the fixed stars, taken several times through our sextant and carefully determined:

Distance	Value
Distance of the comet from Spica	$25^{\circ} 52'$
Distance of the comet from the bright star of the Corona	$27^{\circ} 28'$

From this, by the construction of triangles, comes the comet's longitude $6^{\circ} 24'$ ♀, latitude $16^{\circ} 56'$ north.

On the 24th of November, in the morning between the $4\frac{1}{2}$ and 7 hours, the sky being quite serene except that the Moon was shining, we again observed the same comet; its head was fairly round, and its tail this morning was extended at length, holding its course beneath the middle star in the left knee of Boötes, and the outermost flickering of the tail was, on the following sight, drawn out still farther. Moreover, the magnitude of the comet's head appeared still larger than Spica ♀ itself, although under the high dawn it was not visible for so long.

As for the observations of this day: in the distances from Spica and the Corona, repeated rather often, we observed, at the comet's altitude of 21° — to which $6\frac{1}{2}$ hours correspond — its distance from Spica ♀ $26^{\circ} 52'$, likewise from the bright star of the Corona $24^{\circ} 54'$. From these, the calculation of triangles duly applied, the comet's longitude comes out $4^{\circ} 53'$ ♀, with latitude $19^{\circ} 33'$ north.

Finally, in order to test in some way the parallax of this comet on this day, we afterward continued to take its distance from Arcturus in the same vertical, which was:

Altitude of comet	Distance from Arcturus
$24\frac{5}{6}^{\circ}$	$18^{\circ} 13\frac{3}{4}'$
$26\frac{1}{3}^{\circ}$	$18^{\circ} 12'$
(somewhat later)	$18^{\circ} 10\frac{2}{3}'$

Since we see here that the comet has changed its distances under one vertical with Arcturus very little or not at all beyond what must be attributed in the meantime to the diurnal motion, we easily conclude from this that the parallax of this comet, even at this time, was no greater than before — so that it was carried up far above the Moon, into the upper air. But concerning other observations, from which the parallaxes in this comet are deduced more skillfully, we shall treat toward the end of this chapter.

Furthermore, from the 24th up to the 28th of this month the sky was continuously cloudy by night and in the morning, so much so that the comet appeared less to us.

But on the 28th of November, around the $5\frac{1}{4}$ hour in the morning, I saw the comet in such a position — as far as could be observed in a moment, on account of the intervening clouds — that it made such a triangle with Arcturus and the bright star in the body of Boötes, in which Arcturus occupied almost a right angle. Besides this, the distance of the comet from the bright star in the knee of Boötes was about 2° , by sight, the bright star of the knee lying below and toward the setting side. The extension of the tail inclined a little away from Arcturus toward the east, and the distance between the comet and Arcturus was by then much greater. Now the body together with the tail was fairly large and conspicuous. But after the 7th hour, when serenity had been restored to the sky in that place and the dawn had risen, Arcturus alone was visible to us, the comet having become wholly invisible. From these observations, such as they were, applied to the globe, there comes out, around the 6th hour in the morning of the 28th of November, the comet's longitude $27^{\circ} 45' \underline{\text{N}}$, with latitude 30° north very nearly.

On the 29th of November, around the $6\frac{1}{2}$ hour in the morning, we caught sight of the comet among the clouds, while a strong east wind was blowing. It had been advanced along a great circle from the preceding day, so that Arcturus stood off by about a half-Moon's breadth from the straight line drawn from the comet to

the northernmost and brightest star in the left knee of Boötes, toward the north. Indeed, the comet appeared to have formed a right angle with Arcturus and the star in the right arm of Boötes, the comet itself occupying an almost right angle. But these were momentary observations among the clouds, from which it is established, by [latitude], that the comet's longitude is $26\frac{2}{3}^{\circ}$ $\triangle$, with latitude about 33° .

On the 30th of November, in the morning, when the bright star of Hydra was at an altitude of 20° , to which the time corresponds very nearly to $5\frac{1}{2}$ hours, we turned our attention to the position of the comet, which was arranged in a straight line with the bright star in the left knee of Boötes, in such a way indeed that that line scarcely touched the upper part of Arcturus. The distance, observed once among the clouds, between the comet and the tail of Leo [ϱ] was about 40° , more or less. From these too is given, by [latitude], the comet's longitude 24° [$\triangle$], with latitude 35° north.

Note, however, that on this same day, a little before the 1st hour of the preceding night, around the rising of the comet on our horizon, this comet had cut, by its passage, the path of the other — that is, of the earlier comet — of the year 1607, beneath the body of Boötes, a little below the tropic circle, on which, according to these observations, it had come to rest very nearly. And indeed I marvel that at the same time, when at its rising this morning the comet was almost together with the Moon for us, its tail had scattered unusual rays — beyond the zenith, in fact, namely to an elongation of 104° , that is, as far as the stars in the left hand of Auriga, directly enough from the Sun, while it [reached] midway between those same stars and the unformed star above Orion. This was without doubt done because of the disposition of the subjacent air in our hemisphere, even though the sky was then moderately serene; and because the Moon was nearly together with the comet at its rising, as I said, therefore perhaps that [proximity] had somewhat promoted this unusual stretching of the tail — to say nothing of some hidden sympathy, or ἀντιπάθεια [Greek: antipatheia, "antipathy"], of this comet with the earlier one, while it was cutting across the latter's track.

But afterward, when the comet had become higher, the tail too at once became more contracted, turning itself again and again toward the north with respect to the Sun, and grazing, as it were, the bright unformed star between the tail of the Great Bear [Ursa Major] and the tail of Leo [ϱ].

On the 4th of December, in the morning — some serenity being for the first time restored among these days — the comet appeared to stand above two contiguous stars in the middle of the body of Boötes, in an almost straight line with them;

though to a slight degree, and indeed quite perceptibly, the comet deflected from that line toward the east. It was, moreover, a little farther from the upper of these stars than the said stars are from each other. The tail of the comet was extended along the southern side of the two stars in the square of the Great Bear, in such a way that it was more southerly than that side by about $\frac{1}{2}^\circ$. With this observation made around the 5½ hour, we found the comet in longitude $16^\circ 20' \text{ } \underline{\text{A}}$ and latitude $43^\circ 15'$ north.

On the 5th of December, in the morning at the 2nd hour of the night, I saw the comet as it were in a straight line with Arcturus and the left shoulder of Boötes, but quite perceptibly nearer the shoulder. Likewise, with the star that is in the left forearm of Boötes and the upper of the two contiguous stars in the middle of the body, from which, however, after the fifth hour, the comet deflected quite perceptibly toward the zenith, and this by reason of its own proper motion. The tail of the comet was directed straight toward the southern star in the square of the Great Bear, and reached precisely to it, by sight. The body of the comet was somewhat dark, and the rays darting from the tail were sparse; the comet appeared smaller and redder then than the star in the left shoulder of Boötes. Through the optical glass, however, the cometic body was very livid.

Afterward, some hours having passed, when the altitude of Spica M in our square was $17\frac{1}{4}^\circ$, to which the time corresponds very nearly to the 5th hour, the comet was distant from the bright middle star in the southern wing of [Virgo] $43^\circ 43'$, and likewise around the same time it was distant from Spica $48^\circ 2'$. From these distances, as observations made this morning, while the calculation is deduced from the ratio of triangles with the positions of the fixed stars as intermediaries, the comet's longitude comes out $14^\circ 40' \text{ } \underline{\text{A}}$, with latitude $45^\circ 51\frac{1}{3}'$ north.

For this calculation here proceeds from the demonstration according to the adjoined diagram, in this way:

	Longitude	Latitude
Spica M	$18^\circ 31' \text{ } \underline{\text{A}}$	$1^\circ 59'$ south
Middle star of the wing M	$4^\circ 50' \text{ } \underline{\text{A}}$	$2^\circ 50'$ north

Difference of longitude: $13^\circ 41'$.

Diagram. A spherical-triangle construction for deriving the comet's longitude and latitude on 5 December from two observed distances. From a vertex A at the top, several lines fan downward to a baseline running through points E, F, C, G; an interior point D (marked with a star) lies on a line descending from A, with B

at the left end of the baseline and C (also star-marked) near the right. The construction realizes the chain of triangles BAC, ACB, DCB, ACD, and BAD worked out in the text.

- A (top vertex): the pole of the ecliptic [polus eclipticae]
- B (left, on baseline): the middle star of the wing of Virgo [media alae ♍]
- C (right, on baseline, star-marked): Spica ♋ [spica ♋]
- D (interior, star-marked): the comet [cometa]
- E, F (baseline, left of C): points marking the arc EF of the ecliptic (the longitude difference between the wing-star and the comet)
- G (baseline, right of C): terminal point of the baseline

In the adjoined figure, let A be the pole of the ecliptic; B the middle star of the wing of Virgo, C Spica of Virgo, D the comet.

