Sefer Elim

ספר אלים

[The Book of Elim]

(1629)

by Joseph Solomon Delmedigo (יוסף שלמה דלמדיגו, "Yashar of Candia"), 1591–1655

Translation by Scott Weisman

Ars Astronomica

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Translator Version 2

Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

A wide-ranging Hebrew scientific compendium, framed as a reply to questions put by the Karaite scholar Zerah ben Nathan. It ranges across mathematics, astronomy, physics, and metaphysics through twelve major questions (the "springs" of Elim) and seventy minor ones, followed by appended treatises — dense with geometric demonstrations, worked numeric problems, and astronomical and trigonometric tables, and an early Hebrew witness to logarithms and post-Copernican astronomy. Among the most ambitious Hebrew scientific books of its age. This volume reproduces the Odessa 1864 edition (first printed Amsterdam, 1629).

A note on the translation process

This translation was produced with the translation-pipeline. See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

Translated with the assistance of Claude. The translator thanks the Anthropic team for making this work possible.

[Front matter — binding and library material; no book text on these pages.]

Source — Binding / cover.

Source — Library bookpocket (University of Toronto).

A pasted library pocket. The text at the very top of the page is bleed-through (show-through from the reverse side) of the English stamp below, appearing reversed; it carries no independent content.

Legible printed text on the pocket (English):

> PLEASE DO NOT REMOVE CARDS OR SLIPS FROM THIS POCKET
>
> UNIVERSITY OF TORONTO LIBRARY

Call-number label (handwritten/stamped):

> QB 52 D4 1864 c.1 PASC

[This is modern library apparatus, not part of the 1864 edition.]

Source — Blank flyleaf.

Blank leaf. Faint show-through of Hebrew text from a later leaf is visible but nothing on this page itself is legible or intended as content. [Nothing to transcribe.]

Source — Blank flyleaf.

Blank leaf. Two small ink marks appear near the top: רש) and, below it, ל — apparently a pen trial, shelf jotting, or ownership mark rather than printed text. [?]

Sefer Elim — Front Matter (Title Page and Prefatory Apparatus)

[Editorial note: This opening of the printed volume consists entirely of front matter — the Hebrew title page, the censor/library verso, and two prefatory pieces (the approbation and the publishers' note) set in Rashi script. The two Rashi-script pieces are 1864-edition editorial apparatus, secondary to the work itself; their boundaries are marked below. Both are printed in small, heavily abraded type; spans that could not be read with confidence even at high magnification are flagged [?].]

Title Page

ספר אילם — SEFER ELIM

Hammashle'ach ma'ayanim ban'chalim ["He who sends forth springs into the valleys" — Tehillim 104:10]. To the One on high [*la-tzad illa'ah*] [?] who utters words; and in it [are] **seventy-two** [שתים עשרה … in [?]] [Editorial note: the subtitle plays on *elim* / *ma'ayanot* — "wells/springs." The headline phrase advertises the work's structure of questions; the exact count word here is partly abraded — [?]].

These are five wells [*chamishah ma'ayanim*], comprising lofty questions; [text abraded] *im nata Ha-* [?] — "if He planted" [?] — five [*hei*, =5] comprising exalted questions, awe-inspiring and never-before-seen [*nora'ot mei-olam*], [as of] fierce lions [*aryot*], keen ones [*mecharerei nemerim* — "sharp as leopards"].

The Rav, all-encompassing in Torah, our master, **the wondrous one** [*ha-etzeni* — reading uncertain] [?], posed [these questions] — [namely] **R. Zerach bar Natan**, of blessed memory —

to **the perfect Rav, the philosopher, the physician, the master**, the honorable **R. Yosef Shlomo Delmedigo**, may the memory of the righteous be a blessing, **a man of Candia** [*ish Kandia* = Candia / Crete].

And further [*Ve-od*]: there have been added writings of questions and answers, [text abraded] full of [*me'lot* …] *di'mah* [?] like substance [*ke-geshem*], gifts [*nedavot*] from the Rav [text abraded] [?] [from] **R. Yesh… [?]** and all his disciples — all of them lessons clear, plain, and upright, gladdening the heart [*me'samchei lev*].

It was first printed at Amsterdam in the year [5]389 [1629 = שפ"ט] by the press of the Rav, the learned, the godly investigator, our master **R. Menasseh ben Israel**, of blessed memory, and was [proof]read by his hand.

And now it is printed a second time, with additions, glosses, and [*ha-kinipi'as* / corrections] [?], [together with] a biography [*toldot*] of the days of the author and a transcription of his epitaph [*matzeivah*] — gathered, and also a drawn likeness of his face [*tziyur bidmut panav*].

In the [printing] house of the partners, our masters, R. Meir Greenshpan and his partner, the printer, our master R. Moshe Eliezer [Belinson].

אדעסא — ODESSA

By the press of M. A. Belinson [*bi-defus M. A. Belinson*].

תרכ"ד [(5)1864 = 624 CE]

[Two ink stains / cancellation blots obscure the lower-right quadrant of the page. A pencil notation "1948" appears at the head of the leaf, above the border, in a modern hand — a library/inventory mark, not part of the printed page.]

[Censor approval line, Russian]: *Дозволено Цензурою. Одесса, 10 Іюля 1864 г.* — "Permitted by the Censor. Odessa, 10 July 1864."

[Library call-number, in a modern hand]: QB / 52 / D4 / 1864.

[A pencil notation "1948" appears at the head of the leaf, matching the recto. Two ink blots are present in the lower margin.]

ההסכמות — THE APPROBATIONS

[Library stamp, framed, at head of page]: "תרומות ספרית 'עבריה' ('חובבי שפת עבר')" — "Donations of the 'Ivriah' Library ('Lovers of the Hebrew Tongue')"; "[...] הסתדרות הציונית בלודז' / לספריה הלאומית בירושלים / לודז, תרפ"ו" — "[...] the Zionist Organization in Lodz / to the National Library in Jerusalem / Lodz, [5]686 [תרפ"ו 1926 =]."

[Rashi script — the approbation text:]

The great Rav, eminent in [Torah] wisdom [*ba-Torah*], a scion of "the Chain of Generations" [*Shalshelet ha-Yochasin*] [Editorial note: alluding to the renowned chronicle of that name], well-known in the gates [of the city], the honorable, our master and teacher **R. Yechiel Tzvi Heilperin**, may his light shine — Av Beit Din, together with the great court of law here in the holy community of Odessa, may God protect it [*yitzu*]:

Our masters the printers, **R. Meir Greenshpan and his partner, R. Moshe Eliezer Belinson**, may his light shine, of the holy community of Odessa, have undertaken, with much pains, to bring to the light of the world [a fresh edition of] *Sefer Elim*, the work of the renowned gaon, the investigator and kabbalist, the godly one, the Rav, our master **R. Yosef Shlomo Delmedigo**, of blessed memory, of Candia — set down [originally] only once, printed at Amsterdam in the year [5]389 [1629 = שפ"ט] in the house of the Rav, the godly investigator, our master **R. Menasseh ben Israel**, of blessed memory, and proofread by his hand. From that time until now it has lain as a hidden treasure [*matmon*], [accessible only to]

singular individuals [*yechidim*], who could not obtain it at any price; and many [are they] [text abraded] whose eyes never beheld it [*lo zachu af li-r'oto mei-olam, ve-lo … mi-ziv hadaro*]. [The reason there was no fresh] approbation [for so long] is that no one of the great ones knew [a copy] of the author's gaonic work [*ba … ki lo yada gedolah Rav ha-ga'on ha-mechabber bo*] [?]; and now [the matter] has come up for approbation by the mouths of the wise and the gaonim of [Vilna?] [*Vilnai?*] [?] —

— and so [the court rules] upon this, in our day, to uphold and strengthen the good standing [*tivutah*] of the aforementioned partners over their good labor, the work they have wrought, for the honor of the aforementioned gaon, of blessed memory, and to give satisfaction in this [*lehaspik ratzon*] to all the dealers [*sochrei*] [in books?] [text abraded] who delight in [the works of] the wise of our nation [*chochmei ummenu*], the great sages [*chachameinu ha-gedolim*], of blessed memory — and likewise to give precedence [*lehamlitz*] to the aforementioned partners before the honor of [text abraded] in Israel, in every place and settlement, the generous of our people and the upholders of Torah and wisdom, to support them with their backing.

[Therefore the court decrees:] that no man whatsoever shall set up a [printing-]house of his own kind to compete [*be-machbi… ha-katuv*] in this [work] [?] — that none shall cause to be reprinted [*lo yugras lehamtzi*] [for ten years to come] [*li-vsh"d, kol hezek*]; as is known to every expert in the matter of printing and the engraving of plates [*chakikat ha-tziyurim*] [?] needed for it — [and the prohibition] applies to all the world: that no man set up [a rival edition], so as not to encroach [*lehasig gevul*] upon the aforementioned partners — to reprint this *Sefer Elim*, whether by their full design or in part, whether without commentary or with commentary [text abraded], whether at home or abroad [*be-medinatenu o … le-medinatenu*] — from the day of its printing and for **ten years** [*ve-ad eser shanim*]. And this prohibition shall fall as well upon the buyer [*ha-koneh*]: all this in order not to cause loss to those who do good [*po'alei tov*]; provided only that there be permission according to the civil statutes [*nimusei chukei ha-medinah*] [i.e., the secular copyright law]. And we trust that the remnant of Israel will do no wrong [*lo ya'asu avlah*]; and let him who heeds [this] be pleasant, and a good blessing shall come upon him.

[The undersigned] who hereby address themselves [*ha-mit'addrim*] to the honor of the Torah and the wisdom contained [therein], together, and to the honor of the gaon, author [of the work], of blessed memory —

This day, Sunday, 4 Nisan, [5]624 [1864 = ד' ניסן תרכ"ד], according to the abbreviated reckoning [לפ"ק].

- The words of the humble **Yechiel Tzvi Heilperin**, dwelling here in the community of Odessa.
- The words of **Moshe Zusman HaLevi**, [serving as] judge and teacher of righteousness [*romatz*] here in the holy community.
- The words of **Yaakov Yechiel Michel**, judge and teacher of righteousness here in the holy community.
- And the words of the humble **Shlomo Dov Pilstein** [Filstein] [?], judge and teacher of righteousness here in the holy community, may God preserve and sustain him.

[A round rabbinical-court seal with a Star of David is impressed in the lower-inner corner of the page.]

נאם המביאים לבה"ד — STATEMENT OF THE PUBLISHERS

[Heading, square type:] **The Statement of Those Who Bring [the Work] to the Printing House** [*N'um ha-mevi'im li-veit ha-defus*].

[Rashi script — the publishers' preface:]

To rouse the heart of our brethren, the House of Israel, toward our good undertaking — we now return, and once again set out to publish, [as we have today], the words that we have already sent forth among Jacob [i.e., the Jewish people] by a special announcement [*be-moda'ah meyuchedet*] — to which not every ear has yet inclined; let him now hearken and hear, this time, the voice that calls out at the gateway of this book — testifying to the discerning [reader] that there is real substance [*yesh mamash*] in these words of ours, and that they are worth heeding with full attention [*la-sim be-som lev*]:

[Renowned in Israel, and founded] [*Nora be-Yisrael, ve-yesodah*] [is] this great [book] [under] the name of the Rav, the gaon, the investigator, the godly one [*ha-chakor ha-eloki*], comprehending [a sage] [*chacham*] [who is himself] a comprehensive received tradition [*kollel ha-mekubbal*] [?] — [namely] our master **R. Yosef Shlomo, physician, Delmedigo**, of blessed memory — known and beloved [*chemonus yashar* — reading uncertain] in Candia. His books were [counted] precious [*sifrei … sukim*?]; the [text abraded] upon every sort of [secular learning] [*kol sugei … u-sochrei*] [?], dealers in [wisdom] [text abraded] — they ceased and rested [*chadlu … ha-aretz*]. And in particular [*u-vifrat*] this **Sefer Elim**, the deep and the hidden, which embraces the fruit of every kind of

knowledge [*ha-mechalkel pri kol ha-chochmah ve-da'at*] — no one could measure it [*lo yikleh bo*]. — Many were [the wise] [text abraded] of our generation [*chachamim u-ge'onim*] who recoiled [from it?]; only [a few] merited to see its like at all [*hen lo ratzim zachu af li-r'oto mei-olam*].

Because this book of his was printed only once, at Amsterdam, [in the house of] **Menasseh ben Israel**, of blessed memory, in the year [5]389 [1629 = שפ"ט] — and from then [until now] it has been [*matmon* — a hidden treasure] [text abraded] hidden away in [Jewish] copies [*be-toldot Yehudim*] [?]; and this is the [book's] singular distinction, which [no other] [text abraded] is to be found like [*kammohu … bi-shvat mah*] [?]: whether in its [readings] [*be-tziyut be'alav*] [?], or [other] matters [text abraded]: tens, scores, and yet more [*me'asrim ve-ad esrim … ve-od yoter*]; and the world [text abraded] all [*kol mi*] is as before — and [this] is known to all.

And behold [*Ve-hinneh*]: [men] have already roused themselves there [text abraded], and [some] sons have [text abraded changed?]; the sound of their summons has gone forth before [*ve-kol keri'atam kadmah*], and they have given assurance [*ve-hivtichu*] [text abraded] [that the] remnant of Israel [shall have] an addition [*tosefet*] [?]; for [text abraded] is also taken from us [*nilkach mimmennu*], and [text abraded] no [holy place] [*ve-eyn makom*] [text abraded] — [where] the holy [text abraded] as of old, there is no place [*bli olam, kanah ke-mikdesh be-eyn makom*] [?].

[The remainder of this preface — recounting the manuscript's history, the discovery of additional pieces, the labor of the present edition, the new type and ornaments, and the editors' aims — is set in the same dense Rashi script and is severely abraded across several lines; only fragmentary readings are recoverable, given below in summary with the unreadable spans flagged.]

[Summary of the legible thread of the remaining paragraphs:] We took it upon ourselves [*az lozemt katenu*], the undersigned [*ha-chatumim*] [?], [text abraded] to fulfill the desire of the dear ones of our community [*nedivei kehillatenu*]; and with the special wish of [our] court's leader **[R. Heilperin], may his light shine**, [text abraded] we resolved [*ve-samnu… kol*] to set in order here [*po sheinit*] [text abraded] the book in its proper arrangement [*lefarso le-tachlit ha-siddurim*]; and as our editorial eye [*einei ha-koreh*] [text abraded] [demanded]. **And moreover** [*Ve-af gam zot*]: behold, our hearts were [moved] [*nifteh le-libbenu*] to go and seek out what [other works] have been lost [*ha-ovdim*] to the printing house [*li-vsh"d*] [?]; [we found pieces] [text abraded] that had not been printed [*ha-hayonim ve-dayyek, alah lo nidpas*], [text abraded] until [we] gathered [text abraded] new

type [*tziyon chadash*?] [?]; [text abraded] in their way [text abraded]; and we restored an army [*hechzarnu chayil*] to bring back [*la-gevet*] [text abraded] the labor of the Sephardi-[Spanish type] [*ha-mela'chah ha-Sefardit*]. **And now** [*Ve-attah*]: behold, [the work] is wholly arranged in its order, set [*nug'ah*] in its perfection [text abraded].

[Closing portion:] [text abraded] who [first] began this difficult and beloved labor [*ha-mela'chah ha-chavivah*] — for whom it was not easy [*lo… nichbedah*]; until [text abraded] arose [in print] [*alah … nidpas*]; for [text abraded] he diverted [*chazonek*] [?] [text abraded] [reproducing] the books in your manner [*sefarim ke-darkechem*], [as it is fitting] to gather [text abraded]; — and behold, we have come in only by way of [adding our portion] [*ba-zeh rak be-Hashem himinu li-libbenu*], and we restored an army [*ve-hechzarnu chayil*] to [build up] [text abraded] the dear and generous [men] [text abraded]; the portion [text abraded] [is theirs]; [text abraded]. [And further: the few engravers] [*ha-me'at*] [?] [text abraded] whom we engaged [*she-kannenu lo*]; according to his worth and his value, and his importance [*lefi erkho ke-mohu ve-chivuno*] [text abraded]; and the choice [*ha-mug'at*] [text abraded] of his face [the portrait] (already in our hands) [*be-mukhan kevar be-yadenu*) [text abraded] and the conduct of [his life] [*ve-holdot yamei chayav*] [the biography] — and it shall be a service for [helping] [*la-azor*] [text abraded] [those who deal] [*ha-Sefardi*] [text abraded]; this we hand over [*ha-mosrenu*], that our hands [be set] also to do, and to comprehend the whole [*la-asot ve-lichlol et*] [text abraded] every book [*kol sefer … be-tzun kalar*] [?].

With the help of God [*Be-H' na'aseh ve-natzli'ach*]: a deed shall be done, and may He establish the work of our hands [*ma'aseh yadenu*] for [building up] [*le-harbot*] precious books in Jacob, as [befits] our hearts — to gather the wisdom of our sages, the splendor of antiquity [*chochmat chachameinu… kadosh*] [text abraded], which is shut away [*she-bo nichbedah … selah*] [?].

Odessa, 25 Tammuz [5]624 [1864 = תרכ"ד תמוז כ"ה].

The partners [*ha-shutafim*]:

- **Meir Greenshpan**
- **M. A. Belinson**

תולדת — Toledot (Life-History)

The Rabbi, the Gaon, the Investigator, the Kabbalist, the Divine Philosopher, our teacher and master Rabbi Yosef Shlomo Rofei [the physician] Delmedigo of Candia, may the memory of the righteous and holy be a blessing.

(Compiled from a number of books, and also from this Rabbi-author's own works in several places.)

A city perfect in beauty — and Venice [Venezia] her sister — I have served at the court of the [?] in [?]. [The opening line is set in a more ornate type and is partly abraded.] There was one time, an era of upheaval and unrest. In a time of years that [?], an episode then [?] — for he knew, by his cleverness in every wisdom and science, since he had a powerful arm; and as for matters of [?] he was close, and he was a peddler of nations [i.e. widely traveled], a man among [?]. Therefore he too took to mixing with the family of his kinsmen and to trading with them; for he had the strength to acquire all wisdom and knowledge by a keen spirit and an understanding heart, and he applied himself to every craft and reckoning. Indeed he wrote many books in various languages. — To our astonishment, see, here too:

Whether Candia or Crete — the middle [of the Mediterranean]. — [The island's] circumference is 540 [?], its breadth 035 [?] — [recte: read these figures with care; the inverted forms 540 / 035 appear as printed]. Its [width?] 188 [?], and they add to it a population numbering 270,000, and over them rule [?]: Ishmaelites [Muslims], Greeks, [and] Armenians [?]. The city is called by the name "the chief of Candia" [?]. — For [the island] was beautiful in body, on account of the land of Greece, and it was [?] under the dominion of the regime of Venice [Venezia] (and it [pays] tribute to the Ishmaelites), and it also suckled days of good [?] in the array of war [the wartime situation], for its dispute [?] was between the Venetians and the Ishmaelites; and they took to trading [with] foreign peoples. And in the footsteps of commerce, the sciences too [?] — for from the spring of ancient Greek wisdom and inquiry [?], together with the [?] form of enlightenment [?] they met as one, and were combined [?] in a good [?] portion, and there were [?] men learned and wise, most of whom were [?]; the wise man whose name was [?] was [?] there, and they did not [?]. And in keeping with their native cities' customs (and even though they did not lack a [?] tradition [?] — and

the work [?] inclined toward the tongue of the land [?]) — therefore the precious ones of the families there, [?].

He was the rabbi, the sage R. [Moshe?], a son of the controversy [i.e. an heir to the dispute] of the Gaon, our teacher Rabbi Colon [Maharik], around the year 261 [?]; and the rabbi who debated R. Eliyahu [?], a disciple of the Gaon R. [the chief?] [?]. He brought his composition into the book "Avkat Rochel" of Maran [the *Beit Yosef*] in chapter 64; and it was a dispute [?] of the Gaon Maharik, may his memory be blessed; and [?] — and more. — [And] from the historians: that there was the rabbi R. Sha'ul ben Moshe HaKohen [?], of the [?] neighbor [?], born [in] 229 [?], who occupied himself with his philosophical disputes [?]; and he was a disputing disciple [?] of the rabbi, the sage R. [?], of [the city of] Candia, whose name is mentioned [?], and from there they journeyed to Constantinople [Kushtandina]. And [he?] went out, in covenant with the great sages there — among the company [?] of refugees of Spain [Sefarad] and Portugal [Portugal] — with R. [David?] Yachya [?] and R. Eliezer [?] Alṭonsi [?]. And he was together with the Divine prince [scholar] **Don Yitzchak Abrabanel** — his contemporaries between them, in the year 256 [?]. But the honor of [?] and beyond [?] who were in Candia did not reach the precious honor of the family — for from there came the Gaon, the philosopher, the rabbi R. Yosef Shlomo, the physician — that this author would arise from it [the family]. The family [?] made for itself a name [reputation?] through many books that they wrote and bequeathed — for the family was the Delmedigo family, may God protect it; a great family in Israel, heads of yeshivos, and [?] with [?], greatness and wealth, princes and dignitaries, and merchants without interruption; and from it [came] teachers of peoples in their philosophies [?] in [?] cities — [?] in Florence [?] and elsewhere [?]:

Around the year 5101 [?] [5101 = ה"א ק"א AM; 5101 − 3760 = 1341 CE], the prince [?] went out [?] from the land [of?], settling toward Candia. By the tongue [?] of [?] [a name] — by the tongue of the land [it was] called [?] — among the precious ones of the families there [?] [the family name] Del-Medico [physician], and their families [?].

He was the first of all the sons of these sages [scholars] whose name is recorded [?] for posterity. And he begot three sons [?] of [?] in piety, by the name of "**Abba HaZaken** [Abba the Elder]," who built [?] in Candia [a house] called [in?] German [Alemannic?] [?], and acquired silver and pearls [precious gems] and possessions [?], for he was very rich; and [?] in honor [?] and dignities. — And the rabbi R. **Meirḥam** [?] — died young [?]. — And the rabbi **Shmariah HaIkriti** [the Cretan] [?], or [a Cretan?], by the place of his birth, was a great philosopher, and wrote a

book "[?]" [?] (a refutation [?] and [?]). — And to R. [?] HaZaken [the Elder] there were three sons: the rabbi, our teacher Rabbi **Moshe**, who was a notable [nagid] of Jerusalem, and our teacher **Eliyahu** (the two of them went up to help [?] from [?] the Gaon Maharik [Mahar"i] Colon [Kolon]); and the rabbi, [a man] complete in matters of [?], dignitaries, by name our teacher **Alkanah** [?] [Elkanah] — [?] in Candia. And R. [?] begot his three sons [?]. — And this rabbi, our teacher, R. [?] [?] begot R. [?]: **Shmuel**, [and?] R. [?] **Shmuel**, who [?] his bride [?] pregnant, and she bore him a son, and he named him [?] [Bayan?], who became the Gaon, the third [?], the inner [?] philosopher, and physician complete [in his craft], our teacher **Shmuel Menachem**, may his memory be blessed; and he was very rich, the son [disciple?] of the Gaon Maharam [Mahar"i?] [?] of Padua, and several times he established [presided over] yeshivos of sages: in Talmud and posekim [decisors], and in the [?] of inquiry and philosophy. And he took it to heart [?] to settle the [?] with [?]. He gathered [?] his name and all his money, and he renewed [?] as it is written [in?] R. Eliyahu [?] of [?] [?], that he renewed [?] their [?]; and our community of Candia, that he sat there [presided?] to spread Torah for them. — And the rabbi our teacher **Shmuel** died [childless?], and the rabbi, the Hasid [pious one], the chalUf [?] [aluf, "prince/chief"] our teacher **Abba**, he begot the great rabbi, our teacher **Eliezer**, disciple of the Gaon Maharitz Birav [Mahar"i Bei Rav?] [?], and assistant of [?] [?], may they be remembered for blessing the rabbi R. Eliezer [?] [?] Ashkenazi [?], and [?] [a dispute?] of the great Kabbalist [?], and he disseminated Torah [?] forty-five [?] years [?]. — Then [there] was the rabbi, the Gaon, our teacher **Elyakim Delmedigo** [?], may his memory be blessed, who walked in the footsteps of his fathers [?] — he was a wise man, well-versed, and not [?]. The sons of [?] heard [?] [?], walking in the cities [?]: he was [?] from his death [?] this Gaon, this author [?], may his memory be blessed:

— *Toledot — The Life-History of the Rabbi the Yashar of Candia* —

And the rabbi R. **Shmariah HaIkriti** — brother of R. [?] Abba [?] [?] [?] of [?] [?] [?], the Gaon, our teacher **Elyakim Delmedigo** [?] HaZaken, of blessed memory — who begot the rabbi, the chalUf [aluf], our teacher **Moshe Abba**, and he begot the great philosopher, the [?], glory of the heavens [?] in [?], known in the gates [?] by the name [of] our teacher **Eliyahu**, may his memory be blessed, [a] son of the controversy [heir to the dispute] of the Gaon Maharik [Mahar"i] [?], who was beautiful in appearance and of [a clear?] understanding, [?] of [?] [?], and was [?] of the philosophical writings — and even chose to compile [?] [?] in [?] and in Latin [Latin] — and testified before the Akademi [Academy], [?] his judgment [?]. He [excelled?] in philosophical [?], and therefore his acquirers [admirers] cast

upon him [the title] [?], and from this his disciple "**the prince Pico**" [Pico della Mirandola] is found [?]:

He [Eliyahu Delmedigo] went out [?] to his birth-city Candia [?] in the year 5214 [?] [5214 = ה"א רי"ד AM; 5214 − 3760 = 1454 CE], and disseminated Torah there [?], and disputed [debated] [?], reasoning [?] and [?]; and he was [?] mocking [?] [?] philosophy [?], and the Rambam in his [?]; and [?] in his days [?] [?] (259) [?] [rebellions/disputes] [?] [?] passing [?] for [?] [?] some [?] [?]. — And his sons [?] [?] all the princes of [?] [?] for his honor [?]: were it not for his self-effacement, would the [?] [?] become great? [?] all [?] [?] [?]; but his self-effacement [?] in [?] [?] his honor [?] [?] [?], [?] every philosopher who saw [the?] Israelite [?] [?]. And there were born [?] to the rabbi, our teacher **Eliyahu** [?] [?], three sons. The wise man who included [combined] [the title] philosopher [?] — and his name was the great rabbi, our teacher **Yehudah** [?], [a] Gaon Maharitz [Mahar"i] [?] of Padua; and [?] [the?] investigator of the Frenchmen [?] and assisted [?] R. [?] [Eliyahu?] HaKafsali [?]. And in their request [?] he established a bais medrash [Beit Midrash] [in?] his birth-city Candia. He disseminated Torah there to many, and raised up [?] disciples, great ones of the world; and among them the Gaon R. [?] [?] HaGaon [?], in [whose merit?] [he wrote] many books. — (And it is mentioned [?] [?] [the?] Gaon, our teacher Meir of Klink-Tlinbuinin [?] [Catalano?] in his responsum to the rabbis of [?] [?] Kahal: ["...] [as?] mentioned above 603-676 [?] that R. **Yehudah** Delmedigo [?] supports [?] the [?]" etc.); and to the rabbi, our teacher Yehudah Delmedigo, there were several [?]. And he took from the [?] [?] the heart [?] [?] of R. **Rachel**, wife of the Hasid and the [?], our teacher **Yosef** of Constantinople [Kushtandina], grandson [great-grandson?] of the rabbi, the sage, our teacher **Mordechai** Comțiyani [Comtino?]. And **Yosef** was a man of success in all his dealings, and acquired great wealth in Candia. —

[The opening line is set in ornate type and is partly abraded.] [?] is this great and Yashar [upright] rabbi, the aforementioned Gaon, who [?] [?] his birth-city Candia. And they bore sons and daughters, and they all perished [died] in the dwelling-place of Candia! — The city of his birth, and his distance [?] in the lands of the [?], and not [?] — there is more about it, and as shall be told below:

[Below the chart, the closing prose introduces the genealogical table:]

And the lineage of the family is in this order *(see the table below)*:

Diagram. A brace-style genealogical family tree of the Delmedigo (Del-Medico) family, set as a multi-column descent chart with curved brace-brackets ("⌣"/"⌢") joining each parent to the next generation. The chart reads top-to-bottom by

generation, and right-to-left within each generation (Hebrew order). Birth/death years are given in Hebrew-letter shorthand with "ערך" ("circa") and the *Anno Mundi* fifth-millennium marker implied (e.g. "ערך ק"ל" ≈ circa AM 5130 = 1370 CE). A continuous birth-record line runs across the foot of the chart.

Generation 1 (founder, spanning the full width):

- "השר החלוף כמהו' יהודה" — **The prince, the chalUf [aluf], our teacher Yehudah** (who went out [?] from the land, settling toward Candia, circa the year 5109 [?] [5109 = ה"א ק"ט AM])

Generation 2 (his sons, right-to-left):

- (rightmost) **R. Abba HaZaken** [the Elder], who [settled / was born?] in Candia in [5]228 [?] [בנה בכ"ח בקנדיא — "built in [year] 28"]
- (center) **R. Shmariah HaIkriti** [the Cretan], rabbi and philosopher (circa 5130 [?] [ערך ק"ל])

Generation 3 (right-to-left):

- (right) **R. [?] HaZaken** [?] — [?]
- (right-center) **R. Elkanah** [?] (circa 210 [?] [ערך ר"י])
- (center) **R. Moshe Abba** [?]
- (center) **R. Eliyahu Kreṭinos** [?] [recte: read as "Eliyahu," cf. body text], rabbi and philosopher, "the choice of myrtle" [?] [במינת הדס — "of the species of myrtle"?], the choice [?] of Mahar"i Minz [Maharam Minz? = R. Yehudah Minz of Padua] (died 5243 [?] [נח רמ"ג = died AM 5243])

Generation 4 (right-to-left):

- (right) **R. Moshe** [?]
- (right) **R. [?] Yehudah** [?], disciple of Mahar"i Minz, "the choice [?] of Mahar"i [?] the investigator before [?] the Rema [Rabbi Moshe Isserles?]" [?] HaKafsali [?] (circa 5270 [270 = ערך ר"ע])
- (center-left) **R. Shmuel** [?] (circa 5212 [?] [ערך רי"ב])
- (left) **R. Shmuel Menachem**, rabbi and philosopher (born circa 5260 [?] [נולד ערך רי"ן ≈ 260])

Generation 5 (right-to-left):

- (right) **R. Abba HaSheni** [Abba the Second] (circa 5217 [?] [ערך רי"ז])
- (right) **R. Eliezer** [?] (circa 5320 [?] [ערך ש"כ = AM 5320?])

- (center) **Rachel**, wife of the Hasid our teacher **Yosef**, granddaughter [great-granddaughter?] of the sage our teacher **Mordechai** Comțiyani [Comtino?] (circa 60 [?] [ערך ס' = 60])
- (left) **R. Eliyahu** [?] (rabbi, died circa [?] [רב משנת ע' ש"נ — recte: read with care])

Generation 6 / closing line:

- **Kalfa** [?] [קלפא] (circa [?] [ערך ש"מל — uncertain])

Birth-record line across the foot of the chart:

- "הגאון המקובל מוהר' **יוסף שלמה** דילמודינו נולד כ"ה סיון ה'שמ"ח בקנדיא." — **The Gaon, the Kabbalist, our teacher Rabbi Yosef Shlomo Delmedigo was born on the 25th of Sivan 5348 [ה'שמ"ח = 5348 AM; 5348 − 3760 = 1588 CE] in Candia.** The chart prints (5348 / 1588) ה'שמ"ח, in agreement with the body text — there is no contradiction.

Up to here we sought to learn the [?] of the place and the honor [?] of the families, in a thorough manner [?]. As for the [?] [?] way [?], they have already been [?] upon the position of the stars [?]. — He was [?] near the [?] of [?] of the earth — pursued [?] [?] reversal [?] — and the [?] of a thing was known [?] from its source and its [?], according to what is recounted of him in the books of the ancient genealogists [?], and according to the renown [?] in [?] his printed books (a few of the many) — distinguished above [?] and [?] [?]. — As for the matter [?] of the genealogies [?], it is [?] from this — of the great Delmedigo [?], this Gaon, the investigator, the all-embracing sage and philosopher, the divine [Elohi], our teacher Rabbi **Yosef Shlomo, the physician**, who is called the Yashar [the Upright] of Candia, born on the 25th of Sivan 5348 [ה'שמ"ח = 5348 AM; 5348 − 3760 = 1588 CE], at hour 10 [?] [בשעה 10], in the [city/island?] of Candia. And by [astrological?] signs, war was [decreed?] for him and for his disciples [?]. — He was born under the sign [mazal] "Star of Wisdom" [the planet Mercury], [?] — for [?] [?] the constellations [planets] fell to him for more [?] [?] reasons, in keeping with the [art of?] the [?]. — His features [?]: of [?] stature, his face round, his eyes [?]:

[He was of?] dark [?] complexion [?], and dark [?]; and his old-age [?] hair inclined to white [?]. All his limbs were fine, his height of medium build [?], and his weight [?] therefore [?] between dark and white [i.e. medium-toned]. His body [?] was of one [even] complexion [the weight of his body with the clothes, 140 *litrot* [pounds?]] — [recte: "עם הבגדים 140 ליטרא" — "with the clothes, 140 *litrae*"] — all of him [?]. And even a man [?] who never knew him [?], on meeting him in [?]

would stand and reason [?] within him [?], for the spirit of God was upon him [?] [?], and his great wisdom illumined his face. They [?] [?] his [?]; he was [made?] to bear [tolerate] every toil and sickness, [?] to rest and [pleasant?] to please [?], and to refrain [?] from his measures [?] [?], a delight [?] [?] [?] [?] [?]. As one expounds for all [?] and brings forth [?] and [?]; and he never sat idle [?] for a moment without [?] [?] his body or his mind [?], by his walking [?] [?] in his [?] [?] and [?] in his pacing [?] [?]. Such was [?] [?] his way [?], [?] to [?] [?] to seek [?] in [?] [?] — [?] [?] to [?] [?] [?] — [?] [?] [?].

See, this is the fruit of the womb of the righteous [פרי הבטן לדיקים] — and behold, the blessing of God [?]: a Yashar [upright] root and their [?] [?], for [?]; and the sages [?] in their [?] [?] [?] a calamity of evil [?]. And as a choice [?] of good [?], and to his [?] [?], being like a man [?] Israelite [?] and [?] [?], of blessed memory. — And they hired for him teachers to teach him [?]. They also [arranged] for him to compose [?] all the books [?] of [?] [?] and Latin [Latin] — [?] [?] the learned ones [?] in general — and even composed [?] [?] [?] [?] [?], that they composed [?] [?] in [?] [?] the matters of judgments [?] [?] the Jews [Yehudim] [?] [?] — moreover, his copyists [?] went up to settle [?] [?] [?] with [?] various craftsmen, artisans [?] [?] in his work [craft]; and he was thus [?] [?] after them [?], [?] copy [?] [?] their words [?]. And even [?] in his house [?] for his father — were there those [?] [?] their words [?] [?] without [?] [?] [?]. — Moreover, he reached the year 15 [?] [?] and entered [?] [?] [?] a bais medrash [Beit Midrash] of [?] [?] the great [Gaon] [?] in Padua, and according to [?] [?] [?] he set his [?] first of all upon the study of logic [HaHigayon], and afterwards [?] [?] [?] [?] upon the knowledge [?] and the science of nature [HaTeva]; and he completed [?] [?] his study [?] [?] being a son [aged?] [?] [?] [a youth?], then he began [?] to study philosophy [?] [?] and the [?] inquiry, the [?] [?] the speculative ones [?] [?] and their reasonings [?] [?] from [?] [?] [?]. As [?] [?] [?] the natural sciences brought [?] him [?] [?], the [?] he [?] [?] and learn [?] according to his teacher, the wise man known in [the region of?] Galilee [?] [recte: read with care — "בגלילות" / "בגלילא"]. —

[?] [?] [?] [?]. — His eyes [?] [?] upon traveling [?] thus [?] [?] [?]. — From every place [?] that he saw or knew [?] [?] [?], they say [?], up to as many as **7000 books** in number [?] — and **10,000 [printed?] copies** [?], according to those reckoned [?] in his [?] [?] in [?] [?]; and afterwards, when the rabbi [?] [began?] his [?] [?] [literature]? [?] [recte: "ליטעראטור" / *Literatur*, German "literature"] of [?] books [?], whose like is not [easily] found in the world. — When he came to Candia, he had no [?] [?] [?] [?] — only [?] [?] was his heart in it [?], and the family [?] [?] in [?] of his days, for [were it] not so, would not the [?] [?] [?] in [?] [?]; and his only daughter [?] was born to him, [?] and his [?] [?] [?] Torah in Candia (and see [?]

above [?] [?] as mentioned above). And [?] [?] [his] [?] in [?] Candia [?], [?] passed over [?] [?] [?]; in the [?] [?] [?], lying [?] upon his [?] [?] [?] in the country of [?] [?]. — His mighty spirit [?] [?] never knew [satiety / its fill] [?] from the books and the [knowledge] [?] [?] [?]; and he [?] [?] in them [?] more, until they came [?] [?] [?] his [?] [?] [?]; and his thought [?] [?] [?] [?] [?] in the matter of his [?] [?] [?]. [?] [?] [?] [?] [?] [?] [?]. — His longing [?] [?] [?] [?] to forsake the city of his birth, [?] [?] and to journey [?]; and the [?] [?] [?] [?] forsook [?] [?] [?] [?]. — [?] [?] he went out [?] [?] to draw [acquire?] for himself many [?] books [?], old ones, from the [?] [?] of [?] [?] [?] [?] [?] [?]; and in the city of [?] [?] settled [?] with [?] [?] [?] [?] [?] [?]. — **Ali ben Rachman** [?] [Arabic name] [?] his books [?] [?] [?] [?] [?] [?] [?] [?]; and he was an old man [?] [?] [?] from those [?] [?] [?] [?] disciples [?] [?]; and he was [?] [?] [?] [?] [?] [?] [?] [?] [?] [?]. — When he was born [?] [?] [?] [?] [?] [?] [?] [?] [?] [?] [?], [?] [?] [?] [?] [?] [?]; [?] [?] [?] [?] more [?] [?] [?] [?] [?]. They [?] [?] [?] a man to his fellow [?] [?] [?], and they imposed [?] [?] ten curses [?] [?], and they fixed [?] for them a [?] of [?] weeks [?] [?] [?] [?] [?] [?] [?] [?]. [?] [?] the rabbi the Yashar [?] [to the place...] [— the narrative breaks off at the foot of the folio.]

[The Toledot continues. Folios IX–XII; the 1864 editorial biography of Delmedigo. Set throughout in Rashi script (the entire body of these pages); the catalog of constituent works on folio XII intersperses larger square-type display titles, noted below.]

[The right column opens mid-narrative, continuing the account of his stay among non-Jews.]

a great anger seized him, and he flung [it?] from him; and there were also Jews — Karaites and Samaritans — many of them, all of whom gathered to him to learn something, and they sent their judges, the most senior, to test him with their questions. But the Rabbi the Yashar was still young and tender [in years]; yet he answered them all soundly. The elders of the [Karaite] community then resented him in turn, for they saw his fame surpassing them, and they wished to drive him from their midst. But the Karaites, knowing his worth and wanting to find some pretext against him, said: "There is no substance to this" — [meaning] he had merely consulted some sage in their region; he is still but a youth. [?] So they too sought a charge against him, until the elder of the Talmud [community?] sent for the men who were among his followers and brought them to his house in peace; and the Rabbi in his humility went afterward to the house of the venerable elder, and they reconciled, saying, "Surely this is nothing, [it happened] in error." [?]

"It is folly before the ruler" — and yet no one knew his true worth. From [the lands of] the south his wisdom drew forth, for in his early years [he displayed it];

and they sought him, the elder of the Talmud [community], and honored him before the elders of his people, and made him a trusted authority over the world. [?]

There he was a marvel through all his teaching: "His God made him wise," teaching him as a model for the [whole?] place, and there gathered to him pupils, the discerning ones, listening to his voice. [?]

There was found among the discerning a renowned name, R. Yaakov Abendana [?], for he too was a great scholar in the [secular] learning, and he composed a book *Nifla'ot HaShem* [The Wonders of God] on the craft of mechanics [Hebrew: מלאכת המעכאניק]. For "according to my name, so is my wisdom" — he would not attain nor reach a settled distinction of opinions and beliefs; nor would he yield to disputes. And he conversed with every scholar of heart [?]…

in Constantinople. There he established the foundations, and most of the manuscripts — ancient, choice [?] — he would go [to acquire] for Constantinople. And had it not been that his fame [?] had spread already throughout the whole world, his way [of life] would have continued [thus]; for he had a passion to acquire books and to pore over them [?]…

[The left column of folio IX:]

about four-fifths of a *mil* [?]. He had a great desire [?] to go to Constantinople; and yet we do not know whether he could fulfill it in his whole lifetime — to write books and to compose the like of them. [?] [Margin: "If only he could," etc.]

A place near the Land of Israel, a place [where] his greatness arose. There he settled in Galata [גלאטא, a quarter of Constantinople] — a town near it [Constantinople] on the matter of [?]. There he was bound to the great Kabbalist Rabbi M[?] Y'akev ibn N'siyem [נסתיים, ?], and he took up lodging there. Then it became known to him that the learned men there were "an eye for the learning" [?] — the wellspring of the wisdom of Kabbalah. And he labored intensely [?] over all the books, until they made [him] the most senior [?], and he wrote of his own accord precious works, which were afterward printed [by] the hands of his pupils.

From Constantinople [his journey was] toward [the land of] Egypt [?], by way of [travel and walking?], where [began] this [account of] the holy old Rabbi the wise Moshe… R. [?] Ḥalavi [?] the physician, who sits and is occupied with the wisdom of Kabbalah, "good of name," for many a year, and [conversant?] in books. (In the latter part of his days, when he was advanced in years, he went up to Jerusalem

and died there.) The Rabbi the Yashar added knowledge upon knowledge, the extraordinary increase of understanding that was within him; and he learned every book of the wise, and he pored over them with all his heart and his might, until he became "a master of the deep, and every secret was no secret to him." He became a divine Kabbalist. To the philosophers in their [systems?] he gave [?]... and no man could grasp the depth of his learning. [?] ...And in every place where his foot trod, so too here [in this place] he sustained himself by the craft of medicine; he was a physician of all flesh, and also a physician of the eyes and of the teeth [?], and "skilled at performing." He won himself a great name, and his fame went far afield. By reason of all this [?], a settlement [received him?] in the land of Lithuania [ליט"א], [under?] the prince the Duke [Radziwiłł — ל[י]דזיוו[א]ר], who took him as his physician.

"In the days of his life" [a heading or quote?], something the Rabbi recounted to us [(see *Nivchar* [?], "Maʿaberot ḥ' [8]")] saying: "Did I not make you know that all the years [in?] which I dwelt in Poland and Lithuania, I have not had peace nor rest [?]..."

"Truly [?], in that I have remembered my place to a place" [?] — to relieve a recurrence to the sick, for the cities and the villages are far from one another. So in the villages, scattered [as they are], this matter is distant. [?] Within [?], a man finds the land [too vast] to traverse with their acquisitions, their herds, and their numerous possessions; [he must travel?] only on horseback, [over] swamp and marsh. [?]

But by contrast, he loved books, and "the holy soil" [?] — for the books and the manuscripts were costly; only in dispersed villages [could one find?] the rare among them. So that the land would dwell [?]...[the syntax is broken here] ...only "from the strength of my back" [?], and the wear of time. [?]

He himself [?] said: that all wisdoms are ladders by which to attain the knowledge of the [secular] parts [of learning?], in holiness. [?]

And he railed against the rabbis and the heads of the academies [yeshivos], who waste their time in the study-hall [?] [Hebrew: מחבני בתי-מדרש?], with the multitude and the pupils, "by reason of which" [?]; for there remains "before life" to surrender to opinions of [?] [the year]...[broken]...and from "before me" they will withdraw the matter. So it would have been [?].

He would be set upon the upper [tier?], the eminence of the masters [?], at the very moment of his arrival at the [house of?] study; and had it not been that he came to the house to write down [his] insights and to expound them in books, he

would have squandered his money in vain, and his thoughts would have been "built up" [?]...and the wisdom would be [lost] in his thoughts.

["And he would say":] There he was a crown — a model — for the discerning pupils, listening to his voice, [becoming] workers [?] for him in all [places?]. There were near to him also the men of his own table, those known by name: R. Ezra ben N'sus [?], R. Yaḥ[?] sh[?] ben Yehudah, and especially his pupil "Smobsk" [?] R. Moshe [מ ש ה] ben Meir Mintz [מאיר מיץ], who was a teacher. And on Shabbos day he would teach in the assemblies, Rashi upon the Torah, with his commentaries in pilpul to them. [?]

"And from the mouth of the RABaD [?] — and he revealed the depths of his secrets":

[The indented passage, beginning with a square-type rubric "ואם" (And if):]

And if indeed time has at one moment risen against him as a young lion, and several occurrences and afflictions have crossed his path — by reason of this [?] his face did not fall; and he never spoke of bringing violence [upon himself], for his heart was a heart [like that of] the bull, [which does not] fear before any: he would stand before kings and not be ashamed. [?]

And there came to him a day for the craft of medicine — and God forbid that I should [?] write down speculations [?]. He would learn [an?] occasional Scripture passage, even the laws and the legends of the books of Sefarad [Spain], copying them in [a hand?] sealed and absorbed [?] of the speculations. And all the words of the *posekim* [legal decisors], and especially the *Sha"r* [?] [perhaps the Shulchan Arukh or the Tur — abbreviation unclear], the Maharia [?] — were small in his eyes, for his memory was strong to retain and remember.

In the song and in the metered [verse]: "He set himself a time for the contemplation of the divine sciences, and inquiry and intelligence" [?]. With "Smira" [סמורה, ?], *Milḥamot HaShem* [?], the *Sefer Kuzari* of "Our Rabbi" [Rabbenu Yehuda HaLevi], and the like; and especially "those devoted to strolling [walking]," [the Rabbi] himself was occupied with the wisdom of [secular] sciences, and amused himself with arithmetic [reckoning] upon a writing-sheet. He knew by heart how many books and pamphlets the various authors of generation after generation [had written], deepening... [?] ...So in the land of Poland, where the men and the heads of the academies live, "should they multiply" [?] — there [would be] no good occupation [in?] *Torah*; for good [too] is the Torah; and yet he upholds the laws of the Torah, [for] "good is Torah with [worldly?] knowledge" [Pirkei Avot 2:2, "Torah study together with a worldly

occupation"]. And the knowledge of medicine — "chosen above all knowledges, an arm with a [divine?] might" — [?] makes the heart wise. [?]

"For his only occupation was song [niggun] and music [?]"; and "knowledge [?] is scattered far and wide for the seeker [?]"; for by reason of "love" there is no [?]... [broken]...they have no occupation for [?]. By reason of that, medicine seeks the noble of medicine and trust [?] in their depths. The expounder of the way [?] of the heavens — the philosophers in their generations, and among them the Kabbalah; and the commentator who follows according to the opinion of the philosophers, raising deep matters between the heavens — easy ones, in which there is no... ...the song [of?] "let them sing it" — in their assemblies [?] of testimony, and even "the silent ones in their boundaries." [?] And the [matters] required, "all of them are deep and lofty inquiries," and no one approaches them but a wise man "in [?]." And according to the testimony of his pupil, he was likewise expert [?] in the knowledge of [grammar? — Hebrew: דקדוק?] [the Greek? — אלסכמעטיק, ?] and logic [תשבורת? geometry/measurement — Greek: στοιχεῖα?]...[broken, abraded].

[At the top of the left column, the running head:]

— תולדת הרב יש"ר מקנדיא — (The Life-History of the Rabbi the Yashar of Candia)

[he did not?] place a distant matter that they did not know, nor draw near to confirm [it] truly; and he would set right [?] in his mouth. [?] "Truly it was trustworthy to me" [?] — "[only when] my soul was confirmed" [?]; entirely was [?] in [?]. [?] He was master of [a?] holy tongue — Hebrew [?] — and also the languages of the surrounding peoples. And to those near to him he still [taught?] from the precious books, "and pleasant," to teach them; and especially the books of the [secular] sciences [Hebrew: ספרים הלימודיים, the "mathematical/instructional" books] — the books of the Talmud, or [those of] the Rif [R. Isaac Alfasi], the Tur [R. Yaakov ben Asher], the Rambam [Maimonides], and the books of [Sefarad?] [פרסטו? — perhaps "the books of [the philosophers]"]; in philosophy: Euclid's [אווקלידוס] *Elements* [?] and Galen [גלינוס] on medicine. So that the man would be accustomed to "the well of the depths" with steady hand. And he would set notes [glosses] upon the margins of the books in ink — Hebrew. By the order of the [author? — ספסוקן, ?], they would arrange the notes [in?] additional leaves at the end of the book. And by this he would inquire and rest [?].

A great physician was he, and [yet] he had nothing, [?] for he was not satisfied [?] with [the income from] his understanding as befit his intelligence; and he was burdened with the affairs of his medical practice and the [demands?] of the sick

who surrounded him, [which] would consume his time, and his livelihood was [given?] in his hand "as a salary [?]…[broken]"…by which he would rise and "leap from among" [?]…

[which is] the land of Israel, that they have no field nor vineyard [as their own] possessions — neither lands nor cattle — and their persons are "a prophet for [a sign?] from them." Therefore "we have written" [?] in the land of Poland, where the rabbis and the heads of the academies live, "good of name" and respected; and the editor [?] brought him [?]…[broken]…for there is no occupation of good *Torah* in the city; for good [is] also "her merchandise"; and the world conducts itself with [bias?]. Good Torah goes with "knowledge" [Hebrew: דעת] — "good Torah with worldly knowledge" [Pirkei Avot 2:2 again]. And the knowledge of medicine is choice above all the [other] knowledges, "an arm with a divine might." [?]

In most cases [?], the work of the city is in reckoning and reflection [?], and the like; "it is a covered fountain" [?] of money. And he who has not the resources, by reason of this [?] — most cases [yield] no [?]…[broken]…poverty in his own [?]. And no man saw [?] him "violent from violence, and confusion" [?], for there [was] no business of [?] to occupy them.

[Square-type rubric:] **And so** [כש"ד? — "as we have stated"], medicine seeks from the books of medicine, and the trust is from the sources of Israel; and there are matters distinguished by parts: "philosophy [?] as Kabbalah of the house of the [scholars?]" — and the commentator who runs according to the opinion of the philosophers, fixing matters between [the sciences? — Hebrew: בין הסתים? "between the secrets"?]. There are matters that say there is no [?] [substance to?] them [?]; "let them say of it," in assemblies, "always" — and even the walls in their tongues. And there are matters [requiring] deep inquiries and errors, "let none read them but a discerning man." Indeed, "good in this is the [knowledge?]." [?]

And according to the testimony of his pupil, he was likewise "thoroughly versed [בקיו]" in the knowledge of "the conduct of the dwelling" [Hebrew: הנהגת הבית? domestic economy — Greek: οἰκονομία, transliterated עקאנאמיא], and of "the conduct of the state" [the running of the polis — Greek: πολιτωκή, transliterated פאליטיק]: "not removing a matter they did not know" [?], nor would they consent to believe me [?]; and "my hand was practiced in their mouth, for a true [thing?] is established to me" [?] — "entirely it was a sealed thing in [?]"; he was [taking?] from there the books of the signs of [his?] memory. And he was [committed?] to teach only from old books, written by hand and on paper, and not to alter [from] them ever; and especially the books of the [secular] sciences [ha-limudiyim], such

as the books of the Talmud, or [those of] the Rif, the Tur, the Rambam [רמ"בם — RaMB"aM], and the books of Aristotle [ארסטו] in philosophy; and Euclid [אוקלידוס] [the *Elements*] for the foundations of geometry [Hebrew: יסודות, "foundations" — i.e. the *Elements*]; and Galen [גלינום] for medicine — so that he might be accustomed "to fill the lack [?] in the place of the [scarce?] in steady hand."

And he sent to set in order [?] the notes [glosses] upon the margins of the books — [the rulings?] of the law [Hebrew: דין] — twenty-eight in Bersima [?] [ברסימת ספפוקן, ?] of the Talpurn [?], and to set in order the notes [for] the [editor?]; binding leaves devoted [to be added?] at the end of the book. — And by this they would heal and bring forth [?]. A great physician was he, and [yet] he had nothing [of his own], [?] for he was not satisfied [?] with [what he earned] as his intelligence [warranted]; and he was burdened with the affairs of his medical practice and the [demands of?] the sick who surrounded him, [which] would consume his time, and his livelihood was [given?] in his hand. His habit was: "[only?] four [hours?] of the night — or six hours from sunrise — toward his right" [?]. And immediately [?], "the conductor [?] arises, and 'he draws out' from the works" [?]...

— תולדת הרב יש"ר מקאנדיא — (The Life-History of the Rabbi the Yashar of Candia)

and "set [?] upon the time of [?], the loving, and the abundant" [?]; only "the secrets [?] sleep two years upon his own table." [?]

And by reason of this [?], he was [obliged] to answer "the city" [?] [those who?] continually [posed] to him every kind of question; he would answer them [in?] many letters on every branch of [secular] wisdom — and also in [matters of] *Tanakh* [Bible] and the *posekim* [decisors], as is mentioned [?], by the way, incidentally, in the letter of the wise comprehensive Rabbi R. **Zeraḥ ben Nathan** [ז ר ח ב"ר נתן] (R. Yitzḥak? [ר' יצחק?]), from Troki [טרוקי — a town near Vilna], who sent [it] to the Rabbi the Yashar; for he was reputed for the merchants who came to his town brought him manuscripts [?]:

— Responsa to the MaHaRa"D [מהר"ר ד ו ד — Maharar David], "S.B.L." [סב"ל], on the matter of [?]; "Mukei Yosef" [Munei Yosef? — נמוקי יוסף, "Nimukei Yosef"]; "decisions of laws" to the Maharar Shalom [מהר"ר שלום] — and yet he had no answer [drawn] from his many letters. [?]

But for most of his books, "said to him the Rabbi the Yashar": and he wrote to him about his profound questions in the [secular] sciences — on the matters of the divine Kabbalah, philosophy, and geometry [Hebrew: הנדסה], whereupon all sixty foundations stand and the parts after them. [?]

For a great scholar was he — this distinguished [?] Rabbi — and a companion to those who fear God [?] and to those occupied in *Torah* in the city; he composed a book *B'turah ShaBaT P'* [?] [perhaps a title or a phrase], which is "filled with him always." [?] And he heard the name [of] the Rabbi the Yashar, the great in [the secular] sciences [?]; and so, for the first time, he wrote a letter to him, "to Troki," [a piece set] in measure [meter], its head and its foot [?], as if to say [it was composed in] honor and esteem for the Rabbi the Yashar; and the book and the man "to drip his teaching upon him" [?]. And the heart of the book is "but a little of the chronicles of the days of his life" [?]; and because the man had nothing [?] [to offer], only "the matter [of his?] poverty, with a metered poem [shir] at its end of the same kind as the first," and he composed about him in praise and adoration upon all his greatness; and at the end he wrote it [in?] "the way of the *cymbeln* [?] [the *Sefer HaChasidim*-style cantillation? — Hebrew: עורות טל[ע]הסחב, ?]," to clothe the body [?]. — Then he reasoned in his own mind to go to Lithuania, and he sought to comfort [?] [Vilna — לנא[י]וו] and its surroundings…

[The foot of the left column, in the editor's voice:]

to put up [?] for the editor [?]. (Note: [a marginal note follows.])

His book *Elim* comprises: laws of reckoning [arithmetic] and sixty [?]…[reading: כו' — "etc."], on the [art of?] preparation [astronomical/calendrical computation — תכונה, "astronomy"], on the arrangement of the orbits [the planetary system], on the heavenly bodies [גרמי השמים], on the [art of] medicine [ברפואות, "in [matters of] medicine"], on the craft of optics [Hebrew: מלאכת האלכימותא? — perhaps al-kimia, "alchemy," or "optics"] and the like [וכדומה], on fields [measured land — שדות], on standing crops [קמ[צ]ות], on planting [נטיעות], on animals [ניחושים? "auguries"?], on dreams [חלומות], on [vibrations? — סלצולות, ?], and on the lots [casts] of the eyes [וקסמי חוזי העינים, "and the divinations of the seers of the eyes"], on the recognition of the face [physiognomy — והכרת הפרצוף], and upon all these — on the wisdom of [its] parts [the constituent sciences]. And they asked to make known to us the particulars of the matter…the editorial commentary continues:

in the [matter of?] education [?], from our [?] [Belinson, the editor]. By reason of [?] — for he gives [it as] a perpetual [memorial?], and "withheld for himself" [?] — he wrote on this matter from his understanding [?]: thus says [?] *Sod HaYesod* ["The Foundation of the Foundation"], printed [by] our master [?] [R.?] **Mordechai Kohen** [מ ר ד כ י כ"ץ — "MoRDeChai KaTZ"] of Constantinople [בקאנשטא? — Constantinople].

— And they further requested that he explain to them, and his dispute with the Samaritan [Karaite?] scholar in Caesarea [בקאסיריאה, ?]; and he explained to them his questions. He composed [?] a letter [in which?] he received [reports?] from his pupil R. M[?] Mintz [ר"מ מ[י]נץ? — R. Moshe Mintz], honorably speaking of the matter of [?] [Hebrew: מסלמים? "of completions/perfections"], etc.; and at the end he wrote a poem [shir], metered [טע"פ — "according to the meter"], upon all his [exploits?].

For this the wise **R. Zeraḥ** [ר' זרח] received a letter at length [a long letter].

[Square-type display titles interspersed — the catalog of constituent works of *Sefer Elim* continues. These titles are set in larger square type:]

From the wise, the eminent, our master Moshe Mintz, the pupil of the Rabbi the Yashar [ז"ל], comprising responsa to the Rabbi upon his questions; and he further informed [us] of the chronicles [history] of the life of his teacher (except for [some] of them — he summarized for himself [?] many matters, and named them [?]); and he detailed for him [?]. He received from him [?] the books of the author [Delmedigo], and he explained to him the ten questions [?] [Hebrew: עשר שאלות, "the Ten Questions"] that the son of the Talmud [R. Moshe?] posed to him in Caesarea, [?]. Such is the Rabbi the Yashar [reading: כמל? — abbreviation], as our master [the editor] informed [us] of the matter to the youth [Belinson — נגריידינג? "Greenshpan/Belinson," cf. handoff]: for in order that there be sufficient [?] he traveled [?] to his place, "should they multiply" [?]; for [it was] more famous [?]… for they spread abroad to "cover up upon his acquisition" [?]; "since all the rabbis [?] of the [community?] of the [first?]…[broken]…and many thousands of Israel came down to hear him; and the house of the assembly was filled within it, and they brought [?] his father to [?]. Also [?] [when?] "for the radiance of his medicine was [?] in his house" [?]…

[Square-type titles below — the named constituent works of *Sefer Elim* (the second-level catalog), set as display rubrics interspersed in the prose:]

Sefer Karʾehu [ספר קראהו, "Call him"], **Maʿayan Ganim** [מעין גנים, "Fountain of Gardens"], comprising five [parts? — ס"ה = "in all"]: **Sod HaYesod** [סוד היסוד, "The Foundation of the Foundation"], on [?] the choicest [?] of the systems; **Ḥukot Shamayim** [חקות שמים, "The Statutes of the Heavens"], a return [?] upon [?] [three? — ג'] of the [?]; on the surveyors [?] of the heavens, of [whom] there is no [?] [Hebrew: כו … ג נ', ?]; **G'virat HaShem** [גבורת ה', "The Might of God"], on [the art of?] preparation [astronomy/computation]; **Magen Shamayim** [/ מגד שמים? מגן שמים — "Choice Gift of the Heavens" / "Shield of the Heavens"], "a vessel

pertaining to the [art of?] computation [astronomy]"; **Taʿalumot Ḥochmah** [תעלומות חכמה, "The Hidden Things of Wisdom"], on the wisdom of Kabbalah, and on the matter of fields, of [drawing-out? — כשוף] and of dreams, and the divinations of the eyes [וחוזי עינים, "and the seers of the eyes"], etc.; **Otot HaShamayim** [אותות השמים, "The Signs of the Heavens"], for the judgment of nativities [horoscopes — תולדות], [and?] for [?] times [?]; **Leḥem Avirim** [לחם אבירים, "Bread of the Mighty"], a new commentary [?] [Hebrew: על-ביצירים הדשה, ?]; **Megalleh Amukot** [מגלה עמוקות, "Revealer of Deep Things"], the secrets of nature, and on the craft of [chemistry/elements — סכימיע]; **Mekor Binah** [מקור בינה, "The Source of Understanding"], "the abridgment [Hebrew: סנחקס ?ספקס?] of the Greek tongue" [?] — for the holy congregation [ללהי"ק = "for the holy community"], the [book? — ס'] *Apuristomos* [ס' אפוריזיתוס — Greek: ἀφορισμοί, the *Aphorisms*] of Hippocrates [לחיפוקראטיס]; **ʿEtzem HaShamayim** [עצם השמים, "The Substance of the Heavens"], "a vessel woven through the life of [its?] composition" [?] [Hebrew: בתוך רחיות חריבנו?], "the fifth" [a fifth part? — בנשם החמישי, ?]; **ʿAmudei HaShamayim** [עמודי השמים, "The Pillars of the Heavens"], "spread over all the [astronomers? — הכוכנים = חוכנים, the stargazers/computists]," and answers Copernicus [קופרניקוס] "a youth [?] on the inquiries of the [?]"; **Maʿayan Ḥatum** [מעין חתום, "A Sealed Fountain"], "strange matters" [דברים זרים] for those who hear them, [such] that they would be called paradoxes [פראדוקסס]. From these he [printed?] only [the?] [following four titles, set as display rubrics]:

[Square-type display rubric:]

Sod HaYesod [סוד היסוד], **Ḥukot Shamayim** [חקות שמים], **G'virat HaShem** [גבורת ה'], **Maʿayan HaḤatum** [מעין החתום].

— But the rest of the parts [the editor explains?] he did not put into print; perhaps because the author did not have time [?] to arrange [them] as fit, and according to [?] his arrangement [?]. The great [author?], "at the end of the *Sefer Elim*," said [?]: "I am afraid lest [?] I copy them, and [arrange?] a year — for they are weighty [?] — and I am not able to complete [?] them until the [?]." From the [author's?] labor [?] he was hindered [?]; he [therefore] wrote a long introduction, which is "a precious gift" [?]; and "the rest he left for the Maharar [?] Z[era]ḥ [HaKohen?]," nearly [?]: "There is no equal to him in all his letters that have come to me [?], in the state [?]," "engraved [?] for [?]" — "thoughts there is no end" [reflections without end] he has on them, [made] "in the [secular] sciences [?] [be-chochmat ha-shimush?] from him." His pen [?] is exalted [?], the [whole?] arrangement [?], bringing [?] life [?] in good intellect; "and they put on according

to the measure of his intelligence, and they understood [?] in his [?]," [such] that they were "[as] they were" [?]. And he writes at the end of the *Sefer Elim* on a matter [?]: "[that] there occurred to him several and several prophecies [?] [Hebrew: נבואות, "prophecies"], when he was in [Vienna? — בויניאה?], 'he read [?] a verse [?], and a man [?] arose [?].'" In the year [5]363 [תכ"ג? reading = השע"ג 5373/1613? — see anomaly note] they called him [?] "Moshe is dead, and his heirs [?] [are] established"; and [?] he leaped [?] [as?] "a vessel." Truly [?] he had a son in Constantinople — his name was Moshe — and they [made him?] a son of the children of Israel, "for this is the peace of [?]" and salvation. — And so it was when [his ship?] foundered [?]; the boat sank, and he…[broken at the column foot].

— תולדה הרב יש"ר מקנדיא — (The Life-History of the Rabbi the Yashar of Candia)

the rest of the [titles?] (continuing the catalog above): on land-surveying [?], "**Maʿayan Ganim**, in all five" [a recapitulation of the five-part *Maʿayan Ganim*]: **Sod HaYesod**, on the choicest of the systems [?]; **Ḥukot Shamayim**, "a return [?] [Hebrew: בחור על ג', 'choice upon three']"; "and the surveyors [?] of the heavens, of [whom] there is no [?] — **G'virat HaShem**, on [the art of?] preparation [astronomy]; **Magen Shamayim** [מגד שמים?], a vessel pertaining to computation [astronomy]; and on the matter of [?]… .

Taʿalumot Ḥochmah, on the wisdom of Kabbalah, and on the matter of fields, of drawing-out [כשוף] and of dreams, and the divinations of the eyes, etc.; **Otot HaShamayim**, "for the judgment of nativities" [Hebrew: ישפט תולדות], "**Leḥem Avirim**, a new commentary"; **Megalleh Amukot**, "the secrets of nature, and on the craft of [chemistry]"; **Mekor Binah**, "the abridgment of the Greek tongue" — for the holy congregation, the *Aphorisms* [ס' אפוריזיתוס] of Hippocrates; ʿ**Etzem HaShamayim**, "a vessel woven through the life of [its?] composition," "the fifth"; ʿ**Amudei HaShamayim**, "spread over all the astronomers," and answers Copernicus [קופרניקוס] "on the inquiries of [?]"; **Maʿayan Ḥatum**, "strange matters" for those who hear them, [such] that they would be called paradoxes. From these he printed only four [reading: רק ד', "only four"]:

[Square-type display rubric:]

Sod HaYesod, **Ḥukat Shamayim**, **G'virat HaShem**, **Maʿayan HaḤatum**.

— "And the rest [of the parts] he did not [put?] into print, perhaps because [it] did not suffice [him to] arrange [them], and because the writer [?] [wished?] to separate the parts [?] at the end of [the?] *Sefer Elim*; he said [?]: 'I am afraid lest I copy them, and arrange [them] a year, for they are weighty, and I am not able to

complete [them?] until [?]'...[broken]...in the work [?], 'good in me, and creating [?].'" For "[?] holy ones [?]," he wrote a long introduction, which is "a precious gift, the choice [?] of the [?]" — "to the Maharar [?] [the?] whole," "almost there is no equal to him in all his letters that have come [?], in the state [?]." "Reflections [?] without end he has upon them [?] in the [secular] sciences [?] [be-chochmat ha-shimush?] from him." [His?] pen [is] exalted [?], the arrangement [?], "bringing [?] life [?] in good intellect"; "and they put on according to the measure of his intelligence, and they understood [?] in his [?]," [such] that they were "as they were."

And he writes at the end of *Sefer Elim* on a matter [?]: "there occurred to him several and several prophecies [?], when he was in [Vienna? — בויניאה?]; he read [?] a verse [?], and a man [?] arose [?]" — and immediately [Hebrew: ותכף, "and at once"] "the cabbage [?] [Hebrew: כרוב, possibly 'cherub'] [grew?]." [?]

In the year [5]363 [reading: השע"ג; AM 5373 = ג3 + ע70 + ש300 + ה' = 5373; 5373 − 3760 = 1613 CE? — see anomaly], "they called him [?]" [Hebrew: קראהו? "they called him," echoing the title *Kar'ehu*]: "Moshe is dead, and his heir [?] [is] established; and he leaped [?] [Hebrew: והקף בהקילו] as a vessel." Truly [?] he had a son in Constantinople — "his name was Moshe." — And they [made him? — ויעשהו] "a son of the children of Israel," for "this is the peace of [?] and salvation." — And so it was when [his ship?] foundered [?]; the boat sank, and he [was?]... [broken at the foot; the narrative continues beyond this opening].

He, R. Yashar [Delmedigo of Candia], had himself composed twenty-two mounted instruments [כ"ב מוכנים, astronomical devices] to gather... [text damaged]. He examined and tested all of these, together with the rest of his instruments [?]. The honor [or office] of Sagan [deputy] fell to him in [?], and he too [?]... a precious instrument [?] of brass to know the constellations by day with [?] on Shabbos. — And nevertheless, since these matters were hidden, one might say that he heard them from others, and saw only a little from afar, in fragments, even up to recalling twenty-eight... in his use [of them]. And concerning the matter at hand [?], a witness, [and] the pious will say: For they are not [merely] hints up to here. There they say that the place is to the north [?], and [the word] פס — in [the Romance form] *manorlit* [?]. And yet, since the words of the Torah have no exact measure or definition, neither did anyone instruct him about them in [his] childhood [?], even though [?]... [text fragmentary].

So how strong he was in geometry [בגלילות, the rounded/spherical sciences], even though it is not certain and well-established [?]. And he was weary [?]. It is also easy to understand from what was written above concerning what [?]: that

whoever practiced these acquisitions [skills] here, by means of those who longed [for them], learned this — and as they said, "On the day that I longed to go out [?] in [?]"... For according to my [?], by my own opinion, indeed [?] in his birthplace [?] there was the basis [?], the *kalakta* [?] went to Bnimusa [?]. Therefore I marveled only at his learning. The *Sasim* and *Pasekis* [?], and still more at the sciences, also the *halacha* and the *Murai* [?]. —

2) [Footnote 2] And from the few signs [?] that were disclosed about him — and we did not know their particulars in detail and exactness [?]. Is this not so? The words... in *Kalkra*: "Children of [?] the generation [are] *Sdunai Yedukni Yerukni* [?], and I did not fight with [?] *tultli* [aimlessly?], and over my own ways [?], for I have no redeemer but myself, and who will have pity upon me but myself?" — And as is the way of those who travel to distant places, when I made ready for the inn [the lodging], there was no bread of *manoh* [provision] [?] for my journey [?], and [the verse] "the wise of heart will take [precepts]" [Mishlei 10:8] — but the order [?] of the [Romance: *summakka*] brings out the new [?] for the voice of the spirit [?], "for money answers everything" [Koheles 10:19]. —

3) [Footnote 3] And so it is hard [?], since he found neither a good companion nor disciples to serve him in all his needs, and lacking *gemmonen* [?] — by [?], that He hid himself there; and as he said in *Kalkra*: "For behold, your darkness shall be... darkness; if not, for *manoh romeh* [?] your way went [?]." And the disturbances [?] with [?]. — Has not the youth been brought close [enabled] for the sake of the multitude of the *anasim* [the coerced ones?] — and also those who long for the Torah, all of whom became *re'umlundis* [?], and the learnings... a deep matter, hidden — to say there is no inclination [?] in the wisdom of the Greeks [Yavan] — to be a [?] like [?] and to take good instruction [ולקח טוב, cf. Mishlei 4:2]...

[Right footnote column, fragmentary, partial lines visible: "...to gather...", "...we knew...", "...and we counted...", "...this...", "...the second...", "...the matter...", "...Greenwich [?]...", "...the city [?]..." — abraded marginal annotations keyed to the main text; content too partial to reconstruct.]

and from this there was a path to the divine science [astronomy/metaphysics], and before [it] also to the *Yisrael* of every science and discipline, melting [refining] them — set as a sign for a portion [?]. In the days of the Nazirites, the crown of the [Temple] court was upon them — Jerusalem... — and the sages of the *Techinas* [petitioners?], filling every aspect of it [?], to [?]... its law (as found in the

Talmud), and there was no [?] of its law — how they *yikahu* [take?] *rinas* and the trustworthy *kinas* [?] — for the whole *takchamu* [?] and... it. —

And behold there is among them — even though R. Yashar [Delmedigo] in his wisdom is R. *Arach* [?] — ten [?] in every science and knowledge, and a wonder in *bekanlas* [?]. — And there was in three [?] tablets recounting to the sages distinguished in all the breadth of the world [?], and in all the tongue [?] he gave and recounted (alone), and the order of R. *Hamol* [?], R. *Smachas* who fought [?] in *Binilias* [Venice?] [?], and the chief [?] R. Yaakov *Hachamdarni* [?] in *Kamira* [Candia?] [?] — and *Sfalin* [?]. — R. Yashar attached to the book, saying: For I heard that the holy spirit of the holiness of *Rashbi* [R. Shimon bar Yochai] poured forth upon... portions... your study [midrash], and your *Hozer* [returning one] empties [it] from upon you, and these things — the *darshomekh* [?], etc. — Indeed according to its [hidden] depth [?], for the sake of those engaged with their *parnasimas* [livelihoods] or with the burden of taxes [?], they could not — to weary the inner [reader] from the outer — except by the sages [?].

And R. Yashar fought to say: "Why is it not yours [?] to go upon a path that... has profited [?] — and we have spent days and half the journey, in the storehouse [?], the *Mamon* [wealth] and the labors of the body — so that all who dwell [study] *Yisrael* shall know what is the desire to learn and to investigate [?]," etc. — And how much more... and your darkness [?]. (The above-mentioned three [?]) — For I have established that in all the lands of *Steverki* [?] [Italy?] I do not know... of the investigators and seekers in the disciplines — many times I went up [?] to learn from his treatises [?] of wisdom; for the *Murai* he made [?] never abandoned [?] the burden of the *chatztonos* [?], and I fought [?] with one... the prop [?] of the trial in *nisayon*, and he carried and bore [the yoke of?] learning. —

Indeed, since they think the words of the leaders [?] and *Smachus* [?] are confused — yet like a clenched fist [?] — and they hastened... and how... [?]! But behold, this is the *Tasores* [?], and the *temimas* [the pure?] with the *Masik* [?]; and [the words] did not avail him [recte: did not rise to him] in [the manner of] *Rashbash* [R. Shlomo ben Shimon Duran?] [?]. There is none of this [?], for [?] the *Kanukl* [?]: as is brought there. The reward of the *seturim* [?]... will rest [?]; with the *Maru* [recte: their?] all is not exhausted [?].

And it is depicted [in a portrait] [?] that R. *Mits* [?], who is depicted/engraved, his disciple [?] of *Movosk* [Moscow?] [?] R. *Tabes* [?], rested upon him the spirit of R. Yashar [recte: יש"ר], in *Steverki* [?] [places]... [the matter is hidden].

[Right footnote column, fragmentary: "...Israel...", "...R....", "...comprising...", "...vayifen [and he turned]...", "...veyatzi to...", "...the author [?]...", "...the great [?]...", "...wisdom...", "...the author..." — partial annotations.] [Left footnote column, fragmentary: keyed marginal notes including "...the second...", "...the burden [סבל]...", "...the storehouse [חסרון]...", "...etc. —...", "...likewise [?]..." — too partial to reconstruct.] "...truth [?]; and even though there is no judgment with the *Mekubalim* [Kabbalists], to bring near the wise of *Kofris* [the deniers?] — and these are the kinds [of objection] that arise from a certain quarter; for the believers [Kabbalists] are correct in the principles of the Torah, and [the others'] good ones are aligned — yet a spirit [?] to those whose [?] of *Gorfin Bitzan* lays a *misilla* [highway], dissolving things in [their] portion, etc. And these are to know the matter of [some] souls [neshamot] and the angels — they will search [?] every [?] the Kabbalists, and the proper one his opinion, as he goes after the disciples of his teacher [?]. Are not these the sermons [?] — the chief of which are *yichalem* [?], reward and punishment — and his hand [?] shall not avail [?] to fly to the *dekdukis* [minutiae?]: twenty-eight [כ"ח], heart [לב], anger [רוגז], the clearing [כליון] of eyes, and the reckoning of the soul. — And the *Mata* [?] that occurred... is there for you merit, a portion. Therefore [?], thus *Sarbah Yegtani* [?], and I have read in books, and I did not find in my sermons [?] up to twenty-eight [כ"ח] of the sages..."

And what is *m'tat* [?]? — *vaMaShi"t* (folio 4, דף ד'): "*Yala* [?] recounted [?]..." [Cross-reference to folio 4.]

He found a path to defend the Kabbalah against the philosophers, to gather the truth [?] from each of those known to us. And what [?] from a different spirit moves... and he attributed it to the philosophers — also that they go upon their *Mussis* [recte: foundations / *yesodot*?] — for it is a great principle: that there is no descent to the depth of the authors' opinion from within their books, unless there is a Rav [teacher] who relates [it] to his disciples and explains to us everything concealed in such allusive [*re'umis*] matters as these. And it is known that in his book *Beshamath* [recte: *Behinat HaDat*? — or a work-title] he revealed his pure opinion without flattery [?]... about its words, all this withdrawn a little [?]. The Sagan, the lofty one [?], and the *Smahkadam* [?], by R. *Yashar* [?] alone... depicted in *masuva* [?]. And those *Mitchakmim* [who play the philosopher] [?] by themselves were able to draw [interpret] each opinion of the author toward whatever his way may be [?], and to twist his subject-matter and his heart-meaning [?]. But indeed, whoever the man is — R. Yashar [recte: יש"ר] — for the sons of his generation, the great rabbis who interpreted him gave him preference [?] and called him by all the titles we have heard — twenty-eight [כ"ח] — they

assembled in the name of our teacher [?]: "*Nachon* [the established]," "*Chassid* [the pious]," "*Mechudad* [the keen-witted]," "*Eloki* [the divine]" — as is detailed below; and who is the wise man who will understand? For he did not write this [?] only to glorify and exalt him in his way; and they were precious enough to recommend this for him: the Rim [?] Gaon, of blessed memory, who in his treatises turned aside entirely toward the ways of the philosophers — so it stands to reason — they too with the Kabbalah; and they did not engage with the philosophy except to encompass [refute] the errors of the men of his generation and to bring the lost opinions [back]... in a nearness close to all who dwell upon [study] our Rabbi [?]. — And it is a wonder [?] R. Yashar [recte: יש"ר] did not write, for our [?] is far from one who [?] is near the *Suss* [horse?] [?] — yet he can come close to that which is far. And these are his [?]. (And when I was made *roni* [hidden?] over the words of R. [Yashar]...)

[Tefillin material, main column:] (folio 24, דף כ"ד) In the land of Candia they were accustomed to lay tefillin, as in every land — Ashkenaz and Poland (except that the *Roses* [?] departed [?] who were stricter [?] than the rest of Ashkenaz, as explained above). — (folio 25, דף כ"ה) In the city of Candia, whose name [?] was always established for judgment, the great ones of Israel in every generation laid tefillin from [the time of] *Tamuz* [?]... until the elder Sagan, our teacher R. Eliezer [?], may his Rock keep him — Delmedigo [*Dilmidigo*] of Candia [?]... — he learned [?] from all the Geonim, our teachers... [R.] *Yalmak* [?] of Birtz [?], and *MaHaRaY* [?] [our teacher R....] of *Kanren* [?]; and the custom was annulled [recte: ביטל המנהג]. — Tefillin by the hand of *Smas* [?], the custom for laying [?]... (folio 26, דף כ"ו) R. *Zoher* [?] explains, regarding the interruption [hefsek] between eating meat [?] for the body [?] that is not lacking, only in a new [matter]... and [the words of] the *Tashbetz* (Siman 514) [?], paragraph 415 [?], section 156 [?] — and the principles according to his opinion [?]. — Every matter difficult [obscure] in the Torah that is found in the *Zoher* — *Kotzar* [the brief?], and R. *Las* [?] in a special book. (And it happened [?] that what was said concerning R. *Zoher* adds [?]: And when it took *sisra* [?] in their *masronos* [Masorah?], etc. — and twenty-eight [כ"ח] divides... in its *malvah* [?]. And [the words of] *MS"K* [?]: there are in them inner *kavanot* [intentions] — *Tayin*: a settled matter [?], to this *Bat*, the words of R. Yosef *Sevag* [recte: R. Yosef Solomon? — uncertain], the wise, our teacher R. Yosef *Sevarch* [?] of Jerusalem [Jerusalem] in *MaHa* from him — and the author [?]... the *Maskil* [enlightened one] R. *Sdal"R* [?] — and the *Maggid* [preacher]... thus, folio 26 (דף כ"ו).) —

(folio 28, דף כ"ח) The titles of the philosophers — their words [are] more correct [?]: He spoke against *Aristo* [Aristotle], and explained Plato [the *Falton*] in

particular, that their opinions are like a little... the opinions of the sages of Israel; for were it not for them and our ancients [our predecessors], the inheritance [?] of the Greek systems [?]... (folio 29, דף כ"ט [recte: 30 ,ל"?]) He brings a clarification [?]... of the philosophers, from the sages of Israel in the matters of the angels — that in this way [this path] is a foreign path [recte: דרך זרה] to learn — and as he wrote concerning it: "And we have... what answer shall I give in your honor [?]? This is what is fitting [?] for the science to *l'taks* [?]..."

[Right footnote column, page 3, fragmentary: marginal annotations partly legible — "...truth [?]; and even though there is no judgment...", "the believers...", "...portion...", "...the city...", "from the tongue...", "the Hadar [?]...", "...we fought...", "...and a wonder..." — these duplicate the head of the main column and are keyed footnote lines.]

And R. Yashar further added strength to his disciple, the *Maryyacha* [recte: the מורה / teacher?] — for besides the support [*Samuch*] that he laid as *MeTilkot* [?], moreover the author [recte: המחבר] shall find in several places of the Talmud and Midrashim those that agree with his words, as in the example in *Chullin*, folio 15 (חולין ד' ט"ו) [?]. For my teacher had a Rav to teach disciples [?]; he taught them an explicit teaching [?], and since he expounded at the head of a chapter "the beginning of *deresh*," etc. — its foundation [yesod] is well-established [?] — and *Rashbi"t l"b* [?]... [the matter] is hidden, on which there are *l"b* [thirty-two? — לב] disputes [?]. He shows it as a great measure, and in some places the matter extends to a different Mishnah... rather, this Mishnah they established entirely as a whole teaching, and from another — indeed, an order [?]. The exposition [deresh] to *L'enavolulam* [?] arose, thus, to the Mishnah (and according to *Rashicha"f* [?] — but it is hard to align his words with *Kalra* [?]: an end [?] for *La'arim* [?]). And whoever is older [?] and has entered into all the authors and teachers and reasonings according to the rule of R. Yashar [recte: יש"ר], so that twenty-eight [כ"ח] *yivneh* [shall build] in them for himself, and to particularize in details, in their contemplation [*histaglos*] each according to his own opinion, in matters hidden from the world [eternal matters] — for the opinions are not equal, nor are all the places like one city [?] — and the court [*Hatzer*]... [fragmentary]. — And the matter [?] knows that one knows how to incline at times after his own opinions, to value them [?], to gather and to align every drop [?] from the opinion of *Sisras* [?] and *S'mahosres* [?] to the general communities of Israel [?]. — For who fills them [?] their opinion [?]... [Romance: *sukkonas*?], a reason for a reason, etc.

And R. Yashar sealed his Siman [section] with his words thus: **the essay [המאמר]** that I sent to be written in *Samburg* [recte: Hamburg], and because of [recte: ומחמת] the *Smanapas* [?] of R. *N"L* [?]... my recollections [*zekhirot*] for *Por Ragli* [?]; and I also came *fas* [?] to a new city [*ir chadasha*] in *Glikshtoteh* (in *Holshtein* [Holstein]) — and as it is not [?] — to no city was it [?], nor blessing in it, until I made up my mind to travel from there to *Amsterdam*; and therefore I am [bound] to the readers in the place where I am wakeful to dwell. — And also those who lean toward philosophy will say of me in [their] complaint, "R. Yosef of Candia [Delmedigo] to [side with] the philosophers and the *Ruach* [spirit?]" — indeed assembled... — But behold, I knew that the elders [?] would say "his heart is to the *Razon* [?]," and to the *MaH"l'noganim* [?] he said: "let the place [the verse?] be brought for me from the Scriptures," etc.

And R. Yashar came to the congregation [Kahal] of Amsterdam — in the year הספ"ה [the year-word is printed so as to read like *ha-sefer*/*ha-sap"eh*; as bare gematria 5145 = 5000+60+80+5 = ה'ספ"ה AM (= 1385 CE), which is impossible for Delmedigo, so the ס is most plausibly a worn 5385 = ה'שפ"ה :ש AM − 3760 = 1625 CE. Transcribed as printed and flagged] [?]. It was the year *l"n* [recte: ל"ה, the 35th?] of the days of his life [?], and there were [present] teachers and *Re'em-Tav* [recte: ר"ם ות"ת? — rabbis and Torah-teachers] of the congregation of the Sephardim. And everyone came to him who wanted [?] — what is this wisdom [?] [in which] he walks [?]; and whoever turned aside came to him [?], so that every Shabbos the head [*Resh-Kaf*?] preached a sermon [deresh] in [their] assemblies, and taught order [?] and statutes [chukim] and the knowledge of holy ones [da'at kedoshim] here, until they were astonished there, and there was no more like it [?], and here...

[Menasseh ben Israel passage, main column:] For *Sefer HaSafrir* [the recounting book?] *Bonkayos* [?] of R. Yashar, a wise man who comprehends the inquiry [hoker], the divine portions [astronomical sections] — **Menashe ben Yosef ben Yisrael** [Menasseh ben Joseph ben Israel], of blessed memory — *Bat* twenty-eight [recte: ב"ח, in the year...?], the teacher and arranger [*Sorhas*?] for the congregation of the Sephardim [Kahal Sefardim] — to *L'safitz* [?] the answers, his pure wisdom, in the printing house where they printed [it], gathered in *Amsterdam*; and there remained to R. Yashar — and he set up [established] by the hand of [Romance: *s'olam*] and *d'chalki* — **the *Maʿayan Ganim* [מעין גנים]** (printed there in [the year] פ"ט [recte: 5389 = ה'שפ"ט AM = 1629 CE? — abraded]) [?] — for in most cases his reasoning could not turn away his face empty[-handed]; yet they were not engaged [?] in all of it in his lifetime, and these were aligned [?]... the chronologies [?] *Kadar* [?] for a matter free of *kanf* [?]; and

besides [there were] two precious treatises that he set always before his mouth [?]: the precious book "*Yakar Sa'ach* [?]," like the treatise of **the Rambam [Maimonides]**. And the book *Beshamath* [?] (see there [?]) had a teacher [?] upon it; for the disciple [?] who needs much silver [money], a great hardship [came] upon him from one [side?], and he planted [appointed] his disciple *Talsing* [?] *Bachuto* [recte: בחוטו / in his line?], and twenty-eight [?] in the recounting [*Sefarat*] of his disciple, the wise *Mus* [our teacher R....]... *Tav-Ben-He Min* [?] [recte: a chronogram?] *Nahanu* [?] a matter in him, even *Or HaGadol* [the Great Light?]... their hands [?]; and they said: "Our books that are not... in vain [b'chinam] [are] precious; whoever acquires them, blessed [?] their day — eat [*achlu*], etc." And these are very precious treatises [?]. R. Yashar composed [it] by himself [?], at once [?], in the writing of *Sarpas* [?]. And he stood teaching up to the *Tagonomid* [?]... what *machris* [?] is in the matter of *HaSagnai* [?], to learn for himself [by himself], for I [?] did not change [?], for *Shlomo Almi* [recte: שלמה אלמי?]: predecessors to *Shlomo Zulasi* [my own Shlomo?]. — And there came [were printed] the images [portraits] of the disciples of R. Yashar — the wise [one] comprising our teacher R. **Shmuel ben [recte: במהו"ר] R. Yehuda Leib** [recte: שמואל ב"ר יהודה ליב] of the wise [?], of blessed memory. A great collection [קובץ גדול], *Kaf-Kuf* [?] in the name of **the *Taʿalumot Chochmah* [תעלומות חכמה]** on the wisdom of the Kabbalah, placed in a corner [?], a corner of the house of R. [Yashar] in *Amsterdam*; and the *Malchis* [?] obscure [*afel*] and *chazak* — they requested it from him [?] by hand, and they printed it, all of it from his money [purse]. And it does not appear that... a matter, for they brought it to bind [edit] and repair it as it appears [?], and they printed it again — by themselves, with a refinement [tziruf], after the answers that they discoursed [?], each of the great ones; but his disciple knew the matters that R. Yashar engaged in his ways [?], to honor and glorify [?], the *Meruzim* [?]... and his sermons [drashot] that he ordered upon them [?], all of them, [that] the spirit rested upon him [?] for the congregation of the whole camp [Kahal kol HaMachaneh], set [established] his spirit in order, as it is fitting [?]; thus far (ע"כ — *ad kan*) [?].

[Right and left footnote columns, page 4, fragmentary: keyed marginal lines including references to "the order [סדר]", "Rashbah [?]", "the essay [המאמר]", "and because [ומחמת]...", "the city [עיר]...", "...went [הלך]...", "preached a sermon [דרש דרש]...", "here, here [פס, פה]...", "thus far [ע"כ]" — partial annotations keyed to the Amsterdam-arrival and Maʿayan Ganim passages.]

[The 1864 editorial *Toledot* (biography) continues from the previous folios. The text is set in two columns per page, read right-to-left; it remains continuous

editorial prose, with no שער/פרק/סימן/מאמר rubric. A numbered catalog of Delmedigo's works is embedded within it.]

they came forward and offered to print whatever he asked of them. By contrast, he received from one of the printers many quires of paper [?], yet he did not consent to print at all. He named [this collection] *Nivlot HaChochmah* [literally "the withered fruit of wisdom"], saying that these are mere offshoots [?] — that he had not exhausted himself over them, but had gathered them in passing, along the way, and from the recesses of his thought [?]. From the very outset he had withdrawn from such matters. — [Here] the disciple's words conclude. — His teacher, by contrast, knew his own worth, and so was unwilling to print them. Their cause [?] led [some of] the scholars to err over them, saying: "We have toiled, yet found nothing." [cf. Megillah 6b] By contrast, [the texts in question] are well-ordered and lack [nothing] — for the reason that he composed them at a great remove [?], from the recesses [?] of the philosophers [?], and from the end of the [field of] philosophy.

He honored them with the glory of distinction [?], for in his lifetime he was redeemed [?] in Padua [?], delving deeply into the study of Aristotle. He was unable to endure [?] a matter set down without [proper] intent — and this is true in general. And so it was not settled in his heart to fathom [?] how two kinds [of writing] could come from him — yet this is the lamp [by which] the life of a man's king [?] proceeds: according to the philosophers, the disputants, and the great kabbalists.

The honored elder, Maharar [our teacher the rabbi] Yosef Yaakov [?] son [?] of [?] our brethren, and the men set [over us as] a holy portion: the physician, Maharar Shlomo [son of?] our brethren, of blessed memory, the heavenly one [?]. — Then he began to bring forth from his store [?] new [works] — and so [it was] in the order [?] of his composition. And this is the twenty-seventh [?] book of the rabbi, [composed] for his pupil at Toledo [?]. —

He began to set his heart to [the work of?] his teacher, and printed them without his knowledge in Hanau [?] in the year ה'שפ"ט [5389 = 1629]. [These are] *Nivlot HaChochmah*; [and] *Sefer Yosef* [?], like [?] *Sefer* [?], *Klalei Olam* ["General Principles of the World"], a treatise [?]; and the *Tikkun* [?], with still further gleanings — all of them new [?], on the topic of Kabbalah. — And he printed *Sefer Matzref LaChochmah* ["The Crucible for Wisdom"], and so forth. — And it came to be heard concerning the rabbi YaShaR…

in their language. — Here is what is written concerning [?] YaShaR: "He fell, [unique] as one, in the manner [we have explained?]; — his books were lost from his hand [?] and were not [?] in ink. — Many [?] [things] hung [?] upon his words at Krakow [?], remarked [?]: 'For the [?] honor, all that the rabbi [?] commanded [?] does not [proceed?] of my own portion [?]. This is his way always — to say to me: come, my sons [?]' and I saw, and behold the matters that arise [?] are not [?] [as they] saw [them]. To take from these [?] the appointed treasures [?] — and why [?] did they reverse [?] [his] standing? — and they hid [?] my heart, but they did not [?] wrong me; rather, as they wrote, you have [?] established [?] for [me]; — they did not gather [?], they did not stray of mind [?], and they remained [?] words clothed in [?] all [?]. — Even so, what was honored, how heavy [?] the cup of [?] reproach [?] [from] the deep things; how concealed [?] the deep words — and I do not [?]..."

— I have not [?] [the power?] to set right [what] another [?] has spoiled [?]; only contented [?] is one whose lot it is [?]. My own acquisitions [?] are valued [?] by me; but my [?] is to Beit Yehuda [?], and I designate them [?] for [?] this judgment [?] — for they bind [?] every [?]; the world [?] sees the wicked [?] [and they are] not [?]. They saw [?] my honor [?], and so forth. — And as for what you informed [?] [me], you have nothing for me — neither sons nor acquisitions [?]; — but my strength fails [me] [?] when [my house] is full of books — and there is no longer any precious book in the world [?] which his gaze passed over and did not delight in [?].

And often I have wandered [?] [over] how many [?] places — Pressburg [?], Basel [?], and Frankfurt [?] [?] — in order to see [?] a single small treatise [?]; — and even when the costly vessels [?] were before me [?], they were as nothing in my eyes. And so, too, [it was] with all the books of the entire host [?] [that came] from [?] [my] master [?]: Alexander [?] of Frankfurt [?]; through Kabbalah they came into my hand. — But kinds [?] [of paper?] do not suffice [me] — for his disciples were [as] thorns [?] — it was not [so]. And I, with that [my disposition?], that I take to me [?] half and [?] vex me [?]; this is the meaning of "his roving [?]" — for [thus did] Zercha [?] also climb [?] [and] go forth to *Eretz HaTzvi* [the Land of the Gazelle, i.e. the Land of Israel]. And he [?] introduced [?] a book [titled] *Etz HaChaim* ["The Tree of Life"], and this is the matter...

that contains [?] within it, in the words of the planters [?], and so forth — for in the depths [?] [there are] ancient things [?]. And it is not [a fitting?] [task] to repair [?] what another has spoiled [?]; only contented [?] is one whose lot it is [?]. [Here is] my acquisition [?]: weighing words [?] and exacting [their measure], and

I traveled by way of Beit Yehuda and recorded [?] who would [?] give judgment [?] — for they bind [?] every world [?]: their offshoots [?] are not [?]. — They saw [?] my honor [?], and so forth.

— And further: matters [were] put down [?] without order — neither weighed [?] nor refined [?], and out of order. There is [a way] for you to divide [the work] into chapters or essays [?], and so forth — and I have arranged everything that I wrote, [recording] all my births [?] [productions] in the chambers [?] [of my mind?]. Order upon order [?] — every matter together they shall make known [?]; and so they have set out [?] in Rome [?] — and as many [?] [things] as I have written, [I wrote] only to one another [in private]. — Better [?] is the taste [?] of all who deal in [?] holy books.

— [The disciple's words], like [?] [those] written within YaShaR['s own books]: "He fell, [unique] as one — written [?] in the [?] manner [?]; — kinds [of letters] [?] are erased [?] [as if] by [the printer's] mouth [?], by his pen [?], and not in [proper] ink. — Their words [?] ended in the [?] form of a verse [?], a thorn and a brier [?] — what one [?] did not [?] [in the manner that?] scribes and scholars [?] are wont [?] [to record?] in their book [?]; — and they did not know that out of a flaw [in the type?] came [?] their corruption [?]. — And these are the writings [?] of the rabbi YaShaR, according to what [their] memory [?] [preserves], in the places [where] they are scattered [?] among the books [?] he printed. — (Besides what was assembled [?] by the hands of the author [?], of blessed memory, as stated above.) — They include foreign words [?] and foreign letters [?]; he added to them [?] a tariff [?] of expenses [?], an entrance fee and an exit fee [?]: —

א) Yaar Levanon ["The Forest of Lebanon"], a great treasure [?] which he began to write in his youth, but which he did not complete, since [?] [the printers] withheld [?] paper [from him] [?]. End [of item]. — [It comprises?] four [parts?] of disputes and answers [?], encompassing all the words of [their] disputants [?] in [Scriptural] verse, on thorn and brier [?] [cf. Yeshayahu 7:25], such as no scribe had [yet] arranged; he received [these things] from his teachers, and he completed them with the books of the philosophers — [containing] secrets and hidden things [?] and matters [of nature?]. [There is?] a portion [missing?] from him [?], hidden away [?] [and devoted] to the engraver's craft [?] (and its place is left blank, to be supplied [?] in [its] portion of treatises [?]). —

ב) Be'er Sheva ["The Well of Seven," or "The Well of the Oath"], comprising all the roots [foundations] of all astronomical inquiry [?]. [The student?] of this discipline will fill himself with [knowledge] to the [four] quarters [?] of the compass [?], just as [if] he had learned in several hundred books; — [it] concludes

[?] in the manner of an arrangement [?] [covering] the principles [?] of [several] regions [provinces], [drawn] from the numbered host [the heavenly host?], arranged according to the methods of medicine [?], in the order of study; — [these are] the names of many books. — And at its end [is] the treatise *Tachlit HaAdam* ["The Final End of Man"]; and it is glorified [?] [as] a precious [?] cedar [?] in [its] language and tongue [?] — a book full of wisdom and perfect in [its] beauty, as [in the verse] "perfect in beauty" [cf. Yechezkel 28:12]. And [its arrangement?]: a treatise [?] [bearing] another title, *Be'er Acheret* ["A Different Well"], will mention his [own] earlier [?] things [?], and the wonders and roots of [worldly?] events [?], in [its] other treatise [?]. And it is [?] hidden [?] [from] the [?] [public?]; and [some of] them are hidden away, also to be supplied at the [proper] place [?]. —

And among them they printed the book **Matzav P[i] HaBe'er** [or **Matzav: Pi HaBe'er**, "The Standing-place: Mouth of the Well"] [?], in which he set down dear [things] [?]; in it [?], to clarify the order of theology [?] and the genesis [origins] of all the sciences, together with the books of the surveyors [?] of [the field?] [?], for them; — and at the end of the book, [there is the treatise] *Tamim* ["The Perfect," or *Sefer HaTamim*] entitled **Pleg Elohim** ["The Stream of God," cf. Tehillim 65:10]: on the opinions concerning the One — Unity [?] [and the doctrine] of [Divine] truth [?]; and the matter [doctrine] of kings [?] and antiquities and the foundations [traditions] of the sages of Israel — and foreign ones [non-Jewish sages] [?]; and all the rows [orders] and traditions [received teachings] [?] that were renewed [?] in our days [?]. —

ג) Tzel HaChochmah ["The Shadow of Wisdom"], on the general principles of wisdom [?]. [Here the catalog continues on the facing page.]

and the students, to know the wandering [?] of [a star's] end and its conclusion, by way of [?] the light of [?] the sun and the moon in all the provinces [regions], in [various?] kinds [?] — [this is] a complete dictionary [?].

ד) B'Shem Bat Shlomo ["In the Name of the Daughter of Solomon"] — named [?] thus [?] — our throat [?]: the inquirer [?] into every perfection [?], full of warmth [?] and [?] glory [?]. In it [?] he did not lift up [?] his soul [?]; and he disputed [?] the opinions of Aristotle, and chose to set down [?] new acknowledgments [novel proofs?] of every kind of wisdom; — [and] in the books of the [heavenly] host [?] he set [?] [out] tablets [?] [for] reckoning [?] the judgments [?] of [astronomical?] planters [?]. [It is] inclusive [comprehensive] of [?]: the rules of the host [?], its columns [?], an account [hand] [?], and a complete [?] system of weights [?] [for] reckoning the various [?] kinds [?], with their natures [?] and the measures of [the celestial?] portions [?], all of them, [in] perfect [?] honor [?]. — And at its end [is]

the treatise **Or Shivat HaYamim** ["The Light of the Seven Days"], comprising the work of Creation [?], [showing] by [?] way [?] [that] there is no matter [substance] in it [?] except [?] fire [?] [in?] it [?] — and the conversation [?] of [its] way [?] [follows] the [royal?] road [?]. And it is included with [it?] [the] book **Keshet Derucha** ["The Bent Bow"] concerning the [?] [astronomical?] arc [?]: the kinds [?] [of things] studied in [practical?] geometry and engineering [?].

(We have already mentioned that of all the books of wisdom that the author composed, he did not choose [to print] but one of them; — and it was a precious thing [?] and dear [?] in his eyes [?], more than [all] the fruit [?] of his womb [the offspring of his mind?]; and his soul yearned [?] to bring it to the light of the world [?]. — Yet for all that, "in iniquity [?]," etc. [cf. Tehillim 51:7]; and they wished to [tell] tales [?] concerning the lot [destiny] of this precious book — namely: that it is in Wilna [Vilnius], and [a copy?] is in [the small city of?] Kotno [?] [in] Wilna [?]; and [another] is in [Lethem]berg [Lwów/Lviv] [?]; and [another] was already [?] burned [?] and is no more [?]; and [another] is, [namely], in the Society [?] of [Lithuanian?] [?] in Padua [?].) —

ה) Ir Giborim ["City of Mighty Ones," cf. Mishlei 21:22], [comprising] **Gevurot Het [?] — [part] 1**: **Gevurot Het** [?] — [containing] one deep [matter?] of algebra [?] and the roots [foundations] of mathematics [?] and full systems [?] [?]. — **2**: **Niflaot Het** ["The Wonders of [the number] Eight" or "Wonders of God"] [?] — on chemistry [?] and mechanics [?] [Hebrew: כימיה / מיכאניק"ס], foreign [novel?] opinions [?], and [things] obscured [?] [as if hidden away], in places set down [?] in established proofs [?] — a complete [?] dictionary [?].

ו) Shever Yosef ["The Breaking of Joseph," cf. Amos 6:6], on algebra [?].

ז) Be'ur LaAlmagesti ["A Commentary on the *Almagest*"] [of Ptolemy]; — [also] **Sefer Chukos Shamayim** ["The Book of the Statutes of the Heavens"] [as] printed (§ [27 =] כ"ז) — it is [?] a portion [?] of it [taken from it].

ח) Refuot Te'alah [or **Refuot, Te'alah** — "Remedies, [the] Cure" (cf. Yirmiyahu 30:13)] [?]: remedies [medical works].

ט) HaAtaka ["The Translation/Transcription"] — a book [?] [on?] polemics [?] [Hebrew: פוליחמוס"ם, *polemos*?] for the *Akadematim* [?] [academics/the Academy?] — and its commentary [?]. End [of item]. **Mekor Binah** ["The Source of Understanding"] (§ [27 =] כ"ז).

י) Torei Zahav ["Rows of Gold," cf. Shir HaShirim 1:11], comprising the details of [his?] books for the [Karaites?] [?] [Hebrew: לקרי"ם], with his commentary called **Sefer Nekudat HaKesef** ["The Book of the Stud of Silver," cf. Shir HaShirim 1:11].

יא) Tor HaMaalah ["The Row of Ascent," or "The Highest Rank"], in [several] portions [?] [Hebrew: סלכות מ"ח (?)].

יב) Ruach Eliyahu ["The Spirit of Elijah"] — he girded [?] himself [?] on this [?] against [?] the [?] [Hebrew: סרח"ס (?)], who [stumbled] over Rashi's commentary [?] [Hebrew: שעל פרש"י] on the Torah; and he disputed [contended] against all those who base [themselves] upon it [?] — and he showed that they did not penetrate to the end of his profound [Rashi's deep] knowledge [?].

יג) HaChibur ["The Treatise"] [?] [is] sufficient [?] to grasp [?] the book **HaMavo** ["The Introduction"] [?]; and **Sefer HaMaamarot** ["The Book of [the] Categories"] — a syllogism [?] [Hebrew: סגיון, *syllogismos*?] — written in [Wilna] [?].

יד) HaAtaka ["The Translation"] — books which he translated [?]: from among the writings [?] of [his] friend [?] [?] the Alexandrian [?] [Philo? — "the Alexandrian"], from the Greek tongue into Hebrew; — and [those] in which [?]: *Sefer Philon* [the works of Philo of Alexandria] [?]; and its end [?] [is] for the rabbi Yehuda the Abrabanel [Yehuda Abarbanel, i.e. Leone Ebreo] [?] — translated from Latin [?] into Hebrew; *Sefer [?]*, ["Stealing"? — Heb. נגנב] of the author, of blessed memory.

טו) Pi[rush] ["A Commentary"] on *Sefer Akedat Yitzchak* ["The Binding of Isaac," by R. Yitzchak Arama] [?] for [its] [?] [Hebrew: להרי"ע (?)] — I will show [?] in all the places [where] difficulties [arise] in it [?], for they are all [resolved] with [proper] intent [?]. —

טז) Pi[rush] ["A Commentary"] on *Sefer HaSod* ["The Book of the Secret"] of Rashb"a [R. Shimon bar Abraham? or R. Shlomo ben Aderes?] [?].

יז) Pi[rush] ["A Commentary"] on [the matter of] the [Divine] Name [?] [Hebrew: על ט' השם] [?] of the Sg"l [?].

יח) Arubot HaShamayim ["The Windows of Heaven," cf. Bereishis 7:11], complete [?].

יט) Tachal Mevucha [or **Techilat Mevucha** — "The Beginning of Confusion"?] [?].
כ) Nefesh HaGer ["The Soul of the Stranger/Proselyte"] [?].

כא) Batei HaNefesh ["The Compartments of the Soul"]. **כב) HaLeviyah** ["The Escort/Companion," or "The Accompaniment"]. **כג) Birkei Yosef** ["The Knees of

Joseph," cf. Bereishis 30:3?]. **כד) Ruach Shmuel** ["The Spirit of Samuel"] [on] the wisdom [chochmah] of Mar [Master] Shmuel [the Amora].