Therefore: I. In triangle BAC the given quantities [δεδόμενα, dedomena, "things given"] are: BC; BA $87^{\circ} 10'$, the complement of the latitude of the middle star of the wing of Virgo; CA $91^{\circ} 59'$, the excess of Spica's latitude beyond 90° ; BAC $13^{\circ} 41'$, the difference of longitudes. Therefore is given BC $14^{\circ} 29\frac{1}{4}'$.

II. In triangle ACB, for the angle ACB, the given quantities are: AC $91^{\circ} 59'$; BC $14^{\circ} 29\frac{1}{4}'$; BA $87^{\circ} 10'$. Therefore from these is given ACB $70^{\circ} 37\frac{1}{2}'$.

III. In triangle DCB, for the angle DCB, there are given: DC $48^{\circ} 2'$, the distance of the comet from Spica; BC $14^{\circ} 29\frac{1}{4}'$, the distance of the middle star of the wing from Spica; BD $43^{\circ} 43'$, the distance of the comet from the middle star of the wing. Therefore is given DCB $66^{\circ} 6\frac{1}{2}'$. When this is taken away from the previously found ACB, there remains the angle ACD $4^{\circ} 31'$.

IV. In triangle ACD, for the side DA, there are given: AC $91^{\circ} 59'$; CD $48^{\circ} 2'$; ACD $4^{\circ} 31'$. Therefore from these is given AD $44^{\circ} 8' 24''$, the complement of the sought latitude of the comet; from which the comet's own latitude is $45^{\circ} 51\frac{1}{3}'$.

V. In triangle BAD, finally, for the angle BAD, the given quantities are: B $87^{\circ} 10'$; DA $44^{\circ} 8\frac{2}{5}'$; BC $43^{\circ} 43'$. Therefore from these is given the sought angle BAD $9^{\circ} 50'$, which is the arc EF of the ecliptic — namely the difference of longitude between the wing of Virgo ♍ and the comet. When this is added here to the longitude of the said wing, fixed earlier at $4^{\circ} 50' \underline{\alpha}$, there results the comet's longitude $14^{\circ} 40' \underline{\alpha}$.

These observations, therefore, are to be preferred to those which were obtained on the immediately preceding days only by stretched-thread sightings; and from this the comet's diurnal motion is to be corrected, as need shall require.

On the 14th of December, around the 10th hour in the evening — since for the nine preceding days there had been no serenity in our horizon, but the sky had meanwhile remained almost perpetually cloudy and rainy; then, some serenity having been restored on this night, while the clouds were being driven apart by a rather strong wind — we caught the comet, by sight and by stretched-thread extension, in a straight line with the rear and upper star in the square of the Great Bear and the middle star in its tail; and then in a straight line with the lower and forward star in the square of the Little Bear [Ursa Minor] and the last star of the tail of the Great Bear — yet here in such a way that the comet, as it were, dropped below that straight line by 4'. From this the place of the comet, on the globe, was deduced as longitude about $21^{\circ} \text{ } \mathfrak{M}$, latitude $58^{\circ} 20'$ north.

On this night the comet, with the Moon shining opposite, was very much thinned out, and it stretched its tail a little above the middle star in the tail of the Great Bear, although the trace of that tail was sparse and scarcely perceptible.

On the 16th of December, in the evening around the 7th hour, we carefully observed the comet below the pole, although on account of the splendor of the Moon, shining in the meantime, both its body and its tail presented themselves rather dimly for viewing.

When, then, the middle star of the tail of the Great Bear (which had passed the nadir [Naddir] a little) was at an altitude of 23° through the geometric square, to which the time corresponds very nearly to the $7\frac{1}{3}$ hour, the comet was, by sight, in the vertical beneath the pole itself, as nearly as possible. At the same instant the distance of the comet from Dubhe was $21^{\circ} 28'$, likewise from the antepenultimate star of the bend of the Serpent [Serpens] $19^{\circ} 45\frac{1}{2}'$. The positions of the fixed stars at that time are these:

	Longitude	Latitude
Dubhe in the Great Bear	$9^{\circ} 49' \text{ } \mathfrak{Q}$	$49^{\circ} 40'$ north
Antepenultimate of the bend of the Serpent	$8^{\circ} 10' \text{ } \mathfrak{A}$	$78^{\circ} 23'$ north

From these, by a similar triangular computation together with the foregoing, is given the comet's longitude $12^{\circ} 58' \text{ } \mathfrak{M}$, latitude $60^{\circ} 18\frac{1}{2}'$ north.

On the 17th of December, in the morning, when the bright star of Lyra was at an altitude of $31\frac{1}{2}^{\circ}$ — to which the time corresponds very nearly to the 6 $\frac{1}{12}$ hour — the distance of the comet from Dubhe was again $20^{\circ} 42'$, likewise from the same bend of the Serpent $19^{\circ} 39'$, by observations of ours repeated and determined as well as could possibly be done. From this comes the comet's longitude $11^{\circ} 9\frac{3}{4}' \text{ } \mathfrak{M}$, with latitude $60^{\circ} 39\frac{2}{3}'$ north. The comet was approaching our zenith; its tail was dark, inclining toward Dubhe.

On the 17th of December, in the evening at the 5 $\frac{1}{2}$ hour, the comet was in a straight line with the forward and lower star in the square of the Little Bear and the bright star in the tail of the Serpent, falling short toward the north by about 6'. And it was still manifestly away from the straight line between the bright star in the tail of the Serpent and the middle star in the tail of the Great Bear by about 14'. From this, by [latitude], and on our globe, the comet was found to be at longitude $8^{\circ} 45' \text{ } \mathfrak{M}$, latitude $60^{\circ} 40'$ north. Venus this evening, at its setting, and in $22^{\circ} \approx$ Aquarius, bore an unusual magnitude and brightness before it.

Finally, on the 18th of December — which was the last of those days to which our observations on this comet extended — around the 6 $\frac{1}{4}$ hour in the morning (for Spica was then almost at the meridian), we found the comet to be nearly vertical, but not yet to have reached the straight line that lies between the middle star of the tail of the Great Bear and the bright star in the tail of the Serpent, falling short by at most 4'. Now the comet was nearer the middle star of the tail of the Great Bear by about $\frac{1}{3}^{\circ}$, by sight, or perhaps a little more. There were some traces of a tail, but sparse and short; the body too appeared somewhat dark, tending toward a certain small unformed star between the bright star in the tail of the Serpent and the tail of the Great Bear. It was distant from that star in the Serpent by about 1° . From these observations of this day comes the comet's longitude $7\frac{1}{2}^{\circ} \text{ } \mathfrak{M}$, with latitude 61° north.

There follows a synopsis of the motions of this comet drawn from all the foregoing observations, set down — as in the case of the earlier comet above — in a broad manner and as far as could be done and as the matter required.

EPHEMERIS OF THE MOTIONS OF THE COMET OF THE YEAR 1618

Drawn from the foregoing observations, reduced to 9 a.m.

The table below gives, for each date (in November and December), the comet's diurnal motion along its proper circle, its motion in longitude (ecliptic longitude,

with the zodiac sign), and its motion in declination. Each longitude is given in degrees and minutes with its zodiac sign (♏ Scorpio, ♎ Libra, ♍ Virgo); the diurnal motion and the declination are given in degrees and minutes. In the declination column the first few entries carry a direction letter: **M** = *meridionalis* (south), **B** = *borealis* (north), marking the comet's passage from southern to northern declination.

Date	Diurnal motion on proper circle (G. M.)	Motion in longitude (G. M. S.)	Motion in declination (G. M.)
Nov.	3 20	9 25 ♏	3 20 M
21			
22	3 17	7 55 ♏	0 35 M
23	3 14	6 25 ♏	2 20 B
24	3 11	4 55 ♏	5 30
25	3 9	3 7 ♏	9 0
26	3 6	1 20 ♏	12 0
27	3 3	29 30 ♎	15 20
28	3 1	27 45 ♎	18 10
29	3 0	26 10 ♎	20 40
30	2 57	24 10 ♎	23 15
Dec.	2 47	22 0 ♎	26 10
1			
2	2 38	20 0 ♎	29 10
3	2 24	18 0 ♎	32 0
4	2 19	16 20 ♎	34 20
5	2 16	14 40 ♎	36 30
6	2 14	12 30 ♎	38 40
7	2 12	10 20 ♎	40 50
8	2 10	7 40 ♎	43 0
9	2 8	5 0 ♎	45 0
10	2 6	2 20 ♎	47 0
11	2 3	29 30 ♍	49 0
12	1 59	26 40 ♍	51 0
13	1 55	23 40 ♍	53 0

Date	Diurnal motion on proper circle (G. M.)	Motion in longitude (G. M. S.)	Motion in declination (G. M.)
14	1 50	20 50 $\mathfrak{M}$	55 0
15	1 43	18 0 $\mathfrak{M}$	56 40
16	1 33	14 40 $\mathfrak{M}$	58 10
17	1 18	11 10 $\mathfrak{M}$	59 35
18	1 8	7 45 $\mathfrak{M}$	61 0

We found, moreover, that the angle the comet's track made with the ecliptic was $63 \frac{7}{12}$ degrees. This track — the comet's proper circle — has its northern pole at $15 \frac{1}{4}$ degrees of ♈ [Aquarius] on the surface of the sphere, with a northern latitude of 26 degrees 54 minutes. It therefore cuts the ecliptic at $15 \frac{1}{4}$ degrees of ♏ [Scorpio][?] [the sign-symbol at the intersection is not fully legible]. From this proper circle the comet deviated hardly at all to any sensible degree throughout the whole span of time during which we observed it — except perhaps by reason of the small parallax it had, which is treated next.

On the parallaxes of this comet

It now remains for us to work out properly, from our observations, the parallaxes this comet had, so that from them we may come to know the comet's distance from the Earth and at the same time its size compared with the size of the Earth. This matter not only contributes most directly to establishing the true hypothesis of comets correctly; it also greatly commends the wonderful work of God.

Now, since these observations had to be applied with the very finest precision practised in all of astronomy, they had to be acquired not with childish instruments or with loose conjectures, but rather with instruments of the kind so well built and finished that they keep every single first minute — indeed even the parts of these — as far as possible free of error. Nor will there be any lack of skill in those who must be employed to carry through observations of this sort at the right moment: all of which, by divine grace, are still available in Denmark, and by the king's generosity receive a fresh daily increase. As for which of our observations were selected for this task at that time, we shall rely chiefly on those of 16 December in the evening at hour $7 \frac{1}{2}$, and the next morning following at hour $6 \frac{1}{12}$ — set out above — when in the evening the comet was very nearly in the vertical itself, beneath the pole, and in the morning not far from the zenith above it. For this very reason these observations were made with special care, since our instruments were set up and arranged outside the observatory (which

faces chiefly south), under the open sky. So much so that, had the moon's brightness in the evening not hindered us at all, we know and we promise that these observations of ours were entangled in no perceptible errors — which nevertheless we also

verified so many times over that any misgiving about the moon's brightness could easily be dispelled. But in truth, since the comet's declination — to be drawn precisely from the observations at each of the two times — proved here most apt for the scrutiny of parallaxes, so that afterwards, from the difference compared with the one that meanwhile arises from the diurnal motion, it may correctly reveal what we seek: therefore, from the longitudes and latitudes of the comet already given, first for 16 December in the evening at hour $7\frac{1}{2}$ — namely from a longitude of 12 degrees 58 minutes of ♍ [Virgo] and a northern latitude of 60 degrees $18\frac{1}{2}$ minutes — there results, according to what is taught in the spherics, a northern declination of the comet of 58 degrees $41\frac{1}{2}$ minutes. [Margin: Lib. 2 *Sphæer.* cap. 2.]

Then for 17 December in the morning at hour $6\frac{1}{12}$, from the comet's longitude similarly observed as 11 degrees $9\frac{1}{2}$ minutes of ♍ [Virgo], with a latitude of 60 degrees $39\frac{1}{2}$ [minutes], there arises a declination of the comet of 59 degrees $35\frac{1}{2}$ minutes. So in the span of 10 hours 49 minutes the declination of the comet was visibly advanced toward the north by $53\frac{1}{2}$ minutes. But because the motion of declination was at least 1 degree 25 minutes per day — or at most 1 degree 26 minutes, as is clear from what precedes and what follows — there is given, for the interval of $10\frac{1}{2}$ hours, a proportional daily declination of $38\frac{1}{4}$ minutes. The difference, then, between the true and the apparent declination is $15\frac{1}{2}$ minutes, which is first assigned to the comet's parallax in the first observation, beneath the pole — apart from that tiny amount which would occur in the second observation, since at that time the comet was not far from our zenith.

Nor was the change of declination sensibly varied in both cases by a change of altitude in the same azimuth, where parallaxes are usually properly detected. For our observations, made below and above the pole, were as nearly as possible [in the same azimuth], as will be made clear in the following diagram, where we shall also set the comet's parallaxes before the eyes, and by means of them reason further about the distance from the Earth — and that in the following way.

Diagram. A quadrant-type geometric construction set on a horizontal base line **R ... A ... L**, with the observer/Earth-center at **A**. From **A** a set of concentric arcs is described, and radial lines are drawn to the comet's positions and to the celestial pole. The construction shows the comet at its lowest position (beneath the pole,

M) and at its highest (**E/F**), with the parallax triangles built on the Earth's semidiameter.

- **R** (left end of base line): terminus of the horizon/base line
- **A** (center, on base line): Earth's center / observer's place
- **L** (right end of base line): terminus of the base line
- **B**: point on the Earth's surface (observer's place on the globe), inner semicircle
- **C**: outermost arc — circle of the fixed stars / starry heaven
- **D**: intermediate arc
- **E**: comet's highest position (supreme place)
- **F**: near **E**, on the comet's circle (upper position)
- **G**: point on the comet's circle near **F/Q**
- **Q**: point where arc **G Q** meets the line toward the pole **P**
- **H**: point on the line toward the comet (parallax point)
- **K**: point below **H** on the radial line (parallax of the lowest altitude)
- **M**: comet's lowest position (beneath the pole)
- **N**: point taken as the pole of the cometic circle
- **P**: pole of the world (celestial pole) [**P** is referenced in the text as "P mundi polo"; its mark on the figure is faint]

Let there first be described, from center **A**, the semicircle of the Earth **B**; then the comet's [circle] **D M**; and finally that of the starry heaven, or of the fixed stars, **C L**. Next let the straight line **A C** be drawn to the zenith, and another to the pole of the world, **A P**, which at the same time passes through the point **N**, which is taken as the pole in the cometic circle. Then let the arc **E M** be drawn, in which — following the daily rotation of the heaven — the comet, otherwise motionless, would have been carried round in one revolution with respect to the center of the world (that is, the Earth **A**). But because the comet also possessed a proper motion, nearly toward the north — as set out above from the observations — therefore, by the proportion of the daily motion of declination, while the comet held that same proper motion which it had from **M**, the lowest place beneath the pole, and near **F**, the highest, a little before, we shall find $38\frac{1}{4}$ minutes. But from the observation at

each place the change of declination [is] $53\frac{1}{2}$ minutes; the difference therefore (as above) is $15\frac{1}{2}$ minutes. With these things so posited, let there be drawn first, from center **A** (the Earth's center) through the comet's places set below and above the pole **M** and **E**, the straight lines **A H** and **A F**; then from **B** (the surface of the

Earth's globe) and the observer's place, through the same cometary places, the lines **B K** and **B G**. With the center then set at **P**, the pole of the world, when the arc **G Q** is drawn, **Q H** will be equal to **F G**, that is, to the angle of the comet's parallax existing above the pole **M**, namely **B E A**. And because **H K** shows the comet's parallax at its lowest altitude — since that arc, in the heaven of the fixed stars, is also equal to the comet's **A M B** — therefore, adding **H Q** to **H K**, the aggregate of the comet's parallaxes in both observations is thus assembled, which above we found to be 15½ minutes. [Margin: See Problem 1, cap. 10, lib. 2 *Sphær.*, and its cases.]

Now how the visible altitudes of the comet on both sides — that is, both at **M** and at **E** — are to be shown, is as follows. [Margin: case 1]

First, the lowest declination of the comet, observed at **M**, was 58 degrees 42 minutes, nearly northern. Therefore its complement, 31 degrees 18 minutes, is the arc **P K**; and when that is subtracted from the elevation of the pole of this place, which is 55 degrees 43 minutes, there remains an arc of 24 degrees 25 minutes — namely the sought visible altitude of the comet at **M**.

Furthermore, since the comet's latitude at its highest place **E** is given as 60 degrees 40 minutes, with a declination of 59 degrees 35 minutes very nearly, there is found from this the comet's right ascension of 203 degrees 40 minutes, and afterwards its altitude itself at hour 6 1/12 in the morning of 80 degrees 6 minutes — which, on the meridian, above the pole, would have been 86 degrees 8 minutes. But because of the comet's exceedingly slight parallax at this place, we use the circle of declination for the circle of altitude without any sensible error, together with the altitude found, namely 80 degrees [6 minutes]. Let it be, then, for the discrimination of the parallaxes about **F G**, or **Q H** — that **H K**, or the angle **A M B**, were 13 minutes (the comet's parallax at its lowest altitude). Since therefore in the same triangle **A B M** all the angles are given, together with the side **A B** (namely one semidiameter of the Earth), there is found from these the side **A M** of 240 semidiameters of the Earth, and a little more — which, reduced to German miles at 860 [to the semidiameter], yield the distance

of this comet from the Earth in these miles as 206,400. Therefore, with the side **A E** set equal in quantity to **A M**, and **A B** [equal to] one semidiameter of the Earth; and finally with the angle **E B A** of 170 degrees 6 minutes — which arises by adding the supreme altitude, already measured at 80 degrees 6 minutes, to 90 degrees, that is, to one right angle — by these data there emerges the angle of the comet's parallax at its supreme altitude about **F** of 2 minutes 34 seconds. When this is added to the previously assumed **A M B** of 13 minutes, there is again made

up **Q K** of 15 minutes 34 seconds — which aggregate of parallaxes deviates insensibly from what was observed, even though the angle **A M B** might perhaps have been some few seconds greater than the 13 minutes supposed, on account of some refraction of the comet at its lowest place.