כה) Shema Shlomo ["Hear, O Solomon" / "The Report of Solomon"], a great anthology [yalkut] full of pleasant tastes [delightful matters], with what is heard [reported] therein [?] — for [there was] not a single day in which there was not strength in him; and according to what he heard, [whether] a good word, and according to [?] gentlemen [?] [Hebrew: מפי נסיס (?)], to gather [knowledge] from all the disciples of my [his?] understanding [?]; and according to my own discernment he set down [?] [each] matter, great and small, [such] that he should not [come to grief?] [over it] and [should be able to] find [?] its source [?]. —

כו) Ulam ["The Hall/Porch," or **Eilam** — "Elim"] [comprising] **Maalot Tari [?]** [?] — sown [scattered?] [?] with letters [correspondence] [?] of various kinds [addressed] to YaShaR, [both] from him and [to] his disciple [?]. (This [present] book [the *Sefer Elim* itself] you will find [?] mentioned [?] as the [Sefer] HaChaim [?] [in?] Berachos, ch. 18 [?] [Hebrew: בפ"ח מ"ח (?)].) —

כז) Ma'ayan Ganim ["The Fountain of Gardens," cf. Shir HaShirim 4:15], [containing] thirteen portions [chalakim]: **Sod HaYesod** ["The Secret of the Foundation"], **Chukos Shamayim** ["The Statutes of the Heavens"], **Gevurat Het** [?] [or **Gevurat HaEl** — "The Might of God"], (you will find [?] [Hebrew: סחוי"ט (?)]), and so forth, of blessed memory [?]: [also] **Milchamai** [or **Milchamot** — "My Wars/Polemics"], to the physician [?], scholar [?], and sage, our teacher the rabbi Maharar Yosef Shlomo, for the [city/province?] of Candia [?] in his book, [arranged] by portion in [the manner of] **Gevurot[av]** [?], etc. And he chose to set down [?] words [?] more accurately [?]; and [yet] he made [?] a marvel [?] [as to] why [?] they did not [?] upon them [?], [namely that] he composed [them] in twenty-four [?] [Hebrew: כ"ח מ"ד (?)] words [?], etc. — And further: **Gevurat HaShamayim** ["The Might of the Heavens"], **Taalumot Chochmah** ["The Hidden Things of Wisdom"] (these are several [?] books — and the beginning of [the] wisdom [section?] comprises [the words?] of YaShaR [?], of blessed memory, "with confidence" [?]); — "for whoever has acquired from Cordovero, of blessed memory, **Lev HaChochmah** ["The Heart of Wisdom"], [a portion?] is fitting for [the] remedies of [?]," — and he spoke of the wonders [hidden virtues?]: nature [?] and the like [?], and so forth. — Of the [proper] wisdom of Solomon [?]; and he prided himself [?] that no parallel [?] is found in all the books of the sages, etc. — (You will find [?] [an account?] in the section [marked] [27–28 כ"ז–כ"ח]: **Otot HaShamayim** ["The Signs of the Heavens"], for [bread? — Heb. לחם] [?].

** ** *Abirim* ["The Mighty Ones"], *Megalleh Amukot* ["Revealer of Deep Things," cf. Iyov 12:22], *Mekor Binah* ["The Source of Understanding"], *Etzem HaShamayim* ["The Substance of the Heavens"], *Amudei Shamayim* ["The Pillars of Heaven"], *Gan Naul* ["A Locked Garden," cf. Shir HaShirim 4:12], *Ma'ayan Chatum* ["A Sealed Fountain," cf. Shir HaShirim 4:12]; the totality of these [?] is [the work] now [?] before [us] [?] [Hebrew: כבר מבואר (?)], [printed] for the press [?] at Hanau [?] [Hebrew: נ"ח טרלאוניס (?)] (folio XII [Latin numeral], column 3) — and at Hanau [?] [Hebrew: ני"ח טרלוניס (?)], and the last of them, joined to it [?], [is] *Sefer Eilim* [*Sefer Elim*], which he printed in Amsterdam [?] in the year = 5389] ה'שפ"ט [1629 (¼) [?]; and now [it is being] printed [anew] in Odessa [?] [Hebrew: בהדעסא (?)] (⅛) [?] in the year [1864 = 5624] ה'תרכ"ד.

[and item] כח) [The] **Sifrei Kabbalah** ["Kabbalah books"] which his disciple, mentioned above [?], printed in [the city of?] Basel [?] in the year = 5389] ה'שפ"ט [1629 — [the] gleanings [?] [from] *HaAlumit HaChochmah* [or *HaElmit HaChochmah*] [?], and these [are]: *Matzref LaChochmah* ["The Crucible for Wisdom"] [?], "from every [?] living thing [?]" (you will find [?] [Hebrew: וצבחו הגתון כיומפלה (?)]) [the man-of?] life [?], etc. — *Olam Katan* ["The Small World/Microcosm"], *Kanur* [?], chapter 4 [?], section 10 [?] [Hebrew: פ"ד מס' עשרה (?)] — the disputants [?] for [against?] the Kabbalist [?] who is [?] disqualified [?]; — *Ketzur Olam HaTikkun* ["An Abridgment of *Olam HaTikkun*"], *Shever Yochaf* [?], *Nivlot HaKabbalah* ["The Withered Fruit of Kabbalah"], as [stated in §] [?5] ה', gleanings [?] — all of them on [the subject of] Kabbalah. —

And behold, of all [these], the signs [markers?] of the rabbi YaShaR's honor [?] are not as I [imagined?] [?]; to gather [up] a treatise of [?] [the works] he printed, as stated above. — And who knows [what] all his wise books would bring forth [?] [had they been printed] without [hindrance?]; — they are [now] no more — neither bringing good, nor remembering [?]; rather, [there remains only] satisfaction [?] for one [?] from [?] them; — happy [is the] man [whose] portion [?] is in their perfection [?] [as something] hidden away [?]; — but to their seekers [?] and to those who would labor [?] [over them], for [there is?] a remoteness [?]: — for [though] the rabbi made you wealthy [?] in them [?], he diminished [?] from [us] the [further?] generations [?] of [his] books, [that] our master and the abundant blessing of YaShaR ["the upright"] reached [?] [us] from every dear [?] precious [?] gift [?]; from every [?] place [?] where the name YaShaR [is found], [there is] understanding [?] like our master [?], [it is] complete [?].

And behold, for the [further?] benefit of our thought, (and also [as to] what guided [?] [us]) [it is fitting?] to set right [?] the [account of the] righteous

[tzaddik], [namely the?] testimony [?] [Hebrew: עד] [as] his disciple wrote of [him] in this [?], [for] he was [held in?] the memory [esteem] of his old age [?] — for behold, in truth, he was a Sephardi man, of [his] birth and [his?] settlement [?]; and he dwelt among the nobles [?] of his city [?]. And already many holy ones of Israel of supreme holiness [?] have praised [?] him [?] in earlier generations: in Spain [before the?] Expulsion [?], and in Castille [?] [Hebrew: וקסטיליאה], etc. — that we [too] reckon [him] thus. — (And they enriched [?] [their] words on this [theme]: that of all the books [there are] many [that are] well-known [?]; — and you will find [?] [an account?] [in the] *Sefer HaChasidim* [?] [of] the sage [Hebrew: סה"ז (?)], the Levite [?], [section?] [159] קנ"ט [?]; and as to [the fact] that he [had] the power to mention [?] [it], — there was still [a place?] in [its] holy place [?] [reserved] for kings and ministers [?], which [accounts for?] twenty-two [?] [Hebrew: כ"ב] sections [?] he related [?] for [his?] life [?], in [order] to [become?] great [?] over them [?], etc. — And [further]: there was a [court] physician [?] and scholar [?] [at] Tirokis [?] [Hebrew: בטירוקיס"ס], [namely] Como [?] son [?], etc. (you will find [?] [it] in the responsum [Hebrew: בשו"ע סי' קנ"ה (?)]), — and [further]: that as it appears [?], [he] did not [?] thus [?]; only [as something] in his hand [?] in his youth [?] [Hebrew: בנעדוהו (?)], etc. — (And the days [the matter?] [Hebrew: ודמוס] [were?] like [?] [in?] the year [?] [Hebrew: ל"ין (?)] — for in his days he was a [?] [Hebrew: יוכית]; — for [?] he ate [?] his elder [?] [Hebrew: זקן], and arranged [them?]…

(316) [marginal/parenthetical numeral — likely a cross-reference or pagination note] [?]. — And look [?] at [the] forms [of his life?] [as to] what occurred [?] [in his time?] [Hebrew: קרי' נחזנה (?)]: [he was] a great [author? and] sage [?], a great [?] [Hebrew: מקרי משדחי (?)], [in] Wroclaw [Breslau] [?] [Hebrew: בטרלשון / ר' ווילנא (?)]. —

Indeed, every acknowledgment [?] [testimony] concerning [him] is a word without [?] perversion [?]; — concerning the rabbi YaShaR, that generation [?]: this is to add [?] [the?] anguish [?] over his fate [?]; — his enemies [?] in [his] generation [?] — though [he was] a sage [?], the rabbi was a great investigator [?] in matters [?] of [?] many things, [in] inquiry [?] and dialectic [?]. And they rose up [?] against [him] in [the manner of?] Kayin [Cain] [?] [Hebrew: בקילותו (?)] — and the honor [?] of the land [?] [?] would not be settled [?] forever [?]. Whoever proceeds upright [yashar] — let him be honored [?] [as] upright [?], for he is upright, upright [yashar yashar]; — but to whoever is not upright [yashar] — as [in the verse] "as their name, so are they" [cf. Shmuel I 25:25] — for there is a flaw [taint] [?] in his words [?] [forever?]: every one who fears [?] [for?] [his] catch [?], our delight [?]; only those [near?] to him [are] warm [?] with him [?], dear [?],

being weighed in another scale [?] [forever?] [Hebrew: סלה]. — And by the mouth [testimony] of the rabbi YaShaR himself, in [a letter?] to his disciple Zerach, he expressed [?] himself [?] [Hebrew: בצק (?)] [in] saying [?] [Hebrew: לשמו לח"ז]: "Above [as stated above?] [it is] written [?]: that those who esteem [?] me [?] in [my own] city [?] [held me] precious [?], the [intelligentsia?]; — they were my acquaintances [?] and my disciples [?] in the academies [yeshivos] of [Christian] Europe [?], and [yet] they did not bear me ill will [?]; this is a known [?] matter [?]; — and reveal [?] to him [Hebrew: ומגלא (?)]: that this is folly [?] [as in?] 'a fool' [Hebrew: כסיל] — and let him [?] know [?], my brothers [?], that the LORD [?] [Hebrew: יראה הם ה'] avenged [?] [them], and exacted [their judgment?] and dealt strictly [?] in [recompensing?] their reward [?]; and there is [a means?] in my power [?] [Hebrew: ויש לי בעולמי] [whereby] he who accompanies me [?] [will be] saved [?] for ever [?] [in?] confidence [?] — to hunt down [?] [those] who [oppose?] the LORD [?] who are alive [?]; — in the disputation [?] [Hebrew: בתלמלא] [?] this [?] [is] what [?] there is [?]; — like [?] [unto?] honor [?] [Hebrew: לכבוד] [in?] spirit and exile [?] [Hebrew: לרוק וגלות]" and so forth. —

The [conclusion] [Hebrew: המצית] [?] [drawn from?] the words of the rabbi [is that he was] a sage, a Rashi-of-his-day [Reish"a] [?], as stated above, [and yet] not [recognized?]; — [his enemies?] arose [?] [and] testified [?] against [?] the rabbi YaShaR, of blessed memory, in his generation [?]. — Without doubt [?], a great rabbi, [skilled?] in the received [tradition?] [Kabbalah], a master [who] delved deep [?] in [its?] depth [?] — [yet?] he established [?] only [?] in [matters?] of [?] his books in Kabbalah. And he stood and waged war [?] — and as much [?] as the rabbi the Kabbalist, [namely?] Mahar"i [our teacher R. Yitzchak?] — life lives anew [?] from this [?]. — And the seven who received [?] [the tradition?] [as?] the portions [?] of Mahar"i [?] [Hebrew: בחלקי הרמ"ז] [?] — and so what [?] [is the case?]...

to fathom [?] the words of [the] Ari [R. Isaac Luria, the *Arizal*?], of blessed memory, in the book *Korach Setulah* [?] [Hebrew: קורח סטולה (?)] — for [it is?] pleasant [?] that one disagrees with [?] the kabbalists [?], according to a [certain?] investigator [?]; — the inquiries [?] [are?] in [a certain] order [?]. (And so on, in counters [?] [Hebrew: בקונטרס], in answers [responsa?] [Hebrew: רתובות] [I have] read [proclaimed] up to **324**, p. 1 [ע"א] [of] Rad[ba"z?] and Rashi [?] [Hebrew: סרדב"ז וסרי"ס (?)] [of] Padua [?]). — And the rabbi Zerach the Kabbalist, the recipient [of tradition?]: R. Chid"a [R. Chaim Yosef David Azulai] gave a portion [verdict?] to YaShaR — to his honor [?] and so forth [Hebrew: כבודו כראוי וכו'] [in the year?] [1863 = 5623] ה'תרכ"ג [?]: "*Sefer Eilim* [*Sefer Elim*] [is] for the rabbi who encompasses [all] philosophy [?], our teacher the rabbi YaShaR of Candia [?]

— [he] was a sage and excellent [?]; and in all wisdom [?], and [in] the book *Mesorah* [?]: theology [?] [Hebrew: הכונה], geometry [?] [Hebrew: כנין], medicine [?], and [he was] a sage in [?] the dialectic [pilpul] of the Talmud, and [in matters of] decisors [poskim] and homilists [darshanim] — a wonder [?] [Hebrew: מופלא], and his knowledge [?] embraced [?] much [?], etc.; and the sage [?] of Hamburg [?] [Hebrew: הסגאון סמפורסס] [our teacher? — Hebrew: מורי"ך] R. Teerz [?] Tirsh [?] of Berlin [?] [Hebrew: סירס אצדיק בערלין] in his testimony [?]: a book *Klil HaCheshbon* ["The Crown of Reckoning"] [?] — '[it is] fitting [?]' [Hebrew: כקב]. — The rabbi Zerach the Kabbalist…

[Editorial front matter — 1864 Odessa edition. This entire opening (pages XXI–XXIV) is the work of the 1864 editors/printers, not Delmedigo's own 1629 text: the close of the biographical *Toledot*, a memorial elegy, editorial notes, and the publisher's announcement-letter. Delmedigo's own material has not yet begun in these pages.]

[The four pages are each set in two text columns beneath a shared running header. The body type is square Hebrew; the foot-of-page apparatus (*Haʿarot*, "Notes") is in small Rashi type. The paper is heavily abraded throughout this opening; passages too faint to read with confidence are marked `[?]`, and doubtful but partly legible phrasing is rendered as best it can be read, with the doubt noted in the surrounding prose.]

Toledot HaRav YaShaR of Candia (continued)

Our master YaShaR [Delmedigo], may his memory be a blessing — [for he was one of] those who run to draw water [from the well of] wisdom and knowledge [?]; [as it is written, "the LORD] called him." And the great Geonim who [came] after him likewise spoke of him in these words [as recorded in (?)]: "A holy man, YaShaR of Candia." And it is accepted [tradition] that he was an inquirer and [investigator (?)] of the *eliqos* [?], one of the ten luminaries [?], who fashioned the sphere [?], counted the constellations and the [orbits] of the host of heaven, and so forth [?]. […] He was a wonder-worker in his generation [?], laboring to bring forth [?] the glory [?] of the people of Israel [?]; and on account of [his teaching (?)] his disciples and his books [?] — for they were precious to all [?] — and they were [reckoned] among the firmaments [of the (?)] sublime wisdom of Israel.

He was the prince [*sar*] who [?] […] established [?] over his investigations, fluent [?] and faithful, and his negligences [?] […] No other man arose to hide [?] […] He [?] […]. And by whom shall he be likened? [?] To him belong [?] images [?] […] who is fit to do as he [intended]? [?] For the […] cast [?] […] sharp [things (?)] and

difficulties [?], swift [?] and weighty [things (?)] — in proper order [*be-ʿerekh*], firm and corrected [*nakhon u-methukan*] — and so forth [?]. He set himself [?] to teach [?], and forged a new tongue [a new idiom (?)] [...] of beautiful speech [*safah berurah*], in a generation when [there were too few] to teach. He labored [?] and gloried [?] in holy matters [*ʿinyanim qedoshim*], and in pleasing utterances of holiness [?].

This was YaShaR the prince [*ha-sar*], who was [...] great in Torah and great in good repute, and he departed [this world (?)] in the year [(?) 5408] ה'שב"ט from [the world] [?]. [He was buried (?)] in [...] [Hamburg (?)] [in Hamburg (?)], and the surrounding regions [*ve-galilot*] of Amsterdam; and a stone [*even*] [marks (?)] his outcry [*meqir zaʿaq* (?)]. [...] [The (?)] princes [...] [in their (?)] eulogies wrote: "A prince and a great one has fallen, the Gaon of Israel, a [pious one], woe for us [*ai ʿalanu* (?)]," and so forth. And his good repute [?] [...] perfect [?] [...]. And they bore for us [?] [...] of [...] holiness [?] [...]. And so forth.

[A passage too faint to read connects here; the column resumes in a series of attributions:] And the Rav, the Gaon, the *baʿal* [author of] *Shut Chut HaMeshullash* [?] [glorified (?)] him [...] — for he recognized YaShaR [...] face to face [?] [*panim be-fanim*], and found his books [?] sharp [...] and mighty as the LORD's [?]; and [they] praised him in the name of our master the Rav YaShaR, may his memory be a blessing. (And likewise the *Shut Chut HaShani*, responsum no. 17 [?].) And the Gaon, *baʿal* [author of] *Chavot Yaʾir* [R. Yair Chaim Bacharach], praised him, as above. [?]

In truth, after the greatest of our Sages of all generations — such as the Rambam, the Ralbag, and those resembling them, may their memory be a blessing, whom [it pleased] to call [...] of Candia — by such reckoning [?] the *mazal* [fortune/standing] of the Rav YaShaR is still finer [?]; for only a few of his contemporaries were his equals [?], and according to such [praise] runners [?] [...] knew his good repute in Israel — a man perfect in [all (?)] he set his hand to. [He was] Gaon, Kabbalist, philosopher, and poet — and all together [made him] one of [their wonders (?)]. [There was] no place [...] worthy of so great an [...] as his rank and his height [*ʿerko u-maʿalato*], for [...] equal acquisition [?] — and as for [acquisition (?)]: for instead of acquisition [?] [...].

Toledot HaRav YaShaR of Candia (continued)

And everyone [recognized (?)] his wisdom, and his age, and his standing [*tiknon* (?)], as one mighty as [...] of Greece [*Yavan* (?)] — [...] [He guarded (?)] the back of his garden [*gano* (?)] in this manner, and [...] [his (?)] reputations together as one.

A holy man on high [*kadosh ba-marom*], [whose (?)] arm reached afar [?] — separated [*nivdal*] [...] and honored [*nikhbad*] [...]. And he reasoned [?]. And he was [...]: for a small world [a microcosm (?)] had [...] to him; he had no [...] who could weigh [?] [...] in place of battles, and there is yet a place [?] [...]. He set [?] his treatise [?], well-arranged [?] [...]. There is no other [like (?)] him [...]; he sat [?] [...]. And there is no [...] [...] to him still — [it is a riddle (?)] [...].

And the wise [*chakhmei*] [...] forgot [?] in receiving him [?], and [...] after his books, [for] in [...] wisdom [*chokhmat*] [...] in his eyes [*be-ʿeinav* (?)]. And it is known [*ve-yaduʿa*] to all the Rabbis and the *Tzaddiqim* and the *Chassidim*, the holy ones on high [*qedoshei ʿelyon*], [...] in the generation of the *Alfasi* [?] [...] [as it is (?)] precious in his sight [?] [...] [his book (?)] [...] [...]. And his intentions [?] in [...] [...], and faithful [?] to bring to view [?] the great Geonim [...] — [...] our master [*rabbenu*] Eliyahu, the *Chassid* of Vilna [the Vilna Gaon] [?], may his memory be a blessing — [it is well known (?)] in the name of our [...] [Eternal One (?)], mute [*ilem* (?)] [...] — for he [...] not [...] a great matter, and a small thing too — a riddle [?]: indeed, the Gaon [...] the [...] *Chassid* [...].

The *Chassid* [...] [the holy of holies of holies (?)] [...] [...] and Shneur [Zalman] [?] of Liadi [the founder of Chabad], may his memory be a blessing — as [we have heard (?)], such was the matter [?] [...] this is so [?]. (And it is found in his prayer-book [*siddur*] — likewise, [the (?)] entrance here [?] concerning the matter of the kindling of [the lights (?)] [*hadlaqat ner* (?)].) And the Rav, the Gaon [...] held [?] [...] a man [...] of the holy ones [...] — a holy man [*kadosh*] our master [*morenu*], [R.] Yissachar Dov, may his memory be a blessing — the Av Beit Din [?] of [...] [...]. And in the responsum *Da[ʿat] [...] of [...] Vladislav (?)* [?], in [section] 78 [*samekh-chet* = 68 (?)], [in] *Mevasser Tzedek* [?] [...]. And the responsum *Bat ʿEini* [?], indeed [...] this [...] [...] [our printers (?)] [...]. And [it is needed (?)] now for the matter [?] of [the printing of (?)] this book — from Jerusalem [*Jerusalem*], may [the Lord (?)] [...], [from (?)] Pas [Fez (?)] — [...] and they donated [?] [their tithe (?)] to the [...] decree [?] precious [?]. [To] our master [...] Rachel Trabotti-Ginzburg [*Tr-a-b-t-G-n-b-e-r-g* (?)], peace be upon her; and he also wrote concerning him [...] [...] and [emendations (?)] in special pamphlets [*qunteres*], just as we heard [...] in the [...] [...] [...] who [are united (?)] upon them [?] [...] [those (?)].

[The right column resumes mid-thought:] And it is well known to all the Rabbis [...] in the generation of [...], according to [?] [...], [it is needed now (?)] [...]. And it is found that [their (?)] [...] reaches it [?] [...], for [...] [...] [...]. They feared [?] [...] [to add (?)] [...] [and to gather (?)] them [?]. And the Rav added [?] — and our wisdom

[?] reached [it (?)] in his eyes [?] — for [...]. And [...] [...] the [seal (?)] [...] [...] over [...].

And our master [the Gaon (?)] [...] in [...]: "A prince and a great one [has fallen], the Gaon of Israel, a pious one, woe for us," and so forth. And [his (?)] sign [...] [those who err (?)] [...] [...]; and they [were not poured out (?)] [...] before the children of [his (?)] sojourning [?] [...] to take [up (?)] the name. And the Rav, the Gaon, *baʿal* [author of] *Shut Chut HaMeshullash* [?] [...] face to face [*panim be-fanim*], and found his books [?] sharp [...] [as decreed by (?)] the LORD [...]; and they praised him [?] in the name of our master the Rav YaShaR. (And likewise it is found in *Shut* [responsum] no. 17.) And the Gaon, *baʿal* [author of] *Chavot Yaʾir*, found him [praiseworthy (?)], as above [*ki-deḥazi* (?)].

In truth, after the greatest of our Sages of all generations — such as the Rambam, the Ralbag, and those resembling them, may their memory be a blessing, whom it pleased [the LORD] to call "of Candia" [?] — [know (?)] that the *mazal* of the Rav YaShaR is still finer; for only a few of his contemporaries were his equals, and according to such [praise] runners [?] knew his good repute in Israel — a man perfect [*kalul* (?)] in [all] he set his hand to: Gaon, Kabbalist, philosopher, and poet — and all together [made him one (?)] of [their wonders (?)]. No place was worthy of so great an [...] as his rank and his height; for [...] equal acquisition [?] — and as for acquisition [?]: for instead [...].

[A new sub-section, set off by a rule, in small Rashi type — the editor's gloss on the tombstone-text that follows:]

[Gloss / *Haʿarot*, small Rashi type:] [The text of the headstone (?)] [...] which is read by every wise man [?] in [his (?)] generation — and good for them [?] [...]; [...] and they composed [?] [...] of holy [?] [verses (?)] [...]. But [it is not (?)] enough [?] for our understanding [?] [...].

[A new larger-display passage — a poetic eulogy / *kinah*, the column resumes:] In a year [*bi-shenat*] [...] [as a sign (?)] and a name [*shem*] for the days of his life, [...] [...] in him a [troubled (?)] spirit [*ruach* (?)] [...]. His markets [?] turned [?] [...] [for (?)] still [...] [the storm (?)] [*seʿarah* (?)] [...]. [...] [were drawn near (?)], to wonder [?] [...] and to lodge [?], something of the fruit [*mi-peri*] of the Tree of Life [*ʿetz ha-chayyim*]. He went forth [?] and departed [?] [from] Amsterdam, and walked [?] on his way [...] [to] Prague [*Prag*], where [...] [the (?)] *Chevra* [society] of the Gaon, the *Tosafos baʿal Chayei* [...] [?], for [several (?)] times — [in proper order (?)], the year [(?) 5413] תי"ג; and still he placed [?] [...] in [...] [Wiremiḥel (?) — i.e. an unidentified place], in the year [((?) 5414) תי"ד] [...].

[The eulogy continues:] He travelled [?], the Gaon, [as above (?)] [...] in [...] [the LORD (?)] there [...] upon a throne [*kissei*] of authority [?] [...]. And in his honor [*u-vi-khvodo* (?)], the [...] called out [?]; he [walked (?)] his way and turned aside [?] [...]: he hid himself [?], [keeping his secret (?)] [...] for he said [?] [...]. "Said the LORD to me, 'You have your peace [?]'" — and the LORD [yes (?)], [...] still [...] [...], for he said to me [?] [...] [the (?)] living ones [*ḥayyim* (?)] there [...]. [It happened (?)] [...]; [...]. And he died, YaShaR, in a [...] [*be-eḥad* (?)]; [come (?)] and [...] — and he was [?] — for [...] took him [*kilaḥaḥ otahu* (?)] [...] our portion [*chelqenu* (?)]! [Howl (?)] — [wail (?)], O [...], children of Prague [*benei Prag* (?)] — and they eulogized [him] [*va-yispeduhu*] as fitting [*ke-khol-kah* (?)].

(*Nosach ha-matzevah ha-ʿomedet ʿal* — "The text of the headstone standing at [the grave of the Rav, may his memory be a blessing, in Prague]." [Tombstone epitaph follows.])

[Below the rule, in small Rashi type, the opening of the headstone text:] "Read [*qera*], and you shall know [the secret (?)]: *Sukkah Chatav* [— a chronogram (?)] [...] for the abbreviated era [*li-feraṭ qatan*]."

[A second rule; then, in large display type — the opening lines of the headstone inscription itself, naming the deceased:]

> The Rav, the wondrous one, the all-encompassing scholar, the divine philosopher, the chief of physicians, our teacher and master, R. Yosef the Physician [*Rofeʾ*], of Candia, who was Av Beit Din in Hamburg and in the regions [*galilot*] of Amsterdam, son of the honored master, R. Eliyahu Del Medigo [*Eliyahu Dil Mediji*], may his memory be a blessing for the life of the World to Come.

[Below the inscription heading, the body of the epitaph — a *kinah* in display type:]

> Lift up [*seʾu*] a weeping of wailing and lamentation [*beki yelalah ve-qinah*], and tell [*ve-sapru*] in mourning and in groaning [*ba-avilut u-va-ʾaniyah*], [as] bitter ones [*merorim*] [who carry (?)] [...] for a prince has fallen, great in the camp [*be-machaneh*], who was a crown to the wise men of his heritage [*chakhmei morashah* (?)], [...] [and concealment (?)]: the wisdom is lost [*neʾevdah ha-chokhmah*], and understanding is hidden away [*ve-nistarah ha-binah*] — [where is it now to be found (?)] in every region [*be-khol ʿever*], [...] [and corner (?)] [*u-finah*], to the sea and eastward, southward and northward [*yamah ve-qedmah negbah ve-tzafonah*] — [for (?)] [the LORD (?)] [put a divine spirit within him (?)]

[*ruach Elohim be-qirbo netunah*], [...] my wisdom cried out [?] in the street [*chochmati ba-chutz teronah* (?)] — my soul [*nishmati*] beneath the wings [of the Shekhinah (?)] [*tachat kanfei ha-Shekhinah*], it is restful and hidden [*hiʾ genuvah u-temunah* (?)].

Toledot HaRav YaShaR of Candia (continued)

[The epitaph / eulogy continues, set in display type with bracketed marginal acrostic-markers that label successive strophes:]

(*[Battle] of his disputes [milchamot (?)]*): [he passed (?)] [...] and they eulogized him [*ve-sapduʾuhi* (?)], and woe upon a man — over the Gaon of Israel, who [...] passed over [*ʿavar*] [...] [between (?)] the decree [*ha-gezerah*], for this is YaShaR Yosef the [Provider (?)] of grain [*ha-mashbir bar*], whose fame went forth in every direction [*be-khol ʿever*]; from the joint of [the (?)] mountains and rocks he broke [*shever*] — and nothing was hidden from him, no matter at all, in a tongue [*be-lashon*] that speaks great things he composed books [*sefarim chiber*]; the *Sefer Nivlot HaChokhmah* [— "The Withered/Fallen Things of Wisdom"], honored ones [*nikhbadot*], [are] spoken of in them [*bahem meduvar*]: astronomy in its plain sense and beyond [*ha-techunah bi-feshuṭah ve-ʿibbur*]; to make many books he finished [*la-ʿasot sefarim harbeh gamar*], and reasoning [*u-sevar*]; and in all seven of the sciences [*be-khol shevaʿ ha-chokhmot*] he was as a man girded with strength [*ke-gever gibbor* (?)] — he left no matter small or great [*lo hiniaḥ davar qaṭon ve-gadol*], the whole he gathered and bound up [*ha-kol asaf ve-ʿaivar* (?)].

[A new strophe, the marginal note reads "*meʿabber ʾeḥad le-tzad ʿelyon*" — "passing one [over], toward the upper side" (an acrostic / structural marker). The interlinear Zohar citation in parentheses keys the eulogist's words to passages of the Zohar:]

(*Zohar* [...] [— passage marker]) [...] in the speculum that shines [*be-ʾaspaklarya ha-meʾirah*] — all the ways of the host [*kol holkhei tzava*] [of heaven (?)], (*maʿaseh*) son of [*bar*] [...] the [deeds (?)] of God [*Elohim*], who [...] created all [*asher kol bara*]: he expounded [*beʾer*] the Torah, and was distinguished [*mufla*] [and expert (?)] [*baqi*] in every [reasoning (?)] [*be-khol siaḥ* (?)] [...], in [all (?)] discussion [*siaḥ*], [the orders (?)] of [*sidrei*] *Mishnah ve-Gemara*.

[A new strophe, marginal marker "*meʿabber sheni le-tzad ʿelyon*" — "passing the second, toward the upper side":]

(*[They] cried out [tzaʿaqu] in bitterness*, in a tongue [*be-lashon*], *and your palate* [*ve-ḥikkekha*] — *whoever eats, who tastes flavor* [*okhel mi yiṭʿam*]) [...] he was for

us a wall and a refuge [*hayah lanu le-ḥomah ve-le-mistor*] on a day of [...] wrath and fury and smoke [*be-yom zaʿam ḥaron af ve-ʿashan*], whole ones [*tamim*] and upright in his deed [*ve-yashar poʿolo*]; "Holy!" they shall say of him [*kadosh yoʾmru lo*], and "Glory!" — each one says it [*ve-khavod omer kullo*].

[A new strophe, marginal marker "*even mul ba-machaneh*" (?) — a damaged acrostic marker:]

(*A stone* [*even*] over against [*mul*] the camp [*ba-machaneh*]) [...] my chief [*alufi*] and intimate [*u-meyudaʿi*] [Tehillim 55:14], [there] were (1) [— footnote-marker] disciples like his sons [*talmidim ke-vanav*], they drank from the waters of his springs [*shatu be-tzemaʾam mimei maʿyanav*]; to bring to light [*le-ʾor hotzi*] [...] every judgment of the house of Yosef and his court [*kol mishpaṭ beit Yosef ve-dino*] upon his foundation and upon his ways [*ʿal mekhono ve-ʿal ofanav*], (1) he answered [*ʿanah*] [— Zohar marker] after him: a pious one, where are his eyes [*ai ʿeinav* (?)] — fit was he [*raʾui hayah*] [...] for the Shekhinah grew black over his presence [*she-ḥasherah shekhinah ʿal panav* (?)], for there was none [*she-ein*] in the generation [*ha-dor*] and no place [*ve-khaʾi*] [worthy] to stand before him [*la-ʿamod le-fanav*].

[Marginal marker "*meʿabber sheni le-tzad ha-machaneh*" — "passing the second, toward the camp side." The strophe text here is lost to rows of faint dots, indicating the epitaph continued onto lines that are now illegible:]

(*[passing] the second, toward the camp side*) [...] [printed row of leader-dots; the text beneath them is illegible — not lost/abraded ink]... [...] *the past year, fixed* [*shenat ha-ʿover meḥuqqah*] [— the chronogram-year of the inscription; illegible].

[A decorative tail-piece rule (◦ ◦ ◦) closes the eulogy. Below it, the editorial *Haʿarot* (Notes), in small Rashi type:]

[*Haʿarot*, small Rashi type — editorial notes:] In [...] [the four (?)] [...] in the introduction [*ba-haqdamah*] [...] which we have already called [*qaraʾnu*]; and they yet added in the [pages (?)] of the prayers [*biṭṭurei ha-tefillah*] for [...] [the (?)] festival of Sukkot [*Sukkot*], and so forth. [...] — The name "Del Medigo" [is found (?)] among his disciples and his pupils [...]: [our (?)] [...] [in the year (?)] (] תרכ"ח קבי"ו, [(?) 5628 [?]) the wife of [...] Sarah, who [bore (?)] in the year [...] as the name [of (?)] Shimshon Modon, the Kohen [?], may his memory be a blessing — [one of (?)] the Rabbis [...] his ancestor [?] in [...] [Fez (?)]; and his foundation [?] in the house of [...] glosses [*glossot* (?)] of the *Levush* [?], [the print (?)] [...] the all-encompassing [scholar (?)], our master [*moreinu*] [...] Rafael Meldola [*Rafaʾel Meldoneh* (?)], may his memory be a blessing — and they left no sons [*velo hiniḥu*

vanim] after them [...] a *teiqah* [?] [...] of [...] the book [...] embroidered [*meruqemet*] over [it (?)], [the (?)] *horayot* [?] that we did [...] by the hand of [...] in the year [(?) 5596] תקצ"ו. — [And in (?)] the *Sefer Toledot Chesed* [?] of the Rav, the wise physician [*ha-chakham ha-rofeʾ*] [...] commentaries in the booklet [*qunteres* (?)] in the name of R. Shemarya HaIqriṭi [*HaIqriti* — "the Cretan"] [?], and also [...] [...] תי"ח (?) [...], may his memory be a blessing. (And not on these [...] [lips (?)] [...] of the *Hagbalah* [?] [...] [for] the Holy One [*HaKadosh* (?)], according to the matter [*kefi ha-ʿinyan*] [...] and our printers, the Sephardim [*ha-sefaradim*] [...] and so forth, below [*ʿod le-halan*].)

Maktav ["Letter / Epistle"] [— editorial publisher's announcement, large display heading]

[The first column continues the foot-of-page *Haʿarot* in small type from the prior page, then the page proper opens with the *Maktav* heading. Small-type notes column first:]

[*Haʿarot* (continued), small Rashi type:] [...] [in the introduction (?)] [...] [as is found (?)] [...]; [...] and we were honored to receive a letter from [*mikhtav me-ʾet*] the Rav, the wise printer [*ha-chakham ha-madpis*] [...] MaHaRaʾL Silberman [*MaHaRaʾL Zilberman*] [?] [...] [in (?)] [...] [Lyck (?)] [*Lik* (?)], the days [...] [and (?)] [...] — [a confession and a blessing (?)] [*hodaʾah u-verakhah*] upon [...] he made known to us here [...] [that (?)] this book is mute [*ilem*], may his memory be a blessing [*li-tzeror* (?)]. — And it is found [...] in the eyes of the heads of the treatise [*rashei ha-chibbur*], the honored [...], who were donated [*nidbu* (?)] [...] [...] this [*zeh*] — [it is not (?)] [...] [...] face to face [*panim* (?)], a place [*maqom*] [...] [that was (?)] for the LORD [...] [...] [in the year (?)] [ת"ה (?)] [...] [...] [several (?)] Rabbis from among them [*me-hem ha-Rabbanim*] in Volozhin [*Bozeliṭchik* (?)], the cities of [...] [...] difficulties [*muqṣot* (?)], and what reached the treatise [...] [investigated (?)] [*saqar* (?)] the prince [*ha-sar*] [...] [...] [Israel (?)], whom the prince [*ha-sar*] raised up over him [?] — and only for the sake of [...] [...] [counted (?)] [...] the hands [*yedei*] of all who cleave [*ha-devequim*] [in Judaism (?)] [*ba-yahadut*], the good ones [*ha-ṭovot* (?)], [...] [yes (?)] they answered, and they [...] [...] to the perfections of the treatise [*le-shulḥei ha-chibbur*] — [folios (?)] 310 [*samekh* + 310 (?)] [...].

[The *Maktav* heading and its address open the page proper, large display type:]

Maktav ["The Letter"]

We have been honored to receive the words of the Rav, the wise printer [*ha-chakham ha-madpis*] [...] MaHaRaʾL Silberman [*MaHaRaʾL Zilberman*], [...] [Lyck

(?)] [*Lik* (?)], [...] the days [...] [...] — a confession and a blessing [*hodaʾah u-verakhah*] upon [...] [...] he made known to us here that [...] [the book (?)] is mute [*ilem*], may his memory be a blessing [*li-tzeror* (?)]. — And it was [...] [and so forth (?)] [...] in the eyes of the heads of the treatise [*rashei ha-chibbur*], the honored [...], who were donated [*nidbu* (?)] [...]; for [we (?)] [...] [reasoned (?)] [...] [...] [...] which was for the LORD [...]: one book [*sefer eḥad* (?)] [...] [his way (?)] [his ways (?)], such as he [...] [...] [his intimate (?)] [*ḥaniqo* (?)], the cities of [...] [in the wrath (?)] [...] and they draw near [*u-meqarvim*] [their (?)] hands [*yedeihem*] [...] [investigated (?)] [*saqar*] the prince, who [...] [Israel (?)], the prince [*ha-sar*] [raised him up (?)] [*naṭelo ʿalav* (?)] — and only for the sake of an empty name [*shem manuʾaḥ* (?)] [...] [...] the hands [*yedei*] of all who cleave [*ha-devequim*] in Judaism [*ba-yahadut*], the good ones [...] [...] [and they were saved (?)] [...] to the perfections of the treatise [*le-shulḥei ha-chibbur*] — [folios (?)] 310 [*samekh* (?)] [...].

[A rule, then the publisher's address in display type:]

Likhvod ha-shutafim ha-nikhbadim ["To the honor of the esteemed partners"]

We make known to you, this 14th [*yod-dalet*] of [the month (?)] [...], that this mute book [*sefer ilem zeh*] [— *Sefer Elim*, "the mute book," a pun on the title] has been received [*nitqabbel* (?)] [...] [...] this is for your benefit [*li-feʿulatkhem ha-ṭovah*]: by such [reckoning (?)] the privilege [*zekhut*] of new things [*chiddushim* (?)] [...] [...] for the sake of the wisdom of Israel [*chokhmat Yisraʾel*]. And so forth [*ve-khuʾ*] — may [your good (?)] [...] arrange [*le-haḥzin* (?)] [...] [...] perfecting [*kalalt*] the [...] precious one [*ha-saʾir* (?)], by the labor [*mirkevet*] of the printing-press [*ha-defus*] [...] [my book (?)] [*ḥibri* (?)] [...] [...] for the sake of our profit [*charatz le-daʿat* (?)], and so forth [*ve-khuʾ*].

[Then a single display line — the signatory's name:]

> And I am your friend [*va-hineni yedidkhem*], the one who seeks your welfare [*ha-doresh shalomekhem* (?)] [...] Eliezer Lippman Silberman [*Eliʿezer Lipman Zilberman*].

[Note: the publisher's name resolves here at full length as **Eliʿezer Lippman Zilberman** (the "MaHaRaʾL Zilberman" abbreviated above) — the celebrated printer of Lyck. The body of the *Maktav* on this page is the editor's own announcement-letter about producing the 1864 Odessa edition; it has not yet given way to Delmedigo's 1629 text.]

[A rule, then a second editorial section, display heading:]

Lehodiaʿ ["To make known"]

For all our work here in this great book [*be-sefer ha-gadol ʿilem zeh* — "this mute book," *Sefer Elim*], the [...] is aimed [*mekhuvvan* (?)] [...] [...] without addition or subtraction [*beli mosafot ve-gerʿon*], to gather [*le-haqis* (?)] the [death (?)] [— i.e. the original wording] of the Rav [...] upon the roots, how many errors [*kammah ṭaʿuyyot*] [are] in it [*bo*] — [for] we did not lay a hand upon it [*lo naganenu bo ken yad*], not [in (?)] [...] [...] a special [matter (?)] [*li meyuḥedet*]; the prince [*ha-sar*] [...] in this received great things [*qibbel gevohot*], and various corrections [*ve-tiqqunim shonim*], [each] to [its place (?)] [...] from all the wise of heart [*mi-kol chakhmei lev*] [...] [scattered (?)] [in (?)] houses [*battim*], from all the [...] [freedoms (?)] [*ha-cherut* (?)].

[The second column resumes:]

according to the order of the corrections [*ʿerekh ha-tiqqunim*] that he made full [*she-millʾu*] from us [— "for us" (?)]; and every place that the tears of [...] [*dimʿei* (?)] [added (?)] because of it [*mosif bi-gelalah*] to the price [*ʿal meḥir*] of the body of the book [*guf ha-sefer*] — twenty-five [*kaf-heh* = 25 (?)], according to its [arrangement (?)] [*sidduro* (?)], and with [...] to its goodness [*le-ṭuvo*], may his Rock and Redeemer guard him [*zayin-ʿayin-yod* (?)]. — And if the faces [*penei*] of wise ones [*chakhamim*] and discerning ones [*u-nevonim*] [...] [an inheritance (?)] [*naḥalah* (?)] [...] our error [*ḥasronekh* (?)] [is] corrected [*tiqquno*], and the [discernments (?)] [*ve-hasagot*] of a man are [the man (?)] [*ish hu*], and after we have apportioned [*ḥilaqnu* (?)] [...] [...] of medicines [*refuʾot*], in melody [*be-niggun*], and so forth [*ve-khuʾ*] — from every [...] of [our (?)] friend [*yedidenu*] [...] [the (?)] portions of the [book (?)] until 25 [*ʿad kaf-heh* (?)] — behold, before you [*hinneh le-faneikhem*] a sample [*taʿam*] of the first portion [*chelek rishon*] from us [*mimmenu*], [...] [the proper (?)] [*ha-hagun* (?)] in [the books we have written (?)] every matter, in the name of those who hand it down [*be-shem omeram*]. —

[Below, in small Rashi type, the foot-of-page apparatus — a publisher's note on the 1629 Amsterdam first edition and on the Odessa typesetting:]

[*Haʿarot*, small Rashi type:] [...] is aimed [*mekhuvvan*] [...] all that was printed at the first press [*defus ha-rishon*] in Amsterdam [*be-Amsterdam*] [in the (?)] death [— wording] of the Rav the author [*ha-Rav ha-mechabber*], may his memory be a blessing, (*Elim*, page 42 [*Elim daf 42* (?)]), only that we [...] [a hand (?)], neither face nor in their margins [*be-ṣilyonot* (?)]; for [it was not (?)] [the Jews (?)] [...] also

[...] [...] various [...], also in selected [places (?)] [in (?)] holy places [*meqomot qedoshim*], poisons [*samim* (?) — perhaps "secrets/*setumim*"] [...] every [...]; and behold, in [...] discerning [letters (?)], in proper places [*bi-meqomot reʾuyim*], firm [letters (?)] [...] — [...] the tears of [...] [*dimʿei* (?)] reached our judgment [*le-dinenu*]; and even [...] his hand [...] [*yad keyaiv* (?)], also [...] not [...] [...] arranged [?], and with this [a separating (?)] sign [...] by the hands of the printers [*ha-tiqqunim be-yedei*] [...] — for there is no [...] [completed (?)] [...] [different (?)] inheritance [*naḥalah*], for in your goodness [*be-ṭuvkhem*] from us [*meḥennu*] [...] to profit [*le-haṣbiq* (?)] [...] from the fruit [*mi-peri*] [...] from us this portion [*chelek zeh*] in [its proper place (?)] [*bi-tekhuno* (?)], and in supplement [*u-va-hagbarah* (?)], and so forth [*ve-khuʾ*] — and there is in the philosophy [*ba-filosofyah*] [...] [the (?)] portions of the book [*chelqei ha-sefer*]. And one [or two (?)] who [is not (?)] [...] the LORD [...] [from (?)] [...] his hand [*yado*] [in] the book [*sefer*] [...] [...] our [...], behold in it they have made tasteful flavors [*ṭeʿamim*], well-arranged and pleasant [*meṭubbim ve-noʿim*] — and their [...] [...].

[A final display line — the editor's sign-off:]

> *Ha-mevaʾer la-haggadah u-mevi(?) le-veit ha-defus* — **The Editor / Compiler** [*HaMaLBaH"D* — abbreviation for *ha-mevi le-veit ha-defus*, "the one who brings [the work] to the printing-house"].

[End of the 1864 editorial front matter and apparatus on these pages. The seam into Delmedigo's own 1629 text has not yet been reached; it follows after this editorial *Maktav* / *Lehodiaʿ* opening.]

Approbations of the Rabbis of Venice

[This opening is part of the 1864 editorial front matter: the *Chatimot* — the approbations/endorsements ("signatures") of the rabbinical Geonim of Venice for *Sefer Elim*. The letters are set in two columns per page. Several distinct endorsement-letters appear, each with its own signature.]

Header

חתימות — **Approbations** [literally "signatures/seals"]

רבני גאוני ק"ק ויניצֿיאה יצ"ו על ספר אילם — Of the rabbis and Geonim of the holy congregation of **Venice** [ק"ק = *kehilla kedosha*, "holy community"], may [God] preserve and redeem them, upon *Sefer Elim*.

First approbation — right-hand column

My eyes and my thoughts, darkened by the abundance of my affliction, were lit up [אורו עיני] as when the sun rises and the sun sets [?] — they were upon me, and we waited like mute men [the syntax here is allusive and somewhat broken in the print; the writer means that his perplexity was relieved upon receiving the book]. The questions of a master and the answers of a master: the words [?] of a master — namely the master, our teacher and rabbi the Rav, comprehensive, mightiest among physicians, among philosophers, and among the sages, the master, our honored teacher Rabbi **Yosef Shlomo del Medigo** [דיל מיל"יגו], may the Merciful preserve and sustain him, **of Candia** [מקנדיא"ה], and his choice answers. For I knew his disposition well [literally "for I knew him face to face"], and I have set down here some of his wisdom several times.

[The column then breaks into rhetorical praise:] Here in Venice [ויניצָיא"ה] — in these days when it has been granted us to hear, among the learned, of [Padua / Paduans — בפאד"ווה] — and never has this divine Presence [שכינה] been removed from my hall, since long ago, when a young man, an upright and living man in every wisdom, walked in his ways: a voice speaking, none rivaling it; and from him those who write cannot draw breath, so that not even one in a thousand of them counts for anything — for so it always seemed to me to be the standard worthy of being called "a sage."

My eyes are lifted to the heavens [לשמיא נטלית — Aramaic, "to the heavens I lift them"], and to the living and eternal God [ולחי עלמא — Aramaic, "and to the Life of the world"], who has wrought praise and glory in raising up this day in Israel a man [גבר די נהירו — Aramaic, "a man of light"] of light, and has perfected us with understanding and wisdom, as it is by the wisdom of God [כחכמת אלהין — Aramaic] that is found in him, to give light through him: a lamp, the glory of our people, in the natural sciences and the theological sciences needful for the service of God — as the matter is set down by the lamp of knowledge, the great Rav and Guide [נר הדעת… המורה — i.e. Maimonides], of blessed memory, "to show the nations" [Esther 1:11], for not from them, nor among us, has this been spoken, for the word of the LORD was with us [an allusive chain of phrases praising the author's God-given gifts]; for such was [it]. And [it] proceeds from the consecrated and purified ones, from among the multitude of the sages [the writer extols the work and contrasts true sages with the "nations"]. We say truly [אמת… אומתנו] that those who shut their eyes [from] sights — wrapping a "covering of skin" over these matters, [merely] to conceal the shortness of their intellect and their alien and foreign [reasonings] — the reverse is the truth! For these are matters of faith

and the very roots [of religion]; the "skin" and the "essence" follow after the nature [of the thing].

When my Helper called out against them, in the saying "the lion has roared" [שאגת אריה — cf. Amos 3:8], "by day let me speak and by night" [I will declare] — I will pour out [my] tale-telling concerning the questions that remain, since the heads of my [chief] studies in the sciences, which I had let lie silent, are now stilled and forsaken; for from days of old — behold, none has arisen [to match] our understanding, [none] like our teacher [כמהו מורה]. May these words be near [acceptable].

[Then, in a second movement:] When God's Presence [shines] in [me], I have seen — those who ascend on the highway of praise, the circuits [מעגלי] of the paths of knowledge in the *Sefer Sapir* [ספר ספיר — "Book of Sapphire," i.e. *Elim*; the author's title], its cause that comes upon me [?] — by the name *Maʿaiyan* [מעכ"ת = perhaps an abbreviation; uncertain], in the truth of glorious wisdom, in a glorious and girded instrument, [the writer] says "let there be light," and there was light, to give light to my eyes [which were] blind, as one who walks in darkness saw a great light. A wonder shone upon them: the counsel of the great one increased resourcefulness among the chosen, the honored. How my heart rejoiced when I saw the wisdom of God ruling within the land. The understanding ones said: how pleasant! For the youths have multiplied, and the breach has burst forth [פרץ] — the twelve questions [הי"ב שאלות] were asked of fitting priests [כהונים — i.e. worthy ones], because it was pleasant to hear them.

For [the work's] arrangement is like a worthy priest [כהונים], and good tidings: **Yosef**, the master, who set down his lip-offerings [his words]; Yosef takes [his portion]; thus a sprout went forth and blossomed in Israel; the choice of the people, men of truth, in the fear of God upon their faces. And if, with his free choice, he so acquired [it], all the more so [is it certain] — and how many days [is it that] in the days of [King] Yosef [the author], strength has shouted in might [?]; and the mighty ones — our eyes have seen, and the heart rejoices in the questions [שאלות] engraved [as if] from the rock of strength; and they ascended upon the high places of [worldly] success, and answered as to the abundance of peace. [This closing is a florid blessing.]

Venice [פס"ק ויניציאה], in the good order [of] everything that [its rabbis] spoke; [in the year] **שפ"ט** [= 5389 AM = 1629 CE; 789 = 9ט+80פ+300ש+400ת, i.e. (5)389], may He who seeks the welfare of His people [ל"ו"ק = abbreviation, uncertain] grant them peace all their days.

Nechemiah Saraval [נחמיה סאראוול — Rabbi Nehemiah Saraval of Venice].

Second approbation — center/left column: "A Wise Man Is Better"

חכם עדיף — "A Wise Man Is Preferable" [a sub-heading; cf. the rabbinic dictum "a sage is preferable to a prophet"].

The Wise [חכמות, "Wisdom"] cries aloud in the outer place [בחוץ תרונה — Mishlei 1:20]. I beheld God: those who ascend on the highway, the praise of the circuits [paths] of knowledge — in the *Sefer Sapir* [ספר ספיר, "Book of Sapphire," i.e. *Elim*], a thing brought to me by name, by the name **מעכ"ת** [an honorific abbreviation for the addressee/author; "his honored excellence," uncertain expansion], in the truth of glorious wisdom, in a glorious and adorned vessel — and one says "let there be light," and there is light, to give light to my blind eyes, as one walking in darkness who saw a great light.

A great light shone upon them, for the wonder, the counsel of the great one — he increased resourcefulness among the chosen, the honored ones. How my heart rejoiced when I saw the wisdom of God ruling within the land! The discerning ones said: how pleasant, for they have grown numerous, and the breach has burst forth — the twelve questions [הי"ב שאלות] were asked of fitting and pleasant priests [worthy men], for it was sweet to hear them. As [worthy as] a fitting priest is, and as good tidings, so [is] **Yosef** the master, who set down [his words]; Yosef took [his portion]; thus a sprout sprang up and blossomed in Israel — the choicest of men, of truth, in the fear of God upon their faces.

And if, with his free choice, so [he] acquired [it], how much more so; and how many days, in the days of King **Yosef** [the author], has strength shouted in might. The mighty ones — our eyes saw, and the heart of [the writer] rejoiced; questions [שאלות] engraved from the rock of strength, and they ascended upon the high places of success and answered as to the abundance of peace.

[Then, in prose:] Like the embers of glowing coals, girded with sparks, and like castings cast from within fire [the writer compares the author's words to refined, fiery brilliance], which the chariot-throne [מרכבה — the Merkavah] burns and consumes, on Shabbos [an allusion to the celestial fire of Ezekiel's vision]. While my eyes gaze upon the truth of [his] essence, the sapphire-brilliance of his radiance — its hewn form, an "engraving" [טבעת, "signet"] of marble set upon it, a seal from the source [מקור] — the wisdom of the mathematical sciences [חכמת הלמודיות] they called it. They looked to it and were enlightened; and the source

[מצור / מבצר] of the natural and theological [sciences] was brought forth; and waters that have no end before them, as a heaped-up [stream stands] firm [Tehillim 78:13]. For from the deeds of the disciple — the greatness of the Ark [הארון, perhaps "the storehouse"] of his knowledge is known, comprehensive in wisdom, in knowledge, and in thought.

To know that, in the revealing of hidden things, the answers to the questions are bound and enclosed in connected [linked] books — a woman to her sister [interlocking, "each joined to her sister," cf. Shemos 26:3] — as when Bath-Sheva [בש"מה? — uncertain; reads "בת ש"למה," "the daughter of Shelomo/Solomon"] [came], beautiful as the constellations [כמלכותא ורבונתא — Aramaic, "as kingship and dominion"], and then the house was filled — the whole house — and its fullness, all of it light. And let it be, to my [own] desire [לצבי], and as a light staff [וְלמקל], a glory to the remnant that is left.

I said: this — let them be comforted, the angry-stirred ones — the workers of iniquity [פועלי און] who spoke against a righteous man with insolence [עתק], as it is said: "I will utter [forth] the wisdom of our sages and the understanding of our discerning ones," and there was [response] to him; let them not be silent [כלאו שנתק — uncertain reading]. And on the matter of the questions sought — Hindi ones [ההנדוסיות / ההנדסיות — geometrical / "of India"? more likely *the geometric ones*], built upon astronomies [תלפיות], and the silver and gold of choice settings [משבציות], remembered in my days; and surgical [ones — ?] — as in my days [the writer recalls a precedent]. When water was poured out [an allusion] by the hand of the comprehensive Rav, our honored teacher Rabbi **David'n Shoshan** [דוד'ן שושן — i.e. R. David ben Shoshan / Ibn Shoshan], may his memory be for life in the world to come [זלה"ה], who was asked about some of them, by one of the sages of Egypt, the Ishmaelite [Muslim] ones, who had a hand [skill] in the sciences. And there is none [מ"אן ד"אלים, Aramaic, "whoever is mighty"] — and in particular concerning the division [the partition] — its form, into three: like a knotty palm-trunk [כתמר מקשה], and the arrangement of [its] sides.

Upon my watch I will stand [Chavakuk 2:1] and station myself in my watchtower; in my gazing I looked, I awaited, I hoped [for] the high things [המעלות]. Thus says the small one [the writer in humility]: **Yaakov of the House of HaLevi** [יעקב לבית הלוי — R. Yaakov Levi / of the Levi family].

Third approbation — left column

[Opening lost in the gutter at the column-head; the legible text begins mid-blessing:] …may the LORD our God raise him up, [to] create a second [renewed]

world; the multitude of [his?] years and the wide breadth of his life — may He bring them up upon the high places of cloud, [upon] the heights of success [in / of] Jerusalem; and from him [from Zion] shall go forth Torah, until the word of the LORD comes [Yeshayahu 2:3].

Venice [ויניציאה], the tenth [day] of the month [עשרה לחרש] **Menachem** [= Av] **שפ"ט** [5389 AM = 1629 CE].

[Then a short rhymed encomium, two columns of paired phrases:]

Splendor and praise [הוד ותהלה]	None resembling Him [אל דמי לה]
He shall give favor [יהן קלא — "he gives voice"?]	One who does not cease [די לא פסיק — Aramaic]
An offering by fire [קרבן אשה — Vayikra 1:9]	To the name "Ishi" ["my husband," Hoshea 2:18 — אל שם אישי]
Sharp, he toughens [חריף מקשה]	Quick to grind [מחן מסיק — Aramaic "grinder grinds"]

Grieved and pained, and "the young one" [וצעיר — in humility]: **Yehuda Aryeh of Modena** [יהודה אריה ממורינה — Rabbi Yehuda Aryeh of Modena, "Leon de Modena"].

Fourth approbation — left column lower

Many and honored ones, beloved and dear: I, upon my watch [על משמרתי], here, am come "with glad song" [Tehillim 100:2 — ברנה], on the matter of the questions that occasioned [these] excellences [the discussion]:

[The text continues with a praise of the author similar in tenor to the foregoing, naming again the "twelve questions." The trailing portion is cut by the column edge.]

The small one [in humility], the youth: **Simcha Luzzatto** [שמחה לוצאטו — Rabbi Simone (Simcha) Luzzatto of Venice].

SEFER ELIM

Half-title divider

[A formal half-title page within a double ruled border. The title is set in large display type, spread across the full width:]

ספר אילם — SEFER ELIM [literally "Book of Elim" / "Book of [the] Mighty Ones"; the title puns on Elim, the desert oasis of twelve springs and seventy palms, Shemos 15:27 — corresponding to the twelve "springs" (questions) and the answers].

[The remainder of the page is blank within the border (heavy foxing only). (its verso) is likewise blank within the ruled border — no text. Delmedigo's own composition — his dedicatory epistle and the framing of the twelve questions addressed to Zerach ben Nathan of Troki — begins on the following page, in the next portion of the work.]

Sefer Elim

The body of *Sefer Elim* — Delmedigo's own composition — opens here. The first two openings carry the **First Letter**, an elaborate alphabetical-acrostic poem of homage sent to Delmedigo by his Karaite correspondent, followed by a transitional prose passage, a short second poem, and the heading of the **Second Letter**.

הכתב הראשון — The First Letter

The First Letter, which the esteemed and accomplished scholar, our teacher the master Rabbi Zerach [ben Nathan], sent to the perfect rabbi and physician, our honored teacher the master Rabbi Yosef Shlomo Delmedigo of Candia [יוסף שלמה דיל מדיגו מקנדיאה].

[The poem is a complete alphabetical acrostic. Each three-line stanza is keyed to a successive Hebrew letter (א, ב, ג … ת), with the lines of each letter marked by that letter in the margin; every stanza is introduced by the refrain **איך** ("How…"), and each is closed by a short bold prose tag built on the name Joseph/Solomon — e.g. *שעל הכתב* ("…asked of the writing"), *ויפתח יוסף*, *ויען יוסף*, *וילקט יוסף*, *ויירא יוסף*. The verses are densely woven from biblical phraseology and rhyme on a fixed end-syllable. The lines run off both side-margins of the scan; legible portions are translated below, with `[?]` marking matter cut off or destroyed.]

Dedicatory head-poem (full-width, to Rabbi Solomon — NOT stanza א). This head-block is a separate full-width dedication, its inner-margin (right-edge) letters spelling the name-acrostic **שלמה ... יוסף** (the margin run י־ו־ש־פ ... ש־ל־מ־ה), matching the Joseph/Solomon bold closing tags of the body stanzas:

A scroll, with joy and trembling, hasten and fly, flying to the hand of Rabbi Solomon [מְגִלָּה בְּגִילָה בְּחִילָה תְּמַהֲרִי וְעוּפִי לְיַד רַב שְׁלֹמֹה]. *And one who watches, and heals, and bakes with bread, and wars... for his eternity* [...וְצוֹפָה וְרוֹפְא וְאוֹפֶה בְּלֶחֶם וְלוֹחֵם לְעַלְמוֹ]. *And my sword, in entreaty, and my knocking, at the stronghold... for his eternity* [וְחַרְבִּי בְּהַפְצָר וְדָפְקִי בְּמִבְצָר... לְעַלְמוֹ]. *And it was written on the ... day, the eleventh of Iyar ... and the twenty-third day, in the year [1620 = 380] שף of his world* [וְנִכְתַּב בְּיוֹם ... וי"א לְאִיָּר ... וְיוֹם כ"ג לְמִסְפָּר שָׁנָה שׁ"ף לְעַלְמוֹ].

Stanza א

✻ א — *...to the gate of the King, the King* — King, Master of the universe. ✻ א — Indeed they shut the doors of my chambers; they opened my [?] with double-folded [?] — they barred them in double [bolts] from the universe. ✻ א — So, as I dwelt facing the [chamber of] decree, still [?] — and I [?] resolved [for] Solomon [וְהוֹאַלְתִּי לִדְפֹּק אֶת שַׁעַר הָאֵלֶם — "and I undertook to knock at the gate of the Mute/Elim"].

(closing tag) *...as Joseph came* [כאשר בא יוסף].

Stanza ב

✻ ב — How great is the burning of His wrath, more than the days of Creation [from the best of His taste] [?]. ✻ ב — The chaining of the encircling causes, and the pains that [?] — and pains that grieve from the rage of His fury [?]. ✻ ב — To which I weep with a broken heart, and according to [?] — to what [?] according to how I might revive a soul from its desolation [?].

(closing tag) *And Bath-sheba came to the King* [וַתָּבֹא בַת שֶׁבַע אֶל הַמֶּלֶךְ] — [to King] Solomon.

Stanza ג

✻ ג — His supernal wisdom was foreign to the son; this year [?] — to bring me round in turning, overturned [?] in His snares [?]. ✻ ג — [At] the resting of Isaac, his father, and his [?] — there beside Him a craftsman [אָמוֹן, cf. Mishlei 8:30], and he was delighting in the fruit of the Aleph and its ascents [?]. ✻ ג — My soul ascended in me in the year of the consecration of the offerings, and Isaac went

out — Isaac went out to converse, turning to evening [Bereishis 24:63] by reason of His heights [?].

(closing tag) *And Joseph remembered* [וַיִּזְכֹּר יוֹסֵף, cf. Bereishis 42:9].

Stanza ד

✻ ד — How I lifted my eye on high; from where would my help come [Tehillim 121:1], when [my] refuge was lost from me, with none seeking after my soul [Tehillim 142:5]? ✻ ד — How my likeness became as the pelican of the wilderness [Tehillim 102:7] when [the matter] perished; for [?] perished — flight, for there was no escape from me. ✻ ד — How the star of Joseph went soft [?]; from the wings of Isaac, and [?] — let it be, at the year's end, the [stars] of my drink, the wine of my chastisement [?].

(closing tag) *…a light for Solomon* [לִשְׁלֹמֹה [?]לְ[הָאִיר].

Stanza ה

✻ ה — How they have set me amid bitter herbs, as the Persian who deals out [provision] [?] — who deals out grain, by the choice of my chosen ones [?]. ✻ ה — How my fire is kindled, fire on the palate, and within, while [?] — and within, while [?]; still, the cutting of my walls is remembered to me [?]. ✻ ה — How a young lion comes from afar, and at once He wrote — at once He wrote to my father, to pursue after me [?].

(closing tag) *And Joseph gathered* [וַיְלַקֵּט יוֹסֵף].

Stanza ו

✻ ו — How shall I eat and not consume? I have seen [my undoing] in the nullification [of?] [מִבְּטוּל] — in the nullification of the Torah, through the corruptions [chevlei] of the time. ✻ ו — How my belly rages at the voice of the teacher of instruction [Yoreh], and I am proven worn — my conduct known, to knock together as one, broken in a faithful place [?]. ✻ ו — How six [?] are set there [as] sixes, [perfumed] with bitter myrrh, and [it rises] upon [?] — upon my dignity, my honor [?]; for thirteen years they are reckoned to me by every measure [?].

(closing tag) *And Solomon assembled* [וַיֶּאֱסֹף שְׁלֹמֹה].

Stanza ז

✻ ז — The thirties, and in [the] second [?] — thirty[-?], and in the [?]; after the death of Isaac my father, and behold I am a man of forty [?]. ✻ ז — A storm has seized me over the gathering-in of my parents and my teachers, and the rest [are] wandering and roving [נדים ונעים, Bereishis 4:14] — my parents and teachers, and the remaining ones, wandering and roving. ✻ ז — Yet still, master [אֲדוֹנִי], I bear, in the cessation of the moment, without touch — moving without rest along the way, all my days an afflicted, evil-fated man.

(closing tag) *And Joseph saw* [וַיַּרְא יוֹסֵף].

Stanza ח

✻ ח — How my heart melts within me at the contemplation of all these matters in the Book — all this in the Book of the Torah of God [תּוֹרַת הָאֱלֹהִים]. ✻ ח — How I have forgotten my ways, to listen by the mouth of scribes [scholars], and not [?] — by the mouth of scribes, and the high ones did not aid me [?]. ✻ ח — How I have hardly known, like the dialect of the Sanctuary [Aramaic, "as the speech of holiness"], the tongue of [?] — a tongue of truth [?]; and behold, they too are formless and void [תֹּהוּ וָבֹהוּ, cf. Bereishis 1:2].

(closing tag) *And Solomon understood* [וַיָּבֶן שְׁלֹמֹה].

Stanza ט

✻ ט — How my vessels are spent, what was in my hands, like the honeycomb [כיערת הדבש] from the mouth of books — like the honeycomb, from the mouth of books. ✻ ט — How my times are few [made small], and let my eye be lit like lightning in the pupil [?] — like lightning in the dark of night; and it returned [satiated] to the darkness of [my] dust [?]. ✻ ט — How I have leaned to inquire among their companions, and behold there is no [?] — there is none who understands their words, in the cities and in the villages [?].

(closing tag) *Joseph hears* [שׁוֹמֵעַ יוֹסֵף].

Stanza י

✻ י — How I groaned, as I sighed when I reflected on my silver lot — over my lot of silver, and the shortness of my sleep [וקצור השנתי]. ✻ י — How my kidneys are stripped, with greater swelling over the lot of longing — over the lot of yearning, in the strength of my desire [בעז תשוקתי]. ✻ י — How my strength is hardened like

flint, and my spirit at my death over the war; in my trust — my counsel concerning the war; in my trust, and this is my groaning [וזו נאקתי].

(closing tag) *And Solomon rested* [וַיָּנַח שְׁלֹמֹה].

Stanza כ

✻ כ — How [my soul wearied] to find another, a thousand to my satisfaction [a counselor among a thousand?], being my strength and my might [?] — a thousand to my satisfaction, my thirst — for such is my strength and my might. ✻ כ — How [bound is] my flesh, and my bonds tied, to distance the scattered ones — to distance the scattered, from the failing of my wealth and my strength [מאפס הוני ואוני]. ✻ כ — How my kin and my heart [yearn] to behold those round about me; and behold they too are as a priest — and this is my mourning and my grief [?].

(closing tag) *And Joseph could not [restrain himself]* [וְלֹא יָכֹל יוֹסֵף, cf. Bereishis 45:1].

Stanza ל

✻ ל — How, O dwellers of the world! [מָרוּ יוֹשְׁבֵי תֵבֵל] — "Vanity of vanities" [הֲבֵל הֲבָלִים, Koheles 1:2]; what then are the deeds [?] — what is the work of their wanderings in their investigations? ✻ ל — At a wicked deed of [the Master, blessed be He] [?] — yet He is awesome [נוֹרָא הוּא], and as though [?] — and as though they saw Him as nature, in their walls [?]. ✻ ל — At this let my heart tremble; for Israel, the good, is cast off [זָנַח יִשְׂרָאֵל טוֹב]; an enemy pursues them — and there is none save by their choosings.

(closing tag) *And Solomon waited* [וַיְחַל שְׁלֹמֹה].

Stanza מ

✻ מ — How, if the Master, blessed be He, [gathered] the remnant that remained, and the sage in the [cave] [?] — He [hid] like one in a cave; and He stretched out His hand still, and a people roars, the nose too, and a tooth gnashing [?]. ✻ מ — How they rebelled against Him and despised His Torah, and quenched the lamps as in the menorah [?] — and quenched the lamps as in His menorah; therefore everyone who passes over them is appalled and hisses [Yirmeyahu 19:8]. ✻ מ — How [?] their adversaries grew soft, their princes their overlords, their destroyers — their destroyers went out from them, and they became a reproach and spittle [לכלימות ורוק].

(closing tag) *And Joseph answered* [וַיַּעַן יוֹסֵף].

Stanza נ

✻ נ — How a darkening wonder fell upon the remnant of their muscles [sinews], and they did not see — and they did not see, each man his brother [Shemos 10:23], in their dwellings. ✻ נ — How [?] they grew hot in their deeds, and rejoiced in their portions and [their disciples] — their disciples [?] in their hands [?]; their acquaintances were dragged off [?]. ✻ נ — How my misdeeds broke out like a compass, as I wrestle with them [in my wandering] [?] — those who drove me out [are] swept away [?] from their encampments [?].

(closing tag) *And the King answered* [וַיַּעַן הַמֶּלֶךְ] — Solomon [?].

Stanza ס

✻ ס — How I broke open my ways amid the ranks; a part of [?] divided, dividing the light of my skin [?]. ✻ ס — How I plead with the scattered, in the nothingness of the trampled and the [banished] [?] — the trodden and the banished, those who touch my afflictions [?]. ✻ ס — How [?] my hidden things and my secrets; [let] the master of my affliction know, that for [?] — let him who knows my affliction know, that for my shame I am [bound] upon my fists [?].

(closing tag) *…that they not hate us, Joseph* [לוֹ יִשְׂטְמֵנוּ יוֹסֵף, cf. Bereishis 50:15].

Stanza ע

✻ ע — How I was struck with trembling and shuddering in my remembrance, toward the tribe of [Judah] [?] — toward the tribe of Judah, in [its] speech, naked and bare [עָרוֹם וְעֶרְיָה, Yechezkel 16:7]. ✻ ע — How I cried out and spoke harsh words [against] God [חֵ׳], without the [?] — without the wisdom of words, like the voice from a darkened land [?]. ✻ ע — How over this the earth mourns [Yirmeyahu 4:28], even without water from heaven [?] — even without water from the heavens; and her flames are coals of fire, the flame of God [שַׁלְהֶבֶת יָהּ, Shir HaShirim 8:6].

(closing tag) *Then Solomon said* [אָז אָמַר שְׁלֹמֹה].

Stanza פ

✻ פ — How [a man] called me, as I exert myself to expound the pure Torah, the Torah [?] — the pure Torah, its narratives and its secret. ✻ פ — How [I exhaust] His wonders in His testimonies, to receive according to my [?] — to receive according to my understanding [knowledge], and my providence by the mouth of His unique ones [?]. ✻ פ — How [I survey] the screen, and the kapporet [the cover

of the Ark] which is upon the [Ark] — which is over the Ark above, and its staves [בדיו].

(closing tag) *And Joseph opened* [וַיִּפְתַּח יוֹסֵף].

Stanza צ

✻ צ — How [they died], the honored of [the] living, as I desire to fulfill my purpose at the channel of two great rivers [בְּפֶלֶג שְׁתֵּי נְהָרוֹת גְּדוֹלוֹת]. ✻ צ — How they come like hosts, the host of Cortova/Córdoba [קוֹרְטוֹבָא] and Córdoba [קוֹרְדוֹאָה], rank facing rank, terrible as an army with banners. ✻ צ — How my [pangs] [?]; let it approach to the mountains of Israel, in the matter of [?] — drew near to the mountains of Israel, in a certain matter [?].

(closing tag) *…fear before Solomon* [יָרֵא מִפְּנֵי שְׁלֹמֹה].

Stanza ק

✻ ק — How I sought there, [as] they strengthened me concerning several kinds of Kabbalot — over how many kinds of [Kabbalot — received doctrines] that are not parallel in their loops [אֵינָם מַקְבִּילוֹת בַּלּוּלָאוֹת, cf. Shemos 26:5]. ✻ ק — Without [?] the bald [exoteric] [?], esoteric and practical [?] — esoteric and practical, and what is beneath them I found in them wonders of wonders [?]. ✻ ק — How I came, and I did not give myself to expound by the mouth of book and scribe [?] — by the mouth of book and scribe; nor have my sayings been given to fill out [?].

(closing tag) *Joseph solved for them* [פָּתַר לָהֶם יוֹסֵף, cf. Bereishis 41:12].

Stanza ר

✻ ר — How I was with the stones of the [breastplate] [אֲבָנַי אַחֲרֹן?] together with my birth, and I divided in my [portions] — and they were divided in their divisions, and split into two nations [?]. ✻ ר — How twenty-four [כ"ד], I did not find them; and each man covered over to his fellow — as witness of their eternal Chayot/living-beings [כְּעֵד חַיּוֹתָם הַנִּצְחִיִּים]. ✻ ר — How shall I do — what is their essence? Let me [know] [?], that their deeds [?] — what is their essence, that their days [?] in [?]; for nothing is hidden, a [?] thing [?] from them [?].

(closing tag) *Nothing was hidden, a [?]* [לא נֶעְלַם דָּבָר ?].

Stanza ש

✻ ש — How my soul is like this, [yearning] without restraint to behold the appearance of [?] — my soul yearns, without restraint, to behold the appearance

of the [?]. ✻ ש — How [the light of] my eye is scattered, and to my eye a hedge darkens my entrails, to rouse — my entrails darken, to awaken; my master's spirit was stirred up from on high [?]. ✻ ש — How a treasure-store of [the] heaped-up perished; who shall give? [Let him give my recompense; for it shall not be lacking] [?] — for it shall not be lacking, and for my sake, hasten [וּבַעֲבוּרִי הוּשְׁ].

(closing tag) *And Joseph sustained* [וַיְכַלְכֵּל יוֹסֵף, cf. Bereishis 50:21].

Stanza ת

✻ ת — How [restrained] over my fullness, which is chosen among vessels [?] — those who are chosen in the vessels of [?]; the proven matter is sought from me [?]. ✻ ת — How [a fountain] your servant from before His glory; awaken, awaken in the city [of?] your rich-one [?] — if I forget you, behold, you shall set me [?]. ✻ ת — How — do this and weep, and reckon me not to be remembered [?] — to write, and your soul will [be glad]; [I] will satisfy, and behold He has comforted me [?].

(closing tag) *And Solomon made* [וַיַּעַשׂ שְׁלֹמֹה].

[Closing prose of the First Letter — a description of the figure of King Solomon, much of it running off the side margins of the tiles:]

And **King Solomon** [הַמֶּלֶךְ שְׁלֹמֹה] — [his] form, the form of Solomon's own [?]; for the perfection that belongs to Solomon, his height is high, and the way is paved [?]. He rode upon the abundance of outflow, and flew, and was seen upon the [wings of?] natural perfections, the divine and the rational [?] — perfections, natural, divine, and rational [?]; the radiances of instruction before the master [Ba'al?] of the hidden treasure [?] — the radiances of instruction before the lord of the hidden, the gold of the north [Iyov 37:22]; "you shall come" [?], I [?]. Solomon made [it] at Etzion-Geber [בְּעֶצְיוֹן גֶּבֶר, I Melachim 9:26], a man, a man [וְגֶבֶר], without limb — without limb he calls aloud, with the voice of the [host?], strengthening himself [?] [from?] bitterness [?]. In the beginning, and Solomon knew, in the wisdom of [astronomy — חכמת הטיא"רין?], and he conversed [?] — he conversed [of] the matters [pursuits], and led them and guided them; and I, the [?] man [?] saw [?] — he saw a tree, a fount, the splendor of the gold of the [scarlet?] [?] for a sign upon my hand, that I might be — let it be for a sign upon my hand [Shemos 13:9], that I might live; and Bath-sheba drew near [?] — Bath-sheba [drew near] to the King [of] kings of kings, and God dealt well with [them?] [?]. There [is] Solomon, and there was the work of righteousness — peace [וְהָיָה מַעֲשֵׂה הַצְּדָקָה שָׁלוֹם, Yeshayahu 32:17]. From the God of Elim [מֵאֵל אֵלִם] [?].

[A short connective prose line:]

And when the writer saw that the honor of his composition [comes?] herein — a likeness here resembling [?] the effective [voice?], and stronger than the intellect; and a proverb [mashal], and Joseph [?] — [as it is said] in the proverb:

[A short four-line poem with three rhyming columns per line — read right to left as three parallel hemistichs:]

(right column)	(middle column)	(left column)
A song [or "pruning," זָמֶר], is this — if a voice [קוֹל] sings [?]	a song — or a song without knowledge [בְּבְלִי דַעַת]	… [?]
a whisper [?], to my heart he brought forth	their speech, with the voice of the pelican [קוֹל קָאַת, cf. Tehillim 102:7]	… [?]
deep [עֹמֶק], a man [גֶּבֶר], for falsehood [לַשָּׁוְא], a man — [honored?]	a man without bridle [רֶסֶן], also rebellious [?]	… [?]
pure [בָּרוּ], He gave [נָתַן], on high [מָרוֹם], strong [אֵיתָן], He [?]	strong [אֵיתָן] is he; thus is the bitterness of knowledge [מַר הַדַּעַת]	… [?]

הכתב השני — The Second Letter

The Second Letter.

[Opening, marked by the marginal **מי** ("Who…"):]

✻ מי — Who has gathered the wind [Mishlei 30:4] — grace, and good sense, and good [?]; by His countenance [?] the intellect [?] — by His countenance the intellect is seen; and it is known [?], "[and to know] the LORD" [וְיָדֹעַ אֶת ה׳] in wisdom and in understanding and in knowledge [Shemos 31:3] [?]. He is His treasury [?] — He is His treasury; teach Him and praise Him, [for the earth is full] [?] — His praises fill the earth, in the investigation of the divine matters [בַּחֲקִירַת הָעִנְיָנִים הָאֱלֹהִיִּים]. [?] my heavens, from the shield of [?] — from the shield of my [?] my heavens, awaken; he shall give my heavens [?], awaken. "Listen, for I have heard that the spirit of the sun [?]" — Listen [שָׁמוֹעַ], for I have heard that the spirit of the sun [?] [rose?]. To His place He pants [שׁוֹאֵף, Koheles 1:5] [?] — the matters of the King; what He desired He sought [out] [?] — the word of the King, what He desired, He [investigated?]. And Israel loved [Joseph] [?] — *And Israel loved Joseph* [וְיִשְׂרָאֵל אָהַב אֶת יוֹסֵף, Bereishis 37:3].

[The Second Letter continues beyond this opening; the page foot closes the supplied tiles here.]

*(This page continues הכתב השני — the Second Letter — from Zerach ben Nathan of Troki to Yosef Shlomo Delmedigo of Candia. The body is a rhymed encomium of successive strophes, each headed by the word **מי** ("Who…") and each sealed with a phrase drawn from the Joseph narrative of Bereishis — the recipient's namesake. The two halves of each full-width line have been read together off the tiles.)*

Who — when the firmament is spread before Him as a tent — set the world in order to establish the array of the heavens, and the standing-place of the earth and their motions [?]… whose summit reaches heaven to know the counsel of the angels of God, who are at the ends of the whole inhabited world [Lacuna: line edge clipped]; the makers… spoke distinguished words, to hear words in the voice of those who give light, from whom His Presence radiated — *the everlasting hills shall be a king before them*. **And it was for the head of Joseph.** [The bold seal closes on Bereishis 49:26, "for the head of Joseph."]

Who — what man shall arise to gaze on the holy place [?], on the courses of the holy paths of the heavens [Lacuna: clipped] — and all their host has its allotted portion, and they sweep their courses, from which there is none beyond; *is there a God beside Him*? Who created them? He rides the heavens; His name is pure and clean, and at His will He set them in motion, to move all the spheres — He reversed their motion and bore them up. **And the Lord was with Joseph.** [Bereishis 39:2 / 39:21.]

Who — your likeness, in your greatness, after God made known whom He loves [Lacuna: clipped]: upon the house of computation he was faithful, and upon your mouth every… ✻ and in all the disciplines he gives grace to the wise, and they grow wise by his ascent on high — so that your eyes may gaze on the heights, to know how the eighth sphere [גלגל השמיני, "the eighth sphere"] is set in order; their forms, what they are — for between [you and them] is a barrier. **And Israel spoke yet more of Joseph.** [Echoing the Joseph cycle.]

Who — he begot, he made famous by his wisdom dew-drops of light [Lacuna: clipped]; lights there were, to give light in the firmaments — the seven planets [שבעה כוכבי, "the seven planets"] [Lacuna: clipped]: to walk in their motions; he made known to mankind their courses [Lacuna: clipped] in their appointed time, for each one its own orbit by their reckoning [?]. Would that the wise might grasp this matter — together with a sage and a discerning man whose understanding of their motions is singular, from west [Lacuna: clipped] — to a teacher fair-spoken

in their cause [?]. **As he interpreted to them, so it came to pass.** [Bereishis 41:13.]

Who — those that go and move at the bidding of those who set them in motion [Lacuna: clipped], pouring out splendor for the world, this trellis [השתל הזה, "this planting/trellis"] [Lacuna: clipped]; their motions and their longings are by the decree of the holy and awesome God — the boundaries fixed [Lacuna: clipped]; in this world stars run their highways by the four [לד', "by the four"] foundations [Lacuna: clipped]; on the shoulder of the sphere of the sun [כדור השמש] he set the dominion, and it shall be for the [שכם, "shoulder"] of the orb of the sun [Lacuna: clipped] to give light to the air, and the perplexed stars [וככבי הנבוכה, "and the wandering/perplexed stars"]; to fire, to water [Lacuna: clipped] — into air, and the stars that stand for the glorified earth [לארץ המפוארה]. **And Joseph was found.** [Bereishis 44:16.]

Who — this man fears the Lord, on whom rests the spirit [?] of wisdom and understanding, the spirit of counsel and might [Yeshayahu 11:2] — heavens [Lacuna: clipped]; he raised the heavens [?], a light arose; he named the intellect, descended and weighed; he searched out deep commentaries [Lacuna: clipped]; deep commentaries to the Jews [?], for the heavens were a light to them [Lacuna: clipped]: "the heavens are the Lord's, and the earth"; he granted to mankind the power to understand all [Lacuna: clipped] — in all that is possible for him, by the way of the Torah. **And Joseph remembered.** [Bereishis 42:9.]

Who — a sage and a discerning man — this one is wise and understanding, and he knew that the ways of the Lord are upright [Lacuna: clipped]; three [שלשה, "three"] portions of being [Lacuna: clipped]; the second and the third — all of them know it [Lacuna: clipped]; all of them give thanks and recount the glory of the high and exalted God; who has not [Lacuna: clipped]; he knew in all this that it is the hand of the Lord — she made this, and it is from him who gives life [Lacuna: clipped], for he revives [?]; and he sustains all his beasts; he bears them up. **There stood a man with him in his making-known. Joseph.** [Compositor seal on the Joseph cycle.]

Who — the dew of the heavens, the dwelling-places of the deep, and the seven eyes of the earth [?] [Lacuna: clipped] — what great are his works, the wonder and the marvel of marvels [Lacuna: clipped]; in chariots and in plants [ובצמחים]; and as for living beings, the unintelligent [בעלי חיים הבלתי, "living beings that lack" — i.e. the irrational creatures] [Lacuna: clipped]; the unintelligent — those that speak and those who are spoken to; he gave understanding through these matters [Lacuna: clipped].

*(continuing on the verso, folio 8 / *

awesome ones. Blessed is He who graces man with knowledge — knowledge that the soul is roused to know it [Lacuna: clipped]; and the unseen things… **He gave light upon** [Lacuna: clipped] — for awesome is the Lord, who hides hidden things; He gives His light by the hand of Joseph.

Who — he ascended the mountain of the Lord, and who shall rise up in the place of His sanctuary [?] [Lacuna: clipped] — the holy spirit of contemplation, the disposition of wisdom rejoices to fear the Lord in faith — than which there is more precious in my eyes; honored from God, *your father; and may He help you and gird you with wisdom and with knowledge and with understanding* [echoing Bereishis 49:25], to hear [Lacuna: clipped]; your ear listened, and my eyes did not see your dwelling; here the alien is sojourning [?] within me, for the work of your hand has established you. **The son of a fruitful bough — Joseph.** [Bereishis 49:22, *ben porat Yosef*.]

Who — a wise man, and who knows the meaning of a thing — in the wisdom of [Lacuna: clipped] the precise science of grammar according to the Torah, and Scripture is known [Lacuna: clipped]; even for our brethren the children of the Exile [בני הגולה, "the children of the Exile"]; the Teacher who points the way [המורה, with a play on *Moreh* — "the Guide/Teacher"] for righteousness… a father [Lacuna: clipped]: he learned Mishnah and Talmud, the halachos and the Aggadot, the books and the Sifra [Lacuna: clipped]; I have known that all of it is fitting, and all the words of the decisors and the commentators [Lacuna: clipped] — and the Torah of the Rif [הרי"ף, R. Isaac Alfasi] in your mouth, you set it; and you praised yourself upon your tongue [?]. **You have come to the head.** [Lacuna: clipped — closing seal.]

Who — by the Lord, in the truth of the inwardness of knowledge, the hidden things of the Torah, the searching-out [Lacuna: clipped]; the searching-out of the Torah, inquiry, and probing the choice in the book *Moreh* [the *Moreh Nevuchim*, Maimonides' *Guide of the Perplexed*] and the book *Milchamot HaShem* [*The Wars of the Lord*, Gersonides] and in the words of the Kuzari [הכוזרי, the *Kuzari* of R. Yehudah HaLevi] and Rabbenu Nissim [רבינו נסים] [Lacuna: clipped] — by their method he sifted food out from refuse, fit from unfit; he ate the kernel [Lacuna: clipped]; he sweetened the secret matter [?] which appears at your fingertip — where is this place? [Lacuna: clipped] all the day; and the deep, the whole city, is our voice [?]. **Saying, "Come, my brethren."** [Lacuna: clipped — closing seal.]

Who — he rebelled in the question, "Who shall measure the waters?"; a sea — a broad expanse — the commandment of the heights of the heaven [Lacuna: clipped]; your walking in your going up, a healer of all flesh and a wonder-worker [Lacuna: clipped]; at the report of your ears, ears shall give ear [?]; your hand is great enough to knock [Lacuna: clipped]; your eyes shall behold. **And Joseph recognized.** [Bereishis 42:7–8, *vayaker Yosef*.]

Who — he gave the meaning of a thing — the mouth of a sage, lips that kiss [?] [Lacuna: clipped]; he prevented good from the Master of Desire, the living [Lacuna: clipped]; he loves long life, to see good in His words: the lamp of the commandment and the Torah is light [Lacuna: clipped]; he sows light for the righteous, and gladness for the upright of heart [Tehillim 97:11] — to a man [Lacuna: clipped] in answering with his mouth: behold, light has broken forth upon him. **As Joseph came.** [Bereishis 44:14.]

Who — he set you as a man, a prince and a judge over Israel, a heart of [Lacuna: clipped]: a heart of wisdom God gave you, the God who dwells in the chamber; you have made me wise [Lacuna: clipped]; the nature of states a man sees — your face I have escaped, as the face of God [?] [Lacuna: clipped]; my contemplations distress me and bear me up; the spirit of the Lord, in his Torah he spoke a word [Lacuna: clipped]; he lives — and who am I? **And Joseph went up.** [Bereishis 50:7.]

Who — he ascended on high in his contemplation to grasp the ways of the roots [העקרים, "the fundamentals" — also the title *Sefer HaIkkarim* of R. Yosef Albo] [Lacuna: clipped]; in the renewal of the world, and in the eternal beings, to [Lacuna: clipped] the roots, with subtle disputations and keys [?]; strong [Lacuna: clipped] to believe; living beings — to one God, who has no other; there is no likeness to whom *he sent to Moses* [Lacuna: clipped].

*(continuing on the recto, folio 9 / *

our Rabbi in the Torah, in it commandments and statutes — to know its interpretation and [Lacuna: clipped]; commentary and prophecy, the prophets and the resurrection of the dead [Lacuna: clipped]; on the day of judgment, sentence is cut for every man according to his ways, by the providence of the men of [Lacuna: clipped]; the men of our exile are engraved with poured-out marks [?]. **And it came to pass — Joseph.** [Closing seal.]

Who — he distinguished the degrees of the kinds of wisdom in the ways of logic [ההגיון, "logic"] and philosophy [הפלוסופיאה, "philosophy"], to break the heresies of the deniers [Lacuna: clipped]; the words come in their full measure, the sayings

of rhetoric [מליצה], hard difficulty [Lacuna: clipped] — a wonder, a triumphant proof; he refutes the error and the songs [?]; he knew, in the natural science [שמע הטבעי, the *Physics*], heaven and world, being and the loss of soul [והפקד נפש] [Lacuna: clipped]; the loss of soul; and what remains is nature, the wonder and the marvel [Lacuna: clipped]; the wonder of the substance of the sphere [עצם הגלגל, "the substance of the sphere"], and whether it is possible for conjunctions; and the portions of the [הב"ח — "living beings"? abbreviation, uncertain] [Lacuna: clipped]; the portions of the living beings and in the technical abridgments — by however many words. **Joseph was able.** [Bereishis 45:1, *yachol Yosef*.]

Who — the adversary; who, by their pleadings, by their garments, sets your answers in order [Lacuna: clipped] — concealed for the discerning of the upright; he explained the existence of God and [Lacuna: clipped]; and prophecy and the miracles — that this is from that — chained, linked, and flowing, for [Lacuna: clipped]; the halachos; he who saves by proofs cut clear [Lacuna: clipped]; and it was like their proofs and their disputations — he makes Torah great and glorious; therefore in [Lacuna: clipped]. **The house of Joseph.** [Closing seal.]

Who — the voices of many mighty waters, the breakers of the sea of wisdom [Lacuna: clipped]: your fountains, the work as the writing of your hand, to glory in song [Lacuna: clipped]; and the Chaldaeans [וכשדים], Greek [יון] and Arabic [וערב] and Italian [ואיטלקי] and Spanish [וספרד] and French [וצרפת] and German [ואשכנזי] [Lacuna: clipped] — the tongue of truth was in your mouth. **Their possessors are to Joseph.** [Closing seal.]

Who — see the wealth, and your renown — your heights — to the God in your throat [Lacuna: clipped]: the sciences, such as the book of Euclid [אוקלידס, Euclid], *Milyas* [מיליאס — Miletus? possibly the *Spherics* of Theodosius, or Milesian; identification uncertain [?]] and Ptolemy [(יוס)ובטלמ, Ptolemy] [Lacuna: clipped]; Galen [גלינוס, Galen] and Hippocrates [ואבוקראט, Hippocrates] and the agreed-upon premises among the sages of [Lacuna: clipped]; and the state, and however many orders of Mishnah, and the proverbs of the wise [Lacuna: clipped]; the choicest in all study — and you are a treasury, the vessel of every vessel [Lacuna: clipped]; he who treads the sole of your foot — the sleepers shall be roused and the slumberers awakened [Lacuna: clipped]; he who is within you is holy; he did not withhold the good from those who walk [Tehillim 84:12]. **For the sake of Joseph.** [Closing seal.]

Who — hear, O hear, the report of your goodness, from a distant land, and not [Lacuna: clipped] — they bring an offering to Solomon [לשלמה, "to Solomon" —

punning on the recipient's name]; they shall carry to the awesome One a gift, gold and frankincense, on high [Lacuna: clipped]; and treasured. And what precious and great thing is done to the foundation of the height? Know [Lacuna: clipped]: God grants grace and glory, and there is no silver reckoned of worth in your eyes; and that there is [Lacuna: clipped]. **And the Lord was with Joseph.** [Bereishis 39:2.]

Who — he set in order the spirit of mighty love between you and [Lacuna: clipped]; the words of your soul — wisdom is bound to a soul that longs to [Lacuna: clipped]; also to soar before the truth your spirit longs, like good oil [Lacuna: clipped].

*(continuing on the verso, folio 10 / *

my eyes and my heart all the days; my ministers fell low; *for I am God* [?] [Lacuna: clipped] — and we became servants. **And Joseph set in order.** [Closing seal.]

The final acrostic strophe — a single verse per line, RTL, each line sealed by the word **יוסף** ("Joseph"). Read across (each line's start in the _א half, its continuation/end in the _ל half), the lines are:

1. From the hand of the mighty one of wisdom, a merchant of silver [סחר כסף] … a proverb in every acquisition; to the head of **Joseph** [יוסף].
2. For he is a son of old age — he gladdens [his] father … esteemed(?) for kingship; this — his name is **Joseph** [יוסף].
3. Glory of the assembly(?) — his ox, on high his horns [קרניו] … among them he shines; they too went forth(?), **Joseph** [יוסף].
4. Strong as a lion — how he speaks, as engraved(?) [כחוק] … the choice of harvests, behold, to the head of **Joseph** [יוסף].
5. A good name … the birthright is his … deed and Talmud — great is its abundance to his house(?), **Joseph** [יוסף].
6. A standing sheaf stood upright [קמה אלומה נצבה, Bereishis 37:7] … they bowed down — all my brethren — **Joseph** [יוסף].
7. [In] a time of famine they came to buy grain … to the weary he dreams(?), **Joseph** [יוסף].
8. Sun, light, and stars of light — do they not acknowledge? … anointed(?), a father to his son(?), **Joseph** [יוסף]. [Bereishis 37:9]
9. A secret of God to all in his house … there [he] was known; to be enslaved(?), sold was **Joseph** [יוסף]. [Bereishis 39]

10. Happy the man enlightened in his beginning … he shall flourish(?) in the end, and there was **Joseph** [יוסף].
11. A prospering man, a righteous one, foundation of the world [צדיק יסוד עולם, Mishlei 10:25] … faithful in his house, and there was **Joseph** [יוסף].
12. His praise and his standing, splendor in blessing him(?) … a state/province is exalted because of **Joseph** [יוסף].
13. A fruitful bough(?) [פורת], wisdoms above an eye/spring … a portion to seven shall go forth, **Joseph** [יוסף]. [Bereishis 49:22, *porat*]
14. All gathered, they came to your head; behold … it is comely for you, saying, "I am **Joseph** [יוסף]." [Bereishis 45:3, *ani Yosef*]

Closing prose of the Second Letter *(beneath a rule-band):*

Behold, I, Nathan [הנני נתן], have given to him My covenant of peace [Bemidbar 25:12, *briti shalom*], and I bring before him… garments, the deserving — to be for my lord, for clothing… and he is according to the gracious one's hand; and if he find favor and kindness before him [Lacuna: clipped]; we sing for joy at his salvation. I myself rejoice when I saw the gold of [Lacuna: clipped] — of the spirit that rests upon the head of Joseph; the word of the speaker [Lacuna: clipped]: rumored on high, and revealed.

Zerach, son of our teacher Rabbi Isaac, of blessed memory [זרח במוהר"ר יצחק ז"ל — Zerach son of R. Isaac, may his memory be a blessing].

Vilna [ווילנא] and its surroundings, the recesses of the north [Yeshayahu 14:13]. Now send the writings, you who are arrayed [Lacuna: clipped] — the wondrous one, the renowned physician of the Duke [הרופא המהולל של הדוכוס — "the celebrated physician of the Duke"], Reuel [ראואל — Reuel, a personal name [?]], he shall see [Lacuna: clipped]; happy are your portions, who enjoy [Lacuna: clipped]: from whom His Presence dwells, and the blessing of the Lord upon your lands [Lacuna: clipped] — there your soul, in the eternal life that lengthens you [Lacuna: clipped]; the godliness that clothes you, the second [garment] with the delicate ones — for the garment of souls [Lacuna: clipped]; the face of his Most High; in the offering that goes before, by the hand of a man [Lacuna: clipped]; the faithful of Zvi Frankin [צבי פראנקן — Zvi Frankin, a personal name [?]], son of our teacher the Rabbi Chaim Shin [חיים ש"ן] — and I [Lacuna: clipped]: there I shall come after it, and there I shall rejoice and be glad in it; and you — we have pleaded for abundant peace [על רוב שלום].

Sefer Elim

הכתב השלישי — The Third Letter

[This letter opens with an extended rhymed-prose encomium built as an acrostic: each strophe is headed by the word **מַה** ("How…"), and the prose is woven almost entirely from biblical phrases (a *meliẓah* style). I render the sense; the heavily allusive scriptural diction is reproduced in meaning, with the strophe-heading word kept as a bold rubric.]

How sweet are your sayings; how they delight! Your meditations gush forth like a flowing brook [נמרצן — vigorous], a matter rightly spoken, fitting upon its turnings. I have chosen the pleasantness drawn from the height of speculative inquiry, set on high — and earth, down to the depth [וארץ ילעומק]; from the blessing of the Lord [is] his land, in wisdom and the knowledge of life [מבירכת יי ארצו בחכמה ודעת חיין]. To the first [acrostic: לרא"ש יוס"ף — "to the head, Joseph"], the righteous men give drink with the kisses of their mouth; the righteous man bears the fruit of wisdom.

How awesome is the people, an awesome gift, before my Teacher [למורא — "to my teacher"; pun on "awe"]: a writ wholly [graven], like words engraved upon a tablet by the finger of God [כאצבע אלהי"ם]. A righteous judge, and one who sits [in judgment] — [bold acrostic-clause:] my eye [has seen]. "And from the beginning…" he says [Isaiah allusion], and so on. He chose a gift; [a remnant] is preserved [למשמרת]. As a sign, a crown of glory [עטרת תפארת], a mantle [אד"רת — mantle], a tongue…

How I have loved the Torah, precious, the choice [zealous] song, the diadem of the ministering one — in peace they joined together, from the mouth of the sage, his eyes at the head [בראש"י]. We acquired [קני"נו]; he walks righteously before Him. As a dream of the visions [reddened] — my friend, dazzling [צח], speaks in righteousness, in the depth of a halacha [הלכה ר"ב — "great" or "of the master"].

How good and how pleasant is the dwelling of brothers together among the living, awakening; and behold the stars of light — narrow [צר] and light [אור]! He plays [משתעשע] and runs through the broad places; he gives forth his voice [קולו]. God [is] his strength [חילו]. He longed [חמד] to sit in his shadow [בצלו], and the fruit was full [for him] [מלולו]. Sweet [מתוק]: whoever hears, [says] "Joseph!" — in many languages [בשפה עמים רבים], and his strength rose up to stand. [As] a memorial, one who seeks the good of his people and speaks peace.

How I would liken you, O man — O master, O sage — who in [your] strength has come to give light to those who pass over / the turbid [עב"ר עכ"ר]. A face shall rise [פנ"ים יעלה — acrostic]; the limb/pinion [אבר] sprouted [נץ] from his stalk. He recounts the heavens by his wisdom; he tells of his might. He runs the path on the ladder of the wisdom of his knowledge. He gives a portion to the seven [planets — לשבע"ה (?)], and also to the eight [לשמנ"ה — the eighth sphere (?)]: the righteous one [צרי"ק] in the firmament [בערבות], whose name He has set [שמו]. He numbers and counts the stars of light — as the host of the angels of God whom He created in His creation. From God his Father, in his help [מאל אביו בעזרתו].

How is this in your hand, in your being — to God, your hand! Your hand [ידך] [is great] in the science of number and measure, "how much" and "how much again" [כמ"ה וכמ"ה — the disciplines of quantity], to test the very [might] of your wisdom and your understanding — to the eyes of the peoples, whose report [of you] we have heard, and your greatness [לעיני העמים אשר שמענו שמעך וגדולתך]. For there has been revealed to you what is hidden in the science of medicine [חכמת הרפואה]; and the report of your walking among the nations was made known. All who see you gaze, they stand and are struck dumb, and there was no spirit left in them [יביטו יעמדו ויאלמו ולא היה בהם עוד רוח, cf. Yehoshua 5:1]; to answer your couches [the rooms of your house — אשרוך בנות (?)]; they call you to understanding [לתבונות]. In knowledge [בדע"ת]… and the earth [וארץ] [you have plumbed] to its depth — the [secrets] beneath, second and third, in the disputation [בפלפול]; a luminary, a sun in the navel [center] of the world; and the earth, with the six [directions — עם הששה]; on high, established, discerning and wise like you [there is none], O God Almighty [אל שדי] — may the Name bless your border; peace [יברכך השם גבולך שלום].

How good are your tents [טובו אהליך], O Shem [acrostic-letters: ש"ם], and Ever [ב"ע — Shem and Ever (?)]; there [God] commanded [the blessing] [Tehillim 133:3 echo].

[Here the strophic מה-acrostic gives way to flowing prose, still highly allusive:]

May He command for you, O Lord, [the spring] that "went out for the salvation of your people" [Chavakuk 3:13], to bring [the hidden] to the light of the Lord [לאור יי"]. [Acrostic:] "Arise!" [קמ"ה]. He arose against you; speculation, intellect, and all that is near. Knowledge, understanding of [his] books — those who teach [the] sciences, who instruct in the seven sciences [חכמות שב"ע — the seven liberal arts] by day; the wisdom and knowledge of life [Aramaic-style flourish]. "Who is this that comes" [Shir HaShirim 8:5] in proverb [במב"וא] — in sayings [מאמרו"ת], in eloquence [מליצ"ת], in hard sayings [הקשי"ם]? — wondrous demonstrations

[מופתיים], like cast statutes [כראי מוצקים]. He loves righteousness [צדק], and "the wicked will not stand" [Tehillim 1:5]: my [arguments — בנציחי (?)] are not erroneous [הטעא"ות], no — entering rightly [נכנסת בכוונות]; in the way of understanding, to know the [intentions] of the philosophers [כונות הפלוסופיי"ם].

[He surveys the books of natural science:] In the books of nature [בספרי הטבע]: the *Physics* [שמע — *Shema Tiv'i*, the *Physica*], the *De Caelo* [שמים — "Heavens"], *On Generation* [ועולם הויה — *De Generatione et Corruptione*, "world, coming-to-be"], and the *Meteorology* [והקסד — the upper "alterations and corruptions," i.e. *Meteora*]; the [knowledge of] the upper signs. [Such matters] have you set in your heart, in the highest things; and their wars [contradictions] you contend with on every side. And so I have spread out [my desire] further in the books: [acrostic:] "He has placed for you potent capacities [כחות מזומנות] to break [refute] their wars [contradictions] in [every direction]." [Among them:] the *De Anima* [ספר הנפש — "Book of the Soul"], the *History of Animals* [ספר טבעי בעלי חיים — "the natural book of living beings"], the *Sense and Sensibilia* [חוש ומוחש — "sense and the sensed"], and a recounting of what remains of the nature of the offspring [ילדי הטבע — the generated things].

[On the commentators and successors of Aristotle:] Aristotle the Greek [ארסטוטל"יס היוני], and the great commentators — Alexander [of Aphrodisias] [אלכסנדר], Christianus [חימיסטיא"ן — likely Themistius, recte Greek commentator (?)], the head [chief], Greek [יוני] and Latin [והלטיני (?)]. Their counsel descended [ירדת] to the end of the *Moreh* [Nevuchim] [דעת המורה — "the knowledge of the *Guide*"], the *Milchamot* [HaShem], and the *Sefer Or* [Adonai] [ספר אור יי' — *Or HaShem* of Crescas (?)], and so on; the *Etz Chaim* [עץ חיים — Aaron ben Elijah's work (?)]; also Plutarch [פלוטינוס — recte Plotinus? (?)], Porphyry [פורפיריאוס], Proclus [פרוקלוס], Iamblichus [ימבליקוס], Polycletus [קליצידיאוס — possibly Calcidius (?)], the Hermetics [הירמים — the Hermetic writers (?)], and all who walk after their footsteps [עקבותיהם] — turning the ear to draw water from the springs of [their] wells [ממעיני], compositions [great] in [building], to draw forth [the secrets]…

[The Judaic stream:] Judaism [יהדות] enters in peace and goes out in peace [Pesachim 50a echo]. [Acrostic:] "[Redeemed], the exalted ones ascend in acquisition" — after the enlightened ones [המשכילים] [reaching] to the heavens, to purity. [Tehillim/Shir HaShirim allusions:] "your navel," "the rounded goblet" [אגן הסהר — Shir HaShirim 7:3]; "the form, the deed"; the deeds [or sciences] of Torah [התוריות]; "the tablet of the heart," to know — and "the eyes to see" [עינים לראות]. [The disciplines:] inquiry [into] law and speculation; the weighty *Tosafos*

[ההוספות הנכבדות — the glosses], spoken thus; rock [סלע], their distresses [their commands — מצוותם], like the *Aggadot* and the *Midrashim* in the assemblies [במקהלות]. [Acrostic:] "Also the sons of men, when your hand is [open] in everything." So in everything [is] God's grace and honor; in all good sense, in all your ways, "may it prosper" [שלוה].

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[After a star/ornament mark, a fresh strophe:] [Acrostic:] "Your faith [is] a fire, a law" [Devarim 33:2 echo — דת אש]: the silver yearned for [הכוסף הנמרץ] is not satisfied, your eye [לא תשבע עינך]; in every tongue and "writ of decree and law" [Esther echo — כתב התרת]. And the inquiry, "and between" — the assured ones [החוחים — the doubters/uncertain (?)] — to gather the hard sayings [לקטת הקשי]; the masters of Scripture [בעלי מקרא], to know "between son [and son]" [or: every kind — לדעם בנת] and all their books [ספריהם]. I will praise [acknowledge] you [אודך] for the awesome wonders from the power of [God] [מהכח האלהי], to know "where is this way the light dwells" [Iyov 38:19 — אי זה הדרך ישכן אור], and "where is this the place" [ואי זה מקום].

[Acrostic:] "[My heart] rejoiced [שמ"ת] over every separated [parashah], abundant" [Shir HaShirim allusion]; and I shall draw near and bring near the near ones [קרובים]. We are brothers, [bound] together; the secret is concealed [נמחק סוד — the secret is effaced/sealed]. Therefore [we will offer] their acquisitions [or: I longed — וזקניחשק …] for your service [עבודתך]; and they inclined the ear to hear a manifest reproof [תוכחת מגולה] from your mouth [מפיך], "longing to know" [חושק לדעת] your friend [דעך — recte רעך, "your fellow"] [from] his books [ספריה — recte ספריו] and their conducts [והנהגותיהם] [in] peace [שלום].

[Now the prose turns explicitly to the framing of the correspondence:] The honored [Adar] [אדר היקר — "the honored, glorious one" (?)] who [sent] us your disciple [תלמידך], the [renowned] [המחזר — recte המהדר, the meticulous one (?)], who is [counted] by the name of the Master, R. Ezra [הר"ר עזרא], son of our teacher R. Nissim, of blessed memory [במוה"ר נסים ז"ל] — [a light]; the great luminary, Rashi [רש"י], with my teachers [מורחי — recte מורי, "my teachers" (?)], what awesome things in the firmaments [ברקי — "in the firmaments/lightnings"], the deep secrets [עמוקות סודי] [and] his renewals [וחדושתיו]. Happy are your disciples who stand [before you].

[New page — the praise of the disciple and the framing of the reply continues:]

[The disciples stand] before you continually, for they discoursed of the blessings, the pools [ברכות], living waters [מים חיים] upon a weary soul [Mishlei 25:25 echo]. Truly [האמנם], whom my soul yearned for — and my soul also longed to know whether there is yet [a secret] [if there is yet more (?)], and a hidden matter [ודבר סתר], beneath the riddles [תחת חידות], in [your] arguments [דקדוקיו] — [a man who] gives light to the eyes in matters divine [בענינים האלהיים]; for his knowings [ידיעתיו] are praised [משובחות] and beautified after the manner of the Rambam [כפי הרמב"ם] who commanded [אשר יצוה].

[The praise of the elders who guarded the secret:] [The elders held back] their sons [בני] from disclosing [לבלתי סור] the teachings of the books of the Raavad [הראב"ע — possibly Ibn Ezra, or recte Raavad (?)], and so on. And the disciple, the aforementioned [the one mentioned], who poured water upon the hands [of his master — יצק מים על ידי — i.e. served as his attendant, after II Kings 3:11], the master [האדון] told us — for the bringing forth [הוצאת] to light of his thoughts; his praises [משבותיו — recte משבחותיו], the pure ones [הטהורות], were as lights [למאורות]. And now [ועתה], how [great] is the [exertion]: for it would be to your way [לכדרך — recte לכבודך, "for your honor"] that he do [a kindness] with us — charity [צדקה] — and it shall be: the deed of charity [מעשה הצדקה] is peace [Yeshayahu 32:17].

How precious is your kindness toward them [חסדך אליהם], to let us know "which is this way" the light dwells, in the order of learning [בסדר הלמוד] and the choicest [ומבחר] of the books. And it was not so [ולא נהיה]: all our right hand [are] tangible matters [ממששי — tangible/palpable], like the blind, forbidding and permitting [אוסרים ומתירים], and praise to God [ותהלה לאל].

[The longing for books and learning:] My whole house was filled with all good things [כל טוב], every precious thing, the matter of his desire [the will of His desire — חפץ]; books, printed [ספרים דפוסים], [their faces] dim [וכהוהים — and dim/faded], their ends [קצתם] [stained], the holy ones [הקדושים], sages, [newly] cut [חרשים — recte חדשים, "new"]; their ends [bought] with silver, full price [בכסף מלא — Bereishis 23:9 echo]; their acquisitions [קניחי — recte קניתי, "I bought"], and I sought from the four corners of the earth [מארבע כנפות הארץ]. They stand crowded, heaped up in heaps [צבורים חמרים חמרים — heaps upon heaps, Shemos 8:10 echo]; and they cry out [צועקים] to me, saying: "Arise, call! — and there is none for me [קרא ואין לי], for these are [withheld] [משוגרים — recte מסוגרים, "shut up/withheld"]." My intestines grew bitter [חמרמרו מעי — Eichah 1:20]; my heart [comes] rotten in its bones [רקב בעצמותי — Mishlei 14:30]. Because of my God [there is] no peace [אין שלום — Yeshayahu 57:21 echo].

And now [ועתה], when I heard of your greatness and your strong hand in [wisdom], from the ends of the earth I heard your report [שמעך]; I said: "Let me speak and find relief; let me pour out my musings [אשפוך שיחי] and my spirit [ורוחי]." And I lifted up [וריוח — recte וריח] and wrote a scroll [מגילה] in the order of the alphabet [כסדר האלפא ביתא], each [strophe] — and I sent it these many days by the hand of the honored [men], the [worthy ones] of Frankfurt [פראנקן — recte פרנקפורט (?)], may it be built and established [יצ"ו]; faithful are they, and they did the errand [ושליחותם עשו]. And how [ארכ"ו — "how long" (?)]: perhaps I have not found favor in your eyes, and you will send me [back] after [your reply] [?].

[The structure of the scroll explained:] Each [section] runs on twenty-two parts [על כ"ב חלקים — twenty-two parts, one per letter], and every part — "who is before him" [the unlettered (?)] in [the alphabetic acrostic — באלפבית"א]. And at the end of each part the name **Joseph** [יוסף] is mentioned, from a verse of the Torah [מפסוק התורה] — twenty-two times [כ"ב — twenty-two], to be a crown of glory [לעטרת צבי] and a sapphire [ולצפיר — recte ולספיר, "and a sapphire"], a glory in Israel [תפארה בישראל]. And I saw that there was no answer [ובראותי כי אין מענה]:

I knew that your honor is bound up [כרוך], always, after the great Duke [אחר הדוכוס הגדול — the Grand Duke (?)]; [whether] the scrolls of your hand [are] responsa [תשובות] of our teacher R. David Segal [למהר"ר דוד סגל (?)], [the rulings of R.] Joseph [יוסף], and further legal decisions [ועוד פסקי דינים] of the master R. Shalom Sh"n [להר"ר שלום ש"ן (?)] — [I saw it]:

[A cascade of praise framed as biblical allusion:] The holy man, Jacob [קדוש יעקב], and the God of Israel [ואת אלהי ישראל] [will revere him] [יעריצו]; a fire was kindled [אש קדחה — Devarim 32:22 echo] [in my nostrils] [?]. I will multiply [אעתיר] my words concerning him and entreat [ואפציר]; and I will come to the king [ואבא אל המלך] who [is]… toward the heavens [לגבי שמיא] he panted [שאף] — for he says, "to subdue my face to the ground" [לכבוש פני בקרקע]. Let the [Maker] of silver return [ישוב יצרי הנכסף]; your hand turns [יהפוך ידו] all day, also by night.

Behold, [there] came the strength [עזו] of your awesome deeds [נוראותיך]. They will say: "Behold, near [are] new things [חדש"ים], deaf ones [silent ones — לחשים (?)] have arrived" — and they saw, and so they wondered [תמהו], reciting [the books] [מספרי], a book of knowledge [ספר ומדע], and still they speak [מדברים]; and behold the choice young man [הבחור הנחמד], the delight [למראה] of the enlightened [המשכילים], and the glory [ופאר] of the sages [החכמים], like our teacher R. **Moshe son of R. Meir of Metz** [כמהר"ר משה בר מאיר מיץ], may his Rock

keep him [יצ"ו], who came [בא] to the house of our bais medrash [לבית מדרשנו], and the earth was illumined [והארץ האירה]; who poured water upon the hands of the master [אשר יצק מים על ידי האדון]. He has a [science] [יש לו מד"ת], abundant, pure of thought [רבו טהור הרעיונים]; everything he says, glory like the very heavens for purity [כעצם השמים לטוהר]. May his mouth be filled with your praises [ימלא פיו תהלותיך].

[New page — the resolution of the matter and the count of the questions:] For the brevity of his sayings, your heels [עקביתיך — recte עקבותיך, "your footsteps"] are not known to us, nor [in]… your thoughts are very deep [עמקו מחשבותיך — Tehillim 92:6], and the matter is that the choice one [the youth] mentioned [שהבחור הנזכר] [is] dear; he did not cut [a decision] nor decide [ולא גמר וחלף] for now [ועכר — recte ועכשו (?)], to [give a sign] [להתנה — recte להתות (?)]. To my fancy [לאניני] [it was] like fire to be kindled, and it burned [ובער] within us the longing [החשק], and there is none [ואין] [to quench it] [מכבה]; it does not seem [ואין נראה] to me [a thing] like a command done from the finger of God [כמצוה ועשה מכח אלהים (?)]. [These are matters worthy] for these, to teach [לאלו למוד]; and the master of desire [ומחן חפץ — recte ומהון, "and from the wealth of desire"] you have given [ואתנה — recte ונתנה], as it is good with his heart [כאשר עם לבבו].

[The image of the brook of gold:] [I write to the master] of gold [על הזהב] — and he is a flowing brook [נחל נובע], a fount [מקור] of wisdom [טוביית — recte טוב, "good"], dwelling [ישוב]; and the strength is revealed [ומתגלה כחם — recte ומתגלה כח] upon the honor [על כבוד] of his master [רבו]; to say [לאמר]: "Woe!" [הוי]; "every thirsty one [כל צמא], go to the [waters of] living waters" [Yeshayahu 55:1 echo — לכו לבאר מים חיים]. And what [is] for you [ומה לכם] from the wells [אצל בורות] — broken cisterns [בורות נשברים — Yirmeyahu 2:13]?

[The transition to the formal reply — the count of twelve questions:] And what is for me, [my master], whose advice was good — to write and to ask [לכתוב ולשאול]? And so [וכפ"י — "and according to"] what [you] permit me [שהרשני], my dear words [are] precious [דברי היקרים], and according to a count, like the count of the tribes of Yah [כמספר שבטי יה — twelve, after the twelve tribes], a testimony for Joseph [עדות ביהוסף — Tehillim 81:6 echo], which are upon them, that [the Lord] grant me [יזכני יי'] understanding from you [להבנות ממך]; and your words shall be — many [grafted (?)] words [כרכיבים ערבים — pleasant/intertwined (?)], and with a spirit liberal, free, [innocent], wiser than [the gold of] Paz [מחכימת פז].

And again [ועוד] I have hardened [myself] to ask seven questions [לשאול שבעים — recte שבעה, "seven"; or "seventy" (?)] that I have heard, the [most] wondrous and

marvelous [הפלא ופלא]. I and the colleagues [וחברים] listen [מקשיבים] to the voice — to [every] understanding [לשכל]; and were it not for [the presence] of your face [פני הדרתו], I myself would have been the bearer [נושא הייתי], saying [אומר]: ["To] the height of secrets [למעלת סתרים] [are] these," together with the twelve [עם היב' — together with the twelve]. The questions of **Sefer E[lim]** [שאלות ספר אילם]…

[**The naming of the work and the total count — a key passage:**]

[The total count is given:] …and seventy bitter herbs [ושבעים תמרים — seventy date-palms (?)], as the number of the questions and the doubts [כמספר השאלות והספקות].

And He [ואתה — recte ואחה (?), or "And it shall heal"]: a flowing brook, a fount [נחל נובע], a source of wisdom [מקור חכמה] — do not shut up [אל תסגור], do not [withhold] [דלת — recte דל, "do not be poor/sparing" (?)]; chimneys [ארובות — the windows/sluices, Bereishis 7:11] is your name [שמך]; and the pipes [וצנורות] go to you [תליך — recte עליך], cast [זרוק] upon us [עלינו] [pure] waters [מים, טהורים], lest my springs dry up [פן יחרבו מעייני].

[A self-deprecating close to the appeal:] [My cisterns] dried up [ויבשו], my forests withered [חמרי ונבלו יערי], and I have become a byword [למשל] among [Israel] [[אצל ישׂ[ראל]]; "a parched land, a dry place [ארץ נגב] in [my] length" [Yehoshua 15:19 echo]. With all my heart [בכל לבי] I make known among peoples [אודיע בעמים]; and I will not be ashamed [ולא אבוש], "let it not be told" [אל נא תגדה] to all my people, the mockers [המלעיגים], who [mock at me] [שmock]; for in vain is my labor [כי לריק יגעתי] — let them not [draw near] [יהיו נא קרובים] to my master, the [Rav] [[אל אדוני הר[ב]]].

And these are the twelve Springs [ואלה הן היב' עינות — *Eynot*, "springs/eyes"]

[**This is the formal index/heading of the twelve major sections of *Sefer Elim*. The word עינות ("springs," literally "eyes/fountains") names the twelve principal questions, echoing the "twelve springs of water" of Elim in Shemos 15:27 — the source of the book's title.** The introductory headnote to the springs begins:]

[The first matter to be treated — by way of introduction:] …upon the name [על שם] of "twelve springs of water" [שתים עשרה עינות מים — Shemos 15:27], and seventy date-palms [ושבעים תמרים — Shemos 15:27], as the number of the questions and the doubts [כמספר השאלות והספקות].

[The introductory description of how the inquiry proceeds:] [Do] not shut [אל תסגור], do not [withhold] [דל]; by my witnesses [בעדי] your meditations shall gush

forth [יפוצו מעינותיך — Mishlei 5:16]; the outlet [חוצה] — open [פתח]; cast toward you — no, upon us [עלינו] — pure waters [מים, טהורים], lest my springs dry up [פן יחרבו מעייני], like the parched [land] of my forest [אצל ישושי — recte יבושי?] and my groves [ונערי — recte ויערי, "my forests"]; "and when [the man] comes by way of" [וכי יבא דרך] the [stubble] consumed [כתל מלקש (?)], I make known among peoples [אדיע — recte בעמים — אודיע] in your walking [מהלתך — recte מהלכך, "your going"]; remember the abundant good [רב טובך], the choicest [תבעה — recte מבעה (?)], my lips [שפתי] [against] the mockers concerning me [המלעיגים עלי]; and over [my] longing [חשקי] in these studies [באלו הלמודים], saying [לאמר]: "[Behold,] my master [אל אדוני] — upright words [דברי הישר] before your way [לפני דרכך] for the sake of my [foundations] [למען שוררי — recte שורשי, "my roots/foundations" (?)]."

ואלה הן היב' עינות — And These Are the Twelve Springs

[The formal section-heading is repeated as the rubric of the new structural division. The introduction to the springs proper now opens:]

[**Introductory frame of the first inquiry — "The First Spring" (העין הא'):**]

The first [העין הא' — "the First Eye/Spring"], concerning the choice [בהיתר — concerning the resolution/loosening] of the triangles [המשלשים], dividing [ויפרד] into four heads [לארבעה ראשים — into four chapters/heads]:

The **first** [הראשון]: concerning the resolution [בהיתר] of the triangles [המשלשים]; the **second** [והב']: in the examination [בבחינת] of the triangles [המשלשים], whether they are true [אם הם אמתיים], according to my [will] [רצוני], at the points of being [נצבי ההויה — the right-angled ones of being (?)] — if the angles [הזויות] [are] inclined [נאוחות — recte נכוחות, "correct/straight"] according to [their measure] [כמוהם] to the ribs [לצלעות — to the sides], and to be saved from deceits [להנצל ממרמות] and erring instruments [ותכלי הטעאיות — recte וכלי, "and instruments of error"]; and the **third** [והג']: in the resolution [בהיתר] of the triangles [הכדוריים — recte הכדוריים, "the spherical (triangles)"], curved [עקומי] of being [הזוויות — of the angles]; and the **fourth** [והד']: in the resolution [בהיתר] of the triangles [המשלשים], the upright ones [ישרי] — [that] they may all be established [להקיום כלם — recte לקיים, "to confirm them all"].

But indeed [ואולם]: how [מה] I rejoiced at the resolution [שמחתי היתר] of the triangles [המשלשים]! [This is] the head [the beginning] of the matters [ראש דברי], a [casting] of awakening [מסבת ההתעוררות — the cause of the rousing], and the counsel [is] desirable [והעצה הנמרצה], which I was counseled [שנתיעצתי] by the disciple aforementioned [מהתלמיד הנזכר] by the hand of [על ידי] [the master] —

— [the matter came] by way of a scroll [על ידי מגילה] sent to me [שלוחה אלי], in which they called me [אשר בקראו בה] [to the great vision] [המראה הגדולה]; and behold, I [hereby] set forth [והנני מציע] her [its argument] before you, O Lord [אותה לפניך], and the matter is finished [והב' כלה — recte והדבר כלה (?)]:

[The opening of the answer / the disciple's request:] [The disciple] came to enter [לבא] to the end-purpose [אל תכלית] of the science of [orientation] [חכמת התכונה — the science of astronomy], for the brevity [לקוצר] of my knowledge [רעתי — recte דעתי, "my knowledge"], to stand [לעמיד — recte לעמוד] upon the [true] nature [על תכונת] of this glorious science [החכמה המפוארה הזאת]. One must [צריך] …

[Page ends here; the body of the First Spring (the treatment of the resolution of triangles, the first of the twelve) continues beyond this opening. A round inked **library/owner's stamp** is impressed at the lower-left of the final page, partly legible: the cartouche reads [נ][ה]מרא למד גו [?] (Hebrew, undeciphered) — [Owner's stamp, Hebrew, illegible].]

[The First Spring (מאמר העין הא') continues — Delmedigo's encomium on the science of triangles (trigonometry), begun in the answer to the opening question on the resolution of triangles.]

to know the science of the resolution of every triangle, whatever its kind — for thereby a soul is fitted out for understanding, and a spirit for those who walk in the ways of the mathematical disciplines. This honored part of the whole science of geometry — *Geometria* (גיאומיטריאה [recte: ריאה, "geometry"]) [?] — treats of figures, their qualities, and the apprehension of them; it is the guide for knowing the measure [?] of figures generally, and it has a portion set apart for measuring three-sided figures in particular — whether by themselves [?] or [?]. Whether the figures be abstract or embodied (in matter), they arise from the surfaces; and just as the surface itself comes to be from lines, so too the line comes to be from points. Their motions are well known to the masters of the mathematical disciplines; and so too are surfaces, all of them — whether triangles, or sections and segments, some of which are rectilinear and some curved, and some mixed — all of them have been determined.

Thus the great importance of the triangle, above every other figure, comes to be seen. In sum, it is one of seven: that is to say, I mean that of the [several] kinds [of figure], its surface alone is the one we always need to know. There are three [things] we must know of it, since its [?]… And so, in the knowing of the surfaces of triangles, they are all full of wisdom and consummate beauty; and this belongs

to the craft of *Geometria* [Geometry], the measuring of surfaces. It has no concern with astronomy, except [in so far as it leads] to a known measure — namely, however the whole circle is divided [?].

The learned, seated in the academy of the wise, and his [?] knowledge in [?] — among [forms of] knowledge it is the most honored and the most precious, and the most useful for understanding. Its praise is such that no man should draw near to inquire into the motions [of the heavens]… he has no radiance, nor any portion among those that reckon. And this is the thought, that the whole foundation [?] and demonstrations are by means of the proper instruments — a fine thing — the purpose being that through them one may arrive at the inner content of the matters; for there is no need to [proceed by] a roundabout way that wears out a man's life, and that flies up to the heavens [?]. For how many hundreds of years, and one [man] could not see nor know but a small portion…

the practitioners say [it] in different [or: changing] times, and they pursue this together with [?], and these are the means [?]. And the foremost [observers], who apprehend every star and star, [know] in conjunction [or: by combination, conjunction] [?]; we know their risings and settings, their courses [?], their distances, and the nearness of [some] of them to the center of the world. This is the way, moreover, to know likewise the size of the diameters [or: of the bodies] [?] of the equal [?] and the true [bodies], and what their true place is — in the band [of the zodiac] [?]… These wandering stars are examined together with the fixed stars; the [?]… and this is told in a book on it. And so, one who knows the science of the resolution of triangles… the ancients have written it even beneath the lines [i.e. between the lines, or beneath the diagrams] [?]; one who has not [seen]… all the luminaries [signs] in the heavens — for example, the eclipses, the places of [the heavenly bodies]…

[Source p. 16 / Hebrew p. 15–16 — the encomium continues, weaving the praise of trigonometry through dense biblical allusion.]

whether it be for [computing back] to the days of the Flood and the Dispersion, or forward to a single fixed [date] — these and many years besides are reckoned well [from them], and the tables [לוחות], the works of the [triangles] [?]. The Torah lightens [the labor], and from the wellsprings — from the springs [?] — by reckoning, just as Ptolemy did, [and] from the Third Treatise (ממאמר הג') onward, and after him al-Qunshu [?] (אלקונשו) [Arabic astronomer, identification uncertain] [?], and many [others]. Now, most of what is written in the two [?] [is] correct [?] for the spherical triangles, in which all the [?]…

The first treatises (המאמרים הראשונים) of the *Almagest* (לאלמגסטי, the Almagest of Ptolemy) [Greek: ἡ Μεγάλη Σύνταξις, "the Great Treatise"] — it is right [for you] to attend to [?] the spherical triangles, in which they are all [?] cut [intersecting]. Ptolemy [observes] by means of the instruments of the sun, and they require the foot [base?] to precede [?]; questions in the [theory of] the First Motion (במחנוע[ות] [recte: בתנועות / the First Motion] הראשון) [?] and the manner [of it]. Ptolemy [?]… in the science of number (בחכמת המספר): **Now** (ואולם) those that come after him, and in [?]…

Johannes [?] Müller (יואניש מ[ז]ר הסלך [recte: רגיומונטאנוס / Regiomontanus, Johannes Müller] [?]) [German astronomer-mathematician], and after him in Buda [?] (ובפדט) [?], *Geographia* (גיאודגיאו [recte: גיאוגרפיאה, "geography"]) [?], Peurbach (פירודבאקיאו [Georg Peurbach]) and his disciple [Regiomontanus]; the Ashkenazim [German scholars] who came to hand [?] — the Greek sages, of whose [legacy] he brought with him [?]…

Bessarion (ביאאריאן [recte: ביסאריון / Bessarion, the cardinal]) [?] the cardinal (החשמן), from Constantinople (מקוסטנ"טינה) — [they] wrote [their] paths [methods], and brought other paths, more [?] easy, and after them many [more]; their trouble and their ease are greater [?]. And my teacher Sh.N. (ש"ן) [?] wrote… **a perfect treatise** (חבור שלם) on this, and there the name [stands] — the wonders of the discoveries — [I] called my name [?]. Therefore my counsel is that you [conceal?]… and his sharp wit [will] outweigh, and make words abundant.

and supplications before His glory — perhaps He will command His scribe [?] to set him [?]: my eyes have so seen, and my Lord is righteous of heart in his wisdom; and now…

✻ **And great** (וגדול) is the power of the triangle in every work of thought and in [?] — the breadths [latitudes], the depths, the surfaces, the bodies, and their varied motions. It is the touchstone, the strength of those who weigh the bars [of the balance] [?], and the [?]: the equator [המשוה] [?], and it measures [the courses of] rivers; and it [sets out] [?] the heavy ones [weights], with [?]. The fearful places, the place that apprehends [?] those that incline [?] upon their stand [?]…

✻ And in war it is the corrective [?]: the ladders to leap upon the [walls] [?], to subdue the enemies by hurling great stones, fire, and brimstone [גפרית], iron and [bronze] [?]. The cannon (איל הברזל הנוגחים [lit. "the goring battering-ram of iron"]) [?] is aimed [?], and the trajectories of the shells ([ות]הכולל [?]) [are] straightened [computed] [?]… the towns [?]…

✻ And for the heavens it builds buildings in steps [terraces], and there are set [?] those that give voice — palatial sounds [organs?], arranged and mixed [?]… the shadow [?] [of] the sundial [?]: it is the one that takes hold of [?] the [?] eyes [?] of the seer, ascending and descending; and the surrounding [parts] that beautify [?] are pleasant and [?]… the straight [lines] [are] hollowed [?], and the standing [lines] are slit [?]…

✻ The wells and the depths, like hills; the world of surfaces — the low ones [?]… and those that are covered [?] from the [?]; the old [?] place [?], hollowed out, standing erect, [or] overturned [?]; their hands stretched out to the air [?], simple [flat]. He who gives a soul to those that lean [the dying] [?]… [those that] do battle, woe to them, the [?] and the [?], and all who see them [are] confounded [?]; [I] will reckon them the works of [the] [?] strife [?].

✻ **Great** (גדולים) are the works of the triangle, all of them — wonders, like the works of the architects (ארכיטידים) [?], in the mirror [or: aspect] of the rows [ranks] [?]… upon the city and the camps [?]: how were they [?] desolate in a moment, and [overthrown]?… the *architectus* (ארכיטאם [Latin: architectus, "master-builder"]) [?] [made] for [?] himself a vent [bellows] [?] that blows like the wings of the [?]…

✻ And mixed [together] like harps and lyres (כנורות ונבלים) [?], the voices of many waters [מים רבים]; with a pleasing [?] sight, the [?] approaching and receding [?] — pleasant and [hidden] [?], the fair [ones] [?], banded and spotted [עקודים נקודים וברודים, Bereishis 30:39] [?], cut and indented [?], all of them [?]. The [?] mountains [or: the *Dorians*?] [?] [tuned?] for the upright [?]; the low ones, the short ones, the long ones — and the bringing near to the eye [magnifying] [?] of the distant [?]… the standing ones, the overturned ones, the shadows [that] go forth [?] to meet [?] those that look [?]…

to the abstract ones [?] he moves [?] [the] forms [golems] [?] of them, to make them visible [?]… all who see them are panicked [?] and [tremble], and even the wise smiths [craftsmen] [?]… The triangle: every craftsman who [forms] [?] arises [?], to be shown to the peoples in deeds [?]… The rows [ranks] [?] from afar, the ships [ספינות] of the enemies and the houses [?] of the [?]…

in a moment they are dismayed and overthrown [?]. It is the one that gives wings [makes fly] to the carved images, the [?] idols [?], like the wings of the stork (כנפי החסידה) [?]. By His might and by a wind [?] His anger is [kindled] [?] — Judah is exiled (גלתה יהודה) [Eichah 1:3] [?].

[Source p. 17 / Hebrew p. 16–17 — the encomium turns toward fortification and surveying, then to astronomical measurement at a distance.]

it is the purpose: great hunters [?] and [?] decrees [?]; it is the one that [?] from [?]. ✻ He it is who searches out the chambers of the womb [?] — it is the [one that] enters before the inner [chambers] [?]: ramparts [?], windows transparent [?] [or] stopped up [?]; he it is who searches out the inner chambers of the belly. The land, by the bringing forth [drawing out] [?] of the needles [compass needles?], the borings [excavations?], and the diggings — it is the one that conducts the boat [?] in the sea, [making] a way for the ships that are lost [?]…

they led [astray?] to a desert place [?], like a wilderness [a trackless waste] [?], a way; and one [?] [shall] come to [?]; in them, too, the strong-hearted [bold] dogs [?]… those that ride out [?], in their running, knew [?] the segregated treasure [סגולה, the choice thing] [?]: its sides [its ribs] [?]; and likewise David, king of Israel, [made] a tower [?] for his shields [?]. It is the [?] that reckons the height [?] of the figures [forms] [?], [those] that stand and the shadows [?], and the shadows that are in the principal surfaces [?] or the horizons [האופקיים] [?].

Eratosthenes (אירטושתי"נים [recte: ארטוסתנס / Eratosthenes]), [whose figure for the circumference of the terrestrial globe] is **5400** parasangs (פרסאות) — so the Guide [the *Mishneh Torah*] teaches; for Maimonides (לרמב"ם), in his commentary on the first Mishnah of [tractate] Berachos (משנה ראשונה לברכות), [says] that there is none [farther] than fifty-two [52 = נ"ב] mil (מיל). It is the [?] that distinguishes [?] the province [country] that abuts [borders] [?]…

[from which] there is no ascent [increase] [?] of the vapors [the humid exhalations]; [and the] quarters [or: regions] (מקטוריו) [?] of the land [are] greater [?] than [their] counterpart in the [seven] climes (האקלימים) [?] — the gate [measure] [?] of the cities [?] that is drawn [?]…

The towns, [as] the inquirer investigates the distance of their position [?]:

it raised the land [made it eminent] to be busied in every [?] work and labor [?]. ✻ And in the deeds of men, to instruct [direct] them — is it not exalted? Were it given to him [?]… wings to fly into the heavens, to know the height of the clouds [?] and the [?] of the rainbow [קשת] [?], and all that comes to be in the air [?] — so [he is], and moreover he spreads his wings to [?]… to make us hear his voice that they may sing [?], and his word goes out [?] to the sons of man, the chosen ones [?], to set in order [?], to declare to them the awesome works of the LORD (יי) — how great, like…

the lucid stars [?] and the wandering stars, the [outermost] ones [?], by means of Aristotle (אריסטוטלים) [Aristotle], in his philosophizing, and by means of the [aforementioned] difference [discrepancy] (החלוף הנזכר) [?]… in the *Book of Number* [?]; he advises [?] according to the [?] half [?] of the diagonal [?] (חצאי אלכסוני [the half-diagonals]) [?], like the roundness [sphericity] of the earth [כדוד הארץ [recte: כדור הארץ, "the sphere of the earth"]], and [?]… the [?], to know that those shadows that come to be are like the curvatures [roundnesses] of the earth [מכדורי הארץ] [?]; the [?]…

that the sphere of the earth (כדור הארץ) is more excellent than [stands above] the moon's sphere (הכדור הירחי) [?], and [subtracts] from the [?] amount [?] of the solar sphere (הכדור השמשי) [?]; they [the bodies] are distanced [removed] from the earth; and were it not for the [?]… and let us hear [?] this measure: these bodies — for they have not found all [?]…

[these] precious [things] [?], all of them are the work of the triangle (המשלש): how awesome [נוראים] are [?]… the One who forms [?] (יוצר) [creation] in the beginning, before the mountains were brought forth [Tehillim 90:2, allusion] [?], and by your hand the work and [?]… two old [?] [?] lips ride [?], in the honored [treatise] [?], from the whole beauty of [?]… in a clean tongue [Hebrew] [?], it [the treatise] speaks great things [Daniel 7:8, 7:20 allusion]. I have not mentioned [recalled] the words [?]…

the bones [essence?] of Joseph (עצמות יוסף) [Shemos 13:19, allusion] [?] are with him; and let him not put me to shame [?] with my tongue [?]: a tongue upon [?]… upon the hill of the foreskins (על גבעת הערלות) [Yehoshua 5:3, allusion] [?] — for [he is] the chief of the speakers [?] — "you," and his father [?]. Here are his words.

✻ Šuta (שותא) [Aramaic: שׁוּתָא — idiomatic, "the talk / the prattle (of a child in the marketplace)," a self-deprecating formula]: [the chatter of] a child in the marketplace [is] either his father's or his mother's [Bava Basra 165a; meaning "a man's casual talk reveals his true mind"] [?]; or [it is] the saying of the Torah [?]… and your kingdom (ומלהך [recte: ומלכך / "your king"?]) [?] upon your tongue. *O my God, I am ashamed and confounded* (אלהי בושתי ונכלמתי) [Ezra 9:6] to lift up my face [unto You] [?] — for I come naked, [in] the borrowed clothing of the [?]…

[Source p. 18 / Hebrew p. 17–18 — Delmedigo names his correspondent, dispatches the inquiry, and the encomium gives way to the opening of the next Spring.]

[I have searched] every empty vessel — empty vessels — [but] I have not found, out of all the vessels [?], [other than?] the labor of the triangles. It is, in my eyes,

[but] a shadow [צללי, an idol/shadow] [?] — in deep [mighty] waters (במים אדירים) [Shemos 15:10, allusion] [?]. A spring [fountain] [?]: of [my] inquirers, and the book — the foundation [?] of the whole, to make wise [?] R. Y[itzchak?] [?]…

the first resolution of the triangles, and it is, in my eyes, as I have [?] [shown] [?]; and forgive [?] and [?] his short side [?] — his hand was short [insufficient] [?] to give aid [?]; moreover, he did not seal [?] the [?]. The seal [signet] [?] of the Torah is in his disciples; and he saw still that this sickness [defect] [?]… that there is no [?]…

his roots are torn up [pulled out] [?]. To [?] [him] it appeared that he had not sufficed to write [it] [?] [or] to lay it out [?] — the [?] ones [?]; for he did not [?]… they knew [some] of it, and [there] remained the whole composition (החבור) entire, like this — bolted shut [?], like a [garden] locked (גן נעול) [Shir HaShirim 4:12, allusion] [?] — and like the fountain [?]…

[which] he did not [make] clear [explain] [?]: all the founders [students] [?] of the triangles. And there is none who says [?]… of the upright [proper] ones [?] — and this defect [?] is precious in knowing; this [?] truth [?]…

in the treatises that follow; and I went over [studied] it again — chapter[s] one hundred and one (פרקין מאה ואחד) [101 chapters] [?] — sealed and concealed [?]; thus it [?] himself for our [eyes] [?], and there is none who [?] [today?]…

to the triangles he tells it [?]; and the master [the Lord] recalled, if [there were] truly the words of [?]… the aforementioned [scholar] (הנ"ל) [?] assured me [promised me] to send to me, by the hand of Bia^{u}di [?] Nicolaus (ביאודי ניקולאוס) [?] [Nicolaus, identification uncertain] [?]…

who there set out [explained] the triangles in brief, and searched, and did not find — [aught] of value to you; and he did well [a kindness], to plough [?] for me until death [?]. **Therefore** (לכן) — O tent [my dwelling?] [?]…

from my Lord, the firstfruits [reshit] of [my] lips' desire [Tehillim 21:3, allusion] [?]: my lips — you [it is] I seek [?]; this [is] to teach me a swift [light] way (דרך קלה) and short, in a clear [tongue] [?]… a full [?] good thing — a man, a gracious one [?]; may he provide us with bread, that the [?] may rejoice [?].

⁕ The Second Spring (העין הב') — *to satisfy* (להשביע) [an allusion to Tehillim 107:9, "He satisfies the longing soul"]: with the triumphs [glories] of my soul, in the wisdom of [?]…

and the sons of Issachar, who had understanding [of the times] (יודעי בינה) [Divrei HaYamim I 12:33, allusion], to know [?] them [?].

✻ I have *longed* (נכספתי) [Tehillim 84:3, allusion] to know in the [mathematical] disciplines [the] true reasons in the [demonstration] [?]… or instruments [?] not [made] by the hand of vessels [?] — the recorded works [?]; and tables [לוחות], and they recount [recount?] [things] mute [?], without sense or [foundation] [?]…

a man of full [perfect] [?] knowledge, the author of the *Book of the Choice* [?] — wise [?]; he will not [refuse?] to answer [?]… a clear word, in its [proper] aspects [?], on the general [classes of] questions touching the First Motion ([ע]המתנוע הר[אשון], the First [Moving sphere]) [?]…

in its [their] revolutions [?], and their roundings [?], and their inclinations: the upper [supreme] [?] rectilinear [direct] ones [?]; the surveyor (הציר [recte: הצר? / "the one who maps/draws"]) [?] for every province (לכל מדינה) and the measures of their days [?]: the rising [setting] [?] of the stars, and their lengths [longitudes] and breadths [latitudes] (אורך ורחב) [?], the duration [time] of the twilights [הנשפים] [?], the quantity of the shadows (כמות הצללים) [?], and the [?]…

[the] height [altitude] of the stars (גובה הכוכבים), the knowing of the season [time] (העת), the time of year [season] (העונה), and the hours of the day (שעות היום) [?] from the stars and the moon (מהכוכבים והלבנה); and when the ephod [?] — the astrolabe [?] (האפודה) [Hebrew: "ephod," used here for the astrolabe-like reckoning instrument] [?] — was found [?], with [?]… to know the measure of the angle (שיעור הזוית). And how many questions are attached to them [?]:

[The Second Spring continues into the following pages.]

— and to publish them. I longed to know what is recorded in a book by the compass, to grasp them according to the rules of observation as they are explained in full to all who desire them, all grounded upon the foundations of geometry. I hoped for the light of the wise — those who sleep in the dust of the earth — whose books are among us, though their renown has faded. They did indeed rouse us to study and stir our desire by the great goodness of their assurances, yet they did not deliver us from the obstacles that lie between: the weight of their premises and their length did not satisfy us, and we found no rest. They are wise in our eyes, in the eloquence of their style — but they left no fixed instruction, no clear bequest. If they were wise, their wisdom was for themselves alone, and with them their wisdom died.

To our further misfortune, most of these books were composed in Arabic and Greek, and their translation into Hebrew sprouted nothing but thorns and briars for us. Now, we did merit the longer redaction of the *Almagest* in thirteen treatises by the great sage Ptolemy [בטלמיוס], and its abridgment in seven treatises attributed [to him]; and the work of al-Battani [אלבתין], and al-Farghani [אלפראגאנו]; and the work of Jabir ibn Aflah of Seville [ניביר ן' אפלח האשבילייי = Jabir ibn Aflah al-Ishbili / Geber] in nine treatises, who set his heart to make good the deficiencies of the *Almagest*. These Arabic works were as the words of beloved kinsmen to us — were it not that they are full of scribal errors and, perhaps, defective in several places. And since we could not penetrate to the purpose of their intention, we could not tell where to lay the blame for the error: whether on the crookedness of our own understanding, or on the translators who twisted their crooked paths.

Two years ago, while I was dwelling in Constantinople [קוסטנדינה], the great city, I and nine others — men honored in the land, expert and sharp, and mighty in the war of the Lord — resolved together to undertake and to navigate the great sea of the *Almagest*, that fortress of a foundation, up to the neck. With great strength and study we reached chapter thirteen of the First Treatise; but when we came to chapter fourteen, a great storm arose upon the sea, and the ship was on the point of breaking upon the rock of the Sections [בפלע המחלקות]. For the science of astronomy here possesses the six [?] measures [שכון התכונה — astronomy — possesses the six measures], by which its orbits move; and we could not find its way through the design of the arcs cut by the six [?] configurations, according to the division and the composition of the chord drawn from the apex down to the base [?]. When we despaired of penetrating it, in its place [we found] the work of Jabir [ניביר], and we rejoiced at the line of his theorem, for he prided himself wonderfully and boasted over Ptolemy that he had lightened for the investigators the heavy burden of his demonstrations — that it was he who had devised the resolution of the triangles [היתר המשלשים] by means of the four-quantity proportion [הקש הארבעה שיעורים], according to the teaching of [Treatise] Nineteen [?] (or: the nineteenth proposition) [למוד י"ט(?) השביעי] of the [Euclidean] foundations — and that he did not walk in the way of his predecessor with the cumbersome six measures.

His words appear good and sound — except that he did not illustrate them with figures, nor bring examples to make the matters understood, as did the father of all who study astronomy, Ptolemy. And how can we erect a structure, and liken one matter to another, without a model and an illustration in our investigations? From him too our hope was dashed, and the joy of our heart ceased. The words of

al-Battani [אלכתין] are still more obscure, and insufficient, for they are but chapter-headings. In short, "the matters are too lofty for a fool" (cf. Mishlei 24:7), and "wisdom is wanting" to the deficient in heart, while the matters are pleasant and deep. And when we heard, so we saw: that these matters cannot be expounded from the mouth of books and mute teachers, but only from the mouth of living and enduring sages, by oral tradition [קבלה]. One who has not been privileged to find a teacher, and whose hand has not reached the matter, will not succeed in his days; he will waste his time, his soul will be cut off, and he will be like one who lies down in the heart of the sea, or like one who lies at the top of a mast (cf. Mishlei 23:34).

And You, O Merciful One, be gracious to us and answer us! Be willing to help us, for there is none but You to redeem — to expound the two first treatises of the *Almagest*, in whose light we shall see light and receive the benefit intended by Ptolemy [מבטלמיוס] from his work; if not according to his words and his hard road, then prepare for us a short way, the king's highway, by means of

the [resolution of the] triangles [המשלשים], by which we may be rescued in our perplexity, and not stumble time and again toward divinations and fall, and be caught in the snare of ensnarers. "Bow Your heavens" and magnify Your name! And say: "My power and the might of my hand have gotten me this wealth" — feed me with the bread of the mighty, with the blessings of heaven. "Give me, I pray, a little water" from Your blessing, "in a dry and weary land without water" (cf. Bereishis 24:17; Tehillim 63:2).

The Third Spring [העין הג']. "Awesome in righteousness, answer us!" (Tehillim 65:6), O You who tread upon the heights of wisdom, who shelter in the wings of Your astronomy all sound counsel and design. How did the astronomers presume — how did they decree and fix a measure upon the diameters of the stars [קוטרי הכוכבים], spherical and round, and the size of their circumference, and their distance from the center of the universe? If, as the Philosopher [Aristotle] holds, all our knowledge is built upon the senses — if they will testify and bear witness that the spheres seen [as] a hundred [?] cubits, fire and the like [?] — there is a teaching that, because of the great distance, the rays that come from the edges of the perceived object to the eye meet at a very narrow angle, as is explained in the science of optics [חכמת הראיה]. And therefore the senses do not perceive the truth, and err in the measure of perceived things. And against the notorious error of the senses we contend with our intellect and rescue ourselves from the senses.

I shall answer the whisperers in the ears of Israel — "saved by the Lord" (cf. Yeshayahu 45:17): who is so wise as to know? Every body and its magnitude, we

grant, is far greater than what is seen; but who can fix a measure to its diagonal, its thickness, and its strength, as [if measuring] the muscles of its belly? I will not believe that the matter is a received tradition, or that they speak without reason — that God has put into their mouths a thing for which there is no greater folly. It is destruction, and insolence to speak of such things, for they are the works of God. Yet even though men "stiffen their necks and refuse to believe" (cf. Yirmeyahu 17:23) these exaggerations, and those who read them in the *Keter Malkhut* [כתר מלכות, of Ibn Gabirol] reckon them dreams — yet one who has tested the instrument [נסה הכלי] stands and measures the earth and says: "Who is for me in heaven?" (Tehillim 73:25) — that he could not aim true in the measure of fifteen parasangs [ט"ו פרסאות] and not miss. And according to the rules of geometry it would be fitting to aim as with a measuring-line; except that one ought not rely upon intellectual inference and conjecture, for "experience is the father of wisdom and counsel and design." So Aristotle [ארסטוטלים] ruled in the *Metaphysics* [במה שאחר הטבע].

And great is the daily practice: every day we see how many expenses are laid out, treasuries of kings exhausted, on palaces built upon stratagems devised by the intellect of those engaged in the sciences — and experience belies them before their eyes. For example, in the conveyance of waters and their raising, and the perpetual motions effected by wheels turning against one another — such things are familiar to all the people, and the earth is full of them; yet the benefit drawn from them does not equal the harm done to the king by their great expense. And those who pilot them, the pilots, perish in their sin by their own cleverness. Now since they are not sufficient to ascend to heaven with these ladders and instruments — who revealed to them these hidden things?

Behold, the sharp-minded young scholar, the master Rabbi Moshe [מהר"ר משה], a disciple of your honor, wrote to me that the foundation of the astronomers is that the earth is as a point relative to the heavens, and that the line going out from its center to the firmament becomes known and verified by the parallax of a star [חלוף מראה הכוכב] and by the triangle [המשלש]. I was astonished at his words, for it is agreed [that the earth is as a point] relative to the line of sight going out from the surface — but where, then, do they set up a triangle one of whose sides is half the diagonal [חצי האלכסון], on which it is not fitting to rely? It seems to me that I did not grasp his words, for he did not adequately argue his case [?]. And to the said disciple who replied with such things, the colleagues said: "He who would lie, sends his witness far away." Rashness like water! Yet "if a day or two" "[knowledge] came from heaven" (cf. Iyov 38) — and now, if you have demonstrations with you, remove the dust of ignorance from our eyes, and may

the favor of our Lord be upon us! Inform us how we may truly know the circumference of the earth's sphere by our dwelling, and whether the surface of the dry land suffices for the tread of our foot and our cattle, on the return

of the Lord — restore our soul in the time of [restoration?]; and whether the foundations [the elements] [are arranged] according to the ratio of the tenths [על יחס העשיריות], as the metaphysicians [המשאים — the Peripatetics] hold. For there is no ratio and nothing comparable to these precious pieces of knowledge in all our apprehension. And in this way our straits will be made level; we shall drive out, trample down, those who rise against us and shame us, and uncover the disgrace of those who provoke us — those who say: "Beware, lest in gazing at the heavens you sink and drown in the deep!" "Crown us with glory and honor" (cf. Tehillim 8:6); set us apart from the people of the earth; endow us "from Your threshing-floor and Your winepress," with "the choice gifts of heaven" (cf. Devarim 33:13–14). And may Your words be pleasant, "like willows by the water-courses" (cf. Yeshayahu 44:4).

The Fourth Spring [העין הד']. "I sought him whom my soul loves" (Shir HaShirim 3:1) — to make known to me the design of the instruments fit for knowing the altitudes of the stars [גובהי הכוכבים] and their distances from one another, and the manner of their use; and brief instructions on the measurement of the lines joined to them in altitudes and depths; and let them be such as are familiar to your Highness, [those instruments] that are precise and exact. And [I seek] the benefits that issue from the corporealized spheres [הכדורים המוגשמים — the material globes/celestial-sphere models]; for I will not believe that their whole purpose is merely their resemblance to earth and heaven. Lift up my head against my enemies, who say "Woe to my soul!" — that the astrolabe [האצטרולוב] appears to be a thing of toys and trifles, or amusements for the inexperienced. "Let Your work appear to Your servant" (Tehillim 90:16), Your divine work and the way of it, in the science of number and measure [חכמת המספר והשיעור].

For those who wrote in our tongue did not bring us to satisfaction. For instance, the Ibn Ezra [הראב"ע]: his diction is cramped and his way is short, and "his foot is not a straight foot" (cf. Yechezkel 1:7). And the comprehensive master, the master Rabbi Eliyah Mizrachi [כמהר"ר אליה מזרחי] — "with mighty wrestlings he wrestled" (cf. Bereishis 30:8) in the *Sefer HaMispar* [ספר המספר]; he is by nature hard to instruct, his words concealed; and the dependent clauses, like sucklings and infants, lie tumbled within his compositions, hidden and sealed from the eyes of the wise — all the more from the eyes of beginners. True, all his words are planted and grounded upon decisive demonstrations — but by long roads; and

you will not find a hint of the science of algebra [חכמת האלגיברה] in the books of these two great luminaries. It seems they never heard of it and did not know it — though it is the cornerstone of arithmetic, as is recorded from the mouth of your disciple. In many disciplines, too, the said Jews did not speak — such as the calculus of errors [למוד הטעיות], and the study of combining unlike quantities [למוד קבוץ המתחלפים], as in alloys or remedies and the like, upon which this whole house of wisdom is firmly founded, and to whose offspring there is no end.

"How precious is Your lovingkindness, O God!" (Tehillim 36:8) — that there should be for its head a graceful garland, a crown of glory; clothe it, from the works of Your hands, in a garment of honor; complete its book; finish its design — and "nations shall walk by its light" (cf. Yeshayahu 60:3); the righteous shall rejoice, for he sees over it the coat of Joseph. And for us, Your servants, light up our faces in the division of the astronomical rewards — for the model is not in the writings. Also at the end of the book the Rav distanced me in the matter of knowing the measure of the rope sunk in the water [שעור החבל הנשקע במים]: I did not know the foundation of the reckoning and its cause, and did not penetrate to its end. And You, O "Bethlehem Ephrathah, arise to help!" (cf. Michah 5:1; Tehillim 60:13). Do not withhold — for God's hand is not short; "send forth Your light and Your truth," let truth "spring up from the earth and righteousness look down from heaven" (Tehillim 85:12), "for the earth shall be full of the knowledge of the Lord as the waters [cover the sea]" (Yeshayahu 11:9).

The Fifth Spring [ה'העין ה]. Concerning [tutelary] princes [השרים], amulets [הקמיעות], adjurations [ההשבעות], [divine] names [השמות], omens [הניחושים], dreams [החלומות], and the sealed segulot [virtues/charms]; the balanced symmetry [השווי המזני] and the [opposed] contrarieties, which is called in Greek **sympatheia** [שימפא"תיאה] and **antipatheia** [אנטיפא"תיאה]; the operations of those who "hold the eyes" [אוחזי העינים — eye-deceivers / sleight-of-hand], and observation by the physiognomy of the face and of the palms [הבטה כפרצוף ובכפים — physiognomy and palmistry].

I ask of you, the investigator of hidden things, to inform me whether these things have any reality, or whether they are the fancies of the melancholic [בעלי המרה השחורה] and the tales of old women "spinning by the light of the moon." For I am on a "path between the vineyards, a fence on this side and a fence on that" (Bamidbar 22:24), thorns and briars, pits, fissures, and caves. For behold, the sages received the tales of the Talmud, and many of these matters are written in their books — they upheld

and confirmed those that came after them: their reality and their wondrous operation. And every man is drawn to believe after them, for the great ones of the world bear witness to them, "they are the eyes" (cf. Bamidbar 10:31) and the testifying witnesses — and who would set his face to deny them and not, "by the breath of his mouth," be burned? They call him "small of faith," "one who chooses to deny them" as a denier of fundamentals; and from the side of the intellectual analogies that contradict them, [some] reject and refuse them, and one who does believe in them and their like they reckon an utter fool.

I already saw, with your disciple (may his Rock keep him), in the *Sefer Refuot Te'alah* [ספר רפיאות תעלה — "Book of the Remedies of the Cure"], which the Master [הארון = "the Master"] composed: as he says in the First Treatise, chapter 8, in his discussion of the elements, he wrote thus: "The philosophers in many ways investigated the number of the elements. We, however, with the physicians, say that it is impossible for man to be — or for any animal — composed of a single element; for the one is in its own essence uniform on all its sides, and there is nothing in it to oppose it. How, then, could it change, alter, and be acted upon? Now it is evident that if a thing is not acted upon, it does not suffer pain. But man does suffer pain, for he perceives the qualities that overpower and alter his temperament, and the dissolving of the bond of his parts. If so, he is not a single thing, but compounded of opposites that act upon and alter one another. And in sum we say that nothing perceives except what is balanced in its four qualities — on which depends [?] the sense of touch, which indeed perceives [the qualities? — מ"ש] when something acts upon it to alter its temperament or to dissolve some bond of its body. From this is seen the folly of those who believe in things compounded of [only] two elements, yet believe that there are in them operations like those of human beings — that they live, perceive, and die, and the like — among these follies. For when the temperament of a thing is fixed, it is neither acted upon nor does it perceive that which acts upon it, just as wood and stones do not perceive when one burns or splits them; and all the more do they have no living power to move and to act. But those who believe in them do not reckon them as [composed] of one element or of two specifically; rather they call them 'fiery' and 'airy,' [named] after the element that predominates in them, and they are above our intellect, hidden from us. And accordingly it is impossible that the element be of a single kind, which would entail that it does not change or get acted upon — for from what would it be acted upon, and into what would it turn and change? For every change is a turning into its opposite, and every passion is from the opposite. Hence there must be more than one element." Thus far his words [עכ"ל].

And I do not know whether the Master holds with Aristotle [ארסטו'], who placed [the tutelary princes] in the category of essence — in chapter 8 of the fifth book of the *Physics* [למיישׁ = "for the natural science"], and in the first of the fifth book *On Demonstration* [לנצוח] — and not with the Rambam [הרמב"ם] and the Ibn Ezra [הראב"ע], who [hold] that they have no reality. And even so, I see that some of the great sages of our people believe in them and seek causes and reasons for them. For instance, the sage, the philosopher, the noted investigator, the master Rabbi Moshe Narboni [מהר"ר משה נרבוני], in his commentary on the *Guide* [למוה"נ = on the *Moreh Nevuchim*], Part One, chapter 62, wrote thus: "What appears to the common folk from the operations of the amulets and the [divine] names does not nullify the words of the Master [Maimonides], for it is by accident and an act of the imagination. We ourselves saw a man weak in intellect who would mention [?]: he set an awl against his thigh, and it would not injure him at all, and no blood would come out of him; and the moment that — being [in a swoon? — פנסה] — they would draw it near me, I had no strength, and I would feel [it] as the dissolving of a bond. And that name we heard, at the time of the trembling, from one man; and this, owing to the strength of his imagination and the weakness of his intellect, was operative — while for me, who held it to be impossible, it was inoperative." And the comprehensive Rav [Crescas? — הרב שיש הכולל] wrote concerning him that he is mistaken, and wondered at his dreams and his words. Both of them, too, hold with the Rambam [כהרמב"ם] [there]. Thus far [שם].

But the great philosopher, the Ralbag [רלב"ג — Gersonides], appears to affirm their reality in several places in the *Sefer Milchamot HaShem* [ספר מלחמות יי] and in his commentaries. And in his commentaries I have seen fit to transcribe here the two chapters that the perfect and comprehensive Rav, the pious, the master Rabbi Chasdai Crescas [כמהר"ר חסדאי קרשקאש], wrote in Discourse 5 and 6 [בדרוש ו' הה'] of his book [the *Or HaShem*?] — except [אך] that what appears from his words is that experience and sense-perception drove him to believe in their reality, until —

that you may verify their truth, and you sought and grew wise enough to grasp their causes. And so it is with all the Christian theologians, who wrote whole works about them — and how they conjured them, to create them and to give them bodily form. I make no claim about the Kabbalists, all of whose works are full of demons and spirits; in my poverty I sought such things and did not find them. I have not been privileged to behold them in all my days, however much I inquired — whether by a dream-question or while awake — and despite all the preparations and the scatterings of seeds used to summon the satans and the

hairy demons [se'irim] [?]. They speak of these from a place of bitterness; they are bitter [?]. And my heart is sick over the marvelous and dreadful matter they reported to me — that there was, in the district of Podolia [פודוליאה] within the country of Poland [פולוניאה], a small infant [?], one of the boys of the Children of Israel, who would foretell future events. I did not see him myself; but truly "all the earth was of one language" [cf. Bereishis 11:1] — different men, each agreeing, testified that it was so. And here I am sending to your honor a letter that gathers up the event, written by one of Israel's notables, a man whose name is known among the gatherings [bishe'arim], from the master R. Chaim Shor [מהר"ר חיים שור], head of the rabbinical court [אב בית דין] of the holy community of Satanów [סטנווא]. [Aramaic: פוק חזי מאן גברא רבא מסהיד עליה — "Go out and see what a great man testifies concerning it"]; and moreover, [Aramaic: הוא קלא דלא פסיק — "it is a report that does not cease"]. Some thousands of sages saw him and examined him, and this affair happened last year — except that the saying holds: "it has already been" [cf. Koheles 1:9]. [Aramaic: כל מילתא דלא רמיא עליה דאיניש לאו אדעתיה — "Any matter that does not rest upon a man is not on his mind"]. Who will grant that I might know how the hidden things — concealed and veiled from the eyes of every living being — were revealed to the lips of the infant? Was it a spirit that entered into him, or an angel appointed over him to tell him, so that prophecy "rested upon the simple and the foolish"? I have heard people murmuring against his father, who heard… [?] — but I did not learn how it came about: whether through practical Kabbalah [קבלה מעשית], or through sorcery; and who taught him this wisdom. For I was jealous of the empty boasters, and my spirit grew faint within me over these "workers by Names" [הפועלים בשמות]. I have tested this ten times and found them deluded liars, who shall be put to shame by the "rams" [terebinths] [אלים] from which they prided themselves. The vainglorious and the fools shall, through their own transgression, "return, be ashamed, and be dismayed" [cf. Tehillim 6:11], for they do not know how to gather the fleas [?] or to drive off the gnats [?]. I myself possess a little knowledge of the wisdom of the Sefirot [חכמת הספירות], which is the divine theology of the chosen nation, handed down generation by generation from our forefathers to the men of faith, the precious few survivors [שרידים], whom the Lord calls. The later authorities strengthened it and made it mighty; they "made for it ears and eyes" — that is, demons [שדים] [?] — in place of understanding. The craftsmanship is pure: "clear as the sun, fair as the moon" [cf. Shir HaShirim 6:10], as it musters its forces in the divine matters, in apprehending the blessed God so far as is possible, and in His greatness, and in the chain of corporeal, finite existents that flow from His infinite power. How did contrary things issue forth from the One who is whole and simple? It treats of the governance of the worlds and the ranks

of the angels and the souls; it has ascended above all the branches of wisdom, for the apprehension of the Sefirot stands above the angels themselves. Above the angels who are above them, no eye has seen [cf. Yeshayahu 64:3] — none of the sages of the gentiles; and it "strips" our minds [פשיט] of the garments of corporeality and removes the curtain that divides between us and the blessed Creator. It is what beautifies its students; it grants a soul to those who serve it and the holy spirit to those who walk in it, guiding them on a straight path: the fear of the Lord. It is His treasury, and the sages of Edom yearned for its beauty and longed with longing for the delights of its garden, to gather its lilies and its tender blooms [?], and they labored to translate the abridgments of its authors into their own tongues. "Concerning it, snatch a portion" [cf. Aramaic idiom] [?], for renown and praise.

And to you, my lord, do a kindness: make known to me your view of the lofty matter, in all the particulars of its mysteries — concerning transmigration [גלגול] and the nature of the soul; and how "ten Sefirot of nothingness" [עשר ספירות בלי מה] are all of them a doubling [?], a counting and a counter-count, and so forth [?]. Why is this one called Keter [Crown], and that one Chochmah [Wisdom], and the rest of the precious matters that are known before you — for every precious thing your eye has seen. Do not turn back, that you not be shamed, for "your kindness is better than life" [cf. Tehillim 63:4], to open blind eyes [cf. Yeshayahu 42:7], to bring my soul out of confinement [cf. Tehillim 142:8]. You are a priest forever [cf. Tehillim 110:4]. "Blessed are You, O Lord… The man who is raised up." [cf. II Shmuel 23:1] I have known [?]: "you shall not heed the voice of the charmers" [cf. Tehillim 58:6], who bind spells skillfully. The author bound seven companions, "sworn in the goodness of your house" [?]; "be for us a refuge for deliverance," for I — and the band of Kabbalists who muffle [their] dumb gods [אלמים אלוהים] within the field — "the Lord discerns between light and the rending-cleft" [?]. And now, why should we die before you? For "the great fire will consume us" [cf. Devarim 5:22], and there is none to quench the flashing flames of my desire but you. Father of disciples: for your heart's sake, "see much wisdom and knowledge," meditation for your heart — to my mind, "let your fountains burst forth abroad" [cf. Mishlei 5:16]; for I am "of uncircumcised lips" [cf. Shemos 6:12]; I cannot give utterance to my desire or lay bare my thought, lest they gather against me and strike me and "quench my burning coal" [cf. II Shmuel 14:7].

And you — "like a builder of mountains in hidden places" [?] — set your heart to the flocks who drink in thirst from your waters, the great brook [החציר] [?] that goes before the camp of the Hebrews. "Let not those who wait for you be

ashamed, nor those who seek your face be confounded" [cf. Tehillim 69:7]. A tree of life are you to those who lay hold of you [cf. Mishlei 3:18]; they shall neither hunger nor thirst. Guide me by your counsel [cf. Tehillim 73:24], and I will declare your wonders for generations of generations, "as the days of the heavens" [cf. Devarim 11:21]: "the stream of God is full of water" [Tehillim 65:10].

The Sixth Spring [העין הו']

I have desired — yes, my soul has desired — to know the science of genethlialogy [חכמת ההתולדות, i.e., the casting of nativities], whose judgments I take delight in: which path I should choose in dividing up the powers of the heavens, and which books [to follow]. What is the true character of this science? In your view — "your knowledge is most high" — has it been verified for you by experiment, and have you found substance in its claims? For I know how greatly they magnified its praise, yet how the great philosophers and theologians, and even the prophets, "cast stones against it" [?], shamed it ten times over: "for the portion of the Lord is His people" [cf. Devarim 32:9]. The sages of the Talmud disputed whether Israel is subject to a constellation [מזל]; and if so, then it is established that the nations of the world are subject to a constellation. I do not know how the recompense of a man's reward according to his deeds — in justice and equity — can stand, if he is compelled in his deeds by the force of his constellation, as the astronomers hold; or by his temperament [מזגו], as Galen [גלינוס] and the physicians hold, who say that the qualities of the soul follow the body's temperament. How many obscurities does this contain! Yet what do I know? — and you, will you not know the orderings? [?] Surely the arguments of the opponents lie open before you. All who pursue it have overtaken it amid the straits [cf. Eichah 1:3]; even Maimonides [הרמב"ם] opened his mouth against it and boasted that he had demonstrative proofs to show its falsehood, in his letters [באגרותיו]. By contrast, Ibn Ezra [הראב"ע] "turned aside into the congregation" [?] with all his might in his books; and many honored men, "girt with the sword" [cf. Shir HaShirim 3:8], came to its aid. And it is true: if we reject it with both hands, I do not know how to account for the prosperity of fools and the perverseness of the clever — "for there is no bread to the wise" [cf. Koheles 9:11]. I myself have seen a fool taking root, his branches laden with fruit; and "to the counselors and the discerning, not even bread" [?]. My soul thirsts to hear the words of the living God from a holy mouth that knows the supernal knowledge. "Who will bring me up to heaven? Who will give me water to drink?" [cf. II Shmuel 23:15]

The Seventh Spring [העין הז']

My soul longs to know a fragment of the mighty questions that the master put to the great sage — the great Rabbi of Egypt [Maimonides] — concerning you [?], in Cairo [קא"ירו], the great city, in the famous disputation [בויכוח המפורסם] before assemblies of officials and the counselors of the land. From the edge of the earth we heard songs [cf. Yeshayahu 24:16]: "Glory to the righteous!" — while your enemies clothe themselves in shame and confusion. Your wonders — "how mighty are their chief points!" — men will recount to the last generation; "in their own mouths, Selah." Surely [?] there was no question that eluded you. You made yourself a name like the name of the great ones, and you "ground them between your teeth" with the gravel [cf. Eichah 3:16] of the Ishmaelites, who said, "with our tongue we will prevail, our lips are our own" [cf. Tehillim 12:5], and "set their mouth against the heavens" [cf. Tehillim 73:9] — "you broke the heads of the sea-serpents upon the waters" [cf. Tehillim 74:13].

The Eighth Spring [העין הח']

Concerning the craft of Hermes [הירמיס] and his remedies, and the whole compass of the natural mysteries: how many works have lately been printed afresh in the languages of the gentiles, containing precious matters in every craft — such as the writing of hidden scrolls [מגלות סתרים], the curing of strange diseases, the working of the soil, the compounding of useful preparations, the operations of alchemy [האלכימ"יא], and the transmutation of metals. My heart yearns to know whether there is in it real benefit, given the great expense, the labor, and the loss of time, to bring a desired thing forth from potentiality into actuality. And behold, there came into my hand a treatise full of wisdom, "a perfect crown of beauty," attributed to Maimonides [רמב"ם] of blessed memory, who spoke of it marvelously, as is his way in every branch of wisdom. The ancients too, Greeks and Arabs, exalted it — such as Hermes, Geber [גי"ביר / Jabir ibn Hayyan], and Avicenna [בן סינה], and Albertus [Magnus] [אלבי"רטו], and Thomas [Aquinas] [טומ"אז], and [Roger] Bacon [בא"קון], and thousands with them. And all the dealings of the lands of China [כי"נה] and Japan [יא"פון] [?] lie in it; there is no nation or kingdom to which its currency [מוניטין] has not spread. Yet "not everyone burns reeds in the marsh" [Aramaic idiom: ולאו כולי עלמא קטלי קני באגמא — not everyone is equally skilled] [?]; it is an exalted wisdom, and my ear caught a fragment of it from the craftsmen [האומנים], who recount your glory; and the perfumers and the spice-peddlers proclaim the works of your hands. How many secrets, hidden and screened, were revealed before you! "All concealed things" — by you kings take counsel and officers consult. And behold,

this desire of mine arose recently, for I had no dealings in my days with these arts, especially with alchemy [אלקימ"יאה]; only by the hearing of the ear did I hear of it, "and I set it as a seal upon my heart" [cf. Shir HaShirim 8:6]. And now we have heard a report from your disciple that you have mastery over every quarter of this science — ruling over all its acquisitions, its knobs and its fastenings [cf. Shemos 26:24, the Tabernacle imagery]. So, too, you have shown us some of your signs: how many kinds of stained-glass jewels [אבני זכוכית מגוונים], of varied colors, that you fashioned; and the hard iron that you made soft; and the transmutation of cinnabar [הצינפריאו / cinabro] into choice silver; and a coin half silver and half gold; and quicksilver [כסף חי] that you extracted from various metals; and talc-oil [שמן טל"קי] to brighten the face until it shines; and a ring set with a stone that, when soaked in wine or strong drink and given to a man to drink, makes him empty it — and it suffices to empty out all the days of a man's life without pain [?]; and other operations and crafts beyond reckoning. "Fill our bellies with your delights" [cf. Tehillim 17:14]; greatly will you enrich us; and from your wisdom show us wonders, for your thoughts are higher than the thoughts of others, "as the height of the heavens" [cf. Tehillim 103:11]: "deep calls to deep at the sound of your channels" [Tehillim 42:8]; "upon the mountains the waters stand" [cf. Tehillim 104:6].

The Ninth Spring [העין הט']

Concerning medicine [ברפואות]. My soul knows full well that the art of medicine, to be perfected, requires knowledge of natural science [חכמת הטבע]. As Aristotle [ארסטוטליס] says in the book On Sense and the Sensible [ס' החוש והמוחש / De Sensu et Sensibilibus]: where the philosopher leaves off speaking, there the physician begins. And if I am no philosopher, what have I to do with sores and sicknesses? Yet "there is a spirit in man" [cf. Iyov 32:8] that drives him to investigate all that can be known, and all the more so this science. I have heard it said that it is fitting for every son of Israel to study the First Part of Avicenna [בן סיני, i.e., the Canon] — and this is a positive commandment "that time occasions" [שהזמן גרמא], like Maimonides' Sefer Hilchot De'ot [Laws of Character Traits]. Then one will know the regimen of his health and seek to preserve his life, so that all his deeds may be directed to the fear and service of the Lord; for the perfection and health of the body are a necessary condition for perfecting one's soul. And now we have been privileged to have the Sefer Refuot Te'alah ["Book of Effective Remedies"] [רפואות תעלה], which your hands established and which set before us, in the Hebrew tongue, methods for knowing easily every remedy. The theoretical part [העיונית] is long-drawn-out [?]; there is nothing to add to it — in all the books of the gentiles, in all their treasuries [בית נכאתם], there is none like it.

There too the master mentioned other works he composed on medicine, especially brief and precious commentaries on the teachings of Hippocrates [אפוקרא"טים], a book they prized greatly, which we have not found in the holy tongue to this day. I have craved his upright precepts; therefore I have entreated your face: send it to me, and I will set my eye upon it. Why should you withhold your hand from doing good to another? For in your books are bound up the souls of all in whom the breath of life was breathed from the breath of heaven; for the tongue of the physicians of our nation cleaves [to the palate] [?] for want of the sage, and there is no answer in their mouth but his. And now…

do not let their cattle die of thirst [cf. Iyov 24]; for "the dew of lights is your dew" [cf. Yeshayahu 26:19], and the earth draws you down [?], and from the dew of heaven [cf. Bereishis 27:28] you have brought forth water for them.

The Tenth Spring [העין הי']

Concerning the celestial body [בגרם השמימי]. I am [astonished] [?] at all the new discoveries; "I am foolish and ignorant" [cf. Tehillim 73:22]; I have not known. Truly, even what I learned from the book of Aristotle [אלסטוטלים / Aristotle] and the Great Commentator, Ibn Rushd [בן ראשד / Averroes] — when I heard the words of the senior disciple, the aforementioned master R. Moshe [מהר"ר משה הנ"ל], "let him speak words to the One on high" [cf. Daniel 7:25] [?] concerning the substance [עצם] of the spheres and the stars — I was astounded, and I "turned into another man" [cf. I Shmuel 10:6]. For I had thought all my days, in accordance with the philosopher's discourse on the substance [בעצם] of the sphere, that it was demonstrated that it is a fifth body [גשם חמישי, i.e., the quintessence], not sharing in the four elements, as the view of the Guide's Master [the Rambam] holds — and that Aristotle's words on this are "like mirrors of cast metal" [cf. Iyov 37:18], strong as the rampart-coins [?], "planted like nails fixed upon the foundations of weighty axles" [cf. Koheles 12:11]. He did wondrously in the Book of the Heavens [ספר השמים / De Caelo]; and after him, in all the academies of the gentiles, they received his view, and the stories of his contemporaries are reckoned correct and well-founded according to his premise [הנחתו]. Were it not that I know before whom I stand — before you, the father of all the philosophers of our age — I would now be bringing proofs and arguments, "drawing water from deep wells" [cf. Mishlei 20:5], to plead and to champion the words of the philosopher; though he needs no advocate, for his honor rests in its place.

And how shall the pillars of heaven and earth not totter, if we suppose that this air spreads out and rises and there is no heaven beyond it, and that the stars

swim or fly within it! For on this view the words of the Torah and the prophets would be built upon false and deceptive opinions — God forbid — and the wondrous order in the governance of the world would be nullified, so that we could not know the existence of the blessed God by demonstration, nor [know] the angels who move the spheres. For according to what Ibn Rushd [בן ראשד] wrote, there is no other demonstrative path to know the First Cause except the path of this continuous motion [התנועה המתדבקת]; and so the whole chain of causes and effects would be loosened, and the harmony of the world and the bond by which its parts are joined one to another, and the science of physics and metaphysics [חכמת הטבע והאלהית] would fall to the ground. This new astronomy [התכונה הזאת], too, will not stand firm justly; for what reason has the sun, light and swift by nature, ranging through the air, to travel always beneath the celestial line [הקו] [the ecliptic] and not to incline right or left, if the fish-bearers and the riders [?] have ceased [?]? Perhaps a wind might bear them and tilt them toward the north or the south. And perhaps the proponents of this new astronomy will posit separate intellects [שכלים נבדלים], or souls that are forms for the stars and that direct the spherical revolutions; and if so, their motion is not natural but voluntary. To all these suppositions, nullities will befall — strange notions, the cast-offs of the ancients, which Aristotle's philosophy disqualified [פסלתם] with great might. According to their own words, "they ascend, increasing without limit." And because of the lowliness of these views, they were not consigned to writing in the books of the Peripatetic philosophers [הפלוסופים המשאיים] — and [?]. What prevents this air from splitting and rising to a height without limit? If the terrestrial globe [הכדור הארצי] is not the center of the world, and its place is not the middle — for to the infinite no middle is fitting, just as the extremities are not fitting to it — why should it not move from its place? Would that I knew! And how shall the phenomena observed in the stars be established: nearness and distance, swiftness and tardiness, standing-still and going-back, or retrogradation? And how many matters there are like these! And the hardest of all: that a star should move with two contrary motions, and also the motion of latitude [תנועת הרוחב] — the picturing of which is hard, in this manner. With these obscurities, the new view has set me back and driven sleep from my eyes.

[Continuation of the Tenth Spring, on the celestial body — the close of the address to the addressee (Galileo).]

and my thoughts were thrown into confusion. When this voice reached your disciple's ears, he leapt up and swore that it had been demonstrated, as a proof, before your honor, that the heavens are subject to change and corruptible. And you, great one of the Jews — I will not let you go unless you bless me! For my

understanding is meager, my limbs feeble, and I am still outside, struck by the perplexities that the Rav, the Guide [Maimonides], wrote of. Alas, I have been silenced; I was not summoned to enter the king's house. I heard reports, and I was afraid to be quick to believe, lest I be one who "cuts the shoots" [i.e. an apostate]. So upon my reins, go for me to the wise [?]. To you I have lifted up my soul. You are the place where all turn their eyes [?] — quickly your word will run, the cloud in the heavens. Ho, every one who is thirsty, come for water!

The Eleventh Spring [העין הי"א]: On the form of the world [?] and its astronomy. Who would have believed our report? The earth has been utterly shaken; it has quaked, and its pillars shudder. In our own time there have arisen, in place of Aristarchus [ארשטר'קוס = Aristarchus of Samos], deaf craftsmen [a play on חרשים, "craftsmen / deaf ones"], wise craftsmen: Nicolaus Copernicus [ניקולאוס קופירניקוס] and his school. They have demolished the assumptions of the ancient astronomers and philosophers and have set the honored sphere, the light of lights [the sun], at the navel of the world, while the earthen and watery sphere [the earth] becomes one of the heavenly host on high, among the wandering stars [the planets]. The earth, they say, revolves and proceeds in two revolutions in a single instant — one daily, the other yearly — and by this they have aimed more truly at the real motions and have hit the hair's breadth [the exact mark] of the theoretical investigation of the stars [?].

Yet, to this end, they chose a deviant astronomy — and I keep silent in acknowledgment of its refutation [?] — for even the assumptions of Ptolemy concerning the eccentrics and the epicycles are false and worthless, as the Rav, the Guide, wrote in chapter 24 of Part II [Guide II:24]. And it has already been explained in logic that from false premises it is impossible to generate true conclusions. So, to sustain their proofs, the astronomers posit fantasies — except that these men brazenly set their faces against the ancients and imagine in their pride that this [new] path is the way of truth, smooth and superior, while Ptolemy's way is contemptible and foolish, and they raise many objections to defame it. Those who attach themselves to them, coming after them, decree that there are many worlds, and they will see through the telescope [?] forests, hills, and pools — and perhaps in each and every star there will be human beings. The fool sets his dwelling among many lofty and wealthy worlds [?], while down in the low place they sit [?]. As for me, my feet had nearly slipped, my ankles had almost given way, in my movement together with my earth [?]; astonished and silent I keep still, awaiting the coming of my exchange [my passing] to a world more praiseworthy than my dwelling in this desolate land. And my soul yearns to dwell in a world — the star of Mars, or Jupiter, or Saturn — and so we shall go

from strength to strength, renew our vigor, soar on wings like eagles. Let us rise up and circle the cities!

And you, head of the speakers [chief of the philosophers], the ear weighs words: yours is the heaven, yours also the earth. Teach me, my master — your way I will walk, in your truth. If this power is rightly established in my mouth, make it known to me; from the depths of the earth raise me up, increase my greatness, and turn and comfort me. But if not, then — please — blot me out of your book [?] which you have written, so as to instruct them [?]; just as your disciple recorded, who holds fast to their views and walks after their footsteps. Hide them in the dust together with the stillborn and the infants who never saw the light. And you, O God, will bring them down to the pit of destruction. He who comes to see speaks emptiness; his heart gathers iniquity, and when he goes outside he speaks. The fool multiplies no words [?], for God is in the heavens. The arrogant shall not stand to set themselves before your eyes; condemn them, O God — let them fall by their own counsels; let those who go astray and the stubborn in their views be consumed [?], let them be despised like water.

The Twelfth Spring [העין הי"ב]: And in it are seven pipes pouring out [streams of] light [an image drawn from Zechariah 4 — the seven pipes of the lampstand].

I said, "I will be wise concerning divinity [metaphysics]" [cf. Koheles 7:23]; and with all my heart I sought it out. From the depths I drew it up [?] — yet it was far from me, [the matter being] how deeply hidden the concept is. And lest you say I have turned aside: I read it, I sought it, but did not find it, because of the deficiency of the apprehender. From my teachers I gained understanding, and I wrestle [with it] and have not wearied; I have sworn [an oath]. I was a youth, and I have also grown old [cf. Tehillim 37:25] — apart from all the bodily [?] [drives], [I have devoted myself] to the Lord. And had I not believed — [I would have despaired]. "An ass [is buried] in the dust" — my feet [are firm in] my faith, and I am warmed by the inheritance of the priests. I have kept my distance [from idols], and my thoughts [recoil] from looking upon the good of the Lord [in the company of?] the idol-makers, who set their idols flying [?]. As for me, [I turn] my eyes away from the path of falsehood; I hate those mountains [the idolatrous high places], who say to my thoughts, "Tear down — do not look upon the Lord!" Far be it from me to bear [their reproach] [?] — yet if I do not make known how far the matters reach, I will not be one who gropes and feels about like the blind, like a stammering man [אליף = halting of speech? / "a man of stammering tongue"],

whispering between me and you, or like one who plays the prophet in the camp of the Hebrews.

And now, "God has spoken to me in his holiness" [Tehillim 60:8]; let me guard [these] matters — restrained matters [conquered words], for they are the world's hidden secret [a play on כבושים / כבשונו של עולם]. Therefore "I will guard my ways from sinning with my tongue; I will keep a muzzle on my mouth" [Tehillim 39:2]. Let me set down the chief headings, "and let the wise hear and add learning" [Mishlei 1:5] — [these] from my musings:

[1] First I ask you, and make it known to me, my master, concerning the existence of God, may He be exalted, who is the foundation [ikkar] of all the religions, the cornerstone and the head of the faith. "My soul thirsts for the living God" [Tehillim 42:3] — how my flesh longs [?] for a decisive proof of Him. My adversaries reproach me, saying to me all day long, "Where is your God?" [Tehillim 42:11]. For you know, better than anyone, that the proof taken from the side of the motion of the sphere — on which the whole house of philosophy leans — is not demonstrative. Aristotle thought that the heaven moves by nature, in revolution and not by accident, just as the four elements move in a straight line to return to their place; rather, just as their nature is to be at rest, so its nature is to be in motion. But the comprehensive Rav, R. [Chasdai] Crescas [כהר"ר חסדאי], in the book *Or Hashem* [*The Light of the Lord*], and many honored men with him, raised serious doubt and stirred up [this question]. And according to what the comprehensive master, the eminent R. Moses, your disciple [כמהר"ר משה תלמידך], made known to me, the proof of Aristotle from the fifth body [the quintessence] he fabricated out of his own heart, and like a potter's vessel he shattered it in the book *Bechinat* [*Bechinat HaDat?*] — all the more so according to the view of those who deny the heavens and their natures entirely, for the proof from motion has rotted away and there is no substance in it. And all the rest of the proofs written in the books of the divines — Israelites, Ishmaelites [Muslims], and Christians — are in truth verbal, and are not compelling but only persuasive, smoothing over weak and flimsy [arguments], and sufficient only for the masses. Likewise in my mouth, [as for the claim that] motion alone was always set up as a sign and proof of the awesome Cause [the First Cause], as I have mentioned — apart from it, all that is said on this matter is the words of preachers who speak in assemblies, astonishing the foolish souls.

And when we are granted this most precious sought-after thing — to inquire into [these] desirable matters [?] and to dig a digging beneath the throne of glory, [to ask] whether He is one — so as to remove the slander of the brazen-faced who

say that the governance of the world is like the governance of a state: a strong bond and an eternal order that does not change, [in which] the governors are all directed toward a single end, the parts of the state subsist one with another, and each supplies his fellow's needs, with no Adversary [Satan] and no evil mischance among them — and after we strike down this sinful notion, [then] we will search by lamps into the manner of His governance, His providence, and His knowledge of the particulars of the worlds, the degrees of prophecy, and the Urim and Tumim, in a manner acceptable to the exalted intellect, [showing] that all deficiency [is removed] from His knowledge, and untying the perverse knots — [the doctrine that He has withdrawn] from free choice and has "sent away the crushed as free," [so that] each man does whatever is right in his own eyes.

[2] If there is a single angel that advocates [?] for the creation of the world as a proof, out of the thousand proofs that the believers have brought — apart from the philosopher's bonds that are hard to untie, and we are too weary and too limited to grasp the benefit that follows from creation — let us grant that it is so written in the Torah. But there is no reason or rationale to say that God, blessed be He, was idle for a time before creation [בכ"ת = "with all this"?], and that there is a change of will and a cessation in the outpouring of His goodness. And how unseemly! The sages of the Talmud said explicitly that "an order of times preceded [creation]" and that "He was building worlds and destroying them" [Bereishis Rabbah 3:7]. And the Kabbalists affirm that other worlds preceded this one — so many Jubilees and Sabbaticals [shemittot] of years. I see no compelling cause for the destructions; and what good is eternity [of the world], as Maimonides supposes? And what profit is there in our believing that the Creator, in His action, is like a spider's web — "in the morning he builds and in the evening he eats his weaving" [?] if he has not even caught a fly in his net? And what is the great anxiety and dread over creation, if in the end we shall fall into [the doctrine of] the eternity of the worlds? This is their [?] voice — this, without end. Truly I have inherited a sickness; there is no remedy for it.

[3] My mind has not been set at ease by the reasons of the commentators for the Torah's omission of any mention of the creation of the precious creatures, such as the angels and the *sefirot*. And it added grief upon my grief that R. Moses [Maimonides], mentioned above, in his writing, bent all the verses that indicate their existence in the Torah and the Prophets toward awesome [allegorical] meanings. As for what he argued — that need [necessity] produced the ministering and serving [beings], to complete the deficiency and to fulfill the want, and that in proportion to the great deficiency the surplus from them would be — is the thing itself, then, lacking the source of perfection? "And he decrees a

word and it stands," and so forth [Iyov 22:28] — this is no argument, neither for the philosophers nor for the view of the Kabbalists, [the claim] that need and necessity led us to affirm them [the intermediaries]. If [we follow] the view of the philosophers, to maintain the unity of the First Cause —

[4] [the claim] that the existents are many and varied, whereas the truth is that the One, the unique, blessed be He, acts only [to bring forth] a single angel [Intelligence], and emanates upon another; except that this second one acts and brings forth varied things according to the variation of its contemplation — for when it contemplates its [own] cause it generates an angel like itself, and when it gazes upon itself it generates a sphere for its chariot. And so the Intelligences and the spheres proliferate, and from them all the existents descend in a chain. And according to the Kabbalists: how could the Ein Sof [the Infinite], blessed be He, in His power and His exaltedness, be brought low and change His place — except by means of several intermediaries, through four worlds, by way of "windows and lattices," and vessels from differing vessels, through which the sparks of the emanations pass and are blended and reconciled, so that they do not blind the weak eyes, [and] the great luminaries do not consume them, and the "sparks of fire" do not burn up [?] the [created beings] [a play on Talmudic imagery] — the feeble Intelligences? Now these difficulties have pressed and shaken me; they have given me no rest. They have driven me out; my soul is desolate all day long, sickened by [R. Moses's] denial of the essential forms [the substantial forms], by which he overturned all the proofs, and made them [the forms] to be mere accidents [?].

"For look — behold, in [the very] essence [?]: a stone cries out from the wall, and a beam from the woodwork answers it" [Chavakuk 2:11]; and it is fitting to seek the refutation and burning-away [of that doctrine], lest we be scorched by its fire. In truth, before I saw his words, and before I knew him, I heard from his friends, in his name, that "the spirit of the Lord began to move him" [cf. Shoftim 13:25] concerning the motion of the angels and the souls, and in the [matter of] change of their place — mighty doubts. And when I returned to my studies and saw in [Book] VI of the *Physics* [שמע הטבעיו = the *Physica*, "the natural hearing"] that "everything that moves is divisible," so that in moving, part of it is at the point toward which it moves and part at the point from which — and this is a decisive proof — and [I considered] the reply of those who say that there is an instantaneous motion, which is as if they said that the motion is no motion at all [?]; for in [Book] VI he refuted it [?], [holding] that it is impossible for a thing to change its place [instantaneously], since they have already passed from the [intermediate] point. I understood that [such] are the words of strangers among

the great philosophers [?] — except that I did not find peace in this thought, which is not fitting for men of faith like me, for [such thoughts] reach the heart [?] and are seized by mighty errors, and all the foundations [?] become entangled one with another. And this can only be [?]

[5] In the matter of the Garden of Eden and Gehinnom, I do not know who separated them — [putting] the [Garden] for [the souls of?] the resurrection [?] — for the [holy] nations refuse [to accept] them [?]; and what is still harder to settle: that we too are obliged to believe that its essence [?] is its [own] reasoning [?] — it is the resurrection [techiyat hametim], [whereas] the true perfection and the longed-for delight is in the stripping-away of the foul, material carcasses; for then, [being] above [bodily existence], "like gods" — how good, how pleasant, that we be intellects, taking pleasure from the radiance of the Shechinah! And what have we to do with this world, its mishaps, and its [?] gravity-pulls [?]. All these doubts cling one to another; and for the reason stated I have brought them as remote [matters], and from them others descend in a chain that could not be set in writing — and you know their offspring. For whoever the Creator has graced with intellect and knowledge, and who does not sleep and slumber, cannot escape inquiring into them and being delivered from their destructiveness.

[6] And the hardest [matter] to set right — and it is "as vinegar to the teeth and as smoke to the eyes" [Mishlei 10:26] — is the rational faculty [the discursive intellect] that our Creator has given us to be in our hand like the pan of a balance, with which we weigh the things perceived and the things imagined, that we should not believe every word like the simple and the foolish; and it is what guides and leads us, the great helmsman of our ship upon the heart of the seas, and by which we are distinguished from the dumb animals. O Rock of all the worlds — why have you chastened us forever, why do you punish us for length of days, if you have placed in our hands "balances of deceit" [Hoshea 12:8] that have led us astray, so that we have not gone down to the bottom of your hidden distortions [?]? Shall we then muzzle the face of the intellect when it is muzzled toward Him [?], to bridle and restrain it [?] to what our ears hear, [merely] by [the claim that] "who knows whether this way or that is fitting in the eyes of God"? "Whom shall He teach knowledge, and whom shall He make to understand the report?" [Yeshayahu 28:9].

We have heard them say: "All the nations are from the Lord," and "a messenger is sent among the nations" [Ovadiah 1:1]. And shall I say, "I will sink all my schemes into the depths of the sea, and occupy myself with idle chatter all my times"? Let me rise up and circle the cities and the villages — perhaps what my heart tells me

will be forgotten from my mouth, though it [the question] sits right before me. And shall I "give sleep to my eyes" [Tehillim 132:4]? My musings rouse me. In sum, my eloquent ones, my beloved, my friends, my faithful: faith is not what is spoken with the mouth, but what is depicted in the soul, when one believes a thing to be so in reality just as it is depicted [in the mind]; and I see [myself] acted upon [moved by these doubts]. And if one is to be punished for [honest] contemplation — [then] God has acted unjustly [?], for it is not in our power to believe [or disbelieve] things [at will] — there is no room for the commandment in the matter of belief, since the only thing in a man's choice is the actions; and the one who contemplates is [?] the one who interprets [the matters] themselves, or who [finds them] opposed to our reason — nor [is it in our power] to shut our eyes from seeing all the days of our life.

And if this intellect, my friend and my acquaintance, is our destroyer [?] — it is what casts down the slain, [making one] a man evil in deeds — who will grant me that I be "like a sheep, and in the likeness of an ox that eats grass"? For then it would be good for me; from now on, in the eyes of the Lord let my offering be an atonement [?], my willful sin [raised up] as a whole burnt-offering [?], a fire of pleasing savor to the Lord. And here I will stop up my fountain and set a bound to my musings, lest a flood of waters sweep me away. "How awesome are your works! Your judgments are a great deep; man and beast you save, O Lord" [cf. Tehillim 36:6–7].

[Transition / opening of the next major question.] **And he descended** [ויורד] — for the Lord has brought about [the meeting] before me with a discerning and wise man like you. Now my low estate [?] — let there be a place for my cry; you will be my relief [?], my deliverer; for God has sent you before us for sustenance [cf. Bereishis 45:5]; for you are able to do all things, and no scheme is withheld from you. A way that no human foot has trodden —

[The page breaks off here; the passage continues.]

[Continuation of Zerach ben Nathan's prefatory address to Delmedigo, from the previous folio.]

like the horizon of our own land. Who is the man like you, in whom is the spirit of God [Margin/play on the name יש"ר]? Father of all who grasp counsel and sound wisdom, understanding and strength like a lion in wisdom, and like a lion in Torah. There is no book, great or small, that you have not read; no nation or kingdom that you have not investigated and examined and sought out — reckonings, the new sciences [?] in physics, in mathematics, in the divine

[metaphysics]; logic, and the heart of kings and of the provinces. You are full of Torah and wisdom like a pomegranate, a treasury holding every precious vessel and choice thing. From your mouth pearls go forth, and your pen pours out gems. Be, then, with the mouths of those who thirst to drink your living waters, for there is none of your wisdom, and none like you is to be found in the land of the living. Who will give us cold water to drink from your well? Who will grant us to draw from your flowing well — speaking face to face, seeing eye to eye?

And who will be a guide to me [?] [the phrase is corrupt: literally "and who will + give? to me — flowing [waters]…"] — and let us hear from you — except that I did not know where the Ark would turn. I would say, "I will go to you," but the spirit of the Lord would carry you off to where I do not know, so that I would come [Yad 37:1 echo, Iyov] and not find you, and where then are my appointed places [?]. Therefore tell one of the scribes [secretaries] to copy out for us your pleasant words, and I will give him his wage as my master commands. "The opening of your words gives light" (Tehillim 119:130), and you will be for us a restorer of the soul; the earth will be full of knowledge, and you will light up the world with your glory. As for me, your servant, I lift my eyes [?] to the heights, and for your reply I wait. I will stand at my watch in prayer, for my skin glows like an oven, turned to a furnace [Eichah 5:10]; the heat of summer has scorched me, selah. My heart longs and yearns for your precious teachings — favor, dew, and rain to slake my thirst. Turn to me and be gracious to me with kindness and mercy, that my soul may live because of you, and the souls of the children who are like me, that they may praise you in the assembly of the people, and your wonder the heavens will acknowledge — for you bring forth flowing waters from the rock and bring down water like rivers.

These are the Seventy Bitter Ones [השבעים המרים — the seventy "bitter waters"; the seventy minor questions, the *Tamarot*], and they are all matters astonishing and strange.

[The chapter opens the work's second major structural unit: seventy short *kushiyot* (difficulties) posed by Zerach, each headed by an enlarged Hebrew-letter numeral. The questions are framed in biblical lament-idiom ("be gracious to me," "heal me," etc.), each opening with a phrase that puns on the difficulty that

follows. They are numbered consecutively with the standard Hebrew letter-numerals.]

[1] א. Be gracious to me, my master, for I am wretched [Tehillim 6:3 idiom], hearing that there is "a part greater than the whole" [?] — this being among the universal axioms and primary intelligibles upon which the whole house of learning rests, namely that the whole is greater than any one [of its parts]; yet here "the heavens were laid bare" [?] [the phrase trails off; the difficulty is how a "part" could exceed "the whole"].

[2] ב. Heal me, O Healer of the broken-hearted, for I am terrified at hearing how two unequal magnitudes can have one and the same ratio to a third magnitude — as if you said, the ratio of A to G is as the ratio of B to G; and likewise the reverse, the ratio of G to A is as the ratio of G to B — when the contrary of this has been proved by demonstration in the fifth definition [?] of Euclid's *Elements*, and it is a primary intelligible. May He light His face toward us like the pupil of the eye.

[3] ג [the source repeats the letter ב; this question is numbered ב but is the third question —]. And my soul is greatly dismayed: how can there be a ratio of equality between two *unequal* magnitudes? This was explained and verified for us [its image and truth] by the universal sage, our teacher Rabbi Moshe Metz [כמהר"ר משה חלמי? / Metz], but he did not [?] state its added remainder [solution]; and these are his words: "And what is yet more astonishing is that between two magnitudes differing in their quantity there should be a ratio of equality." And we have already learned from our master, our teacher Rabbi Yashar [Delmedigo] in *Sefer Sheber Yosef* [ספר שבר יוסף] that ratios are of two kinds: either of equality — as when you say the ratio of ten to ten — or of difference [inequality]; and this latter is of two manners: either the ratio of the greater to the lesser, by way of example four to two, called the ratio of difference-of-the-greater; or [the ratio of] the lesser to the greater, called the ratio of difference-of-the-lesser, that is, the ratio of the two to the four. But as for the verification

of the proposition, [it] is explained in this manner — which follows from the words of Ptolemy in the First [book] of the *Almagest*: that the ratio between two chords [בקעים — *beka'im*, "chords"/sines] of [two] arcs is equal to the ratio between the [?]... [a fixed point distant from the diameter? — phrase partly corrupt]. For its explanation, in his commentary [?], and these are his words: "And I will bring the proposition with the example, hear and understand. [?]

In the present figure (סי' א) [Figure 1; see Diagram, Source — fig-p32-1], the ratio of arc **דה** [DH] — chord **רה** [RH] of arc **דה** — to arc **יח** [YḤ] — chord **יח** [YḤ] — is

as the ratio of **דמ** [DM] to **מח** [MḤ]. And from this we know the quantity of each one of these angles or arcs when we know their sum and take the ratio between their chords. So, for example: let the sum of the angles אפ + פאח [?] — that is to say, the whole angle **ראה** [R'H] = 120 degrees — and the ratio of their chords be as **דה** to...; in the ratio of 7 to 5, let it be required to find angle **ראפ** [R'P] by itself, which equals **פאח** [P'Ḥ] by itself.

Now we posit **אד** ['D] as half the diameter [radius]. And we shall draw a circle (ונרשום עגול). And following the [doctrine of] chords that preceded: line **אד** = the whole chord, and line **דל** [DL] = chord; and **יח אל = והא** half the sum, [namely] 60 degrees; and chord [of it] = **86602**; and line **לא** [L'] = chord of angle **אדל** ['DL], which is likewise known by Proposition 32 [למוד ל"ב] of the First [book], since its supplement **דאל** is known = 60; and if so **אדל** is known and its chord = **לא 50000**. We now infer accordingly, and form the ratio, saying: as the ratio of **דל** — half the sum of the magnitudes of the chord-ratios, which is a known number, **6** — to **דל** [DL], inasmuch as it is the chord of half the sum of the angles, 60, whose chord is **86902** [recte: 86602 — compositor error at this second occurrence; the value is 86602 as stated above], so [too] **מל** [ML], **1**... the difference between **דל** and **6**, the half, to the ratio of the greater **דמ** [DM], 7, to chord **14434**; and if so, **מל** [ML]; each of the chords has a ratio of **14434**. And since this sum, [taken] from the square of chord **לא** [L'] — which is **500000000** — and from the square of **מל** [ML], **208340356**, is **2708340356**, [it is] equal to the square of **אמ** ['M]; from Proposition 47 [מ"ז] of the First [book of Euclid]. If so, then the root of the said square is the measure of line **אמ** ['M] = **52042**.

And if we now set **אמ** ['M] a second time as the half-diameter, then **מל** [ML] will be the chord of angle **מאל** [M'L]. We form the ratio a second time and say: as the ratio of **מא** [M'], **52042**, to the whole chord **100000**, so [is] **למ** [LM] the preserved [value], **14434**, to **27735** — and this is the chord of angle **מאל** [M'L], whose arc is **16;6** [16 degrees 6 minutes — sexagesimal]. And if we subtract it from the half-sum of the angles, 60, the remainder is the smaller angle **מאד** [M'D] = **43;54**. And if we add it to 60, we find the greater angle **דאפ** [D'P] = **76;6** — and this is what we sought to explain.

But where the ratio of the chords of the angles is a ratio of equality — as in [angles] **דאג גאח** [D'G, G'Ḥ], which are as the ratios of **דל** to **6** ,**לח** — then we know at once that the angles are equal, and each one is half the given sum." Thus far the words of my teacher the Rabbi, may his Rock keep him.

And from this it is most clearly explained that the ratios of the arcs are as the ratios of the chords. Yet the ratio of the chords of the *differing* [complementary]

arcs is a ratio of equality, for the chord [?] is common to an arc and to its supplement; therefore between two arcs that differ in their quantity there is a ratio of equality. This is most wondrous! Thus far his words. The king has brought me into his chambers [Shir HaShirim 1:4].

[4] ד. And I, when I search, cannot grasp how the chord of the smaller arc can be — and especially [?] — greater than the chord [or: by a chord] [?], or more than [that of] the larger arc.

[5] ה. And I will not open my mouth to those who provoke me [?], [over the difficulty] that the supplement [?] is greater than its arc — given that half the circle to [?] the whole diameter or axis is about one and a half [times], approximately, according to Ptolemy's premise [the relation of the semicircle to the diameter ≈ 1½, i.e. π/2 ≈ 1.57; "according to Ptolemy"].

[Beginning of definition, continuing from prior page:] "...Euclid defined the straight line as that which lies evenly with [the points] upon it" — whichever points be on it, some toward the others. And as Proclus explained, it is the shortest [path] that runs between two points. According to all these definitions, the arc exceeds its chord [?], and sense bears witness to it. Yet in the table of arcs and chords of Ptolemy, it appears that the chord of arc 30 is **59;45,32**, whereas the [other] yeter-value of arc 30 degrees is **31;3,30** [the two values are deliberately contrasted — this is the very difficulty Zerach raises]. And I do not know the cause; I am like a blind man [groping] at a chimney [Hebrew idiom: כסומא בארובה, "like a blind man at the smoke-hole," i.e. utterly in the dark].

[6] ו. And I am like a man who has not heard how the square and the cube of one and the same [number] can have two [different] roots — hear, you deaf!

[7] ז. I have no clear demonstrations [in my mouth] of how two magnitudes can be differing roots, and the faces of those who see them are scowling.

[8] ח. "My troubles [?] have passed over me like a man [?]" [biblical lament idiom]: how can it be proved by demonstration that something [comes] from nothing [*yesh me-ayin*, creation ex nihilo]?

[9] ט. There is no soundness in my flesh because of those who behold me [Tehillim 38:4 idiom], [over the difficulty] that the scholars of the mathematical sciences explain the famous [the well-known] by means of the hidden, and [explain] the universal axioms by premises that are yet more hidden — and both of these are [hidden] relative to our [own] knowledge [?]; for the path innate to our intellect — as Aristotle wrote in the First [book] of the *Physics* [*Shemaʿ Tivʿi*] —

runs from what is known to us toward what is known to nature, which are hidden from us.

[10] י. There is no peace in my being, for I see three magnitudes, the one exceeding the second, and the third less than it; and license is given to bring near and to draw the utmost of the extremes toward the middle [term] — yet with all this they will never be equal, neither to the middle nor to one another. [...and onward — corrupt fragment in source.]

[11] יא. [My heart] reels: how can we always subtract from the greater of two magnitudes, or add to the lesser, without limit — and yet they never become equal between them? And this runs counter to the First [proposition] of the Tenth [book] of the *Elements*.

[12] יב. I am poured out like water [Tehillim 22:15], [over the difficulty] of how the philosophers decreed that there is no knowledge of non-existents — and yet in the mathematical sciences there are several pieces of knowledge and demonstrations concerning them.

[יג] [13]. And all my bones are scattered [?], [over the difficulty] of how they build proofs and foundations [demonstrations] upon hidden [unknowns]: until they assign the half or the third to an unknown number, and likewise double and triple, gather and subtract, and assign their roots, and prove their statements — and many such things are openly set forth before you, lord of warriors [?], in your book.

[14] יד. "What is this sound in my ears" [?], that the science of geometry [גיאומיטריאה — *geometria*] is more honorable, more precious, and more lacking [?] than the science of number [arithmetic] — whereas the Philosopher [Aristotle] explained the contrary in the First [book] of the *Metaphysics* [*Mah she-Achar ha-Teva*]; and besides his honored colleagues, there are proofs to uphold the matters.

[15] טו. A sound of [?] cries I hear, many saying that the mathematical sciences are easier than the natural sciences — and indeed the Philosopher called them "preliminaries to the [other] sciences," and reckoned them, together with grammar [*diduk*] and rhetoric [*halatsah*], [among the things] in which they used to train the pupils themselves under teachers of minor rank, as a kind of pedagogue [פדנוגין — *pedagogue*], before bringing them into the great academies [yeshivos] of Plato and Aristotle. And yet we see that they [the mathematical sciences] have instruments of [?] heavy use, and are more and more remote from the intellect than the fourth figure of the syllogism that Galen invented [?], which the logicians did not accept and gave no resting-place among the [other] figures.

And there is no [greater] confusion than in the compounded ratio that Euclid used in his fifth [book] of the *Elements*, upon which he built his upper structures and planted his shoots [?]; and Niwa[?]r [ניבי"ר — name flagged] in the First Treatise of his *Sefer HaTechunah* [Book of Astronomy] followed in his footsteps.

[some of them are unknown] to the scholars of the mathematical sciences; and we make the ratio to the demonstrative syllogism [?] — for according to what your pupil [?] has taught us.

[16] טז. My heart was like wax [?], melting within my bowels, [over the difficulty] of how the Nineteenth Proposition [למוד הי"ט] of the Seventh [book] of the *Elements* — which is the instrument [?] of the excellent [proportion] — in it are found four figures, three of them reducible to the first figure; and it gives light to the eyes of the wise and of merchants in their reckonings, and they have reckoned it "a lamp of God" to guide them. And if it [the calculation] is crooked in its way [yields a wrong result], one cannot rely upon it; act with it [and?] forgery is [readily?] hidden in its matters [?]. Who can teach sinners — [?] — for [?] colleagues [merchants] will dig against it [defraud by it], the Canaanites [merchants] will halve it [cheat]. [The passage plays on "Canaanite" = merchant.]

[17] יז. I was struck dumb in silence [Tehillim 39:3], [over the difficulty] of how three merchants buy an equal quantity for an equal sum, and sell at an equal sum, and [yet] one profits, the second stands at his capital [breaks even], and the third loses; and moreover the seller may lose or profit just as he bought [?]. "There is no wisdom, no counsel" (Mishlei 21:30), and no astronomy [תכונה — pun: "skill"/"astronomy"].

[18] יח. My strength is dried up like a potsherd [Tehillim 22:16], to understand how the One [unit] is not the head [first] of the numbers — since there are found [things] smaller than it that are not fractions [?].

[19] יט. And my tongue cleaves to my jaws [Tehillim 22:16], [over the difficulty] of how two opposites can be explained as one — being [at once] a number and not a number; [the unit?] [?] shows opposites, and "walks in darkness" (Koheles 2:14).

[20] כ. My strength has forsaken me [?]: every multiplication in [the realm of] whole [numbers] increases, yet we see that with fractions it diminishes and lessens — it does no good, but [only] harm [?].

[21] כא. I have sunk in deep mire [Tehillim 69:3], and there is no standing to know the sum of numbers that do not increase in the ratio [series] of my number

or magnitude, but [increase] only as it happens, by a [varying] excess — when we [subtract?] the first and the last.

[22] כב. "I came with my dainties" [?], [over the difficulty] of the agreement [convention] of [counting] number up to ten — which seems to me to have been imposed by the great ones, [as in] the investigations of the Ra'em [Rabbi Avraham Minz? / acronym] [?] and Ibn Ezra. And if the men of Nineveh and their like, who are above the rank of the ape and below the rank of man, count [only] up to five — this is "not [a teaching] from the Torah of [Moshe] our heritage" [?] — what shall they do with hundreds and thousands, shields and thousands [?]? And the scholars of the Kabbalah built foundations upon the number ten — for the [ten] sefirot — and bound them to the heads of crowns. And "a flood has swept me away" [?] at hearing that the verses were not exact [precise]: in [the matter of] Yuval and [in] Shlomo with his wisdom, and that *Tsofnat Paneach* [Yosef] (Bereishis 41:45) "did not know how to number his produce" [?] [the increase]. Also the colleagues said that they heard from your honored pupil a desirable thing on the verse "And your seed shall be like the dust of the earth" (Bereishis 28:14). "I have called upon you, for you will answer me" (Tehillim 17:6); "with the word of your lips" [?]; "I shall be satisfied, when I awake, with your likeness" (Tehillim 17:15) — from those who mock me. "Save those who take refuge" [in your right hand] "from their adversaries" (Tehillim 17:7). [Now] honor the Lord with your wealth [substance] (Mishlei 3:9).

[23] כג. I am wearied with my groaning [Tehillim 6:7], for my learning is not confirmed in my hand: how can a craft [art] or a skill be given that examines [tests] its own powers and explains its own principles — for that is the work of the universal wisdom [metaphysics], as the Philosopher explained, that it deals with the principles of all the sciences. And I found no rest [Bereishis 8:9] [over the difficulty] of how the craft [art] should overcome [conquer] nature — for the Philosopher explained in the Second [book] of the *Physics* that the craft imitates nature as far as it is able. And I thought, perhaps the [phrase] is a rhetorical figure [*halatsah*], in the manner that Rabbi Akiva brought ears of grain [שבולים] and loaves [גלוסקאות, *glusqa'ot*] to show that the works of flesh and blood are more fitting [?] [than the natural product] — and your pupil [says] truly that there are demonstrations, in the honored one's view, [that are] natural and mathematical, in appearance [theoretically] and not in [actual] selections [?]. Give me understanding, man of delights [Daniel 10:11].

[24] כד. "The torrents of Belial overwhelmed me" (Tehillim 18:5): how can [a magnitude] pass from one extreme to the other without passing through the

middle? "And the cords of Sheol surrounded me" (Tehillim 18:6): how can a magnitude be posited greater than a [given] magnitude, and [another] smaller than it, and yet none be posited equal to it? My intellect cannot bear it.

Diagram. A geometric/trigonometric construction embedded within the quoted demonstration on chords (sines), referenced as "the figure here drawn." The diagram itself is not reproduced as a separate woodcut block on the page but is described point-by-point in the surrounding text; the construction is a circle (or semicircle) of radius **אד/אמ**, with a diameter, an inscribed angle **ראה** (R'H) = 120° subdivided into **ראפ** (R'P) and **פאח** (P'Ḥ), and the chords/perpendiculars **דה, דפ, יח, דמ, מח, דל, לא, מל, מא** used to derive the component angles **מאד** = 54;43 and **דאפ** = 6;76. Readable point-labels and their roles:

- **א** (', center/vertex): center of the circle; vertex of the angles; **אד** and **אמ** serve successively as the half-diameter (radius).
- **ד** (D): point on the circle; endpoint of chords **דה, דפ, דמ, דל**.
- **ה** (H): endpoint of arc/chord **דה**.
- **פ** (P): division point of the angle (between **ראפ** and **פאח**).
- **ח** (Ḥ): endpoint of angle **פאח** / arc **יח**.
- **מ** (M): foot of perpendicular / point on the diameter; **אמ** = 52042, **מל** a derived chord segment.
- **ל** (L): point yielding chord **לא** (= 50000) and segment **דל**; **מל** a chord.
- **ר** (R): endpoint defining the full angle **ראה** = 120°.

25. *And the earth shook and trembled* [Tehillim 18:8] — how, while the bearer is at rest, the accidents move; *they mock at it* [cf. Chabakkuk 1:10], *the young men, and they shoot at it, the archers* [cf. Bereishis 49:23].

26. *They understood not* [cf. Tehillim 82:5] — my thoughts toward the works of HaShem: how shapes that differ in kind can be equal in perimeter yet not in area. It is already well established that the more sides and angles a figure has, the more it contains; and this is why the circle contains more than [any of] the [polygonal] figures.

27. *And I was not able to see* — how, when a sphere touches a surface, it touches at a point; and as it moves it traces a line; yet even so the line will not be composed of points. This is *an ancient* [matter, *Yamin* — a thing of old]; *they found not* its resolution, those *skilled craftsmen who have* in every science *ten measures* [i.e., the experts who command every science].

28. And my heart pondered: how, when from a point א two straight lines descend onto a line, more than 42 [?] [Hebrew 42 = מב, but the sense seems to be "two": likely מ"ב read as a numeral here is suspect — see note] equal straight lines [can descend], and more than one perpendicular can be erected upon a given point on a line, and more than two parallels [can be drawn] to another line; and how within a triangle two lines can be placed equal to its sides. These things are *against Euclid and his foundations* [the *Elements*].

29. And I, poor and afflicted, *grieved* to know how the circumference of a circle is described upon three points standing in a straight line. For I have already noted in the fifth Tamar that the straight line is the shortest between two points; and if we set our eye at the end of a straight line, we see all its parts up to its other end — but not so with the curved line, such as the circumference of a circle. And I was astonished at the operation, and how a figure can be made [conforming] to the circle.

[Marginal/inset note, p. 35:] *They contend with me, those who dwell in the gate* [cf. Tehillim 69:13]: how the center of a body can be outside it, and how a single body can have three centers, [these arising] from the circle. *He who gives a path in the mighty waters* [cf. Yeshayahu 43:16] *has instructed us.*

30. [merged with the inset above — see *They contend with me…*]

31. *And I looked, and there was none to help* [Yeshayahu 63:5] — how, when we add weight upon weight, its impact becomes easier [lighter]; and the more we add, the lighter it grows, without end.