And so we rightly establish the distance of this comet to have been 240 semidiameters of the Earth. Nor indeed was it any less — as the observation of the comet at its beginning, namely on 24 November, also intimates, when it was many times taken in one and the same vertical with Arcturus from this [position]; for, apart from the comet's proper motion, they were found to differ little or almost nothing perceptibly, as I recall mentioning above. Indeed, the same unchanging path of this comet on several occasions sufficiently proved this same thing throughout nearly the whole of its duration. By this distance of the comet from the center of the Earth, then, we can reason out a middling [distance], as established, that it was placed in the heaven below the perigee of the orb of Venus, at about 98 semidiameters of the Earth; and its greatest distance from the Earth surpassed [that of] the Moon roughly fourfold — far from being below it, and, what would be still more absurd, having lodged in the uppermost region of the air. Let the fiction of the Aristotelians, then, depart in the case of this comet too — unless perhaps among those who would rather stubbornly oppose themselves to the truth than detract anything here from their Aristotle.

Now at last we turn to the other point, which concerns the size of this comet in comparison with the globe of the Earth — and that will be bound up in the following procedure.

Diagram. A small geometric figure showing the comet's body as a circle **C E D** with center **B**, and an elongated triangle running down to the Earth's center **A** at the bottom. The line **B A** is the comet's distance, and the circle **C E D** is the comet's visible disk; the converging lines from **C** and **D** down to **A** form the cone subtending the comet's apparent diameter.

- E (top of circle): top point of the comet's disk
- C (left of circle): left point of the comet's disk
- D (right of circle): right point of the comet's disk
- B (center of circle): center of the comet's body
- A (bottom vertex): Earth's center

About the center **B** let the body, or head, of the comet **C E D** be drawn, and from the center **B** let the line **B A** be let fall to **A**, the Earth's center — which line was found above to be 240 semidiameters of the Earth. And because the comet's body,

at its beginning, very nearly attained the size of the spike-star [Spica, ♀] of the first magnitude among the stars, and held a visible diameter of nearly 3 minutes — with these things supposed, let the true diameter of the comet first be sought in this way. In the plane triangle **C B A**, having a right angle at **B**, since **B A** is given as 240 S. T. [semidiameters of the Earth], and likewise the angle **B A C** of $1\frac{1}{2}$ minutes — which is half of the comet's visible diameter **C B D** — therefore, by the conversion of the four [parts] enunciated of our trigonometry, as the whole sine is to **A B** 240 semidiameters of the Earth, so the tangent of the angle **B A C**, namely 4364, is to **C B**, 6 minutes 17 seconds of one semidiameter of the Earth. Doubling **C B**, then, there emerges the whole true diameter of the comet, 12 minutes 34 seconds — of which the diameter of the orb of the Earth is 2 [minutes], or 120 minutes [i.e. the Earth's diameter measures 2 semidiameters = 120' on this scale]. The proportion, therefore, of the diameter of the comet to the diameter of the Earth is [as] number 377 to [number] 3600. [Margin: by the last prop.]

Since the spheres are in the triple ratio of their diameters by the last proposition of *Elements* book 12, therefore, when the individual numbers are raised cubically and the lesser cube divided into the greater, it is found that the comet was smaller than the whole globe of the Earth by 870-fold and a little more; and that it would in turn be surpassed by the Moon's body about 20-fold — seeing that this same Moon is smaller than the Earth by 42-fold and more. [Margin: *Elem.* 12]

On the true size of this comet's tail

As for the true size of this comet's tail, there is little or nothing to treat here, since this was extremely unequal to itself, extending at most to 20 degrees and even beyond, for the rest spreading out from its beginning from the remaining body into the region of the world always opposite to the Sun. But if anyone should wish to investigate this too, let him consult above all *Progym.* book 2 of Tycho Brahe [Tycho Brahe], where he will also find a paradigm of this matter — though indeed under that limitation which our cometic hypothesis above admits, since rarely is a comet placed under such an angle with the Sun and the Earth that it would show us the full ejaculation of its tail. But thus far concerning the situation, motion, altitude, and magnitude of this comet from our own observations.

Now I shall append an account of the same comet observed in Persia, as it was printed in Belgium and has reached us.

[In italic:] *A record of two comets in the city of Spahan [Isfahan], which is the most noble of the kingdom of Persia, and not far from Persepolis toward the west, under*

an elevation of the pole of $31\frac{1}{2}^{\circ}$. From a clause of a letter of García Silva Figueroa [García de Silva y Figueroa], legate of Philip III of Spain and of the Indies, written on the 5th of the Kalends of March [25 February] of the year 1619, to the Marquis of Bedmar, and translated and published at Antwerp in the year 1620, in these words:

[In italic, continuing the quoted letter:] *The conclusion of this letter will be an account of the twin comets which appeared as a spectacle to us during these days.* The first began to shine on 10 November (understand: New Style), two hours before sunrise, and its appearance was observed between east and south. Its color was quite like the smoke that evaporates from very finely powdered sulphur when kindled; its head, as it seemed to me, was in Scorpio. Its size was such as would take up a sextant of the zodiac; its form like a scimitar — as some have supposed — the kind the Greeks call ξιφίας [Greek: *xiphias*, "swordfish" / sword-shaped, here the sword-comet], portending dreadful events. To me this comet seemed rather to present the likeness of a one-year-old palm shoot, not yet spread out, bent slightly at its tip. It held its motion toward the south.

On the twelfth or thirteenth day [duo tertiove decimo die] after the rising of this comet, another appeared, maned or hairy, of common aspect, in color like the star of Venus, and of equal size with it, or a little greater. It took its rising directly from the equinoctial east. And although from the beginning it was far smaller than the first, it nevertheless took on far greater increases day by day, so that it almost equaled the other. It moved itself with its own proper motion together with the *primum mobile*, not far from the vertical line, toward the north. Then, three or four days after its rising, the other vanished. Both, to be sure, had so brief a period that the later one had already ceased to appear by the tenth day mentioned above. It was observed that this one, as it ceased, grew redder. However that may be, if these comets portend evils, at least they will not be long-lasting, like very many others that are seen in the world. Thus he [Figueroa].

From all this, although I acknowledge that the designation of the two comets observed in Persia in the month of November of the year of the Lord 1618 was made by the said Figueroa the Spaniard, and that the earlier one began there on the same day on which Kepler [Johannes Kepler] notes it in Silesia, nevertheless I do not judge that an exact [Greek: ἀκριβεῖαν (*akribeian*, "exactness, precision")][?] [the Greek word here is the adverbial/accusative form, partly abraded] method was applied by the Spanish legate himself, or by those who were with him, to observing the later one — the one we are dealing with — which moreover would have befitted an astronomer. So much so that any reader versed in celestial matters may easily perceive that this royal legate, being intent at that time rather

on other things which he judged would be of greater moment — and excused, by that more pressing care and perhaps at the same time by an ignorance of the art — not only assigns no discrete motion to this comet, but also, contrary to the nature of comets and to common experience in others (as in this one too), makes its magnitude increase successively from its first rising: which, however, on account of the nearness of the Sun in the sphere mostly toward the line [the equator], could have happened. For while this comet was rising for the Persians, it scarcely preceded the Sun by 7 degrees in oblique ascension, so that it would have been a wonder if it could not have been wholly hidden by the Sun's rays — let alone, at its beginning, attenuated — and at length brought more and more into view, until it freed itself entirely from the Sun's rays.

Whatever the case, the opinion that this comet was of less concern to them there is strongly increased by the fact, Figueroa, that in that region — where both a greater nighttime serenity was given, and the phenomenon was, as he says, nearer to the zenith — he did not pursue it by observing for a full 10 days; whereas after it had begun to be seen here, late, in a lower part of the sphere, and after continuous rains, and after it had ceased for him (as he writes), it was for us, over a space of a longer threefold time, likewise — and by others through the intervals which a clearer sky allowed — both noted and observed. For it was impossible that this comet, so enormous, was subject to a parallax such that for this cause it would vanish so quickly for the Persians — since we, faithfully working out and measuring that same parallax above, detected it to be no more than 14½ minutes at the limit of the horizon. And certainly, when the comet was drawn by its diurnal motion to the boundary [horizon] of the Persians, refraction in those regions too restored that whole parallax — unless perhaps the mountains, which are high in Persia, had stood in the way, so that this comet rose into view for them only at a greater bending of time.

Meanwhile, however, from Figueroa's account one may nicely recognize that there was, as it were, another comet — namely the one which Lord Kepler describes, along with some others, from an incompleteness of form — which preceded ours, and that this one began far earlier, namely before the start of the 12th day of November, Old Style: that is to say, around the aforementioned fateful and pregnant aspects of the planets which then occurred — first the quadrature of the Sun and Jupiter, then the trine of Mars and Venus — both of which contribute greatly to the judgment of effects; and what remains for me to treat of this, we shall assign hereafter, according to the weakness of our talent.

CAP. XI.

On the effects, or significations, of these two comets separately, and to be indicated briefly according to the common decrees of the Astrologers.

Concerning the significations of the comets, whose motions we have hitherto laid out from our observations and reduced to order, [if we are] to deal with them more fully and in detail according to the foregoing cap. 8, two ways present themselves. One is more customary and well-worn among the Astrologers in this matter — even while we, thinking differently from Aristotle as to the causes (as above), attribute the generation of comets [to them] — here, by a plausible conjecture, [treating the] effects or signification[s]

we apply to them according to our own judgment. The other line of inquiry leads us into more hidden indications, drawn especially from the differing risings and the contrary motions of these two comets. That I shall treat in the following and final chapter, God willing. But I shall tread both paths timidly and with hesitant step, as these perilous times warn me to do — particularly since the dogmatic, peremptory pronouncements [*decreta praetoria*] of the astrologers (to agree rightly with Ptolemy) have very little place in our science, for many reasons, some of which were noted above; the rest the reader may learn from the proem of Ptolemy's *Quadripartite* [*Tetrabiblos*]. Furthermore — and I confess this frankly — my own nature has always shrunk from predicting events from the stars, and above all from these prodigious ones, lest I be found, like certain others who reap this harvest with greater license, too lavish in such predictions. Indeed I could have wished to refrain altogether from any judgments whatsoever about these comets,

[*right column*]

had not religious duty stood in the way. For these divine portents, sent and directed toward a definite end — namely our amendment — ought to fix themselves in the minds of readers no more deeply than the foregoing reflection on their appearances; and I should faithfully point out a few of the things that have come to pass, drawing them plausibly from the astrological consideration of these comets, while warning that other things are also at the door — prepared, that is, for the ruin of the impious and the impenitent.

Turning, then, first to the rising of the comet of the year 1607, which we found above (plausibly enough) to have fallen on 15 September at hour 15 $\frac{1}{3}$ — that is, in opposition to the Sun's simple motion — we find the sky and the stars, at that moment and with respect to our horizon, in the state that the following chart

expresses. To it we add another, which likewise sets out the configuration of the heavens at the opposition of the luminaries that next follows, so that from the inspection of both a judgment about this comet may be formed.

THEMA COELI [Chart of the Heavens]

[Left chart] Exortus cometae. [The rising of the comet.]

Diagram. A square horoscope chart (*thema coeli*) of the standard early-modern form: a square set within a square, the corners and sides subdivided into the twelve houses by diagonals, with the comet's place and the planetary positions entered by glyph, sign, and degree. The central panel reads: "Anno 1607 / Septemb. / D. H. M. / 15. 15. 24. / Elev. p. 56." (Year 1607, September; day, hour, minute: 15, 15, 24; elevation of the pole 56 degrees.) Reading the houses clockwise from the top:

- Top center (cusp/M.C.): Com[eta] 10 1/4 ♄ (the comet at Leo 10 1/4); below it, Dec[linatio] 53 1/10 (declination)
- Upper left: ♄ 13 (Leo 13); ☉ 0 ♉ (Dragon's Tail at Virgo 0)
- Left (ascendant region): ♂ 12 ♉ (Mars at Virgo 12); ♉ 25 (Virgo 25)
- Lower left: ☉ ☿ (Sun, Mercury); ♀ 2 1/2 ♎ (Venus at Libra 2 1/2)
- Bottom left: ♄ 19 (Saturn 19); ♀ 18 ♏ (Venus 18 Scorpio) [the page repeats Venus here; transcribed as printed]; ☉ 0 2/3 ♏ (Sun at Scorpio 0 2/3)
- Bottom center: ☾ 28 ♏ (Moon at Scorpio 28)
- Bottom right: ♄ 10 (Capricorn 10); ♄ 12 1/2 ♄ (Saturn at Capricorn 12 1/2)
- Lower right: ♊ 13 (Aquarius 13); ☉ 0 ♋ (Dragon's Head at Pisces 0)
- Right: ♋ 5 (Pisces 5)
- Upper right: ♃ 4 3/4 ♋ ♃ ♃ (Jupiter at 4 3/4, with ♃/♃ glyphs as printed); ♋ 25 (Pisces 25)
- Top right: ♉ 25 (Taurus 25); ♈ 19 (Aries 19)

[Right chart] Oppositionis ☉ & ☾ proxime seqq. [Of the opposition of Sun and Moon next following.]

Diagram. A second square *thema coeli* of the same form. Central panel: "Anno 1607 / Sept. / D. H. M. / 25. 0. 43. / Elev. p. 56." (Year 1607, September; day 25, hour 0, minute 43; elevation of the pole 56 degrees.) Reading the houses clockwise from the top:

- Top center: Com[eta] 14 ♏ (the comet at Scorpio 14); below, Dec[linatio] 9

- Upper left: ♎ 26 (Scorpio 26); ♎ 12 (Scorpio 12)
- Left: ♀ 28 (Venus 28); ♐ 14 (Sagittarius 14)
- Lower left: ♄ 12 1/4 (Saturn at 12 1/4)
- Bottom left: ♑ 17 (Capricorn 17); ♉ 1 ♈ (Dragon's Head at Pisces 1)
- Bottom (lower edge): ♈ 15 (Pisces 15); ♌ 11 1/3 ♈ (Moon at Aries 11 1/3); ♃ 2/3 ♈ (Jupiter at Aries 2/3)
- Bottom center: ♈ 23 (Aries 23)
- Bottom right: ♉ 12 (Taurus 12)
- Right: ♊ 14 ☉ (Gemini 14, Sun — opposite the comet, consistent with the opposition heading)
- Upper right: ♉ 26 (Taurus 26); ♋ 17 (Cancer 17)
- Top right: ♊ 18 1/2 ♍, ♋ 28 ♍ (Mars at Virgo 18 1/2, Mercury at Virgo 28); ☉ 11 1/3 ♎ (Sun at Libra 11 1/3); ♍ 15 (Virgo 15)

[resuming main text, left column]

When we contemplate the magnitude, the color, the length of tail, and finally the motion of this first comet at the very moment of its rising, we observe in these the nature of Jupiter, in whose opposition it began together with the mean motion of the Sun; then the nature of Saturn, which it resembled in color, almost in quadrature with Jupiter, and likewise set in relation to the Sun and to Mercury. Furthermore, Mercury himself, the lord of the horoscope, lies hidden beneath the rays, or in the heart, of the Sun; and Mars stands not far distant from the horoscope, in the very trine with Saturn. Finally we note the comet's position, lodged in the eleventh house, from which it afterward bent its course swiftly toward the south, and partly into the order of the signs [eastward through the zodiac]. First of all, the consideration of such things — and most of all the comet's position and motion — plainly intimates to us those harsher winters, almost against the order of nature, especially those at the beginning of the year

[right column]

1608. Among the northern Icelanders and other Norwegians it was far milder and more tolerable than usual; but among us, the Belgians, Germans, Swiss, and even the French — indeed the Italians and Spaniards — it was so inclement, beyond all custom, that not only the smaller animals but a great many people perished of cold in those places. For just as the Sun, rising by the primary motion, drives forward with its rays the searing cold that the vast Indian Ocean exhales, and little by little — as Cardano [Gerolamo Cardano] holds — sows it over the continental world toward the

west, so it is credible that this comet rendered that cold matter, gathered in the air around the north, subject to its own motion, drew it southward with itself, and gradually sowed it over the face of the earth. Hence not only the winter seasons,