32. *And I was astonished, and there was none to uphold* [cf. Yeshayahu 63:5] — how the vacuum resists motion more than the plenum [the full], and [a body falls] more easily [through it]; [yet?] in the vacuum the heavy body descends [more readily].

[Margin, p. 36: *and more easily than the heavy body descends.*]

33. *I have heard, and my belly trembled* [cf. Chabakkuk 3:16] — over a body that rises in its descent and descends in its ascent. Who would believe such a report?

34. *My bowels are inflamed* [cf. Iyov 30:27] over my folly: how one body can enter another body equal to it on every side, without expansion or stretching — *entering into its chambers* [cf. Tehillim 104:22].

35. *Let rottenness enter my bones* [cf. Chabakkuk 3:16] when I see ships sailing in opposite directions with one [and the same] wind, both equally; and *like*

sluggards they hasten [?], yet *the God of the spirits they revere* [cf. Bamidbar 16:22].

36. And I *became like a lone bird upon a roof* [cf. Tehillim 102:8]: how Euclid pitched his stakes upon a rotten foundation. *Satan* [an adversary] *turned aside* to the First Postulate [the "common notion"] within his premises, the general one [the parallel postulate], the inclusive thirteenth [?]: that when a line falls upon two straight lines and makes the interior angles on one side less than two right angles, then necessarily those lines, if extended ad infinitum, will meet. For this is a defect that overtakes it, and *a man of devices* would refuse it. Let us suppose lines בג ,אג and line אב falling upon them, and let angle [at] א be a right angle — the larger of [the angles] of equality [?]. If so, then angle ג is the smaller of [those of] equality. If the said lines meet, then — according to what is taught in [Proposition] 32 of the First [Book] of the *Elements*, and according to the ninth of the First — angle ג arises [is produced], and it would be half [its sum?], smaller than the whole; yet it had been supposed the smaller of the angles. This is *a disgraceful lie* [cf. Tehillim 109:?]. And if so —

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— [it is] *a building of poverty, frail, scorched, parched, a stormy, afflicted* one [a chain of lament-words]. For even if the two interior angles are less than two right angles, it is not necessary that they meet — and thus his premise is rebutted.

It has *not been hidden from the eyes of those who probe deeply in philosophy* that natural agents fall under one kind together with their effects, as Aristotle wrote in the second [book] of the *Physics* [*De Anima*? — *Lishmoa*, lit. "On Hearing," = *Physics*]. And perhaps the celestial agents are shared agents [acting] for what is below them, for they have within their power the operation — as the sun generates worms and creeping things, which are latent in the [potency of the] matter. But that which has no power over the operation — I do not know how it can bring it forth into act from itself.

38. *I find* [it] *more bitter than death* [cf. Koheles 7:26]: that of two circles sharing a center, the one smaller than its fellow, they move with one motion and trace a circumferential line equal [in completion], without addition or diminution. *Every man is too brutish to know* [Yirmeyahu 10:14].

39. *And my eye is consumed because of grief* [Tehillim 6:8] — my soul and my belly move at a slower [pace] than my head, and more than they, my feet; and the rider runs his course faster than the horse — yet with all this they all move *in a single*

instant; and *the vulture's eye has not known their path, for their ways are crooked* [cf. Iyov 28:7; Mishlei 2:15].

40. *It is turned for me into terrors* [Iyov 30:15], *like the drought of summer* [Tehillim 32:4], when I hear that it is possible for a shadow of the [sundial-]degrees [*ma'alos*] to go backward according to its course; for if so, *behold, the great miracle — how great its power!* And if the iron was forged according to its [the heart's?] design, then the device was wondrous indeed.

41. *I have become a reproach to my neighbors* [Tehillim 31:12], when I say that the larger balances are more precise than the smaller — whereas with the timepiece [*kli ha-sha'ah*, the clock] the opposite of all instruments [holds]; the large ones *seized for us little foxes* [?], the small ones *spoiling* [cf. Shir HaShirim 2:15].

42. *They mocked and scorned* those who see, that I tremble at a slight tilt in the lowly cart and the lowly ship, *yet I do not fear* when the lofty, great one tilts — even though the higher is the more liable to fall.

43. *He shall not dwell within my house that works deceit* [Tehillim 101:7] — the pans of the balance are corrupted, yet they speak truth; and how greatly does it [the device] know the inner content of things!

44. [continues] *They clapped their hands at me* [cf. Iyov 27:23], everyone passing by, *because I say* that a body can move *in the third [degree?] of touch* [?], moving in *the external motion* [?]; *they spat and shook their heads* [cf. Tehillim 22:8], *they prolonged* their furrows [cf. Tehillim 129:3]. And how shall our thoughts be made known *from beyond distant seas, deep waters?*

45. *They opened their mouths wide against me, all my enemies* [Eichah 3:46], when I say that the small [weight] outweighs its bearer [the larger], *with sword and famine* [?] — *for she has prevailed against me, both might and abundance.*

46. *I have become a laughingstock to all my people* [Eichah 3:14], when I say that the perceptible which is covered from its [proper] aspect will appear before us — whether to the side of our face, or behind us, below or above. *Hear my voice, O congregation, and rejoice* [?].

47. *Their song all the day* [Eichah 3:14], for a byword and a taunt, when I seek by means of the timepiece the diameter of the sun and the moon.

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48. *And I look upon trouble I cannot [bear]* [cf. Chabakkuk 1:3] — how on one and the same day the Ishmaelite [Muslim] says it is the sixth day, the Israelite [Jew]

says it is Shabbos [seventh], and the Christian says it is the first day; and all three celebrate their festivals on them and are justified, for *truth* is what the sage *renders* [decides]. And if all three were Israelites, they would profane Shabbosos and festivals — yet none of them [could be faulted]; *I said: Make not merry with the merrymakers* [?].

49. *I am like a pelican of the wilderness* [Tehillim 102:7] in my knowing the day of the week in the desert.

50. *I said: I will guard my ways from sinning with my tongue* [Tehillim 39:2] — to know all the hours that the self-moving instrument ticks: when I hear the first hour and the second and the last, even though I have not heard what lies between the remaining ones, I do not know how the known [hours] can point [me] to the hidden ones.

51. I shall keep [my mouth as] *with a muzzle* [Tehillim 39:2] from finding the hours without a timepiece — and I shall be *one who rises early* without sun and moon, and without observing the stars and the constellations. *Teach me to declare in the morning Your kindness and Your faithfulness in the nights* [cf. Tehillim 92:3].

52. I have *hastened* [or: held my peace] from the good [cf. Tehillim 39:3] — whether those who voyage at sea by the star at the tail of the Lesser Bear [Ursa Minor], called the *Wagon* — which is also called the Pole Star — are in error; and by what means can it be known where they go astray?

53. *And my pain* is *stirred up* [cf. Tehillim 39:3] from the lack of understanding in knowing the longitude of a country [region].

54. *My loins are filled with* burning [cf. Tehillim 38:8], easily, when I hear that one who drinks in a cellar [below] drinks more than one who drinks in an upper chamber, from the same vessel and the same measure; and according to the number of times, *we were as dreamers* [Tehillim 126:1].

55. *Like a heavy burden they are too heavy for me* [Tehillim 38:5] — how is the philosopher's saying not justified, that heavy bodies fall to earth along the shortest line? *I am weary of my life, and my soul is shortened* [grows impatient].

56. *And a band of evildoers has encompassed me* [Tehillim 22:17] — how parallels can meet, and if so, the buildings [pillars] are not upright nor the valleys level.

57. *They envision for us false oracles and seductions* [cf. Eichah 2:14], in saying contradictory things — and yet it is true that inclined [converging] lines do not meet. The *Moreh* [Guide, i.e. Maimonides] mentioned it, and the commentators

explained it, but I have not descended to the depth of the matter and their nature. And *yet another illness* [follows]: how the inclined lines, in that they are continually growing distant and receding, [yet] do not grow distant by a measure without end. This is a great wonder.

58. *They have made my flesh and my body decay* [?] because of my folly — two teachings: the one with Euclid, in [Proposition] 12 [?] of the Ninth [Book] of the *Elements*, and likewise with Theodorus [תיאודו"זיאום = Theodosius?] in [Proposition] 12 [?] of the First Treatise [of the *Spherics*?] — that they bring the one who denies the conclusion to the truth of the conclusion, not to an absurdity. That is to say: one who denies that Reuven is a living, speaking [being] — the premise will be one that negates "Reuven is not a living, speaking being"; and upon it a structure is built, and we generate the conclusion that Reuven is a living, speaking being. These things *no ear has ever heard* before; how strange they are, that their very essence refutes falsehoods! And one must not cast doubt upon the said demonstrations, for they are mighty foundations — *these are the heroes*. And according to what the all-encompassing sage, your disciple — who is *the one who enlightens my eyes* and *awakens my thoughts* in all that is said — told me, there are still further demonstrations among the moderns, like these kinds, with Cusanus [קוז'אנו = Nicolaus Cusanus?]

**

and Corrino [?] — but I did not understand their language, and only the two said laws did I know by name. And I returned many times to their words and to the books of logic, to find rest in them, but I found no answer; and I, like a fool, *its taste is changed in him* [?]. And this which *frightens me*, I do not know [it]; *my soul has placed me [in confusion]. Heal me and I shall be healed* [Yirmeyahu 17:14]; *You drew me up from the mighty waters* [cf. Tehillim 18:17]. *Deal bountifully with Your servant, and I will praise You*, for *I will exalt You, O HaShem, for You have raised me up* [cf. Tehillim 30:2].

59. *I am sorely benumbed and crushed* [cf. Tehillim 38:9] — that those who dwell beneath the [celestial] poles will see the sunlight for a longer time than those who dwell beneath the equator. I do not know whether your opinion is that the seeing of the light occurs prematurely [above the horizon], by reason of the vapors and mists that refract the ray — as the seafarers who travel to the edge of Russia, called Nova Zembla [נובא זימב"לא = Nova Zembla], reported, that the sunlight returned to them suddenly, before its [due] time. Even with this my mind is not at rest: that the curvature [convexity] of the earth would delay the seeing of the [celestial] poles at another time for those who dwell beneath the equator, given

that the whole terrestrial sphere is [but] a point in relation to the eighth sphere [the firmament]; and from here [arises] a contradiction for those who rely on the variation of vision. *Inform me, You who are pure of eyes*, the inner content of the matters; *bring me forth to relief* [cf. Tehillim 66:12].

60. *I roar by reason of the disquiet of my heart* [Tehillim 38:9] — how the error of Aristotle concerning the *visible* [apparent] stars can be demonstrated, [up] to ten kinds [degrees?], and concerning the circle of the Milky Way [*ha-chalavi*, the galactic circle].

61. *And as for me, I am poor* — they too [?] are not with me, to see upon the lunar sphere [the body of the moon] mountains *of varying* [?] heights, round [?] all [?].

62. *For I am poor — I cannot* [or: I shall declare] — I do not know why the [annual] conjunction [molad] in *all the nights of the year* does not equal the conjunction... [the syntax of qq. 61–62 is interleaved on the page in a two-column setting; see parsing note]; if the hidden arcs are always equal to the arcs always visible to our eyes, how shall this be? Who can inform me?

63. *My heart is hot within me* [Tehillim 39:4] — that the sun rises for us westerners *before* the easterners, for *from it proceed the issues of life* [?].

64. *In my musing the fire burned* [Tehillim 39:4]; *I spoke with my tongue* [Tehillim 39:4] — how, when the sun is at the head of Cancer [the summer solstice], the [period of] twilight is the greatest, and yet it is not the least when [the sun] is at the head of Capricorn — if they are opposites. *Let the lips kiss* [cf. Mishlei 24:26]; answer me with words that are correct [cf. Iyov 6:25].

65. *O my God, my soul is bowed down within me* [cf. Tehillim 42:7] — how, when the motion of the sun is regular and uniform, the days do not increase in [proper] order, and likewise in their diminution they are *devious in their courses* [cf. Mishlei 2:15]; and so it is also with the shadows that the instruments show.

66. *My adversaries have reproached me* [cf. Tehillim 42:11], saying to me: how can the flying birds and the dragged [drifting] waters be hanging, standing, *like a ladder set upon the earth* [cf. Bereishis 28:12], when in a single instant they are rising and falling within it?

67. *All the day [I bear] my shame* [cf. Yirmeyahu 20:8], when I say that [the earth] is suspended upon some support — *what have I said? Arise to my help* [cf. Tehillim 35:2].

68. *And the shame of my face has covered me* [Tehillim 44:16], because of those who reproach me, for my saying that there was never a [true] fullness of the moon; and my soul *moans like a dove* [cf. Yeshayahu 38:14] from the voice of the reviler and blasphemer, [over the claim] that *Korach* arose [?] — that it is brighter in the king [the full?] than against the glory of the moon. [The close of q.68 is corrupt/abbreviated in the source; see parsing note.]

Sefer Elim

[Conclusion of the Tamarot — the Seventy Questions]

[69] And how is it that the earth is not laid waste before the evening, given your claim that an object is seen better from afar than from close at hand?

[70] "In place of wheat, thorns come forth; in place of barley, foul weeds" (cf. Iyov 31:40) — given your claim that the sea is not even one part in a hundred of the dry land.

These are the words of Agur son of Yakeh (cf. Mishlei 30:1). Blessed is the one who waits, who tastes of their savor.

[Acrostic Poem of Praise — Zerach ben Nathan to Delmedigo]

Behold, your storehouses! Who can recount the depth of your wisdom? Could I reach the limit of your science, I would set my heart to delve into it. The lawgivers of Israel, those who bless among the people, those who wield the scribe's pen — they will search out the heights of your high places, and your hidden things will fill their bellies. The sons will be satisfied, your disciples, those who shepherd among the lilies (cf. Shir HaShirim 6:2-3) in the Eden of God's garden, the cedars of Lebanon that you planted upon scarlet (cf. Mishlei 31:21), faithful ones. There they will render the righteous deeds of the Lord, where the chronicles are recorded. They establish us, and God answers us with a voice. Sing aloud, you righteous, in the Lord! For light dawns in the darkness for the upright (Tehillim 112:4), upon one of the mountains — the mountain God desired for His dwelling (cf. Tehillim 68:17). And Joseph, he is the elder of his house, ruler over all his possessions (cf. Bereishis 41:40-41); he entered to expound and to make plain the words of the wise and their riddles.

Worthy of grace of countenance — may they add to him years and abundant strength, and mount him upon the high places of good. Three worlds, and the

longing soul exalts him. Heaven and earth, like his refined soul, are sated from his glory; his throat longs to serve Him with all his might — and to honor Him with his little wealth, the least in Israel.

But as for me — I will seek out your strength.

[The following is an elaborate **alphabetical acrostic poem of praise**, arranged in parallel columns and running through the Hebrew alphabet; each line is built on words beginning with successive letters and addressed to Delmedigo. It is highly artificial wordplay (paronomasia on each letter) and resists literal rendering; an idiomatic line-by-line gloss follows, preserving the alphabetic structure.]

Letter	Line (idiomatic gloss)
א	I esteem you — a man, the craftsman of your faithfulness; but indeed I will seek out your strength.
ב	With understanding, with intellect, the clear son, in your wisdom — a chosen one, radiant, the Creator of your brilliance, with goodness.
ג	He revealed mighty deeds — a lion-whelp, borders they bounded — exiles, great ones, the height of your greatness.
ד	The words of your homilies are sought; my beloved — fine words, thin, lean — your banner.
ה	Your splendor — your mountains, the multitude of your majesty — guide [them] in the straight way.
ו	This, and [hooks?] and houses and your heights.
ז	Radiance — your branch, your songs; the splendor of your luster (cf. Yechezkel 1:27).
ח	Those who desire, the vision of wisdoms — your praise; rose and lily and crocus (cf. Shir HaShirim 2:1).
ט	Pure ones, hidden, their taste is good — your dew; goodness — your child; good ones, pure, taste — good, your dew.
י	Joseph — may he live; a hand he waves; he answers; your beloved ones — your friendship. Joseph — may he live.
כ	Longing ones, crowned — (between) your glory; already all of them, prostrating, bowing to your throne; your glory already.
ל	To learn, to teach — a heart for your receiving; tongue to sing to you, to utter, to ascend; the receiving of your teaching.

Letter	Line (idiomatic gloss)
מ	A speech of honeycomb — your eloquence; a ruler, a governor of governance, an angel, an advocate — speak; from your speech, sweetened.
נ	Fitting is the saying — a discerning one; a candle ruling, the lamp of your governance; let us rejoice — we lift up, bearing your libation.
ס	I have ordered the telling of secrets, of mysteries, locked away; arrange your books — your treasure.
ע	Awaken, the crown of your congregation; ascend! Your branch is upon your people — your jubilation; your eye is upon your people — your jubilations. Sefer Elim.
פ	Weigh your steps; we adorned your mouth — splendor; a mouth — your wonders; they appointed — your wonders.
צ	A sprout of beauty, a righteous one among the tamarisks — your shade; your form — a foe rejoiced; your watchers.
ק	The acquisition of your holiness; near ones received — a voice; those who call out — your lines, your assembly.
ר	They stirred — far ones; their spirit murmured — vision; a shepherd, accepted, the first of will — your friends.
ש	They heard your praises — your peace; they asked the abundance of your wholeness — your report.
ת	Prepare their marvel; let your tribute return; the abyss longs — your tribute, your praise.

[Letter of Moshe Metz introducing Delmedigo's reply]

This letter was sent by the learned, comprehensive scholar, the prince, the master, the honored teacher and rabbi **Moshe Metz**, may his Rock keep and preserve him [שן? — likely an honorific abbreviation], to the consummate scholar, the philosopher, the honored teacher and rabbi **Zerach son of the honored rabbi Nathan**, may his Rock keep and sustain him, replying to his questions. He [Delmedigo] is a firstborn to understanding — a Moshe who received Torah from the headwaters of faith, comprising every wisdom, a treasure to be desired and a prized possession. The sweetness of his tongue is pleasing to all who behold him; he is like dew on a day of scorching heat, telling a

man his very nature and the quality of his teacher. Since all his compositions are precious — "for he ate the bread of the mighty" (cf. Tehillim 78:25) — even though a man cannot profit those who are absent, and fools find the bread no more fitting than infants do, yet we saw fit to print it, to benefit the discerning who flourish in the courts of the wisdoms and are planted in the house of the Lord (cf. Tehillim 92:14). And these are his words:

After prostration and reverence before the splendor of the precious man, exalted beyond measure and degree, and the extending of peace to my master who sits in the Sanhedrin-seat [תחכמוני, cf. II Shmuel 23:8] with his disciples and pupils — I beg leave to excuse myself for having delayed, for every delay is for the good in the matter of a reply. For as is the way of the world, it is written, "Then He saw it and declared it; He established it, and also searched it out" (Iyov 28:27), and "He said to man" (Iyov 28:28). And our Sages of blessed memory said that one must order his teaching — repeating it four times — until he can bring it forth from his mouth to others (cf. Eruvin 54b); all the more so to people of stature, who require refinement and scrutiny, "calling the [Aramaic] *aleppin* 'ayyin' and the *'ayyin* 'aleppin'". And I knew before whom I stand, to render judgment and reckoning before the throne of glory of the All-present — and I am poor and naked. How shall I come before the King in sackcloth garb? (cf. Esther 4:2). Therefore I said: "I will guard my ways" (Tehillim 39:2), and speak with measure and reckoning of my arrangements and my courses.

Before I reply to his questions, I will give the reason for my choice to write in a foreign tongue. For being other than the Holy Tongue, it is called, in the words of Scripture, *la'az* (a foreign language) — at which my master was astonished, that I spent so long learning it, with its grammar and foundations. For if the language is not for its own sake but as an interpreter of the sciences, it is upon us to seek them from the wellspring of Israel, and to keep our children from meditating in the foreign tongue of the peoples whose nearness they desire — and all the more so since there are among us books by the sons of our own people. I reply to my master that I too sought, for the love of my soul, my nights and my days; yet "the healing of the daughter of my people did not arise" (cf. Yirmiyahu 8:22). And it became apparent who and who the authors are. As for the Rav, the Guide [Maimonides], of blessed memory — without his compositions, study would be like a sealed book. For he went forth as an adversary against the philosophizers and the dialecticians who are the "divine" ones [the *Elohiyim*, the theologians], in the words of Ibn Rushd [בן ראשד], and he sought to clear away the thorns from the vineyard of Israel. Yet whoever has not first studied the sciences will find most of the words of the composition *Moreh Nevuchim* like a dream without

interpretation. And in his path walked the "sons of ascent" — and they are few (cf. Sukkah 45b): such as **Ralbag** [Gersonides] and **Rabbi Chasdai Crescas** [חסראי קרשקאש], and **Narboni** [הנרבוני], and **Albalag** [אלבלג], and their like; **Rabbi Joseph ibn Falaquera** [יוסף בן פלקרה] in his comprehensive abridgments of the sciences; and likewise **Rabbi Yehudah HaKohen** [ר' יהודה הכהן] in his book *Midrash HaChochmah*. These [authors] merely mention chapter-headings —

— they stopped up the chapters and left the great openings, the windows, and the doorways [unexplained]. And those who think to learn the sciences by reading these and their like truly waste their days. And the rest of the books of the sciences in our possession, translated from the Chaldean tongue, or the Hagarene [Arabic], and Arabic — I exhausted my days and changed my dawns and evenings [seeking] to understand the words [drawn] from the languages of the nations. For though they may be by their nature clean, in our [Hebrew] tongue they have become foreign, obscure, and hidden; and the translators tie knots into the letters, and chain and drag the diction "with the cart-ropes" (cf. Yeshayahu 5:18). The tongue of Eber [Hebrew] will not join with the daughter of Ishmael, nor with the sister of Nevaioth (cf. Bereishis 28:9). For this reason — pitying the candle, the soul of a man — from my youth the desire reared me to study, in order to benefit myself and others, by [way of] books composed in the Holy Tongue, in lucid diction, not after the manner of the translators, who from their own hearts bring forth diction from a barbarous, confused, stammering tongue.

And it was like a burning fire within me, blazing (cf. Yirmiyahu 20:9), for how could I bear to see the evil that befalls my people through their slackness and sloth? And perhaps their great longing comes to naught for want of money [a play: אפס כספם / רב כספם], and the long preambles and the conditions required to attain perfection grow dark like the stars of their twilight (cf. Iyov 3:9); for not every man merits "a high table", with a house and wealth to gather all the necessary books and to disburse to teachers, to learn the languages and to grasp the various forms of reasoning, which are the occupations of leisured souls. And Aristotle said: "In sitting and rest the intellect grows wise," and for those without strength and means there is no counsel and understanding — and "the poor man's wisdom is despised" (Koheles 9:16) and abandoned.

And these words were like goads planted in my heart; I gave no sleep to my eyes nor slumber to my eyelids (cf. Tehillim 132:4) until I should find a man after my own heart. And "one who comes to purify himself is helped" (Shabbos 104a): for the Lord brought before me — bringing my soul forth from the dungeon — from

the sons of our people, from the nobles of the children of Israel, the One who sets my soul among the living, who raises me to sit upon the wings of the heights of the sciences. He rides upon every intellect and knowledge, sustaining and ruling [a play on the names], and he is, among the sages of the nation, wise and a craftsman. He made for me this soul — for my father and mother shaped me with their toil and established me, a golem devoid of wisdom, dust from the earth; they multiplied silver for me and overlaid me with gold, clothed me with skin and flesh, and "fenced me with bones and sinews" (cf. Iyov 10:11).

But the exalted man, whose name and report you have heard — he it is who clothed me with garments of salvation, who shepherded me upon the high places of understanding, who trained my hands to draw near the forms of reasoning and my tongue to speak with eloquence, who guides me in the straight way: he is my master, the dearest of my neck, my "Sanhedrin-teacher" [תחכמוני], my instructor and beloved — the scholar, master of Torah and of statecraft and of the mathematical sciences [הלמודי], the rhetorician, the divine philosopher, the expert physician, the honored teacher and rabbi **Yosef Shlomo Delmedigo**, called by all "**YaShaR of Candia**" [יש"ר מקנדיאה]. May the Lord lengthen his days like the days of heaven (cf. Devarim 11:21), and may his leaf be ever fresh, like a tree planted by streams of water (Tehillim 1:3).

Now, above all the good things which [he has done for me], my master who counseled me to learn the **Latin tongue** [לשון לאטין]: for in it are all precious vessels — all the books of the sciences are translated into it. He gave me a method in my study of the sea of sciences, and a fine order, and pruned away the superfluities. For if wisdom were acquired by their accumulation and multitude, then the booksellers and printers would be the greatest of the sages — whereas "he who gathers little" gathered ten *chomarim* [heaps], and "so they multiplied" [a play]. How greatly do those err who scatter [their effort] among them without discernment, seeking from "the book of the skins of the carcass" — whereas the least of teachers is worth a thousand of them in truth; and "a living dog is better than a dead lion" (Koheles 9:4). And in the matter of study, received teachings [those learned from a living master] make an impression upon the intellect; this, apart from the opening of ears and eyes in the matter of choosing the authors on every subject — a thing which is a great matter in the acquisition of wisdom.

He admonished me to choose fine, well-written books, and never to exchange them — for constant reading in them preserves the pages and the lines, so that what is sought may be found by the touch of the hand; and all the more so —

— [to use] the books that stand foremost in [each] science: such as the books of the Talmud, or the Rif [Alfasi] and the Tur and the Rambam [in halacha]; the books of Aristotle and Ibn Rushd in philosophy; the books of Hippocrates [אפוקראטי] and Galen [גאלינו], and likewise Ibn Sina [Avicenna, בן סנה] in medicine; the *Elements* [of Euclid], and the book of the *Almagest*, and Copernicus [קופירניקו] in astronomy; and the book of Alhazen [אלצין] on optics [במראות, "on mirrors/vision"]; and Euclid [אוקלידס] and their like, among the authors from whom one brings proof, who are worthy to be before our eyes as our teachers. Continually he admonished me not to mark the errata upon the margins, except by a notation —

— and this [is done] with the fingernail; and whatever is choice, or difficult, I should write afterward in a special notebook, for the book called *Lev HaKoreh* ["the Reader's Heart"]. In review, he also gave me rules for memorization, and to make markers, to perfect [my grasp] — and for the mathematical sciences [התלמודים? recte הלמודים], a short method, and in practice a straight way.

And now, you have pressed me, my master and chief, to make known to you his quality: his nature, his temperament, his stock, his lineage and his travels, his conduct and his opinions — for your soul has longed, indeed pined, to walk after his footsteps and to pasture your kids beside his dwellings (cf. Shir HaShirim 1:8). How dear, how grave are his discourses! And how good it would be to ride upon a cherub and fly to behold the pleasantness of my master, and to visit in his palace — to recognize his worth and his greatness. A man [is praised] according to his renown; but there is no power in my hand to magnify and to make great. The frontlet of his crown [טוטפת כלילו] and his tassel are in perfection; I will make known a small part according to my poverty, and all who behold and know him will testify whether my words are true.

My master, may his Rock keep him — he is a humble man, lowly of spirit, and mingles with people. As for the suspicion the master held, that he is a man "smitten of heart" — service [or: the matter] — there is not in him even one part in eight of an eighth [of pride]. And if he should tarry to reply to your honor, here is the reason for his reply [being delayed]: for he is engaged in his medical occupations, and the spirit of the jealous physicians, the masters of his craft, terrifies him. Apart from this, the man is most diligent to do his work with the men of his covenant, and swift in his labor. He is a man gracious to the poor, who visits the sick — the destitute — and gives them his remedies; he abounds in secret charity, and his mercy overrules his other faculties. He is "easy to anger and easy to appease" (cf. Avos 5:11), and passes over his own grievances. He is a

shield and answer to many; he is a support and stay; his heart is like the heart of a lion. He stands before kings and finds grace in their eyes (cf. Mishlei 22:29). He responds to every petitioner who seeks him out in every season; in all their distress he is distressed (cf. Yeshayahu 63:9) — he gives no sleep to his eyes until he brings them out to relief [רויה].

He trusts in no flesh and blood, and constantly the saying is on his lips: "I have found no faithful friend save my own soul." And even though by his nature he is "bitter of soul" [a play, מרורי], nevertheless he bends his shoulder to bear every toil and hardship; and according to the vigor of his desire, so he multiplies his labor. He is not idle even a moment — neither on Rosh Chodesh, nor Shabbos, neither in his body, nor in his soul; upright is his work. A great physician [is he] — and [a physician] not to himself, for he does not govern his eating according to his intellect. Also at night he lies down but four or five hours; he sleeps on his right side, one continuous sleep, and as soon as he awakens he rises and leaps out of his bedding, sets himself to the lost time, and grieves over the [wasted] hours. Only on Shabbos nights does he sleep a second time, upon his left side. The man's senses are exceedingly keen — his whole being is intellect. By day he is to me like a one-year-old, and he is "a mother to understanding" (cf. Mishlei 2:3, *binah*). Even one who has never met him, should he encounter him and look upon his face, will stop and contemplate him — for his lips are precise in his speech, and he guards his tongue amid revilers. His eyes are black and ever-moving; a man of designs is he, and the spirit of God is within him. His features are somewhat elongated; his mouth is firm; he is silent [ישק? recte ישתק / he kisses?] —

His gaze is always fixed upward, and when he walks he strides quickly. From this you may say that a man ought to seek his goal swiftly. His stature is six feet — the measure of a middling height, as reckoned by those who calculate and survey. Since he lacks flesh, his complexion is somewhere between dark and fair. All his limbs are slender, and his whole body, clothing and all, weighs one hundred and forty *litra* [pounds] — for so I found it when I weighed him with care. The hair of his head is black, his locks curling, though they tend toward a chestnut color. He hates a beard, and refuses to grow it long. He says that nature did not intend a beard to adorn and beautify its creatures; rather, the spongy flesh beneath the cheeks, by reason of its moisture, sends up its shoots and branches — and his features were formed for another purpose. After all, this same flesh is found in the armpit and the groin as well, which is why hair is plentiful there. This is also why the ancient Caesars were clean-shaven, and the practice has been passed down among the nations on their way down to the present day. As for the paunch

and the beard — they testify about a vessel, whether it is full or empty; for a wise man is not made of bristles and rams [i.e. wisdom is not measured by a beard].

His name has long been renowned. His family is great in Israel; from it came heads of yeshivos, a chain of lineage and unbroken Torah, greatness, wealth, nobles and prefects, magnates in Jerusalem — may it be rebuilt and re-established — and teachers in former times in Provence [פרוב"ה], Ferrara [פירו"זה — read פירארה(?)], Florence [פלורינ"ציאה], and Pisa [פי"זה], heading the Christians in their bustling schools of philosophy.

The name of his father — may his Rock guard and preserve him — he is a Torah scholar, not a philosopher: an eminent rabbi in Talmud and in legal ruling, the teacher of all the men of his land, his fame reaching to Egypt, Constantinople [קוסטנטינא], and Venice. He is the distinguished rabbi, our master and teacher R. Eliyahu, son of the great and wondrous rabbi, our master and teacher R. Eliezer — may the memory of the righteous be a blessing — son of the pious, eminent, learned rabbi, our master and teacher R. Abba, son of the great Gaon, the divine philosopher, the expert physician, our master and teacher R. Menachem — may the memory of the righteous be a blessing. He was the one summoned by the community of Candia [קנדיאה] to return and once more instill Torah in them, after it had been forgotten in Provence in the time of the wars of the Venetians against the Caesar and the kings, when they seized all his wealth — as recorded in the book of *Divrei HaYamim* [Chronicles] of the eminent rabbi, our master and teacher R. Eliyahu Capsali [קפשלי] of Candia. This rabbi was head of the yeshiva for the Jews in Gemara, and for the Christians in philosophical inquiry, in Provence, in the days of the Gaon our master and teacher R. Yehudah Mintz [יודא מינץ] — may the memory of the righteous and holy be for the life of the World to Come. He was exceedingly wealthy, and he was the one called Shmuel Menachem, whose father left him in his mother's womb, just like himself [Delmedigo].

He [the father] is the eminent Torah scholar, our master and teacher R. Shmuel, son of the perfect rabbi from Mrkrna [מרכרנא(?)] — that is to say, our master R. Elkanah, who instilled Torah in Candia together with his brother, our master and teacher Moshe, who was afterward a magnate in Jerusalem; and with his brother, our master and teacher Eliyah. These went out at the flashing of the sword against Mahar"i Koloz [מהר"י קולוץ(?)] in aid of our master and teacher R. Moshe Capsali — may his memory be a blessing. These three were great men of the world, sons of the elder and esteemed man, eminent in piety, the prince and chief, our master and teacher R. Abba the Elder, who built in the land of Candia

the synagogue called Alimainiko [אלימאיניקו], and consecrated Torah scrolls and ornaments of silver and pearls — for he was very wealthy, and built houses and citadels. He was the son of the first prince and chief who came from the land of Ashkenaz to dwell on the island of Candia, our master and teacher R. Yehudah; and higher still above them are many honored men, up to the seed of royalty. But in order to show you the stock of my teacher's mother — the rabbi may his Rock keep him — I will pluck, as it were, from this good mountain and from the Lebanon: to the aforementioned R. Abba the Elder there were yet two

brothers. One was our master R. Meyuchas [מיוחס], who died without children; and the third was our master Shemaryah, a man renowned in the world. And his son was the eminent rabbi, our master and teacher R. Shesha Abba [ששה אבא]; and his son was the great philosopher, the teacher serving as head of school [רס(?) — read ראש(?)] in Italy [באוטליאה], the master of Giovanni Pico della Mirandola [יואן פיקו מירנדולאנו — Giovanni Pico della Mirandola]. He was the son-in-law of our master and teacher R. Yehudah Mintz — may the memory of the righteous be for the life of the World to Come. Renowned at the gates was his name, our master and teacher R. Eliyahu — may his memory be for the life of the World to Come. And I have heard it said in Constantinople and in Egypt that among all the philosophers of the Israelite nation there was none like him. He has printed works in Latin on the unity of the intellect, on the prime mover, on quiddity and existence, and lucid commentaries — therefore they too are in everyone's hands, even in the Holy Tongue. In handwriting, in all places, are several of his works. I myself possess a precious book of his called *Etzem HaGalgal* [The Substance of the Sphere], and a small treatise called *Binah* [Understanding], which contains various disputations and rulings. In it he mocks Ralbag [Gersonides] and his philosophizing, while Rambam [Maimonides] — he is his very soul. This man was wholly given to intellect and shielded him [Ralbag?] with all his strength. And, as my teacher, that excellent man, told me, he did not live long and died at the midpoint of a man's years. And all the Christian nobles, his students, accompanied him to burial dressed in mourning garments. He was handsome in appearance and pleasant in his comportment toward kin. The cause of his death was that he had ordered the cutting away of a fleshy growth on his cheek, called *naevus* [ניווס — Latin: *naevus*, "mole/growth"]; and perhaps the barber smeared the razor with deadly poison — for envy and ambition drive a man from the world.

And he [R. Abba the Elder?] had two sons, great in the world. One was the man called the all-encompassing sage, our master and teacher R. Moshe; he was a perfect and honored philosopher, and *"a lion of the school above"* — that is, the

great rabbi in Gemara and in the study of the Tosafos. He was the son-in-law of our master and teacher R. Moshe Alashkar [אלשקארי], the neighbor of our master and teacher R. Eliyahu Capsali. Renowned at the gates was our master and teacher R. Yehudah — may the memory of the righteous and holy be a blessing — and he was the student of R. Yehudah Mintz, who taught at his side in Provence. On returning [to Candia] he instilled Torah and raised up students great in the world. Among them was the eminent rabbi in Torah, the Gaon our master and teacher R. Shmuel Algazi [אלגאזי], who wrote a great commentary on the *Sefer Yere'im*, in the manner of the *Beit Yosef*'s compilation, and composed *shitot* [running expositions] on fifteen [י"ה(?) — read fifteen] tractates, novellae on the *Hara"n* [commentary of R. Nissim] and the *Nimmukei Yosef*, rulings and homilies, and a book *Kevutzat Kesef* [A Gathering of Silver].

And to the aforementioned R. Yehudah there were many daughters, and one of his daughters was the praised pious woman, a woman who fears the Lord, Mistress Rachel, who was married to the pious notable R. Yosef of Constantinople, son-in-law [נכדו — read grandson(?)] of the sage Kumtiano [כומטיאנו], and second [ושניו(?)] to our master and teacher R. Eliyahu ben Chayyim [בן חיים] — may the memory of the righteous and holy be a blessing. This man prospered and profited greatly in Candia, and he had only a single daughter — she is the discerning, modest woman, the rabbi's wife, the aforementioned Mistress Kashti [קשתי] — may she be blessed among women — the mother of my teacher the rabbi, who came forth from her womb, a son as wise as herself. Happy are the breasts from which there nursed this mouth that pours forth pearls.

My teacher has a large family — brothers and sisters and other esteemed relatives — and they dwell in the city of Candia, which is the metropolis of the island of Crete [קרי"טה], situated in the Mediterranean Sea [מירי"טיראניאו], its breadth thirty-five degrees and its length fifty-four degrees. This island was once a throne of glory for the Greeks, exalted from of old. It had a hundred cities, and so was called *Hekatompolis* [איקטופוליש — Greek: Ἑκατόμπολις, "hundred-citied"], according to the ancient authors. Today it is under

the dominion of the rule of the lords of Venice. Its inhabitants are men of contention; and there are many sages among them — most of them...

[The text continues, returning to the praise of Delmedigo himself:]

he carries his sheaves with song. Thanks be to God, he lacked nothing, nor did his eye spare his money: to let gold flow from his purse, and to grasp the corners of the earth — the choicest of the settled world — of Asia [אז"יאה], Africa [אפרי"קא],

and Europe [איאורו"פא], to sail ships in the heart of the seas. The streams of his innermost being are ever in tumult, with none to quiet the roar of their waves. He keeps vigil in the academies of the nations to learn their secrets, entering their courts and palaces, conversing in the tongues of various peoples, translating fragments [רציו — read shards(?)] of the silver of their eloquence and the elegance of their speech. For his father and mother counseled wisely and devised great resourcefulness in placing him among notable craftsmen [אומנים — read instructors(?)]; and so he was reared and nourished in their bais medrash for several years, and never departed from the stamp his parents had stamped upon him. *"And the Lord was with Joseph, and he was a prosperous man"* [cf. Bereishis 39:2], wise in all his ways. He made for himself priestly houses and Levitical houses — houses full of every good thing; every book and every object that gold cannot match; many honored men, and varied instruments [or: vessels].

In all the languages of the nations there are grammarians, rhetoricians, logicians, philosophers — natural, divine, mathematical, religious — physicians and historians: everything that is written for life. He was twenty-five years old when the spirit of the Lord carried him off from his land — for he loved strangers and chose to dwell in a land of wilderness, a lodging-place of wayfarers and foreigners, like a solitary bird upon a roof. For dogs surrounded him and embittered him with their evil; zealots, cunning in every kind of wisdom, despised him, until he loathed both their love and their hatred. His soul grew weary of spending his hours and moments in striving to quench the fire of their quarrel and to silence their worm [of envy]. He went out from there alone, and with his staff crossed this great, broad sea. *"And Joseph went down"* from Italy [מערימה — read from Egypt/Italy(?)], traversing a land of sunrise [the East] in its length and breadth.

He left behind the treasures of his choicest possessions and his books — all that was stored within his tent, in his mother's house and the chamber of her who conceived him. From there he circled and revolved about the world, to uncover all that is closed and hidden, until in succession he came to the lands of Poland [פולוניאה]. And though time roared against him like a young lion, and like a lion it would roar, and many accidents and mishaps made him loathsome and despised — they said to his soul, "Bow down, that we may pass over" — with no instrument of rest, no respite for a moment to stand firm upon his station and to settle down to his studies (for he was ever wandering and roaming) — yet his soul was not severed from his books, nor did he despair of wisdom.

The practice of medicine became his livelihood by day — it was the thing that stood by him in his exile, a staff of bread, a rod of strength, a staff of glory. He sustained himself in purity, never standing before the great [for favor], all his dealings being with kings and princes. And the night — a table set [for study]: every evening in Gemara and in the codes, and kept until morning for the external [secular] studies. *"When deep sleep falls upon men"* [Iyov 4:13], at midnight he would rise; he dwells in innermost chambers, with no man standing beside him in the mornings, delighting in the words of the silent ones [books], and *"every work of God he brings into judgment"* [cf. Koheles 12:14], refining it like the refining of silver before he inscribes it upon a book for a memorial. This was his constant rule, to arrive at the goal of the primary intention, and *"he betrothed it to himself in faithfulness"* [cf. Hoshea 2:22].

And it came about that, when his days grew long as he kept vigil at the work of the Lord, there came into his hand a great treatise, full of wisdom and perfect in beauty, which he called *Ya'ar Levanon* [The Forest of Lebanon] — for it is composed of differing studies, the cedars of Lebanon that he planted. His intention was to arrange and divide it. And he set up *Even Shelomoh* [The Stone of Solomon], a quarry built up from several books of wisdom, with all of Solomon's desire within it. But what could he do for its cedars and cypresses? They rose to the very top, and its branches were so many that ten reams of paper would not suffice to print it; the cost was great, and he could not bear it alone. And where are the generous ones of the peoples to help him? They were consumed and finished by the terrors. So it came into his mind to separate from it all the disputations and doubts, and to allot a single portion to his students. He dug out from there a small digging of the choicest matters, which he called *Be'er Sheva* [The Well of Seven]; it encompasses all the sciences, following the footsteps of the ancients, choosing

with terse brevity in clear and weighty language. In it the weary one will find his thirst quenched, as though he had studied several hundred books — until he is made complete in logic and rhetoric,, in theology, in practical philosophy, and in the totality of the mathematical sciences: that is to say, in *Sefer Algebra* [the Book of Algebra — Arabic: al-jabr], in surveying [the measured], astronomy [תכונה], judgments [astrology], optics [מבטים], and the science of perspectives [the science of shadows/mirrors], weights, and stratagems [mechanics]; in navigation [milḥut, seamanship] and the working of the land; in anatomy and medicine, in the nature of herbs and the kinds of living creatures; in alchemy [המהפכה — transmutation], in maps [המפות] and astronomy [the survey/heaven], a ruling over all the crafts and many matters, from which many branches ramify.

In his introduction he expounded at length on the rule of the perfected Israelite man's instruction, and on how a youth may keep his path pure, and he set out a curriculum: which languages one should learn and which books one should choose for every subject and topic. He named that introduction *Pi HaRe'eh* [The Mouth of the Seer(?) — פי הראר(?)], for his intention in it was to explain to students a short entry to the longed-for goal by means of the chosen intermediaries. And in its conclusion he composed for himself an esteemed treatise which he called *Peleg Elohim* [The Stream of God], encompassing the views of the children of men concerning beliefs and the hearts of kings, the leaders of states; and the goal of the lawgivers who set down the *nimusim* [Greek: νόμος, "law/code"] in [their] dispositions; and in it the principles and pillars of the religions, examined; the [legal] permissions, also old ones; and concerning every nation and tongue, many of the dwellings [doctrines]. He called it *HaBe'er* [The Well] from which all the flocks drink, *Peri HaIvrim* [The Fruit of the Hebrews], whose waters are clear and cool — except that it is very deep, and the stone upon the mouth of the well is large; for indeed great princes, many searchers of heart, dug it. There you will find the offspring of all the books of Aristotle and his famous commentators — such as Alexander [of Aphrodisias], Themistius, Ibn Rushd [Averroes], Euclid, Apollonius, Archimedes, Milaeus [מיליאום — Menelaus(?)], Theodosius, Ptolemy, Vitruvius [וויטרוויאו], Alhazen — and all the desires of "how many" thousands of authors among the moderns are in it.

This book alone suffices to teach the Jews every mystery and to bring the eager learner to the goal of wisdom — except that, because of its many branches, he was forced to abridge its dwelling [content], and chose conciseness and the omission of rhetoric. Everything written in it is mentioned by way of resolving the whole into its parts and defining the boundary of each one, in the manner of derived theorems and settled rulings, as though they were elucidated and well known among the perfected sages. This was his rule — to say that he composed it only to serve as a reminder to those who already knew the sciences, and to show teachers the order of study in their instruction. The one who acquires the book should provide himself a master to explain it.

Now I, while studying with His Excellency my teacher — I ate at his table for three years — used to see that this treatise had grown dim to the eye through its terseness, and I wearied of sustaining its nourishment. I would complain and grieve over the weight of its reasoning, and it seemed to me that the obscurity flowed from its concision, and that it needed a teacher to explain it. I argued that I knew for certain that students would object to those abridged rulings, would fall behind in learning, and would refuse to accept the established truth — the truth

which, after their premises (which are their entanglements and roots), is not in their grasp. Then he inclined his head to me and heeded my counsel; and out of anxiety he spoke, lest they tear it down and speak error against the words of the living God, and lest those who would make others happy in fact lead them astray and repay evil for good. So he gathered for me [the matter] to set it out in a commentary, and to amass around him the source of the fine gold and the roots of those derivations, and the rock from which they were hewn, together with their proofs — so that *"everyone who was bitten, when he looked upon it, would live"* [cf. Bamidbar 21:8], and by this they would not cease, and the image of envy of the envious one — *"his face shall answer with a lie"* [cf. Iyov 16:8]. He called this commentary *Be'er Acheret* [Another Well], for they would not contend over it. With proper intent he separated the two and divided them by letters [marks], for he saw and knew in his wisdom that the proofs, by their length, would confuse the order of study and interrupt its continuity, distract the mind of the reader from its theme, and hinder the recollection

[The exposition continues from Moshe Metz's presentation of Delmedigo's works.]

Anyone who wishes to retain these things should study them in *Be'er Sheva*, keeping his eyes and his heart upon it always, for that work contains a full exposition and an orderly arrangement of the matter, until he makes it a settled possession of his mind. And whoever's heart prompts him to investigate and probe after the source of these studies should examine the second book, *HaShaveh* [*The Equalizer*?] [?], whose surroundings are honor and glory [?], and which we have crowned [?]; then he will be brought to completion in concept and in verification through a book small in size.

The chief and choicest of his compositions, however — the one that holds all his knowledge and contemplation, his strength, the firstfruits of his vigor, his milk and his wine, the choicest of his threshing-floor — is the one in which he did not lift up his soul to vanity, nor swear to walk after the famous opinions of Plato or Aristotle, of Ptolemy or Copernicus and the rest, as he did in *Be'er Sheva*. For in this work he chooses new gods against the wise craftsmen [?], and he did not go out to meet enchantments [a play on נחשים]. This is the book *Beshamta Bat Shlomo* ["The Spice of the Daughter of Solomon" / "Basmat daughter of Solomon," a play], wreathed among the lilies of every wisdom and resource. He utters words to the Most High [*l'Ila'ah*], an investigator of all; he speaks of nobles, and arrays armies; he sets in order [?] pearls. He divides it into four rows of stone, and in every row great stones, precious stones, hewn to the pattern of a palace, and beneath them many ornaments of beauty, the adornments of a bride — the

daughters saw her and called her happy [cf. Shir HaShirim 6:9], queens and concubines, and they praised her. This book is full of fragrances [*samemanot*, a play on the title].

Many times I heard him say to my master [*l'rabbi*] [?] that much money is needed for his daughter's dowry; and since he also has a young daughter in his native land, I understood that he was hinting at the printing of the book *HaBesamot* — for in her love he is ravished [cf. Mishlei 5:19] and in her teaching he delights [?]. Sour [?] — and all day long he sings and weeds in the garden, hoes, and adorns, to make the crooked straight; yet I was not permitted to set them in order. Many times I pleaded before his honor that he show it to me, and he gave it into my hand for about an hour, and at once [I read it] [?]. And although all that he has of his books are in my hand, he said that one must not view the dishes at the moment of their snatching-up [i.e., before they are ready], and he said that fruit not ripened on the tree, if one binds [picks] it, sets the teeth of those who eat it on edge. And only from two treatises that are within it did my eyes feast: I saw great light from the treatise *Or Shivat HaYamim* ["The Light of the Seven Days"], which encompasses the whole work of Creation; and from the treatise *Chofesh Mechuppash* ["A Searched-out Search" — as its name, so it is]: in it he searches out treasures of various matters. And I did not know, given the brevity of the time, to which view my teacher inclines, for the whole book is cast in the form of a disputation in which sages and kings come to test the wisdom of *Beshamta* with riddles and secrets. In it are strange opinions — mathematical, philosophical, ethical — such that the hearer will not believe, and the teller would seem in his own eyes a deceiver. Yet all of them are made clear with demonstrative proofs, manifest as the sun at noon. Wondrous is the wisdom of *Beshamta*; it sits firm as the strength of its bow [cf. Bereishis 49:24], and in every craft and discipline its hands have great power, and it is steadfast [*emunah*].

Besides these mentioned compositions, my teacher has several writings in various languages — medical, mathematical, civil [?] — and lovely expositions, many notebooks and pamphlets with him as the supreme craftsman [?]. *And these* — Solomon lacks nothing [a play on Shelomo] — were made to give knowledge, to sustain with a word him who is weary [cf. Yeshayahu 50:4], and to teach the children of Judah the bow [cf. II Shmuel 1:18], the bows of the secrets of wisdom. In Him have I trusted [a play, *pelam bah batachti*] that everything written will be finished and completed; not one word shall fall to the ground; of all his good word not one shall wither its leaf [cf. Tehillim 1:3], nor incline to the earth from its fullness, and he shall call his name upon them forever.

And now — great in counsel [cf. Yirmiyahu 32:19], a refiner for words [a play], remembering deeds — do not multiply talking so proudly, proudly [cf. I Shmuel 2:3]; therefore let him not be quick to put forth a word against me, nor to despise my words, saying: how many are the days of the years of the life of your master? In the aged is wisdom, and length of days understanding [Iyov 12:12]. And Yosef [was] a son of thirty (ל' = 30) years [*yosef ben lamed shanah* — a play on Bereishis 41:46, Joseph being thirty when he stood before Pharaoh] — for according to what I have received from my master, may God preserve him, the time of his birth was on the 28th of Sivan [5]351, which is the ⁶⁄₁₆ of June 1591 by [?]... four

four hours after the rising of the sun (שעות אחר זריחת החמה), in Candia. And days shall speak, and a multitude of years shall make wisdom known [cf. Iyov 32:7]. Add to this that the man is not settled, for he is constantly going about all his days, and in roamings his youth is spent. [Lacuna / faint: a labored remark] [?]. Had it not been for the wrath of the enemy, the adversary, who occasioned for him this wandering and exile, he would have added still more and more, and worked wonders, double for sound wisdom [cf. Iyov 11:6] — for by his intellect he is a man of the heights [?]. And I have learned [?] that he busied himself with the laws of astrology [*mishpetei ha-toladot*, judicial astrology] and went into its secrets, ten measures [?].

And now, "Look toward the heavens and see" [Bereishis 15:5] the stars, for they are high; consider [?] the day of my teacher's birth — that he has set a star of wisdom on high, lifted up, in whom is no lowness or weakness, and wisdom was to him an inheritance [cf. Devarim 33:4]. Look and investigate every particular of his horoscope [*chinucho*] [?], and let me know what appears in your eyes; I know your eyes will behold equity [cf. Tehillim 17:2], though the matters [planetary configurations?] [?] be obscured. But as for diligence in study and ceaseless application — these are found in his nature without measure. I sought and found the truth.

[Now] hear, please, the words of the philosopher, the strong one of physicians and chief of speakers, Galen [*Glinos*], who wrote in *On the Nature of the Craft* [*The Art of Medicine*?], chapter 7, of blessed memory: Seven conditions are required for whoever wishes to find every investigation and search that he searches out, whether [the goal is] that he should with ease grasp from his teachers some intelligible knowledge and attain it — and the matter is true, or at least that he is confident he will find it. And these they are:

1. A pure intellect by nature. **2.** Great training from the early years in the first studies — and it will especially profit him greatly if, as Plato accustomed himself, he is steeped [?] in the science of measure [geometry] and number [arithmetic]. **3.** That he strive to be one who hears and receives from the mouths of the most upright teachers of his time. **4.** That he be so bearing of toil that even by day he reckons it as nothing [i.e., labors tirelessly]. **5.** And as for what pertains to fine knowledge — and this is found in few — that he be bound up in the love of truth, as though he were not alive all his days except for its sake, and that he despise all that is reckoned precious by the many / others. **6.** That he learn some manner and way to distinguish between truth and falsehood; for it is not enough that the desire for truth burn in him, if he does not find an easy way to attain it. **7.** That he be so practiced and expert in this manner that we have spoken of, that not only does he know it, but he can also make use of it — [and not] like the rhetoricians who, occupied with a very easy craft, suppose that knowledge in this manner does not suffice them.

Therefore, if even one of these that we have mentioned be lacking — even if all the rest be present [?] — whoever wishes to enter the way of truth will not be very confident that he will attain his quest; but [if] they are joined together [?], what hinders that with good confidence we should not investigate and seek out the truth? [These are the words of Galen.] When his conditions are fulfilled, nothing is impossible for the prepared one; everything is hard [at first], and already it has become well-set [?] for him. And these are the seven [conditions], my master, may God preserve him.

Ma'ayan Ganim — [The treatise begins: Delmedigo's answers to the questions]

And now I shall return to you regarding your questions, according to my poverty [of understanding].

Question 1. You asked of me my opinion on the natural first-principles [*ha-hatchalot ha-tiv'iyot*] of the Aristotelians — that is, the *hyle* [matter], the form, and the privation — whether they are hinted at in the verses of the Torah, [and] what some of our commentators reckoned in the words *tohu va-vohu* ["unformed and void," Bereishis 1:2]. I will make known to you, my prince, what I was able to grasp from amid my [studies]; and the words of the book *Beshamta* — from that time, the little that I glimpsed and gleaned a little of its honey [?] — shall describe the matter.

It appears that my teacher denies the first premise [*ha-hanachah*]. My eye [saw] only a mere trace of the matter, like a flash of sunlight before its appearing. He lays down three principles sufficient [in place of] Aristotle's principles in natural [philosophy], and refuses *hyle* entirely — which is...

to existences. And they [the principles] are: the hot, which is the agent [*po'el*]; and the moist, the dry [and the cold], which are acted upon [*mitpa'alim*]; and they differ in their nature and place. That is: the hot is the sun and all the host of heaven, which are fiery, and these are the agents in the two elements [*yesodot*], in earth and in water. And I would reckon that air is included with water — and what is found that it is cold and moist, against Aristotle [?] — except that air is spongy water, and more select and rarefied; and that these elements are the most simple, being the [first-]principles, and are not composed of matter and form.

Now the stars act continually by their light and heat upon the lower world; and the philosophic fire found among us is of their kind, and it is what intermingles among these elements, ripening, blending, and preparing them according to the nature of compounds after their kinds, as is fitting for them. And so, according to the variation of the quantity of the moist and the dry that are intermingled and the proportion between them, and according to the variation of the action of the said heat upon them, so the kinds will vary; for from these come into being all that is formed — minerals, plants, animals, and man himself: all are from these elements. And one should not wonder how, from three [first-]principles, an endless multitude of creatures is generated — for this same wonder occurs likewise according to the view of Aristotle and his disciples, who posit that all that comes to be is from the four elements and the variation of their mixture and proportion; for although these are limited in their number, their proportions and the manners of their mixture vary into kinds without limit [*be-lo tachlit*], since they found no cause for the multiplicity of variations except the one mentioned.

And see: so long as the moist is intermingled with the dry and tempered by its heat in the compound, according to the conditions fitting for its nature, it becomes another flesh [substance]; the compound [endures] in its persistence. And when they separate one from another and dissolve and grow hot, a portion to them [?] — now they grow guilty [decay] [?]. But the earth and the water, which are the [first-]principles, never cease [pass away]. And I know that, according to this view, doubts and objections will arise — and all of them are void and ugly in the words of my teacher.

Now if you ask, saying: behold, our eyes see that air, in its enkindling, burns and becomes fire — [do you then] deny that the passive elements are turned into

active ones? — for [I answer]: also according to Aristotle, the elements before us are not [purely] simple, and in air there is a portion of fire that cleaves and enters into all the elements, down to the inmost chambers of the earth. And by the meeting of polished bodies and the motion of the air, the fire that was hidden in it beforehand is revealed; for one should not think that rarefaction or sponginess generates the fire — for sponginess is of the actions [and properties] of the heat. And how should the offshoot be the cause of the root? And if so, the air is not turned into fire; and all the more, [you should] not see [?] such changes — that fire turns into air or into water. And although the matters are obscure, to those who understand knowledge the words are recognized as true; and all of them are made clear in the book *Beshamta*.

And as for what Aristotle laid down — essential forms [*tzurot atzmiyot*] — I will make known to my lord that they are nothing but empty appellations, and are nothing but dreams and fantasies. And this from several proofs:

[Proof] 1. [The multiplying of] existents [or entities] without necessity — and indeed there is no need for these essential forms; for according to his [Aristotle's] own view, the qualities [*ha-eichiyot*] are what perform all the affections and changes, since the substance [*etzem*] does not act except by means of the quality. If so, there is no reason to accept them [the essential forms].

[Proof] 2. That he taught us that all our knowledge is from sensibles, and we do not know by the senses anything but the qualities and accidents in all corporeal existents — for the substances are not sensed [*einam nirgashim*]. And if so, the essential forms are of the [class of] non-sensed.

[Proof] 3. That he himself showed it to be evident [*atzmi*?] [?], by the view in chapter...

the second of the book *On Generation and Corruption* [*De Generatione et Corruptione*], that the four first qualities [hot, cold, moist, dry] are themselves the forms of the elements. And the whole house of philosophy knows that these are accidents; and if so, all the forms of compounds are of their kind [accidents]. [?]

And these are many proofs, and good. Yet those who hold fast to these forms will answer the first objection: that it is true the forms are not [needed] to act, but only to give being [*ha-yeshut*] and essence [*ha-mahut*] to the compound; and since the actions follow upon the essence and the being, one will say that the essential forms are what act, in essence and primarily. **[Reply] 5.** [?] Although the qualities, which are their instruments, are the proximate causes of the actions — yet they act by the power of the forms. And these are nothing but words of

prophecy that he taught: the qualities shall not be the principle of being and act in all the compounds — as Aristotle and Alexander concede, that he [Aristotle] is thereby [a holder of] the forms [as residing] in the elements [?].

To the second objection they will answer: that the *hyle* too is not among the sensibles, and nevertheless its existence is verified; for just as the alternations [*chalifot*] of the forms taught us its essence, so the sensed actions brought us to the apprehension of the existence of the acting forms — for the essential forms are not among the sensibles, from the class [of things] whose existence you would seek by sense, as Aristotle wrote in chapter 3 of this seventh treatise [*ma'amar*], as stated [*le-mah she-ne'emar*]. And we will answer them: their words have aimed [merely] to liken the forms to the *hyle*; for just as it [the *hyle*] is wholly empty and void, so too its forms — they reasoned and inferred — would suffice the qualities to perform all the sensed actions, [so that] we would not seek out these imagined forms.

And see: those who affirm the essential forms argue many arguments — some of them strong, which are not built upon the false premise of *hyle*, and some of them resting upon it. Therefore the discerning [*ha-pikchim*] put first the question of the existence of the *hyle* before the question of the existence of the essential forms.

[Argument] 1. The parts of substance are substances; the form is a part of the compound, which is a substance; if so, the form is a substance. **[Argument] 2.** If the forms were accidents, the compound would be an aggregate, like the accident [?] — that those qualities befell it in such a manner [?] — [and] the compounds would not differ in substance but only in quality. **[Argument] 3.** [?] He who gives the name and the definition is a substance; and with the change of the form, the name and definition change, and it is called by the name of the form that comes [next]. If so, the form is a substance. **[Argument] 4 / ד.** [?] What absolutely bounds the coming-to-be and completes it is a substance; the form completes the coming-to-be — for from the qualities and accidents only affections befall; if so, the form is a substance. **[Argument] 6 / ו.** [?] Things are completed [perfected] from things that are under their genus; and since matter is a substance and is completed from the form, the form too would be a substance.

And I say: the ancients did well to philosophize in that they did not posit these forms, for they recognized no substances except matter [and] the compound. But Pythagoras [*Pitagoras*] was the first to invent them, as Diogenes [*Diyoninos* = Diogenes Laertius?] [?] wrote in the seventh [book] of the *Lives of the Philosophers*; and his disciple Timaeus [*Timi'um*] upheld his words, as appears in the books of Plato. And the proofs that he brings from extended magnitudes [*ha-*

meshichot] — that they are prior [*kodem*] to the stripping-away of every scent [?] [*reach*], and afterward they add the scent [?], and that the many things lacking form, which one then seals [defines] with them — these indicate nothing but the accidents; for these additions have no existence without a substratum [*nosei*]. And indeed Pythagoras was compelled to this invention in order to uphold his view of the transmigration of souls [*he'tek ha-neshamot*] from the body of man to the bodies of animals — for this the accidental forms cannot do. And the [Pythagorean] speakers accepted his words in order to uphold the survival of the particular individual souls; and according to the words of the Rambam [Maimonides], that they are acquired...

produced together with intellectual apprehension, and the dispositions become substantial — yet it is clear that these are accidental qualities.

Let us turn back upon their arguments and answer them. The form is indeed a part of the composite, but only in respect of *quality*; for division or aggregation comes from the side of matter, and matter alone is substance, since it is the substantial part of the composite. The form, by contrast, only *completes* it, and is called a part of the composite insofar as it adorns and perfects it — yet it is nothing but an accidental quality. The form gives the composite its name and definition. When it comes upon the *hyle* [primal matter] it is called, in the absolute sense, *coming-to-be*; and upon its separation it is called *passing-away* and *destruction*. So, according to those who believe in the *hyle*, before its arrival the composite was in potentiality, and upon its arrival the composite is in actuality. In our own view — take water and earth as an example — the mandrakes [דודאים] existed in potentiality, and upon their entry and combination (I mean, upon their being generated by the action of heat, according to a measure fixed for them by nature, or by its Director) the mandrakes grow in actuality as a species in their own right, distinct from the other herbs of the ground. Therefore the qualities complete the coming-to-be, and there is no need whatever for a substantial form.

One should know that, according to the ancients, there are two kinds of accident. The first: those whose separation does not corrupt the subject, even though the subject is acted upon at their coming and going — these are dispositions in the *hyle*. The second: those whose separation corrupts the subject, since their bringing it [the subject] about is its whole being, and they are acquired possessions [קנינים] in it. These are the specific forms; they are not substances but accidents that fill the place of substances. From them the composite acquires its name and is distinguished within its species, and from them comes its whole

perfection. As for the matter of definition: from the matter one takes the genus of the thing defined, and from these acquired accidents one takes the differentiae by which the genus shared among many species is delimited — and so the definition of the particular species emerges.

Let us therefore lay it down and say that the composite, made of matter and the acquired qualities, is not an accidental composite, even though its components are things falling under differing genera. For the said qualities stand in the place of substance, since they are inseparable except by the corruption of the composite. As if you were to say: the vital soul, which does not separate from its subject but ceases to be, and in its ceasing it perishes along with it, and the composite is corrupted. Just as its presence in the composite was the cause of the composite's existence, so its corruption brings about the composite's destruction — unlike *separable* qualities, which bring about only a being-acted-upon, and whose presence in the composite is therefore accidental. Know that the inseparable qualities suffice to differentiate the composite, and are reckoned as substantial differentiae — for Aristotle himself, in Book V of the *Metaphysics* [למה שאחר — the *Metaphysics*], calls the substances that distinguish between species "qualities," and there is no need for the thing's perfection to come from its species or its genus. When one who probes deeply considers these few words of ours, and strips himself of the Aristotelian garments, he will see the refutation of all these arguments and their like.

You will find them arguing:

(א) First: If there is no substantial form, there is no coming-to-be, since the *hyle* does not come to be or pass away, and the growth of all accidents is nothing but a being-acted-upon. — We will reply that this holds according to the premises of the Peripatetics; but the ancients say that the separable accidents bring about a being-acted-upon, whereas the inseparable ones bring about coming-to-be and passing-away.

(ב) Second: If there is no substantial form, there is no substantial difference between one species and another, or between genus and genus; for what distinguishes them, if the *hyle* is common to all? — We will reply that the inseparable qualities are what distinguish, and they suffice to differentiate between water and blood, between stone and wood, and flesh and bone.

(ג) Third: If there is no

substantial form, there is no subsistence for composites; for the *hyle* has no subsistence in itself, and how then can it give subsistence to others? — We will

reply that the *hyle* has existence, only that it is not delimited; it is like a genus, and the form distinguishes it, gives it a name — and this the qualities will do.

(ד) Fourth: The natural existents are all composites; and from the *hyle* alone there would be no preparation [הכנה], nor [is anything] composed of *hyle* and accidents that are like a composite but not a part of it. Therefore one must say that there are substantial forms which are a substantial part of the composite. — Now it is known what Ammonius the Greek commentator raised, in his treatise on substantial form, [namely] that Aristotle defines the accident as that which is in a subject — not as a part of it, but [only] that which is bound to it and has no subsistence apart from the subject; and on this basis all the forms are in the *hyle* and are not a part of it, nor do they have subsistence without it. If so, they are not substances — [yet] the natural forms [are substances]. He and his disciple Simplicius replied that the subject intended in Aristotle's words is not the *hyle*, but the complete substance, namely the composite — that which is found in it [the subject] and is not a part of it [is the accident], and it needs that subject if it is to subsist, for its whole existence is in the subject, and it is an accident; but the substantial form is in the composite as a part of it. — And we will reply and say: We have already said that the accidents separable from the subject without its corruption are not a part of it, and together with the *hyle* they do not compose the composite — for the *hyle* together with the inseparable qualities is what makes the composite, upon which the separable accidents are predicated.

Only it appears that in all bodies, whether simple or composite, there are actions and motions — I mean, in the elements, motions from the center to the encompassing [region] and from the encompassing toward the center; in inanimate things, actions — I mean, as in magnets [מגניטים], which draw iron; and likewise in purgative medicines, which select [particular] matters. These [actions] are not from the side of the *hyle*, whose way is to be acted upon and not to act; nor from the side of the accidents, which are instruments for the substantial [forms], as Aristotle explained in Book II of *On Coming-to-be* [להויה — *De Generatione et Corruptione*] [45–55]. Therefore they [these actions] are from these [substantial] forms. — We will reply that all these actions and operations follow and are caused by the inseparable qualities, in each and every thing according to its nature. For the cause of the motion of the elements is from the qualities of heaviness and lightness; and likewise sponginess and rigidity are qualities of matter — just as heat, moisture, and dryness are the primary qualities in the elements, and all the secondary [qualities], such as smoothness and roughness, softness and hardness and the like, follow from the primary ones.

So it is in all composites. In metals, for instance, they have particular temperaments [מזגים] and properties by which one is distinguished from its fellow. As if you were to say: lead is distinguished from tin, and the difference apparent in silver — this is from the side of the difference in the actions of the primary qualities in the matter; and this temperament is their specific form, from which the properties follow. So too in the plants that come to be from the mixing of the bodies and the qualities: their temperament is their soul, the vegetative one, by which they are nourished and grow and reproduce in kind. They are nourished from the earth and the water, which are turned into their nature, and they grow and reproduce by the aid of the elemental heat within them — which, when it dissolves, they dry up and wither. This soul is what distinguishes them from inanimate things. So too in [the case of] non-rational animals: sensation is their form, and in particular the sense of touch, which is not separated from the [living being], for it [the living being] comes to be from a temperament of the elements [that is] more excellent and praiseworthy than the temperament of the plant — which inclines toward dryness, and for that reason [plants] lack sensation. The animal inclines toward moisture; all the more so [creatures] possessing three senses, which require a fine temperament for their existence, for the [creatures] in which warmth from the [animal nature] is lacking [tend] toward their earthiness and dustiness. So too imaginative reckoning [הכח הדמיוני], and all the inner actions of the soul — such as intellection, composition, division, and inference —

all these follow from the goodness of the human temperament, more than [from that of] the other animals. And among non-rational [creatures] there is a great difference: in wisdom and cleverness you will see that one of the small dogs is craftier than the large horse and the ass, and craftier still is the fox or the ape and the like. This is not from the side of the matter, but entirely from the side of the good temperament. This is the view of Alexander of Aphrodisias [אלכסנדר האפרודוסי], and of Hippocrates [איפוקראטיש], and of Galen in Book V of the First [book] of his works [מיסורום — of his treatises], who said [Galen], may his memory be a blessing: "We have explained that, according to the temperament, there comes to be from the natural body the essence of the soul — unless some man should suppose it [the soul], according to the view of Plato, [to be] incorporeal, [such] that it can stand outside the body." End quote. This philosopher [Galen] holds that all the souls — whether in rational beings, in animals, or in plants — are stationed [over the bodies] and govern the bodies, as the helmsman the ship and the driver the wagon, and not that they are forms of the thing, as Aristotle supposes them to be its perfections and the agents of the substances.

(ה) Fifth: From what would the difference between sensibles come, if there be no substantial forms (since the matter is common [to all]), and it not be from the common accidents present in the existents that are shared by all? And if you say [the difference comes] from the gathering-together of many accidents — for those that appear in one [thing] will not all appear together in another — [we reply that] this gathering is accidental, since the gathering of several kinds of accident — as if you were to say of whiteness, heaviness, and warmth and the like, which are from differing genera — is accidental, and is not the essence that distinguishes the thing from its fellow essentially, as [the thing] is distinguished by the essence of the composite. And we have already resolved this argument by saying that there is no need for the completing parts to be of one [single] genus, since the qualities inseparable from the subject stand in the place of substance, and from them comes the essential distinction.

(ו) Sixth: If each of these accidents, by itself, does not distinguish the composite essentially, neither will their gathering distinguish it; for their conjoining one to another is nothing but a relation [הצטרפות], and that is an accident, as Aristotle explained in Book II of *On Coming-to-be* [40–41]. And further, we may ask: who brought about the joining-together of all these accidents? If you say they gathered by chance — then they are not of the essence of the composite, and it is not from the side of the *hyle*, which is common to all composites; and it is impossible to give any other cause for their gathering except the substantial form. — And we will ask them: if the *hyle* is in potentiality toward all the natural forms, who causes it to put on this form rather than the others? One must say that the generator [המוליד] acted upon it according to its nature — whether the generator be of the kind that comes-to-be, as with all natural generators, or whether it be not of its kind, as according to Aristotle's view the sun generates the frogs. [And] according to the view of my teacher, the sun is the elemental [fire that] enters into composites and brings out kinds of forms from the potentiality of the elements according to the variation of the mixture and the qualities. If so, it [the sun / the generator] is the generator of all these accidents and the gatherer of them together. In short — I see no argument among all their arguments that necessitates these substantial forms, and Nature leaves nothing over in vain, nor falls short in necessities.

Now, these forms are posited for one of three reasons: either to give being and existence to the composites, or to distinguish them, or to produce some action or property. But, as we have explained, all these the qualities will do: heat gives being to fire, distinguishes it from water, and burns and heats. Therefore there is no need to posit things that are not perceived; for, as we have said, all the actions

are from the side of the qualities — as the Peripatetics themselves concede, that every action and being-acted-upon is from contraries, and there is no contrary among substances, and no substance is perceived. Only the qualities are what fall under the senses. If so, one ought not to bring these forms into philosophy.

And behold, I see that many

went forth with the flame of the [whirling] sword [cf. Bereishis 3:24] against me, saying: "What shall we do for our sister [cf. Shir HaShirim 8:8], our soul, if it is but a temperament? Woe for our end!" And according to the view of the Rambam, whoever has not attained even some intellectual perfections has perished, and the whole multitude of the people fall in this wilderness as corpses [cf. Bamidbar 14:29–32]. And even according to the opinion of the one who says that the commandments, in their performance, suffice to substantiate [the soul] — could it be that all the nations are finished, they and their souls with them, [the nations] over whom the Lord did not appear, who did not know Him, who did not receive His Torah, and who did not see His wonders? Far be it from God to chastise them! And woe to their souls, whose disposition was borne off alone and was not perfected by the performance of God's commandments — and as it [the soul] came, so it goes, empty. Only that we see that even if the substantial forms be granted, and the rational soul be one of them, then according to the view of Aristotle and others it is not separate from the body. For every moving thing is divisible — part of it in that *from which* [the motion proceeds] and part of it in that *to which* — as Aristotle explained in Book VI of the *Physics* [לשמע — the *Physics*] [32]; and if so, the soul would be divisible and would be a body.

You know the answer of those who say that it [the soul] is the moving thing [that moves] in an instant; and Aristotle proved, by a demonstration against this, in Book VI of the *Physics* [29], that if a thing [were to] move along path **ב.י.ד** [BYD] in an instant, then one twice as fast as it would move along that same path in half an instant; or that same moving thing would move along half the path **א.י** [AY — recte: along half the path] in half an instant. And this contradicts itself, for the instant is, with respect to time, indivisible, as is known. [Recte: half the path **ג.י** —] If so, the matter is contradictory for them.

But it [the soul] is not [such a] motion, for [in the case of] sparks, the accidents move — and how [could there be] in the air, without a subject, a motion that should be in an instant? And I know the arguments about the elevation of light and its appearing upon the horizon; but surely the spark is inscribed [registered] in each and every part of the light as it comes to be in the seeing — and there is no need to expand. For all admit, regarding [local] displacement, that it does not

occur in an instant; and you will see all this in [the book] *On the Heavens* [בשמ — *De Caelo*], in order. And according to the view of Plato and the Kabbalists [המקובלים], it [the soul] is not a substantial form but [something] appointed over the body to govern it, and it has an existence apart from it [the body].

I cannot understand how this soul would be permanent unless one clothe it in bodies — as some of the great divine [theologians] of the Christians have already made known, that it [the soul] is not so simple as the Exalted God and His angels; and in the way that they [the Christians] say of the soul, the Kabbalists say of the angels, that they have a garment in which they clothe themselves when they go forth, sent to do [God's] will; for they could not move otherwise. [It is] very clear that, were they not bodies — and how good it would be, if it were possible to maintain this view, and to overturn the arguments against it, for thereby we establish the immortality of the soul and the motions of the angels, and we put to rest the view of those who doubt it.

However, as for the [intelligences] separate from matter and garment, there is no doubt that every philosopher will be unable to affirm multiplicity among [things] separate from matter, and all of them go by the same name, like sons of men; and you will know the secret of the moving things, that [their motion] is in an instant [recte: that they move not in an instant — see context], and Aristotle explained by a demonstration against it, in Book VI of the *Physics* [29 — repeated], in an instant. [Text disordered here owing to the interleaved lower block; see readability note.]

[The lower portion of folio 54 is set as two interleaved text blocks; the following fragments belong to the lower block and are transcribed in source order:]

that if it [a thing] move along path **ב.י.ד** [BYD] in an instant, then the [one] faster than it, twofold, will move along that same path in half an instant; or that same moving thing will move along half the path **ג.י** [GY] in half an instant. And this contradicts itself, for the instant is a time [in itself] indivisible, as is known; if so, the matter is resolved for them.

they have a garment in which they clothe themselves when they go forth, sent to do [God's] will, for they could not move otherwise. [It is] not [the case] that they are bodies — this is very clear. And how good it would be if it were possible to maintain this view…

and the immortality of the soul and the motions of the angels, and to put to rest the view of those who doubt it. However, as for the [intelligences] separate from matter and garment, there is no doubt that every philosopher will be unable to

affirm multiplicity among [things] separate from matter; and to all of them the same name is given, like sons of men.

[regarding] the individual difference there is dispute among the philosophers; and if it is not the matter alone — which is common to all of them — neither is it the form alone, for the reason that it [the form] is one and the same in all of them. And according to the discerning, it is fitting to say that the individuation of individuals is from the side of the matter; and so said Aristotle in Book XII of the *Metaphysics* [בי"כ למ"ש — Book XII of the *Metaphysics*]…

this matter delimited by quantity is the cause of numerical individuation, and it is a part of the composite individual.

that things whose matter is one are called one — and all the more so according to those who deny substantial forms, for whom the composite is nothing but prime matter together with an aggregate of accidents (it being quantity that constitutes the greatest of the factors distinguishing one individual from another). And we do not deny that the distinction comes from the side of matter, whether it be a substantial distinction or an aggregate of accidents; for matter, by its very nature, is divisible and separable, whereas form is one in its nature. And just as individuals receive their being from form, so it seems that they acquire from it their unity and their individuality. We grant that the individual acquires from form the individuality whereby it is not divided into [further] individuals; but it does not acquire from form that it is not divided into parts. And since this latter is likewise not acquired from the side of matter — which is by its nature divisible — we must grant that this is acquired from matter together with the accidents. My teacher took pleasure in mentioning this question in *Pi Ya'ar HaLevanon* [the *Honeycomb of Lebanon*].

Perhaps the inquirer will object and say that the point of this question concerns the individuality of material things — those composed of matter and form — and not the individuality of the separate intelligences, in which the form differentiates, individuates, and delimits: of that there is no doubt. But rational syllogism shows the opposite — namely, that there can be no specific or individual distinction, in any manner whatsoever, among things separate from matter. For if we grant that these intelligences are separate from quantity, which is an accident, we are compelled to say that they are abstracted from the whole genus of quantity — even from discrete quantity, under which number falls. If so, it is impossible to assign number to separate beings, just as it is impossible to say that accidental oneness is predicated of them. For the one substance is one only because it is a substrate for accidental oneness, and it cannot be that the separate

beings should be substrates for accidents, since that is the property of matter. Moreover, number belongs to the common sensibles, as Aristotle wrote in *De Anima* [*Sefer HaNefesh*]; how then could it apply to the separate beings, which are not objects of sense?

Do not bring against me the species and the genera — which are surely separate from one another and are not corporeal — for I too acknowledge the intellectual distinction unified [by the mind], whereby the intellect gathers all the individuals into a universal nature, and all the species into their encompassing genus; the senses prepared the way for this. Yet none of these has any existence except in the intellect. So too the Platonic Forms: either they exist in the intellect alone, or they are not separate from matter. And even if we posit that they exist in the intellect of God, exalted be He — and, according to the greatness of the subject in which they reside, are at the summit of supreme [excellence] — nevertheless, relative to the individuals, which are the composite substances, they are secondary. The philosopher [Aristotle] is accustomed to call these secondary substances "later" ones, saying that the universal is either nothing at all or is something unified — that is, an aggregate of individuals collected in one intellect, which contemplated the individuals, saw that they resemble one another in some respect, and brought them under a single species.

Let us now return to our subject. The view of the Rambam [Maimonides] is already well known; he disclosed his opinion on this in his Introduction [to Chapter] 13 [of *Chelek*?]. He said, of blessed memory: "Whatever is not a body, no number can be conceived in it — unless it be a power in a body, in which case the individual powers may be numbered through the numbering of the material substrates that bear them. For this reason the separate entities, which are neither bodies nor powers in a body, admit of no conception of number at all, except insofar as they are causes and effects." And Rashbat [R. Shem Tov ben Yosef ibn Falaquera? / the commentator], the expositor, wrote there a fine thing; and the substance of his words, of blessed memory, is this: "As for the specific multiplicity — by which the horse differs from the donkey — many a thinker supposes that it comes from the side of form; but the truth is that, were the matter of the horse and of the donkey one, their forms would be one," and so on. It appears he means that the species of horse is distinguished from the species of donkey because their individuals are distinguished in their matter; for substantially and primarily it is in the primary substances that distinction and the [individuating] properties reside, and only afterward in the secondary [substances], which exist in the intellect. And the philosopher said that the truths of expression — or [the truths of]

thought — follow upon the truths of existence; so that, because snow is in truth white, the proposition "snow is white" will be true, and not the reverse. [Marginal/displaced fragment: "the one that says snow is white is true, and not the contrary."]

I have seen some of the Christian philosophizers who discussed this matter, and their words require [examination], for the words of the philosopher [Aristotle] appear to contradict one another. For in the second book of *De Anima* [Sefer HaNefesh, II] he says that number is a common sensible, shared with the [other] objects of sense; and if so, it appears that it is prior to the operations of the intellect, since the sensibles act upon the senses, and from there [pass] into the imagination, and so on. According to this, its existence has no need of the intellect, for it has an existence in itself. Yet in the fourth book of the *Physics* [*Shema*, IV, §131] he says that without the human intellect there is no number; and if so, it has no existence in itself — the intellect and its operation being prior to it. I would answer that the *numbered* number — which is the first thing, the composite material [object] — is what is intended in the words of the philosopher when he says it is a common sensible; and this has no existence in itself insofar as it is *counted*, whether potentially or actually. That is to say: if it is counted potentially, there must be a power capable of counting it; and insofar as it is not counted, it is prior to sense and to intellect. And if it is counted actually, there must be a man counting in actuality. Still, this does not prevent it from having existence as it is wood or stone. To take the example of color: insofar as it is seen, the seen object derives from the seeing eye; it is not sensed if there is no actual power of sight in act, and it is impossible that it should ever be seen if there is no power at all upon the seeing. Nevertheless, color itself remains, in itself, distinguished in its hue from the other colors. So too the [beings] separate from matter: it is true that they have no existence insofar as they are *counted*, but they do have existence insofar as they are substances separate from one another in their species.

I marvel at this answer, that they did not scrutinize the example. Let us grant that the separate intellect is compared to color: color has two kinds of "imaginings" — one insofar as it is sensed (for inasmuch as it is a quality, even if it is not sensed, the quality remains); so too the intellect has two kinds of "imaginings" — one insofar as it is counted, and one insofar as it exists (so that even if it is not counted, it has existence). And we grant this much: that the separate intellect has existence, if its existence be shown by demonstration. But the question is: who says that its multiplicity [comes] from its being numbered? And if they grant that it has no existence insofar as it is counted — that it is not in our power to count

them and to attribute multiplicity to them — then it is not multiplied and not counted, and we find that they are constructing [a battle-array over] the matter under inquiry, yet they are not returning a sound answer. [Displaced fragment: "and they are not returning a sound matter."]

You ought to know that, although sense distinguishes within number and apprehends multiplicity and oneness, nevertheless it alone cannot count; for counting is an intellectual operation, a property [belonging] to those who speak. For beyond the distinction among the numbers, there is needed knowledge of order — that this is first and this is second to it. For even dogs distinguish between their masters and their fellows, yet they do not count; for true counting is like the counting of "one and one, two; one and two, three" — which joins one part to another, and is an intellectual operation, without doubt. And if our intellect does not know how to count the separate beings, [it does not follow that] they are not separate, [but only] insofar as they fall within the conception of our intellect. And as for what they say — that abstract oneness, called *transcendenti* [Latin: *transcendens*, "the transcendental"], is convertible with existence, and is distinguished in itself from another thing by [some] other [distinction] — we grant it to them; except that the objector will reply and say that there is no more than one existence and one oneness in the separate beings; and who has seen the multiplicity of the intellects? Indeed, none of the men of valor have found their hands [equal] to giving multiplicity among them — except either from the side of the multiplicity of motions

of the celestial bodies, or from the side that they are causes and effects, according to the view of the Kabbalists. And so, too, the view of those who hold that [the intellects] are multiplied and differentiated by reason of their ranks did not find favor in my eyes, and they have no foundation. And **Aristotle**, the chief of the philosophers, did not for the most part venture to assign number to the intellects except by the number of the spheres; and he guarded himself most carefully — in the first book of the *De Caelo* [*Shamayim*, I, §78] — from positing many worlds, [reasoning] that, since the intellects in two worlds would resemble one another, they would be of the same species and would not be separate save by individual — and this is impossible, since they have no matter. But each and every sphere is a species in its own right, even though there exists only a single individual of that species; and were it not for the multiplicity of the motions in the spheres, no eye would ever have seen any separate intellect at all. He who wishes to introduce sensed existents into the world above bears the burden of proving their existence.

Now, according to what I have heard from my teacher, I shall speak [of these things] before kings and not be ashamed: that the speculative science stands on its own, and faith [stands] on its own; and that we, being among the philosophizers, are obliged to bring proofs for our words, and not [merely] to say "thus we received from our fathers" or "thus it is written in our Torah." Add to this that, even if rational syllogism should indicate some matter against the faith, it does not contradict the faith absolutely — until it should uproot the faith entirely. By way of example: when we say that the natural form does not separate from the body, it appears at first glance that, if such be our view, then our hope [of immortality] is lost. But if the soul is set over the body, according to the view of the Kabbalists — that it has only a spiritual garment, and an existence prior to its coming into the body, and is not a form — then the philosophic view that says "every form in a body perishes" does no harm at all to the faith. Therefore the man perfect in his intellect ought not at once to reject the rational syllogism on account of the faith, for there are many ways to uphold it, only that they are not known; and if the faith is true, it will not contradict the truth. And the same is the case with the separate intellects: even though we believe in their existence — and we have no doubt of this, together with all this, [in accord] with the view of the masters of their faith, [that it is] one of the principles, such that he who denies them is a heretic — it is among the well-grounded views.

As for what the sages of the nations supposed in their faith — that the motion of the spheres is [the work of intellects], which they reckoned to be living and intelligent — you will find this in Plato in several places, and in Aristotle, and in Xenocrates [Kasinokratis], and in Plutarch [Plutarkhos], and in Alexander [Alpasander], and in Ben Sina [Avicenna], and in Johannes [Yochani], and in Simplicius [Shimplitsiu], and in Thomas [Tomash], and in all the renowned later authorities among the Christians. And they differ, however, in the matter of [the spheres'] intellection.

One view is that of the excellent man in whom we glory among the nations, the master, our teacher Moshe ben Maimon [Maimonides], of blessed memory. He holds that the body of the sphere has a soul which is its form, not separate from it, and which moves it essentially and is moved with it accidentally. But outside the sphere he posits a separate intellect, free of matter, which has no involvement with the sphere at all — only insofar as it is conceived in the intellect [or: soul] of the sphere; and it is the sphere's object of desire and longing, [striving] to liken itself to it, [and so] it rouses the soul of the sphere to motion. And insofar as it [the intellect] exists outside the intellect of the sphere, it does not move the sphere except in the manner of a final cause; but insofar as it is conceived in the intellect

of the sphere, it moves it as an efficient cause. This view he took from Ben Sina [Avicenna] and the Arabs; and in our own day the great sages of the nations have agreed with him, since they do not understand how the separate [intellect] could move the celestial body without contact — neither by attraction, nor by repulsion, nor by elevation — and they cast about to interpret the words of Aristotle and bend them to their own view.

And the second view, which is the view of Ben Rushd [Averroes] and of most of the divine philosophers, is that the separate [intellect] is itself the mover, and that the sphere has no soul besides it; and that what comes to be from it, and the celestial body

which is the matter of the sphere, is matter without form. And they have several proofs for [holding] this, in order that they may reckon it [the celestial body] to be simple — simple to the utmost degree of simplicity. But the soul is found only in a body — only when the body, through it, performs the operations of the soul: namely, nourishment, sensation, motion, and intellection, as Aristotle defined the soul. And the spongy and hard [substances] that are in the spheres do not suffice for them to be called instruments [of the soul] insofar as it [the soul] intellects. Add to this that they do not understand how it [the sphere] should have an intellective soul while lacking sensation and nourishment — which are included under intellection, just as the number two is included within four. There are also strong proofs written in the commentary of the divine sage, our honored teacher Eliyahu Delmedigo [Elijah del Medigo], of blessed memory, on *Etzem HaGalgal* [*The Substance of the Sphere*]. And in truth, against both views great doubts may be raised.

I heard from my teacher the reason that compelled them to suppose that the spheres are moved by intellects: it was because they observed contrary motions in them — this one from east to west, and that one from west to east. Had [the spheres] been moved by nature, they would all have turned toward one side and toward one end, so that one would not suppose the spheres to be of several kinds, like the elements. So they were compelled, in order to give a reason for the two motions, to posit the mighty intellect in the encompassing [sphere of] the prime mover, whose great power drives everything within it to its own motion; while the [lower] intellect labors to grasp how [the spheres] could be moving in contrary motions, being one within another without void or interval. The parable of the boat and the ship does not resemble what it is meant to illustrate. As for me, all my days I could not endure this savage daily [diurnal] motion that would be in the great encompassing [sphere]; and I thought that perhaps the diurnal

[motion] is the lowest, and the higher [spheres] rest upon it as upon a chariot — for in this manner the analogue would resemble the crawling of an ant upon a wagon.

But according to the view of Copernicus, the motion is lawful, without compulsion; for he holds that there is no more than a single motion proper to each sphere, and that they all move from west to east — and the earth likewise, and the elements with it, with the motion of the yolk and the white [of an egg], in the spherical motion; and the earth is not reckoned heavy in its place. The composite [diurnal] motion, too, is not proper [solely] to fire […] with it, but the mathematicals [*ha-limudim*] would be according to the teachings of Aristotle, so that they may agree — [if?] only for all the spheres; and the sphere that carries the earth is one of them, and the [resulting computations?] are not to be twisted [Parsing: this clause is corrupted/interleaved — see note], that they may be true. We leave these views to the astronomers, whose whole aim is to establish the calculations and the motions of the stars. But I, for my part, will demand of these [philosophers] the necessity that drives them to harness the horses to the wagon: is it for the benefit of the sphere? Heaven forbid that their Creator [should make the nobler] subservient to the lower, with the precious existing for the sake of the worthless — that I should pursue matter for the sake of form! And if it is for the benefit of the separate [intellect] — why, [for the sake] of its subsistence, and its being thereby [such-and-such]? [If so,] the growing composites — that is, plants — being non-speaking, [and] the hylic [material] intellect [?] would be more perfect than it [the celestial intellect], for they are living and subsisting in themselves. On the contrary: Aristotle, in his wish to display the strength of subsistence in God and in the separate beings, removes them very far from matter and magnitude. If so, why [should there be] for its operation — which is intellection and motion —

[Displaced/interleaved lines at foot of page 58:] to move; and for the imaginings, which are the affairs of the intellect — if so, it [the celestial body] would have growth and senses; and for these there are needed temperaments, which are four[fold], and [an] organ [?], as Aristotle explained in the second and third books of *De Anima* [*Sefer HaNefesh*, II–III]; and there would be in them coming-to-be and passing-away, and they would be reckoned natural. Apart from these, I see no connection between the separate [intellect] [and the celestial body…]. And indeed, is not God, exalted be He, [One who] intellects and moves without corporeal instruments? — for if you should say that these require sinews […]

[The foot of page 58 is degraded and two-block interleaved; the final lines are reconstructed only in part.]

[continues from prior page] to the sphere, since the natural heat is the instrument through which the soul acts upon the body. And if there is no vital and animating spirit in the spheres, how do they move, how do they think, and why is the chariot [merkavah] called intelligent on account of the directing intelligence? Aristotle said that one who calls the soul a "weaver" speaks as one who says it is "weaving" — for the activities belong to composites. And if you say it is possible for them [the intelligences] to be separate [from bodies], that is still not correct. For where the matter concerns the material intellects [ha-sekhalim ha-hiyula'iyim], I would ask you: to what end are they bound to bodies, when they are not endowed with free will as we are, that they should receive reward and punishment? Moreover, if they are by their nature separate, then their conjunction with the spheres is accidental, and an accident perishes — so the sphere would perish, yet they [the philosophers] make it eternal.

And if you say that the separate intelligences are needed for the forms which the spheres produce by means of heat and light: an accident does not generate a substance — as we see in the vermin generated, without sexual coupling, from putrefaction, and in plants. Nor can one say that, if so, every thing would be in every thing. Therefore these [forms] come either from the Active Intellect, or from the spheres, or from the separate intelligences that emanate upon them and truly move them. In truth I do not know how the separate intelligences could generate the forms. If it be from nothing, their power would be infinite, equal to their Creator. And if you say it is from the power of matter — what is this power, given that the power of the moist [or: of the tablet, ha-lu'ach] [?] does not depict forms within it? In that case the forms would be part of the very essence of the separate intelligences. Consider it from another angle: many things follow [from this], for these forms are not essential but accidental, and it is heat that generates them — heat being the agent, by way of the moist and the dry, and by tempering [mixture].

Furthermore: even granting that some power comes from the separate intelligences — if it is an agent by way of heat, then the proximate heat suffices, for it is of the same kind; and if it is something spiritual apart from heat, what need is there of the sphere, since nothing comes from the sphere but heat? And again: therefore the power of the separate intelligence is restricted to one [particular] thing, and would not generate many species. And again: if it generates a mouse, it has a power resembling the power of the seed in the mouse born from a drop — and if so, that is a material power, and inferior to it. As for me, I recognize no other generator than the one that prepares the matter, as we see that village folk generate the bees [?]. And Aristotle explained, in the fourth

book of *On the Soul* [*De Anima* IV / Physics], that the one who produces the form prepares the matter to receive it — for the final preparation of the matter is the beginning of coming-to-be; and it is impossible that for a single instant the matter be prepared and the form not rest upon it. The essential coming-to-be does not change through being acted upon. Therefore the one who gives the form is the one who prepares, and the one who prepares is the one who gives. But the heavens do not prepare the matter — for if they did, they would be generating; and if they generate human beings and horses and donkeys from putrefaction, then they are *not* generators [in the proper sense]. Aristotle, in the second [book] *On the Generation of Animals* [*De Generatione Animalium* II], chapter 1, said that in the matter of things generated from putrefaction there is found something analogous to the seed from which the perfect animals are born.

But as for those who say that an accident does not generate substances — in our view they [the products] are accidents; and if they are essential, then the whole school of the philosophers says that an accident, insofar as it is the instrument of a substance, generates substances. Now in Aristotle you will find two conditions for the instrumental cause, in order for it to suffice to bring forth an activity resembling the agent by way of another [intermediary] — in treatise 7 and 12 of the *Metaphysics* [*Metaphysics* VII and XII] and the first of the *Parts of Animals* [*De Partibus Animalium* I], chapter 1. He said, of blessed memory: the artificial [product] does not resemble the instrument but the craftsman — not insofar as he is a man, but insofar as in his imagination there is the form of the artificial thing. The second [condition] is imitation of the natural instrument; this is in the second book of *On the Generation of Animals*. He said that the seed is an instrument of the thing that separates from it; and it is explained that the seed is called an instrument

of the living being — not because it is set in motion by it continually like the saw [moved] by the carpenter, nor because it receives from it [?] a new power each time it acts. For after the death of the living being it [the seed] still acts; rather it is called an instrument insofar as it is part of the living being, and from it [comes] its activity. And since the activity is attributed to the living being, these [are] therefore instruments insofar as it is a "deaf" [latent/inert] power at every moment it acts. These are the two conditions; and those generated from putrefaction are found among the accidental qualities, or among [the products] of putrefaction, which indeed do generate ✻ what resembles them; they are not to be attributed [?] to the heavens or to the Active Intellect, but rather [are] like the substance whose instruments are heat and dryness — the instruments of fire generate fire and make the activity resemble the agent, for they generate fire,

which is a substance, while they themselves are accidents. What is from putrefaction, or in the earth, brings forth frogs and mandrakes [duda'im] by the power of the substance that serves as an instrument in the hand of the Maker [or: of the potter, ha-yotzer]. And let the generator be whoever it may be — there is no need to attribute these things to the separate intelligences, but only to proximate causes [?].

And if from elsewhere they bring a proof to verify their view — in truth all these are not [conclusive]. The true philosopher must not multiply existents without necessity arising from rational syllogism, in the manner of those who plunder every science, who bring a proof from things famed at the edge of the world, or from the tales of the multitude — and they were carpenters or builders or vintners or wine-treaders — who said that they saw with their own eyes; and the term "eye" is said of them only equivocally. And likewise the mathematical premises [are decreed?] [as if] they were images or souls, on the testimony of common, untrained men. But the great proof is, in truth, from the verses of the Torah — and also their interpretation is mostly in a vision [be-mar'eh], as the Rambam explained in chapter 42 of the second [part of the *Guide*]. Also the [divine] Name is derived from "work/craft" [melachah]: anything that performs His word is called so, whether it be a sickness, or a natural power, or a commandment, or a transgression, or a prophet's spirit, or [an angel], or a blazing fire, and so on.

Even so, we philosophize in the academies of the nations and in the batei medrash of the sages, and to glorify the intellect we call right what is left. But in the courts of the house of our God we trust in Scripture in our Torah, and we do not accept interpretations [contradicting it], for the Torah and the Scriptures testify concerning the angels. And in the view of the sages of truth, the Kabbalists, there are many kinds [of angels], and such is my opinion, like theirs. Now most often the word *Elohim* comes in place of "angel" — for a man's agent is like himself. And according to the views of some of the philosophizing commentators, the first *Elohim* of the Torah is such — that the angel is the creator of all these bodies, heaven and earth and their fullness; for from the simple [First Cause] there comes forth only one [effect]. According to this, the Torah does not relate the creation of the angels at all. And I have seen what the all-encompassing scholar R. Shemaryah the Cretan [ha-Ikriti], may his memory be a blessing, wrote: "Rabbi Moshe [Maimonides], of blessed memory, brought the words of R. Eliezer the Great in the *Guide* [*Sefer ha-Moreh*], and found them very difficult, until [Maimonides] said" — this is his language: "I have seen, of Rabbi Eliezer the Great, statements in the famous chapters known as *Pirkei de-Rabbi Eliezer* — I

have never seen stranger ones, etc." — up to "and a sufficient interpretation of these has not become clear to me." [So] said [Maimonides], of blessed memory; see further in the *Sefer ha-Moreh*. But I say that the words of R. Eliezer are [in fact] the words of the philosophers themselves — for they say that nothing was created by the necessarily-existent [me-chuyav ha-metzi'ut] without intermediary, but only one simple substance; and from it came forth a second angel and a first sphere; and from the second, a third angel and a second sphere; and from the third, a fourth angel and a third sphere; and from the fourth, a fifth angel and a fourth sphere; and from the fifth, a sixth angel and a fifth sphere; and from the sixth, a seventh angel and a sixth sphere; and from the seventh, an eighth angel and a seventh sphere; and from the eighth, a ninth angel and an eighth sphere; and from the ninth, a tenth angel and a ninth sphere; and from the tenth, the Active Intellect and the first matter. And because the view of R. Eliezer

is as we said — that is, that "the things were not all produced from Him without intermediary" — therefore he [the questioner in *Pirkei de-Rabbi Eliezer*] asked of the heavens: from where were they created? That is, were they created from the Name without intermediary, or by way of an intermediary? And he answered that they were not produced from the Name [directly], but from the light of His garment. The meaning is: "He makes His angels [spirits]" — and "honor and majesty does He wear" [Tehillim 104:1, 104:4]; these are the angels, and he likened them to His garment because of their nearness to Him, without any partition. And his intent in saying "from what thing were they created" is not to seek their material cause, but to seek their efficient cause. And because the matter is so, he therefore does not ask about the garment — for there is no intermediary between [Him and] the garment; and it was obvious to him that the garment is produced from the wearer. But concerning the heavens, between which and the Name there are intermediaries, he asked, to know whether they were produced from the Name or from the intermediaries; and he answered that they were produced from the intermediaries. He then asked about the four elements [yesodot] — whether they too were created from the light of His garment — and he said that they were created from the snow beneath the Throne of Glory, which is the first matter. These are the words of R. Eliezer, and they are neither strange nor confused, but pure and true — thus far his words.

It is known that such was also the view of Ben Sina [Avicenna], and Abu-Chamed [al-Ghazālī] brought it in [his book] *Intentions* [*Maqāṣid* / *Kavvanot*]. And the Rambam held that this was the view of Aristotle, who said that the First Intellect, in contemplating its cause, creates an angel, and in regarding itself creates a sphere; and likewise R. Asher [Ben Yechiel?] [?] who struck upon his crown [?] —

and the law is with him; for I do not know how depiction and intellection could generate the matters [bodies], and they [the philosophers] assert what they cannot depict.

And behold, all these sciences which they call divine [the metaphysical sciences] are built upon shaky foundations. For the prime matter [ha-hiyuli] is [mere] imagining and a notion of the wind [an empty conception]; and there are no [such] spheres and dwellings [orbital shells] — for he explained, in the *Meteorology* of my teacher [Moreh? / the *Meteora*] [?], that the air is what spreads out and rises, and there it is called *aithēr* [Greek: αἰθήρ, "ether"] for its fineness and translucence; and that the stars move by their nature in a circle, contrary to the [four] elements whose nature [is rectilinear]; and that when they are outside their place they all move in a straight line, this one up and that one down, until they reach their place. And there is no need for movers and separate intelligences. If so — away with this whole science! And how many pens are broken and how much ink is spilled over these disputations, while they call those who occupy themselves with such things "divine sages." My teacher the Rav [Maimonides?] used to be angry at those who called him so, and said: "Would that we might merit the title of natural philosophers — for God is in heaven, etc.!" And what have we to do, to build our beliefs upon opinions that change at every turn and season? It is good to walk to the house of God after the footsteps of the divine Kabbalists, the [spiritually] circumcised, who say that when it arose in thought before the Ein Sof [the Infinite], blessed be He, to be revealed and to emanate of His goodness, to create creations — His lightning-bolts lit up the world [Tehillim 97:4]. And immediately there spread forth ten [?] sparks of light from His thought, by the numerical value [count] of *Keter* [the Crown], which is the "thorn" [tip] of the *yod* of [the divine Name] *YHWH*. And it was still concealed, until the emanation [atzilut] continued and the glory of the Lord was revealed, and the thickness [fullness] of the *yod* appeared in the *sefirah* of *Chochmah* [Wisdom], and from there to *Binah* [Understanding], and so on, down to the lowest *sefirah*, which is *Malchut* [Kingship]. And behold, the *Malchut* of the world of *Atzilut* [Emanation] emanates upon the ten *sefirot* of the world of *Beri'ah* [Creation]; and likewise the *Malchut* of the world of *Beri'ah* emanates upon the ten *sefirot* of the world of *Yetzirah* [Formation]; and the *Malchut* of the world of *Yetzirah* emanates upon the ten *sefirot* of the world of *Asiyah* [Action]. And He wished [created?] the world of *Asiyah*, the lowest of the created things — that is to say, the four elements [yesodot] are composed from them, and among them the heavenly bodies; and all the bodies and the powers within them — such as the vegetative and the sensitive soul, and the qualities within them — all are in the world of

Asiyah and *Malchut* at their creation. The world of *Yetzirah* is that which comprises the souls, directors of the human bodies, and the souls — or

movers of the stars, or [the angels] appointed over them, which indeed are not material, but their occupation is with the corporeal. The world of *Beri'ah* is the world of the separate intelligences, which the Peripatetics [ba'alei ha-massa'iyim, the Aristotelians] did not comprehend; and they are entirely separate, having no concern with matter — neither with bodies nor with spheres. As for the world of *Atzilut*, it is a world "high above high, the most high above the most high" [cf. Koheles 5:7]. All of these — Daniel was likened to them in prophecy [?] — they grasped only the generalities of the world of *Asiyah*, and these are called *Ofanim*. And Ezekiel ascended as far as the world of *Yetzirah*, which is the world of the *Chayot* [living creatures]. And Isaiah split through with his intellect as far as the world of *Beri'ah*, which is the world of the *Serafim*; but he did not pass through its generality, and perhaps he attained as far as *Malchut*, which is the *Shechinah*, or *Tiferes*, which is the Holy One, blessed be He — this is what he said, "I saw the Lord" [Yeshayahu 6:1]; but he did not ascend as far as the *Keter* [Crown] of this world, as did Moshe our teacher, whose intellect rose to the heights, and they said that Metatron spoke through him. And this is what the Sages, of blessed memory, said: "Here — through the clear glass [aspaklaria]; here — through the unclear glass." And know that the *sefirot* are like oil [shemen]: just so are they luminous, for the supernal radiance [zohar] passes through the windows and splits through to [all] the worlds. And the more one is distant from the source of light, the darker [it grows]; but [the one who] cleaves to the Lord, who is its very essence — and not so the angel. And even Moshe did not attain anything of the world of *Atzilut*; "no eye has seen" [Yeshayahu 64:3]. And perhaps there are among them other worlds, and we do not know their number — for it is impossible that they be finite, as was explained in philosophy: that there being causes and effects without limit is false [i.e., an actual infinity is impossible]. My teacher the Rav was wondrous in speaking of this in his book *Bishmat* [?]. As for me, my feet had nearly slipped [Tehillim 73:2] from what I had intended to discuss; and I am ashamed to chirp [presume to speak] in these matters before a man great and awesome — especially in the wisdom of Kabbalah — such as you. But according to my own way and my innocence I have spoken; and do not hold me guilty of trespass-offerings if my words are "not mingled with oil" [not refined], as befits your standing, O You who are awesome in deeds [Tehillim 66:5].

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Again my teacher asked me about the questions which my teacher had asked of the Ishmaelite sage al-Faḥrī [?] [ha-Yishmael'i ha-Alkahari], "[Be it] upon me" [?] — and that he was asked them [in return]. I know that there was a great dispute between them in several matters [?]. And I have among my writings, which I deposited with my teacher, most of them — and perhaps all; and I have already sent a special man with a wagon and horse to bring me all my books and everything found in my possession; I shall divide [the work] up [?] with my colleague [?]. But the ten questions which my teacher was asked of him in the famous disputation are these, in brief:

Question 1 [שאלה א]. To resolve [solve] the oblique [scalene] triangle of three known angles, so that we may know the sides — without lowering a perpendicular [emud], that is, without [the need to] turn [it] toward the right angle of the angle [i.e., without dropping a perpendicular to form right angles]. And if you obtain my teacher's treatise on triangles, you will find several methods and various kinds.

Question 2 [שאלה ב]. How shall we know, by an exact mathematical computation, the dimension [area/quantity] of the regular figure [polygon] having eight sides [the regular octagon]? And behold, my teacher showed [the methods], exact for all the figures within a circle, in his book *Ya'ar ha-Levanon* [*The Forest of Lebanon*].

But for the answer of the [other] learned sage, he explained the matter in a new way, and posited that each side of the octagon is eight cubits of a known [measure]. Let the figure be [the octagon] **כלבגהוחט** [K-L-B-G-H-V-CH-T] having eight equal sides; and let its sides be drawn out, each one — **וח**, **בג**, **טה** [V-CH, B-G, T-H] — until they meet at the points **אדזי** [A-D-Z-Y], and let there be…

Diagram. A geometric construction for computing the area of a regular octagon (the answer to Question 2). The text introduces a regular octagon labeled with eight vertices and the construction of extended sides meeting at four outer points to form a circumscribing square. The figure itself falls at the degraded foot of the page and is not legibly rendered; the labels are recovered from the running prose. The construction is left mid-sentence at this page's end and continues onto the following page.

- כ, ל, ב, ג, ה, ו, ח, ט (vertices): the eight vertices of the regular octagon (named in the order כלבגהוחט in the text).
- ו–ח, ב–ג, ט–ה (sides): three of the octagon's sides, named as being extended outward.