[*left column*]

but even the spring seasons (since at the same time Saturn had occupied its own domicile [Capricorn], from which it was most effective) lay buried under great cold and frost. Among other things, these without doubt caused those high grain prices among us, in Germany, and elsewhere — prices that also lasted for several following years, when those immoderate winters brought dry summers along with them, besides various diseases, both acute and chronic, especially unaccustomed fevers, which from that time poured into our regions as well, on account of the trine of Mars [and Mercury]. In this number are to be reckoned the continued fever [*synochus*] and especially that quartan ague which so wretchedly afflicted our countrymen and the Germans from then on. Nearly the same follows from the chart of the next full moon. For in this revolution too the comet occupies nearly the same position as before; and Mercury, lord of the horoscope, was thrust down as a cadent planet into the third house; and finally it was admitted into the ninth house together with Mercury and the Sun — which greatly harmed religion, so far as it is fitting either to learn or to predict this from the stars. Indeed the position of these planets from that quarter also portends ill-omened deliberations concerning it [religion] and other affairs of state. As for the remaining condition of the commonwealth throughout the world — which in these last ages of the world has, by divine justice, on account of men's various sins, and especially the lust of princes to rule, been turned to war (according to Christ's own prediction) and for the most part torn apart from charity, concord, and peace —

[Margin: Matth. 24, ver. 6, & seqq. (Matthew 24:6 and following)]

[*continuing*] we mark in both charts no obscure signs of wars, especially among the Muscovites, Tartars, Indians, and Turks. That these things are true you will find from the reports that have reached us — namely that those peoples for whom the comet (according to the precepts given in chapter 8) was at those times either vertical or otherwise in the tenth house were harried by wars and disasters. Let one read the recently published history of the Ruthenians, [to learn] what disturbances those three false Dmitrys [*Pseudo-Demetrii*] stirred up in the meantime, and what slaughter and killing of men they wrought in Russia, Livonia, and Poland.

Furthermore, around the year of the Lord 1610, in Tartary — where the Great Khan had died a little before — a vast war broke out, namely among those peoples for whom this comet was as it were vertical at its rising. For when, the Khan's son being excluded, the brother of the latter tried to draw the empire to himself, the matter was decided by war, in which the Khan's son is said to have been victor, and in which up to 40,000 men fell. Two years before, in the regions of the Turkish empire over which this comet culminated in the chart of the opposition of the luminaries — namely at Aleppo and Caraman [*Curmania*] — a very great tumult arose. The pasha of Aleppo broke out in rebellion and defection, until the vizier-pasha, asserting himself as the avenger in the name of the Turkish emperor, only barely emerged victorious after three enormous battles.

[*right column*]

As for us, however, although this comet was nowhere plainly vertical, neither in these charts nor outside them (for nowhere did it attain a greater declination), nevertheless, since it had begun its course and its beginning from Leo — a sign which, among others, is by Ptolemy reckoned trinal for the nature of these men, the Germans, Danes, and Swedes — for that reason too wars extended themselves to this region, chiefly on account of the directional decree that the conjunction of Saturn, Jupiter, and Mercury had produced in the year of the Lord 1603 at the beginning of winter, at 10 degrees of Sagittarius — one, namely, of the trinal signifying signs; likewise on account of the new star found in the following year, namely 1604, at 18 degrees of Sagittarius. For when we consider the comet at its rising and its motion through the ecliptic from these places, we have from that the space, on either side of the trine, in which the comet, from 10 1/4 degrees of Leo, makes no perceptible interval from the earlier point at 10 degrees of Sagittarius and the later at 18 degrees of Sagittarius — the difference of which, by that direction, prolongs the comet's significations for about 8 years. But at these intervening times — and because it had its greatest stimulus from the fiery conjunction of the superior planets — there fell the flood through Britain, the disturbances and wars through Germany, and indeed these northern realms. For scarcely four years after the comet's rising, that two-year war broke out between the Danes and the Swedes; and its end too was occasioned by the death of Charles, king of Sweden [Charles IX], who had given great occasion for that war, when in the following year — namely 1612 — the comet by its directional motion reached the horoscope of this [war] at 7 degrees. That same year was fatal to our most serene and never-sufficiently-praised Queen ANNE CATHERINE [Anne Catherine of Brandenburg].

Furthermore, when we produce this comet's significations by the reckoning of the Moon's course, we find that it completed nearly 4 signs in their consequence [eastward] by its motion — which likewise intimate to us a duration of the significations over eight years. Within that span not only the evils we enumerated above worked themselves out, but most of all those of which Mercury was the signifier, cast down and afflicted in both charts. To this not only the Venetian war with the Archduke of Austria attached itself unhappily, but most of all the beginning of the rebellion and defection of the Bohemians and the adjacent provinces, chiefly (as they suppose) for the cause of religion — when, that is, just as the Poles before them (and with quite the same outcome), these men too declared themselves, all too prematurely, vindicators of the free exercise of religion. On which matter, that I may judge nothing to be disputed for the present — except this one thing alone, namely that God finds a middle way by which He will show Himself, at the time appointed, to be the sole defender and avenger of His Church — so it is firmly to be held that rebellion will scarcely ever have success unless it is one preceded by so great a tyranny as may move heaven and God Himself

[left column]

to promote thence into a just vengeance by prayers and tears.

And so much for the significations of the comet of 1607 considered in themselves, apart from any comparison. There follow those which we judge can likewise be deduced from the latter [comet], according to the measure of our ability.

On the significations of the comet of the year 1618 separately. [De significatis cometæ anni 1618 seorsim.]

Since here too an occasion has been given us of recalling the phenomena of the comet of the year 1618 likewise to the beginning of its rising — which beyond doubt had fallen in the quadrature of the Sun and Jupiter, namely

[right column]

on 11 November, one hour before noon by our reckoning, at the restitution [renewal] of the courses of both the Sun and Jupiter (since indeed the Persian observation anticipates this time by at least one day, namely when the comet could first be seen by them owing to its nearness to the Sun) — accordingly we shall fit below a celestial chart for the first rising of this comet, and then add another, drawn up for the time of the opposition of the luminaries immediately

following, namely on 21 November in the next night, so that here too we may proceed more rightly and more readily in the judgment of the effects.

THEMA COELI [Chart of the Heavens]

[Left chart] ad Exortum cometae. [For the rising of the comet.]

Diagram. A square *thema coeli* of the same form. Central panel: "Anno 1618 / Novemb. / D. H. M. / 11. 11. 0. / antemerid. / Elev. p. 56." (Year 1618, November; day 11, hour 11, minute 0, before noon; elevation of the pole 56 degrees.) Reading the houses clockwise from the top:

- Top center (M.C.): ♍ 29 ☉ (Scorpio 29, Sun); to the right, Com[eta] 25 (the comet at 25)
- Upper left: ♄ 12 (Saturn 12)
- Left: ♐ 2 (Capricorn 2)
- Lower left: ☾ ♋ 14 1/4 (Moon and Dragon's Head at 14 1/4); ♀ ☿ (Venus, Mercury); ☊ 25 1/4 (Dragon's Tail at 25 1/4)
- Bottom left: ♒ 19 (Aquarius 19); ♃ 29 ♒ (Jupiter at Aquarius 29)
- Bottom (lower edge): ♈ 19 (Aries 19); ♉ 14 (Taurus 14)
- Bottom center: ♉ 28 (Taurus 28); ♄ 5 ♊ (Saturn at Gemini 5)
- Bottom right: ♊ 12 (Gemini 12)
- Right: ♋ 2 (Cancer 2)
- Upper right: ♌ 19 (Leo 19)
- Top right: ♂ 15 ♍ (Mars at Virgo 15); ☿ 11 ♏ (Mercury at Scorpio 11); ♌ 19 (Leo 19)

[Right chart] ad Oppositionem ☉ & ☾ proxime seqq. [For the opposition of Sun and Moon next following.]

Diagram. A square *thema coeli* of the same form. Central panel: "Anno 1618 / Novem. / D. H. M. / 21. 12. 55. / Elev. p. 56." (Year 1618, November; day 21, hour 12, minute 55; elevation of the pole 56 degrees.) Reading the houses clockwise from the top:

- Top center (M.C.): ☾ 9.47 ♊ (Moon at Gemini 9°47'); below, ♄ 4 ♊ (Saturn at Gemini 4)
- Upper left: ♊ 23 (Gemini 23); ♋ 26 ☊ (Dragon's Head at Cancer 26)
- Left: ♍ 3 (Virgo 3); ♌ 4 (Leo 4); ♂ 29 ♍ (Mars at Virgo 29)
- Lower left: ♍ 25 (Virgo 25)

- Bottom left: ♎ 15 (Libra 15); Com[eta] 8 3/4 ♏ (the comet at Scorpio 8 3/4)
- Bottom (lower edge): ♏ 13 (Scorpio 13); ☉ 9.47 ♐ (Sun at Sagittarius 9°47'); ☾ 23 (Dragon's Tail 23)
- Bottom center: ♐ 2 (Sagittarius 2); ♀ ☾ 26 ♑ (Venus and Dragon's Tail at Capricorn 26)
- Bottom right: ♑ 4 (Aquarius 4); ♃ 10 1/2 ♒ (Jupiter at Pisces 10 1/2)
- Right: ♒ 3 (Pisces 3); ♒ 25 (Pisces 25)
- Upper right: ♈ 15 (Aries 15)
- Top right: ☾ 9.47 ♊ (Moon at Gemini 9°47'); ♄ 4 ♊ (Saturn at Gemini 4); ♉ 13 (Taurus 13)