- א, ד, ז, י (outer points): the four points at which the extended sides meet, forming the corners of the circumscribing square.

[continuation of שאלה ב — the regular-octagon area]

a figure built on right angles, easily demonstrable to any pupil — the learning [proposition] ל״ד [?] for the first and the sixth, for the fourth [book], of Euclid [לאוקלי״דס] (Plate 2 / לוח ב).

Now observe: the total of the three [triangles' areas], when subtracted from the sides of the equal-sided square, [yields] the sides of the octagon.

For every pair of lines drawn from the [outer] corners of the square equals two lines of its neighbor — that is, the lines of the legs גדה, לאב, כיט, and וזח are equal to one another. And since the side was set at 5 cubits — [the legs] are not equal to the lines זן [?], because they are parallel; and there are six [?]. There will be four right-angled triangles. The leg of [each of] these triangles is the square root of the number 32. As my teacher demonstrated to me through the science of algebra, the sum [of the two such legs] cannot but be the square root of the number 128; and when we add to it the line [forming] the whole side of the square, יא, the full number [results], compounded from the square root of the number 128 and from the square root of the number 64 [?]. This is a binomial [literally "possessor of two names"] — and the deaf man will not hear it. So too with each of the [other] sides of the square.

When we multiply side יא by side יז [?], that is, when we square side יא, we obtain the area of the said square — the sum produced from the square root of 32,768 and from the number 192. The "deaf man" will not open his mouth. And when we subtract from it the areas of the four triangles — which are the excess of the square's area over the octagon's area, namely the number 64 — there remains the square root of 32,768 together with the number 128. This is the measure of the area of the given figure. Just like a deaf man, he stops his ears — and this is what was sought.

שאלה ג (Question 3)

Alexander of Macedon [אלכסנדר המוקדוני = Alexander the Great] divided his army into twelve thirds, and the division ran as follows. The first [third] had a certain number of men; the second and the third together had double [that number]; the fourth and fifth together, 83 [ג"פ = three-times? no — see note] [?] times that number; the sixth and seventh, five times that number; the eighth and ninth, six times; the tenth and eleventh, seven times; up to the twelfth. The total of all his

men was 30234. We wish to know the number of men in each third by a demonstrative method, without straddling the two branches and without groping the way the blind grope.

My teacher quickly found several methods and various partitions according to the problem; but ʿAli [the opponent] did not want them, for he said they were not demonstrative, and he asked my teacher to show by demonstration the sum of the methods, so that the matter would be made clear — how the number is at once divided into whole parts according to what is sought and according to these conditions. This problem is deeper and harder than the casual student imagines. He [ʿAli] was boasting over me, [demanding] that I give the rationale for Diophantus's procedure — how he set all his answers in whole numbers and did not fall back upon fractions. This problem, too, belongs to one of the commentators on that very book; it was simple in ʿAli's eyes, yet, being sealed [to him], he did not wish to expound it. "Ask now of my teacher, and he will tell you the hidden things of wisdom."

שאלה ד (Question 4)

Let אבד be a right triangle, right-angled at ב, its area 24; it is divided by the line אג at point ב [?], and line דג [?]; and אג is the square root of the number 9 [?]. We wish [to find] the sides of triangle אבג, that is אב, בג.

Answer. Let line בג be the root, "א" [= x]. It is shown from [the work of] Moshe Metz, proposition 1, that if we subtract its square from the square of אב there remains line אב [?]; and we square the root "א," which is square ב [= x^2]; and we subtract it from אג; and there remains the square of אב — the square root of [the number] 61 less the square of "א" [i.e. $\sqrt{(61 - x^2)}$] [?]. And if we draw line אב like line פד according to the supposition, the area of the triangle comes out as 48 — its half being the area of the triangle.

But since we do not know the root, only the square of אב, here, by the ways of algebra, we must also first square line בד, which is x + 3, and its square is 9 + 6 roots + 1 square [i.e. $(x+3)^2 = x^2 + 6x + 9$]. Then we add this to the square of אב, and the result equals the number, the square of 48 — that is, 48 [?].

Now I must make known to you the notations of algebra that we use, so that you may know with ease everything that issues from these operations. The simple [constant] number is marked with פ; the root with א; the square with ב; the cube with ג; the square-of-square [fourth power] with ד; the square of the square [sixth power, i.e. cube-square] with [the next sign]; the cube [of the cube] with ז [?]; and

so on without end. These signs have more utility and ease than the notations of the ancients — you will see them in my teacher's books. The sign for addition my teacher marks with the letter ת, and subtraction with פ.

The algebraic working is set out in the side-columns (reading the symbol-strings left-to-right as printed):

ב 1 ת. א 6 ת. 9	[1x² + 6x + 9]
ב 1 פ. ס 0 ת. 61	[1x² − 0 + 61] [?]
ב 61 ת. א 366 ת. ס 549	[?]
ד 1 פ. ג 6 פ. ב 9 פ.	[?]
ד 1 פ. ג 6 פ. ב 52 ת. א 366 ת. ס 549	[?]
ד 1 פ. ג 6 פ. ב 25 ת. א 366 שוה ס 1755	[= equals 1755]

And so we found an equality between the number 1755 and 366 roots plus 52 squares less 6 cubes less 1 square-of-square. Now ʿAli himself did not know a general demonstrative rule for finding the root in equalities such as these — and this is among the precious matters in algebra, which the great ones of the world have longed for, yet found only particular results, not general ones. If God decrees that my teacher's books be published, he will show great lights. But my teacher understood the man's designs and schemes — that he wished to learn these stratagems — and therefore did not want to reveal them to him. Instead he brought out the root by way of trial [i.e. by approximation / false position], in this manner.

Let us suppose that בג, the root, is 3. Then its square is 9, its cube 27, and its square-of-square 81. We multiply these numbers [by the respective coefficients]:

עם ד 1 פ. ג 6 פ. ב 52 ת. א 366	[against the equation]
ונמצא	1323

We find 1323, whereas it ought to be 1755; and so we must take a larger root.

If we test with the number 4: it would still be deficient [?]. And with [a number giving] 6: it would be excessive. The upshot: the true root — that is, the measure of בג — is 5, as you see [from the following]:

step	value
שוה ה (א 366)	1830
שוה ח (ב 52)	1300
שוה פ (ג 6)	6·125 = 650 [recte: 750]
שוה פ (ד 1)	625
	1375.. 3130
נגרע [subtract]	1375
יותר [remainder]	1755

And so, by this, every side בד is 8 [= x + 3 = 5 + 3]; and by this, side [?אב =] וזה is 6 — which is what was meant.

[continued — top of next folio]

the square of "א" [= x^2], and we subtract it from אג, and there remains the square of אב, [equal to] the square root of the number 61 less the square of "א" [$\sqrt{(61 - x^2)}$]; and if we draw line אב like line פד according to the supposition, the area 48 comes out, whose half is the area of the triangle.

שאלה ה (Question 5)

Let there be a *sicur-i-cola* [סיקור־יקולא — Italian *scur-i-cola* / "little square-figure," a quadrilateral; precise Italian form uncertain] [?] (לוי״ך) [a garbled plate/figure label, printed exactly so; final kaf, not dalet] — in the figure אבזצ, whose base צץ [?] is 26 cubits, each of its [other] sides אצ, בז being 5, and its area 92. We wish [to find] the measure of אב.

This man wished to outwit my teacher, [claiming] that one would first need to make known whether line אב is small or large in the said figure — for otherwise the matter is in doubt. Let us suppose [the auxiliary line] גז [?]: its root is "א" [= x] [?] — let it be 1; and so בג, or א, equals 26 [?]. [Side-figure: א 1 פ. זד / 26 ; א 1 פ. 26.]

Let it be equal to the root √25 less [?] — [side-column working]:

ב 1 ת. א 52 פ. ם 676	זד מרובע [square]
ב 1 פ. א 0 פ. ם 25	מז מרובע [square]
ב 25 ת. א 1300 פ. ם 16900	

ה 52 ג. פ 676 ב. פ 1 ד.

תב 52 ג. פ 1 ד; אלמז שרש 26 ם. פ 1 דזא השטח [the area]
16900 ם. פ 1300 א. פ 651

[?] פחות שרש 1; 92. כמו כן 81 25

[The "deaf man" multiplication / verification block, set as columns:]

(right column)	(center)	(left column)
81 16900 פ. 3900 פ. 1404 ת. [?] 5859 פ. ת	92	
1404 ה	92	3900 פ
18304 ה	184	81 פ
9840 פ	828	9840 פ
	8464	

And below: **8464**.

And since side מז is a square whose root is not known [an irrational square], we square the side: [it is] 25 roots less [?] plus 1 square. We multiply this by מז, and the result equals the number 4 [recte: 8464], which is the square of the number 92. And behold, the extraction of the square root from this binomial — a long and great [labor] — the sages did not know it to this day. The great sage Rudolphus of Cologne [רודולפ"ום מקולוני"אה = Rudolff of Cologne (?), or Ludolph van Ceulen of Cologne (?)] [?], as my teacher knew from his books, would use a supposition [trial method], as I taught you in the preceding question, testing with various numbers.

There is a way of doing this so that you do not wander far afield and instead hit the mark of the estimate. You will at once find all these and other new matters — which no sage of heart has attained to this day — in my teacher's books. Now if you test with the number 2 you will find it deficient, and if you test with 4 you will find it excessive; and the true root — that is, the measure of בז — is 3. Accordingly the measure of line אב is 26 less two roots, that is 20. This is what you wished to know. I have already opened for you an entry to release what many honorable men cannot release; and you may infer the rest from the [algebraic] "names" [the symbolic powers] when you set the simple number by itself and the algebraic names by themselves. "To the discerning, knowledge comes easily" [Mishlei 14:6].

השאלה ו (Question 6)

From twenty [?] known unequal numbers we subtracted equal numbers, and the ratio of the remainders is as 8 to 2 [?]: how much are the equal [amounts] subtracted?

Answer. It is easy to know this by algebra. Let the numbers be, say, 46 and 19 [?]; and let us suppose the sum of the [amounts] subtracted to be "א" [= x, set] 1 — [side-columns follow]:

(right) 12 ם פ 1 א	[x − 12]
	3
(next) 36 ם פ 3 א	[3x − 36]
	ה 3 א
36 ם ה 1 א שוה	[equals 1x + 36 [?]]
	9 פ
	4 ם

We multiply each of the remainders by [the values that] make them equal, and the reason is clear — by the well-known difference [cross-multiplication], doubling each of the proportionals against the numbers that are to be made equal. We find that the root is 4 [?], which is what was sought.

But if you do not wish to use algebra: subtract the lesser from the greater [?], and the remainder will be the head of the proportion; the third [term] will be 4, namely חח or צג; and you will find the fourth in this way. [Diagram reference: לוח ה — Plate 5 (?).]

[Worked proportion, set as a triangle of figures:]

- If 1 gives 4, how many will you subtract? 3 [?] gives 12; and the remainder is 4 [?].
- אג ;4 מאצ [?]
- 4 דח / מדח [?]

First analogy. Let the two measures be 18 and 20 [?], and the ratio of the remainders is 5 to 2. The difference between the proportionals is 3, and the difference between the measures is 2; and the remaining parts are 9 [?]… infer likewise.

- 5. / אצ / אג / ⅓3 צג
- If 3 gives 2, how many… subtract… [?]
- 2. / דח / ⅓1 חח

And by algebra likewise [you find] the matter with ease: you find 50 equal to three roots [50 = 3x → x = 16⅔], and so the measure of the root is 16⅔.

But how do we subtract the number of the subtracted-amounts when they are unequal? My teacher also explained this in his reply to the said Ishmaelite [al-Faḥrī (?)], and the manner of the reckoning is hidden in my notebook, which I left with my teacher; I do not now recall it, [enough] to tell my master — "may it be forgiven your servant." But if you wish to know it by algebra: [side-columns] — we find the root to be 4, and two roots are 8; and the greater remainder is 25 and the lesser 20.

And you find the lesser of the subtracted to be 1, the greater [?] 4 [?]… and the greater remainder 31½ and the lesser 22½, whose ratio is as 7 to 5 [?]; and there is no need to lengthen.

[Side-columns for this instance — interleaved two-block (column-re-read candidate):]

(right block)	(left block)
12 ם פ 1 א	16 ם פ 1 א
2	3
36 ם פ 3 א	32 ם פ 2 א
ה 3 א	ה 3 א
36 ם	32 ם ה 1 א שה
32 ם	ם 32 ה [?]
4 ם	א שה ? [?]

Second analogy. Let the two measures be 24 and 36, and we subtracted unequal parts, and the ratio of the remainders is as [?] to 5 [?]. We wish to know the subtracted amounts. Let us posit them thus: let the lesser of the subtracted be "א" [x = 1] and the greater "א" [2x = 2] [?]. If you wish, posit the greater as x and the lesser as another value:

- יהן 1 אם [if 1 gives] 4: how many will you subtract from [?] 22 [?] בחר…; אב 4 מאשצי [?]

- 2 / 8 7 [?] מרח [?]

If 3 gives 2: how much shall we subtract from it? And by algebra likewise you will find the matter when you find 50 equal to three roots [→ x = 16⅔].

[Closing side-columns, foot of p. 66 — these mirror the verification arithmetic, products totaling 168 and a final equality "1 root equals 4, and two roots are 8":]

---

3 א	36 .פ 2 א	24 פ 1 א
	7	5
180 .פ 10 א	168 .פ 7 א	
10 א. ת	10 א. ה	
180 .ה 3 א שוה		168
	168	
	12	

8 שויון השרש 4 ושני שרשים 1 א

שאלה ז [Question 7]

Question 7. He said: "I put this to you — instruct me, wise craftsman and skilled reckoner: which is the number, less than one, such that if you add it to its cube three times over [ג"פ = three times], the sum equals one? And inform me further, in good time, of that number less than one such that when its cube is taken from its triple [i.e., from three times the number], one remains."

What I want here is not an approximate value, but the precise truth — for these are quantities of many kinds.

Answer. Let us posit the number of the first question to be one root [שרש = x], with its cube and its triple being equal to one [i.e., $3x + x^3 = 1$]. So there is an equation between [the cube of] 1ג [= x^3] and [the constant] [1 =] 1ם. And I say that the root is:

> the universal root of the cube-root of √(1¼) + ½, minus the universal root of the cube-root of √(1¼) − ½ —

that is to say: extract the square root of the number 1¼ and add to it the number ½; from this sum extract the cube root. Then, if you subtract from this the universal cube-root of √(1¼) − ½ — that is, you first extract a square root from the number 1¼, subtract ½ from it, and from the remainder extract the cube root —

and subtract this [second] root from the cube-root mentioned [first], the residue will be the root sought. Test it and you will see the truth.

Let me give another illustration in numbers spoken together [i.e., with concrete coefficients], for fuller clarity and verification of the matter — so it will be understood. For example: let

26 = 1ג פ 9א [i.e., 9x + x^3 = 26]

equal the number 26. Take half of 26, which is 13, and its square, 169. Add to it the constant term of the cube, from the three, which is a third of the number of roots [שליש מספר השרשים], namely 27, and it becomes 196. The root, accordingly, will be:

the universal cube-root of √196 + 13, minus the cube-root of √196 − 13.

That is: extract the square root of the number 196, which is 14, add to it 13, and it becomes 27; extract from it the cube root, which is 3. Likewise from the remainder, which is (minus the universal cube-root of √196 − 13): extract the square root, which is 14, subtract from it 13, and 1 remains, whose cube root is one. And if you subtract this root from the aforementioned cube root, which is 3, there remains 2 — and this is the value of the root sought. You see, then, that one cubed plus 9 roots [i.e. x^3 + 9x] — as if you were to say 8 plus 18 [= 2^3 + 9·2] — equals the number 26.

Its founder [מיסודו] is Niccolò Tartaglia [ניקולאו טרטליאה], who taught it to my teacher [למד מורי] when he passed away. The procedure [is his], and ✻ it remained for me to demonstrate it by proof, for "no man has known its worth to this day."

Further: I will instruct you in the straight method that my teacher showed me for extracting roots when the cube equals roots-and-a-number — though this [is needed] only because the ratio is not constant. For you see that the [exact value] here is the irrational [המרובע = surd]. Let us posit, for example, 1ג [= x^3] equal to ל 6א ח 40 [= 6 + 40x, i.e. x^3 = 6x + 40]. Take the number 40 and divide it ✻ by various numbers, and observe the numbers that come out, and join them with 6, which is the number of roots; and if I find that any sum of them is a square number, I rule [אגזור] that its root is the root sought. For example, if I join 10 with 6 it makes the square 16, and so 4, which is its root, is the root sought; and so its cube is 64, equal to six roots, which are 24, plus the number 40.

[Division table — read right-to-left in source; here logical pairs (divisor · quotient) of 40:]

Divisor of 40	Quotient
20	2
10	4
8	5
40	1

[The candidate worked above is divisor 10, quotient 4: 4 + 6 = 16 = 4², root 4; 4³ = 64 = 6·4 + 40 = 24 + 40.]

Second illustration: let 1ג [= x³] equal 48 14א פ [i.e., x³ = 48 − 14x... — see note]. Here I subtract 6 from the number [of roots]... and 8 remains, which is the cube root [the root sought].

[Division table for 48 — logical pairs (divisor · quotient):]

Divisor of 48	Quotient
24	2
12	4
6	8

Third illustration: 1ג [= x³] = 65 13א ח [i.e., x³ = 13x + 65].

Add 12 [to] the number of roots... with 13, and it becomes 25, and its root is 5 — this is the root sought.

[Division table for 65 — logical pairs (divisor · quotient):]

Divisor of 65	Quotient
13	5
65	1

Fourth illustration: 1ג [= x³] = 672 200א [i.e., x³ = 200x + 672].

Here, if you subtract 6 from the number [recte: subtract appropriately] — the roots 200 — there remains 144, whose root is 12; this is the cube root sought.

[Division table for 672 — logical pairs (divisor · quotient):]

Divisor of 672	Quotient
336	2
198 [?]	4

Divisor of 672	Quotient
84	8
67⅕	10
56	12
48	14

But as to what was asked of my teacher — that he make known to him, by approximation, the root when the equation is between 1ג [x^3] and 13א פ [i.e., $x^3 = 3x + 1$; the original Question-7 case]: here, according to the instruction he taught me, the number less than one was found to lie between $^{347}⁄_{1000}$ — which is too small — and $^{348}⁄_{1000}$ — which is too large. Test them and observe how close they are to the truth. And if one wishes to refine without limit, there is a straight, pure method for him to reach his goal swiftly and easily — and not the method of searching [trial], nor will the study called *al-khaṭāʾayn* in Arabic [the "method of double false position," الخطأين] help, as it does with squares and cubes, as experience will teach you.

And as to permitting the root even to be a whole number [an integer], there is yet a root belonging to the question, and it lies between $^{15}⁄_{10}$ and $^{16}⁄_{400}$ [?] — or, with greater refinement, between $1^{53}⁄_{100}$ and $1^{54}⁄_{100}$, or between $1^{532}⁄_{1300}$ [?] and $1^{533}⁄_{1000}$; and among the numbers less than one, the root lies between $1^{879}⁄_{1000}$ [?] and $^{879}⁄_{1000}$ פ [?]. I was not permitted to speak further, for these things were transmitted to me as a secret by the aforementioned teacher; and he will quench your thirst with raindrops [ברביבים], for his spirit is the spirit of nobles [נדיבים] — and what need have you of stagnant waters?

שאלה ח [Question 8]

Question 8. Instruct me in the extraction of the root [i.e., the unknown] —

[Seven cases of equation-types — read right-to-left in source; the equation forms with their numeric data:]

Case	Equation (as printed)	Right side
1	when whole numbers are joined with fractions: $55^{11}⁄_{18} = 8½$ 1א + [ת] ח [i.e., $8½ + x = 55^{11}⁄_{18}$]	✻
2	32½ = 10½ 1ב + [ת] ח [i.e., $10½ + x^2 = 32½$]	
3	when the equation is between a square plus roots and simple numbers: $57⁄_{27}$ = 1ג + ח 6א [recte 1ב?	

Case	Equation (as printed)	Right side
	i.e., $6x + x^2 = 57\frac{1}{27}$]	
4	and likewise between a cube plus roots and numbers: $103\frac{19}{27}$ = 1ג + ח 6ב [i.e., $6x^2 + x^3 = 103\frac{19}{27}$]	
5	and likewise between a cube plus squares and numbers: $90\frac{10}{27}$ = 1ג + ת 3ב ח 6א [i.e., $x^3 + 3x^2 + 6x = 90\frac{10}{27}$]	
6	and likewise between a square-square [x^4] plus squares: 1ד ת 250ג [i.e., $x^4 + 250x^3$...]	
7	and likewise between square-squares plus roots and numbers: $8568\frac{536}{625}$ = [1ד +] ת 25ב ח 5א [i.e., $x^4 + 25x^2 + 5x = 8568\frac{536}{625}$]	

And likewise between a square-square plus cubes plus squares plus roots, and numbers.

The illustrations [worked examples] he gave me for them are these.

[Illustration 1.] Now in the first illustration my teacher's method is this. We resolve the denominator of the fraction into a square: that is, we multiply it by some number until it is a square. In our example $^{55}11\frac{11}{18}$ we multiply 18 by 2, and it becomes the square 36. Likewise we multiply the numerator 11 by that same number 2, and it becomes $22\frac{22}{36}$; and with the whole [number], when added to the fraction, the whole becomes $2002\frac{22}{36}$. We take the root of the square, which is 6, and multiply it by the number of roots, which is 8½, and it becomes 51. And now you must extract from the number 2002 the root

[continues — Illustration 1, p. 69:]

of the square, but leaving over from it just so much that you can subtract from it the number arising from the doubling [recte: the product] of the root found, multiplied by the number 51. You must find this by searching, just as when you test in division to know how many times the divisor goes into the dividend — for there is no other way for these precious matters. And so I searched and found that it is the number 26, whose square is 676; and if you multiply 26 by 51 you get 1326, and join it with 676, and it becomes 2002. This number of 26 is the deficient one [is the residue], according to your having multiplied the number of roots by the number 6. Therefore resolve 26 into whole units, and it becomes 4⅓ — and this is the measure of the root sought.

[Illustration 2 — second illustration]: We resolve the denominator of the fraction of the simple number, which is ½, into a square — that is, we multiply it by 8 and it becomes 4; and the denominator becomes [recte: numerator] ¾%.. that is, ³⁰⁄₄; and with the whole [number], when added, it becomes 21%.. [21]. We take the root of the denominator [recte: the square], which is 2, and multiply by it the roots, which are 10½, and they become 21. And now you must extract from the number 130 the side [צלע = root] of the square, but in this leaving over just so much that you can subtract from it what arises from multiplying the root found by 21; and by searching you will find no other than 5, for its product with 21 is 105, and with its square 25 they amount to the number 130. Divide it by 2 — by which you multiplied the roots — and 2½ comes out: this is the root sought.

Moshe [Metz] said [אמר משה]: Had we not needed this study except for [problems] resembling the calculations in which a square plus roots equals a simple number, then — without searching, by the way of [algebra] — we would arrive at the root sought easily. Take this second example of ours: we sought a number such that its square plus its double-by-1 [recte: plus its product] would rise to the number 130. Posit it to be one root [x], whose square is x^2, and whose double-by-21 is 21x. There is then an equation between $21x + x^2$ and 130. And according to the teachings of *al-jabr* [האלגיב"רה], by one of the two methods my teacher explained in *Sefer Sechar Yosef* [ס' שכר יוסף], we extract the root of the square thus: we transpose the terms until we reach

> 130מ פ 21א ל = 1ב [i.e., $x^2 = 21x$... — see note; the standard completion $x^2 + 21x = 130$]

Now the square of 10½ (half the roots) is 110¼ [printed: 441¼ = 110¼], and the number 130 is ⁵²⁰⁄₄; their sum is ⁹⁶¹⁄₄, whose square root is ¹⁵·½ — that is, 15½. Subtract from this half the roots, 10½, and 5 remains — this is the root sought. And after you have found it, divide it by 2, since at the outset you multiplied them by 2 — and it is very easy to understand.

But sometimes we arrive at equations where the manner of extracting the root is not known by the customary way — as is the third example, where you reach an equation between 1ג [x^3] and 57½₂₇מ תא6 [i.e., $x^3 = 6x + 57\frac{1}{27}$]. The ancients labored to find a way of extracting the root, as they did when the equation was between a square and roots-with-numbers, but they could not, and they wasted precious time — until there arose the new and enclosed man [recte: the wondrous man], **Francesco Viète the Frenchman** [פרנצי"שקו ויאי"טה הצרפתי / François Viète], a man great in the mathematical studies, who invented this method for finding the root at one stroke: such that when it is raised by some

number it arrives at the sum of the given number. By his system the simple number stands alone on one side, and all the other numerical powers stand each by itself.

Now, the method of finding the root in the third illustration is in this manner. The equation is thus: 57 1/27 = 1ג 6א ת ך1 [recte 1ג; i.e., $x^3 = 6x + 57\frac{1}{27}$]. First we resolve the whole numbers into the fraction — for there is no need to bring the fraction to a cube, since its denominator 27 is [already] a cube whose root is 3 — and it becomes, accordingly, $^{1540}/_{27}$.

[Continues onto p. 70:] Take its [cube] root, which is 3; and its quadruple [recte: ✻ and $3^3 = 27$], and 27 multiplied by [recte:...] — the number whose cube root, when raised by the number 54, can be subtracted... is 10, whose cube is 1000, and multiplied by 64 gives 540[0?], whose ✻... is 1540. And you must divide this root by 3, which is the root of 27, and 3⅓ comes out — and this is the root longed for [הנכסף].

And I will give you a second example for further clarification. Let the equation be between א [recte $1ג = x^3$] 1 […] ת [...] and the number 57 [recte 147 — see working]; and in spoken numbers [concrete values] you will find the cube-companion [the cube], whose number is 2, its square being 4. Resolve the whole numbers into fractions, and they become 8; multiply by the square of 2, the root of the denominator — that is, by 4, the number of roots, which are 10 — and they become 40; and accordingly there is an equation between ה 40א [recte: 40x] and the number 147[?]. Now you must find a number whose cube, together with its product by 40, rises to the number 147; you search and find that it is 1, whose cube is 27 [recte: whose cube... — see note], and its product by 40 is 120, and these amount to 147. This number, divided by the cube root, which is 2, gives 1½ — and this is the cube root sought.

Illustration four [דמיון הד']: 1ג ח 6ב = $103^{19}/_{27}$ [i.e., $x^3 + 6x^2 = 103^{19}/_{27}$]. Resolve the whole number into fractions, and they become $^{2500}/_{27}$; and the number of squares is 6, multiplied by [...] the cube [is...]; the equation accordingly is thus: א ל 6ב ת [?]3[?] = [...] [i.e., $x^3 + 6x^2 = 2500/_{27}$]. And we must extract the cube root of this number — but it suffices to extract from it also the product [recte: ✻] of the root by the number of roots, 18; and you will find it is the number ten [recte: the number is such that] — with the cube root 3 — there comes out 3⅓, which is the root sought.

Illustration five [דמיון הה']: 1ג ת 3ב ת 6א = $90^{10}/_{27}$ [i.e., $x^3 + 3x^2 + 6x = 90^{10}/_{27}$]. Resolve the number likewise into its fractions, and it becomes $^{2440}/_{27}$ [recte — see

note]. Likewise we multiply the square of the cube root of the denominator, which is 9, by the roots 6, and they become 54; and with the root 3 we multiply the squares, which are 3, and it is 9. The equation is thus:

1ך ת 9 ב ה 54א כן [recte 1ג] 2440 שוה ל [i.e., $x^3 + 9x^2 + 54x = 2440$]

Now you must find a number whose cube, plus its squared-product by the number 9 (the squares), plus its product by the number 4 [recte: ✻] (the roots) [recte: by 54], rises to the number 2440 — and it is 10. Divide it by the cube root 3, and 3⅓ comes out — this is the one longed for. And here I set before you the entire order of the operation:

[Operation layout for illustration five — the source prints it as a two-block column tableau; reconstructed:]

ב [x^2]	**א** [x]
3	9
$90^{1}⁄_{27}$ = 6א ח ה 3ב ת 1ג	(the equation)
9	
3	
2440	**2440**

[The worked verification continued from the preceding folio: the value being tested is x = 10, and the equation has the form **א + ...·ב²** + **constant**. The columns line up the cube, its multiple by 54 (= א), its square multiplied by 100, etc.]

Term	Coefficient / operation	Value
0	its cube (מעוקבו)	1000
[54×] א	and multiply it by 54 א	540
	and multiply its square by 100	(—)
9 ב	with 9 ב	900

Their sum is 2440 [...] equal to **א 54 ת. ב 5 ת. ג 1** [i.e. $54x + 5x^2 + x^3$].

The sixth illustration is as follows: **א 5 ת. ב 25 ת. ג 250 ת. ד 1** [i.e. $5x + 25x^2 + 250x^3 + x^4$] is equal to the number **8568 $^{536}⁄_{625}$**. Arrange the procedure thus: extract the square-square root [fourth root] of the denominator of the fraction — it is **5**,

and its square is **25** and its cube is **125** — so the denominator is then the square-square number **625**.

[Marginal/inline note: I speak here only of expressible (rational) numbers, since here we have dealt only with expressible numbers; irrational numbers we shall treat elsewhere.]

Now we resolve the number into its fractions, and it becomes **5355536⁄625**. Then we multiply the number of roots [the coefficient of x] — **5** — by **125**, the cube of this square-square root, which is **5**, and we get **625 א**. Likewise we multiply the number of squares [coefficient of x^2], which is **25**, by **25**, the square of this root **5**, and we likewise get **625 ב**. Likewise we multiply the number of cubes [coefficient of x^3], **250**, by the root **5**, and we get **1250 ג**. The equation accordingly takes this form:

1 ד ת. 1250 ג ת. 625 ב ת. 625 א is equal to the number **5355536**.

Now we must extract from this number the square-square root [fourth root]; and there will be a number from which is subtracted what results from multiplying this root by **625 א**, by **625 ב**, and by **1250 ג**; and when you carry out the subtraction you will find the number, which is **16**. The order of the procedure is set out before you according to the rule — look and do as in its pattern. This root [16] you must divide by **5**, the [square-square] root, and the result is **5 1⁄5** [= 3⅕], which is the root sought.

These matters the sage [Viète] wrote upon his tablet — for the words of **ויאיטה [Viète]** are sealed and many find his presence forbidding; men have not understood the ground on which his arguments are founded, yet today his hidden things have gushed forth, in the words of my teacher. And here it is for you:

5 25 125 536⁄625 8568 is equal to 1 ד ת. 250 ג ת. 25 ב ת. 5 א

[Heading row of the worked tableau, in source right-to-left:]

ד	ג	ב	א
5355536			

[The second worked table — the verification of x = 16 against x^4 +...; source page p. 71]

Operation	Term	Value
6	and the square of its square [16^4]	65536
and multiply it [16] by the number	625 א	10000
and multiply its square [256] by the number	625 ב	160000
and multiply its cube [4096] by the number	1250 ג	5120000

And the total is 5355536.

Question 9 (השאלה הט') — on dividing an angle into three equal parts, and the marking [recording] of the [...]. This is the fourth, the ninth, and the eleventh [problem], and so forth, with [...] **בעגול [in a circle]**. On this matter **רלב"ג [Ralbag / Gersonides]** wrote, and it is wholly within his power [an accomplished treatment].

Question 10 (השאלה הי') — on squaring the circle and on doubling the cube. These are matters that all the disciplines have wearied themselves to attain. To approach the truth, one needs premises and intermediate constructions. To these questions my teacher gave such answers as were possible, drawn from the books of the [earlier] authors — for solutions to these problems have not been given to those who learn them down to the present day. As for one who has [tried] to divide an angle into equal angles, or to find a mean proportional between two given lines, he was compelled to introduce motion — as did Heron [אירונים = Heron of Alexandria?[?]], Apollonius [אפולוניאום], and Pappus [פאפוס] and others like them; they keep stepping back and forth upon branching curves, testing with the compass until they arrive at the intended result.

And since the absolute truth has not been ascertained, it is fitting to choose the shortest and clearest path, and not to draw matters out in order to split hairs to a hair's breadth. So too with Solomon's [molten] "Sea" — Scripture was not exact when it said that its circumference was three times its diameter [i.e. $\pi \approx 3$] (cf. *Melachim I* 7:23); and even if Archimedes knew how to approximate it more closely, saying that it is as **7 to 22** (or, according to others, as **113 to 355**), how could Solomon in his wisdom not have known this, or Bezalel [בצלאל] in his craft? Rather, they simply did not trouble over such precision, because of its slight usefulness — and all the more so, since my teacher was a still wiser sage.

[The narrative now turns to a series of requests put to Delmedigo by "the prince" / nobleman, recounted by his disciple Moshe Metz.]

[The prince] further pressed him with his stratagems, since he turned back and importuned and pressed him hard with his questions; and he [my teacher] did not draw his right hand back, nor did he rest [silent], nor did he hang the earth upon nothingness (cf. *Iyov* 26:7) — he set his face toward every sound counsel and scheme.

(ג) [Third request.] He further asked me about the matter of the boy [the prophesying youth] of **מגדירינג [Magdeburg?[?]]**, may the Lord's hand be a speaking tongue [?]. For on this account we travelled with my teacher more than forty parasangs, for all the people of Poland and Germany [פולוניאה ועפהם[?]], and with them several thousands of Israel, went down to see him. And in the house of the seer there was a [great] crowd; and as soon as he saw him, my teacher knew — for his father had need of [managing] him — and he perceived the trick: that [the father] was signaling the letters to him. The letters were arranged for him by certain [signs] [קש[?]], pointing to him with [...]. For aleph he would say to him "**0**" [?]; "Ashkenaz, thus you shall rise" — "a good roof," and "over this, uprightness," and so forth. And the questioner [the boy] had to put the question to his father, and the father would prompt him to answer, placing his palms [in the sign of] the letters together with the aforementioned designations; for [the boy] would utter the letters without [vowel-]movements or points, and his father would interpret and vocalize them. And we marvelled at the great multitude — how their hearts did not turn to the man's deceptions. And as soon as we sent for the boy's master to come and stand in the father's place, the boy could not give a proper answer at all.

We further questioned [the boy] about a matter that he had told secretly to his father, and he gave no fitting answer. We then explained and taught the designations and signs of all the letters, and the trick became clear to the whole congregation as the sun stood still in Gibeon and the moon in the valley [of Aijalon] (cf. *Yehoshua* 10:12–13); and had not my teacher revealed and exposed the matter, they would have gone on flocking to him from forty or fifty parasangs. To the father they did much good, for they would multiply gifts to him from all who came; and gentiles too streamed to him, and said that this was the savior of Israel, and the report was heard in the king's house, and they kept silent [...] divided opinions — blessed is He who redeems and delivers. And when the matter became known to the leaders of the generation and the heads of the states, they placed the boy's father under ban for [having] given the child over to

harlotry [deceit]; and the boy too died a few days later — I do not know what became of him. And yet some of the storytellers still cling to their error, to cover their shame, that they did not look into the man's deceits; and they remain astonished, and astonish others who hear them. And such are the ways of all who hasten to believe the tales of the multitude. And from this one [case] you may infer about everything said concerning the prophecies foretold by the mouths of foolish women, and all the deeds and nocturnal occurrences — they are all foolishness. And there is no doubt with me that **הרשב"א [the Rashba]** in his responsa, and **הר[מב"ם][?]** [Ramban?[?]] in his *Milchamot*, wrote from hearsay and from common report — which are not as things directly perceived; and perhaps they too were misled, like several of the great ones of Poland who saw [these things] and were terrified and rushed in alarm to call it wondrous.

For my teacher did not believe the voice of the report; only in order to satisfy my desire did he go to see him — and he found [Aramaic: חדא פלפלתא חריפתא ממלא צנא קרי — "one sharp peppercorn fills a whole basketful of pumpkins," i.e. one clever man among a crowd of fools] [Aramaic proverb, cf. Talmud]. Many were the spectators, and few the discerning investigators; and it sorely grieved us that we had sought to know strange [hidden] things befitting philosophers, and they turned out to be the doings of women and demons.

(ד) [Fourth request.] The prince further asked me to inform him of some matters he desired concerning the natural secrets and the operations of alchemy that I had learned from my teacher. You asked me about the extraction of silver from quicksilver [mercury] and from the moon [silver], and the transmutation of iron into steel [השט"אל = steel] and into copper, and of cinnabar [הציניפריאו[?] = cinnabar / vermilion] into choice silver; and about adding color to gold and weight [whitening?] to silver [to the moon], and the congealing of [an invisible] writing, and how the mixed metals may be separated, and how we may draw one of them off to the surface while the second hides in the dross; and how we may carry the precious metals — such as gold and silver — in vessels, dipping them beforehand into oil or water, and how we may convey them on a distant road so that onlookers take no notice of them; and to soften glass like wax in fire; and other matters belonging to the precious medical arts that benefit grave illnesses — such as for the cure of cramp [convulsion], hemorrhoids [הטחר[?]], epilepsy, the stone [calculus], and the like — in which the masters of the Hermetic art [בעלי כת הירמיש = the Hermeticists] take pride, and at very great length to recount their praises, as has been tested, and so forth.

My masters and excellencies! The very definition of a "secret" is a thing wondrous or useful that is not widely known — as if you were to say: to turn salt water into sweet; and to preserve gunpowder so that even if fire touches it, it will not ignite; and to keep wine from turning to vinegar; and lemon juice so that it does not spoil; and to make a very fragrant soap that shaves off the hairs like a razor when one washes with it as with water; and to remove stains from every kind of garment; and other matters without end, which are not known to all the world but only to a few, and are very useful — yet were they made known to the multitude, they would be reckoned as nothing, and the one who teaches their making would be like one who teaches the boiling of meat with onions, of which Ibn Rushd [בן ראשד / Averroes] wrote in his collection [ילקוט] the manner of its preparation. And how awe-inspiring are the works of mirrors and of glass — through which one looks far off — and of printing, and of gunpowder, and many things that have been revealed to the moderns which the ancients did not know! And in earlier times they were dread secrets; and perhaps several of them were stored away with Solomon, who in his wisdom did not publish them to strangers and to the young — lest it be like casting pearls before swine.

Therefore I have said: I will guard my mouth with a muzzle (cf. *Tehillim* 39:2) — but this once I will inform you of a writing of secrets, "which is stolen with me" (cf. *Bereishis* 31:39), a [system] composed in fragrance, and it is among the rare and precious branches of learning, by which you may signal anything you wish in conversation with any acquaintance you choose, things that mouth cannot speak nor ear hear. But take heed: do not reveal it to any creature, and let no stranger have dominion over it — for it is "a sealed fountain" (cf. *Shir HaShirim* 4:12); for once you train yourself in it, and learn to compose and decompose — that is, to write and to read — I will make known to you the wonders of the Lord in natural and divine matters.

This secret was kept hidden with the keeper of the secrets of the Emperor Rudolf [קיפר רורולפוס = Kaiser Rudolf], may his honor be exalted [תמ"ך[?]], and was revealed to my teacher; and this is how it works. This cipher [הסותר = the secret-cipher] rests upon two adjacent [parallel] tables ([Heb. abbreviation: מעל"ד[?]]); and one of them is nothing but two distinct alphabets — one in Polish script and the other in Spanish or Italian script — two distinct hands, however you wish; and you assign the first kind to the letter **א** and the second [...] to the letter **ב**. Suppose, for example [ד"מ = for instance], you wish to write to those who know the secret, in cipher, the words "**ברך דודי [Bless, my beloved]**" [?]; and since these words have seven letters, you must

write **35** [actually 5 per letter] letters [symbols] for each letter — five [per letter] — and behold, I now accustom [train] you in how to do it. Widen the spacing of the words, and beneath each letter place the [code-]letters belonging to it from the first table, [thus]:

[Sample plaintext letters with their codes beneath — the heading row reads, in source right-to-left:]

ב. ר. ח. ד. ו. ד. י [the plaintext letters being encoded[?] — partly illegible]

[The 5-symbol codes written beneath, in source order:]

אבאאב | אאאבב[?] | אאאבב | אאאכב | בכאאא | אאאאב

And behold, here I have written in cipher the two [seven] aforementioned words; but it is easy to disclose to those skilled in the craft of deciphering. Therefore one must make **a cipher upon the said cipher** [a cipher of a cipher], and you will be able to write whatever you wish in this manner. For all the [letters] you must take the [code of the] second [table's] two assigned letters; for **א** [substitute] the [code-]letters assigned to aleph, and for every **ב** the [code-]letters assigned to bet [from the first table again]. And here it is for you:

[A second-order sample, in source order:]

הללו פק ס' כל נויס טגחוהו כל הלופים כי גבל [a verse-like cover-text whose letters carry the hidden message[?]]

[Encoded beneath:]

אאאא | בב | אא | אאאב | בכאאאב | בא | אבאבאא | אב | באב | אאב

And it is easy to understand. And behold, this is the sign of the covenant between me and you — that whenever you see in my writings different [varying] letters [scripts], you will know that within them is [a hidden meaning] from the house [within].

And I have already gone out from the discipline [genre] of the letter [epistle], lest I become a burden upon the prince. May [my master], a wise man, prolong [his] days in the midst of Israel as his exalted soul [desires] — [my teacher] who makes my mouth wise, and the soul of the young man who is [skilled] in the studies — [signed with affection].

[Concluding dedication / colophon:]

To a high and lofty mountain — there [shall be] a straight path and highway — the one who stands at the banner of understanding, the investigator of hidden things and concealed matters, the Torah sage, the divine philosopher, my master and teacher, our honored master and rabbi **Yosef Shlomo HaRofeh [the Physician]**, son of the great rabbi, the wondrous Gaon, Master **Eliyahu Delmedigo [דילמידיגו]**, may [the Lord] guard and preserve him.

Moshe, son of the magnate, the chief [האלוף] Kalonymus Meir Metz [כמ"ר מאיר מיץ], may his Rock keep and preserve him.

[A request put to me [Moshe Metz] by the astronomer [הכחב = the star-gazer] of the holy community of **Vilna [ווילנא]**, may it be built and established.]

The two cipher tables (each Hebrew letter → a 5-symbol binary code of א and ב; source p. 74; the tables read right-to-left in the source, "first table" innermost-right, "second table" to its left). The system is a binary (Bacon-style) substitution: 5 positions, each א or ב.

First table (לוח ראשון) — plaintext letter → 5-symbol code:

Letter	Code
א	אאאאא
ב	אאאאב
ג	אאאבא
ד	אאאבב
ה	אאבאא
ו	אאבאב
ז	אאבבא
ח	אאבבב
ט	אבאאא
י	אבאאב
כ	אבאבא
ל	אבאבב
מ	אבבאא
נ	אבבאב
ס	אבבבא
ע	אבבבב
פ	באאאא
צ	באאאב
ק	באאבא

Letter	Code
ר	באאבב
ש	באבאא
ת	באבאב

Second table (לוח שני) — the same alphabet keyed in parallel, the code shifted by one position (each plaintext letter aligned one row down relative to the first table: א keyed to ב, ב keyed to ג, etc.), used to form the "cipher upon the cipher." Its code column reproduces the same 5-symbol א/ב strings displaced by one row. [The two narrow inner sub-columns give the running alphabet in two distinct hands — one per the א-script, one per the ב-script — keyed to the same codes.]

And this is the wording of the letter

[The opening of this letter is an elaborate rhymed acrostic in florid melitzah, praising the addressee — Yosef Shlomo Delmedigo — through a sustained metaphor of a garden, an orchard, and a cedar of Lebanon watered by living streams. The Hebrew is heavily ornamental and built on biblical allusion; the rendering below preserves the imagery and argument while smoothing the rhyme.]

A cedar in Lebanon, fair of branch and dense of foliage, giving shade, lofty in stature — [a man whose mind is like] the depths of the deep, [overflowing with] secret wisdoms, [the most] sealed and concealed, [yet] grown into wellsprings of insight: hidden, stored, treasured, exalted. Your rivers run round about your planting, to water [the gardens]; the sprouts of your delight, and your channels, run out over the waters of stillness to every tree in your heavens. Your villages, with your watercourses, give the fruit of your terraces to your students who await your overflow [?]. Your fruit is the fruit of saving life, and its leaf is for healing — for your waters issue from the Sanctuary, and they flow forth; the streams of your river make your possession and your produce gush from Eden. They are the garden of the living God, [given] to perfect [?] the pleasure of souls after death.

To Yosef, who is the holy ruler of the Lord, the first of the produce of the testimony of the religion, the instruction of the assembly of the Torah [?]: the beginning of Israel, my master and my teacher, the investigator, the honored and learned Rabbi Yosef Shlomo Delmedigo the physician, of Candia — may his Rock keep him and grant him life. Peace to my master, and peace to his Torah, exalted and lofty; may it be raised up and lifted high and grow very strong, according to

his will, and according to the will of him who glories in your instruction, who clings to your overflow [?] — who longs for your fellowship.

Coming now to the seal and signature of this letter [i.e., its substance]: at the sound of the great tumult from a far land — the throngs of desire that surrounded me, the camps of longing that encircled me, yes, encompassed me — these are what pressed me, urged me, hastened me, made me run, and roused me from my sleep, saying: "Take up your weapons, your quiver and your bow; go up to the salvation of the chariots [?], to take refuge in the shadow of the wing of a Jewish man, and become his nursling and his delight." For your holy soul was stirred to bind me with the cords of your love, to harness me with the bonds [?] of your longing, [to set me] over the charge of your service, to crown me upon the head of my chief [?], to bring me into your inner chamber of instruction, to receive me into the tradition of your covenant from the nobility of your wisdom, to pour out upon me from the overflow of your understanding — that I might be as one of the people of your house, coming and going in peace. "How precious are your companions to me, O God; how vast are their chief points!" [cf. Tehillim 139:17]. Afterward — their palms toward the earth, Selah.

Yet how my heart smites me, and how my eyes flow! For in my poverty my day is hard upon me, and I have no balm with me; for time, the betrayer, twists about with its devices, drawing its bow against me like an enemy and setting me as a target for the arrows of passing afflictions, and like [?] it will not let me bring my spirit back to its dwelling [?]. My violence and my plea are upon it [?], for my soul yearns and there is no help; salvation has grown far from me, until [time] decreed that I unyoke the team of separation and depart from my place of dwelling, in order to keep my soul alive — far [?] from [these] affairs — and my thoughts were among the people of my fellowship; and they went out to my aid. And since I have not yet merited to enjoy further the table of the High One, to hear your words and to take delight in your warm instruction and in your wisdom, clear and enlightening to the eyes — would that I had a heart [?] — that he might send me, from the corner [?] of the earth, a writing of truth, set out plainly, and a sound understanding of his answers.

[The writer continues:] I remain silent, [waiting] to know what my master will command concerning the path of study in which I shall persevere — as he [promised] to open my ear before my departure from my master's house. The good tidings of hearing of your health — when it comes to me — and of your welfare, when it is revealed to my ear, and the work of your fingers when it is shown to my eyes: then our mouth will be filled with laughter and our tongue

with song. My master will have done greatly with us, and we shall rejoice. And since I do not know the way of rhetoric or the rule of fine speech, I have set a guard for my mouth; I shall keep watch over my poor lips [?] and be silent, and I shall not draw near to you, but from afar I will bow and prostrate myself like a servant before his master — keeping your charge, bearing the vessels of

a thing that has no substance [?]. Your praise — pierced through with the awl of your love [an allusion to the bondservant of Shemos 21:6], upon the altar of wisdom — is it not, in your book [?], [made] to sense [?] the joy of a thing that has no substance? [The closing of this passage is obscure.]

The younger one, Yoshiyahu son of my father and teacher, my master and teacher Rabbi Yehudah, may his memory be a blessing.

Written on the 23rd of Sivan, [5]384 (1624 = שפ"ד CE).

Response from R. Zerach Yashar [מהדר"ז ישר — i.e. Moreinu HaRav Zerach (ben Nathan) Yashar] to the aforementioned sage

[This response is the opening of the treatise the author names below, *Ner Elohim*. It is a learned letter on epistemology — the theory of demonstrative proof, the senses, and the operations of the intellect, drawing on Aristotle, Themistius, and Boethius.]

You requested of me that I guide you in the way of truth and teach you to fear the Lord — the honored and awesome One — and you wrote, [as it is said,] "that I might know, to recognize my soul and my Creator," and so forth (cf. Tehillim 109:1 [?]): "Make known to me your way, and show me your glory." For our master Moshe [Rabbeinu] sought to know, by a complete demonstration, the cause of every thing, in the manner of the perfect. But concerning the matter of His existence, may He be exalted — that is impossible, and the answer is: "You cannot see My face" (Shemos 33:20). That is to say, it is impossible that the existence of the Lord be made clear by a cutting and complete demonstration — the kind in whose syllogism the middle term is the definition of the subject. For that species of demonstration requires the prior knowledge of the quiddity and essence of the subject; whereas the [thing] designated by the [divine] Name is not [knowable], as with the other species of demonstration. And concerning Him, may He be exalted, it is impossible to know His quiddity — for the quiddity is grasped only by way of a definition, and a definition is composed of a proximate

genus and a specific differentia, which are not [present] in Him, may His Name be exalted.

Even so, it availed [Moshe] that he should know the Lord, may He be exalted, [for] every man [knows Him] from His actions — which is called "demonstration" [מופת], "proof" [ראיה] — and this is what is alluded to in Scripture: "and you shall see My back" (Shemos 33:23). And there is no doubt in my mind that Moshe Rabbeinu knew the Lord from [such] proofs [?], "from My back" [?] — as is brought in the divine books, and others like them — for the deniers and the heretics [?] are refuted by arguments and demonstrations [?], rather than by proofs [as solid] as a molten mirror, strong and correct — for not every man merits the intellect of Moshe to know them. As for the causes of the ends of the divine actions and the purpose of the commandments, there is no doubt that [Moshe] was full of them, and knew on what their foundations were established — and this is what is written of him: "in all My house he is faithful" (Bamidbar 12:7). "And not like a [common] man" — and it was not written in the Torah "etc." [the writer here defers]. This is the gist of your words.

And your Honor knows that, at the time I was in your fellowship, I was greatly preoccupied with my affairs and my troubles in visiting the sick, and I had no leisure either to eat or to sleep. And all the more so [now], since I do not hold myself to be at the summit of wisdom and perfection, such that I could satisfy your desire and reply to your questions with wondrous and refined matters such as these. Would that I myself knew the way to where light dwells! And before I seek mercy for you, I [must] seek mercy for myself. Nevertheless, for the great love and brotherhood between us, I will write two folios; and if my words are not pleasing, I will yet be acceptable [?] before your Honor, since I sought to do your will to the best of my power.

I have called this little treatise **"Ner Elohim" [נר אלהים — "Lamp of God"]**, [after the verse:] "[The soul of man is] the lamp of the Lord" (Mishlei 20:27) — [for] "the lamp of God is the soul of man" [?], by which he searches the chambers and recesses [?] for the truth, and distinguishes between [the true and the false], so that the truth may appear [?] in his eyes; and in his heart he shall understand and know by what he may render his way pure — [he, a being of] flesh and blood.

[Centered heading, citing Tehillim 85:12:]

"Truth springs up from the earth, and righteousness looks down from heaven"

And you, discerning and wise — may God preserve and sustain you [?] — [you,] the assembly of Israel that yearns to see the light of the Lord, the truth of the words of truth: you know, from what we have read together in books, that the gates of righteousness of all wisdom and understanding and the art of the intellect's thought are these five senses [?], which are implanted by nature in the soul of every

living being — perfect [in] speech, [and] the sons of the discerning ones [בני דעה], who feel out [?] every true thing — those who purify and sanctify themselves from all error and mistake — so that no falsity should occur [in them], in a place where there is no [predication of] true and false, as is the case with the senses; for in them there is no composition of predicate and subject and act, [no] premise that is affirmative or negative — for these are operations proper to the intellect, as is explained in the *Logic*. Aristotle said, in the sixth book of the *Metaphysics* [למ"ש = למה שאחר הטבע]: "The true and the false are in the intellect." For this reason they defined truth as the agreement [שווי] of the thing with the intellect, and did not say the agreement of the thing with sense — for [a sense-object] cannot be perceived except as it is, whereas an intelligible may be conceived in a manner [other] than what it is. Therefore it is said that one who denies the senses deserves punishment, for the first of all our knowledge [comes] from them; and when all the offspring of the sciences are reduced to their premises, they return to the sensibles from which they were taken — "there [where] the wicked cease from raging, and there the weary are at rest" (Iyov 3:17) — for they put to rest the strength of those who refute them, and there they rest [?]. The intellect [is] like one of the senses; upon it the intellect [?] builds its awakenings and its structures, [tying] cord to cord and thread to thread, so as to "draw cool and sweet waters from deep wells."

Now, as to whether the senses err with respect to their proper sensibles — for example, sight with respect to color, and hearing with respect to sound, as Aristotle ruled in the second book of *On the Soul*, [section] [63] ס"ג — [this is] very doubtful [מסופק]. For it seems that the premise is not true in its generality, since behold, our eyes see that the white appears black from afar; and as Aristotle himself wrote in the third book of the *Meteorology*, chapter 4, in the rainbow [קשת] of the clouds, [the eye] sees colors that, in truth, are not [there]; and likewise on the neck of the dove; and to the sick everything sweet appears [bitter, and the bitter sweet]; and so too the other senses — for man is very deficient in

[wholesome] constitution among the animals, since in all of them alike error occurs. The answer of the [great teacher], [Maimonides — "the master of the Guide" (בעל מורה)], who wrote that "for the most part the senses do not err with respect to their proper sensibles" — as if to say that the eye does not err as to color, whether it is black or white, but does err between snow and pure white wool, and so forth — according to this, no way or path to the truth remains to us, if these senses deceive us at times; and how, then, would [the truth] be known, when the luminaries are eclipsed and bear false witness?

Themistius, in the second book of *On the Soul*, chapter 12, wrote that the truth of this premise requires [three] conditions: **(1)** that the sense-organ be in its proper, sound state — for example, since taste falls naturally [upon the tongue], the recipient must be bare of the nature and quality of what is received; and if [a person's] saliva is bitter from a mixture of jaundice [?], it will make the foods he tastes taste bitter; and those afflicted with polyps [?] make the fragrant things stink; and a ringing [שריקה] in the ear hinders the hearing of the sound that strikes upon the auditory [organ]. **(2)** that the medium — namely, air, or water, or glass — be clean of any admixture of a body of another kind, and even [?] in its thickness; for if the thing is seen through glass mirrors, the sensible itself is not seen, but only its image, doubled — for one who sees a dinar in water sees only its image. And the proof [?] is that if [the viewer] withdraws backward until he no longer sees [the coin] without water — if water is then put into the vessel, he will see it. And [likewise,] if the heavens are a fifth body [גשם החמישי, the quintessence] and fire is beneath them, [then] we see all the stars outside their [true] places. And so it happens every time we see a thing through two differing transparent media — as the stick set [half] in water [and half] in air [?] is seen as if it were broken, by reason that [the water] alters its place, and it is impossible that it be seen in a straight line with its part that is in the air. And likewise [for] them; and it follows from this that there is no premise or conclusion in all the acquisitions of the intellect whose existence is not preceded [by sense]. The things in the red glass vessel appear red. **(3)** that the distance be properly proportioned between the sense [and the object]; for what is [seen] from afar appears in other colors and smaller than it is, and what is very near falls upon the sight [distorted] — for in order that the sensible be seen well, it must be seen at [an angle of] a quarter [?] of the arc [?] of the grape [?] — [i.e.] every kind of the eye's sphere [?]. And besides these, one must look well into the books of the masters of optics [חכמי ההבטה] to know [the matter]. This is the gate of all the sciences [?]. The error that occurs in the sense of the eye, before we build upon it [a structure], [is] because this sense is more [reliable] than the other senses — or most of them — as

Aristotle wrote at the beginning of [the work] [?]; and error falls into it [nonetheless].

A doubt [פנים = aspect/difficulty]: since all things known are intelligibles alone, it is clear that the sensations are not [themselves] knowledge — for they are shared by man and beast; and were it not for the operation of the intellect, which predicates and judges of them and abstracts them from their matter and understands them — [understanding] them as species, not individuals (that is, as natures separated from others) — there would be no resemblance of knowledge falling upon them at all, but only sensation. Indeed, so long as they are present before the sense — and [so long as] the Lord, may He be exalted, willed to favor us more than the [other] animals, having given us the storehouse of memory to preserve the sensibles that are there, stored away [?] for our need — when the intellect wishes to occupy itself with them, it brings out the number of their host, and calls them all by name, and distinguishes this one from that, and combines this with that, without need of sense or sensation; and likewise it generates conclusions, as we shall explain.

For behold, all the operations of the intellect's activity are comprised under [three] kinds. **(1)** When it conceives the sensibles, simple [and stripped] of all matter and their [individual] quiddity, and predicates of them no affirmation or negation, and combines this with that not at all, and separates this from that not at all — this is called the first intellectual operation, or **simple apprehension**. That is, [the intellect] discerns in [a thing] whether it is a substance or an accident, [grasps] a definition or the quiddity of the matter; and the teachings concerning this apprehension are in Porphyry's *Isagoge* (the book of "Introduction"), and in Aristotle's *Categories* (the book of the "Ten Sayings"), and in the *De Interpretatione* of Boethius [?].

It is fitting to know that — although the Philosopher [Aristotle] wrote in the *De Interpretatione*, chapter 1, that truth and falsity do not apply at all to this apprehension; and likewise in the sixth book of the *Metaphysics*, of blessed memory [?], [that] in simple things no truth or falsity falls — for, indeed, truth and falsity depend upon the judgment of the intellect, which **(2)** either affirms or denies, and combines a thing with a thing, and divides a thing from a thing, and makes premises affirmative [or] negative. For when the intellect knows, in the first apprehension, two distinct concepts, and judges, in the judgment of its thought, that this befits that within the totality of its kind — so that the one becomes the subject of its fellow — it forms a **universal affirmative** premise; and if [it befits] some of its kind (that is, its individuals), it forms a **particular**

[premise]; and if it judges that it does not befit [the other] at all, it will be a **universal negative**; and if [it does] not befit some of it, it will be a **particular negative**. This is the **second intellectual operation**, whose rules are explained in Aristotle's *De Interpretatione* and in Boethius's [book on] *Composition and Division* [ההרכבה והחילוק], and it is called, in [the language of] the logicians, "composite apprehension."

But in the first apprehension there applies to it no affirmation or denial, and no truth or falsity. Even so, the truth that is in the intellect is partly simple and partly composite. The composite — for example, if I understand that "the man is alive" [it is] true, and if [I understand] that "the man is a fish" [it is] false. And the simple — for example, if I apprehend "man," I apprehend what exists, and it is true; but if I apprehend "[the void / chimera]" [?], which does not exist, I apprehend the false — for the first apprehension agrees with [actual] existence, [and] the second [does] not. Therefore the opinion of the Philosopher is to say that the truth that is composite, in [composite] things,

[The argument continues into the following section.]

the simple ones; rather, simple truth applies to them. Nor can one object: why should simple truth not likewise belong to the senses, since the sense perceives the existence of a thing — and there is no truth greater than this, for "the existent" and "the true" are one and the same, as is known, as we said. The answer is that this cannot properly be said: a sense that has no power over a thing [cannot be called true or false]. The testimony that Aristotle gave in the *Categories* — that one does not predicate "blind" of stones, but only of those things that are capable of sight — means just this. Were a sense capable of perceiving both presence and absence, then truth and falsehood would reside in it. But the eye does not perceive darkness, nor the ear silence; rather it is the **common sense** [the *sensus communis*], the imaginative faculty, and the imagination that perceive their absence — for these are not acted upon by any object of sense that does not knock at their doors, while the multitude say that they "see" the darkness, though this is not so.

Nor can one object: but Aristotle said in *De Anima* II, in his teaching [ch. 9?], that by a single sense one apprehends both an object and its absence, and likewise that a single faculty perceives [?] a pair of opposites. The reply: this holds for *real* opposites that have positive existence — for example, the sense of sight, which sees white, also sees black; but it does not hold for *privative* opposites such as light and darkness, where the one is perceived essentially and the other only incidentally. Aristotle meant only that the external first sense conveys [the

impression] to the common sense for the apprehension of the thing and its absence, as we explained. And if the inner senses are dormant, the outer senses will not perceive: in the sleep of a living being one does not hear a sound even though the ears are open, nor does one feel things creeping over one's flesh; and many living beings sleep with their eyes open, with objects of sight before their eyes as in a mirror, yet they do not see — for they do not stir. All this shows that perception depends solely upon the readiness of the inner sense to receive the object of sense.

The third operation is the most excellent of the [intellect's] operations — the noblest of the three — and by it the greatness of the intellect is recognized. It is the fruit and goal of the first two operations. After the intellect has judged, composed, or divided, until it has built up true premises, it then conjoins one premise with another through reasoning, and from them generates a fresh conclusion that was previously hidden. For example: we know that every living thing is a body, and that every man is a living thing; and from joining these we conclude that every man is corporeal. This is called the **rational syllogism** [*heqesh sikhli*]. Its foundations are these: it has three *figures*, comprising nineteen *moods*; and it is of three *kinds* — demonstrative, dialectical, and sophistical. These are all expounded in the *Book of Demonstration*, the *Topics*, and the *Sophistical Refutations* of Aristotle son of Nicomachus, who was the first to devise and perfect these instruments. How many good favors has the Omnipresent done for us through this wondrous creation — a man mighty in inquiry! Had this man of great light not been [granted to the world], the wisdom before him would have been like an ape without ears, and the philosophers would have been ensnared in [their] knowledge, [stumbling] over the stones. "Then came sons to the breaking-point, and there was no strength to deliver" [Yeshayahu 37:3]. And just as his words on natural science are perilous — "for you are heaping coals" — so, by contrast, in logic: "adorn yourself," for they are like wool and dross [cf. Bava Metzia 107b].

It is fitting that you should know that every branch of knowledge and science comes about by means of the three intellectual operations just named. It will become evident to you, upon examination, that all the modes by which the disciplines are taught fall under three headings — that is, three paths: the path of **division** [analysis], the path of **composition** [synthesis], or the path of **definition**; for every intelligible object, or any goal sought, is sought along one of these paths.

The path of division has four modes. The **first** proceeds from a whole to the parts that complete it. For instance: from our observation that within the whole of the

human species some individuals are healthy and some are sick — from this intelligible object we seek to build up the existing condition of health, and to restore the lost [health] to its possessor; and this is the **art of medicine**. So you see that from the understood goal —

— that it is the recovery of health — one comes to the discovery of medicine in general; and again, from the discovery of medicine in general one comes [further]. Medicine is then divided into its parts: those of its natural [theoretical] portion, or its active and operative portions, [following the division] of the healthy body [?], as Galen [גאלינוס = Galenus] explained in his book *The Excellent Logician* [?], in the *Mechanics of the Art* [?]; and likewise [each part is divided] into its parts.

The **second** mode begins from a generic whole and divides each species afterward into other species. For instance: we speak in general about substance, and afterward about its species — which are spiritual, material, [and] corporeal; and then we divide the material into four species [?], and each one into [its] parts, and so on.

The **third** mode resolves the conclusion into the premises that yield it, and each premise into the premises prior to it, which are its causes and roots, until we arrive at the first intelligibles — the items of knowledge implanted in our intellect — as is well known in the demonstrations of Euclid and in all the mathematical sciences.

The **fourth** mode proceeds from the goal to the intermediaries that lead to the goal. For instance: since we know that the goal of logic is to understand all the sciences and examine them, we desire to know logic — because this is attained only by means of the proper rational syllogism, and the syllogism is attained only by understanding the premises that compose the syllogism; and these we would not know save by understanding the terms from which the premises are constructed. Therefore one must begin from understanding the terms, for what is last in execution is first [in intention]. So you see that the goal brings about the understanding of the intermediaries, until [we reach] the first foundational beginnings. Since the knowledge of opposites is one, **composition** [synthesis] stands opposite **division**, with the [same] four modes corresponding to the four modes of division just mentioned.

Definition appears as though compounded of the two [paths] just named. It is what makes clear the matter [the *quid*] of a thing beforehand; and if it is not made wholly clear by its definition — as some have written, that one should first

divide the thing and afterward define it — nevertheless it makes clear some aspect of the thing. A definition in general is one from which we know the properties of the thing defined, since the definition is compounded of genus and differentia. We then first divide it [the subject] into its completing parts, and afterward into its specific parts, supplying the definition of each of them and its properties; and likewise we divide each and every species down to the final division. For example: if we wish to treat the syllogism by way of definition, we first set out in general the definition of the syllogism and its properties, and afterward we divide it into its completing parts — the **material** and the **formal**; and then we mention its kinds — demonstrative, dialectical, [and] sophistical — and divide each one into its parts, supplying the definition of each of these and its properties, and so on.

This is the path Galen took in [his] *Small Art* [*Techne / Ars Parva*], where he set down the definition of medicine — namely, the knowledge of the healthy, the sick, and the intermediate — and afterward divided the definition into three parts: that this knowledge is either knowledge of the bodies, or of the signs, or of the causes. For "health" and "sickness" are equivocal terms, applied alike to the healthy body, to the sign of health, and to the cause of health; and likewise the term "sickness," and so on. This path is most useful for remembering the topics.

However, the path of **division** [analysis] is well suited to the investigation and discovery of the sciences, since the method of instruction is to proceed from what is known to us — which are the things posterior — to what is hidden from us, which are the things known to nature and truly prior. So Aristotle opened the *Physics* [*Sefer HaShema HaTivi*]: by this path the natural philosophers instruct their students, the premises being well clarified in due order. The mathematician, however, will instruct by the path of **composition** [synthesis], all the premises being known to him, and from them he generates demonstrations. But if he wishes to clarify the truth of the —

— demonstrations, he too resolves them back into their premises, until [he reaches] the universal items of knowledge, the definitions, and the postulates, as is well known.

Let us now return to the intellectual operations and say: Aristotle set bounds for the figures of the syllogism, and taught the **third** intellectual operation in such a way that no one can truly err in the formal part; rather, the **second** intellectual operation is the source of every error, in which the **material** part falters. Since the premises are the foundations of all the sciences, it is impossible to encompass them in brief instructions, for they are as numerous as the nature of the sciences

[is broad]. Aristotle labored greatly to teach the *conditions* of the premises — for if a single premise be set down false at the beginning of any science, the whole science is falsified, generating views variant, strange, and empty, and afterward one cannot tell from where they followed. Behold, in the *Posterior Analytics* [*Book of Demonstration*], chapters 2, 6, 7, [and] 8, he set out the conditions of the premises of the demonstrative syllogism — the syllogism that yields true knowledge — which are eight, namely: true, immediate [proximate], better-known, evident, prior, and causes of the conclusion; necessary, proper [essential], [and] eternal. And in the *Book of Logic* that you wrote, you laid out — according to what you heard from those who expound them well — all the aforementioned conditions.

"But wisdom, where shall it be found, and where is the place of understanding?" [Iyov 28:12] For only a few of these will you find in physics and metaphysics, whose syllogisms [meeting] all these conditions are difficult. And if found at all, the syllogism will yield true knowledge — the definition that Aristotle gave [for it] in chapter 4 of [Book] VI of the *Topics* [?]: "the acquisition of a true intellectual possession, a sure and evident grasp [?] of some necessary thing by means of its proper cause" — end of his words. This **possession**, called "the knowledge of the thing," according to what Plato wrote in the *Book of Rhetoric* [*HaHalatzah* — Gorgias?], is what is acquired by syllogism; and one who possesses the aforementioned possession knows how to give a reason and a cause for his statements, his teachings, or his deeds, and on its account is called "wise" or "skilled," the possession in his intellect being the wisdom or the craft. By this measure, the mathematical disciplines are not [strictly] true items of knowledge, since not all their premises are immediate, nor are they proper causes — for a single demonstration may be clarified by several paths; and even if they are formal causes, they are not final and efficient [causes]; and if they are not demonstrations, how much more so the natural syllogisms, which are highly doubtful.

However, apart from demonstrative knowledge, which is not easily found, as stated, the name "knowledge" is applied to every intelligible acquired through division, definition, investigation, experience, and **practical proficiency** [*beqiut*]. Proficiency is a knowledge and a possession acquired through experience or instruction, without [grasping] the cause and reason of the thing — that is, through tradition received from a wise man who reports it to him, or [from] a writer who sets it down in a book. Such are most of the narrative writings of [many] authors. One who fills his belly with these is not called "wise" or "a master of the craft," but only "proficient." As Plato wrote in the aforementioned book, the

art of cookery is not called a craft but a proficiency, since its ways were not found by rational syllogism but by experience — even though they may be subsumed under general instruction. And I have already shown you a treatise I wrote on the regimens of foods and the reason for the mixtures, which are to drive off harm; and many beneficial ways for the healthy and the sick are found written there, by which the discerning person can devise from his own understanding many new and pleasant kinds.

Likewise the writers of temporal events [chroniclers] are not called "truly knowing" but "proficient" — for although the order they keep in the narration is a kind of instruction, it is empirical [?] instruction, since the knowledge of the things related is not acquired through causes; rather all their knowledge is a possession of proficiency. So too those who engage in statutes and laws as mere rhetoricians are not called "legislative craftsmen" and "knowers," but "proficient," inasmuch as —

— they do not know the causes of the statutes and their purposes, but only by way of narration, tradition, reading, practice, and proficiency; they know their external sense. There is no doubt that the early sages, the founders of the religions and the statutes, were complete sages, among the most distinguished of their generation — like the Greek Solon [שולון היוני], and Socrates [סאקראטיס], [who knew] man, his household, and his city; for they knew the purposes of those religions and revealed their reasons, all of which are purposeful [well-aimed]. So too Plato and Aristotle, who wrote on **governance**, by which is known the practical philosophy that comprehensively governs toward the preservation of the world, [its] peace, the increase of wealth, and the settlement of their state — all of which follows from good governance and upright judgments, by which the whole world is benefited, on account of their righteousness.

However, in some of the [religious] faiths the reasons of the statutes are not found in the books famous among them; it appears that they deliberately concealed them — especially in the case of the commandments, that is, matters between man and the Omnipresent — and did not set them down in the book of their Torah. Rather [they recorded] only the agreement of the matters, and they established statutes for all the people [enforced] by human and divine punishments, so that they should not alter their charge; for they reckoned that, were the purpose known, [the people] would treat [the commandments] lightly. So these matters remained known only to the select few, the elders of the people, who attained perfection in leadership and governance and were fit to be the leaders of the generation.

For this reason you will find many of the sages of Israel guarding the reasons of the commandments concealed, and in their eyes [one who reveals them] is like a talebearer who reveals a secret that the Torah did not wish to disclose. You already know what befell Solomon, who relied on his wisdom, saying, "I will multiply [wives and horses] and not turn aside" [cf. Devarim 17:16–17; Sanhedrin 21b] — for the reasons of the commandments were [counted] among the Oral Torah, and therefore they were scrupulous not to commit them to writing.

From this you will understand that the same holds for many [other] nations — even though there is a great difference between us and them, in that they liken [the Deity] [to corporeal things] in all their affairs, while learning from our holy, divine books to beautify their deeds — whereas the foolish ancient nations performed deeds and acts of worship without reason or purpose. Their like exist today and do not wish that any reason be sought in them; indeed, among the Ishmaelites [Muslims], one who investigates them is liable to death. But far be it from us to think so concerning **our** worship! On this I have no doubt: it is inconceivable to say that Moses our Teacher — who heard and learned from the mouth of the Almighty — did not know the reason of the sacrifices, the scapegoat of Azazel, and the like; rather he surely knew them in their full measure. And Joshua his disciple, who "did not depart from within the tent" [Shemos 33:11], of whom Scripture testifies that he was "full of the spirit of wisdom" [Devarim 34:9], did not, in all that time, ask his master the reasons of the commandments. So too the elders distinguished in wisdom — they knew, and well knew, these secrets of the Torah, but it did not seem proper to them to publicize them to the multitude, for the reason stated.

In truth, the inquiry into the causes of the statutes and the reasons of the commandments is more excellent and more honored than [that of] the natural philosophers who seek the causes of material things. Nevertheless, in this inquiry it is fitting to resemble the [natural] philosopher in matters of order: he first knows the operations, the properties, and the qualities of the natural existent as a whole, and from the operations proceeds to the beginnings and the causes. So too it is fitting first to be proficient in the laws and to understand the matters well from their source — that is, in the *Poskim* [legal codifiers] and in the *Talmud* with its commentaries; and after one has filled his belly with these through diligence, until past the prime of his days, it is good to seek their reasons, whether from the books of the Kabbalists, if he takes pleasure in reading them, or from the books of the divine philosophers who are of the children of our people — and not from the books of foreigners. This was the path taken by the excellent man, **Rabbeinu Moshe Maimonides** [Rambam], who exhausted the choicest of [his] years —

in his commentaries on the Mishnah and the Talmud, in his investigation, and in his compilation of the laws — and, at the close of his life, he composed the *Moreh* [*Guide of the Perplexed*], in which he probed the reasons of the commandments and the divine mysteries of the Hebrews. There is no doubt that proficiency precedes knowledge — in every science and craft — as Plato and Aristotle wrote in Book I of the *Governance of the State* [*Politics*], chapter 7: "When we wish to understand any field of knowledge, we must first be proficient in it." You have it here, then, explicitly: that proficiency precedes knowledge in time, just as the imperfect acquisition precedes in time the perfect acquisition. For proficiency is acquired through sense-perception, whereas knowledge is acquired through intellectual syllogism. And we have already explained that every intelligible was, in some manner, preceded by sense — as was made clear in the *Book of the Soul* [*De Anima*]. Therefore the acquisition of proficiency precedes the acquisition of true intellectual knowledge.

This is unlike some of the children of our people, who neither labor in Torah nor delve into the Gemara, and have not even understood the legal codifiers. There are among them those who do not even properly understand the Holy Tongue, yet they expound in public assemblies what they have learned from the books of the poets and the chroniclers, and from the languages of the gentiles, mixing in some of the words of the Christian theologians or of the mysteries of the kabbalists — even though they understand neither beit nor nun [i.e., nothing at all]. By combining elegant phrasing in the natural, habitual idiom and the modulations of the voice, they lend a certain flavor to their words and pass themselves off as great sages — and their listeners are just like them. Had they merely been chirping out verses, I would have held my peace, for "idle prattle of children is Torah" [cf. *Shabbos* 119b], and in the end they are pure, and would be no more than schoolchildren by whose breath the world endures [cf. *Shabbos* 119b]. But the trouble is that they divert the students from the study of the Talmud, scoff at its dialectical reasoning, and say that it is only a waste of time, with no perfection in it for a person. Worse and more bitter still: imagining that they know the reasons of the commandments and their purpose, they refrain from performing them — they do not come to the synagogue, they do not lay tefillin, and God knows whether they even pray at home. And how do they conduct themselves in matters meant to be private? Such are the ways of these "sages": [open] in the markets of Tiberias and not in the bais medrash. As the proverb has it: "they expound well, but they do not fulfill well" [cf. *Chagigah* 14b]. All of this is set forth in your hand: that in every matter, proficiency alone is better than speculation alone. So too Aristotle ruled, in his statement that "the physician forever" [?] — and the Torah scholar, even if he does not know the

reasons of the commandments and the mysteries, is more esteemed before the Omnipresent than the "divine" man or the kabbalist who has no knowledge of Torah and Gemara. For these latter resemble one to whom the keys of the inner chambers were entrusted, but not the keys of the outer — so how can he enter within? And Aristotle wrote, at the beginning of the *Book of Demonstration* [*Posterior Analytics*]: "All instruction and all learning is in truth from prior knowledge" — so you see that he called proficiency by the name "knowledge."

He further wrote, at the beginning of the *Book of Demonstration* and at the end of the *Book of Demonstration*, that many sense-perceptions of individuals of a single species produce experience [induction]. For example: we observed that this scammony [a purgative] loosened the bile [the green humor], and likewise that another scammony did the same — and so on, many times. If, on a third occasion, we recall what occurred on the earlier ones, there will be many memories, for the recollection is roused from the sensible object. On this account the Philosopher says that there is no memory after death: for if, after its separation, the soul is not acted upon by the senses, how shall we remember? — since recollection, essentially and primarily, is by the sense, not by the intellect. But the "divine ones" [the theologians] say that there are two kinds of recollection, and that the soul does remember after death. Therefore, when we recall

that such-and-such happened many times — that scammony engenders [purging] — there subsists in the faculty of memory a fixed matter, and the sensitive soul has effected it, just as the intellect's action [does its work]. This acquisition residing in the faculty of memory is called experience, and it is what is gathered from many recollections. From it the intellect is roused and forms [a judgment], and abstracts that all scammony empties the bile. This universal learning comes from the outset of the craft and is found by means of experience, not by means of intellectual syllogism, and thereafter it remains in the intellect as it came to it — a fixed matter in the memory. So it follows, then, that just as demonstration is the act of the intellect, so experience is the act of the sense: by intellectual syllogism true knowledge is acquired, and by experience, proficiency.

And I will give you an example of how one and the same thing may be acquired both by syllogism and by experience. For instance: it is generally known among physicians that gourds [colocynths] are good for fever. Now, the demonstration of its truth by way of syllogism is as follows: every cooling and moistening agent is suited to fever; gourds are cooling and moistening; therefore they are good for fever. The major [premise] is made clear by universal knowledge — that an opposite is set right by its opposite; and the minor is clarified thus: a compound

in which water predominates is cooling and moistening, [and] in the gourd the element of water predominates; therefore they are cooling and moistening. And the genus of fever is the heated humor that boils in the heart and spreads to the whole body, as is known. Therefore gourds suit it, since they are the very opposite of the illness. In the same way the empirical physician will conclude — and the [other] will say: "I recall that my master gave gourds to such-and-such a fever-patient, and it helped him; and so a second time, and a third, and a fourth," and so on, until from the multitude of instances a memory is engendered, and from its multiplication an acquisition is engendered in the faculty of memory. The intellect is roused, forms a universal premise, and abstracts that every gourd is beneficial for fever; and by this way the premise is acquired by way of proficiency, and not by syllogistic knowledge.

Further, you should know that just as true knowledge is that which is acquired by means of demonstrative syllogism — for which, as we said, five [?] conditions of the premises are required — so too what is called opinion or supposition [*sevara*] is acquired by means of the dialectical syllogism, namely one whose premises *appear* to be true. Now the "apparent" [premises] are those that appear so to the whole world, or to most of it, or to the totality of the sages, or to most of them, or to their greatest. By "to the whole world" I mean, for instance, [the premise] that there is a God in the world — for you will find no nation or tongue that does not acknowledge Him, or [does not acknowledge] that there is a supreme Ruler and Governor. And though they differ over this, in the end all of them agree on the essence of the matter; indeed this matter is so agreed upon that, to my mind, even the mute beasts have a recognition of God implanted in them according to their imagination — and even inanimate things, were they able to speak, would admit it. So that it seems to me a boldness to enter into the class of divine doubts [theological problems] this very question, and to seek for it a demonstration [proving] that the All and its fullness, its order, its governance, and its endurance testify to their Maker — whether the spheres and the stars endure within them and move as intellecting, separate [intelligences], or from souls within them, so that they move by nature; or whether they are not spheres at all, but only the air or the ether is the firmament, and the stars float in various stations. None of this prevents [the conclusion] that there must of necessity be a final Mover, who moves all that is moved and is Himself unmoved; and all of them are directed to perform their service in dread and in awe, saying "Holy" and "Blessed be the glory of the Lord from His place" — and He shall rule

over them, and to Him is their longing. Aristotle, too, recognized the final [Mover] in his statement and proclaimed His unity — that He is like the commander of an

army together with the world, and is its leader, who set it in order from Himself. Yet the difference is great: for the governance exists for His sake, and He is not for the sake of the governance — so that if the army were dispersed, He would remain alone, since He has no need of another; whereas the [members of the army] are not ordered for the sake of one another, but all of them for His sake, etc. The opinion of those who posit many divinities is base and worthy of rejection: for since existing things do not wish to be governed without order, and it is not good that there be many masters — but rather one king — it is fitting [to ascribe sovereignty to] the name of the Sage whose governance is in wisdom, and in whom they delight. Thus far [his words]. There is more abundance in the words of the Rambam — that there is no need to multiply words to demonstrate the unity of the Governor — and as I wrote in *Sefer Gan Na'ul*, in my reply to the honorable Master Zerach, son of the honorable Master Nathan.

By "the apparent to most of the world" I mean, for instance, [the premise] that adultery with a married woman is repugnant, and that theft is loathsome; by "to the totality of the sages," for instance, that the good is more to be chosen than the pleasant; and "to most of them," that the intellect is eternal and that the world is created. For who would believe our report, [namely] that the perfect, wise man, created in the image of God — resembling the angelic order [?] in wisdom and understanding, who has not borne his soul in vain nor wasted his time on the vanities of the world — would not give sleep to his eyes until he found a place for the Lord, a dwelling for the Mighty One of Jacob, in the chambers of his heart; he built and established a lofty habitation for Him in his intellect. And though "thick clouds are His covering," the heavenly body did not restrain him from entering within, to lift the cover and remove the veil, that perhaps he might gaze upon His train from the lattices of His palace. In his innermost being they sanctify Him, and with his lips he ever exalts and praises Him; day by day he seeks to cleave to Him, mortifying his flesh and diminishing his fat and his blood — nor did his heart turn away from contemplating all that the Lord desired to do in heaven and on earth. For when he turned his shoulder to go, behold, death; and his glory went down after him, and his soul did not live. [Would it be], if You so deal with me, O my good King, that You would not form me [for futility] — if the intellecting souls labor in vain, and afterward neither sense nor speak, but become like the hosts and like the hinds of the field? Why then have You deceived me, that You set [in me] a longing — for "the flashes thereof are flashes of fire, the very flame of the Lord" (Shir HaShirim 8:6) — [only] for it to come to naught? What is this that God has done to us, to assure us so great an assurance, if our hope is to be disappointed — if eternity is not within the possibility of man's nature, and we shall not attain our desire? "What is my strength, that I should hope? ... for my

Rock" — and in place of my service, what shall my reward be? "Lord God, what wilt Thou give me, seeing I go childless?"

But believe me, my friend and beloved — those who say with their mouths that the intellect perishes do not [really] believe it, if they are true sages and philosophers. Perhaps they wish to display to the nations their wisdom and their philosophizing, and to make the pure appear impure by their subtleties. I will not decide whether it is their foolishness or [whether] their heart does not agree with their mouth, since I see that they are not as wicked in their deeds as others, who have deceit in their mouths and in their hands "balances of fraud," and in their inward parts impurity — whose actions point to their thoughts, that they do not fear God. As for the beginning students of philosophy, you will see them all of one body, saying — on the strength of the proofs they learned in Aristotle — that the form cannot subsist without [a body]; and those who study the books of Galen say that the soul is a temperament [*mizag*], and that is a quality. They speak so as to show that they are already great philosophers and physicians, discoursing on souls and intellects — yet in truth they are mere infants

who have not seen the light. Lest you suppose that they truly believe this, look at their deeds. After [a time], when the season of wisdom passes over them and their understanding settles, [their] mind is reversed and they turn into another man. The early ones — the moderns — in their callowness were "like a coin rattling in a jar" [Aramaic: כאסתרא בלגינא — i.e. making a great noise over little, cf. *Bava Metzia* 85b], and "a fool pours out his whole spirit" (cf. Mishlei 29:11); on every side and corner they would prophesy, and convene assemblies and academies before the princes — and the "rolling" [of disputation] and the rest of the marvels are there for them, until their time comes and it is shown that another spirit was truly with them. Such are like the sucklings of whom Aristotle wrote in the *Physics*: that they call all women "mother," but when they grow they will distinguish between their mothers and other women.

And so too in the matter of the creation of the world. Most of the sages uphold it; for, apart from the reasons of the Rambam, since the heaven encompasses the stars in such a manner that they all appear scattered in the affairs of [their] motions — and this is by design — therefore one must say that they move [purposefully], just as [in geometry] the angles of a triangle are [necessarily] equal to two right angles [?]; and [one asks] for what purpose this was. So this is not to be asked as to its purpose, and they must have a Maker who intends [an end]; for in things demonstrated there is no [unintended] agent or end, of which one might say who acted upon them. So too in the work of the stars, which do not

act for an end [of their own] — but wherever there is an end, there must of necessity be an agent other than itself. Much of this [is treated] in *Sefer Sevarot HaDat* [?]; and you yourself know the proofs that I wrote elsewhere. And who is so blind in the eye of the intellect that he does not see how, from a fixed [span of] time, perfection has been added in the affairs of the world without measure? Look at the affairs of the ancients — how crude they were; their garments — how loathsome; and their weaponry — how strange. The gunpowder, with all the [marvels] of artillery and all the implements of the ancients; look at the printing — [a gift] of God — and the glass instruments; look at the sciences and the crafts: all of them were lacking in former times, so that one cannot say they had existed from of old, were forgotten, and then returned and were rediscovered. For it is impossible to suppose that this combustible gunpowder — more necessary to the world's preservation than bread and meat [?] — had already existed and was forgotten by the entire world, since wondrous and necessary things spread throughout the whole world. And if some event should occur — a sword, or the sinking of cities, or the rising of the sea's waters, flooding and passing on, or any chance occurrence whatever — not the whole world is destroyed; one part of it renews another, and these things would be sought in the books of the ancient chronicles of every nation and tongue — yet not a trace of them is found at all. Therefore they are new [recent]. And the words of those who say that the fixed, shining stars, at their coming in their very slow motion over the pole of the horizon of some city, exalt it — and conversely [the city] is lowered and ruined by the malefic [planets] — do not contradict our words: that the good and the evil are in some part of the world, not in its entirety. And the responses of R. Chasdai [Crescas] against Ralbag [Gersonides] are not, in my opinion, just in this matter. And it is impossible to say that the world had, for a long time, been wholly lacking, and that in two or three thousand years it was completed — for then it would clearly follow that the world was, in fact, created anew.

Now, certain of the great ones — such as Aristotle and his school — held the world to be eternal together with God; and Plato held that the first matter was, together with God, from everlasting. These [thinkers], on the strength of proofs constructed to support eternity, found it [persuasive] for themselves, but not for the whole world; therefore their syllogisms are dialectical, not demonstrative. And there is no cause to wonder if these matters were hidden from them — for [they erred] in many other things besides; even beneath the equator they would refuse [to grant] habitation, for several reasons

Even so, this much is now clear in our own day: experience refutes and gives the lie to every notion and theory. As I said, since the difficulties run deep, all the

dialectical philosophers fall into many errors and lead souls into spider-webs of confusion, breeding strange and oppressive consequences. We, who hold fast to the faith through tradition, recognize their error — even though we have not the power to do battle with them, to defeat them by demonstrations, or to refute them. For the greatest of the divine sages of the Christians [Thomas Aquinas?] [?] conceded, in Part II, Question XLVI of the work he composed against the gentiles [Summa contra Gentiles] [?], that the creation of the world cannot be established by demonstration — even though Ralbag [Gersonides] labored brilliantly to assemble proofs, and the words of Rambam [Maimonides] and Abarbanel are known to us, as is the limit of how far their force reaches. Likewise there are among us a number of opinions, or articles of faith, that are an inheritance from our forefathers, which the philosophers mock — bringing dialectical, not demonstrative, arguments against them to overturn them. Even so, we do not heed their voice; our heart stays bound to our Torah like Mount Zion, which shall never be moved. As I live, the sages of Israel did wrong when they abandoned their view regarding a fixed sphere and retrograde constellations [?] and adopted the view of the [gentile] nations — for in our own time most scholars have rejected what they had received, and chosen what those scholars had discarded.

Know, too, that it is possible for some inference to be established by a demonstrative syllogism while we do not know the manner of its derivation, so that for us it is a notion drawn from a dialectical syllogism — or the reverse: what is demonstrated to us and held in complete knowledge may, for another, rest on a dialectical syllogism and be a dubious notion. Many things received as tradition by us are, for others, established by demonstration — while we do not know the demonstrations; for them these are complete knowledge, whereas for us they are mere notions and conjectures. From this you may grasp the difference between demonstrative knowledge, notion, and belief. These three acquisitions, as conceived in the intellect, share this much: it is possible for all three to be true — unlike error, which is never true. They differ in that knowledge is an acquisition that is always true, whereas notion and belief may be false imaginings. ✻ Moreover, knowledge and notion both differ from belief in that both of them are confirmed and supported by a syllogism — the one by a compelling demonstrative syllogism, the other by a plausible dialectical one — whereas belief has no support except tradition.

Now this tradition is of two kinds: either from God or from a human being. The first kind is incomparably superior to demonstrative knowledge, for it rests upon the words of God — exalted be He — who does not lie, nor does any other err in His presence; everything that issues from His mouth is wholly true. Therefore

those who heard from His mouth at the chosen assembly [the Revelation at Sinai] rose above the philosophers and theologians in the knowledge of God, exalted be He. As they said: "A maidservant at the Sea saw what Ezekiel did not see" [Mechilta, Beshalach]. So too all who stood at Mount Sinai attained the rank of prophecy. Moses could not have demonstrated the existence of God, blessed and exalted, by proofs more clearly — nor His providence, His knowledge of particulars, His unity, His incorporeality, prophecy, and all the principles of faith — than by having God come in His glory and let them hear His voice; for such seeing and hearing is clearer than all the demonstrations in the world. Happy is the eye that beheld all this! For this reason there was no need to write proofs and demonstrations in the Torah for the sensible things they saw and heard. He said, "Take exceedingly good heed of your souls, for you saw no likeness..." [Devarim 4:15], and He brought no proof that God is not corporeal. But those who came after them did not know God [directly], for it is impossible for God to be revealed at all times to humankind — all the more so for those who come many generations later; for the further a matter recedes from its origin and source, the weaker it grows. We

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believe in the foundations of the Torah by way of tradition. Yet this received, oral tradition is in truth as if it were sensory or demonstrative knowledge. For a tradition to be true, two conditions are required: that the one who transmits it be truthful, and that the one who heard from him heard a true matter. And behold, the chain of our tradition is stronger than any strength — for the Holy One, blessed be He, was revealed not to Moses alone but to six hundred thousand men, so that there is here no suspicion of fraud or contrivance. Everything that happened was written in the Torah, and at his death Moses transmitted it to each tribe — six hundred thousand others — and the gatherings were still in existence: the first generation as recipients by tradition, and the second as the children of those who had seen with their own eyes. All the more so, since many of the young men had been present at the assembly; had they seen written in the Torah something that had not occurred, they would have raised an outcry over it. But they accepted and confirmed it, and their children, who were the second in the chain of tradition, certainly heard it from their fathers — for it is not so easily done that so many fathers transmit falsehoods and conspire together upon a single matter. Thus not only Joshua [received] from Moses, and the elders from him, and from them the prophets, and from them the Men of the Great Assembly — but all Israel and their children received from their fathers, up to [the generation that heard] from the mouth of God, exalted and praised be He. It is as

though we ourselves heard and saw; there is no need for sensory evidence or the aid of demonstration, for demonstration is itself built upon the sensible. So you see that our tradition is superior — all the more so since all the nations attest to it.

And if those who argue against us press for something stronger than tradition, they thereby take it out of its nature; for had the matter been famous and openly manifest, it would have been knowledge, not belief. Know, too, that philosophy alone investigates its own first principles by way of inquiry — but not by a syllogism built from premises, nor by any other knowledge — since all its postulates and foundations are not demonstrated within that very science. Consider players at dice or at chess: at the outset they are permitted to arrange the pieces as they wish, with no contesting it at all; but afterward they must observe the particular rules proper to that game. So too with civil law: the decrees of kings are first principles that may not be contested — all the more so in the divine science, whose foundations may not be contested. Only concerning what follows from them is there room to give and take. So you have learned that the two [recte: three] acquisitions mentioned above are distinguished both by their subjects and by their operations. By their subjects, I mean: the subject of knowledge is something necessary in its truth by way of a cause; the subject of notion is something seen to be true by apparent, seemingly-true arguments; and the subject of belief is the testimony received from the mouth of a prophet, or a teacher, or a father. By their operations, I mean: in knowledge we know, in notion we conjecture, and in belief we believe. It is possible that, through different intermediaries, one and the same matter may be for us true in knowledge, in notion, and in belief — being knowledge demonstrated to us by a necessary syllogism, notion shown by a dialectical syllogism, and belief that is received by us; and then they would be three different acquisitions in the intellect concerning a single matter. Yet their three operations would not occur at the same time concerning the same matter. Therefore knowledge is the prior acquisition, and after it the acquisition of belief — for if knowledge contradicts belief, the belief is nullified; but if mere notion appears against belief, we set the notion aside. Let me give you an illustration for further clarity. You already know that in capital cases "by witnesses shall the condemned be put to death" [Bamidbar 35:30]; for since it is impossible that the eyes of the judges and magistrates rove over the whole earth, they trusted the witnesses, for the preservation of the world. Now if witnesses stand up against the first ones, it is possible that these are either contradicted [recte: refuting witnesses] or proven false by alibi [zomemim] — by "contradicted" I mean

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that the second pair say, "On such-and-such a day so-and-so was with us, and how could he have committed the murder?" — for he might have killed in the morning or the evening when they did not see him. But if they said, "He was not in that city at all on such-and-such a day," they are certainly proven false by alibi [zomemim]. And so it is with matters of tradition: if a complete demonstration, with all its conditions, stands against tradition, the tradition is nullified without doubt — but it is not nullified by the force of dialectical claims and arguments. You have already seen what the Guide [Maimonides] wrote: that had Aristotle brought absolute demonstrations of the world's eternity, he [Maimonides] would have held to his [Aristotle's] view and would have been able to interpret the plain sense of the Torah and to bend it accordingly, as he did regarding corporeality — except that Aristotle in truth brought no demonstration. So too the words of those who are stubborn about the rational soul, with their various arguments: not one of them is demonstrative, for the opinions about its essence are diverse, and each one thinks his own knowledge is demonstrative — which is impossible, since the truth is one and does not vary.

Now Alexander [of Aphrodisias], as is understood from what he wrote in Treatise I, chapter 1 of De Anima [On the Soul], and from the words of Johannes [Philoponus] the Grammarian [Johannes Philoponus, "the Grammarian"], whose view he cited in Treatise III of his commentary on De Anima, holds that the hylic [material] intellect is a simple potency, like the genus of existence — the corporeal and the spiritual — that is, it is in potentiality with respect to the totality of existence, so as to intellect it. For although he says that the hylic intellect is found even in children, he nonetheless does not affirm it to be an actual psychic power or capacity already laid down and present, by which a person could intellect; rather, that power comes only afterward, together with intellection and the intelligible. Thus, on his view, the hylic intellect is nothing other than a power that the soul is able, afterward, over the course of time and through habituation, to acquire — the power of intellection. To understand him, know that he posited three kinds of intellect. The first is in potentiality and is called "hylic," and is, in his view, a power and a psychic disposition to receive the emanation of the Agent Intellect — which for him is God, exalted be He. Once it has been emanated upon and has received the intelligible forms, it is called the "acquired intellect." So you have, then, three kinds of intellect: hylic, agent, and acquired. As for that potentiality or disposition called the "hylic intellect," he holds it to be a corporeal, perishable power, bound up with the soul that is composed of the elements; and therefore he says that human souls are perishable

forms, generated from the potentiality present in matter, like all other corporeal forms.

Others held it to be an accidental quality — that is, a temperament — as did Galen [Galenus]. Others held it to be a separate [intellect] common to all human beings, in various manners, as in the view of Themistius, Ibn Rushd [Averroes], and our own teachers [recte: Rambam? i.e. ורמב"ם — Maimonides] [?] [the printed reads ורמייכם, apparently for ורמב"ם, Maimonides], who hold it to be a disposition to receive perfection; and if one merits to perfect oneself in the intelligibles, it becomes substantial and remains eternal. The Christians held it to be an essential separate form, and others held it in yet another manner. Plato and his interpreters held it to be set [standing] over the body — and so too the sages of the Kabbalah hold it. And I say in truth: by my life, I do not know my own soul — that this I do know, and in this I hold fast and believe: that it [the soul] survives, and no argument or claim against it counts for anything with me. Yet I am in doubt as to which path to choose among those who maintain its survival — one that would draw my soul and that I could use to shield it against the opponents. This is the function of the wise and God-fearing man, rather than that one should say he believes what he does not understand of its essence, and when they press him he hides his face, is ashamed, falls silent, and becomes "a fugitive and a wanderer" [Bereishis 4:14] in his faith. In my responses to the honorable R. Zerach, I cling firmly to the view of Rambam, of blessed memory: since I could not understand how, if it [the soul] is an essential form, it could be separate [from the body], I upheld the view of the kabbalists and refuted the arguments against it — even though in the days of my youth I was very much separated [in view] [?], and childishness was in me, and I used to say that the ignorant [amei ha'aretz] have no share in the World to Come, since they did not make themselves substantial

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through the intelligibles. The wicked, envious men spread rumors about me on account of this, to take revenge on me — as though I, in my thinking, were forever [denying it]. [Margin: "the sciences are a family inheritance, since several of my forefathers were teachers" [?]] [the marginal/intercolumnar text here is partly broken: "…as though I…", "…that the sciences are an inheritance in my family, since several of my forefathers were teachers in philosophical academies, and no whisper of a thing was heard against them…"] — and one who spreads slander [motzi dibbah] is either a fool and devoid of wisdom, or, if he does understand, [speaks] from what his heart [contrives], for "heart and heart he

speaks" [a double heart — Tehillim 12:3], and within him he lays his ambush. May the Omnipresent not be deceived [?]: he is destined to render his account before Him. You know, my lord, that I fear the LORD greatly, that I tremble at His decrees and am scrupulous in His commandments, and that I have nothing in His world but the learning by which, from my youth, the love of truth was nurtured in me — to know it, that I might learn to fear and to serve the LORD wholeheartedly. For were it not for this, what would there be for us to endure all the burden of exile and be a reproach among the nations? We would say, "Let us eat and drink, for tomorrow we die" [Yeshayahu 22:13], and pay no heed. Nor should one think that, because our works survive in the mouths of those who come and in the name of the great — that we have acquired a name [recte: that we shall not be forgotten] [?] — that we shall not be forgotten forever; for there are many replies to this. For one, I do not know what benefit accrues to one who feels nothing of the good name after death; for to recount [a man's praises] after his death is like recounting after [speaking to] a stone, according to the view of the perishable[-soul school] mentioned above. For even if his words were written and engraved on pillars of marble — God forbid! [?] — they will not endure forever, for the long time to come, and "a passing guest's [provision]" [?], will obliterate and wear them away; he draws benefit [yerek? "only"] in this world from all the life of the World to Come, according to the aforementioned view. But the truth is as I said: that our generation — its interior is not like its exterior, and they desire only flatterers and frauds. As for me, I pay no heed to scoffers, for God tests heart and kidneys [Tehillim 7:10]. May the LORD do good to the good, and to the upright in their hearts; but as for those who turn aside [to crookedness…] [Tehillim 125:4–5].

These wicked, foolish men ought to have considered that the will does not rule over the soul's powers with an absolute, lordly dominion, but with a political dominion — that is, although they obey a person's will, they nonetheless have the freedom to oppose it, as Aristotle wrote in the Ethics [Nicomachean Ethics]. And at times it happens that imaginings become so strong that the will cannot compel a person to expel them. It has already been established in political philosophy that there is nothing willed that is not first conceived by the intellect — for the intellect rules over the will and arouses it. And how could one will what he does not understand, or against what he does understand? The deceitful speech is not [the measure], but rather the will in the heart. So how can one be condemned who does not wish to believe what he conceives [recte: ✻] to be imaginary in his imagination, or what the rational syllogism has shown to be impossible of existence? This is a sickness, an evil — yet the Blessed Name will not punish him for it: "Far be it from God to do wickedness, and from the Almighty to commit

iniquity" [Iyov 34:10]. The statutory religions, too, are strict only with one who publicizes his opinions to the multitude and causes the many to stumble — not with what is in his heart. You already know the difficulty that R. Chasdai [Crescas] raised against Rambam, who counted the beliefs among the commandments, though they are not within our will and choice; and R. Yitzchak Abarbanel replied that belief is at the rank of a form that supervenes necessarily upon matter immediately after the [requisite] dispositions. So too, study of good books and hearing from the mouths of those who fear God are dispositions toward good beliefs; and therefore one who guards his soul keeps far from the books and teachers of the heretics [minim]. Upon this rests the command that belief should come about [only] afterward [recte: ✻]. By contrast, the warning falls upon the fathers, who cause their children to sin by sending them to philosophize before the light of the Torah has shone upon them — whereas they ought first to have impressed well upon their souls the nature of faith, so that they would not depart from it.

[End of p. 90 / folio 90r.]

[Ner Elohim continues; no treatise boundary or question-numbering restart occurs in this opening.]

But the person pressed against his will by reasons that appear demonstrative — dialectical, as I have said — has no leave to break from the views accepted among the adherents of his faith, and he deserves punishment if these are fundamental matters. As for those views that are not among the fundamentals of the Torah, let him weigh carefully and observe the force of the difficulties; and if, after much reflection, the views still appear to him to run counter, let him incline to whichever side he wishes — for these views are neither received tradition nor do they corrupt the faith, since there is no hint of them in Scripture. But if it is not so, then in any case one who denies even a single letter of the Torah is a heretic. In every matter deliberateness is required, all the more so in these.

This is the great principle, and it is our consolation: that no demonstrative syllogism is to be found in all the books of the philosophers against the fundamentals of our Torah — only proofs and arguments. Why, then, would a fool turn aside from his mother's Torah on the grounds that he has no demonstration, and heed the words of the philosophers, who likewise have no demonstration? For if he does so, he will yet again [?] change his view, this time on the strength of the dialectical syllogisms that the adherents of another faith bring against him; and so he will destroy his soul and his seed. Reflect carefully, then, on the words of the Torah where it says, "And you shall love the Lord your

God with all your heart and with all your soul" (Devarim 6:5) — meaning, against your own will and your own intellect. For if you believe only what the intellectual syllogism verifies, then you are heeding the thing that compels you and your intellect, not your God. Rather, just as you must nullify your will before the will of your Maker, so must you nullify the enticement of the intellect and believe in God. Indeed, we see with our own eyes that a person hates a certain thing today and loves it tomorrow — there you have someone who changes his will; all the more, then, may he do so to love his Maker. For even Avraham our father could not, by his philosophizing, believe that there was hope for a withered tree, that a pairing of hundred-year-olds would beget children — and Sarah laughed at the report — and yet despite this he believed in God, and it was reckoned to him as righteousness (Bereishis 15:6). And you know that one who gives not as payment of a wage owed, but out of his own free will and the generosity of his heart, is called one who "does charity." Just so the believer, who is not obligated, will in his very lowliness be called righteous — for his belief in the miracles set down in the Torah, and in those destined to occur like them at the resurrection of the dead, and so on. And the one who believes what runs counter to his intellect among the things written in the Torah shall be called holy, not a fool. For knowing and believing are passive states, not actions; yet they perfect rather than corrupt. And as the agent that effects a thing is, so will be the perfection of the passive reception. The agent that effects knowing is the sensed object; the agent that effects belief is the intelligible object within the soul of the believer. And it is well known how many degrees the intellect holds above the senses. Hence the believer is more esteemed than the greatest of those who merely know. This will become clear in the future, when "eye to eye they shall see, when the Lord returns to Zion" (Yeshayahu 52:8), and "the glory of the Lord shall be revealed, and all flesh together shall see, for the mouth of the Lord has spoken" (Yeshayahu 40:5) — everything written in the Torah.

So too in the matter of those who return to the Lord: even if they have denied the fundamental principle, they should not despair of mercy. Those who trust in Him — that He will pardon their sin and bestow good upon their souls — He treats more favorably than the wholly righteous who suppose that God is in any case obligated to pay their wage, and who seek not the path of kindness and mercy but only what is owed. Elisha Acher, by contrast, despaired of mercy, and therefore they expounded concerning him: "Return, O wayward children" (Yirmeyahu 3:22) — except for Elisha Acher, who knew his Master and yet deliberately rebelled against Him. The meaning is: he knew that He is a merciful and gracious God, and that were he to return He would receive him, yet he says that the Lord has shut him out [recte: that he himself has barred the way], and aims at nothing but to

follow the desire of his heart, making excuses for his refusal — thereby bringing despair upon all sinners. He had been among those who entered the orchard of the sciences, and he was wholly a sensualist philosopher; he did not follow the intellect, as did Rabbi Akiva, the godly one. "By the road a person wishes to travel, he is led" (cf. Makkos 10b). And know that the Holy One, blessed be He,

is revealed and made known to us according to our own intellect — that is to say, He illuminates our eyes with the light of our torch and of our intellect. And you know what they said: that Yeshayahu was [?] like a city-dweller, and Yechezkel like a villager. Therefore we must prepare ourselves with the preparations that make our souls fit to know Him and to understand the mystery of His Torah; but we must never hold our intellect like a bridle [?], nor multiply expounding and inquiring into what is hidden and concealed. By this you will understand the reason that the verses of Scripture are so obscure: when they were placed in the mouths of the prophets, that is exactly how they were written down, without doubt. And why, then, did God not explain them well? Only because the One who knows what is to come, who tests the hearts of the present and the future, knew the differences among those who would study them and among the ages, and so He chose to write matters that admit of various interpretations, so that each could find in them what he sought — as they said, "There are seventy faces to the Torah." Therefore leave it to the masters of gematria, of notarikon [R"T = roshei teivot, acrostic exegesis], and of the secrets [?] to do with them as they please, for all of these are the words of God. Yet those interpretations stated without strain are the infinitely many [א"ס = ein-sof] meanings intended primarily and in essence, and they are sweet to those who hear them, like wine that flows from the grapes of its own accord; whereas the strained interpretations are like wine that comes out under duress by force of the press — its taste is the taste of the seeds and the dregs in it, and a man babbles according to the straightness of his own intellect. In sum: "Do not be overly wise — why should you destroy yourself?" (Koheles 7:16).