[resuming main text, left column]

Although we hold the rising of this comet only by conjecture, so far as the time is concerned — as we argued above with probability that it began in the quadrature of the Sun and Jupiter — nevertheless, from the inspection of the chart erected for that moment, I find such an agreement, not only with the following celestial position established at the opposition of the luminaries, but also with the things that have happened since the rising of this comet, that I now less doubt that its rising fell at the prescribed time. Survey with me, I beg, kind reader, whoever you are that are an astrologer, the chart of this appointed rising. Do not the malefics Saturn and Mars there so grip, entangle, and draw into their own party Mercury — who is near the M.C. and is the disposer and dispenser of affairs — that through this configuration of Mercury, and indeed of the malefics and of the luminaries, the worldly commonwealths incline to nothing but every ill-omened thing in their counsels and actions? And all the more so because Mars resides here in the eighth place — the domicile of death, namely — and exchanged with Venus; while Saturn stands not far from the cusp of the sixth house, which is the house of diseases; and these malefics too regard each other with nearly a quadrate aspect. I add that Saturn is here lord of the horoscope, who also afflicts the Moon, found in her own fall. As for the comet's own placement in this chart, what shall I say — it that has scarcely yet emerged from the Sun's rays, and is besides exalted in the tenth place and set in the domicile of Mars? — which itself signifies [an effect] upon the earthly luminaries of the world,

[right column]

namely emperors, kings, and princes in the commonwealth; and it has extended itself to them thus far, and I judge will extend itself hereafter; and no less to those who wish to prove themselves prelates and protectors of the Church, if it shall be

permitted to prophesy anything from the stars about this divine economy, since the Moon and Venus have been invited into hospitality by the malefic Saturn. Moreover the comet itself, near the cusp of the eleventh house and so close to the Sun, besides the other evils that comets are wont to announce everywhere, also intimates in many ways treacherous counsels against pacts and friendships.

Surely whoever is an astrologer, and does not utterly deride the decrees of the ancient sages, will scarcely doubt that such ominous things were portended through this comet — things which in that same year 1618 and the next following came to pass in the departure of magnates from the commonwealth. For the first of these, among others, was fatal to Maximilian [Archduke Maximilian III of Austria], and likewise to the wife of the emperor;

[left column]

[the latter] was fatal to the emperor Matthias himself, and to Anne, queen of Great Britain [Anne of Denmark] — a heroine assuredly most deeply missed in this season. And to add something further concerning the emperor: he departed [this life] on 10 March of the year 1619, when Mercury, lord of his natal horoscope, had migrated — now while the comet was blazing — into the eighth house, which is named the house of death, the mid-heaven and the comet likewise passing a little afterward into the place of his natal horoscope. Consider besides the actions that fell upon Germany and the whole Empire, chiefly on account of religious discord, since this comet referred its rising to the quadrature of Jupiter and the Sun, and was not far distant from the Sun at its rising — in one domicile, namely, with Mercury (Mercury, I say, fraudulently invited by the malefics, and, as I said, taken in with the domiciles exchanged). The things, therefore, that perhaps could have been governed by wholesome counsels were hitherto carried on by the worst — so much so that within these few years they have been the causes of the slaughter of almost countless men. The same chart, erected for the opposition of the luminaries, intimates this too, where Mars in the twelfth house nearly occupies the horoscope; by which again Mercury, in a cadent place and as it were in a quadrate aspect, is afflicted — being, namely, in his long ascension — where again the luminaries are afflicted by the sinister aspects of the malefics, who themselves too are placed in unlucky positions in this chart, since Saturn is found in the house of religion and Mars in the house of imprisonment [the prison-house, the twelfth]. But since Saturn is more sluggish and more long-lasting in producing and defining effects, it is certainly to be feared that the evils which the present comet signifies will not be dissolved and cease before the conjunctions of the superior planets have passed — which will

occur in the year 1622 in [Leo], and in the next [year] in a fiery sign — whence ill-omened and evil outcomes are wont to issue, here both as to floods of waters and as to motions of wars afterward, evils which seem to wish to extend themselves into the year 1624, a year which indeed I, with several others, judge will be grievous to the world, if not fatal. For in the meantime, unless all things shall have been composed by divine mercy and singular providence throughout the Roman Empire, and some peace erected and confirmed, you will see horrendous wars and upheavals, which will at the same time set the Turk in motion as well, because of this comet culminating over his dominions and realms at its first rising.

[right column]

But would that we too, the men of the North, be not touched by the decrees of this comet in these deplorable upheavals, nor be entangled in them at these intervening times — when, that is, the comet shall have been drawn back by direction into the antecedent [signs] to nearly the beginning of Libra, and afterward, around the end of the year 1623, when (near the end of the second decan of Virgo) the comet had become vertical to us. For among other portents which are reported to occur in these times here and there among us, to rebuke our complacency, there is also that horrendous eclipse of the Sun which on 11 May of the year 1621 was seen among us — and indeed in a greater degree than among those farther south, on account of the increase of parallaxes. Such an eclipse threatens evils of this kind, or the like, when in the last degrees of Taurus — the sign and domicile assigned to luxurious Venus — the luminaries were joined in this eclipse, the Moon about to enter Gemini, a sign that belongs to the domicile of inconstant Mercury, who indeed is the signifier of various upheavals in the Empire and commonwealth, even as to laws and disciplines, since Mercury is commonly held to be the director and standard-bearer of scribes and learned men. Furthermore, from the sinister aspect of the malefics, and likewise from the comet's retrogradation through the signs of the zodiac, besides the febrile and other diseases which still continue among us, that pestilential plague too invaded the men of our climate in the year 1619 and the next, when the earth, meeting the comet under that elevation of the pole where it [the comet] most inclined to its final resolution, seethed forth from itself a Mercurial kindling-matter, and by its motion promoted it toward the east — whence first a part of Norway, then the greatest part of Denmark and some of Sweden, afterward Gotland, Prussia, Livonia, and Poland, under this same climate (which was a wonder), were wretchedly afflicted by this plague; and yet at the same time they were so successively freed from it by the most merciful God that you would recognize this

plague to be nothing but a fatherly admonition concerning the saving repentance of men. Would that it might so fall upon us hereafter, that — the filth and complacency of our sins cast off through earnest penance — we might be held worthy to escape these evils otherwise destined to come, and might stand with divine grace and favor before the Son of Man.

But as for what concerns the fruits of the earth, the grain prices, the weathers, likewise the diseases and other evils of this kind to be further expected from this comet's decrees — these surely will then most of all be aggravated, and will seem destined to be harsher, when the comet, in the last sign of its disappearance, namely Leo, together with the conjunction of the superior planets in the same sign (of which above), shall have exerted its powers — powers which seem reserved for the year 1623, and most of all 1624. For although this last comet began at [its] rising — so that thence, according to Ptolemy, there would be an indication of its earlier-ripening effects to follow — nevertheless from the magnitude both of its body and of its tail, and likewise from the constancy and duration of its motion, our judgment is confirmed, both as to the magnitude of the events and as to their more long-lasting duration up to the terms we have named. And so, thus far the significations of these comets have been considered separately, in whatever manner. It now remains that, as a conclusion to the treatise undertaken, we throw together a few things about what they have in common.

CHAPTER XII.

On what the significations of the two comets described above have in common — arising from the same efficient causes, from the contrariety of their motions, and from the nature of the signs or constellations through which they were carried; and further, on two other comets that arose at nearly the same time as the last one. From all of which, before anything else, the detestable complacency of mankind in these latter times is exposed.

Besides the things set forth in the preceding chapter about the significations of these two comets — significations that come onto the world's stage successively, as the times bring them — I suspect that there are still other matters surrounding these comets which will, beyond all doubt, marvelously increase what they signify.

First, then — apart from God himself, supremely good, who brings forth celestial portents of this kind, like everything else, to a sure and good end, as was clearly stated in chapters 4 and 7 — the very origin of both these comets, or, if you

prefer, the efficient natural cause, is to be attributed to the two brightest and most powerful luminaries among the wandering celestial bodies, the most kindly of all in fostering things below: namely, the Sun and Jupiter. For we have argued, with reasonable plausibility from the timing and other circumstances, that the earlier comet began at the opposition of these two bodies, and the later one at their quadrature aspect. From this it follows briefly — to sum up everything in this chapter — that the purposes for which these two comets were sent by the supremely good and great Creator, to give warning and to signify, are not very different from one another. Moreover, since the same significations are drawn from the temperament and nature of these planets according to the astrologers, it is evident that the one — Jupiter — signifies, among other things, religion and justice above all; while the other — the Sun — signifies kingdoms, empires, worldly majesty, and riches. The decrees of both comets extend alike to the violation and assault of these things, when God shall permit it on account of the sins of men.