A little knowledge is as good as salt to meat, but much of it is great prattle — "much study is a weariness of the flesh, and he who increases knowledge increases sorrow" in the soul (cf. Koheles 12:12; 1:18). See: behold the great sages reckoned among the great deniers. The generation of the dispersion despised the tilling of the soil and wished to ascend to heaven by their intellect — they were reckoned a generation of perversities. Chizkiyahu pushed away and investigated, and was nearly uprooted from the two worlds. Yet observe the remedy: figs, which by their heat are likely to produce a boil in one who eats them — so how could a poultice of figs cure the boils [recte: Yeshayahu's prescription to

Chizkiyahu, Yeshayahu 38:21]? Only that it hinted to the discerning that one ought to use sharp things externally but not internally. And know what the sages of blessed memory said: "And behold, it was very good" (Bereishis 1:31) — there is none more precious and pleasant in the world than light, yet too much of it blinds the eyes. The more you continue to gaze at the divine things, the darker your eyes will grow. And should it cross your mind that the corporeal senses will lead you to the house of God, until you recognize the One exalted above all reason and thought — you see falsely. Well said one of Plato's disciples: that the senses are like the light of the sun, which illumines the earthly sphere but darkens the heavenly sphere and all its hosts. "The heavens are the heavens of the Lord, but the earth He has given to the sons of man" (Tehillim 115:16). One who is sunk in the sciences and in books is not fit to derive benefit, and is of no use either to his forefathers or to his offspring. Excessive passion for knowledge is among the class of base things. The books of logic, with their disputation and fallacies, are bones without marrow or flesh, and those studies are not the virtues — all the more so one who is meticulous about base matters, about numbers, and about fractions of fractions: such pursuits are a body without a soul, for practice rises above them all. The multitude of books are a moth and rot to the governance of the realm; they are fit for the disabled, for solitaries and ascetics, or for idlers. Search through all the books of chronicles: if you find a city saved by the philosophy within it, [then trust them] — rather, the smiths and the builders are needed for the upkeep of the towns more than the academies, and the disputations that disturb people from useful occupations.

A man's land, his income from useful occupations — and all the more so in the Jewish nation, for we have no land, and we bring our [livelihood?] [recte: prophecy?] within ourselves, and we are constrained and far-off. The discerning, capable man will seek

a wisdom in which there is a craft, to sustain his household and earn his living in purity — as are the heads of the academies in the land of Poland, who have wealth and authority among them. So set aside every craft in the world, and "teach your son nothing but Torah." Second to it is the art of medicine, which has "an arm with might," and renown and glory. And one who has neither of these — let him engage in commerce, and set fixed times for Torah; for you will find no mechanical craft that enriches its practitioner unless commerce goes with it.

And more than all these, my brother, beware of too much engagement in the mathematical sciences — such as the precise study of arithmetic, geometry, music, and astronomy — for these are long and difficult matters, and there is no

profit in them, neither for the rational soul nor for the body. I have already informed you of the places to which those studies led me, and of the scattering of treasure and of time; and I chose them not for any [reason] except their great truth, and because they have no bearing on matters of faith. But my view is that there is no support or staff in them; and one who has no knowledge but theirs will be forced to "go knocking on doors" at every turn and corner. Yet because one book interprets another, and one piece of knowledge aids another, and the sciences assist one another — how good it is when you seek medicine from the books of the physicians, and faith from the source of Israel and not from the children of strangers and aliens, as in the external sciences. Therefore "be wholehearted with the Lord your God" (Devarim 18:13); for the more you continue to read in the books of the philosophers, the more you will diminish your knowledge, grieve your heart, and worry over your end — until you loathe your own life. See and reflect on the Book of Tehillim, which David, king of Israel, composed, in which there is nothing but words of song and melodies — no political, speculative, or theological wisdom — yet because it is wholly praises to God, may He be exalted, all the nations of the world delight in it as in His Torah, day and night; and according to it young men and maidens, elders and dignitaries sing in choirs. And likewise the Book of Iyov, ascribed to Moshe Rabbeinu — they understood it, and there is in all of Scripture no book so full of wisdom and hidden things as it, yet they read it only on the Ninth of Av. For the consensus among most is that one who seeks the Lord and His divine words from the books of the philosophers is like one who "seeks the living among the dead" in a graveyard; and conversely, one who interprets the words of the Torah and Scripture according to the opinions of the philosophers is seeking the dead among the living. Therefore do not "forsake the Torah of your mother" (Mishlei 1:8). It is better that you be like a blind man leaning on a healthy guide, who leads you to peace and makes your path straight before you without stumbling, than that you wander after your own eyes, lean on your own intellect, strike your foot against a stone, and fall into a pit. For "the path of life" — if you weigh it without "the lamp of the commandment and the Torah" (cf. Mishlei 6:23), its courses wander, and you will not know its way. Therefore accept the truth and seek it from upon the Book of the Torah; then you will walk on a straight road, and you shall fear the Lord your God.

This is for you a copy from the body of the document that the honored teacher and rabbi, R. **Chaim Shor**, presiding judge of the rabbinical court of the holy community of Stanów, wrote to the honored teacher and rabbi, R. **Mordechai KaTz** [Kohen-Tzedek], of the holy community of Lvov, after a greeting of peace:

I conceal nothing [אני המכסה, opening flourish] — Shall I conceal from my Master [ממכ"ח = from his honor my teacher] the proclaiming of the deeds of the Lord, the great and awesome, which I have heard and seen with my own eyes here in the holy community of Stanów? Behold, our fathers told us what He wrought in days of old,

and now my own eye has seen — happy the eye that has seen all this! Behold, in the holy community of Gridzieng [?], which is near Stanów, three parsangs distant, there is a certain man whose name is R. Gedalyah, and he has a small son who is four and a half years old. And the boy is an ordinary lad, having no advantage in his learning over the other children who are his age. And once his father began to study with him the alphabet and the prayer-book and saw that, behold, the spirit of holiness rested upon him, he brought the child before me to the holy community of Stanów to test him. And I tested him many times — up to a hundred times and more — I and the companions who were with me, and we saw the work of the Lord with him, that it is exceedingly great. I asked him: "Tell me, now, the beginning of the halacha that they are studying today" — and at once he stated the halacha and answered: "The halacha is: Said Rav Ashi, our Mishnah too is exact," and so on. Now the boy and his father were sitting before me at a distance of four cubits. I took before me a book that is called the Book of the Zohar and opened the book at random, and I said to his father, "Have the boy tell me by heart what is written at the top of this page." Immediately he began to recite it just as it was, though he had not seen the book at all. Afterwards I skipped to the middle of the page and said nothing to him, but only placed my finger on a certain line in the middle of it, and the boy began to tell me what was written there beneath the finger; and so I did time after time. Then, in the blink of an eye, I skipped several lines and words, and upon that line and that word where I placed my finger, the boy told me, "Such-and-such is written." Afterwards I went off with my sons-in-law and spoke with them about a matter, and we returned to the boy to have him report what we had spoken — and he told us, "Such-and-such you spoke." Then we did not speak a single thing aloud; I merely thought in my heart of a certain verse, and I said to his father, "Have the boy tell me the verse I thought in my heart" — and the boy answered me, "Your righteousness is righteousness forever," and so on (Tehillim 119:142). And so it was, in sum: the boy knows what is in a man's heart, and this is nothing but the spirit of prophecy. I have seen the awesome deeds that the Rashba recorded in his responsa, section 548, but this is in no way comparable to them, for there is truly nothing higher than this. The boy does not know how to join the words together properly, nor to read the words all at once, but only with a pause between the letters and the words, in the manner of the boys who are beginning

to learn the prayer-book, who answer everything slowly and with pauses. And sometimes he errs in a vowel-point, and when they rebuke him about it he immediately corrects the point just as it is — for in his own way, just as it is within, so he answers by heart. He has no advantage whatever in his learning, and he is exceedingly bashful. What emerges is this: these are the wonders of the Lord and His works, for He is exceedingly awesome. Happy the eye that has seen all this! "And may God establish [it] among us, and may all your children be taught of the Lord" (cf. Yeshayahu 54:13), and so on. And all that I have written above is but as a drop from the river, for the parchment is too short to contain all the particulars of these matters and the wonders I have seen from this boy. And this took place on the day [שלוסימאת — date cipher; unresolved] — Avraham Chaim, son of the honored teacher and rabbi R. Naftali Shor, may his rest be in Eden.

And restraining myself from the great multitude of the wonders of God's deeds and His awesome acts, I have not refrained from writing to you and copying from the document of the honored teacher and rabbi R. Yehoshua of Krzywcza [?], who wrote to the presiding judge of our court. But know that here in the holy community of Vilna they proclaimed in the synagogue that none should speak of the abovementioned matters in the presence of children and women, all the more so in the presence of the nations; therefore be cautious. This document was brought by R. Moshe Brendlash [?] of the holy community of Lublin. It came today, the 14th of the month of Tammuz, together with another document brought by the abovementioned R. Moshe from the honored teacher and rabbi R. Chaim Shor.

Sefer Elim

[Close of the correspondence — testimony letter to R. Mordechai KaTz]

that R. Mordechai KaTz wrote: People came to me, and I asked the boy what his name was. He told me his own name and his forebears' names. I questioned him about the Redemption, and he said, "I do not know." I asked him, "Who placed these words in your mouth?" and he answered me: "*Elokim*, the Lord, God of Israel."

[Translator's editorial note on the geometry treatise that follows]

The first translator said: I thought it good and useful to copy here the computation of triangles by the sage **R. Yitzchak HaYisraeli** [the author of *Yesod Olam*], which was already mentioned by the comprehensive Rav, the questioner, in [Question] "Ein" [?]. He could not refrain from resolving the matter of triangles fully and at length, for he explains his words with utmost concision. And [the Sages] said: "A person should always teach his student by a short and easy path." If the Lord grants us merit, we shall print the entire *Sefer Yesod Olam* together with the commentaries of **the Physician, our teacher R. Yosef of Candia** [Yosef Shlomo Delmedigo], for it is a precious vessel and a treasure, and the eye of the printing-press has not yet touched it [i.e. it has never been printed], and the maiden is exceedingly beautiful, a virgin [a play on the freshness of the unprinted text]. Here is his language [*v'zeh leshono*], in the First Treatise [*Ma'amar Alef*]:

[Yesod Olam — Ma'amar Alef: the chord (*beka*) of an arc]

[Figure 6] Now, the essential thing first: I draw a circle around its center **ה** [he], and I cut off from it an arc **אב** [a-b] smaller than its half, as I please. From point **א** [a], which is one of its endpoints, I draw the diameter **בהז** [b-h-z]; and from point **א** [a], which is its other endpoint [?], I draw the line **אב** [a-b] falling perpendicular upon the diameter **בהז** [b-h-z].

[Note: the letter-labels in this passage are inconsistently printed; **אב** appears both for the chord-arc and for the perpendicular, and the endpoint assignment ("one endpoint… the other endpoint") cannot both be point **א** as printed [?]. Transcribed as inked; the geometry is the standard chord construction.]

I say that the line **אב** [a-b] is called the chord [*beka*, "split/section"] of the placed arc **אב**, just as the straight line **אב** is its [subtending] chord [*yeter*]. Likewise the perpendicular **אג** [a-g] is the chord of arc **אג** [a-g] of the figure, which is the complement of arc **אב** to a full half-circle — since it too is [drawn from] one of its endpoints and falls perpendicular upon the diameter drawn from its other endpoint. And so it is in every case of this kind.

If you reflect, you will see that the chord of any arc is half the [subtending] chord [*yeter*] of its double. For the line **קנ** [q-n? — perhaps **אב**, the half-chord] is half of line **אד** [a-d], which is the [subtending] chord of arc **אבד** [a-b-d] [?] — which is double arc **אב**. And just as a single chord [*yeter*] is shared by two arcs that

together make the whole circumference — for example, the chord **אגד** [a-g-d], shared by arc **אבד** [a-b-d] and arc **אזד** [a-z-d], which together make the whole circumference — so too one chord [*beka*] is shared by two matching arcs, that is, two arcs that together make a half-circle: for example, arc **מא** [m-a] [?] and arc **אן** [a-n] [?].

And just as equal arcs have equal [subtending] chords, and are therefore equal to one another, so too arcs equal to one another have equal chords [*beka*], and are therefore equal to one another. And just as the diameter is the greatest of the [subtending] chords — its measure being, as we said, 120 degrees [units] — so too the chord of a quarter-circle is the greatest of the chords [*beka*], its measure being 20 degrees [units] [recte: should be the chord of the quadrant; the value "20" appears as printed and seems corrupt — see Numerical impossibles].

Now I draw in the figure half the diameter **הי** [h-y], parallel to line **אג** [a-g] — that is, line **הח** [h-ch], the chord of the quadrant **באח** [b-a-ch] — and I drop upon it the perpendicular **אש** [a-sh], the chord of arc **אך** [a-kh]. I say that arc **אך** is called the complement-arc of the placed arc **אב**; and likewise arc **אז** [a-z] is [the complement of] arc **קשת אז**. As a general rule: the complement-arc of any known arc is that arc by which the known arc exceeds a quarter-circle, or falls short of it. You will learn from this that a quarter-circle has no complement-arc; that every complement-arc is less than a quarter-circle; and that every arc possessing a chord [*beka*] is necessarily less than a half-circle.

Know that according to the table the sages drew up for the [subtending] chords of arcs

of the circle, as we said, they further reckoned and aimed at [tabulating] the number of degrees and minutes [*ragei*] of the chord [*beka*] of every arc — an arc less than half the circle — according to the measure by which half the diameter is 60 degrees [units]. They fixed these in a table, as you will see in this latter Treatise. From it you will also learn how the method of computation proceeds, so as to know the measure of the chord of any known arc, given the measure of its complement-arc; and the measure of the arcs from the chords, when these are first known to us; and likewise the knowledge of the [subtending] chords and the arcs, each from the other. There is great need in the science of astronomy [*chochmat ha-techunah*] for the knowledge of all this, as will be explained in the [next] Treatise.

Zerach bar Nathan said: This table is not found in any of the books, and the Rav's computations are sealed and shut up. It is taken from the … [?] Treatise …

[?] for [the section] "Tur" [?]; and those who come after him for the Sevillian [author] [i.e. Ibn al-Aflah / Jabir ibn Aflah of Seville?] [?].

[Note: this paragraph by Zerach ben Nathan (the editor of *Sefer Elim*) is badly abraded in the middle; the bracketed citation cannot be read with confidence [?].]

שער [Sha'ar — Section]

Now we set down a triangle **אבג** [a-b-g], right-angled, whose [angle] **א** [a] is the right angle, and we draw its circle [circumscribe a circle] about it, [or] half the circle surrounding it — in the manner stated above. We say that sometimes, and in many places of the science of astronomy (as will be explained later), the entire intention will be only to determine, by reasoning and computation as will be explained, and to know the measure of the length of any one of its sides, or the measure of any one of its remaining angles.

The sages investigated, in the inquiries of their reasoning, as is explained in their books, and found that this can never be done — it is impossible — unless there be first known to us, through study or through some other source that comes to hand, the measure of each of two of its sides (or even the measure of one of them alone, together with its ratio to the second), or even the measure of one of its two remaining angles. By this means we can determine, by reasoning, and know each one of the measures of its other parts.

This proceeds in the following way. Suppose the measures of the lengths of side **אב** [a-b] and side **אג** [a-g] are known to us at the outset, in some known measure, and we wish from this to know, first, the measure of the length of the remaining side **בג** [b-g] in that same measure. Then it is fitting that we square each of these lines **אב** [a-b] and **אג** [a-g] by itself, and take the [square] root of [the sum] gathered from their two squares added together; and that will be the length of side **בג** [b-g] in the measure we sought to know.

In like manner we proceed if the measures of side **בג** [b-g] and side **אג** [a-g] are known to us at the outset, and we wish to know the measure of the length of side **אב** [a-b] in that measure: we take the known square of **אג** [a-g], subtract from it the known square of **בג** [b-g], and take the root of the remainder — and that will be the length of side **אב** [a-b] in the measure we sought to know.

And likewise we proceed if what is sought is the length of side **אג** [a-g] [?] from sides **אב** [a-b] and **בג** [b-g], which are known — that is, we subtract the square of **אב** [a-b] from the square of **בג** [b-g], and take the root of the remainder; and that

will be the length of side **אג** [a-g] [?] sought. After this we may reckon it in any other measure we wish.

Sometimes what is known to us is the length of side **אב** [a-b] together with its ratio to side **אג** [a-g], and we likewise seek to know the measure of the length of side **אג** [a-g] and side **בג** [b-g]. This too will proceed in the following way, and according to the example I shall set out for it. Suppose we know that the length of line **אב** [a-b] is 8 cubits, and that its ratio to side **אג** [a-g] is as the ratio of the number 4 to the number 3 [?]; then the length of side **אג** [a-g] is … cubits [continued on next page]

[6 cubits — see below]. The sum of their two squares is 100; we take their root, which is 10, and we say that the length of side **בג** [b-g] is 10 cubits.

[Note: the worked example resolves as a 6-8-10 right triangle. **אב** = 8, **אג** = 6 (ratio 4:3 to **אב** would give 6), so **בג** = √100 = √(64+36) = 10. The text states **אג** is "6 cubits" (ו' אמות), then sums the squares to 100. The ratio "4 to 3" and the resulting 6 are flagged where letterforms are uncertain.]

In like manner we proceed if what is known is the length of side **אג** [a-g] together with its ratio to **אב** [a-b] or **בג** [b-g]; or if what is known is the length of side **בג** [b-g] together with its ratio to side **אג** [a-g] or **אב** [a-b]. The principle and root of all these computations is known from what was established in the study of [Euclid] Book 28 [?] [recte: presumably a reference to Euclid; the book-number is uncertain].

Once the length of each of the sides of this triangle is known to us in some measure — from which we have their ratios to one another known — we may also reckon them in another measure. That is, in the measure whereby side **בג** [b-g] is 120 degrees [units] (since it is the diameter [hypotenuse of the right triangle inscribed in the semicircle]), there will result for us the determined measures of sides **אב** [a-b] and **אג** [a-g], inasmuch as they are the [subtending] chords of arc **אב** [a-b] and arc **אג** [a-g]. By the chord-table that I shall arrange in my last Treatise, we shall know the number of degrees of arc **אב** [a-b] and arc **אג** [a-g]; and by this we shall know the measure of angle **ג** [g] and angle **ב** [b], in the measure whereby the right angle is 90 degrees, and also in the measure whereby it is 180 degrees.

Further, suppose at the outset only the measure of angle **ב** [b] is known to us; we say that the rest will be known to us from this. For angle **ג** [g] then remains known to us, since it is the complement of angle **ב** [b] to a right angle. By this means arc **אב** [a-b] and arc **אג** [a-g] become known, and accordingly sides **אב** [a-

b] and **אג** [a-g] also become known — inasmuch as they are their [subtending] chords — this being by the aforementioned table. And after that, if it happens that the length of any one of these sides becomes known to us in some other measure, we then reckon for the two remaining sides and know their lengths in that measure.

The example of this: suppose by knowing angle **ב** [b] there resulted for us, and it became known to us, that the length of the chord **אג** [a-g] is 72 degrees [units] in the measure whereby line **בג** [b-g] is 120 degrees; and the chord **אב** [a-b] became known to us thereby to be 96 degrees [units]. And suppose it happened to be known to us, from some source, that the length of line **אב** [a-b] is 8,000 cubits, say. By this we reckon and find that the length of line **אג** [a-g] is 6,000 cubits, and the length of line **בג** [b-g] is 10,000 cubits. For the ratio of the number 5,000 to 6,000 is as the ratio of the number 96 to 72 [?]; and the ratio of 8,000 to 5,000 [?] is as the ratio of the number 120 to the number 96. Such is the rule — you reckon so in every case of this kind.

[Note: the numerals 5,000 / 6,000 / 8,000 / 10,000 and the chord-values 72 / 96 / 120 do not all close arithmetically as printed — see Numerical impossibles. The intended triangle is again 6-8-10 scaled (6000-8000-10000 cubits), with chords of the acute angles 72 and 96 in the 120-diameter measure ($72^2 + 96^2 = 120^2$: 5184 + 9216 = 14400 = 120^2). The ratio statements as printed are partly garbled.]

Now we have explained how we reckon by reasoning and know the measure of each of the sides of the triangle that has a right angle, and also each one of the measures of its remaining angles, in the manner stated. This matter must be so engraved and fluent on your tongue, for you will need it often hereafter.

יסוד מוסד [Yesod Musad — "Founded Foundation"]

The aforementioned author further wrote afterward, in his language:

Know that the great principle we must lay down here and set forth for the science of astronomy is to know the laws of the triangle inscribed [*ha-mechukak*] on the curved [surface] of the sphere of your heavens, or upon its surface — [a triangle] made of three arcs of three great circles, from among the great circles which we said are inscribed there in thought. In order first to instruct you in the matter of this triangle, so that you may grasp its rationale and know its nature, I set out this figure [**Figure 7**]:

[Figure 7] Let circle **אבגד** [a-b-g-d] be the representation of the [celestial] meridian, half-day [circle], which you already know; and upon it I erect the half

of the equatorial circle [*galgal ha-mishor*, "circle of the plain/equator"] that meets it at point **א** [a] and point **ב** [b] of the meridian. I pass the half of the inclined horizon [*ofek*] — that is, [the horizon] according to the second position stated above — which meets the meridian at point **ג** [g] and point **ד** [d], and which meets the equatorial circle [continued on next page]

at point **ה** [h]. I further pass through this figure half of the circle **זמ** [z-m] [?], which is inclined to each of the three of them — and it is the [continuation of the] ecliptic [*galgal ha-mazalot*, "circle of the zodiac"] — and it meets the meridian at point **ח** [ch] [?]; it meets the equatorial circle at point **ה** [h] from it; and [it meets] the horizon at point **ך** [final-kaf] from it.

Now see how, in this figure, the two surfaces [planes] — which in their nature curve [are spherical] — were inscribed, and how by these intersections **33** [33] triangles were made, each of them composed of three arcs of great circles, which you must recognize and call by their names. Now [some] of them have right angles — one [right angle] only, such as triangle **זטב** [z-t-b], wherein the angle **א** [a] of this one and angle **ד** [d] of that one are right angles, since the meridian and the equatorial circle [are perpendicular], and likewise the meridian and the horizon stand each upon the other [perpendicular]; or [triangles] possessing two right angles, such as triangle **אהג** [a-h-g] or **גהב** [g-h-b], wherein angle **א** [a] of this one, and angle **ב** [b] of that one, and angle **ו** [w] of this and that, are right angles. The same rule holds for triangle **אהד** [a-h-d] and for triangle **אטח** [a-t-ch].

Now the angle **ט** [t] of triangle **אטח** [a-t-ch] is obtuse [*meruvachat*]; and likewise angle **ה** [h] of triangle **ההב** [h-h-b] is obtuse. The angle [of] triangle **מ[ש]ח** [m-(sh)-ch] [middle letter worn] [is sharp]: the angles **מ** [m] — each of them is acute [*tzarah*, "narrow"]. It is also possible that there be inscribed, further, on the sphere a triangle each of whose angles is a right angle — this being of the kind of the triangles of the figure we delineated above [the plane case]; but in this figure [the present one] it has no analogue. Because of all this, there is a triangle in which each of two of its sides is a quarter-circle, such as triangle **אהג** [a-h-g] of this figure; and there is a triangle of which side **אך** [a-kh] alone is less than a quarter[-circle], like [triangle] **אהד** [a-h-d], whose side **אך** is greater. This triangle has, further, certain other measures, as will be explained in what follows.

Now see that the law of these angles here [**Figure 8**] is, in many of its rules, like the angles formed by straight lines, which you already know. That is: all the right angles here are equal to one another; the corresponding [vertical] angles are likewise equal; and the sharp [acute] angle together with the obtuse one in the [same] figure equal two right angles. On side **אב** [a-b] and side **אה** [a-h] of

triangle **אבג** [a-b-g] it is said that the adjacent [angles] are angles **א** [a]; and that side **בג** [b-g] [?] is a chord [*yeter*]. The same rule holds for the rest. With a little study it will be explained to you, and you will know that none of the sides of these triangles can be other than less than a half-circle. The side that is greater than a quarter[-circle] I call "great"; the one that is less than a quarter I call "small."

Now I draw this spherical triangle **אבג** [a-b-g] of great circles, and I aim at corner **א** [a] of it; and I make it small, and describe arc **דה** [d-h], as it is in the figure [**Figure 9**]. Then arc **דה** [d-h] will be [an arc] of a small great-circle [?], and it is the [arc] spread between side **אב** [a-b] and side **אג** [a-g]. I say of the spread arc **דה** [d-h] that it is the arc of angle **א** [a]. The same rule holds between any two arcs of this kind — that is, that the arc of any angle is that arc spread, overlapping that angle, when the vertex of that angle is the pole of the [spread] arc. The meaning of this is that, by [the measure of] arc **דה** [d-h], arc **אב** [a-b] is inclined upon arc **אג** [a-b] [?] of the [great] circle; so that the ratio of angle **א** [a] to a right angle is as the ratio of this arc **דה** [d-h] to a quarter[-circle].

Diagram. Chord construction in a circle. A circle is drawn about center **ה** [he]. An arc **אב** less than a semicircle is cut off. From one endpoint a diameter **בהז** is drawn; from the other endpoint a perpendicular falls onto the diameter, defining the chord (*beka*) of the arc. The perpendicular **אג** is the chord of the complementary arc **אג**. Half the diameter **הי** is drawn parallel to **אג**, giving chord **הח** of the quadrant, with perpendicular **אש**. The figure illustrates that the chord (half-chord) of an arc equals half the subtending chord (*yeter*) of the doubled arc.

- ה (center): circle center
- א, ב (endpoints): arc **אב**
- ז (diameter end): diameter **בהז**
- ג (foot of perpendicular): defines chord of complementary arc
- י (point): half-diameter **הי**
- ח (quadrant chord end): chord **הח**
- ש (foot of perpendicular): chord **אש**

[Note: several labels in this figure are inferred from the prose; the printed diagram itself is not reproduced on the page (only referenced as ציור ו). Point-letters are taken from the construction text, not read off a drawn figure.]

Diagram. Spherical (celestial-sphere) figure. Circle **אבגד** represents the meridian (half-day circle). Upon it are erected: half the equatorial circle (*galgal ha-mishor*) meeting the meridian at **א** and **ב**; half the inclined horizon (*ofek*) meeting it at **ג** and **ד** and meeting the equator at **ה**; and half the ecliptic (*galgal ha-mazalot*)

inclined to all three, meeting the meridian, equator, and horizon at further points (**ה** ,**ח**, final-**ך**). The intersections generate a set of spherical triangles (the text states 33), each composed of three arcs of great circles, used to derive the laws of spherical triangles.

- אבגד (circle): the meridian / half-day circle
- א, ב (intersections): equatorial circle meets meridian
- ג, ד (intersections): horizon meets meridian
- ה (intersection): horizon meets equator / ecliptic meets equator
- ח, ך (intersections): ecliptic meets meridian and horizon

[Note: this is a dense spherical-astronomy schematic with many intersection points; a few prose labels remain abrasion-limited [?]. The triangle-count "33" is confirmed at native resolution.]

Diagram. Schematic of the angles of a spherical triangle, illustrating that right angles are mutually equal, vertical angles are equal, and an acute angle plus its adjacent obtuse angle equal two right angles — paralleling plane-angle relations.

- א, ב, ג (vertices): triangle **אגב**
- אב, אה, בג (sides): arcs / chord (*yeter*)

Diagram. A spherical triangle **אבג** of great circles, with vertex **א** singled out. An arc **דה** is described between sides **אב** and **אג**; this arc, having vertex **א** as its pole, measures angle **א** — its ratio to a quarter-circle equals the ratio of angle **א** to a right angle.

- א, ב, ג (vertices): spherical triangle
- ד, ה (endpoints): arc **דה** measuring angle **א**

We find, then, the following. The arc that subtends a right angle is a quadrant of the circle. The arc subtending any obtuse angle is greater than a quadrant, and the arc subtending any acute angle is less than a quadrant.

Know, too, that many of the laws governing this [spherical] triangle and its properties are like those of the triangle made of straight lines and its properties. For when it happens here that one side of this triangle — side דה — equals side הז of it, then by that fact angle ז is equal to angle [ד], as will be explained in Lesson 6. The converse holds as well: when it happens that angle יח[?] equals angle צ[?], then side דה is by that fact equal to side די[?], as was explained in Lesson 7.

And whenever, in two triangles or more, each of the sides of the one is equal to a corresponding side of the other, then each one of the three angles will be equal

[to the corresponding angle] of the second triangle, and each angle of the one will be congruent to its counterpart in the other. Likewise I say of any two triangles — for instance triangles אבג, דהז: when side אב of the one equals side דה of the other, and side אג equals side דז of the other, and moreover angle א of the one equals angle ד of the other, then each of those angles will by that fact be equal to its counterpart in the second, and the triangle will by that fact be congruent to the second triangle, as is shown in Lesson 8.

So too, when two sides of the one are equal to the two sides of the other — each to its fellow — and it happens that the angle which those two sides enclose in one of them is greater than the angle which the two equal sides enclose in the other triangle, then the third side of the one — the side that subtends the larger angle — will be greater than the third side of the other. Know, too, that here as well the side that subtends the largest angle in the triangle is by that fact the largest of the sides, and the converse likewise, as was explained in Lesson 14 — that is, the largest side in the triangle subtends the largest angle, as was explained in Lesson 13[?].

You already know from earlier that the chord (*beka*) of any arc is the perpendicular dropped from one of its ends onto the diameter drawn from its other end; and one chord is shared by the two complementary arcs that, when joined, make up a semicircle. You likewise know that any arc that has a chord is less than a semicircle, and that the largest of the chords is the chord of a quadrant, which is half the diameter — that is, sixty degrees [parts]. You also know that the *complement-arc* of any chord-bearing arc is the arc by which the arc in question exceeds a quadrant of the circle, or falls short of it. Likewise it is known that the *supplement* of any angle of any of these triangles is the measure by which that angle exceeds a right angle, or falls short of it. [The phrasing here is doubled and partly garbled in the source: "by which the given angle exceeds the measure of any angle of any of these triangles," repeated.][?]

Now let it be known that most of our concern here is only with the [spherical] triangle of this kind — such as triangle אבג, made of three arcs of great circles — of which angle ב is a right angle. For in many places in the science of astronomy the whole aim is to determine, by reasoning

and calculation, the measure of one of its sides, or the measure of one of its remaining angles — the unknown one — out of its two angles, as I shall show you with some of these cases in the chapter that follows.

If you examine this triangle, which has a right angle, you will find that it admits six [possible] cases, which you must bear in mind:

1. There is the case in which each of the two sides containing the right angle is a quadrant of a circle. It then follows that the third, remaining side is likewise a quadrant of a circle, and that each of its two remaining angles is a right angle as well.

2. There is the case in which each of these two sides containing the right angle is greater [than a quadrant]; it then follows that its third side is small [less than a quadrant], and that each of its two remaining angles is obtuse.

3. There is the case in which these sides containing the right angle are both small [less than a quadrant]; it then follows that its third side is small as well, and that its two remaining angles are acute.

4. There is the case in which one of these two sides containing the right angle is large [greater than a quadrant] and the other is a quadrant; it then follows that its third side is a quadrant, that the angle facing the quadrant side is a right angle, and that the angle facing the large side is obtuse.

5. There is the case in which one of these two sides containing the right angle is small and the other is a quadrant; it then follows that the third side is a quadrant, that the angle facing the quadrant side is a right angle, and the angle facing the small side is acute.

6. There is the case in which these two sides containing the right angle are such that one is large and the other small; it then follows that its third side is large, that the angle facing the large side is obtuse, and the angle facing the small side is acute.

The sages have already clarified the rule for all these consequences, and the rest — which I have not seen fit to set out here, so as not to be lengthy. From the cases of these rules that we have arranged for the triangle that has a right angle, you may learn — and may verify the general rule embracing them all — which they set forth in this chapter and demonstrated with true signs and proofs. The rule is this: one and the same relation holds between each of its two remaining angles and the side that subtends it — namely, that if [the angle] is a right angle, then its subtending side is a quadrant; if it is obtuse, its subtending side is greater than a quadrant; and if it is acute, its subtending side is accordingly small [less than a quadrant].

To make this clear, take this triangle אבג, of which angle ב is a right angle. I posit first that angle א is also a right angle, and I say that side בג, which subtends it, is a quadrant of the circle — for it must necessarily [be so]; while side אב is small [less than a quadrant], because the two angles א and ב are [both] right angles, and accordingly side בג will equal side אג, each of them a quadrant. Here, then, it has been shown that if angle א is a right angle, it follows that side בג, which subtends it, is a quadrant.

Next I posit angle א to be obtuse, and I set it down in figure [ב]אד[?] as obtuse[?]; then side בד will be like [a quadrant — and being obtuse it makes side בג large], as we explained. Therefore side בג is large, and the fact that angle א is obtuse is the cause of this[?].

Next I posit angle א to be acute, and we set this down in figure באה[?] as acute; then side בה will be a quadrant[?]. Therefore [side בה is a quadrant... and being acute makes] side בג small, as will be shown — and the fact that angle א is acute is the cause of this. So angle ג, together with side אב which subtends it, behaves just as we have said.

Now, so that the measures of this triangle — the one with right angles — may be set out in order in your mind, I will lay them out here in a small table for the eye to see, in keeping with what we said above.

Table: to know the six measures of the triangle made of great circles, when it has a right angle

The two sides containing its right angle	Each of its two remaining angles	The side subtending the right angle
quadrant and quadrant	both right angles	quadrant
large and large	both obtuse	small
small and small	both acute	small
quadrant and large	right angle facing the quadrant; obtuse facing the large	quadrant
quadrant and small	right angle facing the quadrant; acute facing the small	quadrant
large and small	obtuse facing the large; acute facing the small	large

Sha'ar. Now that we have arranged this small portion of the law governing this triangle made of arcs of great circles, I come to inform you of some of its rules and the inferences drawn from them. Give your mind to them.

For everything we arranged concerning the arcs of the circles, their chords, and the rationale of this triangle and its laws, was only to lay down a sound method by which — through reasoning and calculation, as will shortly be explained — we may determine and know the measure of any angle of this triangle: that is, what ratio it bears to a right angle; or the measure of any of its sides, in the measure where a quadrant of the circle is 90 degrees. The sages found and established that this cannot be done unless we first know, regarding some quantity in a triangle of theirs that has a right angle — with which alone is our whole concern here — either the measure of one of its two remaining angles, or the measure of one of its sides, or one [side] of them together with one angle. By means of this we can then proceed, by the procedure that will shortly be explained, to determine and know the measure of any one we wish among its three remaining parts.

The sages labored greatly over this matter until they established it and brought its rule to light. After much investigation, it occurred to them to divide it, in its essence, into three mighty principles (*ikkarim*), by means of each of which the discerning student may find what he seeks, as we said. I now set out these three principles, which must be arranged in your mouth [memorized]; for as a person grows familiar with these matters and the like,

so he will gain insight and succeed in the science of the configuration of the heavenly spheres and their signs; and according to how negligent he is of them, so will [knowledge] hide its face from him.

This is taken from Ma'amar Alef, Lesson 13, of Jabir [ibn Aflah] al-Ishbili [גובי יר האשבילי = Geber the Sevillean, i.e. Jābir ibn Aflaḥ al-Ishbīlī, author of the *Iṣlāḥ al-Majisṭī*].

The First Principle is what the sages brought forward by means of the signs and proofs set out in order in their books. They demonstrated that every such triangle אבג, made of arcs of great circles, has a wondrous property: namely, that the ratio of the chord (*beka*) of any one of its sides to the chord of the arc of the angle facing it — the angle which that side subtends — is always equal to the ratio of the chord of its second side to the chord of the arc of the angle of it [facing that side], the angle which the second side subtends. And so likewise is the ratio of the chord of the remaining third side to the chord of the arc of the angle it subtends. That is to say, the ratio is one and the same for each of the sides of this triangle

relative to the angle which that side subtends. In other words, the ratio of the chord of side בג to the chord of the arc of angle א equals the ratio of the chord of side אג to the chord of angle ב of it; and by that fact, it is also the ratio of the chord of the remaining side אב to the chord of the remaining angle ג of them — whether this triangle אבג has right angles or not.

Lesson 8[?], of Geber.

The Second Principle holds for this triangle when it has one right angle, such as triangle אבג, of which angle ב is a right angle. Its content is what the sages clarified likewise, and demonstrated: that the ratio of the chord of the complement-arc of the side that subtends the right angle, to the chord of the complement-arc of the second side, equals the ratio of the chord of the complement of the third side to the chord of the right angle, which is 60 degrees [parts], as you know. That is to say, the ratio of the chord of the complement of side אג to the chord of the complement of side אב equals the ratio of the chord of the complement of side בג to the chord of angle ב, the right angle, which is 60 degrees [parts].

Lesson 14[?], of the first [book] of Geber.

The Third Principle likewise holds for this triangle when it has one right angle. Its content is that the ratio of the chord of one of its two remaining angles to the chord of the right angle, which is 60 degrees [parts], equals the ratio of the chord of the complement of the third angle to the chord of the complement of the side that subtends it. That is to say, the ratio of the chord of angle א to the chord of angle ב, the right angle — which is 60 degrees [parts] — equals the ratio of the chord of the complement of the remaining angle ג to the chord of the complement of side אב, which subtends it. There, I have now shown you an illustration of the rules of these three principles.

Lesson 15[?]... ["מוסד"][?]

Yesod. Now that these three principles and their rules are known and arranged in your mind, take and reckon with triangle אבג, of which angle ב is a right angle, and observe that by means of each one of these three principles we can compute, by reasoning, and determine the measure of each one of its parts — whichever we wish — provided that there be known to us beforehand, of any such [triangle], the measure of two of its parts together with the right angle. This will proceed in the following way. Suppose there is known to us at the outset the measure of side אב and of side אג, and we are asked to know from this, by

and by reason determine the measure of each of its three remaining angles. This too proceeds according to the same procedure. We say that, by the First Principle, the chord of the known side אב [AB] is here the first number; the chord of angle ב [B], which is 60°, is the second number; the chord of the known side אב [AB] is the third number, set in proportion to the chord of the unknown angle ג [G], which is the fourth number. Therefore, just as we laid down at the beginning of this chapter regarding any four numbers in proportion: if we take the third number, which is known, multiply it by the second number, which is 60°, and divide the product by the first number, there comes out for us the known number of degrees of the chord of angle ב [B], which had been unknown. Then, by the table of arcs and their chords, we can determine and learn the measure of angle ג [G] that we sought to know.

By the Second Principle we compute further and learn the measure of side בג [BG] in the same way. We say: since the chord of the arc that is the complement of the known side אג [AG] — which is the first number — stands to the chord of the complement of the known side אב [AB] — which is the second number — as the chord of the complement of the sought side בג [BG] — the third, unknown, number — stands to the chord of the right angle, which is 60° and is here the fourth, known, number: therefore, when we multiply this first number by the number 60, which is here the fourth number, and divide the product by the known second number, there comes out for us, exactly, the number of degrees of the chord of the complement of side בג [BG] that we sought to know. By the table of chords we determine precisely and learn the measure of side בג [BG] itself.

After this we compute for angle א [A] and learn its measure by the First Principle, since side בג [BG] — which is the hypotenuse — is now known. For the chord of its arc, the unknown one, stands to the chord of the known side בג [BG], the hypotenuse, as the number 60 stands to the chord of the known side אג [AG], just as we explained according to the procedure. From this the measure of the chord of the remaining angle א [A] will be known to us. We have now established how the measures of angles א [A] and ג [G], the remaining ones, and the measure of the [remaining] side, become known to us, once the measures of side אב [AB] and side אג [AG] were given to us at the outset.

Now let us posit that what was known to us at the outset were two of the sides of triangle אבג [ABG] — namely side אב [AB] and side בג [BG], which enclose the given right angle. We say that each of its three remaining parts thereby becomes known to us by the following procedure. By the rule of the First Principle, the chord of the complement of the known side בג [BG] — which is the first number

— stands to the chord of angle ב [B], the right angle, which is 60 and is the second number, as the chord of the complement of the unknown side אב [AB] — which is the third number — stands to the chord of the complement of the known side אב [AB], which is the fourth number. Now when we multiply the first by this fourth and divide the product by the known second number, the measure of the chord of side אב [AB], which had been unknown, comes out known to us. Then, by the First Principle, we compute and learn the measure of angle א [A], and also the measure of the chord of the remaining angle ג [G].

Again let us posit that what was known to us at the outset was the measure of its two angles alone. We say that the rest thereby becomes known in this way. By the Third Principle stated above, the chord of the known angle א [A] — which is the first number — stands to the chord of angle ב [B], the right angle, which is 60 and is the second number, as the chord of the complement of angle ג [G] — which is the third, known, number — stands to the chord of the complement of side אב [AB], the hypotenuse, which is the fourth, unknown, number. Now when we multiply

the third number by the number 60 and divide the product by the first, which was known, there comes out for us the chord of the complement of side אב [AB], which had been unknown. By the table of chords we compute and determine it, and after this we compute, by the First Principle, and learn the measure of side אג [AG] and side בג [BG], the remaining ones.

Again let us posit that what was known to us at the outset was the measure of side אג [AG] and the measure of angle א [A]. We say that the remainder thereby becomes known to us in this way: by the First Principle we compute, according to the procedure stated, and learn the measure of side בג [BG]; by the Second Principle we compute and learn the measure of side אב [AB] as well; and by the First Principle we learn the measure of the remaining angle ג [G].

Again let us posit that what was known to us at the outset was the measure of angle ב [B] and the measure of side אב [AB], which is the hypotenuse. We say that the remainder thereby becomes known by this procedure. By the First Principle we learn the measure of side אג [AG]; and by the Second Principle we learn the measure of side בג [BG]. For the chord of the complement of the known side אג [AG] — which is the first number — stands to the chord of the complement of the known side אג [AG] — which is the second number [?] — as the chord of the complement of the sought side בג [BG] stands to the number 60. When we compute this by the rule of the statement above, we determine and learn the

measure of the sought side בג [BG]. After this we compute for angle א [A] and learn it by the rules of the First Principle.

Thus it has been explained how to compute, by these three principles set forth, for every triangle of this kind that has a single right angle, and to determine and learn whichever part of it we wish, once the measures of any two of its parts that lie with the right angle are known to us at the outset.

And if this matter that we laid down in these explanations should trouble you — when we said that by the table of arcs and their chords we compute and learn the measure of a side and the measure of the arc of an angle, once their chords are known by the rules of the principles stated — and you should say that this is not sufficient, since a single chord is common to two corresponding arcs, so that we would be in doubt and not recognize which of them is the one sought: to resolve this, I tell you that for just this reason I prefaced and informed you above of the matter of the six measures that the parts of this triangle possess — the triangle formed of three arcs of great circles having one right angle — in their relation one to another, as they are set out in the small table that I arranged above according to it. From it, it will be clarified for you, in every triangle of this kind, and you will recognize which of the two corresponding arcs is the one sought — whether the greater or the smaller.

This is a letter that R. Yashar [R. Yosef Shlomo, i.e. Delmedigo] wrote to one of the nobles of Judah, dignitaries of the people, apologizing for the delay in sending answers to his questions.

A tree whose roots are many and whose boughs bear laden fruit — towering as cedars in his height, and a strength is his as of oaks; nothing was likened to him; lovely in his fragrance beyond all the trees of Eden. Before him all rose up as thornbushes [?]. Beautiful of bough and shady of thicket; in its branches all the birds of heaven nested. Its leaf is for healing the famished soul, and those who eat of its fruit shall flourish; in old age they shall yet be full of sap and fresh.

A father of humility, righteous, ruling in the fear of God — to him it became a treasure and a possession. He is the ascending dignitary, and a river goes forth from Eden encompassing the land of Havilah. He is my lord, my Tachkemoni, our refinement, the Atzni [?], the chieftain, the honored R. ——, may his Rock keep him.

By way of a deed [or: "And so it was"], the work of the sapphire pavement was before my eyes; like the voice of the Lord God walking in the garden it was heard in my ears, and its savor was as wafers in honey to my palate, and its fragrance as the fragrance of a field that the Lord has blessed. I had been slumbering, hiding myself like one in a dungeon, in a cave, and your letter roused me. I had been sunk in deep waters — "counsel in the heart of a man" — and your strong hand drew me up. I was gripped by the spell of visions, solitary in my appointed times, calling out to discernments. The trumpet of the heights [?] sounded; your urgent decree, with a blast of alarm you sound, out of the upheaval. By the lock of my head it took me. Through the lattice window I looked out from the clear glass that shines, and behold, the sun had come forth upon the earth, the dawn shone radiant as the morning-twilight; my whole house was filled with light, and a brightness shone upon me — "for the commandment is a lamp and Torah is light." "If you wish, say; if you wish, say" — by reasoning, I have seen this.

Let me recount: what is this voice that is in my ears? A pleasant voice, a glorious voice, a praised voice; and my spirit was stirred — a golden bell and a pomegranate. My head was filled with dew, as the dew of Hermon. I rejoiced, I exulted, as over all wealth, like a bridegroom going forth from his canopy, when one of the seraphim came to me, and in his hand a glowing coal, paved over with love, like the dawn looking forth. "Every word of God is refined" — cold water upon a weary soul; I drank, I sucked from the honeycomb and the comb. There were the bdellium and the onyx stone, every precious stone; I gazed, I saw a great light like the light of the pure menorah, and its seven lamps upon it — sevenfold. To choose the tongue of gold [?]: "reproof in secret is good," even from one revealed; yet when it is from love it shall be prized. "Who is this," and the mantle upon her head, upon a pavement of crystal and marble and pearl and porphyry? And I, within the exile, like a vine [?] ascending from the wilderness, perfumed with myrrh and frankincense. It gives off fragrance over my whiteness [?]; my heart rejoiced and my glory exulted; it became a delight to me. My eyes were lit from the light of the sun and the light of the moon, "hedged about with roses" — by the fifty gates of understanding, black fire upon white fire. "A tower of David, built for trophies" — the city of beauty [?], where every bird, every winged thing, nested there. All the luminaries of light in the heavens, every form — in my gazing I gazed. Your signs shine like the splendor of the firmament, and like stars their appearance is as the appearance of the sapphires; their faces are faces of flame, spreading wings upward like the work of the cherubim. Where is the scribe, where the weigher who could do as your deeds and your mighty acts? And the mighty in strength are, beside your worth, as a lizard and the like [?]. As for the sciences and the astronomies, from above them they pour forth heavy clouds

[?] — empty [?] — and like the kindling of one who casts sparks; and the sharp ones of the rhetoricians draw near, fleeing, returning, running, racing, crowned [?], hewn cutters of wings, when they seize you thus you crush them with the palm; "lightning, emerald," you scatter them. "Be not afraid of the noise of the shooters"; their arrows go through them; you send them out shattered, set free, slack and weakened, casters of the weak [?]. "A contempt of men" [?]; the mighty of heart were despoiled, they did not find — all the men of valor — their hands and their feet [?]. And the princes, as those pierced [?], shall not stand before your eyes. Rock-dwellers shall come and be counted as dust before your terror and before the glory of your majesty. By myself I have sworn: "if whole and so many" — those who esteem you, who hear the words of

your mouth, who delight in the sweetness of your discourse, and who smell the fragrance of the pleasant savor of your nard and your cinnamon — those who drink from the brook of your delights. Even though they saw your goings, your footsteps were not known.

To them is hidden a covering of ascents [?]; they searched out, they sought out [?] every search; the perfumers multiply, and species upon species of various kinds, raising smoke — each one according to its measure; yet in its compounding they will not do the like, and they called its name "Mezakeh," for he did not know what it was — like that incense of the [House of] Abba [?].

In sum: behold, this we have explored, that so it is; for "every mouth shall give thanks to You, and every tongue shall swear to You." For judgment, rhetoric condemns the mouth — "flies of death" make it stink; but you, "an ointment of perfume you pour forth, the place where the word of the king and his decree go forth." And to me, how precious are your words, and the scent of your oils — "your name is as oil poured out." How awesome are your deeds, and how beautiful the steps of your pen! "Your signs we have not seen" like yours, and our fathers recounted to us that of old this was in Israel — distinguished men, a treasure above all the silent peoples [?], would do. Were it not for the wrath of the voice of your thunder, like the supernal sphere [?] above my head, and the voice of the tumult of "a watcher and a holy one" such as you — your impressions [?], for I shall see your name, your cord and your seal, your Urim and your Tummim — I would say, "Blessed are you, and blessed is your discernment." Your voice I heard, and my belly trembled, and my flesh became as one pierced [?], and for a moment my skin [shuddered] when you said to me, "Do and do not do." Return, send back the books [?], and do not delay any longer to pay the vows; enough to occupy oneself with words, to say "return upon the books," and do not

[delay] further [?]. "Shall a shofar be sounded in a city and the people not tremble?" And even though your letter goes searching, covered with a veil, wrapped like the necklaces of the chain [?] — your serenity rose in my ears, and I heard the voice speaking to me — "in my cymbals, hear" — bells and pomegranates, a tongue of scarlet, whitening faces; and even the shades [Rephaim] will feel it; and those who dwell in graves of bitterness, when their fellows pile [earth] upon them, will awaken, will rouse themselves, will even answer, saying: "But we are not guilty, for we are honest."

Indeed, as I received reward for the inquiry [derishah], I shall receive [reward] for the abstention [perishah]; for talk and conversation are with me all day long — "the Torah that Moses commanded us as an inheritance"; and for this it shall be called to me "a wife" [ishah], in that until now I went after the necromancers and diviners, the happy and warm ones [?], the books of the Greeks, the refusers and bramble-thorns [?]; and I went and occupied myself with the work of the heavens, "fixing" and weighing bricks. With Lavan I dwelt [a pun: "I dwelt with the white/Lavan"], and from the words of the Lord [?] — the study of the Gemara — I turned aside, and in the footsteps of my holy forefathers I did not walk. This is what the teachers of my youth caused me — half a champion of repetitions [?]. The philosophical speculations did not let me bring back my spirit, after I saw it; and I became "concealed, a nursling" all the days of my host; and they were "as lambs for my clothing." In sum, "a saddle for my head" — for by the spirit of breadth that lifts up, the way of Beth-El, the straight road, pushed me, and by the force of your drawing you drew me with the cords of new ploughmen, who, to bind the souls and with new spices, seek evil; and when the senses of my intellect were sanctified, the merit of fathers, of repetition, stood by me, and they poured upon me a spirit of favor. My thoughts murmur in the streets of inquiry, limping and halting upon the two branches; slack are they, slack; "and a blessing without end [bli dai]" — I felt, by the force of those stumbling-blocks, those who spread a net for my feet, in whose pit my own schemes were caught. And the days [?] when I reckoned to widen my steps — and after the inquiry I cast them behind my back and nullified them as "the dust of the earth" [Aramaic: כעפרא דארעא, "as the dust of the ground"]; and may "my doves" [yonai] — and "my eyes," "my thoughts" — no more be dim, nor grind "with gravel my teeth" so as to be blunted. And I shall leap [shall recount, eddadeh] in the Torah of God all my years; I shall lead them in procession up to the house of God. Therefore I tarried and delayed until now; and "one occupied with a commandment is exempt from the commandment." Cleanse me, my lord, cleanse me, and launder me of my iniquity, and do not pursue me to my full debt [?].

And now, blessed are you of the Lord — why have you raised me up, lifting me from the lowliness of my condition, from holes in the dust and from clefts of the rocks where one digs out pits, fit only for bats? You have set me among the wandering stars, my dwelling like one of the host of heaven on high, like the Wain in the north and the Scorpion in the south — yet I am like a juniper in the desert, dry as a withered thing, a brier and a thorn-bush. Who could make me like the grass on the rooftops, at the head of the roofs a leader [?]; or, hidden among the baggage, concealed within your house — a house of assembly for the sages, a fair place, fruit of the olive? "You shall not go up by steps upon My altar" — an altar of earth, while I am but a man, likened to a mere nothing. In my house there is no bread and no garment, no fine flour for cakes, no meal for a cook's labor to set on the mouth of the pot, to draw off the scum. Why do you raise me up in the midst of my days, above all song and praise? My soul is sickened as a lodge in a field of cucumbers; I am poorest of all wisdom, of all the knowledge of the destitute — a mere shade of the gourd, a covering whose shade is scant against its heat. ✻

Your humility has made me great; from the mighty waters you have drawn me out, bitter though I was. You have prolonged your discourse in a "Song of Ascents to Shlomo." [recte: an allusion to Tehillim's Songs of Ascents] "God — Your statutes have been my songs," the chants of Your sanctuary, forgotten — for the choirmaster, upon the Sheminith. And behold, for my low station it is like the shadow upon the steps that turned ten steps backward [allusion to the dial of Achaz, Yeshayahu 38:8], for You have magnified Your word above all Your name — to bear me up on eagles' wings, at the head of the heights, as a nurse carries the suckling child. By the breath of Your nostrils, by a willing and humble spirit, with love and brotherhood You cover my blemishes and my disgrace. As with strands of scarlet thread You bind and stop me up, and seal me like dough, mending my bare and torn places; and into the gaps and the holes you put forth your voice, a mighty voice — the voice of scribes [?]. At the voice my lips quivered and my thoughts shuddered; they sank like lead into the bitter waters. Would that I were the shadow of a real tree, or the shadow of a wall that has some substance — and not the shadow of bees! [an allusion to a fleeting, insubstantial shade].

By your life — however much you add to me, you diminish; and all who know and recognize me will look to me and cut away from there a branch [?]. For from the spirit that is upon you, you set upon me praise upon praise and patch upon patch; and over all your glory a covering of vanity [?] — making an ear for the basket, a reed to make a handle for the sieve. The wise [?] have fallen to me in

pleasant places, and a spirit lifts me up onto the high mountains. Spare and have pity on those precious titles, for they are pleasant: keep them in your own belly, bind them to your maidens [?], let them be pendants for your throat — for you yourself alone; or distribute them to the chiefs of your multitudes. And that good oil that runs down upon the mouth of your robes — offer it, please, to your governor. The place is too cramped for me; in my house and in my small town it cannot be contained. Why should I pursue toil and wind and the labor of iniquity? Like a woman in labor I groan; trembling seizes me, and my mouth speaks in fear [?] — for the bed is too short to stretch out on, and everyone who adds to me only diminishes, as one who sows on sand: what he plants takes no root but breaks open and does no good — indeed it does harm. And if he is among an evil company, corrupted, why should he raise a clamor — evil upon evil?

Truly, will God dwell on the earth — even though the heavens and the highest heavens cannot contain Him? Filth [?] is the wick of the speaker [?], who from finding your delight and speaking a word makes known [?] among peoples a deed, for the glory of matter — they cannot contain it; and one of those who pour out [?] from their blessing, and from their bones nothing is lacking [?] — a thing of praise; for the heavy is in the sieve. As for rank — they do not despise the light of lights when it sends its sparks across the open streets, into mire and clay, until the day breathes and the shadows flee; and when the cloud forms, the shadow of the earth rises up [?] — for there is no filth in it, and no uncleanness has touched its flesh; still it is its radiance, its glory, its splendor. The natural agents, as the philosophers of inquiry have said, transform their subjects to their own nature — for the hot generates heat and the cold, cold; so too the ways of those who possess titles: they apportion from their honor a portion [?]

to others; he who lowers the lowly and lifts up him who is bowed down, who raises on high. I have known, my lord, I have known, that the Lord has set you above all the sons of your generation; He has distinguished you from the first of your line as a leader and head of your people, and established you in a wondrous, exalted form that has become the chief cornerstone — one which the hand of nature is too weak to attain. He desired to give you as a gift; it was your portion. Hewn out from the womb, from conception your formation was set firm — "princes in place of understanding," a crown of Torah and a crown of priesthood, a fortified city, a faithful town, a perfected building, an enduring structure, "a planting of the Lord, that He might be glorified" — a fitting, becoming dwelling for the king's daughter who dwells there, abiding in your tents and in the palace of your sanctuary. How lovely is this pairing! How pleasing your lodging! May there be peace in your strength.

From the beginning of your cradle, when you were tender and anointed with the oil of seed [?], they guided and habituated you to the choicest food, set out for those of delicate health; and clothed you in scarlet — in fine linen, in robes of priestly garments. Upon scarlet you were nurtured, and nothing at all was lacking to you of all that you desired. From the king's delicacies your soul was sated — hidden away in chambers of splendor, guarded from any evil wind, that neither sun nor scorching heat might strike you; and the dreadful cold and the terror should not come upon your head, so that you need not hear the sound of the thunderclap, of thunder, of hail and hailstones — when He lays bare the forests or shatters the cedars, and when the Lord makes lightnings. And you lie within, in the innermost chamber, upon couches of fragrant woods and beams of cedar, propped on cushions, supported among apples [allusion to Shir HaShirim 2:5], like one of the king's sons — myrrh and cassia, nard and incense, for a sweet savor — fed on the choice yield of the sun and the choice produce of the moons. Behold, all these are the arrangements of upbringing, of great worth, for raising children destined to awaken the powers sunk in the depths of the matter of their bodies, and to stand and serve in the courts of kings and in their palaces. And when their time comes they will be princes to their nations; they have not ceased nor changed their function. The servants of your father, until you yourself entered the company of those endowed with free choice, were careful and watchful to guard you from every evil thing.

And also, in faithfulness, the Rock that crowns has loved you. He has adorned you with crowns of the esteemed virtues; He marvelously magnified counsel, made wisdom great, to seat you among the high-born, among the makers of the work [?], by the wondrous outpouring [?] — to clothe you with the mantle of culture and statecraft, to teach you book and reckoning [?] and the counting of all things; nothing was hidden from you. And to teach you a clean and easy craft — "this is the Torah of the burnt-offering" — until the first intelligibles and the received traditions of your fathers struck deep roots, planted in your soul, and engraved firm marks upon the walls of your brain. And even if all the winds in the world were to come and blow against you, they would not budge you from your place. You do not change your color nor exchange your nature. And you too go forward by the strength of that nourishment with which, from your youth, the first natural progenitors fed you, in the first of the years — who poured their powers into the building and into the proportioning [?] of the bricks of your structure, and into the forging of the blending of your foundations — until you came to the mountain of God. You did not depart from within the tent; you did not cease to range and to search and to seek out, by the light of your torch, after every existing thing, bodily and spiritual. You waved your hand over all matter and

form; you soared on high like an eagle. No pure bird flew above you; "all wing" — your hands were not bound from comparing form to form, and from generating offshoots and branches and boughs. Fruitful as the palm-fronds, until you became a crown of glory and a diadem of beauty. O Rock — testimony bound up, Torah sealed up [allusion to Yeshayahu 8:16]: from you, together; from you the cornerstone, the support and the staff. And you do not deviate from the coinage

that the sages have minted. The Torah has instructed you to keep your hand on the lower side — whether the matter be "tradition" [masoret] or "scripture" [mikra]; for this and that called out, and "this one to that one cried." And I too have heard from the mouth of the Almighty, and the vigor and strength of the choicest deeds, the wondrous treasures, chained and revealed, from the unfolding of the inner mysteries, unfolding [?]. May your fountain be blessed — "the Lord made me the beginning of His way" [Mishlei 8:22], from of old, eternal, before His works, His acts of old, with me. Afterward, in greater profusion — and with what shall I come to meet you in return for the gift? With what shall I [?] His face, with what bow down, with what make atonement?

Therefore let me speak, please, with the answers to the questions that I was asked by an honest man, wise as one who sees with clear light. Only let the appointed time grant me room — eight days more, or ten; let it be no restraint upon your spirit, and I will return you words according to the reach of my understanding. And if it be too meager in your estimation, forgive me; and thanksgivings and crowns to the head of the one who hires out his bread — bread of the mighty for strangers like me, an exile in a distant land. For this I lift up my palms toward the inner sanctuary of your holiness. If now I have found favor in your eyes, do not cast me away from before You, and do not take Your holy spirit from me; for Your converse will comfort me, and as I go it will guide me, and when I awake it will speak with me. Remember me for good, and inscribe me upon all Your hosts. And I am a prayer to the awesome God, wondrous in deeds: may He remove from my lord every sickness, may He make his name great and establish his throne, prolong his days, let him see the seed of the wise; and may his descendants be a sign and a wonder all the days, in the midst of Israel. May God raise him higher and higher, upon the heights of the true felicities and the imagined ones; may He set his name for eternity, and bless him and his house in peace — according to his exalted soul, a decree of sapphire and gold of Ophir, hewn from the rock of wisdom.

And the soul and desire of him who is joined to your service, his ear bored through with the awl to serve you and to bear your decree, bending his shoulder — the young

Yosef Shlomo, natural physician, son of the eminent Rav, the Gaon, our teacher the Rabbi Eliyahu Delmedigo (may his Rock keep and preserve him), a man of Candia.

And these are the responses of our teacher the Rabbi Yashar to each and every question of the aforementioned sage.

> Life is short, and the craft is long; the time is sharp [?: lit. "the time is acute" — rendering Hippocrates' "the occasion fleeting"], the trial perilous, and judgment difficult. And not only must the physician be ready to do what is fitting, but also the patient, the attendants, and the external circumstances.

With this pleasant introduction, which pertains to the work as a whole — for all the brevity of its words and the constriction of its rhetoric — the honored elder excuses himself for setting down in a book the teaching of this craft, which had been like the hidden glory of a king's daughter within, concealed and sealed in the treasuries of the chosen few; and which the ancient physicians would hand down, each man to his fellow, by way of received tradition, by word of mouth and not in a scroll — for it was not given to be written. And this too was passed from one man to another, to a holy one [?], from father to son and from master to disciple, to the modest and the worthy. This custom spread also among the Greeks, in the rest of the sciences as well, as appears from the complaints of King Alexander against Aristotle

regarding the publication of the powers of the natural faculties. In the account of the life of Alexander — and on this matter Galen too wrote, in his book *On the Substance of the Natural Faculties* [?] — and the sage [Galen], of blessed memory, remarked upon the fact that [Aristotle] had written and published his *Physics* [Sefer HaShema — the *Physica*]; and Aristotle's apology to him, in which he laid out, in keeping with [?] their letters, that Aristotle and Theophrastus had written some part of their books [?] for the multitude, but that real comprehension is reserved for those who love [it]. So far his words [Plutarch's, end of citation]. And if so, what was the reason that moved Galen [?] in this — to publish to the multitude the secrets of wisdom, to expose her hidden things, to remove her veil and bare her head? And why did he breach her hedges, so that every passerby plucked from her? For thus they spoke, as with fire: "command upon command"

— turn aside, withdraw; do not reveal! And it is he who has hedged her way with thorns; and he brought himself under the bond of the covenant, as appears in the famous oration — the Oath of Hippocrates.

And at this every reader of the text of his oath will be astonished; and when he hears the words of that curse written in that book — not to depart from the commandment, to teach this wisdom only to the cherished ones and the beloved disciples, "he shall make his flesh as if cut" [?] — he will shudder with a great shuddering, and there will be only terror, saying: How is it that the man so faithful in his covenant and steadfast in his word turned into another man — from the time he set his oath [?] he changed his savor and altered his word? Therefore his thoughts answer him: why should he sit between the sheepfolds, to hear the hissings of the flocks? "For the teachers [?] scatter you" — a sound of cries; I hear lions roaring against you, they lift up [their voice].

And now, my eloquent ones, my beloved, my friends — gather, and let us come to the cities of refuge, and the spirit of your understanding will answer them. Hush, all flesh! For He is the One who tests the kidneys; exaltation of God is in their throat, and a two-edged sword of the lips, kissing — returning right words, bringing forth pearls, saying: Hear, hear my word, and know my produce. For the mouth of those who speak falsehood shall be stopped, and she has answered that I am in the right, before all who know law and judgment. The discerning shall shine forth, in the goodness of the purpose of my intent — to multiply my proofs and my trials in the midst of the earth. And those who come after me shall receive my blessing, for I have grown old and gray, and still I have not attained to grazing the herd [?] in this craft. I turned and saw that it is broad and long in proportion to the days of our short years. Let not the wise man boast in his wisdom — even the swift in his work and the perfect in his understanding — that he could, of himself, begin and investigate and inquire into its roots and foundations, and enter its buildings and its upper chambers, spacious upper chambers, and examine the holes and crevices, from foundation to coping, until he reach the goal of its perfection in all its parts, general and particular. For the way is too far for him, and before he brings forth a flower and ripens almonds, his days are done and he grows faint. You will find toil spent for emptiness, for confusion, and for nothing — many remedies, but no healing — unless he be aided by the books of those who preceded him in time, which serve him as a peg in a faithful place, by whose light he sees light, and the heavy burden is eased from upon him; and every great and hard matter they shall judge, and he searches in their saddlebags and finds well-ordered things,

refined and correct. Upon their words he too builds new buildings and makes foundations [?]; and so it is done. And as for those who study it, generation after generation, and its sages — the honor of the sages — those who come later inherit ten measures more than the ancients. For when new and old stand together, in the multitude of years they make wisdom known; and it is known that it is easier to add than to invent, in every kind of resourcefulness and counsel. This is the substance of the chief of his words, the truth made plain from the brevity of his sayings. And indeed, were it not for the desire implanted in the heart of the generous-minded to benefit those who come after them with precious compositions — in which a light shone in the darkness for the upright — and were it not that a measure of envy, or of sloth, as

the ancients would refrain from committing their knowledge to writing, so that their wisdom died together with them; or else in every generation we would have remained like dreamers, binding empty sheaves, stripped bare and groping like the blind at the gates of the sciences. Mankind as a whole would be a wild ass of a man, his flocks and his thousands all of them brutish, blind one leading another. The people would wander, each gathering according to his own strength, and the craft would be divided into five great parts. These are the five daughters of medicine — that is, the natural, the etiological [study of causes], the symptomatic [study of signs], the therapeutic, and the preservative [hygienic]. They would become wives, and every one who bore them sons and daughters; and a man of understanding would fill his hand from them, according to what the fathers who begot them had written in their books. And it would become a gathering of all the beliefs, opinions, and experiences, and serve as a sign and a wonder for the many.

Do you not see how many great and honored men spent all their days on the craft of dissection, learning every particular and part and limb of the living creature and its usefulness — while others, in the knowledge of the simples [simple drugs], went on and entangled the whole world, God forbid, from the Scorpion to the Bull, from the Ram to the Scales [i.e. ranging across the zodiac, from herb to herb], and were as eyes for us? Others were occupied solely with the compounding and preparation of medicines — those called apothecaries (and many of them); others solely with the manual craft called כירורגי״אה [chirurgia, "surgery"]; and so with its remaining parts. This one is reckoned eminent in the theoretical part, that one is expert in the practical. Go out and learn from them; "go, eat your bread" [cf. Koheles 9:7] — if you are able to gather them all together at once into your own collection in full. And who does not know in all of this that they demand the whole of a man's mind and heart — that he meditate on the

discipline of each and every part of them day and night, if he wishes to rise in the net of his study and to acquire it [as a possession] in actuality? It is impossible for one man to be made complete in them all. And if this is so even when the matters lie arranged in books before him, as in this time of ours — when the river of wisdom has overflowed all its banks — how much the more so if he had to discover and devise them himself!

This great man is the first in time and in rank. "And whatever was precious his eye beheld" [cf. Iyov 28:10]; he is generous of eye [bountiful], and he is blessed — he who shook out our bosom and emptied [poured out] the trainees of his thought, out of the great grief and wrath he felt toward the physicians of his time. "The toil of fools wearies them" [cf. Koheles 10:15] — to raise up the fallen booth of the craft and to mend its breaches, [breaches made] by reason of the strayers, the sinners, the practitioners of mere experience — those called אימפי״ריצי [empirici, "Empirics"] — who, through the corruption of their intellect and the evil of their deeds, did not know how to gather the particulars under a single genus and to generalize them. They saw [only] disconnected things [?], multiplying words in idle talk; and if they did glean a sheaf, two or three grains, even these were in their hands like scattered plaster-stones [cf. Yeshayahu 27:9]. The foundations were demolished, until they made the glorious craft "a vineyard to be kept" [Yeshayahu 27:2] — and "the vines of the city" [?] became "a measuring line for it, and a trampling, and a stone of burden" [cf. Zecharyah 12:3], "a thing of scorn, divorced from her husband" [?] — for some hundreds of years. "And it went up like a [foul] woman, and its stench rose," for the wicked students profaned its splendor. But now its time has come, "a time of love" [cf. Yechezkel 16:8], its new feast and its restored praise; it has put off the garments of its widowhood, and it has set out [?] its nose-ring and its ornament. "In its palace, all of it says 'Glory'" [cf. Tehillim 29:9], and it rejoices and exults, for the Lord has widened its border with the coming of its redeemer. This man has consented to make it splendid and great — to record and to set down in a book select, brief words, hewn out clearly like channels — words spoken in truth and righteousness; all the sayings of his mouth, there is in them nothing twisted or crooked, such that the small as well as the great, the discerning together with the disciple, may learn them by oral instruction. And his eyes and his heart were upon them all the days, for the matter is very near to him, by reason of their brevity and their pleasantness; and so in every sound counsel he shows himself [skilled]. For one is not called knowing except…

[except according to] the man, his nature, his acquisition, his body and his soul and his habit — and not according to the count of his years and the season of the

year, or the climate [region]. Not in vain [is it called] συνήθεια [synḗtheia, "habituation"]. And in general, in his words [the author] wished to explain the strength [position] of the rational physicians, who rely on reasoning and on experience and not [on experience alone]; as our father in medicine [Hippocrates] wrote.

[There follows] from among the universal physicians, those called μεθοδικοί [methodikoi, "Methodists"], who say to the disciple [?]: "Do this and that" — for they could teach a man medicine in half a year, and with great ease; that there is no need to investigate and inquire into the nature of the illness, nor [into] the place of pain, but the knowledge of the illness alone suffices — as we have explained elsewhere concerning their refutations. And already one of the physicians, called Prosper Alpinus [פרוספיר אלפינ״ש], in our time, ordained in a book… "its skin was raised, the kids [were prepared] by his hands, and the smoothness of his neck" [cf. Bereishis 27:16, the metaphor obscure here] [?] — the whole purpose of his intention in this was [the brevity of] the study, and "of greater age [?] I would prefer" — to attain it and to confirm his words. He said that the time, that is, passes and goes by, and there is none [to stop it]; as the sword is sharp, [time] cuts in the blink of an eye, so that a man cannot…[?] those that are called [?]… [now] external [factors], now internal, and [the body] is not in one state, [not] continually fixed, stable, and steady. And on account of this it is very hard, and it would have been fitting for them to say "this is so" [?] — because of the little harm [?] that befalls them from error in such operations…

their illnesses, and the manner of curing all the diseases according to their ways. But indeed it came into his hand [?] what he…[?] to renew the old craft that had been forgotten for some hundreds of years; and just as he touched one [?], all his vessels [tools] became, in the eyes of all who heard him, like the dead of the world [?], and he did not succeed in drawing and persuading the disciples with the pleasantness of his words. He clothed [?]… [the craft is long compared to the days of our years — seventy or eighty years] — and it [the craft] needs [a long time]…

[The text here is laid out as a continuous argument that the craft is long compared to a human lifespan:] for the craft is long [cf. Aphorisms I.1] compared to the days of our years — seventy or eighty years [cf. Tehillim 90:10] — and it requires a longer, longer time to attain. And as to the confirmation of his words [Hippocrates' words], he said: "Time, that is, passes and goes by, and there is none [like it]" — sometimes to take proof, and to build universal teachings [generalizations] upon their like, in order to know how to conduct oneself at the

coming of the [right] time and season. For there is no understanding nor temporal permanence for them [the bodily states], since the body receives the changes from its [shifting] causes — the external and the internal — and is not continually in one state, fixed and steady. And on account of this it is very hard to attain this wisdom except through length of days and the experience of "an Eden of times" [a long span] given over to varied practical applications. Added to this is that all knowledge and learning is in truth settled by one of two ways, according to the opinion of the rational physicians — the δογματικοί [dogmatikoi, "Dogmatists / Rationalists"]: either by the way of experience and trial, or by the way of inference and rational judgment.

Now experience — it is known how dreadful and dangerous it is, on account of the eminence of the subject and its importance. For man is [like] the tree of the field, or like clay in the hand of the potter [cf. Yirmeyahu 18:6]. The physician's ways and his work are not like the work of a smith of iron, who forms his tool with charcoal and with hammers; nor like the carpenter of wood, who stretches a line, marks it out with a stylus [cf. Yeshayahu 44:13], makes it with planes and shapes it with the compass, or like one who works a craft upon stones, upon wood and metal and earthenware vessels — which can be cooked [?] in. Rather, his occupation is with this body [creature], this delicate and tender one, which is patterned by the compass of God; and it is like a harp and a lyre, abundant of strings, interwoven, hollowed boards, holes upon holes and hollows upon hollows, arranged of old in proportion and reckoning. "And Armon shall dwell upon its judgment [proper order]" [?, cf. obscure wordplay], for he who blunders in it [the body] like a tool [vessel] of hasty knowledge, twisted and stripped of the wisdom of music, will tear off the strings and stop up the holes and break down the hollows; he will spoil the harmonious proportion and corrupt the sound of its sweetness and its pleasantness. And how hard is its repair, even for a wise man who hearkens to my voice — "voice upon voice" — and who ascends by the steps of wisdom, an expert craftsman, [like] Asaph and Heman. So too is the bodily body, the dwelling-place of the intellect, the sanctuary of the Lord. When [the physician] wishes to explain that the rational physicians lean upon both inference and experience, and that man's hands are given [?]...

to add and to give life, whether openly or in secret, and there is none who delivers from his hand; nor will iniquity befall him in this matter, and there are none who restrain or judge in arbitration [the obscure phrase here suggests the physician acts without accountability] [?]. And how easy is the corruption, and how hard the repair — for "man does not know its [the body's] value" [cf. Iyov 28:13], nor the proportion of its music; nor will the hearing of his ears prove it

[the physician's case]. [If] the physician acts as the [mere] μουσικός [mousikos, "musician"] does — behold, the offense in it is small; but "the folly of man perverts his way" [Mishlei 19:3], and a soul is led off [destroyed] when "a man lets loose an ox and it dies" [cf. Shemos 21:35]. Far be it from a man who hates [?] — and "an eloquent man of words is tested" [?] — that he should, untroubled and not knowing from sight or from hearsay, [follow] every fancy that rises upon his spirit, and not have his eye spare the poor and needy, just as one would not [spare] the gourd [קיקיון, cf. Yonah 4:10] for which he did not labor. Should a man treat the children of men [?] [like the gourd]? Every science and learning is not worth [the price of] harm [done] to one of the honored persons — even though from it he might come to know how to benefit several men. Not so should one act, to ruin one [person] merely in order to mend others! And moreover: "blind upon blind" [?] — for "in the image of God He made man" [Bereishis 9:6]. And the Messiah [the anointed], the God of Jacob, said [?]: "The Lord is good to all, and His mercies are over all His works" [Tehillim 145:9]. And the mouth of the righteous Galen [גלינוס] [shall utter] wisdom, who said: that which experience has not confirmed in their purity [proof] is dangerous and ruinous; for experience without reasoning is, in truth, for the patient destruction and annihilation.

And the second tool with which the physician makes use is rational inference and judgment — and to God it belongs. For who has done these things [cf. Yeshayahu 41:4]? Wisdom decides — whether in the matter of determining the fitting operation [treatment], or in the matter of the prognoses and the contingencies that will come, whether for death or for life. For every wise of heart who adds knowledge will the more come to be in doubt about all these things; for it is on this account that the sects multiplied, and the opinions diverged, and the calculations were confounded in this craft more than in any other. All of this points to the difficulty of arriving at the truth in it — whereas, by contrast, there is no disagreement at all in the wisdom of number [arithmetic] and measure [geometry], by reason of the clarity of the truths of the demonstrations in them. For God is true [and] truth does not contradict truth; rather, [only] the apparent truth, the imagined, which leans upon false premises and deceptive proofs, contradicts the true truth. And now, if you are a man of [great] spirit, multiply your host and the reasoning of your heart; go out, you yourself, and do battle with one of the opinions — refute it [or prove it] with demonstrative proofs, and "distinguish between the holy and the profane" [cf. Vayikra 10:10] — for you do not know which is right, this one or that, or whether both are equally good according to what appears.

But the sect of the practitioners of experience [the Empirics] will not accept the word "judgment" upon rational inference, for in their opinion this is "a tool that does not prosper" [cf. Yeshayahu 54:17]; rather, [for them] the desire is for selection [trial] in matters of medicine, and the knowledge and distinction of the useful from the not-useful. And in their words [there is something], as the elder [Hippocrates] says, and his words are right: that for the newborn child whose navel-cord is cut and who is washed clean in water, and salted with salt and swaddled in swaddling-bands [cf. Yechezkel 16:4] — and after all these, [whether] he died or was healed — who [can say] what was the cause out of all these? "Oh that I might know!" [cf. Iyov 23:3]

And now [the author proceeds] to explain the reasons that moved him [Hippocrates] to write in the manner of the ἀφορισμός [aphorismos, "aphorism"] — that is, [a form] comprising all the special properties of a matter in few words. This is on account of the breadth [length] of the craft in proportion to the years of man. For indeed brevity is good and useful — both for beginners, that they not sink in the sea of studies, [and so on]; [it is useful], all of it, both for keeping [retaining] the studies and for recalling them after forgetting. He [the author] added, saying, that all that is written is right and true: not one thing will fall, will fall short [?], of all his good words that he spoke. Only this, that…

if it does not have the proper rest by reason of the [absence of? the position of?] [?], and whether it faces toward the north or the south, or [some other quarter] [?]; that is to say — that it is always fitting for the physician to liken the operations of the craft to the good operations [processes] that occur in nature.

[It is fitting that] a defect, perversion, or deficiency should occur not from the side of the physician, nor from the side of the patient, nor from the side of the attendants and servants, nor from any external cause. For if the physician is not diligent through his slackness — [then] the contingency or the fever drags on, and through his sloth he falls short of doing the duty laid upon him and all that is needful for the patient's benefit; or [if] the patient does not put a bridle upon his appetites and transgresses the physician's command; or if the servants err in their commission, in the matter of cooking [preparation] and the whole regimen required for him, in the manner of the slothful — [as are] our servants; or [if] an east or sea wind has come before [it], with winds harmful to the patient according to the nature of the illness; or [if the place] is too airy and desolate, or noisy — the braying of an ass, the barking of dogs (this is harmful), [or] weeping, or women chattering and telling tales; or [if] he has emotional passions and troubles of the heart — agitation and sorrow and anxiety and wakefulness and

[excessive] sleep, and a broken sighing; [if] his cow has been carried off [?], his field flooded, his ship sunk, and the like — it is possible that these are a cause of hindrance, so that [the cure] does not arise [succeed], and it will not be as written in this Book of the Teaching [Torah], and the studies will belie [seem to contradict] the promise of [the physician's] performance — even though they [the teachings] are upright judgments and true instructions. But when these [hindrances] are absent, there is no room to doubt or to cavil at them, for they are all truth and faithfulness.

[Display passage — a quoted aphorism, in larger type:]

> **2.** In the cases of disordered bowels and of vomitings that occur of themselves — if any of [the matter] requiring to be purged is purged [?], it is beneficial and [the patients] bear it easily; and if not, the reverse. So too with [artificial] evacuation: if it is of [the kind] that should be done, the evacuation made is beneficial and [the patients] bear it easily; and if not, the reverse. Therefore one must consider [carefully] the region, and the season, and the years [age] of life, and in which illnesses [evacuation] is needed or not.

[Commentary resumes, regular type:]

[As to the question] whether in all the studies [aphorisms], or in most of them, Hippocrates did not intend [a fixed sequence of] how much he begins with and how much he concludes, nor to write matters connected one after another — for all the chapters [aphorisms] are separate matters, as we have said.

At the beginning of his words [the first aphorism], one should not believe that he opened without a cause and a reason, [following] the division of medicine. As is known, under the genus of the medical craft there are three species — namely, the manual art [surgery], the regimen of diet [nutrition], and [the administration of] drugs — called χειρουργία, δίαιτα, φαρμακεία [cheirourgia, diaita, pharmakeia]. And why did he not begin with one of the studies of diaita, [or with] the cheirourgia? Even though it appears that his whole intention in the First Discourse [Section] was to speak of the regimen of nutrition — for in this he is occupied from the beginning of study [aphorism] 4 [ד] to the end of study [aphorism] 19 [י״ט] — what is close to reason is this: that since, as the Philosopher [Aristotle] wrote in the second [book] of the *Physics*, chapter 22 [סי׳ כ״ב] [?], the craft imitates nature so far as is possible, and just as is customary in nature — all of which is what is intended in this study — he was therefore compelled to begin from it [nature] at the head of his words. And at the beginning of the craft's

instruction — also because his intention was to begin from the part [concerning] the order of the regimen of nutrition, in truth. Added to this is that, since the correct order of study is to begin from the most universal of the studies — as the Philosopher made known in various places — and this study [aphorism] was universal [in scope], being well among the universals, he began with it, to build upon it and to bring forth derivative conclusions. And after [the fact] that in this study he spoke of quality, it was [now] fitting to speak also of quantity; and this he does in the study coming after this one. And after that, he immediately begins to speak of the regimen of nutrition — for this was the principal aim of his intention in this Discourse, as we have said. But indeed, as to at what time it is fitting to give…

of medicine, and how the evacuation should be carried out, the final teachings of this treatise [of Hippocrates] will explain. The upshot of his words, then, is that this teaching consists in laying down a great general principle which the physician should always set before himself, looking into it as into a mirror in all his operations. Namely: the physician must prepare himself well and observe what is done by nature, learning from it [how to treat] the patient — for what nature does is good, and so the physician too should do it deliberately and as required; and what comes of itself and proves harmful, he too should guard against doing it with his own hands.

To clarify his words, understand that the maxim which says that *art imitates nature so far as possible* means [it imitates nature] in what nature does of itself, for then nature acts toward an end — as the philosophers say, "God and nature do nothing in vain but act toward an end." Likewise art acts toward an end, and the way of an end is to be good, always, according to the intent of the one who acts toward it, as Aristotle explained in *Physics* II, ch. 23, and in *Ethics* I.

Yet we also know that sometimes [nature] errs to its own harm. For the nature of the individual aims at the preservation of the individual, and exerts itself with all its strength and intelligence to produce a male offspring for the world — yet not [primarily] to preserve the species, as the universal nature aims at the preservation and benefit of the whole, caring nothing for the good or harm of particulars, since that is the way it [the universal nature] is turned. By contrast, nature, when constrained as if compelled, does harm to its subject; this is not intended by it but comes about incidentally, as in the case of diseases.

For this reason the physicians did not seek in them [diseases] a final cause, but only the three [other] causes. To illustrate: in pleurisy [the disease of the rib], the matter is the membrane lining the ribs, the form is the inflammation or swelling

with fever, cough, and shortness of breath, and so on, and the proximate efficient cause is the blood. This [disease] came about there by necessity: nature, forcing the impurities of the head [to discharge] because of the fullness of blood, [acted through] the spirit-faculty — which is one of nature's servants — to lighten the head of its burden, and it [the faculty] thrust the matter downward, and it fell where it fell and produced a grave disease. This disease was not made by the intent of nature but incidentally.

So too the agent acting by intellect: as long as it acts by its own will, it always acts toward a good and beneficial end; but if it acts under compulsion, it may produce harm. This is what [Hippocrates] meant to hint at by his precise wording: "[the evacuations] that occur *of themselves*" — and he did not say "by nature," for the phrase "of themselves" includes both what is intended and what is not intended by nature. This suits his meaning well, as though he said: art ought to imitate nature in what is beneficial, since that is done as an end, and not in what is harmful, since that is not intended by it and comes about incidentally — except that he too, in Aristotle's manner, brought the syllogism without its major premise, since it is self-evident, as was likewise mentioned in the words of the Philosopher in *Physics* II. And the whole of the syllogism is thus:

[Aphorism] The physician must imitate in his operations the operations that occur of themselves, and guard against the harmful ones.

In diarrheas and vomitings that occur of themselves, when what ought to come out comes out, it is beneficial, and the patients do not feel much weakness; but if what should not [come out] comes out, it is harmful, and so on.

Therefore the physician ought to evacuate what is fitting, and not the reverse.

That is to say: if the white [bile/phlegm] is what is causing distress — as in [?an]p"rkh [?Greek term, possibly ἀνάρρηξις or a related term] — evacuate the white and not the yellow [bile], and do not induce vomiting [?]…

of blood; let blood, or diminish it in this manner. So too with salt-water, when [?]… the patient [?] to bury [?]. And [?] evacuate the yellow [bile], and a fourth-part the black [bile]. Likewise the fullness of [?] and the medicines specially suited to evacuate them. [Hippocrates] alludes to this in what he teaches concerning this: he did not write "the [physicians] should reckon by *quantity* in those who are purged, but [only] that whatever needs to be purged should be purged"; and at the end of this aphorism he likewise wrote "whatever needs to be sucked [drawn off], let it be sucked" — beneficial, and they [the patients] bear it easily; and if the reverse, [they bear it] with difficulty. So too in aphorism [?]… 50

[ɔ'], all of which indicates that the concern is not with the *quantity* but with the *quality* of what is purged. Therefore he preceded [the others] with the words "which" [masc.] and "which" [fem.], as well as with the word "to suck" — for he did not say "to evacuate," since purgation is properly the drawing out of the matters that distress by their *quality*, whereas evacuation is the expelling of those that distress by their *quantity*.

After he completed the syllogism, he answers a questioner's query: how, then, do we know which of the matters are the distressing ones? For in truth, by and large they are known and recognized to the sight of the eye, since as the matters within the body are, so is the color [shown] on the skin. But matters whose visible appearance lies deeper than the skin, and which are not spread uniformly throughout the whole body so that the sight of the eye sees them as different from what is within — how shall we know their quality?

He says that, besides the fact that each matter has its own distinctive signs that indicate its presence and abundance in the body — as is explained elsewhere in medicine — also the nature of the disease, the season, the country, and the years of one's life will give testimony to this, so that by many witnesses a person can evacuate the distressing matters without fear and with confidence. See [Galen's] *De libra* [the book "On the Balance"], and the book *M[?]Q* [?title]. An illustration of this: which sign [indicates] an abundance of yellow [bile]? Observe — in which body or climate does one dwell? If it [the climate] is hot, [as in] Cairo and the other countries that have no great latitude beyond the equator [literally "the equalizer of the day"]; and whether it is summer; and whether [the patient is] in his prime; and whether he has a tertian fever, or erysipelas, or jaundice — for all these indicate the generation of yellow [bile]. So too, by this illustration, [observe] for the white and the black [bile], until one can confidently evacuate according to what the signs — all or most of them — indicate. See the commentary of the second [?]disciple [?student/commentator] on the fourth treatise, for he is rigorous about this, and there he expanded the discussion.

And behold — "by [the testimony of] two witnesses a matter shall be established" (Devarim 19:15) — according to the two teachings mentioned, and from them infer the rest. For in this manner I have explained the whole *Book of the Aphorisms* [*Sefer HaAphorismos*] and the *Book of the Predictions* [*Sefer Hagadat HaAtidot* = *Prognostics*] of Hippocrates, briefly, according to my translation from the Greek tongue, since I had not seen it in the Holy Tongue [Hebrew] to this day.

I have also composed a *Book of the Remedies of the Sick* [*Refuot HaAleh* (?)] comprising all of medicine, theoretical and practical, together with surgery

[*chirurgia*], dietetics [*diaeta*], and pharmacy [*pharmacia*] — that is to say, the operations of the apothecary and the practice and all that is needed for this craft — and this is sufficient to make several physicians, for I sifted it out of many hundreds of books. And if you have any books that are yours, send them to me so that I may translate them, for I cannot bring them forth from under my hand.

Further, you asked me, the Ark [?Aharon — i.e. "you, Aharon"], to inform you of my opinion on several matters — most of them hidden from me and unknown to me — such as the operations of the [Divine] Names, both of holiness and of impurity, and the deeds of [?]...the Princes [שרים] and the eye-deceivers [conjurers], and so on. Others have already preceded you in asking, in particular the universal scholar, the honorable rabbi...

Zerach son of the honorable Rabbi Natan, who pressed me greatly with his questions. So I wrote, to satisfy his wish, thirteen [י"ג] essays, and I called the book *Ma'ayan Ganim* [*A Fountain of Gardens*].

In the first essay I explained the resolution of all triangles, which are the foundation of the science of astronomy; I called it *Sod Kishon* [*The Secret of Kishon*].

In the second essay I explained the first two treatises of the *Almagest* and most of the questions of the *De Motu* [?"the [book] of the moved/the mover"] — the first; I called it *Chukos Shamayim* [*The Statutes of Heaven*].

In the third essay I clarified all the choice material [?] that [is found] in the rest of the *Almagest*, and the *Book of the Revolutions* of Copernicus, and the book of *al-Bitruji* [אלכיתין — *al-Biṭrūjī / Alpetragius*] on the distances and thicknesses of the spheres, and the measures of the globes of all the stars, and other valuable matters; I called it *Sefer Gevurot H'* [*The Book of the Mighty Deeds of the Lord*].

In the fourth essay I taught the construction of several instruments pertaining to astronomy and their uses, and other agreeable matters; I called it *Meged Shamayim* [*The Bounty of Heaven*].

In the fifth essay I explained all the devices of the eye-deceivers [conjurers], their instruments, their preparations, and their tricks — so that one will no longer be amazed at the sight of his eyes [i.e., at what he sees] once the matters and their roots are revealed to him in my book. There too I spoke about the science of Kabbalah, its beginnings, its foundations, and how far the hand of those engaged in it reaches, and the greatness of its power in divine contemplation, and concerning the matter of the Princes [angels], sorceries, and dreams with their

interpretation, and the science of the hand [palmistry] and the face [physiognomy], and other matters — all clarified there; I called it *HaAlumot Chochmah* [*The Hidden Things of Wisdom*].

In the sixth essay I spoke about the judgment of nativities [astrological natal judgments] and what I found in it worthy of taking to heart; I called it *Otot HaShamayim* [*The Signs of Heaven*].

In the seventh essay I spoke about various delightful questions, and there I taught a new algebra; I called it *Lechem Abirim* [*Bread of the Mighty*].

In the eighth essay I spoke about the craft of alchemy [*alchimia*] and the secrets of nature; I called it *Megalleh Amukot* [*Revealer of Deep Things*].

In the ninth essay I brought the *Aphorismos*, the book of Hippocrates which I translated from the Greek tongue into the Holy Tongue; I called it *Mekor Binah* [*Fountain of Understanding*].

In the tenth essay I refuted Aristotle's proofs concerning the fifth body [the quintessence / fifth element], one by one, [as] written in [his] *De Caelo* [*Book of the Heavens*]; I called it *Etzem HaShamayim* [*The Substance of Heaven*].

In the eleventh essay I set in the scale-pan of a balance all the hypotheses of the astronomers concerning the spheres, and explained the advantages that attend each one of the opinions and the benefits on the one hand, and the necessity that compelled Copernicus to posit that the earth moves; I called it *Amudei Shamayim* [*The Pillars of Heaven*].

In the twelfth essay I explained divine matters concerning the principles of the faith; I called it *Gan Naul* [*A Locked Garden*].

In the thirteenth essay I explained several matters strange to hear, called *paradoxa*; I called it *Ma'ayan Chatum* [*A Sealed Fountain*]. Because of the multitude of disturbances [troubles] I have not yet been able to translate and arrange them, for they are with me [still] as matter without form, and I cannot send them until I am free of my affairs in the coming winter — may it come upon us for good — in the long nights of Teves.

But this I will inform you: I am not like others, even though they are wiser and more discerning than I — for I do not affix [my name to] and write of anything that has not been verified and established by demonstration; whereas they write books against it [the demonstration], and sometimes this is from their deficiency, in that they do not understand it at all, and they regret it in the end, that they did

not know it. Nor, on the other hand, do I, like the simple, believe everything. I sought to know all these sciences from their roots, from the mouths of the distinguished craftsmen and the experts available among them — each one of the masters of his own craft — and I attached myself to them constantly, in every city and country. I did not hold myself to be a master, nor did I dispute with them in the manner of the young who chatter, but I was attentive, receiving their words…

[receiving] from them what was honored [valuable], and afterward, back in my house, I would act as a sieve and a strainer, sifting their words, [keeping] what one cannot [keep] without [some] chaff; and by this my mind was settled, and I found that I had more strength than [I would have had from] the [mere] hundreds of books I read in them. For I saw and recognized which is better — between practice [hands-on use] and reading. And though many mock my judgment of astronomy, I have found in it the fifty [נ'] gates of understanding. And in the science of physiognomy, one skilled in the craft of drawing — to know the proportions of the limbs of the human body — will recognize the dislocated, the deformed, and the abnormal, and will understand from the deficiencies of the body the deficiencies of the soul. He who wishes to be complete in this science must be skilled in the physiognomies of all living things, and in natural philosophy and medicine.

However, the marks of physiognomy and of the hand [palmistry] have no known natural reasons for us; but experience — if it is among the things [that lead] to complete knowledge — testifies to the truth of much that is said of them in the books of *chiromancy* [chiromantzia]. As for *physiognomy* [?] [the readings of features] and their like, they are more remote from reason; therefore you will not find among them, among the true sciences, many treatises.

Dreams are, to my mind, among the wondrous matters, and in the book *HaAlumot Chochmah* I investigated them at length. But for now I will inform you only of this: that many [things] testify to its truth, which became famous in the world for its astonishing nature. For when I was in Venice in the year [ה]עש" [5370 AM = 1609/1610 CE], there was a Jewish man awaiting his son from Constantinople, and he wrote to him and warned him not to come by way of the sea, since the roads [routes] were beset with robbers and pirates; but he [the son] paid no heed and embarked on a ship. The father did not know [of this], yet one night he dreamed, and they were reciting to him the verse "And Moses cried out to the Lord, and the Lord showed him a tree" (Shemos 15:25), and at once he was alarmed and awoke — for his son's name was Moses. When he told the dream to the sages of Israel, they said to him that it was a dream of salvation; and so it was,

for the ship was wrecked and he [the son] was saved by means of a wooden plank [literally "a plank/ark of cedar"] which he grasped until he was brought to dry land.

I myself know of many prophetic dreams that I dreamed and that came true for me, in which I saw matters truly astonishing. And I have a reliable sign: that whenever I see my teacher, my father — the Gaon — in my dream, some distress will befall me on that day, but I will be saved from it. This has been tested by me more than a hundred times; therefore I fast over it [such a dream] even on Shabbos day — for although "the wounds of a lover are faithful" (Mishlei 27:6), a miracle does not happen at every single hour; therefore I take an oath by the dread of my father. Perhaps it is because I left my homeland and did not occupy myself with serving and honoring my teacher, who labored with me in his own person and with his money, and taught me Torah and wisdom and a craft, and made for me this soul [i.e. gave me my livelihood/being], and "seated me among nobles" (cf. Tehillim 113:8) — [perhaps for this] this matter befalls me as a punishment, and it is a distress mingled with consolation.

And there are signs by which to distinguish the prophetic dream from idle matters, as I explained there.

However, the properties [virtues] ascribed to minerals, herbs, and animals — few of them are true. For who would believe that the *remora* [echeneis], a small fish a handspan long, as we have seen, could halt a ship from sailing; or that the *adamant* [diamond] could prevent the *magnet* from drawing the iron near it to itself? And the like of these are many — matters full of vanity — with which the books are filled. I myself have heard so very many [such claims] and seen so very few that are true, that I resolved not to affirm or mention anything except what I have tested and verified several times. But what I have not tested, or what has not chanced for me to see — I neither affirm nor deny…

[Text continues]

nor did it occur to me — neither concerning *kishuf* (sorcery), *lachash* (whispered incantation), nor the *shedim* (demons) and their like — that I [?] saw [them]; rather they are [reported] from Egypt and onward to here, and what is said is [merely] received [tradition]. There is no clear demonstration that those who deny [these things] would say what they said [?]; they [the powers] cannot harm or help, for it is all empty and vain. And the proof: they are honored as "demons" [?] — no, rather they are not such things; nor [are they known] by sight or sensation, but only by hearsay. These are novelties [or: "these were new ones"],

which their forefathers did not know — if so, it was not as it once was [?], for they were [held] in the manner of fools of the world. And "your blood be upon your own heads," for it was a beginning of mischief among them.

Likewise R. Abarbanel [Don Isaac Abravanel], in his commentary on the [Book of] Avot [?] — [no:] in the manner of [his] forefathers and many. With them, [the matter] is already [known], and you already know [it]. As he explained, regarding [a certain matter] he raised difficulties, and brought it [?]; and so too the Radak [R. David Kimchi] wrote, and so wrote the Ra'avi [Ibn Ezra?] [?], that they are called *se'irim* ("goat-demons") according to the imagination — in the likeness of the *se'irim* and *chayyim* [?]. Just as the one bitten by a mad dog [imagines he sees dogs everywhere], so the imagination corrupts its likeness [perceiving such forms]. And as I said, it never befell me to see them [demons] in my days.

I have seen many lands [literally "provinces"]; I have crossed deserts and forests — yet for all this, I do not rule [?] that there are many things hidden from us, a thousand thousandfold more than the things known. [Therefore] it is fitting [?] for the sages of the Talmud, the kabbalists, and the Platonists who spoke at length [?] in their accounts; these are not foundations of the faith. And if [?] [there] were [something to it]... [As for the fact] that I publicized [these matters], and as one who understood, to speak about them — the wise man, the kabbalist, the rabbi [?] — [enough!] why prolong? Inasmuch as I have already written about it in the said book ✻

But here, what one looks upon — that he [the seeker] should find favor in the eyes of [the discerning] [?] — in amulets and incantations, to drive out spirits [shedim], that these matters are not foundations of the faith — and if it were so [necessary], I would have published them; and as one who has had the privilege to understand, [enough]. [And] enough, R. Meir Gabbai [?], in the *Sefer Avodat HaKodesh* — and I will not prolong; for the reward of what I have written in the said book ✻.

But what awaits him [the reader], until he brings them forth, to enlighten [?] — may you delight in HaShem, and you shall find rest for your soul — and you will return [?], and so be at peace [for it]. To copy [it] out for you, in writing, [bringing you] happiness; and you shall understand the fear of HaShem, and find [it] [?]. The knowledge of God; and many words — "go now and be at peace" (cf. *Shemos* 4:18).

> *He who is preserved [is] for your service, Yashar [R. Yosef Shlomo Delmedigo] of Candia.*

Written on the 23rd of Sivan [in the year] 381 [381(5) = שפ"א AM = 1621 CE], here in the holy community of Lublin.

And so we have merited to complete — in peace — Sefer Elim

In the year HaShPaT [389(5) = ה'שפ"ט AM] [?], the 8th [day] of the month: "Ani le-dodi ve-dodi li" ["I am my beloved's and my beloved is mine," Shir HaShirim 6:3].

(First printing)

(Says the editor [א"ה = אמר המגיה]: We found this [text] written on the reverse of the title page of the Sefer Elim from which we printed — and we did not withhold the good thing from its readers, and it shall serve as a memorial also for its first owner, who spoke in praise of the author, in words fitting for one of whom they were spoken.)

[The body below is in Rashi script, set in two columns; it is the inscription of a former owner. Right column first (RTL), then left column.]

[Rashi script — right column:] I examined, according to the shortness of my knowledge, the Sefer Elim — which "is great" [?] — and behold, I was as a dreamer toward [its] wisdom. Look at the kind of book it is! Who is this? "HaShem of Hosts, He is the King of Glory" [*Tehillim* 24:10] — He who is great [?]. And he [the author] took perfumed oil from the perfumer-apothecaries of all of them [the sciences], that all may live and grow wise. There is none like him, and behold from his mouth goes forth the upright instruction [?] that his heart pours out, like the well-spring of the world. Indeed, he gathered up all his words in [right] judgment; in the eloquences of language he drew out the depths of speech, the ornaments of praise [?]. And [if] I did not know — [if] I had not heard true testimony of Joseph [the author] — his name would be a marvel to him [?]; and [there is none] like him in the world. Far be it from me, in my inward parts, to comprehend; would that He set me [?] [to] understand — if my mother had only given birth to my masters [?]. And were he to say to me, "Did not all my comparisons and my investigations [?] make me known…" — what shall I answer? For according to my likeness and my yearnings [it is] hidden what is between him and [?] — there is no judge yet [for it]. Who can [explain] the chastisement [?]? Behold a youth, for I knew not [?]; let one speak — who shall bring me [?] — to be silent, to give voice to the lord [?] with strength, over the swift waters, [running] to draw [from] his wisdom in his garment, and also in his rank [?]. The young lions [scholars] who hated [?], and were thus — what were they [?]: wise men and scientists, aged and venerable [?]; and behold the multitude [is] melted away like wax, and could not [?] take their lodging in the

depth of his understanding — he went on and grew [?]. Indeed I am poor, lacking the forms [the learning], a "city lad" all my days and forever [?]; yet I roused my spirit here, in [my] dwelling, and Shaddai gladdened [me] with [these] forms. And I knew [?]: surely [thus] spoke the wise man with lofty compassion [?] [End of right column]

[Rashi script — left column:] [that the wise man], for the sake of [his] wisdom, set it so as to enlarge [for it] the opening, that there be no [hindrance] [?]. Let Isaac [Yitzchak] laugh, and let him gird and crown [him] like vessels of my nation [?]; and let your lips be [adorned], placing [?]. Before every counsel he [the author] is a mighty hero, indeed up to a "prince of peace." He fashioned me [?] with [his] hand and formed me, and my masters and my discerning ones [?]: I have known that he can [do] all [?], and nothing is withheld from him — to him be greatness [?].

According to the smallness [of him] who lacks possessions and silver — who shall give that he begin [?], and may he begin and renew teaching upon teaching — Yitzchak Eizik, son of [the deceased] R. Shimon Rofé [the physician] Wallich [וואליך], may he live [?], of the holy community of Koblenz [קובלענץ]. I wrote [this] in the year 457 [457(5) תנ"ז AM = 1697 CE] by the short reckoning.

[Rashi script, continuing in the left column — a praise of the author:] He [the author], from beginning to end — in all the milk of his bottles [the riches of his nourishment] — sealed off [mastered] all the disciplines of wisdom; and they are exact [?]: the *Sevara* (logic [?]), the *Binyan* (construction), the *Mispar* (arithmetic), *Handasi* (geometry), *Kavvanah* (intention) [?]. And [Joseph] held sway over them all, encompassing all the wonders of his Maker; who can withhold [?]? Therefore in a parable did his mouth open, and he produced the matter. For the sake of [his] honor: he is a nature [a force] from one to one — there is none [like] the master of the matter; and according to his understanding he spoke [each] thing.

[Then a metrical poem, each line ending in an acrostic syllable spelling out a name/word — read with the bracketed end-rhyme:]

> The grammarian-speaker who teaches truth, the [aforesaid] — *"yenei"* [ינ"י]. The words [the *tevot*], the eloquent advocate, exceedingly — "what shall I say?" — *"tzenei"* [צנ"י]. Refined [?], a prince, a sage of number, as they reckon — his name shall be set — *"henei"* [הנ"י]. [In] measures [shiurim] of geometry, an arranger and astronomer — may it be set up [?] — *"kenei"* [קנ"י]. Who will grant, and [who] will consent toward me, that I might know these — *"ani"* [אנ"י (= "I")].

*

[Footnote at foot of page, marked by an asterisk:] And here a place was found by us to make known to the honored trustees [of the funds] for the library of the government school here in Odessa [ב"ה אדעסא], who graciously did us the kindness of lending us the copy of the Sefer Elim — the [only] complete copy found in their library, the venerable Madame Rachel Trachtenberg [רחל טראכטענבערג], peace be upon her [?] [recte likely: the lady who funded it] — in order to make the copy [print-ready] for the purpose of the printing, since there is no second copy of it here. This was a great kindness — for our benefit, for the benefit of Hebrew literature, and for the honor of the author's name, of blessed memory. May his name be blessed in the mouth of all who study his book — to generation after generation!

This page is a full-page plate of geometric and astronomical figures, headed **ספר אילם** ("Sefer Elim") across the top center. Ten figures are arranged on the page, each captioned **ציור X)** ("Figure X") with a Hebrew-letter index. The plate carries the construction figures keyed to the mathematical demonstrations of the work; the labeled points (א, ב, ג…) are the demonstration's point-names. No running prose accompanies the plate.

Diagram — Figure א. A large circle with center **א**. From the center, radii and chords are drawn to numerous points around and within the circle. A horizontal diameter runs through the center to **ב** (right) and **ח** (left). Chords and lines connect the center to upper points and to two points on the lower arc.

- א (center): center of the circle
- ב (right end of horizontal diameter): point on circumference
- ח (left end of horizontal diameter): point on circumference
- ג (top): point on upper circumference
- פ (upper left): point on circumference
- ד (right, upper): point on circumference, joined by chords
- ק (lower middle-right): interior/construction point
- ה (below-left of central cluster): interior point
- צ (lower-left on circumference, below ח): point on circumference
- י, נ, ס, ר (small cluster of interior points just above center): construction points marking intersections (top→bottom י · ס · נ, with ר to the right)
- [A short dotted radius descends from א toward the lower arc; foot label not legible] [?]
- [lower-arc points ז / ש referenced but not confirmed] [?]

Diagram — Figure ב. A square with its four corners cut off by chords, producing a regular octagon inscribed in the square. Each of the eight cut/edge segments is labeled **8**. The square's corners and the octagon's vertices are lettered. Inside the octagon stands the value **√(32768 − 128)**. Along the top edge, three sub-segments are labeled **√32, √64, √32** (right-to-left); the left edge is labeled **√32**.

- א (top-left corner of square): vertex
- top edge sub-labels (right→left): 32√ · י · 32√ · כ · 64√ · ל (the points י, כ, ל mark the octagon's two upper vertices on the top edge)
- ט (right side, upper): octagon vertex
- ה / מ (right side): octagon vertices [ה lower-right]
- ב / נ (left side): vertices (ב the left edge, labeled √32)
- ג (lower-left corner): vertex
- ז (lower-right corner): vertex
- ח, ו (bottom edge): octagon vertices; bottom segment labeled **8**
- Interior value: **√(32768 − 128)** (the area / a derived length)
- The eight corner-triangle legs are each marked **8**.

Diagram — Figure ג. A right triangle. The right angle is at **ב** (lower left). The vertical leg rises to **א** (top). The horizontal base runs from **ב** through **ג** to **ד** (lower right). The hypotenuse from **א** to **ד** is labeled **√[?]** (a square-root expression, the radical legible but its radicand abraded); along the hypotenuse the value **24** is marked. The short base segment **ג–ד** is labeled **3**.

- א (top): apex
- ב (lower left): right-angle vertex
- ג (base, middle): point dividing the base
- ד (lower right): foot of the hypotenuse
- hypotenuse: √[?] with **24** marked along it
- base segment (ג–ד): **3**

Diagram — Figure ד. A parallelogram (slanted), with a dashed rectangle completing it to the right (the bounding rectangle of the construction). Inside is written **√8464** with **92** above it (92 = √8464). The two slanted sides are each labeled **5**; the base (**ד–ג**) is labeled **26**.

- א (top left): vertex
- ב (top right): vertex
- מ (top right, dashed extension): corner of bounding rectangle
- ז / ד (bottom left): vertices [ז the lower-left corner, ד adjacent]

- ג (bottom, middle-right): vertex
- י (bottom right): vertex of dashed rectangle
- slanted sides: **5** (left), **5** (right)
- base: **26**
- interior: **92** / **√8464**

Diagram — **Figure ה.** A vesica/lens construction: two horizontal axes (upper through **א...ג...ז**, lower through **ד...ה**) crossing a central vertical at points **ג** (upper) and **ה** (lower). Three arcs span the figure: a small left lens between **א/ד** and the center; a large arc above labeled **3**; a large arc below labeled **2**; and a small arc at upper right labeled **1**.

- א (left, upper axis): endpoint
- ד (left, lower axis): endpoint
- ג (center, upper): crossing point
- ה (center, lower): crossing point
- ז (right, upper axis): endpoint
- arcs: **1** (small, upper right), **2** (large, lower), **3** (large, upper)

Diagram — **Figure ו.** A circle with an inscribed rectangle. A horizontal chord/diameter runs from **ב** (right) through interior points to **ז** (left). The rectangle's corners and several interior intersection points are lettered.

- ד (top, right of center): point on circumference
- ש (top left): point on circumference
- ב (right, on the horizontal): vertex / circumference
- ג (interior, right of center): rectangle corner
- ר (interior, center): intersection point
- ז (left, on the horizontal): circumference
- ט (interior, lower center): point
- א (bottom, middle-right): vertex on circumference
- ח (bottom): point on circumference

Diagram — **Figure ז.** An astronomical/cosmological schematic: a circle (a celestial sphere) crossed by a slanted diameter. The diameter is labeled **המזלות** ("the zodiac [belt]"). Inside the circle is the legend **הקו הנקרא קרסה** ("the line called *karsa*") and **'אופן וגלגל וכו** ("epicycle and sphere [orb], etc."). A vertical diameter runs from **א** (top) to **ב** (bottom).

- א (top): pole / top of the sphere

- ב (bottom): pole / bottom of the sphere
- ה (center): center of the sphere
- ד (lower left, end of slant): point on circumference
- ג (upper right, end of slant): point on circumference
- inscribed labels: המזלות ("the zodiac"); הקו הנקרא קרסה ("the line called *karsa*"); אופן וגלגל ("epicycle and orb")

Diagram — Figure ו [second, lower]. A semicircle with an inscribed right triangle (a "3-4-5"-type construction scaled to 6-8-10). The diameter **ג–ב** is the base; the apex **א** sits on the semicircular arc, giving a right angle at **א**. Sides are labeled: **6** (left leg, ג–א), **8** (hypotenuse-chord / right leg, א–ב), **10** (base, ב–ג). Below the semicircle a separate straight ruler-line runs from **ד** through **ג** to **ה**, divided by tick-marks into equal units.

- א (top, on arc): right-angle apex
- ב (lower right): base endpoint
- ג (lower left): base endpoint
- base (ג–ב): **10**
- left leg (ג–א): **6**
- right leg (א–ב): **8**
- ד … ג … ה (lower ruler line): a graduated scale of equal tick-divisions

Diagram — Figure ט. A spherical "lune" / pointed-arch figure (a curvilinear triangle with a tall pointed apex): apex **א** at top, the two long curved sides sweeping down to a curved base; **ב** marks the foot. (The figure carries only the apex **א** and the foot **ב**; the ג/ה labels formerly listed here belong to the abutting Figure ח.)

- א (apex, top): vertex
- ב (foot): base endpoint

Diagram — Figure ח. Two adjacent spherical triangles (curvilinear triangles, each with three arc-sides), set under a decorative brace. The right-hand triangle has apex **ד** (top) with base corners **ה** (left) and **ג** (right). The left-hand triangle has apex (top, cut off on the scan) [?] with base corners **ב** (left) and the shared corner with the right triangle.

- Right triangle: ד (apex), ה (lower left), ג (lower right)
- Left triangle: apex [?], ב (lower left), shared lower-right corner
- [A brace ⏞ is drawn above the pair.]

Diagram — Figure י. A spherical triangle subdivided by internal arcs (great-circle arcs drawn from one vertex). Apex **א** at upper right; the two outer sides curve down-left to **ד** (left) and down to **ב** (lower); internal arcs from **א** cross the base, marking points **ב** and **ג** along it.

- א (apex, upper right): vertex from which arcs radiate
- ד (left): base vertex
- ב (bottom): base vertex
- ג (along base): subdivision point of the base arc

The fourth leaf of this opening is blank — a foxed/spotted blank verso with no text or figures.

Sefer Ma'ayan Ganim

(The Fountain of Gardens)

Every one who thirsts, come for water; come, bring yourselves water, you men of wisdom and understanding — for they have crossed [over]. You islands of the Kittim, look and see; you of Kedar, send word and consider closely how rare a thing this is: such a small book! Every one who thirsts, come for waters; the sciences have given of their own faithful waters, removing the [barrier of] language for the Greeks and [bringing] the discernment of the elders.

It comprises the everlasting answers, "built for armories" [cf. Shir HaShirim 4:4], to the questions of *Sefer Elim* — which contains seventy palms and twelve fountains [cf. Shemos 15:27]. The depths cover them over; his words are sealed and locked — fixed, and made to thunder. His mighty deeds are beyond reckoning. This is the opening [given to] Yisro, and he answered us; and every hidden thing the powers of reasoning have not obscured — [the topics] mathematical, natural, and divine: the sciences [tekhunot].

And this is the teaching of the complete man, the all-embracing master, the philosopher learned in Torah, the man of **Candia**, the expert physician, our honored teacher and master **Yosef Shlomo Delmedigo** [Yosef Shlomo Delmedigo], may the memory of the righteous be a blessing.

It was first printed in **Amsterdam** in the year שפ"ט [(9ט + 80פ + 300ש) = 5389 1629 ;389 = CE] by the abbreviated reckoning, at the house of the rabbi, the sage, the divine investigator, our honored teacher and master **Menasseh ben**

Israel [Menasseh ben Israel], may his memory be a blessing, who edited it himself.

And now it is printed a second time, with added glosses and corrections, together with an account of the days [i.e. the life] of the author, a transcription of his gravestone inscription, and also a depicted portrait of his face.

At the dispatch [i.e. expense] of our master and teacher Meir Grinshpan, and his associate the printer, our master and teacher Moshe Eliezer Belinson, may his Rock keep him.

Odessa

At the press of M. A. Belinson. [1865 = 5625] תרכ"ה. [recte: the title page of the Odessa edition; the colophon below dates the censor's permission to 1865, though the imprint year is commonly cited as 1864.]

[Censor stamp, Russian: "Permitted by the Censor, Odessa, 29 January 1865."]

The Words of a Wise Man's Mouth [cf. Koheles 10:12]

The renowned rabbi, our teacher R. Eliezer Lippman Silberton, may he live, editor of the newspaper *HaMaggid* [Eliezer Lipman Silbermann, founding editor of *HaMaggid*]

[Rashi script: How greatly our heart rejoiced and our soul exulted in our spirit when this precious work — long buried — was renewed in its youthful vigor and shone forth once more from the gloom in which it had lain, as I noted recently in our paper. For its author all but exhausted the whole vocabulary of the Hebrew language; his words are nearly all weighty, so that it would have been superfluous for me to add anything to my earlier praise of it, were it not for one thing. In my judgment, to do this work the honor it deserves — and it is precious and wondrous, a work whose worth is well known, its science marvelous in the eyes of those who walk its ways, and many have come to draw from it as one draws from a wellspring of wisdom and knowledge — its author drew into this marvelous work nearly the whole of the speculative scientific literature available to him. All the great men of Israel who were learned in the sciences relied upon this all-embracing author — "from him take counsel" — for his words of wisdom are far-reaching and profound, beyond [those of] his fellows.

But for a long time now this precious work has been hidden away, the property of the few who possessed wisdom and knowledge and were happy [to have it]; and not every seeker of wisdom and knowledge and of [moral] correction would find it. So this work fell into oblivion, until at last two honorable men girded themselves like mighty heroes for the sake of wisdom and its supreme benefit, and undertook the worthy and honorable task of reprinting this precious work — and they invested no small expense in the work. And because the cost was great, both in money and in the great labor [it entailed], the honored men decided to print only one part of this book, with their eyes turned toward the lovers of wisdom and the seekers of what this great author, of blessed memory, left us, that they might lend a hand in this great matter. They therefore made the proper and fitting offer to print, at the expense of the said honorable men, the first part [as a trial] — [costing] some 1,000 rubles in silver — so that, through it, the remaining parts too might be printed in time, [God] willing. They sent their request to every wise heart, for the honor of the wisdom of Israel and so that we should not be lacking, on this honored count, in anything that the [other] nations possess; for many of theirs sit and — if they do not know this very work that their printers issue and from which they themselves boast — they "swallow" it [out of envy]. Were it not so, [the words of] our holy tongue would have ceased; and were the science-books of our people [printed in Hebrew], the science would not be sold off into the hands of [those who] print books as a perpetual reproach — but instead "let their hands grow slack" at the small profit, until the precious book [is lost], while everyone else in their eyes [grows rich]…

[The Rashi-script passage continues with a dense, allusive argument urging support for printing science in Hebrew; several clauses are heavily abbreviated. The thrust: the honored printers of our people, mindful of the honor of Israel, took it upon themselves with the right hand of skill to bring [this work] to press, undertaking the first part. May it be God's will that the man who has begun, together with the publisher, find the strength to print the whole precious work to its end, as he is "a light to the gladness of the Hebrews in general."]

The first part now before us is printed on good paper and properly edited; and according to the corrections [made] in the author's writings, of blessed memory, in good taste and discernment, by the work of our friend the printer, the honored man, our master R. M. A. Belinson — who took no trouble too great, and labored to gather from various authors and from the science-books of the author everything bearing on his biography and the particulars of [his life]… [text damaged/abbreviated] — *(HaMaggid, No. 41, year [1865] תרכ"ה).*

And these are the words of the editor [HaMevalbeh"r] a second time:

[Rashi script: Here too there is room to justify [our course] before our honored people, the buyers of *Sefer Elim* now in our hands: as to why we have so far printed only this one part — the truth is that all beginnings are difficult [cf. Mechilta] — "the way of Yisro." Secondly, because the work is costly, and to it precisely the words of our people apply, that we should multiply the honor of the science in our hands. But this precious work — its printing is costly, and it "swallows much money," indeed, if it does not [recoup it]. And may every one who lends aid to good works be blessed by Heaven.]

To the Eye of the Reader

[Footnote marker:] *The printer [says]:*

As is fitting, since the time is short and the work is great and the means [of attaining] the mathematical, natural, and divine sciences are hard to come by, and the workers in the holy work are lazy and few — for with the loss of [our] rest and our sovereignty the sciences were lost from us; our signs we have not seen, and there is none among us who knows how long [this will last] — "Where is one who can weigh? Where is one who can count?" [cf. Yeshayahu 33:18], where is "one who utters great things" [cf. Daniel 7:11]? — when I heard that the complete sage, our honored teacher and master Yosef Shlomo Delmedigo (may his Rock keep and preserve him), had composed several books, among them this work, the very crown of beauty, "built for armories," with a thousand bucklers hung upon it [Shir HaShirim 4:4] —

I, the man "established on high" [cf. II Shmuel 23:1], at his honor's request made several times that he hand it to me to print — for "the good, the more it is good, deserves to be made known, to be of benefit and to confer merit upon the many" — and when he heard my plea, and [the pleas of] the many who were with me, "Yosef could no longer restrain himself before all who stood about him" [cf. Bereishis 45:1], and "Yosef made himself known to his brethren" who longed for his light, and "gave forth his voice in sweet melodies" to make his gates heard at "the gates of Zion, renowned" for his decisions [halachos].

But since the distractions of the time were upon him and his mind was not settled enough to set his eye upon it [for proofreading], and many of Yosef's words are sealed up and his subtleties — "who can understand?" [cf. Tehillim 19:13] — it is no wonder if there should be found in this book some error, since the pen has

slipped past it, for it came into my hands as faulty quires, "a brick upon brick" [i.e. in disarray]. Therefore I beg before all who know law and judgment: do not blame me, and do not "ascribe to me sin and iniquity" [cf. Iyov 13:23]. And may God, in His mercy, grant me the merit to print other books, as is in my heart; "and may the pleasantness of the Lord our God be upon us, and establish the work of our hands upon us; yea, the work of our hands, establish Thou it" [Tehillim 90:17].

Thus speaks he who "wrestles in the dust of the feet of the wise" [cf. Avos 1:4],

Menasseh ben Yosef ben Israel, may his soul rest in Eden.

[Footnote:] *) The printer. [keys the opening word "אמר" above]

The Author's Preface

I was **seized** by the cords of vanity, [one] of those who dwell in [the world of] decay, [a man] among the brambles braying and roaring; beneath the thornbush — woe is me, afflicted! "The sons of man — not their heart — did he draw"; he flees like a gazelle, he soars like an eagle, [it] sees. And I, in the midst of the exile, "Woe is me, that I have sojourned [in Meshech], that I dwell [beside the tents of Kedar]" [Tehillim 120:5], at the very edge of the land — toward **Lithuania** [or: Lijuvania], "a land driven and remote," far in the frozen north, distant from the land of Adam, the city of my repose — I lay among the burning [coals]; I was broken, my pot of grief, sick at heart and pained. "Stumbling at the knees," "perverse in their ways" [the men], [I] groped at noon as in twilight; warming myself from oven and stove, yet "upon the wing of abominations a desolator" [cf. Daniel 9:27]; "like a sluggard who buries his hand in the dish and is too weary to bring it back to his mouth" [Mishlei 19:24; 26:15]. The children of the age hemmed me in like the mire of the streets; they crushed me, they trampled me, they shattered and broke in pieces my thoughts, the very pillars of my heaven; my reflections were undone. "As when they that gather grapes have passed by" [cf. Ovadiah 1:5], — the sycamore-fruit-gatherers — gathering [scattered] lights, licking the dust, pursued by "vessels of darkness," bastard-born; like one waiting for the sun, [hoping] for its light — but it had gone astray, "as one who sees." I was bewildered at the courses of the days, "priests of the days" — "like the days of the mad," with terrors; "and all of them like the deranged" — should I look, [I find] "no light"; for it gives no shining; it keeps silent, weary...

And behold, two scrolls — and upon their head [was] a rolled thing like the appearance of a radiance, "like the gleam of *chashmal*" [cf. Yechezkel 1:4] — upon a single reed, rising "in a tempest and a storm." Their way and their path [were] "to the lightning of voices" [cf. Tehillim 77:18]; [they were] sapphires, mantled, and like a divine wind hovering over the face of their couch, [their] gold of grace and glory close-set. At the openings of my "sheaves" [or: standing-places] I trembled and I was astonished at the vision — for I saw **divine [beings]** rising, a great light "like my drawings" [cf. Yechezkel 8:3?], "daughters of kings" [in] my preciousness; the "perfect [ones]," the "beautiful" [cf. Shir HaShirim 6:9]. And behold, wings like the wings of the stork; like the children of the *shaf* [a bird], "they lift up [the ephah] flying like birds in flight" [cf. Zechariah 5:9]. And behold, they were clothed in garments — the robes and the mantles — and their adornment [glimmered with] light; "walking and mincing as they go" they tripped along, "and with their feet they tinkled" [cf. Yeshayahu 3:16], amid maidens dancing. And in their hands, "to the four winds of heaven spread out" [cf. Zechariah 2:10], crown-stones gleamed; "Ayish, Kesil, and Kimah" [the Bear, Orion, and the Pleiades — cf. Iyov 9:9] they covered. Running and racing, and not resting, clothed in "splendor and majesty." And **I heard** the sound of their wings, "like the sound of many waters" [Yechezkel 1:24], like the flame of "the whirling sword" [Bereishis 3:24] — and the cherubim — "trembling overcame me" [cf. Iyov 21:6]; I did not know myself; my soul "set me [among the chariots]" [cf. Shir HaShirim 6:12]; there it held me fast. I quaked and was afraid; "the hair of my flesh bristled" [Iyov 4:15] like the dread frost, and "bastards [waters] from the cold" [cf. Tehillim 147:17]; and from beneath their arms I "dug a tunnel" [cf. Yechezkel 12:7], and "I fell upon my face" [cf. Yechezkel 1:28] and hid myself "in the cleft of the rock" [cf. Shemos 33:22]; I was struck dumb in silence, and it became a shelter over me.

And behold, "a third-born heifer" [cf. Yeshayahu 15:5] — "Bare and ready thy bow" [Chavakuk 3:9] — and she said, "Strike the capital, that the thresholds may quake" [Amos 9:1]; and it was to me "like a needle in living flesh," like a sharpened goad rousing my drowsy thoughts and my swathed [reflections of the] heart. And it did not avail me, until I was caught in "the net of rhetoric" — like a foolish shepherd, for between their lines they would shine; from the east they would give light; they roused me and awakened me with love; and "sister of peace," beloved ones, they slaked me — and "lying down [in] guilt, they drew me, surrounded me, breathed me in" — and the eyes that had "bewitched me," these "made me bold." And behold, their sheaves stood about and bowed down [cf. Bereishis 37:7]. "I adjure you, O daughters of Jerusalem" [Shir HaShirim 5:8] — turtledove, dove, and pearl [Peninah] — "What do you seek?" [cf. Shir HaShirim

6:1] — "Who are these that fly like a cloud" [Yeshayahu 60:8], that come in the meadow [reed-grass — *ba'achu*] [cf. Bereishis 41:2], "and like bees they swarm" — and they said: "A man of [our] camp, whose appearance is the appearance of the Lion-City [Ariel] — we are sent to you, to feel and to see all that is in your house, your vessels, and all that is found in the treasuries of your palace. Peace be to your host!"

Author's Preface (הקדמת המחבר) — continued

[The preface continues in Delmedigo's allusive, biblically-saturated style. The Hebrew weaves a dense fabric of scriptural quotation — Ezekiel's chariot, the Song of Songs, Isaiah, Psalms — into a single self-portrait and an extended hymn to the mathematical and philosophical sciences. The translation renders the sense of each phrase while marking phrasing whose reading is genuinely doubtful; the merkavah-style allusions are conveyed in prose with the underlying biblical references noted where they are clear.]

and the seers, whose eyes look into the mirrors of the hosts — and my soul went out as they spoke with me, and I was left desolate in their midst. The cluster of henna [Song of Songs 1:14] that yields fine sayings, to one who would meditate and discourse; he also formed it — to one who would breathe out the breath of life. The teachers work a wonder: a furrow incised upon the clay he beds down; in place of the brier the cypress comes up, and in place of the [thorn the myrtle, cf. Isaiah 55:13].

And I — a man bowed low, a worm and a son of dust — I raised up [my dwelling] like the depths of the earth, and time burst upon me breach after breach. My soul lodged in a small upper wall-chamber and in the lowest annex, made desolate as a hut [cf. Isaiah 24:20]; and yet, how He bore me up on eagles' wings and brought me to His holy mountain, to the mountain of myrrh, the hill of frankincense [Song of Songs 4:6]! "The stone the builders rejected has become the chief cornerstone" [Tehillim 118:22]. How are you fallen from heaven, O shining one, son of dawn! [Yeshayahu 14:12] — for you compose elaborate compositions, [magnifying] one matter beyond all bounds, and without knowledge you multiply words; and over a small city you have built great siege-works, and forests of fortresses and towers, and rocks of refuge for the conies, you have set like the high mountains for the wild goats.

[The next clause is heavy with worn allusion; reading uncertain:] …He bowed down, [he crouched [?]], and there is none to shame [his] handiwork [cf. Judges

18:7]. The work of fullers, the work of foxes. I said to the boastful, "Boast not" [Tehillim 75:5]: extol Him who rides upon the heavens [Tehillim 68:5], for your praises have crossed over [all bounds]. The heads of [my people [?]] — my princes — would lay me [low]; my flesh cleaves to the dust [cf. Tehillim 44:26], my soul; and the songs sent to me have lifted my crown, removed my mantle too, taken my veil [cf. Song of Songs 5:7] from upon me.

You have wrapped shame upon me, for you have made me speak and have exalted me above all blessing and praise [Nehemiah 9:5], and you have seen me as the choicest of mankind, and you set a crown upon my altar — yet my altar is too small to contain the burnt-offering. Your arrows are sharpened, and like the wounds of a faithful friend [cf. Mishlei 27:6]. And you are girt with a two-edged sword, and like waters poured out on the ground [cf. 2 Samuel 14:14], your hands, [open] above, are not bound from doing the deed of a man — and feats too wondrous for me, which I did not know. The letters questioned me, "Even if the praise of God is in your throat" [cf. Tehillim 149:6], "and nothing is hidden from before your eyes; not one is missing [cf. Yeshayahu 40:26] — let them all be written in your scroll. Are not cedars in your store…" [the reading here is obscure]. The cedars did not darken you [Yechezkel 31:8], and bundled goods are in your market [cf. Yechezkel 27:24].

Let them be, [as a crown] for my chief patriarchs — your couch — the seed blessed of the Lord [Yeshayahu 65:23], and the crown shall be for Chelem, for Toviyah, for Yedaʿyah, and for Chen ben Tzephanyah, as a memorial in the Temple of the Lord [Zechariah 6:14]. And now — let your livestock go forth and pasture [cf. Yeshayahu 30:23]; your harvest and your tears shall not fail. For you have stripped off the coat of many colors from upon yourself; and to your wax-tablet and to your bow and to your throat, give no [room for] the beast to live; your bull — remove from me your [oxen [?]] for your sake. And the dripping of honey, the words of pleasantness — bring them to your forest! [cf. 1 Samuel 14:26–27] May the King bring you to His chambers [Song of Songs 1:4]; do not bare your forest, and let your terror cover [you] no more. For each man according to his praise — and a sending of portions to one for whom nothing is prepared [Nechemiah 8:10]; forever they shall perish; in their entirety they shall be as though they had never been [cf. Ovadiah 1:16], for they are not his.

For the sound of the shofar I have heard; my soul moans like a dove [cf. Yeshayahu 38:14] over its disgrace, saying: "What will be taken of you, and what have you here [Yeshayahu 22:16], that you have gone up entirely upon your roof [cf. Yeshayahu 22:1], that you prepare your spices and bind up your delicacies?"

You set me like a furnace upon the mountains, like the mountain of God [Tehillim 68:16] — this is Sinai; and Mount Sinai was wholly in smoke, because the Lord descended upon it [Shemos 19:18]. What profit is it that you speak of those who flutter upon their beds, those who scatter pearls with vows [cf. Amos 6:4–5]? And you have clothed me in garments of state and seals, like men of stature, as though I had wisdom tenfold [cf. Song of Songs 5:10]. But these garments are Saul's armor, and they do not suit dwarfs [cf. 1 Samuel 17:38–39]; I cannot go in these, for I have not tested them. Yet because I drew and drank waters from the upper pool [cf. Yeshayahu 7:3], from the summit of Senir and Hermon, from the summit of Amana [Song of Songs 4:8] — I have not defiled the dwellings of [my] smoke [reading uncertain] from the day I was on my father's knees; in faith I called Humility "the city of encampment," I cut a covenant with her, and I betrothed her to me

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in faithfulness; in the statutes of [the law] she rests, and she became for me a hind of loves [cf. Mishlei 5:19]. And as long as I grew up and increased in wisdom, my heart was not haughty [Tehillim 131:1], and I did not turn to the proud [cf. Tehillim 40:5]; I did not loathe my standing among the beloved; and though I was chosen, I did not act as a youth before me [reading uncertain]. From the dens of lions [cf. Song of Songs 4:8], from the family — I, a lofty one [reading uncertain] — the evil report [cf. Bereishis 37:2] I heard; the vineyard of the Lord of Hosts [Yeshayahu 5:7] keeps watch, and there a wheat-hedge is set, and the dominion increases [cf. Yeshayahu 9:6], and from a great city — [there are 365 [?] [ש"ה may be an abbreviation; uncertain]] and from the scribes, Tyre the crowning [cf. Yeshayahu 23:8] — my heart was not haughty for this, to spoil the ranks. I did not pretend to authority, and I did not sit like Deborah the palm [cf. Judges 4:5], and I did not put incense in my nostrils.

[The author here turns to humility and his love of the disciplines:] Famed for refining silver [cf. Mishlei 26:23], lying between [borders [?]], sleeping in the inner chambers' track — a man wrestling in the dust of the feet of the sages [cf. Avos 1:4]; I hold a man dear, finer than pure gold [Yeshayahu 13:12], and a man who learns from every man [cf. Avos 4:1] — even though his might be from Edom [reading uncertain]. And I, by my nature, love the civic community, leading my flock after the desert of the multitude, and there is my hidden treasure, my wandering, in the ways of my people. After publishing the conception in my morning [reading uncertain], after the [scattered] thought — and other deep matters, of which it was said, "the glory of God is to conceal a matter" [Mishlei

25:2] — I made them as garments for my clothing, and behold, they lie hidden within my tent, under my pillow. I shall feed them to the strong ones whom the Lord calls — "to my beloved," my brothers and companions, may they live.

And now — how honored is the day you betroth, when in the heavens you dyed embroidered garments [cf. Judges 5:30]; and there are twelve fountains and seventy palms [Shemos 15:27], and at our doors all choice fruits, new and old [Song of Songs 7:14]: the henna-blossom opened, the pomegranates budded [Song of Songs 7:13]. And that which I tasted of the choicest of the firstfruits — your honeycomb dripped [reading uncertain], your honey; and from your sweetness it sustained me. And it warmed my schemings at the peeled rods [Bereishis 30:37], where you watered with your waters; I was like a king bound in the troughs [Song of Songs 7:6], kneeling, falling, lying between the soles of your steps.

My ear heard, and it took it faithfully [cf. Iyov 4:12], that your precious soul longed and even pined [Tehillim 84:3] to descend to the depth of inquiry — the wings of a dove sheathed in silver, and her pinions in yellow gold [Tehillim 68:14]. Your gold came to me — the gold like a mixture of refined gold, neither dimmed nor running [reading uncertain] — "like a warrior to run his course" [Tehillim 19:6], and "the glory of the Lord has shone upon you" [Yeshayahu 60:1]. For behold, the heavens and the heaven of heavens cannot contain You [1 Melachim 8:27]; and you speak words to the Most High [cf. Daniel 7:25], and among the stars you set your nest [cf. Ovadiah 1:4]; your dwelling and your nest [reading uncertain], in your storm of spirit, your soul has crushed itself with longing — a great love, the love of wisdom, which by the name "philosophia" was long ago set by the mouths of the wise, the comely one.

And your thoughts shall mount up on the pinion of eagles [cf. Yeshayahu 40:31]; they shall seek out heights; from it search out, searching; they cross over the recesses of the heart of men of your front [i.e., the bold-faced]. Whoever eats my learning — natural and divine — their soul abhors them [the food], as though to eat a poor man's [bread] in the storehouse of one who probes and examines, his eyes [set] to pick fault, winking the eye, giving sorrow [cf. Mishlei 10:10]; their multitude turns against him. But had they been seeking for the glory of God, and to keep the charge of the Torah, not to turn aside — to the houses of those who learn from the toil of the Gemara, they would go out equipped, hastening — [I have made silent [?]] my walls, a possession; for these, these are the body of the Torah; these are the ones that bring to life [in the World to Come [פהע"ה ≈ פה העולם הבא?, uncertain]].

The chief and choicest of our time — we draw out like this; and in the times set apart for strolling and pleasure, the orchards, when others stray in the gardens and the parks, I occupy myself with the external [i.e., secular] sciences, and I laugh, in measure, over the smooth surfaces [i.e., geometric figures], in the hours not fit for [Torah] study, and at the time when others sport at dice. Except that I see that behold, darkness covers the earth [Yeshayahu 60:2], and behold many now are common folk [literally "people of the land"] — for though the breadth of your land is full of yeshivas and batei medrash, is not the learning but small in proportion to the multitude of persons? And they all alike pursue sustenance and bodily provision — burdens of vanity and rejected things; even the choicest among them,

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those who knock at the doors of the Lord, the gates of Zion that are distinguished in halacha — behold, their whole aim is to be teachers, judges, or heads of yeshivas. This is a grievous sickness [Koheles 5:15]: their goal is to make the precious vile and the essential subordinate; the purpose of their effort is foreign, their service is alien. And these too — through wine they have erred, and through strong drink they have gone astray [Yeshayahu 28:7]. And at the time of harvest, at the turning of the year, they looked, then were astonished [Tehillim 48:6], that the precious treasures, esteemed in their eyes as pearls, turned in their hands to glass. And after they had consumed the precious times on journeys to [poets' gatherings [?]] and to feasts, in foolish counsel — and they understood neither the external sciences nor the Torah — they became adversaries to astronomy, and provoking and deceiving, [misleading] their students and corrupting their vineyards, saying, "The Lord has no desire in the [strength of a warrior" [Tehillim 147:10] — in sharpened, precise rhetorical figures and reasonings, nor in the measures of the disciplines and the calculations of the astronomers, who like the Philistines [are] iniquitous; and like sucklings — the filth of Aram [Syria] — the wisdom of the Greeks lies in the bosom of a strange woman, upon a scarlet [couch] of loves, embracing the dunghills of philosophy — deaf serpents, Kittim and Dodanim — as though all the delight of religion were in feasting and music. They mock the proud; contempt is for the comely ones of the Greeks [reading of these last clauses uncertain].

And yet, indeed, that which is called by the name "external wisdom" — she is the daughter of my father; she is a good teaching [Mishlei 4:2], and balm of God who graces our intellect, by which we have substance, and with whom is the fountain of life [Tehillim 36:10]; in His light we see lights — after the covering is stripped

off [cf. Yeshayahu 25:7], when the filthy garments are removed from upon us [cf. Zechariah 3:4], He who clothes us in scarlet with delights [cf. 2 Samuel 1:24].

[Here begins the survey of the individual sciences:]

If [it be] logic and what is allied to it from the craft of grammar and rhetoric and poetry — to understand the twenty-four [24] holy books [the Tanakh], whose rhetoric is constricted and the path of whose poetry, as written, the bird of prey does not know [cf. Iyov 28:7]; and to refute those who dispute against us with disputatious, deceptive syllogisms built upon the verses — and what simpleton would turn in here to do battle with them without a two-edged sword? These are the learning of the desert; and with them the smith and the locksmith [cf. 2 Melachim 24:14] shall declaim in a smooth tongue [reading uncertain]…

And if [it be] the mathematical sciences, which raise [us] up to sit [and contemplate] the disposition of the spheres — man knew not their value and their true station until this day; and every man whom his heart has borne up builds structures and prepares ladders according to his way, with wondrous and mighty stratagems, that perhaps he may attain to their disposition and the cause of their motion. They are the numberings of "the glory of God" [Tehillim 19:2], and they testify to the work of the Lord, for He is awesome — and the separate intellects and their existence.

[Geometry:] That which humbles [us] to look upon the wisdom of measure — how great are the works of geometry [הגיאומיטריאה, Greek γεωμετρία], which sets a bound to the broad earth, to its length and to its breadth! It ascends by degrees to know the height of the firmaments, and the dwelling of the stars; it descends to the depths of the sea, and upon the mighty waters it sets [a path]; it makes straight our steps in her courses, and it is not too hard for her to bring forth a land from its place. Every precious thing my eye has seen in her, for she has borne many sons: stereometry [השטיריאומיטריאה, Greek στερεομετρία, "solid geometry"], mechanics [המיכניקה, "mechanics"], [the science of weights / hydrostatics [האישורופיקה, ≈ Greek for the "even-weighing"/hydrostatics; reading and identification uncertain]], perspective [הפירשפיקטיווה, "perspective/optics"], and their like — for the spirit of God is within her, and upon her is a staff of strength, a rod of glory.

[Arithmetic:] The wisdom of numbering [computation] — to open blind eyes in the reckonings of the expenditures of kings and provinces, and the count of their wanderings; were the whole inhabited world full of them, and like the drops of the sea and the sand upon its shore, and the stars of heaven and their

constellations — yet she raises her stature above all the chambers of the soul, and the whisperings, by means of a precious instrument and a treasure of kings.

[Algebra:] The wisdom of algebra [האלגיברה, Arabic al-jabr], "a well that the sages of Charish dug" [cf. Bamidbar 21:18]: how many good benefits has the Omnipresent [bestowed] upon us! This inheritance is weighed against all that He has bequeathed us in His righteousness; and may it be for us

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[a light] to the eyes, to bring to light every hidden thing — by the torch of this lofty and exalted wisdom; without her I would be like a deaf man who does not hear, and there my soul's dwelling would be [silent / like the deep [reading uncertain]].

[Music / harmonics:] And if [it be] the wisdom of song [music], to understand the measures of sounds and the harmonic ratios — to know how to bend the weary [voice [?]], to bear the sound to the ends of the isles [coastlands], as is hinted at by what is in our hearts through the tongues of the nations; and she dwells in the King's palaces. When the spirit of God [i.e., melancholy] terrifies them [cf. 1 Samuel 16:14], they are soothed by song and music; the waves of their bitterness [are stilled]. Indeed, the final purpose of the mathematical sciences, added to all the good things mentioned and the benefits — is to train our intellect to strip the forms of things from their matter; and this is the path to the wisdom of theology [metaphysics], as Ptolemy the sage wrote in the preface to the book of the Almagest [ספר האלמאגסטי, Ptolemy's Almagest].

[Physics / natural science:] And if [it be] the wisdom of nature [physics] — she is the mother of all; from her we learn the principles, the causes, the forms, the qualities, and the properties that come to be from the four elements; to ask, to know those that are compounded, "and from whose womb came forth" [Iyov 38:29] the springs that flow out, dispersed throughout the world; to know the disposition of [their composition]. The healing of the body [medicine] spreads its wings up to the firmament of heaven, and over all that appears in the air it stretches; the plants and the fishes and the fowl, the creeping things and the beasts, and the form of their parts, and the benefits of their limbs. And [God] gave it in her heart to multiply the species by the compounding of diverse substances — in trees, and in [garden-beds [?]], and in pigments; and to strip the lofty compounds of their form, to revive them; and from the lion's strength and the horse's carcass [reading uncertain] she will engender, like a [smith] blowing on coals [cf. Yeshayahu 54:16] — he rejoices and is glad when there is revealed to

him a revelation of secrets. And in general, her occupations and her inquiries are many; her words gather the smiths and [refine [?]] the compounds in metals, and beneath the bronze she brings beautiful gold.

[Psychology:] [Her] account is the knowledge of the rational soul, its faculties and its operations — her knops and her flowers [cf. Shemos 25:31]. And if the contemplation of each and every one of the existent things brings a man to recognize the Creator — to exalt and lift up the Lord of all good, the cause of all — how much more so, for "from my flesh I behold God" [Iyov 19:26], our Maker, and to praise Him, since all His judgments are a great deep [Tehillim 36:7].

[Ethics / governance:] And if [it be] the wisdom of ethics [middot], like the rules of governing oneself and one's household and one's state — see, [I am the head of] this: it is the upright path that a man should choose for himself who wishes to perfect his soul [cf. Avos 2:1]. And let me call upon faithful witnesses for myself: the sages of the Talmud relate of the sages of the Greeks that they would come from Athens to Jerusalem to hear the teachings.

In former times this was so in Israel: all wisdom and astronomy — as from Sinai it was given — to Moses it was a portion. And in the days of the prophets, men of faith, the courses of the luminaries — wives shining with light — and rhetoricians and poets and physicians and natural philosophers were many among the Hebrews. Also the wisdom of theology — the speculative Kabbalah, the living bird — found a home, and was a tradition handed down to the dear ones of Jerusalem, the high-born; and the wisdom of the mathematical disciplines had its nest, and song — its nestlings — set [their nest] among the altars of the Lord of Hosts; in the courts of our God they put forth blossom, in the days of King Solomon.

Behold, today, through our iniquities and the iniquities of our fathers — through the length of the exile, the disgrace, and the poverty — the stars of our twilight are darkened; our spirit has grown faint, and our wisdom has decayed. Our signs we have not seen [Tehillim 74:9], and there is none among us who knows how to do the work of heaven — preparers, and an architect [to set] bricks [reading uncertain]. [He shall make known] and they shall stop up the springs of astronomy; and they have darkened the [windows [?]] in the [latticed] heavens. Have not the sciences been reckoned as foreign in their eyes? The moon is abashed and the sun ashamed [Yeshayahu 24:23]; they go about as mourners, and there is no brightness for them; and the mathematical disciplines, which were bowed down before them like children [held] in their bonds…

[The preface continues; this opening's prose runs without interruption into the following page.]

today they whisper against them [the sciences] like foreigners, denying them; the sun has become for them a burden, and the moon a weariness [cf. *evel*, "mourning"]. As for the moon and the stars of heaven and their constellations [*kesileihem*, "their Orions"] — who has counted this multitude? None, save one man of Venice, a prince and a great man among the Jews, whose name is known in the gates [Mishlei 31:23], beyond the door and the mezuzah — his name be remembered — beneath oak and poplar: the comprehensive Rav, our honored teacher Rabbi Simcha Luzzatto [Simone Luzzatto], may his Rock preserve him. He knew every part of the mathematical sciences. He entered into their chambers, and the dripping honeycomb of his forests [cf. Shmuel I 14:26] was sweetened on his palate — yet he was found wanting in one matter [?]. Among his works there was a lodging-place [?], though I do not know whether he composed in it a treatise on geometry [*tashbores*] or on astronomy. Second to him, from the land of Egypt, in the city of Cairo, I came to know an honorable and wise man, a mighty man set up as a chief, the master, our honored teacher Rabbi Yaakov Iskandari [the Alexandrian], a man with spirit in him, who added an embellishment of his own — forty demonstrations [*limudim*, "theorems"] upon the first six Books of Euclid, like firmly planted nails, well-founded in proofs.

And as for you [the addressee of the answer-volume] — in the third Book, your wisdom did not fall short in my eyes; precious is the splendor of your silver and your form [?]. I did not believe the voice of the report from a distant land, until I saw your words — for you came to dig up this lean land of mine, and to test me with riddles, though you have no need of me. From the very walls of your house and the pavement of your letter your mastery in every branch of the tenfold wisdom [?] is recognized. Apart from you, together with those two golden pipes [*tzantarot*, cf. Zechariah 4:12] mentioned above — for I swear, in all the lands through which I have traveled, in cities and villages where the choicest of Israel are scattered, I have not known among my people anyone who investigates and inquires into the mathematical sciences. Many times I tested the sons of the Rabbanites and the Karaites, my students, [to see] whether they would succeed and persevere with constancy — for I said, "Let me make a help to match me" [cf. Bereishis 2:18], to lighten from upon me the burden of many computations. But not one of them did I find able to bear it — even though my tongue's portion drew them with charm, until they were yearning and knocking at her [wisdom's] doors. In one small moment they turned to despising her; in their eyes she became a menstruant [*niddah*, an object to be cast off] — they would discard her

and trample her, breaching her hedge and tearing down her fence. What fault did the students find in me, that they drew far from me? Is she not a pure creature, the fear of the Lord, a warm treasure-house, with the Lord our God? She has no business with hidden things [*nistarot*]; she touches the statutes of the Torah and the foundations of faith, and the yoke of the ban of the Rashba [R. Shlomo ben Aderes] and his company — who held back students from [studying] philosophy whole — never came upon her. Rather, her affliction is nothing but the foulness of heart that arises from the length of the preliminaries and the shortness of their intellect in grasping the precious mathematical sciences, many and weighty, which she requires — drawn from the science of number and of measure, which are the wings of astronomy and the ladder to ascend heavenward. These have been granted only to those who eat the manna, who drink spiced wine, the choice and the fragrant, the perceptive — not to the base intellects that creep upon the earth, who eat the bread of meanness, the grain with its chaff and its dust they eat, together with the beer of the barley set apart for the donkey-like [students], which deranges their understanding; they reel and stagger like a drunkard [Tehillim 107:27]. Concerning the likes of these the Rambam, of blessed memory, wrote in chapter 2 of [his commentary on] Rosh Hashanah, that they understand nothing of the celestial sphere [*galgal*] except as much as the ox and the donkey understand; and the lack of their knowledge made for them this strength, that they recount

in the disgrace of the mathematical sciences, and let darkness be upon them, a covering over folly and deficiency. Well did the wise man say: founding [conclusions] upon the loss of the premises, before encompassing what is conceived of them, is falsehood — it is striving in error and in blindness. And one should not wonder at a man distinguished and outstanding in wisdom such as yourself, lofty in your intellect above all the people, if they set their eyes against you, so that arrogant speech goes forth from their mouths — for I saw that you were blessed in these matters [?], lifting your eyes heavenward and investigating every end. And if I should search the whole land — like one who gropes in pits upon pits [where they are] hidden, and their elders are dreaming, drowsing — concerning them the prophet calls out: "Woe to those who rise early in the morning to pursue strong drink" [Yeshayahu 5:11], and the rest, "and wine is at their feasts, but the deed of the Lord they do not regard, and the work of His hands they have not seen" [Yeshayahu 5:12]. And it is written: "Lift up your eyes on high and see: who created these? He who brings out their host by number" [Yeshayahu 40:26], and the rest. And it is written: "All that is called by My name, for My glory I created it" [Yeshayahu 43:7]. And it is written: "And God has done it, that men should fear before Him" [Koheles 3:14]. And what honor or greatness

is rendered to the Creator of the world — He who is choice above all wisdom — and where, then, is the fear, the service, or the prayer, in a place where there is no recognition of [His] deed? "But all the deed of the Lord is for His own purpose" [Mishlei 16:4]. As my soul lives, if there is iniquity here, [it is] in the men of the upper chamber, who, like me, are found in all His handiwork that He has made laboring in vain and for nothing — who is there that has heard such a thing? For I speak of [these] excellent men [?], and I see that there is no man who lays it to heart, and I am appalled, for there is none who understands [Yeshayahu 57:1], with a student [?].

And I, too — my feet had almost turned aside [Tehillim 73:2]; in my exile I forgot my wisdom and all the books of my wealth and my settled life, and I did not take, when I went out from my place, even three husks of carob. From the day I came to the land of **Poland**, my silver was spent on vessels of gold [?]; the fire blazes within me — many waters could not quench the love [Shir HaShirim 8:7] of knowing precious things, new every morning. By her [wisdom] my mind was cooled, and my heat passed and was gone. "Go down to the city, go down" [Aramaic: *zil b'karta, zil*; cf. Ketubot 17a] in its custom, and "go out and see what the people do" [Aramaic: *puk chazi mai ama davar*; cf. Berachot 45a] — lest they say of me that I did not learn wisdom and the knowledge of the holy ones. This is their foolish way [Tehillim 49:14]: to say, "Surely there is a spirit in man" [Iyov 32:8], but all his strength is in the external sciences, and in Torah speculation his hands and feet found nothing — and many things like these you [pl.] have prolonged for your own purpose. And though from my youth the pain of Gemara study was leaped over [neglected?] [?] — for all my fathers sat in the yeshivos, disseminating Torah, and they raised up many students, and my family is a crown of glory in Israel and a diadem of splendor — even so, the [mathematical] disciplines I marveled at: to study them in study [Talmud], and in their love I would ever stray [cf. Mishlei 5:19]; but to the whole of the [external] sciences I said, "Go, go, O Nazirite, [they say,] go around, go around; do not draw near" [cf. Bava Basra]. Matter and forms were unknown to me [?], unnumbered. And how shall I come with the mighty deeds [Tehillim 71:16] and go out time after time and bestir myself? For she [wisdom] was with me, and I sent her away, and my strength departed from me and was no longer with me as in time past — in the [mathematical] wisdom. And as for my hand [?] — because one does not refuse a great man such as yourself: here, I come to you in the thick cloud, out of the abundance of the burdensome bodily affairs that test me, and the sick who hope for the salvation of the Lord and for His mercies through my hand. These are nobles and officers, whose youths and elders rise early to my doors in wagons, moving me from city to city; with glory and majesty they crown me, with the

smoothness of their lips, and with the abundance of their gifts they draw me with the cords of their love. And through their folly [?], the divine speculations are cut off and the precious times consumed; they nullify all my mighty counsels, which are truly in their season [?] — it was [my intent] to compose books in the language of the Hebrews, comprising all learning and knowledge, to teach the sons of Yehudah bow and battle [cf. Shmuel II 1:18], that they not be ashamed when they speak with enemies in the gate [Tehillim 127:5], in every sound wisdom and design. But behold, the affairs of my time have driven me from rest, and from my thoughts they have ceased [?] — the waters became a desolation. The philosopher said, in the *Physics* [*Shema Tivi*, "the Natural Hearing" = *Physica auscultatio*], that wisdom is acquired in settlement and in tranquility, in calm — for many are the slain that wandering has felled [cf. Mishlei 7:26], and mighty are all those whom mental derangement [destroys], [from] house to house divided and place to place; a soul that wanders [from] the way diminishes more than three…

matters [Aramaic context], it is better to speak of "Shabbos rest and refreshment" [cf. Shemos 31:17]. "And these are the generations of Noah; Shem…" [Bereishis 6:9]. I know, my lord, I know [cf. Bereishis 48:19] — but my wisdom did not stand by me to heal this sickness, and in all my books of remedy I did not find balm for my wandering. For as long as I am upon my ground, my body's woundedness [?] is upon me, and this murky inferiority that dwells within us — a disgrace it is to us; necessary mishaps overtake him, and who cares for me besides myself? For he has no redeemer but me — all the more so an isolated man [*ishon*, "little one"] like me, without brother or kin; for I am a sojourner in a land not mine. And all who see me and know me will bear witness and testify that, if ever in my life I entered an inn, it was on the way of those who travel to the far reaches of the earth. For I have never taken pleasure in a war of commandment, permission, or obligation, like one who busies himself drawing forth dough from the house of the baker [Aramaic: *ke-mai de-asik chala mi-bei karba*; an idiom for fruitless labor] [?]. Praise to God, who has dealt bountifully with me: I bring in guests, and my students eat at my table — "He gathers, He will gather Yaakov" — those who go wandering from city to city; my house is opened wide; I have scattered freely [cf. Tehillim 112:9], given to the needy, and to the sick of my people I have given my bread; my waters are sure. And though I am poor, the Lord of Hosts has left me a remnant [cf. Yeshayahu 1:9] as my portion — my spirit, the intellective spark, the offspring of the star [*kochav*, Mercury?] — He is the One who has shepherded me from my youth, and in every way my foot has trodden He is in authority; He guides me, and seats me among nobles, restores my soul, leads me and supplies me in the paths of righteousness for His name's sake [Tehillim 23:3],

lest I perish in my poverty, in the evil of my fortune. And it is a byword on my lips, to take up my parable [cf. Bamidbar 23:7]: "Let me speak, that I may find relief" [Iyov 32:20]:

> If, like a multitude anointed, He shelters under His wings — A righteous one [*tzaddik*] with greater strength, in my generations / nativities [*toldotai*] [Hebrew letter ה in margin] Found, as it is written, "then before the sun [shall his name be continued]" [Tehillim 72:17] and the rest [the poem cites it] — my name, and a light for my generations.

for the stars from their courses [cf. Shoftim 5:20] have magnified the heel [trouble] upon me; and who can mend what the celestial array has twisted [cf. Koheles 1:15]? The philosopher decreed that in the place where the intellect is weak, the powers of fortune grow strong, and the reverse. And so [said] the sweet one of the songs [David]: "The Lord preserves the simple" [Tehillim 116:6]. The sages of the Talmud stopped their mouths [silenced their detractors] in saying: "Not every man merits two tables" [Berachos 5b]. All the more so, the multiplication of excessive possessions — they are among the impediments to perfection. For the herdsman has returned to being a master, and the keeping of the peace of the flock is the whole purpose of the herdsman, and nothing else. Concerning this the wise man said [Koheles 5:12]: "Riches kept by their owner to his hurt" — for the end of money is to be scattered, and the end of cattle is slaughter. Yet the absolute lack of honorable, mathematical possessions [?] will, without doubt, bring a man to shortness of spirit, to weariness and folly; and "the wisdom of the poor man is despised" [Koheles 9:16] — for it is impossible that the poor man should occupy himself with all sound wisdom. In sum: silver answers everything [cf. Koheles 10:19]; it is what abases and also exalts. Happy is he whose hand is full of it, that it may be a help to him for attaining wisdom — and the more there is, pursue it to destruction [?].

And now, after begging your pardon, let me enter with your honor into matters [like] thunder [cf. Iyov], an open rebuke [Mishlei 27:5]: why did you not come to me, a hundred or two hundred parasangs, by wagon — when I have crossed seas and rivers and deserts to see all the inhabitants of the world and the dwellers of the earth, in the lack of my money and the affliction of my body? If my strength were the strength of stones [Iyov 6:12]! For these sciences are not acquired with money alone, but with mighty exertions of effort. Alas, my heart! And as for my flesh [?] — how much we would have multiplied to study and to investigate

those things that fall in their pasturing [?], and so in the sciences. And [there is] the book of the Almagest of Ptolemy; the orbits [*hakafot*, "epicycles/revolutions"]

of Niccolò — that is, Copernicus [Nicolaus Copernicus]; the book of al-Battani [*ha-Albateni*] and his like; al-Farghani [*Alfarnagu*]; Tycho [*Tikoni* = Tycho Brahe]; and all who came after them — into its depth I have not descended, and I have against it bundles of doubts, like silkworms [*ka-rimah*, "like maggots"?] built up [?]; nonetheless I did not find [resolution]

in the commentaries on the Torah, and the commentators, and the authors; and I would be one who beautifies — when the pearl [?] is lost, and crushed like the foundations [?]. "[Their hands were as] sapphire stones" [cf. Shir HaShirim 5:14], and "like a hammer that shatters the rock" [Yirmeyahu 23:29] of disputation, I would bring forth the fire of religion [cf. Devarim 33:2] — all the longing of Israel — at the sight of your face; and we would return to investigate in all the books of Plato and Aristotle, and the words of Alexander [of Aphrodisias] and Ibn Rushd [Averroes] and Themistius, and the rest of the commentators who came after them, their reasonings and their arguments and the doubts that fall in their opinions. And so too, in the sciences, we would be observing the stars by means of great instruments, by which the secondary fractions [the finer divisions of arc] are discerned; and we would be reckoning the eclipses of the luminaries and the courses of the wandering planets [*meshartim*] according to the foundations of each one of the astronomers. I have in these matters wondrous abridgments [*kitzurim*] and tables [*luchot*] such as you will not find in [any] writings; and from there we would descend to the earth, to know all the particulars of the places of the inhabited world — its length and breadth, their lives and their affairs — and all that is comprised in geography [*ge'ografiah*]. And [there is] the science of those who work in the great waters, who go down to the sea in ships [Tehillim 107:23]; and from there, in the science of the [mechanical] contrivances done with water — of those that float upon them [cf. floating bodies] — and the work of weights [*mishkalim*, hydrostatics/statics] and its vessels, and the awesome operations made necessary by the fleeing of vacua [*berichat ha-rekot*, the abhorrence of the vacuum]: all that is written in the great books, the inquiries of the mind of Archimedes [*Archimedes*], Vitruvius [*Vitruvio*], Hero [*Heroni*, Hero of Alexandria], Pappus [*Papos*], and those who came after them. And from there we set forth on the sea of the science of optics [*ha-habatah ve-ha-mar'ot*, "vision and mirrors"] — the deceptions of sight, and the strange instruments: some that magnify and enlarge and alter the perceived objects, and some that conceal and diminish them in many ways, and some that burn, sending their sparks far off — their rays go forth and melt even the metals set opposite them. And we would be investigating the qualities of the formation of colors, and the rainbow that is in the clouds; and the whole book of Apollonius [*Apolonios*] and our song [?], and Witelo [*Vitloni*], and Alhazen [*Alatzin*]; and we would be examining the science of shadows

[gnomonics], and the hour-instruments of the gnomonica [*gnomonika*, sundials], and the totality of the science of measure, and all the mechanical crafts [*mikhaniki*] that depend upon it, and the deep problems in it, and the science of number, and the hidden treasures of algebra [*ha-algibra*] — to understand the tenth Book of Euclid and its benefits, and the book of Diophantus [*Diofuntush*] and its commentators. And there together we would sweeten the secret in the science of alchemy [*ha-alkimia*] and the working of the earth, and in several hundred wondrous properties [*segulot*] found in simple bodies, and the quantities and qualities of every inanimate, growing, and living thing; and in the medicines and the compoundings, and in the vessels and the limbs of man and beast — their form and their benefits, as an interpreting angel [*malach melitz*] among them, and an intellect that does not wander among their melodies [?]. And from there we would ascend by degrees into the science of the divine [theology/metaphysics], and it divides into two heads: part of it built upon natural science with the addition of faith, and part of it received, wrapped in [precious] sapphires — some of them awesome — the words of the man of God, RaShBI [R. Shimon bar Yochai] and his disciples. I have heard that the spirit of his holiness shone forth in your study-house and filled [?] upon your tongue, so that you pour gold from your mouth and cause the oil of the perfumer to flow [cf. Koheles 10:1] — the words of the Book of the Zohar — in your expositions. And I, to this day, among the Egyptian souls [the people of Egypt], have known that with you it [the Zohar] is "like apples of gold in settings [of silver]" [Mishlei 25:11]. And behold, heaven is my witness, that when my heart inclines beyond its place [cf. Iyov], "who will give me" [Tehillim 55:7] a wing like a dove? I would fly off and dwell with you, and you would bring me to abundance [Tehillim 66:12] — for these matters have not been granted to be written, except as a whisper among the modest ones. And even in my own [private] roof-chamber [?] I am afraid to bring forth from my mouth a whisper and a secret; and I turn this way and that [Shemos 2:12], and I see…

הקדמה — Preface (conclusion)

complete. How could I set down in a book everything needed to satisfy your wish as I would wish? For the bed is too short to stretch out on [cf. Yeshayahu 28:20] — from the rafters of the ceiling down to the foundation — and a single hour of conversation face to face is worth more than all the rolls of a [whole] scroll. Who is the scribe in these towns? They have run out [?]. The man of the land who knows how to copy out the demonstrations and to draw the figures according to their proper form and the numbers according to their pattern — and who will not

pervert his way, blind the eyes of the clear-sighted, or distort the words of the righteous? Indeed, I have disclosed to your honor the reason I delayed until today, when I found a scribe — a discerning young man, of superior abilities among the masters of Scripture, a man skilled in his craft. I said: "Perhaps my beloved will lodge among my chief men, [helping] me with the rest of my affairs and all my labors." Here I am, here I am at your service: I have bent my shoulder to it; I have turned away from all my other business; and to do your will I have applied myself. By your command I took a full handful of the precious time of those who [must] be created [?], and yours, my lord, is the pardon if the handful does not satisfy a lion of your stature.

And so today I come to the **First Source** [העין הראשון]. The substance of your words — to inform you on what ground the master who authored *Yesod Olam* [Sefer Yesod Olam — R. Yitzchak Israeli] founded the three principles of the spherical triangles, which he introduced without [demonstration?] [?] — and the perfection of the rule of the solution of every triangle, since it was not explained in his words as it ought to have been. By the requirements of this inquiry it is fitting to call this treatise of ours *Sod HaYesod* ["The Secret of the Foundation"], and it has four corners, each corner divided into stones [headings] comprising various lessons, some upon [others, building course] upon course.

In truth, the solution of triangles — I mean, knowing the measures of the angles and of the sides once we know some of them — is the foundation of foundations for the whole body of the mathematical sciences. All our objects of inquiry are investigated by means of this science. The ancients, mainstays of the world, greatly enriched the methods of solving them. So Theon [חיאוניש — Theon of Alexandria] wrote, in his exposition of the First Book of the *Almagest*, that Hipparchus [איפרכוס — Hipparchus] wrote twelve books [on the subject], and Menelaus [מיליאום — Menelaus] six, and Ptolemy [תלמי] surpassed them all, for he clarified the solution by laying out a few sharp, concise lessons. With Ptolemy and his aforementioned commentator [Theon], it was easy to make use of the method of the intersecting arcs and the three ratios [?] across the breadth [?] — for which they all [used] it; but in our eyes there is great toil and labor in it, and error comes easily. Now those who came after them gave wondrous counsel and increased sound wisdom: each opened for himself a doorway to the solution. And I trust in Hashem to instruct students in a shorter path, in clearer language than all the [other] authors. When you set these lessons of mine — which are two or three kernels — beside their lengthy books, you will know that the words are true. This I shall begin to do with the help of Hashem, the shield about me, my glory and the one who lifts my head [cf. Tehillim 3:4].

סוד היסוד — *Sod HaYesod* ("The Secret of the Foundation")

הפנה הראשונה בהתר המשלשים הכדוריים נצבי הזוית — The First Corner: On the Solution of Right-Angled Spherical Triangles

R. Yashar of Candia [Yosef Shlomo Delmedigo] said:

The honored sage R. Yitzchak the Israelite [R. Yitzchak Israeli], in *Sefer Yesod Olam*, Treatise I [מאמר א], in coming to explain the solution of spherical triangles, set out three principles in advance. He took them from the First Treatise of Jabir [גיבִיר — Jabir ibn Aflah], that is, Aflah the Ishmaelite [Muslim] of Seville [אפלח הישמעאלי האשבכילי — Jabir ibn Aflah of Seville], who composed in the Arabic tongue and comprised in nine treatises everything written in the *Almagest*. The first principle is taken from [his] lesson 13, the second principle from lesson 15, and the third principle from lesson 14 — for this author was the first to abridge the path of Ptolemy and to invert the six measures attributed to the lesson of the [golden] lesson [the "golden" theorem — i.e. the *regula sex quantitatum*, Menelaus's theorem], familiar to the merchants of the land and which won a good name among the astronomers.

Now the first of these principles is a lesson of great strength and mighty power: it is the cornerstone in the solution of all the spherical triangles. He [Jabir] demonstrated it by proof and based it on the lesson he had set before it earlier, in lesson 12.

And being an Arab, he used in that proof the compounded ratio that Euclid expounded in the lesson [proposition 23] of [Book] V [actually Book VI] of his Elements — and he goes on at length and confuses the minds of the students. But I have explained it by a fresh proof, easy to picture, since I derived it from the strength of the last lesson, from chapter 12 [recte: Book I, proposition 12 of Ptolemy?] — the last, [traced back] to the first [proposition] of Ptolemy. To Him it is fitting to give thanks forever and ever. And you will find it, should you seek it, in *The Book of the Complete Solution of Triangles* [ספר התר המשלשים השלם], which comprises every precious thing of this craft; I named it *Ahavat HaShamayim* ["Love of the Heavens"], and [also] in our commentary on the *Almagest*. But here, in this abridgment, I shall bring you a clear method — and that Jabir's proof, whose book you possess, is true and faithful and not to be doubted at all.

Now this principle alone, that is to say: that **in every triangle the sines of the sides are proportional to the sines of the angles opposite them**, by way of illustration:

In triangle [אבצ] before us (Figure A):

> sine of angle [א] : sine of [side צ?] = sine of angle [ב] : sine of side [אצ];
> and likewise sine of angle [צ] : sine of side [אב]

— this alone suffices for the solution of the triangles required by Jabir's method. He needlessly brought the two other principles; and so did R. Yitzchak, who walks in his footsteps, and the sages like them — for these are branches of the first principle, and they follow and ramify from it. The [other] two principles add no clarity or ease at all and are as though they had never been. And since they follow directly from it — why should we multiply lessons that we cannot remember by heart, [so that] we should need books?

For from the first principle [the others] are explained in this manner. In the figure before us let there be a triangle [אבצ], right-angled at [angle צ], and its two [other] angles, at [א] and at [ב], acute. Accordingly the arcs [אב] and [בצ] are smaller than a quarter-circle. (Had the angles been obtuse, or the sides large, you would take the remainder up to half the circle, and the proof is the same — there is no difference to one who attends carefully.) Extend its three sides up to quarter-circles, namely [צג], [צכ], [בי], so that arc [יבג] is perpendicular upon [צבב — i.e. side צב extended]. And according to what R. Yitzchak wrote in his aforementioned book — and which we [have shown] elsewhere — angle [ז] [?] is a right angle, and arc [יכנ] is a quarter; and arc [כז] [?] is the measure of angle [אצב], and arc [כי] is its complement; and arc [בב] [?] is the measure of angle [י], and [וצב] is its complement; and arc [אי] is the complement of side [a — אצ?].

Now if we apply the first principle and say: as sine of arc [אנ], which is the complement [of אצ?], is to sine of [י], the complement of [צ?], so sine of arc [אי], the complement of side [אב], is to sine of [כ], the angle that is upright [the right angle] — that is, the whole sine [the radius]. And conversely, by the manner of lesson 16 of [Book] V of the Elements: as sine of [אכ], the complement of [אצ], is to sine of [אי], the complement of [אב], so sine of [י], the complement of [צב], is to sine of the right angle — and this is what the second principle intended.

By this same path, for the demonstration of the third principle: as sine of angle [כאי], equal to angle [אצ?], is to sine of [יכ], the complement of angle [צ], so sine of angle [כ], the upright [right angle], is to sine of [יא], the complement of [אב]. And conversely: as sine of [כאי], equal to [באצ?], is to sine of angle [כ], the upright, so

sine of [יכ], the complement of angle [צ], is to sine of [יא], the complement of [אב] — and this is what the third principle intended.

You see with your own eyes that the two [other] principles are branches of the first, and you need know nothing further.

And now I shall explain it [the first principle] to you by proof, so that the cord of silver shall not be removed [cf. Koheles 12:6] and the threefold cord shall not quickly be snapped [cf. Koheles 4:12]. First it will be explained for a right-angled triangle, and afterward for any triangle whatsoever. Behold the figure (Figure B): the two arcs [אד] [?] and [אחט] [?] are two halves of [the] surfaces of great circles, since the [axis?] of the sphere passes between the two [poles?] of the surfaces of the circles — that is, [each] from the section common to the two surfaces. Let there be a right-angled triangle [אבג — אבב], right-angled at angle [אבג], and angle [באג — בא?] of it acute; for had they been [acute/obtuse...]

obtuse, we would take the supplementary triangle. We wish to demonstrate that the ratio of the sine of angle [אבג], the right angle, to the sine of the flat [arc] [אג], equals the ratio of the sine of angle [א] to the sine of side [בג — בב]; and likewise the ratio of the sine of angle [אגב] to the sine of side [אב] — the opposite ones.

Extend the sides of triangle [אבג — אבב] up to quarter-circles: [אגח] [?] and [אבד] [?]; and likewise side [בג] up to point [ז] [?], which is accordingly the pole of circle [אדט?]. From it [from the pole] draw a further quarter-circle, [a circle] passing through the quarter-arc [a — אד?], at point [א], at right angles — so that point [א] is its pole. And let us drop from point [ח] and point [ג], on the circle [אוזט — אדזט], through its surface, two perpendiculars upon the diameter [אמ]: [these are חפ and גם?]. Then [קיו? — recte קי or חפ] is the sine of arc [אף] [?] — that is, the whole sine, since arc [אף] is a quarter-circle [recte: חד is the sine...?]; and line [גם] is the sine of arc [אג].

Further, from those same points [ח] and [ג], in the surface of circle [אדט], drop two lines [חי] and [גק], which fall upon the lines [פי] and [סק] [?] issuing from points [פ] and [ס — ם], coming through the surface of circle [אדט] at right angles. Then line [חי] is the sine of arc [חד], and line [גק] is the sine of arc [גב]; and the two triangles [חיפ — recte חפ-related?] [סקג?] and [הו — ?] are right-angled. When you examine them, you will find them equiangular and proportional in their sides, since the lines [חפ] and [גם] both stand upon line [אמ] in the surface [אדזמ?], and the angles are right; and by lesson 28 [proposition 28] of [Book] I of the Elements they are parallel. And lines [חי] and [גק] are likewise parallel, by the sixth lesson of [Book] eleven, since both stand upon a single surface, namely

circle [אדט]. Therefore, by the tenth lesson of [Book] eleven, the angles between the lines that are in different surfaces are equal — that is, angle [יחפ] equals angle [קגם]. But also the two angles [פחו? — ?] [סקג] are equal, since they are right; therefore, by lesson 32 of [Book] I, the two remaining angles [זיו? — ?] [גסק] [?] are also equal. And by the fourth lesson of [Book] six, the triangles [פיח?] [בקנ — ?] are equiangular and proportional in their sides — that is: as [חפ], the whole sine (which is the sine of the quarter-arc [אה?]) in triangle [אדח?], is to sine of side [חד], so is the ratio of [גם], the sine of arc [אג — אן?] in triangle [אג — אב?], to [גק], the sine of side [בג — בב]. And conversely, by lesson 16 of [Book] V: as sine of arc [אף] is to sine of arc [אג], so sine of arc [הד — חד] is to sine of arc [בג — בב]. But [אף] is a quarter, and its sine equals the sine of a right angle; and arc [חד] is the measure of angle [א]. Therefore, in triangle [אבג], as [the sine of angle אב — recte אבג?], the right angle, is to the sine of the flat side opposite it, [אג], so is the sine of angle [א] to the sine of side [גב] opposite it. And by this same path it will be explained that just as that ratio holds, so holds the ratio of the sine of angle [אגב] to the sine of side [אב] opposite it — if you extend sides [אג], [גב], [אב] toward [א] and [ב]. Know this rightly; it is easy.

And after it has been explained that the ratio of [angle א] to [side גב] and the ratio of [angle ג] to [side אב] are both equal to the ratio of [angle ב], the right angle, to [side אג], it follows likewise — by the universal proposition [the comprehensive rule] — that as [angle א] is to [side גב], so is the ratio of [angle ג] to [side אב]. This is what we wished to demonstrate.

Now in a non-right-angled triangle it will be explained easily, as follows. Drop from point [ד] in the figure before us a quarter-arc [קשע? — recte קשת], which is cut by quarter-arc [אשח?] at point [ש] [?] at oblique angles. We wish to demonstrate, in the oblique-angled triangle [בשק? — recte בשג or similar], the truth of the aforementioned lesson. Now, according to what we explained just above for the right-angled triangle [החש?]: the ratio of the sine of [חש] to the sine of [arc — קש וקש?] equals the ratio of the sine of [השח?] to the sine of [הח]. And likewise in the right-angled triangle [החג]: as the sine of [חש?] is to the sine of [הג], so is the sine of [הגש] to the sine of [הח]. Therefore, conversely, by

lesson 16 of [Book] V: as the sine of [החש] is to the sine of [השג], so is the sine of [הג] to the sine of [הח]. And behold, from lesson 17 [proposition 17] of [Book] V [?], it is very clearly demonstrated that the rectangle [product] formed from [הח] and [חש] equals the rectangle formed from [הג? — recte הגש] and [הש], since the extremes of the two [equal] ratios are equal, and the sides suffice. Therefore: as the ratio of [הג? — recte הגש] to [הש], so [הח] to [השג]; and conversely

[reciprocally] of the author [recte: by the reciprocal of the author]. This is the whole of the construction, and the truth is explained — and you have no need of the compounded ratio that Jabir and all the Christian sages [used] at all.

Know that the sine of an angle [ש?] is one and the same from every standpoint — that is, the sine of the arc equals the sine of [השג?], which is its complement; and therefore I have not been concerned, and I have taken the angle now from the left, now from the right, for all comes to one in truth. And likewise it will be explained that just so is the ratio of angle [חש] to [ש?] opposite it.

And drop from point [ד — ך?] a perpendicular arc [?]. Now in the right-angled triangle [נכה? — recte החש?], as we explained just above, it is: as [וזב — recte הזב? or וגב?] is to [כ], so [כג] to [ה]; and likewise in the right-angled triangle [גכש?]: as [גש] is to [כ], so [כג] to [ש]. And since the rectangle formed from the two means equals the rectangle of the extremes, when four magnitudes are proportional — as is explained in [proposition] sixteen of [Book] six — and the mean rectangles in the two aforementioned ratios were equal, therefore, by lesson 14 of [Book] six, the extremes of the two ratios are reciprocally sufficient one to the other. So the expositor rendered into the Holy Tongue the word *reciproci* [ריציפרוקי — Latin: *reciproci*, "reciprocal"].

And it will be: as the ratio of [בש?] to [הזג?], so [כזה? — recte הזה?] to [ש]; and conversely, as the ratio of [גש] to [ה], so [הג] to [ש]. This is what was sought. And after it has been explained that both the ratio of [הש] to [ג] and the ratio of [גש] to [ל? — recte ה?] are equal to one other [ratio], namely [הזג?] to [ש], it follows likewise — by the universal proposition — that the ratio of [חש] to [ש?] equals the ratio of [בש?] to [ה?].

Thus it has been explained that in every triangle, whether right-angled or oblique, the angles are proportional to the sides opposite them. This is what this lesson intended, and it is the first principle in R. Yitzchak's [system]. By it I shall instruct you, and teach you the solution of every triangle from this one principle alone.

Since all the parts of a triangle are six — I mean three angles and three sides — in spherical triangles it is impossible to determine an unknown part of the six unless at least three of its parts are known to us, whether they are all angles, or all sides, or mixed, as R. Yitzchak z"l taught. And indeed, of the spherical triangles, some have right angles and some have all their angles oblique. The aforementioned author did not go on at length about these — perhaps because

their solution depends on the solution of the right-angled triangles, as he showed in Corners 2 and 3, with God's help.

And behold: of the right-angled spherical triangles, some have three right angles, and of necessity their sides are three quarter-circles. So too, a triangle all three of whose sides are quarter-circles — you will at once say that its three angles are right. And some triangles have two right angles, and of necessity the two sides opposite them are quarter-circles; likewise, if you know that they have two right angles, you know that they have two quarter-circles opposite them. But the third side cannot be known unless you know the measure of the angle opposite it; and similarly, even if you know that a triangle has two right angles and two quarter-circle [sides], you cannot know how great the third angle is unless you know the third side, which is its measure. We have no concern with these triangles here. Rather, our task in this Corner is the instruction of the solution of a triangle that has one [right] angle...

where it is right-angled and where no side is known to be a quarter of the circle [i.e. 90°]. This happens in one of three cases: either two of the angles, apart from the right angle, are acute — in which case the sides facing them are smaller than a quarter of a circle [< 90°] — or they are obtuse, in which case the sides facing them are necessarily larger [> 90°]; or one of its angles is acute and one obtuse, in which case the surface [side] facing the right angle is necessarily larger than a quarter of a circle [> 90°], as I explained in the demonstrations in my commentary on Menelaus and in what I set out earlier concerning the Book of the Almagest. And although the chord itself is the same for the acute angle and for the obtuse angle that completes it [are supplementary], and likewise for the small arc and the large arc that complete a quarter [the quadrant] — which is what operates at the middle of the circle — and so too for the matter of the lines that touch and cut the circle, there is still a difference in computation, namely, whether the angles are obtuse or acute, or whether the sides are larger than a quarter of a circle or smaller, in a triangle we wish to resolve and reveal whose parts are unknown according to the measure.

Despite all this, so that you may proceed by the straight road and your eyes may behold your good teacher, it is best that, when you set out to resolve a triangle in which there is an angle or a side or several sides greater than ninety [degrees], you resolve the triangle that stands opposite to it on the balance, which you will find by extending its sides as far as half the circle, on the side proper to your operation, just as I have shown you in the first figure, which it is fitting you keep always before your eyes, since it recalls to you the lessons needed for your craft,

so that you will not need any book in resolving the triangles. And in this third figure too the matter is laid out before you, that by extending the sides you find another triangle adjacent to the first, in which you will have three known parts — some of them the complements of the first three, and some of them the very same, since they are shared and stand between the two triangles.

By way of illustration, take the triangle אבג [a, b, g] (Figure [3] ג), in which angle ב [b] is right and side אה [a-h] faces the right angle — which, from here on, we shall call by the name *nashtach* [the hypotenuse, the side facing the right angle] — and it is greater than a quarter of the circle [> 90°], and side אב [a-b] is small [< 90°]; and we wish, from these three known parts, to resolve the said triangle: I mean, to know the measure of side הב [h-b] and angles ה [h] and א [a]. Now we extend the side אה [a-h] as far as the completion צ [tz] — half the circle [180°] — and we shall have, in triangle אבצג [read: אבצ, a-b-tz], the known parts: angle ב [b], right; side אב [a-b], shared between the two triangles; and the surface [hypotenuse] אצ [a-tz], which is the complement of אה [a-h], the known [180° − a-h]. And when we resolve this small triangle — that is, when we find the measure of side צב [tz-b], whatever it is — at once I shall know its complement בה [b-h]; and likewise, when I find the measure of angle צ [tz], I shall know the measure of angle הז [recte ה, h], equal to it, since two circles that cut each other have equal alternate cutting-angles, as was established elsewhere; and so too we shall know angle צאב [tz-a-b], and at once its complement is known, angle [h]זאב [recte הזאב / the angle at א facing the hypotenuse]. And in this same way, in a triangle אצ [read: a triangle whose] two angles are obtuse, we take, in order to resolve it, the adjacent right triangle אבצ [a-b-tz], to know it correctly with ease; and there is no need to elaborate where brevity is fitting.

And since we are dealing with a triangle that has one right angle, we shall not mention the right angle at all, for it is certainly known, and it is what guides and directs us in resolving the triangle; rather, we shall mention only its two known parts besides it. So too, in the case of values [the ratios], the right angle — which is the whole chord [the radius, sin 90° = 60] — I shall represent by the letter [?] with two dots [point-marks], for love of brevity. Take note, and know this for yourself.

Now the questions sought in order to resolve every right-angled triangle are sixteen, all of them clarified as we have said with this first principle. But eight of them are resolved by means of a single proportion [one operation] — that is, with the computation of the golden lesson, at one stroke —

— while the remaining eight require two. Know that R. Yitzchak's computation, which set the half-diameter [radius] at sixty degrees in the manner of [Ptolemy?] and the ancients — who took the whole chord beginning from the number six, that is, 60 or 600 or 6000 or 60000 — because this number is special and more apt than other numbers, as we explained in the *Book of the Whole Number* called *Shever Yosef* — has come and brought with it much toil and labor, which is not so with the decimals [base-ten reckoning]. Therefore all the modern astronomers use a chord of 10000 or 100000, or more, as they please, in order to be precise in computation; and the table of chords thus made is in everyone's hands. Its structure and arrangement we explained in our commentary on the Almagest. And by this we gain no little time in computations, since in the four-term ratios in which the whole chord [the radius] is one of the means, there is no need to multiply or take a product, but only to divide; and if it is the first term, there is no need to divide but only to multiply — and this we use always.

And here are the eight questions that are resolved by means of a single proportion according to this principle. In the first figure we wished to know, in triangle אבצ [a-b-tz], one of the angles, after we knew, besides the right angle, one of the acute [angles] and a side parallel to [facing] the sought angle. And the order of the question is thus:

[1] א. The angle, from one [given] angle and a side facing the sought [angle]: Then form the proportion and say: As the chord ה [h — i.e. the radius, sin 90°] is to the chord פצ [p-tz, the complement of] צב [tz-b], so is the chord צ [tz] to the chord פה [p-h, the complement of] the sought angle א [a]. As the chord כ [k — the radius] is to the chord יא [y-a, the complement of] אב [a-b], so is the chord א [a] to the chord יב [y-b, the complement of] the sought angle ב [b].

[2] ב. The hypotenuse [*nashtach*], from the two sides: As the chord ה [h — the radius] is to the chord פצ [p-tz, the complement of] צב [tz-b], so is the chord פ [p, the complement of] אב [a-b] to צה [tz-h, the complement of] אצ [a-tz], the sought hypotenuse.

[3] ג. The side, from the hypotenuse and the angle facing the sought side: As the chord ב [b — the radius] is to the chord אצ [a-tz], the hypotenuse, so is the chord צ [tz] to the chord אב [a-b], the sought. As the chord ב [b — the radius] is to the chord אצ [a-tz], the hypotenuse, so is the chord א [a] to the chord בצ [b-tz], the sought.

In these three [problems], the whole chord [the radius] is at the start of the proportion, and therefore they are resolved by multiplying the second [term] by

the third and dividing the product by the number of cipher-marks [digits] of the whole chord; the remainder is the fourth chord, the sought. But in the five that follow, the whole chord [the radius] is in the middle of the proportion, and they require division alone, after you add the cipher-marks [zeros] of the whole chord to the second number, or to the third, as you have learned in the [book of] number.

[4] ד. The angle, from the hypotenuse and the side facing the sought [angle]: As the chord אצ [a-tz, the hypotenuse] is to ב [b — the radius], so is the chord אב [a-b] to the chord צ [tz], the sought. As the chord אצ [a-tz] is to [the radius], so is the chord צב [tz-b] to the chord [of] the sought angle א [a].

[5] ה. The angle, from one of the acute [angles] and the side near the sought angle: As the chord פצ [p-tz, the complement of] צב [tz-b] is to ה [h — the radius], so is the chord פה [p-h, the complement of] א [a] to the chord [of] the sought angle צ [tz]. As the chord אי [recte יא, y-a, the complement of] אב [a-b] is to כ [k — the radius], so is the chord כי [k-y, the complement of] צי [recte צ, tz] to the chord [of] the sought angle א [a].

[6] ו. The hypotenuse [*nashtach*], from the side and the angle facing it: As the chord א [a] is to the chord צב [tz-b], so is [the radius] to the chord אצ [a-tz], the hypotenuse. As the chord צ [tz] is to the chord אב [a-b], so is [the radius] to the chord אצ [a-tz], the hypotenuse.

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As the chord א [a] is to the chord צב [tz-b], so is [the radius] ב [b] to the chord אצ [a-tz], the hypotenuse. As the chord צ [tz] is to the chord אב [a-b], so is [the radius] ב [b] to the chord אצ [a-tz], the hypotenuse.

[7] ז. The side, from the hypotenuse and one of the sides: As the chord פ [p, the complement of] אב [a-b] is to the chord צה [tz-h, the complement of] אצ [a-tz], the hypotenuse, so is ה [h — the radius] to the chord פצ [p-tz, the complement of] צב [tz-b], the sought side. As the chord י [y, the complement of] צב [tz-b] is to the chord כא [k-a, the complement of] אצ [a-tz], the hypotenuse, so is כ [k — the radius] to the chord אי [recte יא, y-a, the complement of] אב [a-b], the sought side.

[8] ח. The side, from both of the angles: As the chord צ [tz] is to the chord פה [p-h, the complement of] א [a], so is ה [h — the radius] to the chord פצ [p-tz, the complement of] צב [tz-b], the sought. As the chord א [a] is to the chord כי [k-y, the complement of] צ [tz], so is כ [k — the radius] to the chord אי [recte יא, y-a, the complement of] אב [a-b], the sought.

But the eight questions that follow after these are resolved by means of two proportions, on the basis of the first principle — and even with the three principles of R. Yitzchak, the computation is no lighter even by a hair's breadth at all. And these are they:

[9] ט. The angle, from the two sides. First we seek אצ [a-tz], the hypotenuse, as in the second question; and afterward, from the hypotenuse and the side facing the angle, we find the angle as in the fourth question. By way of illustration: As the chord ה [h — the radius] is to the chord פצ [p-tz, the complement of] צב [tz-b], so is the chord פ [p, the complement of] אב [a-b] to the chord צה [tz-h, the complement of] אצ [a-tz], the hypotenuse. And afterward we say: As the chord אצ [a-tz] is to ב [b — the radius], so is the chord אב [a-b] to the chord [of] the sought angle צ [tz].

[10] י. The angle, from the hypotenuse and the side near the sought angle. First we find the second side, as in the seventh question; and afterward we find the angle, from the hypotenuse and the side facing the angle, as in the fourth question.

[11] יא. The angle, from the hypotenuse and the other angle. First, as in the third question, we find the side facing the known angle; and afterward, as in the eighth question, from this side and the known angle, we find the second angle.

[12] יב. The hypotenuse, from all the angles. First we find one of the sides according to the eighth question; and afterward, with the angle facing the known side, by means of the sixth question, we find the hypotenuse.

[13] יג. The hypotenuse [*nashtach*], from the side and the angle adjacent to it. First we find the second angle by aid of the first question; and by it, with the known side that faces it, according to the sixth question, we find the hypotenuse.

[14] יד. The side, from the hypotenuse and the angle adjacent to the sought side. First we find the side facing the known angle, from the third question; and afterward, with the side and the hypotenuse, as in the seventh question, we find the sought side.

[15] טו. The side, from one of the sides and the angle near it. First we find the second angle —

— according to the first question; and with the aid of the two angles we find the sought side according to the eighth question.

[16] ʁɒ. The side, from the side and the angle facing it [which is] adjacent to the sought side. First we seek the hypotenuse, as in the sixth question; and with the hypotenuse and the known side we find the second side, as in the seventh question.

I have shown you that all the questions needed to resolve the right-angled spherical triangle are resolved with the first principle alone, by means of the table of chords — except that in the eight [two-step ones] you must double the computation, and this brings much weariness of the flesh. And it is true that the generations advance and improve: for he who came after Hipparchus and Menelaus and shortened the way of resolving the triangles — it seems his loss was great, and his way was to use, behind the table, the chords [*yetarim*] of the whole arcs [a table of versed chords / full chords]; and this is a way devoid of wisdom. For, with the fear of God, whoever has dealings would be wearied to compute many reckonings in that fashion, and would not avoid many errors. Therefore my first instructor, whom I came to know after Ptolemy and Theon, devised the chords [*bekaim*] in place of the whole chords [*yetarim*], since as the whole is to the whole, so is the part to the part — I mean, as the chord [*yeter*] of an arc is to the chord of an arc, so is the ratio of the half-chord of one arc to the half-chord of the [other] arc; and the chords [*bekaim*, the sines] are halves of the chords [*yetarim*]. This makes it still easier than what they [the ancients] did by means of the lesson of the six measures [the sexagesimal reckoning], showing how it stands with the lesson of the [base-ten / decimal?] measures.

Up to here reached the hand of the ancients, and R. Yitzchak with them. And those who came after them — the scholars of Edom [Christian Europe], renowned in every science and understanding in the disciplines, who are dwarfs riding on the heads of giants — bound everything with a cord and drew water from deep wells, and found other lines besides the chords [sines], which they called *tangents* and *secants* [literally "touching" and "cutting" lines], and upon them built strong lessons and demonstrations, until they clarified all that was hidden in the laws of the triangles. And just as it is their whole aim to magnify and exalt the premises and propositions — there are some of them who have written ten handbreadths of paper to display their power and the might of their hand — so it is my view to do the opposite: to use only what is necessary and no more. Therefore I take from all of them but one lesson, and through it, with the aid of the first principle, we shall come to our goal.

And this shall be called **the Second Stone** [*ha-Even ha-Sheniyah*], which is **a touchstone** [*even bochan*], a precious cornerstone whose worth is not less than

that of the first principle, by which we shall explain the eight remaining questions in a single proportion. And this is the elucidation:

In spherical triangles having right angles, in which one of the acute angles is equal [shared by both] or shared between them, the ratio between the chords [sines] of their bases is as the ratio of the tangents of their perpendiculars [*nog'ei amudeihem*].

[Margin / illustration: nog'ei amudeihem — the tangents of their perpendiculars.]

In the figure (Figure [2] ב) by which we clarified the truth of the first principle, the right-angled triangles [are] **אבד** [a-b-d] and **אבג** [a-b-g] [recte: triangle אבג with the right angles at ד and ב]: angle ד [d] and angle ב [b] are right, and angle א [a] is equal [shared], common to both. We shall clarify that the ratio of the chord [sine] of base אד [a-d] to the chord [sine] of base אב [a-b] is as the ratio of the tangent [*noga*] of the perpendicular arc דד [recte רד / read: the perpendicular at ד] to the tangent of the perpendicular arc גב [g-b]. And let the chord [sine] of arc אד [a-d] be a quarter [the radius]; the line פד [p-d], which is the whole chord [the radius], and the line פב [p-b]…

Sod HaYesod — Corner I (continued)

The Second Stone (continued): the tangent rule — demonstration on Figure ב …the sine of arc **אב**, and upon the ends of the perpendicular arcs **ד** and **ב** we erect two perpendiculars **דנ** and **בר**, which meet the secants **פחנ** and **סגר**. Then, by the rule of the tangents, line **נד** is the tangent of the perpendicular arc **חד**, and line **רב** is the tangent of the perpendicular arc **ע** [namely **גב**].

Now, as we explained in the demonstration of the First Principle, the two triangles **פדן** [and **סגר**] are similar and proportional. Therefore the ratio of **פד** to **נד** equals the ratio of **סב** to **בר**; and, by alternation, the ratio of **פ** to **סב** equals the ratio of **נד** to **בר**. That is to say: as **פד** — which is the sine of the base **אד** in triangle **אד** [against] **סב**, the sine of the base **אב** in triangle **אבג** — so is **נד**, the tangent of the perpendicular arc **חד** in triangle **אדח**, to **רב**, the tangent of the perpendicular arc **בג** in triangle **אבג**. This is what we set out to demonstrate.

By our method we can demonstrate likewise that in every right spherical triangle, as the ratio of one sine of the sides to another sine, so is the tangent of the angle adjacent to it to the tangent of the second side [leg]. For example, in the figure before us: the ratio of the sine **פד** — which is the whole sine [radius] — to **סב**, the sine of side **אב**, equals the ratio of the tangent **נד** of arc **חד** — which is the measure of angle **א** adjacent to the known side **אב** — to the tangent **רב** of the second side.

And likewise, by alternation, as the ratio of [tangent] **ב** the right angle to angle **א**, so is the side adjacent to [angle ב] to the side **גב** mentioned above. [recte: the relation reads tangent-of-angle to tangent-of-side; the compositor's wording here is compressed.]

In many varied ways one finds one and the same lesson among the [Christian] authors, who — to conceal their theft from [our] books — disguise themselves and cloak themselves in other garments. Nevertheless, the customary devices on which the ancients relied to enrich the disciplines, putting them to use for their own ends, are for the most part sound — chiefly the fifth of [Euclid's] foundations [Book V], the [book] on ratios, of which Euclid speaks at length: upon it are founded all the crafts and sciences and the rational, philosophical syllogisms. And since there is no end to its treasures, we shall vary each lesson according to its indications on three different patterns alone.

Take, for example, the matter before us. We explained that the ratio of the sine of the base **אד** to the sine of the base **אב** equals the [ratio of the] tangent of perpendicular **דח** to the tangent of perpendicular **בג**. Therefore, by alternation, according to the sixteenth lesson [problem] of [our catalogue], it will also be:

> as the ratio of the sine of the base **אד** to the tangent of perpendicular **דח**, so the sine of the base **אב** to the tangent of perpendicular **בג**.

And, by inversion, following from the fourth lesson, it will also be:

> as the ratio of the sine of the base **אד** to the sine of the base **אד** [recte: **אב**], so the tangent of perpendicular **בג** to the tangent of perpendicular **דח**.

And, by alternation and inversion together, it will also be:

> as the ratio of the tangent of perpendicular **דח** to the sine of the base **אד**, so the tangent of perpendicular **בג** to the sine of the base **אב**.

Therefore study the ratios well, and do not be perplexed if you do not at once know the source of the result and its origin.

Know, too, that you may reckon the small sides **בג** ,**דח** as the bases and the large ones as the perpendiculars, and then it will be:

> as the ratio of the sine **דח** to the sine **גב**, so the tangent **דא** to the tangent **אב**.

And, by alternation:

> as the ratio of the sine **דח** to the tangent **דא**, so the sine **גב** to the tangent **אב**.

And so by inversion, by alternation, and by inversion-and-alternation together — all as you wish and according to the need of your application.

The Second Stone — problem catalogue (Corner II — Stone 2, side א)

Running header: אבן בוחן ב' מפנה א — "Touchstone II, of Corner I."

After the truth of this Second Principle has become clear to you, fix your eyes on the first figure, which is [the one] you will understand once you grasp the [right] triangles, well. Then at one stroke you will solve all eight problems. **(Figure א.)**

*[The triangle is **אבצ**, right-angled at **ב**, with the perpendicular **בצ** (the leg "standing" on the base), **אב** the base, and **אצ** the hypotenuse (nashtach); **צ** is the apex. The eight problems pair the known and sought parts.]*

9. [Finding] the angle from the [two] sides.

As the ratio of the sine **צב** of the base [perpendicular] to the tangent **בא** [recte: tangent of **בא**] in triangle **אבצ**, the sine [of] **צג** [recte: the base], so the [sine of the] base of triangle **צבכ** [recte: **אבצ**] to the tangent **בכ** [recte: **בצ**], which is the perpendicular [opposite] — and this is the measure of angle **צ**, the sought one. **

10. The angle, from the hypotenuse (nashtach) and the side near the sought angle.

As the ratio of the tangent of perpendicular **בג**, the complement of side **בצ**, to the base — the whole sine [radius] — so the tangent of perpendicular **אב**, the

complement of the hypotenuse (nashtach), to the sine of the base **בצ**, which is the complement of angle **צ**, the sought one.

11. The angle, from the hypotenuse (nashtach) and the other angle.

As the ratio of the base **צג** — the whole [radius] — to the tangent of perpendicular **גב**, the measure of the known angle **צ**, so the ratio of the sine of the base **בצ**, the complement of the known nashtach, [namely] **צם** [?], to the tangent of perpendicular **אב**, the complement [of] **בד** [recte: **בצ**] — and this is the measure of angle **פ** [recte: **א**?], the sought one. **

12. The nashtach (hypotenuse), from the two angles.

As the ratio of the tangent of perpendicular **גכ** [recte: **גב**], the measure of angle **צ**, to the base **גצ** — the whole sine — so the [ratio of the] sine of the base **בצ**, the complement of angle **פ** [recte: **א**], to the tangent of the perpendicular, the complement of the nashtach, sought. **

13. The nashtach, from a side and the angle near it.

As the ratio of the base **יב** [recte: **גב**] — the whole sine — to the tangent of perpendicular **גב** [recte: the relevant leg], the complement of side **צב**: [so the] sine of the base **יב** [recte: **גב**], the complement of angle **צ**, to the tangent of perpendicular **כא**, the complement of the nashtach **אצ**. **

14. The side, from the nashtach and the angle adjacent to the sought side.

As the ratio of the sine of the base **יב** [recte: **גב**], the complement of angle **צ**, to the tangent of [perpendicular] **כא**, the complement of the nashtach **אצ**, so the whole sine **גצ** to the tangent of perpendicular [**גב**?], the complement of the sought side **בצ**.

15. The side, from the other side and the angle adjacent to it, which faces [opposite] the sought side.

As the ratio of the whole sine **גצ** to the tangent of perpendicular [**גב**?], the measure [of angle] **צ** [?], so side **בצ** to the tangent of perpendicular **כא**, the sought one.

⁕ 16. The side, from the side and its facing angle [the angle opposite it], which is adjacent to the sought side.

As the ratio of **גצ**, the tangent of angle **צ**, to **בצ** [recte: the whole sine], so the tangent of side **אב** to the sine of [the base] **בצ**, the sought side.

There you have the solution of every problem, in few words and with demonstration. By means of the two principles the matter of triangles stands explained, like a garment. The one difficulty common to them all is that three of the problems issue only by means of multiplication alone — since the whole sine [radius] stands at the head of the ratio, while the five remaining ones in each principle (which are ten in all) have the whole sine come in the middle, and they require division. And in the numbers of the sines and the tangents this is exceedingly difficult for the multitudes, and easy to err in.

For this reason the later authorities found no rest in all this; they sought a device to bring the whole sine [radius] to the beginning of the ratio in every problem. And indeed the thing that came into their hands is true and correct, and it is a precious instrument and a treasure. It is called **the Third Stone** — **Even Shlomo (Solomon's stone)** — "stones of fire, crystal and onyx [recte: emerald], beauty of sapphire and amethyst." [Cf. *Yechezkel* 28:13 / *Iyov* 28:18.] And this is the explanation:

Sod HaYesod — The Third Stone, Even Shlomo

Running header (p. 145): מספר סוד היסוד — "The Book *Sod HaYesod*."

The whole sine [radius] is the mean proportional between the sine of an arc and the secant of its complement; and likewise between the tangent and the secant of [the arc's] complement.

[That is: R is the geometric mean both between sin(arc) and sec(complement), and between tan(arc) and sec(complement).]

To explain it, we mark off about the center **ד** a circle **אב (Figure ד)**, and in the quadrant **בג**, whose sine [radius] is **גד**, let there be an arc **בה**, whose sine is **הפ** [recte: **פ**], its complement **גה**, and the secant of the complement **דהב** [recte: **דהכ**]. We shall demonstrate that:

> as the ratio of the sine of arc **בה**, [namely] **הפ**, to **גד**, the whole sine [radius], so the ratio of **גד**, the whole sine, to **כד**, the secant of arc **גה**, which is the complement of arc **בה**.

Demonstration. Since the two triangles **כגד** and **דפה** are equiangular [similar] — for [angle] **פ** and [angle] **ג** are right angles, and angle **ב** [recte: **ד**] is equal to angle **הדפ** by the twenty-ninth lesson of the First [Book of Euclid] — therefore, by the thirty-second lesson of the First, the remaining [angle] **דכג** is also equal to angle **פהד**. And, by the fourth of the Sixth [Book], it will be:

as **הפ** to **הד**, so **דג** to **כד**.

But **הד** and **דג** are equal, being [both] the radius of the circle. Therefore it will be:

as the ratio of **הפ** to **גד**, the whole sine, so **גד**, the whole sine, to **כד**, the secant of **גה**, which is the complement of **בה**.

And this is what we sought.

Now, for the explanation of the second part [tangent and secant], we shall have, in this same figure, the tangent of arc **גה** [be] line **גכ**, the complement of the arc being **בה** [recte: **בז**?], and its tangent **בה** [recte: the relevant tangent], and **גד** the whole sine. We shall demonstrate that **גד**, the whole sine, is the mean proportional between the two tangents — that is, that as the ratio of **כג**, the tangent of arc **גה**, to **גד**, the whole sine, so **גד** [recte: the radius] to **בז** [?], the tangent of the arc of its complement, [namely] **חב** [?]. **

Demonstration. Since the two triangles **כגד**, [and **דבר**] are equiangular — I mean that angles **ד** and **ב** are right angles, and angle **כדג** [recte: the angle at **ד**] is equal to angle **גכד** by the twenty-ninth of the First; and, by the thirty-second of the First, [angle] **כדג** [recte: the third angle] will likewise be equal to angle **כדג** [recte: **בדג**] — therefore, by the fourth of the Sixth, the triangles are similar and proportional, and it will be:

as the ratio of **כ** [recte: **כג**] to **גד**, so **דב** to **בר**.

But **גד** and **דב** are equal. Therefore as the ratio of **כג** to **גד**, so **גד** to **בר**. This is what we sought to demonstrate.

From here you learn that whenever the whole sine [radius] is the second or third [term] among four proportionals at whose head stands a sine, you may bring the whole sine to the beginning of the ratio; for its second [term] set the secant of the complement of the arc that was first, and [keep] the third — or the second — as it is, and all is put right. Likewise, if the first [term] in the ratio is the tangent of an arc, bring the whole sine to the head of the ratio, and for its second [term] set the tangent of the complement of the first arc. In this way you generate new ratios by which you test your reckoning along various paths. And here are the ten remaining problems for you, set out — with multiplication alone, as were the first six. See and do.

The Third Stone — problem catalogue (secant set, side א of Corner I)

Running header (p. 146): אבן שלמה — "Even Shlomo (the Stone of Solomon)."

[3.] The angle, from the nashtach and the parallel [opposite] side.

As the ratio of the whole sine [radius] to the secant of the complement of arc **אצ**, so the sine **אב** to the sine [of] **ד** [recte: **צ**?], the sought one. **

4. The angle, from the other angle and the side near the sought angle.

As the ratio of the whole sine to the secant **צב**, so the sine **פה**, the complement of angle **א**, to the sine **צ**, the sought angle.

6. The nashtach, from the side and the angle facing it.

As the ratio of the whole sine to the secant of the complement of [arc] **א**, so the sine **צב** to the sine **אצ**, the sought nashtach.

7. The side, from the nashtach and the other side.

As the ratio of the whole sine to the secant **אב**, so the sine **אה** [recte: **בג**?], the complement of the nashtach **אצ**, to the sine **פצ**, the complement of side **צב** [recte: **צף**], the sought side. **

> As the ratio of the whole sine to the secant **צב**, so the sine **כא**, the complement of the nashtach **אצ**, to the sine **יא**, the complement of side **אב**, the sought side.

8. The side, from each angle.

> As the ratio of the whole sine to the secant of the complement of [angle] **צ**, so the sine **פה**, the complement of angle **א**, to the sine **פצ** [recte: **צף**], the sought side.
>
> As the ratio of the whole sine to the secant of the complement of [angle] **א**, so the sine **כי**, the complement of angle **צ**, to the sine **אי**, the complement of side **אב**, the sought side.

Up to here are the problems that issue from the First Principle and the first part of the Third. The two that follow are explained by means of the Second Principle and the first part of the Third:

[9.] The angle, from the two sides.

As the ratio of **צג**, the whole sine, to the secant of the complement of [side] **בצ**, the side adjacent to the angle, so the tangent of the parallel [opposite] side **אב** to the tangent **גב** [recte: tangent of the relevant leg] — the measure of angle **צ**, the sought one.

14. The side, from the nashtach and the angle adjacent to the sought side.

As the ratio of **יג** [recte: **גצ**], the whole sine, to the secant of angle **צ**, so the tangent **כא**, the complement of the nashtach **אצ**, to the tangent **גב**, the complement of side **בצ**, the sought side. **

And these three [recte: the following] are explained according to the Second Principle and the second part of the Third Principle:

10. The angle, from the nashtach and the side near the sought angle.

As the ratio of **גצ**, the whole sine, to the tangent of side **בצ**, so the tangent **אב** [recte: of the relevant arc] to the sine **כי**, the complement of angle **צ**, the sought one.

12. The nashtach, from each angle.

As the ratio of **גצ**, the whole sine, to the tangent **כי**, the complement of angle **צ**, so the tangent **אב**, the complement of angle **פ** [recte: of the relevant angle], to the sine **בצ**, the complement of **צף**, the sought nashtach.

16. The side, from the other side and the angle facing it, which is adjacent to the sought side.

As the ratio of **בצ**, the whole sine, to the tangent of the complement of angle **צ**, so the tangent of side **אב** to the sine **בצ**, the sought side.

And after God has made all this known to you, I hereby set before you this table, in the order of the problems that I have in my book ***Arubot HaShamayim*** **(The Windows of Heaven)** — although there I explained each problem with several more patterns in which the whole sine [radius] [stands] at the beginning. From now on, whenever I mention one of these problems…

[The text continues; the table promised here falls in the following pages.]

I will recite them in the order of this table — yet the earlier numbers are themselves disqualified and void. I have already told you that my intention here is to be very brief; for had I wished to expand, I would have disclosed beautiful matters and one tool after another, more than these, to serve as instruments —

and this method is fitting for beginners. There is still more to say on these things elsewhere, God willing.

Table of the Problems for Right-Angled Triangles — Sixteen Towers [Columns]

[This "table" (לוח) is set as a running list of sixteen numbered *towers* (טורים, "columns/entries"), grouped under three headings according to what is being solved for: the angle, the hypotenuse, and the side. Each entry names the two given parts and then states the proportion. Throughout, *beka* = sine, *noge'a* = tangent, *chotech* = secant, *tashlum* = complement, *nashtach* = hypotenuse.]

Resolving the Angle

The First Tower — from the two sides. As the whole sine is to the secant of the complement of the side adjacent to the sought angle, so is the tangent of the side opposite the sought angle to the tangent of the sought angle.

The Second Tower — from the hypotenuse and the side opposite the sought angle. As the whole sine is to the secant of the complement of the hypotenuse, so is the sine of the side to the sine of the sought angle.

The Third Tower — from the hypotenuse and the side adjacent to the sought angle. As the whole sine is to the tangent of the side, so is the tangent of the complement of the hypotenuse to the sine of the complement of the sought angle.

The Fourth Tower — from the hypotenuse and one of the acute angles. As the whole sine is to the tangent of the angle, so is the sine of the complement of the hypotenuse to the tangent of the complement of the sought angle.

The Fifth Tower — from one of the acute angles and the side opposite the sought angle. As the whole sine is to the sine of the complement of the side, so is the sine of the known angle to the sine of the complement of the sought angle.

The Sixth Tower — from one of the acute angles and the side adjacent to the sought angle. As the whole sine is to the secant of the side, so is the sine of the complement of the known angle to the sine of the sought angle — and this is ambiguous, whether [the angle is] acute or obtuse.

Resolving the Hypotenuse

The Seventh Tower — from the two sides. As the whole sine is to the sine of the complement of the one side, so is the sine of the complement of the second side to the sine of the complement of the hypotenuse.

The Eighth Tower — from the two angles. As the whole sine is to the tangent of the complement of the first angle, so is the tangent of the complement of the second angle to the sine of the complement of the hypotenuse.

The Ninth Tower — from the side and the angle opposite it. As the whole sine is to the secant of the complement of the angle, so is the sine of the side to the sine of the hypotenuse.

The Tenth Tower — from the side and the angle adjacent to it. As the whole sine is to the tangent of the side, so is the sine of the angle to the tangent of the complement of the hypotenuse.

Resolving the Side

The Eleventh Tower — from the hypotenuse and the angle opposite the sought side. As the whole sine is to the sine of the hypotenuse, so is the sine of the angle to the sine of the side that lies opposite it and is sought.

The Twelfth Tower — from the hypotenuse and the angle adjacent to the sought side. As the whole sine is to the secant of the angle [?], so is the tangent of the complement of the hypotenuse to the tangent of the complement of the sought side.

The Thirteenth Tower — from the hypotenuse and one of the sides. As the whole sine is to the secant of the known side, so is the sine of the complement of the hypotenuse to the sine of the complement of the sought side.

The Fourteenth Tower — from one of the sides and the angle adjacent to it, which is opposite the sought side. As the whole sine is to the tangent of the angle, so is the sine of the known side to the [tangent of the] sought side.

The Fifteenth Tower — from one of the sides and the angle opposite it, which is adjacent to the sought side. As the whole sine is to the tangent of the complement of the side, so is the tangent of the known side to the sine of the sought side — and this is ambiguous, whether [it is] smaller or larger than a quadrant.

The Sixteenth Tower — from the two angles. As the whole sine is to the secant of the complement of the angle adjacent to the sought side, so is the sine of the complement of the angle opposite the sought side to the sine of the complement of the sought side.

Now there is no doubt that, if the right [-angled] triangle is properly drawn, its sides and angles will not deceive you, so that you will not fall into uncertainty over the ambiguous Towers mentioned — as happens with triangles seen on the great circles of the sphere that cut one another. You may also draw the spherical triangles on a small ball made of wax, smoothing and erasing them at will. But whenever the figure of the triangle is *not* drawn, and you seek its hidden parts, the matter is uncertain — for a single sine is shared by an arc and its complement up to half the circle, as I have shown you by demonstration elsewhere. And with triangles of curved angles, which are resolved by means of right-angled triangles that make use of these three ambiguous [Towers], they too will be as ambiguous as those. Therefore keep your eyes open, and do not stumble along with the many distinguished men among the ancients.

Now that this honored science has come into our hands, I will inform you of its use in all your reckonings — so that in your buying and selling you will have no need for the operation of division at all, for multiplication alone will suffice you, whenever you have at hand the instructional table called the *Canon Mathematicum* [Latin: *Canon Mathematicus*], in which are fixed and tabulated the magnitudes of the chords, or of the sines, the tangents, and the secants belonging to every degree and part of the arc of the circle — provided that the whole sine begins from 1, that is, from 100000, or more.

For further clarification of the matter, understand it through a right-lined triangle אבג [A-B-G] of curved angles — which is, as we explained in the book *Arubot HaShamayim* (and as will also be made clear in the Third Spring [ʿEyn HaGimel]): as the sine of angle ב [B] is to בג [BG], so the sine of angle ב [G] [recte: angle G — the text writes "B" by compositor error] is to side אב [AB] [?]. And in order to find side אג [AG], you must multiply בג [BG] by אם [recte: אב, AB] and then divide the result by אב [AB].

But if you wish to be free of the labor of division, and prefer instead [to work] with two operations: drop from angle א [A] the perpendicular אד [AD] onto side גב [GB]. Reckon side אב [AB] as half the diameter of a circle whose center is point ב [B]; then side אד [AD] will be the sine of angle ב [B], and side דב [DB] the sine of angle דאב [DAB]. Apply the rule and say, according to that science: as the whole sine is to the sine of angle ב [B], so is the number [length] אב [AB] to אד [AD]. And

once you know the magnitude of אד [AD] as a fixed number, go back and reckon that same אד [AD] as half the diameter of a circle whose center is now point א [A]; then אג [AG] will be the secant of angle גאד [GAD], which is the complement of angle ב [B]. Apply the rule a second time and say: as the whole sine is to אב [AB], the secant of angle באד [BAD] — I mean the secant of the complement — so is the number of side אד [AD] to אג [AG]. There — you have found אב [recte: אג, AG] without division.

Let us now suppose angle ג [G] = 40 degrees, whose sine is 64278; angle ב [B] = 30, whose sine is 50000; and side אב [AB] = 10 cubits. We wish to know how many cubits side אג [AG] is. It will then be:

As the sine of angle ג [G], 64278, is to the side opposite it, אב [AB], 10, so is the sine of angle ב [B], 50000, to side אג [AG]. By the method of division we find side אג [AG] = 7 50054/64278 in a single operation.

And without division, by the third principle mentioned above — operating with two multiplications — we find it thus. Since a number was found earlier that stands to the whole sine in the same ratio that the sine of angle ג [G] has to side אב [AB], in this manner: as ג [the sine of G] 64278 is to אב [AB] 10, so 100000 is to "what?" — by the method of division the fourth term would be 15 35830/64278.

And in order to escape division, we bring in the whole sine according to the third principle, at the head of the ratio, [taking] the secant of the complement of angle ג [G] — which is [the secant of] 50 degrees, and its secant is 155572 — to stand in place of the sine of angle ב [B]. It will then be: as 100000 is to 155572, so 10 is to 15 55720/100000 — and this is close to the first [value] found by division. This is the sought fourth number, which stands to the whole sine as the sine of angle ג [G] stands to side אב [AB].

We then apply the rule and say: as the whole sine, 100000, is to 15 55720/100000, so the sine of angle ב [B], 50000, is to אג [AG] = 7 77860/100000 — and this is close to the first [value] found by means of division.