The **second** point is the contrariety in the motions of these comets. They passed through the same zodiacal signs, but the earlier one ran from north toward south and east, with a more impetuous and irregular motion (though almost continuously beneath a great circle on the sphere); the later one, by contrast, directed its course from south and east toward the north, with a slow, more measured motion, likewise beneath a great circle. From this, again, it seems possible to infer that the affairs which these comets touch — whether of religion, or of empire and rank — will strive in opposite directions and be torn apart; so much so, indeed, that those earlier disordered motions, which signified those premature and unwholesome plans, can hardly easily be settled again, without enormous floods of upheaval (such as this sea of empires stirs up of itself), once the vials of divine wrath — which these comets surely represented — have been sent down from heaven for that purpose.

Not crime, nor a fierce lust to reign, can avail to compose the troubled waves that must be stirred; by prayer, by tears, must the wrath of the Godhead be appeased.

The **third** point: when we consider those constellations in the sky through which these two comets were carried — and especially Boötes, that is, Arctophylax [the Bear-keeper], beneath which above all they directed their courses in opposite directions. The earlier comet moved obliquely beneath this constellation, beginning its course at the thigh of the Great Bear and ending in Serpentarius [the Serpent-bearer]; the later one, more directly, ploughing through this very

star-figure, did not leave it until it was let down, from the extreme left hand [of Boötes], into that empty space left between the tails of the monstrous beasts — namely, the Serpent and the Great Bear — taking its rise from the star of Scorpio, and so cutting across the path of the earlier comet beneath the right thigh of Boötes, near the tropic circle (of the summer turning of the Sun, that is). From this we may surely gather that God does not without reason now make use, for our admonition, of such celestial figures in displaying the courses of these comets — figures that were once handed down even by the ancient astronomers and worshippers of the heavens as the errors of fables.

For although the interpretation of these monstrous constellations — beneath which our comets either took their rise or into which they faded — may be sought elsewhere, in Manilius, Aratus, and other writers, but especially in Natalis Comes [Natale Conti] and Henricus Decimator [Heinrich Decimator], this one thing I assert about the star of Boötes: since he was appointed by those ancient astronomers to keep watch over the Bear (that is, the north), and yet is found slow — too ox-like — in this office of his (because of the slow daily turning of the stars about it), he is therefore now stirred up by these cometary envoys and their swift motions, so that, returning to his office and pricking up his ox-like ears, he may in some way point out what God wants of the world — and especially of these northern regions, on account of their utter complacency — and what God threatens by his just judgment; and may, as it were, indicate it plainly with the hand by which the last of these comets was carried directly, ascending a little higher into the north. This too, I say, beyond all doubt, it pleased God to make known to us in this way and through this star, now that men, too complacent, have neglected to attend dutifully and carefully to his will, which he has clearly revealed in his Word — so that no one may be found ἀναπολόγητος [Greek: anapologētos, "without excuse"] when that great day, for which the children of light are already panting, shall shortly dawn.

Meanwhile, however, for the impenitent and those who persevere in their complacency, God does not seem willing even to suspend the deserved punishments, much less to remove them entirely; but rather, besides the other penalties by which he will justly requite their sins, to hand over the hearts of men to dreadful contentions and quarrels — above all from the lust to reign — in such a way that, the means and counsels of peace having been wretchedly despised and set aside, they tear one another to pieces, after the manner of those very beasts among which these comets passed: that is, the beasts from which they arose and among which they at last vanished.

Fourth — and this too is worthy of notice — in this same year 1618, a little before the rising of the last comet (a thing I do not know was ever done before), two other, more monstrous comets appeared, in nearly the same prison-cells, or regions of the sky, from which these two we have so far discussed were sent forth: one in the north, beneath the Great Bear, as it were the herald or guardian of the coming of this great last envoy; the other in the south, as though it had equipped and given strength to that same great envoy before his departure southward from it. About both of these, let Master Johannes Kepler be read, whose verdict on this last comet was also recorded — indeed, his dissertation against Seneca on behalf of the Ephorus — namely, that one comet was split into two, or departed into two parts; with an omen added of the Turkish empire becoming wider and, as it were, divided into two, of which the one would embrace the southern world and the other the northern. May God in his mercy turn this omen away from us — the outcome of which he might otherwise justly permit to be brought down upon us by Turkish tyranny on account of our sins — unless, putting aside all delay, we set aside sloth and disobedience toward the long-suffering of the most kindly God (like a serpent to be cherished in one's bosom), and henceforth walk in his commandments and persist constantly in prayer, so that we may behold the light in his light [Margin: *Ps. 36*], and detest from our soul the works of darkness.

And if we do this — and are turned to our God maturely, from our inmost heart, without pretense — we shall not doubt that he will destroy the enemies of his Church, and grant her, rightly founded here in the sound Word and the true use of the sacraments, halcyon days yet further among us according to his promise; and that he will at the same time graciously preserve the magistracy and the state, which now rely solely on the divine promises and protections. But if otherwise — surely no one will doubt that our complacency must one day be requited by the gravity of punishment? — since the histories of every age, even among the heathen nations, but especially among the people of God, show this; about which, among other books, the books of Kings should be diligently read and pondered in the holy Scriptures.

But now, to bind myself to concluding this treatise on the new phenomena with such a παραίνεσις [Greek: parainesis, "exhortation"], we ought to know that all the errors and sins among men — whom God created from the beginning in his own image and for a life perpetually better — flow chiefly from two sources: namely, ignorance, or lack of knowledge, of the true God, and unbelief in believing him according to his revealed Word. Of these, the former — that it may not hold us blinded together with the chief parts of the world — is reason for us

to give eternal thanks (though never sufficiently deserved) for God's good will toward us. But from unbelief — which does not know how to put faith in the promises and warnings of God in his Word, but rather, while divine patience and long-suffering last (by which, with fatherly affection toward us, we are invited to saving repentance), doubts that these are true — there has arisen that detestable complacency among the people, which spreads itself far and wide through every kind of vice among men. So much so that, if only for the sake of slaughtering this one beast and exchanging it for a mature and truly penitent vigilance, divine portents occur everywhere — in the sky, in the air, in the waters, on the earth, in human beings, in animals — opening the eyes of mortals, tweaking their ears, and stirring all their senses: in case there should be any who, before the final fates of the world and the ruin of all things, are willing to believe that God reigns in the world; and that Christ, whom the Father set as King over Zion [Margin: *Ps.* 2], the most powerful King of all, will rule the tumultuous and unruly nations with a rod of iron, but will overshadow, as with a shield, the obedient nations that flee to the throne of grace.

Blessed, therefore, are all those who, while still in time, bid farewell to those worldly enticements which Satan holds out, and by which he everywhere baits men in this age — or rather, who curse them, as most destructive to their own salvation and that of their people; and who rush, rightly (that is, without offense or scandal), into the embrace of the Son of the most high King — he who is himself also, according to his nature, virtues, and merits, our supreme King and priest. They constantly fix their minds on his coming, which without doubt presses on shortly; they earnestly persist in the renewal of their minds, so that when this King — perhaps sooner than expected — shall come to judge the world, our lamps may be burning and we may be found worthy to be received, with this our Bridegroom, into the eternal dwellings, where, all fraud and worldly sorrow being shut out, joy and peace shall reign forevermore.

Come, therefore, Lord Jesus. Yea, I come quickly. Amen.

THE END.

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[No printed work-text on these four pages. The *Astronomia Danica* itself ended with FINIS pp. 527–528 were blank. These final four pages are the trailing physical binding matter of the volume. There is no errata leaf, colophon, or index.]

- ** ** — Blank trailing leaf (free endpaper / flyleaf), recto. Aged, foxed laid paper; no printed text. A few scattered ink specks and one faint reddish mark in the left margin, none constituting a legible inscription. The dark leather turn-in of the binding is visible along the right (fore-)edge.
- ** ** — Blank trailing leaf, verso, showing the gutter and the green-stained text-block edge at left. No printed text. A handwritten library shelfmark in pencil/ink appears near the lower center: BC 74 2877 [?] (library shelfmark / accession number — not work content). A faint pencil notation appears at the very top edge, illegible [?].
- ** ** — Inside back cover / pastedown (the leaf pasted to the inner face of the rear board), with the leather turn-ins of the binding visible along the top, right, and bottom edges. No printed text. Discoloration and a paste/glue stain at the upper right where the leaf adheres to the board; offset ghosting visible. No legible library marks.
- ** ** — Back cover (exterior of the rear board): blind-tooled (blind-stamped) brown leather binding with a paneled design — a multiple-rule frame enclosing concentric decorative roll borders of foliate/scrollwork ornament around a plain central panel. Raised bands visible on the spine at right. No text. This is the physical rear cover of the book.

[End of the volume. No further leaves follow.]