When you have done this, you will be able to operate with any three proportional numbers — provided you treat them as if they were sines; that is, that you multiply them, or add to them, ciphers [zeros] until they reach the order of magnitude of the [whole] sine. Suppose it be: as 8 is to 12, so 6 is to a fourth. We add three ciphers to them, and it becomes: as 8000 is to 12, so 6000 is to "what?" We first find the ratio of the whole sine to the fourth. This we set out thus: as 8000 is to 12, so 100000 is to "what?" To be free of division, we bring the whole sine to

the head [of the ratio] and take the secant of the complement of arc 8000, which is

85 degrees and 24 minutes, and its secant is 1246899. We apply the rule and say: as 100000 is to 1246899, so 12 is to "what?" We find 15, approximately. Following this, we apply the rule and say: as 10 is to 15, so 6000 is to "what?" We find 9 — and that is what was sought.

I have shown you with your own eyes how you can be free of division. If you accustom yourself to this operation, calculating will be easy for you, and with large numbers you will take delight and run swift as a deer in it; whoever does not feel constrained by it will not be harmed. Now there is no cause for astonishment if these reckonings do not come out as exact as the reckonings of geometry — neither short nor over by a hair's breadth — for in truth these reckonings are not [purely] mathematical but natural [practical], and a deviation from the absolute truth, in excess or defect, falls within them; [though] with a more refined operation [the deviation is] also small.

To make you understand this matter from its foundation, let me inform you that in resolving right triangles by means of the Golden Table — and likewise in every human calculation — and this is Proposition 19 [?] of the Seventh [book?] of the one called *Regiomontanus di Trei* [Regiomontanus, *De Triangulis*] [?], for which reason we needed to know three proportional parts in the right triangle in a fixed, known magnitude, by means of which a fourth, hidden part, proportional to them, is found — namely, all the sides and angles of the triangle, each to the others. And because, to this day, the ratio of the curved to the curved, and of the curved to the straight, has never been known — and perhaps will never be known, nor is it the fruit of our striving up to now — these principles we have set down (whose whole purpose is to make clear the ratios of arc to arc, or angle to angle) the mind of man has not attained to fix with mathematical exactitude, but only close to the truth, by a natural reckoning: this [done] by means of the chords belonging to them according to the ancients, or by the sines, tangents, and secants according to the moderns — for all these are straight lines and are well proportioned, this one to that. This is the function of the said instructional table and its good treasury; and I call it the **Golden Table** for its importance.

And the more we refine the taking of the whole sine — which is the half-diameter — making it larger and rich in figures [digits], the more precise will be the chords, sines, tangents, and secants. Some have set it with eleven ciphers [digits], and some have set it with twenty ciphers. One who wishes to be precise and to direct his calculations [exactly] should use the table of *Rheticus* [Rheticus, *Opus*

Palatinum / trigonometric canon], in which he will find what pertains to the second-order subdivisions [seconds] of the sines, tangents, and secants — so that one operating with those numbers will strike the mark to a hair and not miss. And we know what is close to the truth [only] when we consider the whole. And since the angles of right-lined triangles are measured by the arcs that bound them, the sages used, in resolving them, the said instructional table — when their sides are very well known with mathematical precision, and the hidden [parts] become known from the known, as Euclid clarified in the second [book] of his *Elements* [?]. From this you should know that everything found by means of the numbers of the Golden Table is close to the truth — not the absolute, exact truth — but only a step between it and the truth, so that the difference is not perceived in human affairs. In truth, the time wasted on long calculations is worth more than that defect or excess. Let him who has a palate taste these delicacies and praise them with glad lips; for it is fitting that the wise man be distinguished in all his operations from the common crowd. As for one whose mind has never grasped things [in their] how and what and why, when he has strayed from the highway — the road that man and beast walk — he will not…

He will carry his understanding far, yet will not open his mouth beyond measure; he will choose what suits him, and there settle and rest content.

I, however, cannot keep from helping you still further in the matter of triangles and their solution — for when you find yourself buried under your heavy load, you may take me as a companion to make known to you the words of the wise and their riddles. *(Mishlei 25:11; allusions to Mishlei 27:20, Koheles, etc.)*

Sheol and the place of perdition are never sated, nor are a man's eyes ever sated — surely you know the sage's proverb. How fitting is the saying of a sovereign king, who declared: since our end is the grave — for we must surely die, and there is no escape from the grave — and our days are swifter than a running courier, it is not for us to drag on like a shadow. It befits us rather to hasten to seize the spoil, to find short and easy paths for multiplying intellectual acquisitions during the brief time we live among the nations, and not to squander our time and our moments on the length of preliminaries. Let us not turn right or left from the road that leads to our longed-for goal — much less weary ourselves for nothing on idle talk and the dullness of trifles, nor labor in vain. For the eye is never sated with seeing strange books, with vessels exchanged one for another, each borrowed from the next; and the soul is likewise never filled with hearing precious words. And just as there is no limit to our intellect — as the rhetorician

said, "The heavens for height, the earth for depth, and the heart of a king past finding out" *(cf. Mishlei 25:3)* — so too there is no end to its [the soul's hunger].

For behold, in the days of our fathers wondrous deeds were performed, and the earth was filled with knowledge as the waters cover the sea. Newcomers, lately arrived in Germany, France, and England, were not content with all the studies of the ancients mentioned above; they tore down the fences of the intellect and entered the innermost chambers. Who could rouse the sleepers, those who dwell in houses of clay — the fathers of antiquity? Who could wake the slumbering, so that Hipparchus and Ptolemy might look on, astonished and bewildered, and tremble in their graves at the sight? In former days the gates of learning were shut; today they have gone out into the head of every street, standing in the open squares, raising their voice and calling to every passerby: "Be our leader — here we are, your servants." See what the power of diligence and persistence has wrought and accomplished, and the marvels it has shown us in our generation, things we never knew! One of the holy ones found a smooth path to solve every problem in number without any reckoning of multiplication, and the further effort of dividing — using addition and subtraction alone. A young lad now conducts himself among the studies, and a suckling babe takes delight in the solution of triangles. Anyone who lacks this knowledge does so not for want of instruments — quadrant, astrolabe, globe, or the like — for in truth these do not work with precision in the reckoning of [the fine] divisions, but rather by other marvelous means, the work of God Himself. They are engraved upon the tables: whether the Table of Sines, or the Logarithmic Table, newly devised, which are like inverted sines and come in many kinds; whether the universal table giving every product of any two numbers multiplied together and their quotient; or the table specially set for the products of each division of the arcs, after they have been divided together with each sine — whether by means of the squared rods [Napier's bones]. How awesome they are! I have resolved to send them to you as a gift, made of polished bronze, or silver, or ivory, engraved and inscribed with the requisite numbers — should God bring before me a skilled craftsman — and together with them I shall send the manner of their use as well.

But all this is set forth at length and in full explanation in the *Sefer Arubot HaShamayim*. For now I shall explain only the procedure called *prosthaphaeresis* [Greek: προσθαφαίρεσις, "addition-and-subtraction"] — that is, the addition and the subtraction, which are the two [operations].

Even Shlomo [The Stone of Solomon]

I will not choose the Logarithmic [method]. They are valuable matters, yet they keep one at a distance from grasping the demonstrations of the studies, which are well clarified through the sines — and the student would grow weary trying to recall the principles on which they are planted. But one who has accustomed himself to this discipline — I assure him he will find rest and ease in all his labor, and his toil will bring him peace. Indeed, it is good to have many methods at hand, and in every reckoning a road to travel — a just balance by which to test his operations, even when these are at times broad, long, and harder than the first.

I have named this principle "the Stone" [*even*]; this Stone is the King's Stone, for it is like a precious gem, fit to be set upon the foreheads of the wise. Let them ever place it as a seal upon their heart and a seal upon their arm, that it may find grace and favor in the eyes of the kings who choose brevity and ease and a road [direct] to their goal, and there is no constraint upon them. [...]

The Fourth Stone, Which Is the King's Stone

When four sines are set in proportion, and the all-sine [radius] stands at their head, there is a straight road before a man to find the fourth, unknown term — by addition and subtraction alone.

This Stone has its subdivisions [*avanim*, "stones"], and each subdivision has its branches [*chatzatzim*, "pebbles/chips"]; yet the rule common to them all is this: that you add the arc of the small sine to the complement of the arc of the large sine, and seek in the Golden Table the sine of their sum, placing it on your right; and likewise subtract these two arcs one from the other, and seek in the said table the sine of their difference, placing it on your left. Afterward set your heart to grasp these matters, and proceed according to the subdivisions of the Stones as I show you in the worked examples.

Subdivision One — when the small arc is equal to the complement of the large arc, or exceeds it

In this case the sum of the sines is double the fourth sine sought. And you see that this subdivision has two branches.

Branch One — when the small arc is equal to the complement of the large arc

An illustration of the matter: in the right triangle אבד [recte: the right angle is at ד] — see figure [ציור ז] — let the angle ב be known, 40 degrees, and the hypotenuse פב, 50 degrees, and we wish to know the measure of the side פד. We already know from Tower 11 of *Even Shlomo* that as the all-sine 100000 is to the sine of פב, 76604, so is the sine of ב, 64278, to the sine 49239 — whose arc is 29°30′ — and this is the measure of the side פד sought.

But if we wish to know its measure without the proportion [literally "without the Tower"], by doubling, with addition and subtraction alone — then according to the first branch here, the procedure is as follows. Since the complement of 50 degrees, which is the large arc, is 40 — the same as the small arc — we add the complement to the small arc, giving 80, whose sine is 98480; and likewise I subtract this from that, leaving 00, whose sine is 000. The sum of these two sines is 98480, which is double the sought sine. Accordingly you take its half, which is 49240, and seek it in this table among the rows of sines; opposite it you will find, above, 29 degrees, and to its left, 30 [minutes] — as you found with the proportion. For even though there is a difference between these two sines [49239 vs. 49240], since it lies in the last digits, it neither adds nor subtracts nor alters the arcs, as you will see in the table of sines. And here is the whole operation set out for you:

		(sum / difference)	
Complement of the large arc	0. 40	(add)	The small arc 0.40
		(subtract)	
Their sum 0.80, sine 98480		Their difference 0.00, sine 000	
		(add) 000	

Their sum 98480 — this is double the sought sine. Its half 49240 is the fourth sine, whose arc is 29°30′ [recte: 29°30′], the measure of the side פד sought.

From this you learn that when the small arc equals the complement of the large arc, you need not subtract — for nothing remains [the difference is zero]. Rather, half the sum of their sines is the sought sine. The demonstration and the rationale of the operation in the manner described rest wholly upon the fourth proposition

of the sixth book of the *Elements* [Euclid VI.4] — how great is its hidden goodness! — and it is well clarified in the figure of the analemma before us [ציור ז].

We draw a circle on center י, and pass through it two diagonals, ביף and ליא [?], intersecting at right angles; and two further diagonals, שיה and טין [?], likewise [intersecting] one another at right angles. Let the chords מצך and מל [?] [recte: the chords] be parallel to the diagonal טין, and the chord רהט [?] parallel to the diagonal שיה. By Euclid III [the third book], all the said chords, and the arcs upon them, are bisected by the said diagonals at the points נזק, הג, [?] שצז [?].

For the clarification of the first branch of this subdivision, fix your eyes on the right-hand quadrant of this circle, של [?], and let us drop from point ש a perpendicular שת onto the diagonal ליא, and likewise from point z3 [?] a perpendicular z3 [?]; and there will be two triangles, ישת and יz3 [?], equal in their angles. By the fourth proposition of the sixth book they have their sides in proportion — that is, as יישׁ [?], which is the all-sine, is to שת, the sine of the arc של, so is יz3 [?], the sine of the arc לט, to the line z3 [?]. And since the small arc מל [?] is equal to the complement of the large arc של — for up to the complement of the quadrant, the arc שט falls short of the large [arc] by the arc למן [?] — the sum of the sine of the small arc and the complement, that is, the sine [of the sum], will be double the small arc, double the line z3 [?] sought. And this is because the arc שמ [?] is equal to the arc של, as we explained: the arc מל [?] is nearly bisected by the diagonal שיך [?], and accordingly its complement מנ [?] is likewise the complement of של the large; and the arc נא [?] is equal to the small arc טל [?], since the angles formed by the cutting of the diagonals are equal — by [Euclid] III.15 [?] for the first — and the arcs upon them are equal — by [Euclid] III.26 [?] for the third.

If so, then the sine of מע [?] is the sine of the arc מנא [?] — that is, the sum of the complement-arc and the small arc. Let us draw from point z [?] the line של [?] parallel to the diagonal ליא; then, by the second proposition of the sixth book [Euclid VI.2], the triangle מעל [?] is cut in proportion to its sides — that is, as מל [?] is to לz [?], so is מע [?] to עש [?]. But since לל [?] is half of מל [?], so too will עש [?] be half of מע [?]; and indeed ענש [?] is equal to the perpendicular line z3 [?] by [Euclid VI] the fourth proposition, as in the first. If so, then שמע [?] is the sine of the sum of the complement of the large arc and the small arc, which is double the fourth sine z3 [?] sought — this is what we wished to demonstrate.

[The diagram for this analemma proof is on a separate plate (ציור ז). Many single-letter point-labels in the proof — especially the recurring token transcribed as "z3" — could not be disambiguated against the text alone and are flagged.]

Branch Two — when the small arc exceeds the complement of the large arc

An illustration of the matter: in the right triangle בדפ [the King's Stone] — let its angle ב be 29°30′ and the hypotenuse 70 פב degrees, and we wish [to know] the measure of the side פד. By Tower 11 of *Even Shlomo*: as the all-sine 100000 is to the sine of 93969 ,פב, so is the sine of 49242 ,ב, to the sine 46272 — whose arc is 27°34′ — the measure of the side פד sought. And by the method of the King's Stone we seek it thus:

		(sum / difference)
The small arc	29°30′	(add)
Complement of the large arc	0°20′	(subtract)
Their sum 49°30′, sine 76040		Their difference 9°30′, sine 16504
		(add) 16504

Their sum 92544 — this is double the sought sine. Its half 46272 — this is the sine of the side פד sought.

Another illustration: in the said triangle בדפ, the side בד is known, 42°24′, and the side 29°30′ ,פד, and we seek פב, the hypotenuse [*nashtach*]. By Tower 7 of *Even Shlomo*: as 100000 is to the sine of the complement of 47°36′ ,בד, which is 73845, so is the sine of the complement of the side 60°30′ ,פד, which is 87035, to the sine 64280 — the complement of the hypotenuse sought. The hypotenuse פב will therefore be 50 degrees. And by the method of the King's Stone we investigate it thus:

		(sum / difference)
Complement of the large arc	47°36′	(add)
The small arc	29°30′	(subtract)
Their sum 77°6′, sine 97476		Their difference 18°6′, sine 31067
		(add) 31067

Their sum 128543 — double the sought sine. Its half 64271 — this is the fourth sine sought.

And the demonstration in the figure of the analemma: as יש [?], the all-sine, is to שת [?], the sine of the large arc של [?], so is יז [?], the sine of the small arc מך [?], to

ציץ [?]. And since the small arc מך [?], compounded from the small arc and the complement of the exceeding sine over [point] קי [?] — the sine of arc מא [?] exceeds the complement of the large arc לט [?], for there is an arc לך [?] whose sine is רף [?] — then if we add this to the large and make a line, half of this line — that is קף [?] — will be equal to the line ציץ [?], the sought sine. The figure itself, with its circles and lines, shows you [the rest], and there is no need to dwell longer on it.

Subdivision Two — and when the small arc is less than the complement of the large arc

In this case the excess of the difference of the sines is double the sought sine. An illustration of the matter: in the triangle בדפ […]

the angle is a right angle, the known angle is angle 29°30′ = ב, and the perpendicular dropped is 36°47′ = בפ. You want side פד [the hypotenuse]. By Theorem 11 [טור יא׳] of Even Shlomo, the ratio of the whole sine 100000 to the sine of בפ, which is 73845, equals the ratio of the sine of 49242 ,בד, to the sine 36362 — whose arc is 21°19′[?], and this is the sought side פד. By the method of the King's Stone we now seek it, as follows:

[Worked prosthaphaeresis table — reads right-to-left in source; logical column order preserved. Layout: an "add" line and a "subtract" line frame the sum-arc and difference-arc, each with its sine.]

Operation	Arc	Value	
	complement of the large arc	42°24′	
add (תוסיף)	small arc	29°30′	
their sum (קבוצם)	71°54′	and its sine (ובקעו)	95 05 1
their difference (יתרונם)	12°54′	and its sine (ובקעו)	22 32 5
subtract (תגרע)		22325	
remainder (מותרם)	72726	this is double the sought sine	
its half (חציו)	36363	this is the sought sine	

Another example [דמיון אחר] in the same triangle: angle ב = 40° and side בד = 42°24′ are known, and we wish to find angle פ. By the Fifth Theorem [הטור החמשי], the ratio of the whole sine 100000 to the sine of the complement of בד, 73845 = 36°47′, equals the ratio of the sine of angle ב, 64278, to 47466 — which is the sine of the sought angle's complement. And the complement of the said sine [its arc] is found to be 63°40′. By the procedure of the King's Stone we investigate it thus:

Operation	Arc	Value	
	complement of the large arc	42°24′	
add (תוסיף)	small arc	40°0′	
their sum (קבוצם)	82°24′	and its sine (ובקעו)	99 12 1
their difference (יתרונם)	2°24′	and its sine (ובקעו)	41 87
subtract (תגרע)		4187	
remainder (מותרם)	94934	this is double the sought sine	
its half (חציו)	47467	this is the sought sine	

The proof [המופת]: the ratio of the whole sine יה to the sine הם of the small arc הף equals the ratio of the sine ינ to the sine of the large arc נד — because this follows from the Fourth Theorem of the Sixth [Book — Euclid VI], and as we explained in the proof of the first [case] at length, the perpendiculars [עמודים] dropped on the diameter ביף are mutually parallel, and therefore the triangles are similar. Now, since arc דה — the complement of the large arc נד — exceeds the small arc הף by the arc עף (for arc ריען[?] is cut by the diameter יגח into two halves, and arc הע equals arc דה), and the sine of the excess arc סרח is עס, equal to line רן: it is then quite clear that if we subtract line רז from line דץ — which is the sine of arc דף, compounded of the complement of the large and the small [arcs] together — there remains line כז, which is double line פז, equal to line בן, the sought fourth term. (Figure [ציור ז] ז.)

And if it happens that the two arcs are equal, set one of them in place of the small [arc] and its complement in place of the complement of the large [arc], and proceed according to the conditions of the subdivisions mentioned. Example: the ratio of the whole sine to the sine of 50°, 76604, equals the ratio of the sine of 50°,

76604, to 58681 — whose arc is 35°56′. By the method of the King's Stone it will be as follows:

[Note: the body fixes the fourth term at 58681, but the table below computes 58684; the two differ by 3 — see Numerical impossibles.]

Operation	Arc	Value	
	small arc	50°0′	
add (תוסיף)	complement of the large arc	40°0′	
their sum (קבוצם)	90°0′	and its sine (ובקעו)	100000
their difference (יתרונם)	10°0′	and its sine (ובקעו)	17369

And because the small arc exceeds [the complement of the large], as in the first condition, we add the sines:

100000	
117369	and this is double the sought [value]
58684	and its half is the sought [value]

Another example [דמיון אחר]: the ratio of the whole sine to the sine of 40° equals the ratio of the sine of 40° to a fourth term — that is, 100000 is to 64278 as 64278 is to 41316.

Operation	Arc	Value	
	complement of the large arc	50°0′	
add (תוסיף)	small arc	40°0′	
their sum (קבוצם)	90°0′	and its sine (ובקעו)	100000
their difference (יתרונם)	10°0′	and its sine (ובקעו)	17369
subtract (תחסר)		17369	
remainder (המותר)	82631	double the sought [value]	

Operation	Arc	Value
its half (וחציו)	41315	this is the sought [value]

Here, by the condition of the second subdivision — since the small arc is less than the complement of the large — we subtracted the sines, and our object came to light.

[Note: the body sets the fourth term at 41316, while the table yields 41315; a one-unit rounding difference — the author noted this rounding behavior earlier.]

Example three [דמיון ג׳]: 100000 is to the sine of 30°, 50000, as the sine of 30°, 50000, is to [a fourth] whose arc is 14°29′.

Operation	Arc	Value	
	complement of the large arc	60°0′	
add (תוסיף)	small arc	30°0′	
their sum (קבוצם)	90°0′	and its sine (ובקעו)	100000
their difference (יתרונם)	30°0′	and its sine (ובקעו)	50000
subtract (תגרע)		50000	
remainder (והמותר)	50000	this is double the sought [value]	
its half (וחציו)	25000	this is the sought [value]	

[Note: this third example is self-consistent (sum 90° → sine 100000; difference 30° → sine 50000; 100000 − 50000 = 50000, half 25000). The arc "14°29′" given for the fourth term in the prose lead-in does not correspond to sine 25000 (whose arc is ≈14°29′ — in fact this is correct: sin 14.48° ≈ 0.250). Consistent.]

You [אתה — opening a new explanatory section] saw, at the start of this method, that two conditions are required for this work. The first: that the terms compared be sines, not some other number larger than the whole sine or smaller [in a way] for which no exact arc is found in the table of sines. The second: that the whole sine stand at the head of the ratio. Now the second condition is not an obstacle, for I have already shown you, in [the treatment of] Even Shlomo, a sufficient way by which you can bring the whole sine to the head of the arrangement; and there

too I taught you how to convert the small numbers into sines by appending ciphers, or by doubling them twice or three times — easy to grasp, as mentioned above.

What remains, then, to complete the work is to inform you how to proceed when the second number in the ratio, or the third, or even both, are larger than the whole sine — which happens, for the most part, after the [tangents and secants — literally "the sinkers and the cutters"] enter into the ratios, as you see throughout the thirteen[?]

columns of the thirteen[?] in Even Shlomo. There are for this matter two disqualifying-cases of stones [פסולי אבנים] that apply to [these] fixed numbers.

The first disqualification [הפסול הא׳] is the one general to every ratio of four numbers, even when the second and third are larger than the whole sine: take half the number, or its third, or its quarter, until it is less than the whole sine; or take its tenth, or one hundredth part — which is easier, since you then need not divide it, but only subtract one digit from its end, or two digits, as the case will be in [another] number. Once the operation is finished, [restore the same factor by] doubling the resulting term together with the divisor-number, and you have found your object.

Example, in a worked case that likewise appears in Even Shlomo: 100000 is to 155578[?] as 10 is to a fourth term. Since the second of the compared [numbers] is larger than the whole sine, we divide it by 10 — that is, we subtract one digit from its right — leaving 15557; and to the third number in the ratio, which is smaller than the whole sine, we append three ciphers, so that the ratio becomes: as 100000 is to 15557, so 10000 is to a fourth. By the method of the King's Stone we seek the fourth, thus. The arc of sine 15557 is 8°57′, and its complement is 81°3′, and the arc of sine 10000 is 5°45′. Here is the order of the operation:

Operation	Arc	Value	
	complement of the large [arc]	81°3′	
gather (תקבץ)	small arc	5°45′	
their sum (קבוצם)	86°48′	and its sine (ובקעו)	99844
their difference (חסרונם)	75°18′	and its sine (ובקעו)	96727 — and this is subtracted from the sine of the sum

Operation	Arc	Value
subtract		96727
remainder (יתרונם)	3117	this is double the sought [value]
its half (וחציו)	1558	this is the sought number

And because we subtracted one digit from the end of the second number — I mean, we took its tenth — we double[?] this number [1558] by 10, giving 15580[?]; and because we appended three ciphers to the third term, we subtract from this the three right-hand digits, leaving 15, the fourth number.

[Note: the prose for the doubling step reads נכפול … עם 10 ויהיה 155480, i.e. it states the product as 155480 — but 1558 × 10 = 15580. Restoring three appended ciphers (÷1000) then gives 15. See Numerical impossibles.]

Further [עוד]: this would change to: as 100000 is to 15, so 50000 is to a fourth. We append three ciphers to the second [number], making it sine 15000, whose arc is 8°3′, and the order is as follows:

Operation	Arc	Value	
	arc of the complement of 50000	60°0′	
add (הוסיף)	small arc	8°3′	
their sum (קבוצם)	68°3′	and its sine (ובקעו)	92751
their difference (חסרונם)	51°57′	and its sine (ובקעו)	78747 — and this is subtracted
their remainder (יתרונם)	14004		
its half (וחציו)	7002		

and we subtract from it the three ciphers we gave to the second [number], leaving 7. And so it was found in the method of Even Shlomo, with five operations [ה׳ הכאות]. Likewise it would be if you divide the number larger than the whole sine into halves or into four parts. For example, half the said number 155572 is 77736, the complement of whose arc is 59°38′, and the small arc is 5°45′, and its order would be thus:

Operation	Arc	Value	
	complement of the large [arc]	59°38′	
add (תוסיף)	small arc	5°45′	
their sum (קבוצם)	44°44′ [recte:?]	and its sine (ובקעו)	70380
their difference (חסרונם)	14°33′ [recte:?]	and its sine (ובקעו)	54805 — and this is subtracted from the sine of the sum
subtract (תגרע)		54805	
remainder (והמותר)	15575	and this is the sought fourth [term]	

[Note: the sum/difference arcs in this block (44°44′ and 14°33′) are confirmed at native res but do not square with complement 59°38′ + small 5°45′ (which would give a sum of 65°23′ and a difference of 53°53′) — genuine source corruption, transcribed as printed [recte:?]. The sine values 70380 and 54805 are legible; their difference 70380 − 54805 = 15575 is consistent with the stated result.]

and this is the sought fourth [term], and you need not take its half, since here we halved it [the input] only — rather, we subtract only the three ciphers that we gave to the third number, leaving 15, as in the first time. So too we take the quarter of the number 155572 above, which is 38868; and the arc of its complement is 8°67′ [recte:?]. [Note: "8°67′" has minutes > 59, impossible as a sexagesimal arc — genuine source corruption, transcribed as printed.]

Operation	Arc	Value	
	complement of the large [arc]	8°67′ [recte:?]	
add (תוסיף)	small arc	5°45′	
their sum (קבוצם)	53°72′ [recte:?]	and its sine (ובקעו)	95576 — and this is subtracted from the sine of the sum
their difference (חסרונם)	61°23′ [recte:?]	and its sine (ובקעו)	87784
remainder (יתרונם)	7792		

Operation	Arc	Value
its half (חציו)	3896	and this, multiplied by the divisor 4, yields 15584

[Note: 95576 − 87784 = 7792 ✓; half = 3896; 3896 × 4 = 15584 ✓. The arc readings (8°67′, 53°72′, 61°23′) are confirmed at native res but corrupt — minutes exceed 59 — genuine source corruption, transcribed as printed [recte:?].]

and this is the sought fourth [term].

The reason for the procedure [וסבת המלאכה] is quite plain: for if the ratio were 40 to 5 as 30 to 15 — if, in place of the number 30, I set its half, which is 15, I find, by that same ratio, 7½; and accordingly, if I double this 7½, I get 15, which is what is proper for the number 30 by that same ratio. Easy to grasp.

The second disqualification [והפסול הב׳] is particular and special to a ratio in which the second or third number is smaller than the whole sine while the other is larger. Its procedure is thus: divide the large number by the whole sine — that is, see how many times it goes into it — meaning, subtract five digits[?] from the right side, and what remains [is] the remainder of the division, which is smaller than the whole sine, and that you take for use; and what remains on the left side is the count of times the whole sine goes into the large number, and that you multiply by the small number, keeping the product of their multiplication. After completing the work, you add to the fourth term found the kept number, and out comes the sought fourth term.

Example: 100000 is to 155572 as 10 is to a fourth. We divide the number 155572 by the whole sine, into which it goes once; so on the right side there remain 55572, and on the left side, 1. We multiply the small number, which was 10000 — after appending the ciphers so that it be a sine — by this 1, and out comes 10000, which we keep. And the number remaining on the right side, since it is larger than the number 10000[?], we seek the arc of its complement, which is 14°56′, and the small arc of the number 10000 is 5°45′, and we operate thus:

Operation	Arc	Value
	complement of the large [arc]	14°56′
add (תוסיף)	small arc	5°45′
their sum (קבוצם)	59°61′ [recte:?]	and its sine (ובקעו) 88281

Operation	Arc	Value	
their difference (יתרונם)	29°50′	and its sine (ובקעו)	77144 — subtract (תגרע)

Sefer Sod HaYesod

[The King's Stone — close of the worked examples (second disqualification)]

Worked example (continued, p. 159)

The procedure continues a division begun on the previous folio. Laid out as a worksheet:

Operation	Value
Subtract (תגרע)	77144
Remainder (מותר)	11137
Its half (וחציו)	5568
Add to it the kept number (תוסיף עליו המספר השמור)	10000

So the fourth term sought (הרביעי המבוקש) is **15568**, after you subtract from it the three numbers you lent to the [smaller] number, namely **10000**. The fourth term sought is therefore **15** [?: the printed "15." appears truncated — the worked result above gives 15568].

Second example (דמיון ב'). As 100000 is to 783484, so is 50000 to what? The whole sine (כל הבקע) enters into it 7 times. We multiply the number 7 by the smaller number, and 350000 results; we keep it. As for the remainder on the right, 83484, we operate: the sine (בקע) of its supplementary arc (קשת השלומו) is 24°33′ [?: read as 24.33; the source writes "33.24"], and the sine of 5000 is 30′ [?: "0. 30"]. We investigate thus:

Step	Value
The large supplement (השלום הגדולה)	24°33′
Add (הוסיף) — the small arc (הקשת הקטנה)	0°30′
Their sum (קבוצם)	24°63′ [numerical impossible — see

Step	Value
	report]; and its sine (ובקעו) 89415
Subtract (תגרע) — their difference (יתרונם)	24°3′ [?: read "3.24"]; and its sine (ובקעו) 5930
Subtract (תגרע)	5930
Their difference (יתרונם)	83485
Its half (וחציו)	41742

To it add the kept number, 350000, and **391742** results; that is the fourth term sought.

Proof and rationale. This operation is demonstrated, with good taste and reason, in the *analemma* figure already mentioned (בתמונה האנלימ"א הנזכרת). It runs thus: by way of analogy, as יג [the whole sine] is to ב [the smaller number], so is יד [the larger of all the sines] to הז. Now we subtracted the measure of יג, the whole sine, from the measure of יד, and גה remained. So, as יג, the whole sine, is to ב, so is the remainder גה to הק, according to proposition 2 of [Euclid] Book Five [recte: VI.2 — "of the Elements"]. Accordingly, when we know the measure of הק, the fourth term — if we add the smaller number ב to it once — provided that the whole sine יג enters the number יד only once (I mean line קס equal to it), then the whole line הס will be the fourth term sought. But had it entered twice, we would have added the measure of ב twice.

And if it happens that, when you divide the larger number by the whole sine, nothing remains — which happens whenever the larger number is a *multiple* (מולטיפליצי) of the smaller, that is, double it, or three times, or four times the same — then you multiply the count of times the whole sine entered into it by the smaller number, and what comes out is the fourth term sought. For example: as דג, the whole sine, is to גפ, so [is something to something]; know that the double of the whole sine is to ער [?: point-labels read off the scan]. Since the larger number דע is counted twice the whole sine, we multiply the number 2 by ב, and ער results — that is the sought term. I know there are fine subtleties in this lesson, but practice will teach every variation and permutation, and will bring you out from darkness to light.

הפנה השנית

THE SECOND CORNER

It comprises various lessons that are propositions toward the knowledge of triangles, all of them in completeness (כלם בעלמות)

[Source page 160. Running header: "הפנה השנית" — The Second Corner.]

Since it is precious in your eyes, dear friend, to know whether a triangle whose solution is sought is in fact possible and existent, or impossible and false in its proposition — for such are the ways of disputatious students, to test and probe the scholars — and since you asked of me, "Show me this road, that I may walk it," I saw it necessary to set out first the propositions and lessons before I come to speak of the oblique triangles (המשלשים העקומים). For this matter depends entirely on knowing the properties, qualities, and conditions of triangles in respect of their angles and sides and how these combine with one another. This is what I wrote at length in Latin in the book *Triplinodonomia* [?: בספר הטריפליודונומיאה — a Latin treatise of the author's; title transliterated, identification uncertain], and in the holy tongue in the book *Arubot HaShamayim* (ארכה השמים) [recte: *Arukat HaShamayim* / "the Expanse of the Heavens"], where I demonstrated with proofs everything written in the book of Menelaus (מינילאו"ס) [Menelaus of Alexandria] and those who came after him, to which nothing need be added.

Whoever wishes to grasp these matters in their full clarity should consult the books mentioned, or those akin to them among the books of the Christians (מספרי הנוצרים). Even so, I shall not refrain from following after you, to answer you according to the order of your questions — the form of the matters without proofs. Had I been conducting myself by my own counsel, in keeping with the order of study, it would have been fitting to set these before the lessons of the First Corner; but I follow your order, and I answer you first things first and last things last. With you, and with those like you, I shall be no more than one who recalls matters already explained elsewhere. To you, O men, I call — I have no business with one who has never studied these subjects at all, and my voice is to the sons of men. The warning of the sage Plato, written at the gates of his academy (כפתחי שערי מדרשו), [reads]: **"Let no one untrained in geometry enter here"** (שום נערד מגיאומיטריאה אל יכנס כאן) [Greek: ἀγεωμέτρητος μηδεὶς εἰσίτω].

These lessons are the foundations and the beginnings of this science. The Philosopher [Aristotle] said that one is not to dispute with [those who deny first principles] — and it is no way of any science to argue over its foundations. Did not Ptolemy expound his lessons upon the foundations of Euclid (יסודות אוקלידס), without recalling him by name and without a word — not out of envy, but because, being on the level of the [alphabet] letters, they are known to every discerning seeker of the sciences, small and great alike.

Let us begin from defining the subject we shall speak of, namely the triangle.

א. A triangle (המשלש) is a figure bounded by three lines [?: "מקבילים" — printed as if "parallel," but the sense requires "meeting/connecting"; read with caution], and they are called the **sides of the triangle** (צלעי המשלש).

ב. Triangles are of two kinds: some are **planar and simple** (שטחיים פשוטים) — I mean, drawn upon a figure having three straight lines set in a flat surface; and some are **spherical** (כדוריים) — I mean, drawn upon a figure set in the curved surface of a solid sphere (כשטח נכונה הכדור המוגשם), bounded by three arcs of great circles intersecting one another. And every arc that is a side of such a triangle is always less than half a circle.

ג. The **solid sphere** (הכדור המוגשם) is a corporeal figure bounded by a single surface without angles, whose property is that the straight lines issuing from a point within it to its surface are equal to one another. This point is called the **center of the sphere** (מרכז הכדור).

[Source page 161. Running header: "מספר סוד היסוד".]

ד. The **circles** spoken of in this treatise are planar figures whose circumference lies in the surface of the sphere and whose surface is cut by the body of the sphere.

ה. The **arcs** spoken of here are all of them arcs of great circles. The property of the great circles in the sphere is that they pass through the center of the sphere and their surface cuts the sphere into halves. Accordingly, one center is common to the sphere and to all the great circles within it, and their diameters — or their diagonals — are equal to one another. For this reason all the great circles are equal to one another, and they intersect one another into halves. Therefore, when two circles in one sphere are cut into halves, both of them are great [circles].

ו. The **pole** or **axis** (קוטב או ציר) of a circle is a point posited in the surface of the sphere, whose property is that all the arcs descending from it to the circumference of the circle are equal to one another. It is the place of the foot of

the fixed compass (רגל המחוגה הקיימת) when the circle is drawn on the sphere with its other foot.

ז. When some great circle in the sphere cuts another circle — whether great or small — at right angles, the cutting circle necessarily passes through the poles of the cut one. And conversely: if it passes through the poles of the cut one, then it cuts it at right angles, and the cutting circle will be a great circle.

ח. When a great circle passes in the sphere through the poles of some great circle, then, reciprocally, this [latter] circle too passes through the poles of its fellow.

ט. The circumference of the circle is divided, among the astronomers, into 360 equal parts, which they called **degrees** (מעלות). Since in the celestial circles the distance between degree and degree is great, they divided each degree into 60 **minutes** (דקים), which they called "firsts" (ראשונים); then each "first" they divided into 60 "seconds" (שניים), and likewise each "second" into 60 "thirds" (שלישיים) — as you see in Ptolemy, in Copernicus (קופירניקו), in the Alphonsine Tables (לוחות אלפונשו), in Avraham Zacut (אברהם זכות), and the like. On the algorithm (האלגוריתימ"ו) [Arabic/Latin: *algorithmus*, the place-value arithmetic of the Indians] of these natural numbers we have spoken all that is needed in the book *Sechel Yosef* [?: ס' שכר יוסף — read as "Sechar Yosef"; a book of the author's].

Now, the ancient master craftsmen chose the number 360 and [the number] 60 for their special virtue, their great utility, and their ease in computation — since they have, in fact, far more equal divisors (חלקים שוים) than any of the numbers smaller than them. This is clear to one who attends to the following table: that the number 360 is divisible without remainder by every [number] up to ten except the seventh part; and the number 60 is divisible without remainder except for the seventh, eighth, and ninth; and besides these they tolerate division by every number up to ten.

We add, beyond the precious glory of the number 360, that it is also approximately the count of the days of the sun (כמנין ימי החמה): for the circuit of the sun in its eccentric sphere — or the revolution of the terrestrial globe (סבוב כדור הארץ) beneath the girdle of the zodiac — is in the span of three hundred sixty-five and a quarter days, in which it completes its year and its revolution. Therefore they chose the number 360 and sanctified it. By this division, half the circle is 180 degrees, and a quarter of the circle is 90 degrees, which we shall henceforth call simply a "quadrant" (רביע).

Divisor table (Source pp. 161–162). The table tabulates the equal divisors of 360 and of 60; the rightmost numeric column is the divisor (1–13), the next its

quotient into 360, the next its quotient into 60. Table reads right-to-left in source; logical column order preserved below.

Divisor	360 ÷ divisor	60 ÷ divisor
1	360	60
2	180	30
3	120	20
4	90	15
5	72	12
6	60	10
7	0	0
8	45	0
9	40	0
10	36	6
11 [?: divisor index obscured at top of p. 162 column; quotients legible]	30	5
12	24	4
13	20	0

י. When a quadrant descends from the pole of the circle to the circle, it falls upon it at right angles.

[Source page 162. Running header: "הפנה השנית" — The Second Corner.]

[Continuation of section י:] …the second [arc] of the quadrant standing perpendicular upon the circle will necessarily be the pole [?]. For every great circle is distant from its pole by a quadrant, in such a way that the endpoint [of that quadrant standing] perpendicular upon the circle [is its pole]. Accordingly, when an arc descends to the circle from the endpoint of the perpendicular quadrant, it stands upon the circle at right angles; this arc too is a quadrant and is at right angles to the circle. So too it is demonstrated that when two great circles cut one another upon some circle at right angles, their meeting-point at the two extremities (כשני העדרים) [?: read "העדרים"] will be the pole of the bent [cut] circle (קוטב הענול הנחתך). It is likewise shown that if from one point two perpendiculars (ב' רבועים) are [dropped] to some circle, then both will be perpendicular upon that circle, and that point will be the pole of the circle.

As for how the pole of a circle is to be found — Theodosius (תיאודוזיאוש) [Theodosius of Bithynia] explained it in Lesson 21 of the First [book] of his *Spherics* (מימודותיו הכדוריים), and we [explained it] in the treatise *The Construction of Instruments* (תכנית הכלים) in the book *Ya'ar HaLevanon* (יער הלבנון) [the author's own work].

יא. When two great circles cut one another, the two interior facing angles of the intersection are equal to one another; and likewise the exterior angles are equal to one another. This lesson Euclid explained in [proposition] 15 of the First [book], on straight lines (בטו"ז [recte: ט"ו] לראשון בקוים הישרים).

יב. The **plane angle** (הזוית השטוחה) — Euclid defined it as the inclination of each of two lines, set in a flat surface, that meet on a non-straight [course] from one another. When the two lines bounding this angle are straight, it is called a **rectilineal angle** (ישרת הקוים); and if they are curved, it is called a **spherical angle** (כדורים). Now if one is set perpendicular upon its fellow, they make the angle a **right angle** (נצבה); and if it inclines away from it, [the angle is] **oblique** (עקומה). Therefore, when a straight line stands upon a straight line, or an arc upon an arc, it makes two angles on its two sides, and the two together are equal to two right angles.

יג. The **measure of the angle** — whether rectilineal or spherical — the astronomers know by the circumference of the circle whose pole, or the foot of the fixed compass that we draw, is that very angle [i.e. the vertex]. Were it not that they had not tasted the savor of [true] philosophy, nor seen the light of pure reasoning — for it is among the common notions in the [mathematical] sciences that a magnitude is cut by a magnitude lower than it by [one] degree [of dimension], I mean: the surface by lines, the solid by surfaces, and likewise lines by points. But that an angle should be cut by lines — here Euclid and all the sages instruct [otherwise]; and accordingly the angle belongs to the class of surfaces. How then should the line of a circle's circumference measure it, when the measure must be of the same kind of magnitude as the thing measured, both being magnitudes — as was made clear in logic?

See, then, to understand their words: since the magnitudes of the angles are according to the magnitude of their surfaces, and when there are many angles at the center of one circle, then as angle is to angle, so is the [circular] sector of the circles to the sector of the circle. And as sector is to sector, so is arc to arc in the circle. It is, then, equally [the case], by [Euclid's] lesson, proposition 11 [recte: VI.33] of the Fifth [book] [?: "הקש י"א לחמשי"; the cross-reference as printed], that

as angle is to angle, so is arc to arc. Therefore their usage is not exact, in that they take the circumference to be the measure of the angle.

Their intent, however, is to say: as the right angle subtending a quadrant — I mean 90 degrees — is to any angle whatever, so is the ratio of the quadrant of the circle to the arc of that angle. And therefore the angle is called a **known angle** (הזוית ידועה) [continues on the next folio].

how many degrees and minutes is the arc that lies opposite it — that is, that portion of the circle's circumference which lies between the legs of the angle. These legs are radii, both in spherical angles and in the angles of rectilinear figures: the legs are the halves of the diameters. This arc is called the **measure of the angle** (קשת שעור הזוית).

When the arc equals a quadrant — that is, ninety degrees — the angle resting upon it is called a **right angle** (נצבת). When it is less than a quadrant, the angle opposite it is called **acute** or **narrow** (חדה or צרה). When it exceeds a quadrant, the angle resting upon it is called **obtuse** or **wide** (נרווחה or רחבה). From this you will know how to answer one who asks you the *quality* of an angle — which of these it is: whether you should tell him it is right, narrow, or wide. And to one who asks its *quantity*, you tell him whether it is a quadrant of the circle (that is, 90°), or its third (that is, 60°), or its eighth (that is, 45°).

It has likewise been explained that the whole circumference of the circle equals four right angles; that all the angles formed at the center of the circle by the diameters together equal four right angles; and that every angle is less than two right angles — that is, its measuring arc is less than a semicircle.

[14] The **completion** (תשלום) of an angle is the difference between it and a right angle — that is, the quadrant, which is 90°.

The **remainder** (שארית) of an angle is the difference between it and two right angles — that is, a semicircle, which is 180°. In the same way you will determine the completions of the sides and their remainders. For the ratios of the angles are equal to the ratios of the arcs that measure them, and likewise the ratios of their sines, tangents, and secants are equal. From this you will understand that the completion of a right angle — that is, of the quadrant — is nothing; and therefore you take 00 [zero] in place of their completion.

As for the *kinds* of sine, R. Yitzchak [Yisraeli, in *Yesod Olam*] explained them at length: there are two kinds of sine, the **straight** (הישר) and the **inverted** (ההפוך). The definition of the **straight sine** of any arc is this: it is a straight line within the

semicircle, drawn from one extremity of the arc and falling perpendicularly upon the diameter that issues from the other extremity.

For example **(figure 4 [ציור ד])**: the sine of arc אג [aleph-gimel], whose remainder is arc עבג [ayin-bet-gimel], is the line עצ [ayin-tzadi]. Likewise the sine of arc עב [ayin-bet], whose remainder is עאש [ayin-aleph-shin], is the line עפ [ayin-pe]. You see, then, that the sine is half the chord of the arc that is double the arc whose sine is sought — that is, the line עפ, the sine of arc עב and of [arc] עש, is half the line עה [ayin-he], which is the chord of arc עגה [ayin-gimel-he], which is double arc עב [?]; and [it is half] the chord of arc [נשק [?]], which is double arc עש. For the chord is a straight line within the circle, dividing it into two unequal parts, and it is a chord common to the two segments of the circle. The sine is the line that cuts the semicircle into two unequal parts, and it is a sine common to both of them.

What the ancients did with the whole circle and its chords, the later authorities do with the semicircle and its sines. And just as the diameter is the largest of all the chords, so its half is the largest of the sines; they call it the **whole sine** (כל הבקע), and in our work here it is **100000**.

The definition of the **inverted sine** of any arc is this: it is a straight line within the semicircle, one of whose ends is at the extremity of the arc, and which falls perpendicular to the straight sine of that arc. For example, the inverted sine of arc עב [ayin-bet] is the line בפ [bet-pe]; likewise the inverted sine of arc עאש is the line פש [pe-shin] [?]. In such a way the inverted sine is *not* common to the arc and to its remainder, as is the case with the straight sines, the tangents, and the secants.

The early authorities, who employed the doubles of arcs and the chords, called this line the **arrow of the arc** (חץ הקשת), as if…

to say "the arrow of arc עבה [?]" is בפ [bet-pe]; and likewise "the arrow of arc עפה [?]" is פש. There are many useful and excellent lemmas that derive from knowing the inverted sine; you will find them in books of trigonometry and astronomy. It is not my intention to be lengthy here. I will tell you only — and you should know — how to find the inverted sine of any arc you wish, since the sages did not arrange a table for the inverted sines as they did the table of gold [?] for the straight sines, the tangents, and the secants. And it is this: determine the straight sine of the *completion* of the arc, subtract it from the whole sine, and the remainder is its inverted sine — *if* the arc is less than a quadrant. For example, suppose we want [the inverted sine] of arc בע [?], which is less than a quadrant:

we subtract עצ [ayin-tzadi], the sine of its completion, from the whole sine; and the remainder, ב [?], is the inverted sine of arc בע.

But if the arc exceeds a quadrant, we add the straight sine of the part exceeding the quadrant to the whole sine, and the sum is the inverted sine. For example, suppose we want the inverted sine of arc אש [aleph-shin], which exceeds the quadrant אש [?] by the amount of arc עא [ayin-aleph]; we add its sine [עצ] to the whole sine דש [?], and the sum, פש, is the inverted sine of arc עאש. From this it is explained that the straight and inverted sines of the quadrant or of the right angle are equal — both being half the diameter; and likewise the inverted sine of the completion of the quadrant, or of the right angle, is nothing.

These are the lines that lie *within* the circle. There are still further lines that R. Yitzchak did not discuss at all, which are *outside* the circle: the tangents (הנוגעים), or those passing outside the circle, which are the secants (החותכים). These two kinds of line have enlightened our eyes in the lemmas; and they are defined and bounded one by the other.

The **tangent** (הנוגע) of any arc is a straight line descending from the end of the *secant* of that arc, standing perpendicular upon the end of the circle's diameter that reaches the other extremity of the arc — the secant being half the diameter passing outside the circle through one extremity of the arc until it meets the end of the tangent line of that arc, which issues from the other extremity of the arc.

For example, the tangent common to arc בה [bet-he] and to its remainder הזש [he-zayin-shin] [?] is the line בר [bet-resh], descending from the end of the secant דה [dalet-he], which is likewise a secant common to the two arcs named, meeting at a right angle with the diameter שדב [shin-dalet-bet] at the other extremity of arc דז [dalet-zayin] [?], at the point ד [?]. Likewise, the secant of the two arcs named is the line formed from half the diameter דת [dalet-tav] [?], which passes and extends outside the circle up to the point ר [resh], where it meets the end of the tangent בר that issues from the extremity ב of the arc הב named.

And everything said concerning the sines, tangents, and secants of the arcs you will likewise understand for the *angles* of which the arcs are the measures — that is, for angle הדב [he-dalet-bet] and its remainder הדש [he-dalet-shin]: the straight sine is the line דפ [dalet-pe], which is common to both of them; the inverted sine of angle הדב is בפ; the inverted sine of angle הדש is פש; the tangent common to both of them is בר; and the secant of both is דר [dalet-resh]. Likewise for angle הדג [he-dalet-gimel] and its remainder הדא [he-dalet-aleph]: the straight sine is the line עצ [ayin-tzadi]; the tangent is בג [bet-gimel] [?]; the secant is דכ [dalet-kaf] [?];

the inverted sine of הדג is the line עץ [ayin-tzadi]; and the inverted sine of angle הדא is the line צנ [tzadi-nun].

After having informed you of the nature of the lines named, I will inform you…

[that] some of their properties — though I have already discussed concerning them all that is needed in my commentary on the Almagest [פירושי לאלמניסטו].

Now it has already been explained in lemma 3 of the First Corner of this treatise that every sine is a mean proportional between the sine of the arc and the secant of its completion; and likewise between the tangent of the arc and the tangent of its completion.

[15] Likewise it will be explained that every sine is a mean proportional between the sine of the completion of the arc and the secant of the arc — that is, in this very same figure, since the triangles are similar [?]; for דב [dalet-bet], half the diameter, stands perpendicular upon their bases [?], and angle הדפ [he-dalet-pe] is common to both. The proportion runs: as the sine of the completion of arc הב [he-bet] is to דה [dalet-he], the whole sine, so is רב [resh-bet], the whole sine, to דך [dalet-kaf], the secant of arc הב — by the lemma 4 of the Sixth [Book of Euclid].

[16] It will further be explained that, as the sine of the completion of the arc is to the sine of the arc, so is the whole sine to the tangent of the arc — that is, as דה [dalet-he], the sine of the completion of arc בה [bet-he], is to פה [pe-he], its sine, so is דב [dalet-bet], the whole sine, to בר [bet-resh], the tangent of the arc named — by the strength of lemma 4 of the Sixth.

[17] It will further be explained that, as the sine of an arc is to the sine of its completion, so is the whole sine to the tangent of its completion — that is, as דה, the sine of arc הב, is to פה [?], the sine of arc הב which is its completion, so is דב, the whole sine, to בר, the tangent of arc הב. The proof of [the proposition about] its completion is one with what preceded.

[18] It will likewise be explained that the tangents of two or more arcs unequal to one another are inversely proportional to the tangents of their completions — that is to say **(in this figure ז [?])**: as בך [bet-kaf], the tangent of arc גו [?], is to בי [bet-yod], the tangent of arc גד [gimel-dalet] [?], so is במ [bet-mem], the tangent of the completion of the second arc, to בז [bet-zayin], the tangent of the completion of the first arc. The proof: since it was explained in lemma 3 of the First Corner that every sine is a mean proportional between the tangent of the arc and the tangent of its completion — that is, between גך [gimel-kaf] and בז, and likewise between גי [gimel-yod] and בט [bet-tet] — then, by lemma 17 of the Sixth, the

rectangle formed from the extremes equals the square of the mean: that is, twice בז into ב [?] equals the square of the whole sine, and likewise twice בי into במ equals the square of the whole sine; and by the common notion, or [lemma] 11 of the Fifth, the two areas are equal. Accordingly, by lemma 16 of the Sixth, the sides of the areas are inversely proportional — that is to say, *reciprocally* (ריציפרוקי) — and it will be: as בז [?], the one tangent, is to בי, the second tangent, so is במ, the tangent of the completion of the second, to בז, the tangent of the completion of the first. This is what we wished to prove.

[19] It will further be explained that the secants of different arcs are inversely proportional to the sines of their completions — that is to say: as אך [aleph-kaf], the secant of arc בך [bet-kaf], the first, is to אי [aleph-yod], the secant of arc בך [?], the second, so is דך [dalet-kaf], the sine of the completion of the second, to צך [tzadi-kaf], the sine of the completion of the first. For every sine is a mean proportional between the secant of the arc and the sine of its completion, by lemma 15 of this Corner — that is, between אך and צך [?], and likewise between אי and דפ [dalet-pe]. The area formed from twice אך into צה [?] equals the square of the whole sine, and likewise the area formed from twice אי into דפ, by [lemma] 17 of the Sixth; and by lemma 16 of the Sixth their sides are inversely proportional — that is to say: as אך, the secant of the first arc, is to אי, the secant of the second arc, so is דפ, the sine of the completion of the second, to צך [tzadi-kaf], the sine of the completion of the first. This is what we wished to prove.

[20] It will further be explained that, as the sine of an arc is to its tangent, so is the tangent of its completion to…

the secant of its completion — that is to say, as צה [tzadi-he], the sine of arc בה [bet-he], is to בז [bet-zayin], its tangent, so is גך [gimel-kaf], the tangent of its completion, to חא [chet-aleph], the secant of the completion. For, as the completion-secant חא [?] is to גך [?], so, by [lemma] 4, etc.; and likewise, since אב [aleph-bet] equals it, [as] אן [aleph-nun] is to מד [mem-dalet] [?], by the Sixth; and just as this proportion is, so too is the proportion of גח [gimel-chet] to חא; it will then be, by [lemma] 51 of the Fifth, in proportion: as צה is to בז, so is גח to חא. This is what we wished to prove.

[21] From these examples it will also be explained to you that, in every right-angled rectilinear triangle in which the perpendicular [stands upon] the base — that is, the side opposite the right angle is taken as half the diameter, that is, the whole sine — the sides adjacent to the right angle will be the straight sines of the acute angles opposite them.

For example, in triangle אשך [aleph-shin-kaf], in which the base אק [aleph-kaf] is like half the diameter [of] a circle drawn around it בהג [?]: side אש [aleph-shin] is the straight sine of the opposite angle אזש [aleph-zayin-shin] [?], since angles אהש [aleph-he-shin] and אצ [?] are equal — 29 [?] to the first [?] — and the line הצ [he-tzadi] is the straight sine of angle ראצ [resh-aleph-tzadi] [?], as was explained in lemma 14 of this Corner. And if so, then אש [aleph-shin], which equals it, is also the straight sine of angle זקאצ [?]; and this [?] is what we wished to prove.

But if one of the sides adjacent to the right angle is taken to be half the diameter — that is, its measure is the secant of an arc, one of whose [extremities lies at] the center of the circle, אש [?] for example — and the side אש is half the diameter: then the second side שק [shin-kaf] will be the tangent of angle שאק [?] [?] at the center, and the base אק its secant. And had we placed angle אזקש [?] at the center, with side דקש [?] [as] the diameter, [then] אז half the diameter [?] would be the tangent of angle אש [?] and the base אה [?] its secant.

From this you will know how to vary the proportions of the triangle sought — by means of three different proportions and three different procedures — as we shall treat this matter more fully, God willing, in the Fourth Corner of this treatise. And it is explained from this that, in right-angled rectilinear triangles, the proportions of the sides of the angle are as the proportions of the sines, tangents, or secants of the angles opposite them.

[22] In a spherical triangle having one right angle and two acute angles [צרות]: the perpendiculars of its sides, [extended] up to the semicircle, will meet at a right angle, and the triangle corresponding to it will be one with two obtuse angles [רחבות]. Conversely, if the triangle has two obtuse angles, its corresponding triangle will have two acute angles. But if, besides the right angle, it has one acute and one obtuse angle, the perpendiculars of the sides of the acute angle will meet at an acute angle, and the corresponding triangle will likewise have two acute angles. In a triangle all of whose angles are obtuse, every two sides of the obtuse [angle] will meet at an obtuse angle, and the other two angles of the corresponding triangle will be acute. And in a triangle all of whose angles are acute, the perpendiculars of every two of its sides will meet at an acute [angle], and the remaining [angles] of the corresponding triangle are obtuse. Conversely, if it has one acute and two obtuse angles, the perpendiculars of the sides of the acute angle will meet at an acute [angle], and all the angles of the corresponding triangle will be acute. By "the sides of the angle" I mean the sides between which the angle lies.

[23] In every kind of triangle, every two of its sides, however chosen, exceed the third…

It follows from this that, in spherical triangles, if the sum of the three sides does not exceed a quadrant, the third [side] will necessarily be less than a quadrant; but if their sum does exceed the quadrant, there is nothing to prevent the third from exceeding the quadrant as well. [The phrasing here is compressed; the sense is that the relation between the sum of two sides and a quadrant governs whether the third side is less than or greater than a quadrant.]

24. In every triangle, the greater side subtends the greater angle within it, and likewise the greater angle is subtended by the greater side. It follows from this that the smaller side subtends the smaller angle, and the smaller angle the smaller side.

25. In every spherical triangle, the three sides taken together are less than the whole circle—that is, less than 360 degrees. It follows from this that, if two of its sides together equal three quadrants—that is, 270 degrees—or exceed them, the third side must necessarily be less than a quadrant.

26. In every spherical triangle, the three angles together exceed two right angles, yet they are always less than six right angles. It follows from this that, if two of its angles equal two right angles or are less than them, the third angle must necessarily be obtuse. Likewise, in the case of the present triangle, that is, [when] the sum of the angles is more than two right angles—and if not, the triangle is spurious.

Now in a rectilinear triangle, since its angles equal two right angles, if two of them are equal to a right angle or exceed it, the third must necessarily be acute. Similarly, if it is right-angled and isosceles, each of the acute angles will be 45 degrees; and if it is equilateral, each of its three angles will be 60 degrees.

26. [thus printed; a doubled siman—this lemma should be 27] In every spherical triangle, if one of its sides is extended and stretched out: should the sum of the two remaining sides together equal half the circle, the exterior angle will equal the interior angle opposite, which rests upon the extended side. But if they fall short of 180 degrees, the exterior [angle] will exceed the interior opposite angle; and if they exceed [180], the exterior will be smaller than the interior opposite angle. Conversely, if the exterior equals the interior opposite angle, the two other sides will together equal 180 degrees; if it exceeds the interior opposite [angle], those sides will be less than half the circle; and if it falls short of the opposite [angle], the two remaining sides will exceed half the circle. In rectilinear figures,

however, the exterior is always greater than each of the interior [angles] opposite it, since it equals the two opposite interior angles together, as stated in lemma 32 of the same [Book One of the Elements].

28. When the three angles of one spherical triangle equal the three angles of another, their three sides will also be respectively equal, each to its counterpart—that is, the side subtending the angle equal in both. This does not hold for rectilinear figures: there it can happen that the sides of the one are proportional to the sides of the other, which suffices to make the angles equal, but it is not necessary that the sides be equal, as is shown in the figure of lemma 20 of Book Six of the Elements [recte: Euclid VI.4 (equiangular triangles have proportional sides)]. In any two [spherical] triangles, however, whose sides are equal, the angles too will be equal, each to its counterpart, namely the angle lying between the equal sides.

29. In any two triangles in which two sides of the one equal two sides of the other, and the angle between the equal sides is equal in both, then the third side too, as well as the two remaining angles, are equal to one another, and the whole of the one triangle equals the whole of the other. But if the angle between the equal sides of the one is greater than the angle between the equal sides of the other, then the third side of the former must exceed the third side of its counterpart.

30. And if two angles of the one triangle equal two angles of the other, and the side lying between the equal [angles] is likewise equal in both, then the two [other] sides as well as the third angle of the one are equal to the two sides and the third angle of its counterpart. You see that this lemma is taken from lemma 26 of Book One of the Elements, of which it is a portion; yet the lemma as a whole does not suit all spherical triangles—only those that are right-angled, where the side equal in both triangles is the one stretched [adjacent] to the right angle. For in other triangles, even when the side lying between the equal angles is equal in both, it is possible that the triangles are nonetheless different from one another; and on this Copernicus stumbled—or his typesetter—in lemma 4 of Book One of his *De Revolutionibus*. But this is an absolute and indubitable truth: that if two spherical triangles are right-angled, and two of the oblique angles of the one equal the two oblique [angles] of the other, and the side stretched [adjacent to the right angle] of the one equals the [corresponding] stretched side of the other, then both the two sides and the angle of the one are equal to the two sides and the angle of the other—for in that case this angle is a right angle in both.

31. And if two angles of a spherical triangle equal two angles of its counterpart, and the side subtending the angle equal in both is equal: should the sum of the two sides subtending the equal angles exceed 180 degrees, or fall short of them, then together with the two sides of the one, the third angle of this one too equals the two remaining sides and the angle of its counterpart. But if their sum equals 180 degrees, this is not true in every case. On this too the wise Copernicus failed to set his mind, in lemma 12 of Book One.

32. When in two spherical triangles two angles of the one equal two of the other, and the two sides of the third angle of the one equal the two sides of the third angle of the other, and the pole of the third side does not fall on that third angle, then the third side of the one equals the third side of the other, and likewise the third angle of the one equals the third angle of its counterpart. But if the pole of the third side falls on the third angle, this lemma is not true; and here too Nicolaus Copernicus erred, in lemma 12 of Book One of his *De Revolutionibus*.

33. When in two spherical triangles one angle of the one equals one angle of the other, and likewise the two sides of another angle in this triangle equal the two sides of another angle in the second triangle, and each of the remaining angles is either both less than a right angle or both greater [than a right angle]—then the third side of the one equals the third side of the other, and likewise the two remaining angles of the one equal the two angles of the other. But if each of the remaining angles is less than, or greater than, a right angle [in opposite senses], the lemma does not hold true; and on this the wise Copernicus stumbled, in lemmas 8 and 11 of Book One of his *De Revolutionibus*.

34. In every isosceles triangle, the perpendicular dropped to the third side from the angle between the two [equal] legs bisects both the third side and the angle.

35. In every spherical triangle, the sum of any two angles you please is less than the difference between the supplement of the third angle and the whole circle—that is, 360 degrees; yet it is greater than the difference between the third angle and half the circle—that is, 180 degrees. From the end of this lemma it follows that, in every spherical triangle one of whose sides is extended outward, the exterior angle is less than the sum of the two interior opposite angles—unlike the rectilinear case, where the exterior equals the two interior [opposite angles], as shown in lemma 32 of Book One of the Elements. It is likewise clear that, in every spherical triangle, the difference between the sum of any two angles you choose and the whole circle exceeds the difference between the third angle and half the circle.

36. In all isosceles triangles, the angles at the base are equal to one another; and when the legs are extended, the angles that are their remainders, beneath the base, are also equal to one another. It follows from this that every equilateral triangle is also equiangular. For rectilinear figures this is shown in [Book] One of the Elements [recte: Elements I.5–I.6], and for spherical [figures] it is easily demonstrated as well; the full demonstration of all this is in *Sefer HaShamayim* [the longer work]. Likewise, in every triangle two of whose angles are equal to one another, the sides subtending them are also equal—Euclid demonstrated this only for rectilinear figures, in Book Six of Book One [recte: Elements I.6]. It follows from this that every equiangular triangle is also equilateral.

37. Of any two arcs of the same kind, the difference between them equals the difference between their supplements, and likewise the remainder of their sum equals the sum of their supplements. But if they are of different kinds, then their difference equals the sum of their supplements, and the remainder of their sum equals the difference of their supplements.

When the sum of two sides of a spherical triangle equals half a circle, the sum of the two angles upon the base will also equal two right angles; if it is less, the sum of the angles too will be less than two right angles; and if it exceeds, the sum of the angles likewise exceeds two right angles. Conversely, if the sum of the two angles upon one side equals two right angles, the sum of the sides opposite them also equals half a circle; likewise if [the angle-sum is] less, [the side-sum is] less, and if greater, greater.

39. In a spherical triangle all of whose angles are acute, all of its sides too will be less than a quadrant. But not the reverse: it is not necessary that every triangle whose three sides are less than a quadrant should have all its angles acute, for it is possible that one of them is right or obtuse while the remaining ones are acute.

40. A spherical triangle that has two equal acute [angles]: the sides opposite them too will be less than a quadrant. And if they are unequal, the side subtending the greater of them will be greater than a quadrant. But if they are unequal and there is the further [condition] that the third angle is obtuse, it is necessary

that the side subtending the more acute of the angles be less than a quadrant; the [other] sides, however, may be a quadrant, or greater, or smaller.

41. In every spherical triangle whose sides all exceed a quadrant—or [where] one [is a quadrant] and the rest exceed [it]—all its angles are obtuse. But the converse does not hold, for it is not necessary that a spherical triangle all of whose angles are obtuse should have all its sides necessarily exceeding a quadrant, or one of

them a quadrant and [the rest] exceeding [it]; for it is possible that two of them exceed [a quadrant] and the third falls short of a quadrant.

42. In every spherical triangle one of whose sides exceeds a quadrant while the two others are less than it, it is impossible for it to have a right angle.

43. In a spherical triangle one of whose sides is a quadrant, the second less [than a quadrant], and the third exceeding [it]—the angle opposite the [side] that exceeds will be obtuse, while the other two will be acute. But if each of its sides is less than a quadrant, the angle opposite the greatest of the sides may be right, obtuse, or acute, while the other angles are necessarily acute. And if each of two of the sides is less than a quadrant, but the third is not less [than a quadrant], the angle opposite the greatest of the sides will be obtuse, and the remaining [angles] will be acute.

44. In a spherical triangle two of whose angles are acute, if the side opposite one of them is less than a quadrant, the third angle will be obtuse and the side opposite it will exceed a quadrant.

45. In a spherical triangle two of whose angles are obtuse and equal, the sides opposite them too will exceed a quadrant.

46. In an isosceles spherical triangle: if its legs are quadrants, the angles opposite them at the base are right angles; if they exceed [a quadrant], [the angles] will be obtuse; and if they fall short of a quadrant, [the angles] will be acute. Conversely, if the angles upon the base are right, the two legs too will be quadrants; if [the angles are] obtuse, [the legs] exceed [a quadrant]; and if acute, less than a quadrant. It follows from this that, in every equilateral or equiangular spherical triangle: if the sides are quadrants, all the angles will be right; if [the sides] exceed [a quadrant], [the angles will be] obtuse; and if less, acute. Conversely, if the angles are right, the sides will be quadrants; if obtuse, [the sides] exceed [a quadrant]; and if acute, less [than a quadrant].

47. In an isosceles triangle, if its legs are quadrants and the angle between them is right, the base too will be a quadrant—being an arc the measure of the right angle. If [the angle] is acute, [the base] will be less; and if obtuse, [the base] will exceed a quadrant. Conversely, according to the base, the said angle will be right, acute, or obtuse.

48. When each of the two sides of a spherical triangle is less than a quadrant, and the third is greater than each of them, the perpendicular dropped from the angle between the two smaller sides

it falls inside the triangle. Likewise, if there are two acute angles, or if their sides are narrow, the perpendicular that descends falls inside the triangle. And there is a fine point to examine here in this teaching: it is possible for the perpendicular to fall outside the triangle, but then it falls inside the triangle adjacent to it, and you then operate with what remains of it. [This is treated] in *Sefer Arvot HaShamayim* [the author's own work, "The Vault of the Heavens"], where you will find the figure clearly drawn. But if one of them is acute and the other obtuse, the perpendicular descending from the third angle falls outside the triangle. The manner in which the perpendicular falls in rectilinear (plane) triangles was explained by Euclid in his two foundational [propositions], and we have studied it there.

[מא — see numbering note a] A triangle in general is designated either by its angles or by its sides. By its **sides**: in that some triangles are **equilateral**, having three straight lines or three arcs equal to one another; some are **isosceles**, having two straight lines or two arcs equal to each other while the third differs; and some are **scalene**, composed of three straight lines or three arcs none of which equals another. By its **angles**: some have a right angle. With spherical triangles it is possible to have two or three right angles — unlike in rectilinear triangles, where it is impossible to have more than one right angle. Some triangles have an obtuse angle: this occurs in plane figures in two ways — either two acute angles and one obtuse, in which case it is named for the latter and called **obtuse-angled**, or three acute angles, in which case it is called **acute-angled**. But with spherical triangles the matter varies according to type: some are entirely acute — that is, all three of their angles are acute; some are entirely obtuse, all three of their angles being obtuse; and some are mixed, and this in two ways — either two of their angles are acute and one obtuse, or two obtuse and one acute. Each and every one of these types has its own rules and laws that must not be transgressed, as has been demonstrated.

Now, the writers have agreed by convention to call the side opposite the right angle the **nashtach** (hypotenuse), and the two sides bounding the right angle the **legs of the right angle**. If one of them is the base or seat, the second will be the perpendicular (upright). I have already shown you that you are at liberty to make whichever of them you wish the base. In an oblique triangle, all three arcs are called by the name of "their sides," or "their base" — likewise [extended] to the third.

[53 — נג] A spherical right-angled triangle: if its three angles are right, then its three sides are also quadrants, and all its parts are known. One that has two right

angles — then its two corresponding sides (I mean the hypotenuse and one of the legs of the right angle) are quadrants; and as for the third angle, if it is in fact right, the third side too is a quadrant; if obtuse, the side exceeds [a quadrant]; if acute, it is less than a quadrant. As for one having a single right angle with two acute angles, each of its three sides will be less than a quadrant. One having a right angle with two obtuse angles — its hypotenuse will be less than a quadrant, but each of its [other] sides greater than a quadrant. And one having one right angle, one obtuse, and one acute — each of the sides of the acute angle in its totality, as well as the hypotenuse, exceeds a quadrant, while the second leg is less than a quadrant. And it is clear from this that…

in a spherical right-angled triangle, if the leg is a quadrant, the angle opposite it is also right; and likewise the hypotenuse is a quadrant. If the leg is less than a quadrant, the angle opposite it too is acute; and if it exceeds a quadrant, the angle opposite it is obtuse. But if the two legs are greater than quadrants, the angles opposite them are obtuse; and if they are less than a quadrant, the angles are acute. As for the hypotenuse: it is always less than a quadrant. But if one of its legs is greater and one less [than a quadrant], the greater corresponds to an obtuse angle and the lesser to an acute, while the hypotenuse will exceed a quadrant — and conversely.

[52 — נב; see numbering note b] As for the right [angle being] a quadrant: if the angle adjacent to the hypotenuse is right, then likewise the corresponding leg [is a quadrant]; if it is obtuse, the leg corresponding to it exceeds a quadrant, and [if it is acute,] the leg adjacent [or: opposite] is less than a quadrant. Likewise, if the hypotenuse is a quadrant, then at the least one leg will be a quadrant; and if it exceeds a quadrant, one of the legs will be in excess — namely the one corresponding to the obtuse angle — while the second, corresponding to the acute angle, will be less than a quadrant. But if the hypotenuse is less than a quadrant, the legs will either both exceed a quadrant (if the opposite angles are obtuse) or both be less than a quadrant (if the corresponding angles are acute). It follows from this that every spherical triangle which has one side a quadrant must necessarily also have a right angle; and further, [it must have] a second side a quadrant, or else all three of its sides are quadrants — and accordingly its three angles are also right. For it is impossible for a spherical right-angled triangle to have only a single side that is a quadrant; rather, at the least, [it must have] two quadrants.

[52 — נב] In a spherical right-angled triangle, all three of its angles together are less than four right angles — meaning that its two oblique angles are less than three right angles, which are 270 degrees. [See numbering note c]

[54 — נד] In a spherical right-angled triangle, each of the oblique angles exceeds the supplement of the second angle, but is less than the difference by which the supplement of the second is distinguished from a semicircle — or from two right angles. [text faint here]

[55 — נה] In a spherical right-angled isosceles triangle: if the equal angles are acute, each of its legs is greater than half a quadrant — that is, exceeds 45 degrees; and if [the equal angles are] obtuse, each of its legs is less than a quadrant and a half — that is, does not reach 135 degrees.

[56 — נו] In a spherical right-angled triangle, if it has one acute angle, the arc that descends from the second oblique angle inside the triangle to the side corresponding to it — which lies between the right angle and the acute angle — is less than the hypotenuse, but exceeds the side corresponding to the acute angle. But if the said triangle has one obtuse angle, then the arc descending from the second oblique angle to the side corresponding to it, lying between the right angle and the acute angle, exceeds the hypotenuse, but is less than the side corresponding to the acute angle.

[57 — נז] In a spherical triangle, if it has one right angle and one acute angle, and the side between them is a quadrant, then the angle corresponding to that side is also right, and likewise the hypotenuse is a quadrant. If the side between them is less than a quadrant, the opposite angle too is acute, and the hypotenuse is less than a quadrant. If [the side between them] exceeds a quadrant, the angle opposite it is also obtuse, and the hypotenuse too exceeds a quadrant — but the side corresponding to the acute angle is always less than a quadrant. And conversely.

[58 — נח] In a spherical triangle in which one of its angles is acute and one right: if the angle…

opposite the side between them is right, then the side between them too is a quadrant, and so is the hypotenuse. If [the opposite angle] is acute, then both the side and the hypotenuse are less than a quadrant; and if obtuse, both the side and the hypotenuse exceed a quadrant.

[59 — נט] In a spherical right-angled triangle that has no side a quadrant: the angle opposite the side less than a quadrant is acute, yet it exceeds that side; and

the angle corresponding to the side greater than a quadrant is obtuse, yet it is less than the side opposite it.

[60 — ס] In a spherical right-angled triangle, if the hypotenuse and one leg exceed a quadrant, the second leg is less than a quadrant.

[61 — סא] As for the spherical oblique-angled triangle: it is either simple — that is, [composed] of three acute angles, in which case its three sides too are less than a quadrant; or [composed] of three obtuse angles, in which case [the sides] will at the least exceed a quadrant (since the third [side] may possibly be a quadrant, or less, or more — nothing compels it either way); or it is mixed — meaning it has two angles of one type and the third of another type. Now if the equal angles are acute and the third obtuse, it is possible that all the sides are less than a quadrant; or that two are less and the third, corresponding to the obtuse angle, is a quadrant or exceeds it; or that one side is less than a quadrant and the two [others] are in excess, but the greater of them is the one corresponding to the obtuse angle; or that one is greater than a quadrant — namely the one corresponding to the obtuse angle — the second is a quadrant, and the third is less, these [latter two] corresponding to the acute angles. But if it is composed of two obtuse and one acute angle, then the side corresponding to the acute angle is less than a quadrant, and the remaining ones exceed a quadrant.

All of these have been explained, with their figures and their demonstrations, in *Sefer Arvot HaShamayim*; and there you will prepare yourself in all these teachings, and have them all written out in your own book and kept ready by you, if you wish to be successful and swift in your craft. I will inform you of a small portion of their usefulness in this discourse, and they will be sweet to your palate — for there is nothing empty in them, and there is no teaching you possess that does not have its place and its time and season in the sciences.

[62 — סב] When you study these teachings, the nature of each and every side will be revealed to you, and likewise the hypotenuse and the kinds of angle, in every right-angled triangle — apart from certain triangles in which, from knowledge of a side together with its corresponding angle and the right angle, you seek the three remaining parts, concerning which doubt arises as to whether they are less than a quadrant, or right, or in excess of them. These are the cases [treated in] the sixth, ninth, and fifteenth columns [טור] of the First Corner, where I warned you that their nature must be known from another source — for the computation will reveal to you only the magnitude, and that [remains] in doubt. Let me now explain the matter to you from its foundation.

Let us suppose, by way of example **(figure [9] ט)**, a spherical right-angled triangle אבג [A-B-G] whose legs of the right angle are known, and we wish to know the nature of its angles א [A] and ג [G]. We know, from teaching 51 [נ"א] of this Corner, that the angles are [each] according to the nature of the sides opposite them; and since its legs are known to us by hypothesis, we know whether they are right, acute, or obtuse; and from that same teaching we likewise know the nature of the hypotenuse — whether it is a quadrant, or less, or more. Likewise, if the hypotenuse is known, and side אב [A-B] which corresponds…

to angle ג [G], we know the nature of angle ג [G] from teaching 51 [נ"א]. And likewise, from knowledge of the hypotenuse, we know the nature of side בג [B-G] from the second teaching, [namely] teaching 52 [נ"ב] of this Corner; and when we know the nature of the side, we know the nature of angle א [A] from teaching 51 [נ"א]. So too, if side אב [A-B] is known along with the angle א [A] adjacent to it, then with the aid of angle א [A] we know the nature of the side opposite [it], בג [B-G], from teaching 52 [נ"ב]; and with the aid of אב [A-B] we know the nature of angle ג [G] from teaching 51 [נ"א]. And likewise, from that same teaching, once we know the type of its two legs, we know the nature of the hypotenuse.

[If] the hypotenuse [is known] together with angle א [A], then from the nature of angle א [A] we know the nature of side גב [G-B] corresponding to it; and likewise, from knowledge of the nature of the hypotenuse, we know the nature of the second side אב [A-B] from teaching 52 [נ"ב]; and from these we know the nature of angle ב [B]. So too, if we knew its two oblique angles א [A] and ג [G], we would know the nature of the sides from teaching 51 [נ"א], and from there too the nature of the hypotenuse.

But if side אב [A-B] is known together with the angle ג [G] corresponding to it, you are not permitted to verify the magnitude of angle א [A] unless you know which type it is — I mean, whether it is acute or obtuse. This is made clear by the figure before you **(figure [5] ה)**: that in two different triangles you may find three parts equal to one another. That is to say, in the small triangle אבג [A-B-G] its three known parts — namely the right angle ב [B], the acute angle ג [G], and side אב [A-B] corresponding to it — are [also] known in the large triangle אבה [A-B-H], since angles ג [G] and ה [H] are equal (from teaching 11 [י"א] of this Corner), side אב [A-B] is shared between them, and angle אבג [A-B-G] equals angle אבה [A-B-H], both being right.

Therefore, even when you find, by the directions of the columns of the First Corner, the unknown quantity sought, you would still be in doubt whether it is less than a quadrant or exceeds it — I mean, whether it belongs to the small

triangle or to the large one, whose two remaining parts differ from one another. For in the small triangle the oblique angle sought is angle באג [B-A-G], whereas in the large triangle אבה [A-B-H] the oblique angle sought is באה [B-A-H], which is the supplement of angle באג [B-A-G]. Likewise the side sought in triangle אבג [A-B-G] is arc בג [B-G], whereas in triangle אבה [A-B-H] it is arc בה [B-H], its supplement. And likewise the hypotenuse sought here is אג [A-G], whereas in the large triangle it is אה [A-H] — because the sines, the chords [or: tangents], and the secants are common to arcs and to their supplements, as I explained in teaching 14 [י"ד].

It is therefore incumbent upon you to seek to know, from another source, the nature of the angle sought in the sixth column, or the side sought in the fifteenth column, or the hypotenuse sought in the ninth column of the First Corner. And do not stumble along with **Johannes Regiomontanus** [יוחני ריניאומנאנו = Johannes Regiomontanus] in teaching 27 [כ"ז] of his Fourth Book *On Triangles* [*De Triangulis Omnimodis*, Book IV, prop. 27], nor with **Nicolaus Copernicus** [ניקולאוס קופירניקו] in the fourth teaching of Book I *On the Revolutions* [*De Revolutionibus*, Book I], for these two giants supposed that knowledge of a side together with its corresponding angle in a right-angled triangle suffices to determine the three remaining parts — which is not so, as your own eyes have seen, and no stranger's. And what we have said and explained concerning right-angled triangles has its place also in oblique-angled ones, which are converted into right-angled ones by means of the perpendicular. And when the result is in doubt — that is, when one makes use of the 6th, 9th, or 15th column of the First Corner — then the parts of the oblique [triangle] composed of them will also be in doubt, since composites change with the accidents of the simple [components] that compose them. However, I will inform you that you need not know the nature of all three unknown parts — I mean, whether the second angle is acute or obtuse, and likewise whether the side and the hypotenuse are less…

or the cosecant. If you know the quality of any one of these, that knowledge will lead you to know the quality of the rest, provided you remember the lesson. [The columns] [?51] נ"א, [2] ב, and [?52] נ"ב of [?] the Corner — I have, in accordance with their conditions, set this table before you (left side). And in the two columns of doubt I have placed marks for you, together with points that indicate the knowledge of the quality. The mark [3] ג indicates that it is less than a quadrant; the mark [4] ד, that it exceeds [a quadrant]. And the letter נ stands for "erect/right" [נצבת], ק for "small/acute" [קטנה], and נ for "large/obtuse" [גדולה]. These are the three categories: the small triangle indicates "less," the square indicates "more than a quadrant" — and these are the parts sought. So once you

know three parts plus the quality of one more of the unknowns, you immediately know the rest as well. Understand this.

[63] סג. As for spherical triangles in general — I mean those in which we do not know whether they have a hypotenuse [נשטח] or not — there are other ways to know each one of their angles by its quality. Do not suppose that knowledge of the sides alone will at once bring you to knowledge of the angles, as it does in plane (straight-line) triangles. For in a plane triangle, if the square of the largest side equals the squares of the two remaining sides, the one angle is right and the two others acute; if it is less, all three are acute; and if it is greater, [the one angle] is obtuse and two are acute, as is explained in Euclid's [Elements] I.47 [Pythagorean theorem] for the first case, and I.12 and I.13 for the second. But because the spherical figures differ from these in their foundations, the laws of the sphere are different.

Now fix your eyes on the sides enclosing the angle and see whether they are of differing kind [i.e. one greater, one less than a quadrant], and whether the base [התושבת] does not exceed a quadrant. If so, the angle between them is acute. But if the sides are of one kind and the base is not less than a quadrant, the angle will be obtuse. However, if the sides are of one kind and the base is less than a quadrant, see whether the sine of the complement of the base exceeds the product of the sines of the complements of the two sides multiplied one by the other, after dividing it [the product] by the whole sine [the radius]. If so, the angle is acute; if less, obtuse; and if equal, right.

Here is its explanation. Suppose in your mind that the angle sought is right. Then, by the seventh column [of the First Corner], it will be: as the whole sine is to the sine of the complement of the first side, so is the sine of the complement of the second side to the sine of the complement of the hypotenuse, which is the base. See whether the arc of the complement of this found value equals the base. If so, the assumption that the angle is right stands well. But if it exceeds, this indicates that the base is less than a quadrant, and the angle facing it is therefore acute; while if it falls short, the base is in excess, and accordingly the facing angle is obtuse.

But when the sides enclosing the angle are of differing kinds — for instance, one greater and the other less than a quadrant — and the base exceeds a quadrant, see whether the sine of the complement of the base exceeds the product of the sines of the complements of the two sides multiplied one by the other, after dropping five digits from the right. If so, the angle is obtuse; if less, acute; and if equal, right. The procedure is made clear by the said seventh column. You can

shorten it and use the method of Ibn al-Malik [?], and you will not need to multiply, since the two numbers are sines and it is quite easy — there is no need for any reckoning at all. You will find these lessons set out and explained in the book *Orchot HaShamayim* [The Paths of the Heavens] [Delmedigo's own work], along with many such things, in an excellent manner. I have no leisure to elaborate, for I did not set out to compose a treatise but only to reply in the manner of an epistle — to pluck two or three rose-petals, things that draw the soul, brief words for the discerning, and a gift to the wise. Would that these laws and treasures that I have written in this brief form be kept before your eyes; bind them upon the tablet of your heart, that they may suffice you and light up your paths and your ways…

and may they give light, so that your foot does not stumble in these precious works, in which sharp-witted scholars have become entangled — as happened to ʿ**Ali ibn Riḍwan the Egyptian** [ʿAlī ibn Riḍwān], who was a teacher in the academies of the great city, **al-Qāhira** [Cairo], a man held in honor and high standing among the Ishmaelite [Muslim] notables. While I was there, I came as a [poor scholar with] books, and he was tranquil and at ease, settled in his house "resting upon his lees" [Tzefanya 1:12]. But I poured myself "from vessel to vessel" [cf. Yirmeyahu 48:11], with riddles and deep questions in the sciences, and I kept myself hidden and concealed from him, for I lacked a [reputable] day. I went about as an exile, yet he pressed me hard until I was embarrassed, and I was compelled to meet him face to face. We agreed upon a day, and there each of us handed the other six questions, and we fixed a term of two weeks. On the appointed day there was assembled there a great gathering of the Ishmaelite notables, and also of Jews — Rabbanites, Karaites, and Samaritans — counting some [?] in all; they gathered there to see "whose word would stand." It distressed me greatly that the judges' decision had fallen out that I should answer first all his questions, and submit them in explicit writing and set them before them, while he would come after — for I was a youth and tender, whereas he was already old, "a great and awesome name."

But the thing I had thought would be a grievous evil turned out for me to be a salvation and a great deliverance. For the first question he had asked me was founded upon a chaotic, foolish, and absurd premise. This was its form: [there is] a spherical triangle אבג with oblique angles, where angle ב is 150 degrees, angle ג 140, and angle 50 א, and one wishes to know the magnitude of the sides without resorting to the device of right-angled triangles. Now, as I have taught you, in lesson 5 [?], the angles of a spherical triangle always exceed two right angles. This is clear in the figure before you (figure 11 [ציור יא]): each of the angles between

the arcs [of great circles] is greater than the angle between the straight chords, which are a part of them. Moreover, it is known from Euclid's [Elements] I, in its foundations, that the angles of a straight-line triangle equal two right angles; and therefore the angles of a spherical triangle exceed [them]. This too was not hidden from him, for it is the twentieth proposition of Book I of **Menelaus** [Menelaus, *Spherics*], and the book is found in everyone's hands, in the Holy Tongue and in Arabic, in Egypt.

True, the angles of the triangle he asked me about do exceed two right angles — but he did not take to heart that its supplemental [triangle], which is its residue, does not have angles even of two right angles. For the residue of angle ב is 30, the residue of angle ג is 40 degrees, and angle ד equals angle א — by lesson 11 [?] of this Corner — and it is 50 degrees; the whole [sum] is 120 degrees only. And therefore, just as its residue is false, so its triangle is impossible and counterfeit. I, supposing he intended to mock me — to test me — said to him at the very outset, before the examiners, that his question was disgraceful and that I had no business with it; and the Lord set straight his whole design, yet he did not concede, saying: "I too intended only to test you." But in truth he had not grasped the deficiency; rather he laughed and shook his head at me, saying that the question was good and sound. And this was my apology — that I did not know its superfluity [?]. So I said to him: "If so, then you yourself state the magnitude of the sides." Then he was clothed with pride and said in his foolish heart: "Now I will arise, now be exalted!" And I whispered into the ear of one of the notables that he should send for the book of Menelaus. After he had finished speaking and held forth for three hours, I opened the book for him and asked him whether he had ever read it; and I showed the whole assembly that he was erring in the very rudiments and foundations. All the people looked to him and watched him until his face was covered with disgrace. The day had already turned and the sun set low, and he withdrew, sullen and "sated with contempt," driven back to his house, and his soul all but departed; while I…

was trembling even more than he, on account of the throng of Ishmaelites, lest they set their eyes upon me — for we are "bowed down" under their hand in our many sins. The notable sent several men after me, and they brought me home safely. From then on my name went out in Egypt and my praise [spread] among the nations. Do not let it enter your mind to say that he was an empty, hollow man; for "by my head," his head was full of wisdom, as you can see from his questions and his stratagems. Afterward I went to his house and humbled myself before him, and I said that I knew it had been an inadvertent slip on his part, since I recognized that he was a man of designs, and that I wished to learn from

him as one of his pupils. He too afterward embraced me and honored me before his elders, and showed me great kindness. The upshot: whoever enters into disputes with one lesser than himself will come to folly.

[64] סד. When you drop a perpendicular arc from one of the angles of an oblique-angled spherical triangle to the base beneath it, or to the segment of the base extended outside the triangle, then the angles between the perpendicular and the sides are proportioned to the angles at the base — the right to the right, and the left to the left. Likewise the sides are proportioned to the segments of the base, and likewise the angles at the base to the segments of the base, each with the one adjacent to it. Now, besides the three parts of this lesson — which Regiomontanus already demonstrated [Regiomontanus, *De Triangulis*], and which I expounded by another method in the book *Chukos Shamayim* [The Statutes of the Heavens] [Delmedigo's own work] — I have here added two more parts. I lay them before you in brief.

The first part (figure 12 [ציור יב]): In triangle אבג, from whose angle א a perpendicular arc אד is dropped to the base בג — whether it falls within or without, it is all the same in its proof — I wish to show that as the sine of angle באד is to the sine of angle דאצ [?], so is the sine of the complement of angle ב to the sine of the complement of angle צ.

The proof: By the sixth column of the First Corner, as the whole sine is to the secant of side אד, so is the sine of the complement of angle צ to the sine of angle דאצ. Likewise the sine of the complement of [angle] ב to the sine of [angle] באד. Therefore, *ex aequali* and alternately: as the sine of the complement of angle ב is to the sine of the complement of angle צ, so is the sine of angle דאב to the sine of angle אצ [?]. And this is what we wished to show.

The second part: In it I wish to show that as the sine of the complement of side א [?] is to the sine of the complement of side אצ, so is the sine of the complement of arc בך to the sine of the complement of arc צך.

The proof: By the fifteenth column of the First Corner [recte: tenth — see anomaly], as the whole sine is to the secant of אך [?], so is the sine of the complement of אצ to the sine of the complement of צך; and likewise the sine of the complement of אב to the sine of the complement of בך. Therefore, *ex aequali* and alternately: as the sine of the complement of אצ is to the sine of the complement of אב, so is the sine of the complement of צך to the sine of the complement of בך. And this is what we wished to show.

The third part: In it I wish to show that as the tangent of the complement of angle ב is to the tangent of the complement of angle צ, so is the sine of דב to the sine of דצ.

The proof: By the fifteenth column of the First Corner, as the whole sine is to the tangent of side אד [?], so is the tangent of the complement of the facing angle ב to the sine of side דב; and likewise the tangent of the complement of angle צ to the sine of דצ. Therefore: as the tangent of the complement of [angle] ב is to the tangent of the complement of [angle] צ, so is the sine of דב to the sine of דצ. And this is what we wished to show.

[65] סה. It will also be shown that as the sum of the sines of two arcs (or angles) is to their difference, so is the tangent of half the sum of the arcs to the tangent of half the difference. Let them be in the figure before you…

Diagram. A geometric construction for the law of tangents (lesson 65). The text describes: angle דאפ together with angle פאח [?], whose [sides] are beneath the arcs דה [recte: דפ — see below] and פח, whose sines are דה and [?]. The chord subtending the two arcs is divided by a diagonal שפ between the two arcs at point מ, and a perpendicular is dropped from the center to the said chord. The construction draws a line רל equal to line למ, so that angle ראל equals angle מאל [?], because [the line] from ראמ is the mean between the noted angles (their difference being halved) together with the perpendicular אלג. A circle קל is drawn so that line לך is the tangent of half the sum of the noted angles or arcs, and line [?] is the tangent of half [the difference]. The triangles of the angle דט [?] are similar to one another.

- א (center): vertex / center point
- ב (point): division point on the diagonal
- ג: point on perpendicular line אלג
- ד, פ, ח: points on the arcs (arcs דפ, פח)
- ה: foot of sine דה
- ל, מ: points on the chord and perpendicular (למ, רל)
- ר: endpoint of constructed line רל
- ק: point on the drawn circle קל
- ך: endpoint where לך is tangent of half-sum

[the construction proceeds]. The line drawn diagonally שפ, lying between the two arcs at point מ [?], is dropped [perpendicular]. Angles פז [?] and ראפ [?], which lie beneath the arcs דפ and פד [?], whose sines are דה [?] and די [?]. And the chord subtending [the angle] from the center to the said chord is the perpendicular אלג.

We take a line רל equal to line למ, so that angle ראל equals angle מאל, because [the angle] ראמ is the mean between the said angles — which is their difference, divided into halves — together with the perpendicular אלג, and likewise the arc דפ. And if we extend a line to the midpoint of the diagonal, we trace a circle קל; then line לך is the tangent of half the sum of the said angles or arcs, and line [?] is the tangent of half [the difference].

Now, since the triangles of the angle דט [?] are similar to one another, it is plain — by Book I [of Euclid] — that since the sines יח ,דה [?] are parallel to one another, it will be, by [Elements] VI [Euclid VI.4], that as יח is to חה [?], so מח is to מ[?]. And by the composition and division of ratios that Euclid mentions in Definitions [Elements] V.14 [?] and 15 [?], for lesson 5 [?], it will be: as the sum of the two sines דה ,רז [?] is to their difference, so is הח [?] [is to] מם [?] and מד [?] — I mean, the whole line דח is to their difference, which is line רמ; and likewise its half דל to the half רל. Accordingly, the ratio of the sum of the two sines to their difference will be as דח is to רמ, that is, as the tangent of half their sum of angles or arcs (which is לד [?]) to the tangent of half the difference (which is ל[?]). This is what we wished to show.

In this very same manner the lesson is proved when the sum of the two arcs exceeds half the circle [semicircle], provided each one separately falls short of it. For the sine, by one and the same [principle], is common to the arc and to its residue, as I have already taught you.

From this lesson of Ptolemy's school [בית תלמי] there follow two very useful matters. **The first** is: when you know the ratio between the sines of two arcs, and you also know their sum or their difference, you can determine each one of them individually, in its own magnitude.

For the explanation of the first part: let the ratio of דק [?], the sine of arc דפ, to יח [?], the sine of arc הפ [?] — which is the ratio of דמ to טף [?], as we explained — be like 86 to 34. And let the sum of the two arcs דפ, פף [?] be 80 degrees. Then the sum of the sines, דך [?], will be 120, and their difference, רמ, will be 52. The tangent of [half] of בך [?], [for] half the sum 40, is 83909. Then the fourth proportional, the sought value, will be 36360 — and this is the [value of] line לך, the tangent of half the difference. By the table of tangents you find its arc to be 1959, and this is called "the arc of addition and subtraction." For if you add it to the 40, [which is] half the sum of the arcs, the greater arc דפ is revealed: 5959. And if you subtract it from half the sum, the smaller arc פד [?] is revealed: 20[°] 1[?] — so that the sum of the two totals 80. This is what we wished to show.

And since "as the whole is to the whole, so the part is to the part" — by lesson 5 [?] applied to lesson 5 [?] — you may take half the sum, דל, and half the difference, רמ, which is ל; and it will be: as 60 is to 26, so is 83909 to the fourth [proportional]. Or you may resolve them into smaller [numbers], as Euclid explained in lesson 2 of [Book?] V [?], together with the division — and they are, in our example, as 30 is to 13, so is 83909 […]. And with large [numbers] you may use the method of Ibn Shalmoh [Ibn al-Malik / ibn Shlomo?] to be free of the division.

For the explanation of the second part: by way of example, suppose you knew the ratio of the sines of the arcs, and the difference between one arc and the other — then each part of the arcs will easily be known to you of itself. We assume, in the said figure, that we know the ratio of the sine of arc דפ to the sine of arc פח…

as 86 is to 34. We thereby know the difference of the arcs, which is angle ראם [R-A-M], namely 58°39′, and its half, which is arc פג [P-G], 19°59′. Their tangent will be 36360. We now form the proportion: as 120 is to the sum of the sines of the angles' difference, so the sine of half the sum of the arcs is to the sine of half their difference, 36360. Here the third term is unknown to us. Following Proposition 19 of Book VII of the *Elements* [Euclid VII.19]: when you multiply the first by the fourth and divide by the second, the third comes out known — 83909, which is the sine of half the sum of the arcs, and its arc is 40°. When you add to it 19°59′, the known half of their difference, you obtain the greater arc דפ [D-P], 59°59′; and when you subtract from them the half of the difference, you obtain the smaller arc פח [P-Ch], 50°01′ [?: printed "50 1 פח"]. This is what we set out to demonstrate.

Here too the choice is in your hands, if you prefer, to make use of the lesson of *Even HaMelech* and dispense with the division.

[The First Corollary] For the **first corollary** [as] well: just as, by the two parts of the first part — that when you know the sides of a spherical triangle you also come to know the parts of the base lying between the sides and the perpendicular, each in its own magnitude — [so too the **second part**] is that if you know the angles at the base, you will likewise know the parts of the base in themselves.

Illustration of the first part. In triangle אבצ [A-B-Tz], let אב [A-B] be 66°01′, side אצ [A-Tz] 36°51′, and side בצ [B-Tz] — which is the base — 87°04′. You wish [to find] the parts of the base produced by the perpendicular אד [A-D] — I mean the

magnitudes of בד [B-D] and דצ [D-Tz]. First, then, we investigate the quality of the angle opposite the greater side אב, which is angle צ [Tz]. By the lesson of §63 of this Corner you know it is acute; and *a fortiori* the angle ב [B], which lies opposite a still smaller side, will be acute. By the lesson of §48, the perpendicular אד falls within the triangle; and since the base בצ is less than a quadrant, *a fortiori* each of its parts בד [and] דצ [D-Tz] is less than a quadrant. By the lesson of §37, then, the difference of their sum from 180° equals the sum of their complements — I mean 92°26′ [?: printed "26 92"], which is the remainder of base בצ. So this sum is known to us, and we further know the ratio of the sines of the complements of these parts בד [and] דצ, which is as the sines of the complements of the sides אב [and] אצ, as explained in Part 2 of the lesson of §64. Thus, by the first corollary of this lesson, the magnitude of each of these parts בד [and] דצ is known. I lay the whole working out before you in due order:

	sine of complement of אצ	half-sum of their complements	arc between the parts of their complements
	80020		
sine of complement of אב	40647	26°28′	18°58′

As the sum of their sines, 120667, is to their difference, 39373, so the sine 105255 [of the half-sum] is to the sine 34347 [of the half-difference]. When you add this arc to the half-sum of their complements, 28°… [?: column value], and subtract it therefrom, you obtain 26°65′ [?: printed "65 26"; minutes >59 — source-corrupt], which belongs to the smaller part דצ, because the complement of the smaller part is greater than the complement of the greater arc. So the smaller [part] will be 30°27′ [?: printed "30 27"], which is the complement of בד; and when you subtract each of them [from a quadrant — text reads "מ"של" = "for example"]…

Second illustration. In that same triangle אבצ let us posit 81°19′ אב [?: printed "19 81"], 50° אצ, and בצ the base 122°25′ [?: printed "25 122"]. We wish [to find] the magnitude of בד [and] דצ. By the lesson of §63 we find angle צ [Tz], opposite the greater side אב, to be obtuse [?: printed "צרה" = "narrow/acute," but context here later treats it; reading follows §63 result]…

and if so, *a fortiori* angle ב will be still more acute, and by the lesson of §48 the perpendicular אד falls within the triangle. Since in each of the two triangles formed [the angles] אדב [and] אדצ [A-D-Tz] are right, the angle [at the base] is less than a quadrant by hypothesis; *a fortiori* each of the sides בד [and] דצ is less [than

a quadrant]. By the lesson of §37 the difference of their sum from 180° equals the sum of their complements. This sum is known, namely 57°35′ [?: printed "35 57"]; and the ratio of the sines of their complements is likewise known, being as [the ratio of] the sines of the complements of the sides אב [and] אצ, as explained in Part 2 of the lesson of §64. Thus, by the first corollary of this lesson §65, we learn the magnitude of each part by itself — I mean the measure of בד and the measure of דצ. You proportion and compute thus:

	sine of complement of אצ	half-sum of their complements	arc between the parts of their complements
	64278		
sine of complement of אב	15097	28°47′30″	18°48′30″

As their sum, 79375, is to their difference, 49181, so the sine 54956 is to the sine 34050. When you add this arc to the half-sum of their complements and subtract the result from a quadrant, you obtain the smaller part דצ, 24°42′; and if you subtract it from [the half-sum] and subtract the result from a quadrant, you obtain the greater part בד, 80°01′ [?: printed "1 80"]. Everything is clear before you.

[Third] illustration. Let side אב be 130°, side בצ 95°12′ [?: printed "12 95"], and אצ 50°. We wish [to find] the parts of the base, ב[ד] and דצ. By the lesson of §63 we find angle צ obtuse and angle ב acute, and by the lesson of §48 the perpendicular אד falls outside the triangle. Since the arcs בד [and] צד [Tz-D] are of different kinds [one obtuse, one acute], their difference equals the sum of their complements, by the directive of the lesson of §37. The difference between בד and צד is known, namely 95°19′ [?: printed "19 95"], by hypothesis; and so the sum of their complements is also known. The ratio of the sines of their complements is likewise known, being as the sines of the complements of the sides adjacent to them, by Part 2 of the lesson of §64. Thus, by the first corollary of the lesson of §65, the magnitude of each of the parts בד [and] צד is also known. The proportion is as follows:

	sine of complement of אצ	half of the sum	arc of addition and subtraction
	64278		
sine of complement of אב	64278	47°36′	00°00′

As their sum, 128556, is to their difference, 0000, so the sine 109514 is to the sine 00000. As you were taught to add and subtract: when you add the arc found — which is 00 [?: zero, the degenerate case] — to the half-sum and subtract it therefrom, you obtain 47°36′, which is the complement of the two parts. Since the one [part] exceeds the quadrant and the other falls short of it, subtract 47°36′ from 180° and there remains part בד, 137°36′ [?: printed "13736"]; and likewise subtract 47°36′ from 90° and there remains the smaller arc צד [Tz-D], 42°24′. For example:

Note that whenever the complement of the one side, being less than a quadrant, equals the remainder of the greater side from a quadrant, the sines will be equal and nothing remains of their deficit. You then need neither to proportion nor to seek the [further] sines; rather, subtract half the arc of their sum from a quadrant, halve the remainder [?: "ומחצו ענול" = "and make it a circle/round it" — reading uncertain], and the sought parts will be revealed to you. The principal use of this procedure depends on the lesson of §37 and the understanding of it; so let your eyes be upon it, and there is no need…

to prolong matters with illustrations and waste time.

Illustration of the second part. In triangle אבצ let the base בצ be 122°25′, angle ב, 29°53′, and angle צ, 40°. You wish to know the measure of בד [and] דצ, each by itself. Their sum is known by hypothesis, 22°25′ [?: printed "22 25"; expected base-related value — flagged]; and the ratio of their sines is also known, by Part 3 of the lesson of §64, being equal to the ratio of the sines of the complements of the known angles. Thus, by the first corollary of this lesson §65, we find each of the arcs by itself. We proportion and compute in this manner:

	sine of complement of ב	half their sum	arc of half the difference
	174022		
sine of complement of צ	119175	30°12′61″ [?: minutes/seconds >59 — source-corrupt]	18°58′00″ [?]

As their sum, 293197, is to the difference, 54847, so the sine 181961 is to the sine 34058. And this arc: if you add it to the half-sum, you obtain the greater part בד, 80°01′ [?: printed "1 80"]; and if you subtract it therefrom, you obtain the smaller [part] דצ, 42°42′ [?: printed "42 42"]. This is what we set out to demonstrate.

When, however, the perpendicular falls outside the triangle, then בצ is the difference of the parts of the base, not their sum. This practice and exercise will suffice you. It is fitting for the student to take various triangles and prove the parts, and so to test and examine the lessons mentioned until they are fluent on his lips and he is expert and reliable in them, and his hand is ample to him, like the work of a skilled craftsman.

§66. Among the precious discoveries that the moderns made is a method of transforming one triangle into another that corresponds to it in its parts. By this, the proofs are made easy, since the conditions of one [triangle] suit its fellow — as I shall make known to you some of its uses here, and some in the Corner to come, God willing. Let there be, in the figure before you (figure 14 [ציור יד]), triangle אבג [A-B-G] whose three angles are known. We wish to invert or transform it into another triangle corresponding to it, such that the sides of the second are equal to the angles of the first, and its angles equal to the sides of the first.

We posit, then, that angle א [A] is 102°53′31″, angle ב [B] is 30°, and angle ג [G] is 59°34′. We take the remainder of the greater angle א, which is 77°06′29″; this will be the greater side פמ [P-M], and the smaller angle ב, 30°, will be the measure of the smaller side פמ [?: same label as greater side — the two sides should be distinct point-pairs; one is likely פט (P-T); flagged]. And [the complement of] 59°34′ will be the measure of the third side ממ [M-M; ?: doubled label uncertain]. Thus we have transformed triangle אבג into triangle מפמ [?: M-P-M, label sequence uncertain].

Conversely: in triangle טפמ [T-P-M] let angle פ [P] be 50°, angle מן [?: M-N] 26°22′20″, and angle [at the third vertex] 120°. We wish to transform it into another triangle. We take the remainder of the greater angle [120°], 60°, and make of it side גב [G-B]; from its angle מן, 26°22′20″, we make side אג [A-G]; and from its angle פ, 50°, we make side אב [A-B]. The triangle will be אבג, its transform. From here it is very clearly explained how the angles were changed into sides and the sides into angles.

The reason for this operation is made clear in the figure before you. Suppose, for example, [we have] triangle אבג. You extend side אב on both its sides לב [?] and describe a circle אבסד [A-B-S-D]; likewise extend the sides בג [and] אג to half the circle, so that they are cut [intersect] with the first circle at points עם [?: A-yin-M] [and others]. Set each of the angles א[בג] as a pole and describe great circles — I mean לפטם [?] upon point א, דמטי [?] upon point ג, and שפצ[…] [?] upon point ב.

I mean to make clear that by means of these circles a triangle פמש [?: P-M-Sh] is described corresponding to triangle אבצ. For arc אל… [?] is an arc of [a quadrant]… [text partly abraded] and arc בצ [is] of quadrants. If you take the common part between them, אצ — I mean [the part lying] between the quadrant א[…] — [it gives] the measure of angle מ [M], equal to side אב. Likewise, if you take the common part נה [?: N-H] between the quadrant… [it gives] arc אה [?: A-H]; [and] אג… arc [the measure of] angle ט [T], equal to side אג. And likewise if you take the common part גר [?: G-R] between the quadrants… you derive this and say: בר נז [?] will be ב[…], the measure of the remainder of angle… equal to side… and arc דן [?: D-N] will be the measure of the remainder of angle פ [P], equal to side גב. By this route compute and say: since side פמ equals arc צר [?: Tz-R], [it is] the measure of angle ב; and side ממ [?] [equals]… the measure of arc נז [?: N-Z]; angle צר [?] is the measure of [arc] נז [?]; לקש the measure of angle ג; and side פמ equals arc לה [?: L-H], the measure of the remainder of angle א. In these triangles the angles are exchanged for sides, one for the other. All is clear before you.

Members of triangle אבג		Members of triangle פמט [?]	
א [A]	102°53′31″	its remainder	פט [P-T] 77°06′29″
ב [B]	30°00′		פמ [P-M] 20°00′
ג [G]	59°34′		מט [M-T] 59°34′00″
בג [B-G]	60°00′	its remainder	מ [M] 120°00′
אג [A-G]	26°22′20″		ט [T] 26°22′
אב [A-B]	50°00′		פ [P] 50°00′

This is the method of the [other] authors, but it does not satisfy me, for they all alike reckon it a necessary condition of the transformed triangle that one of its angles be obtuse and two acute — as their figure, which they drew, and the general rule they gave, and all their illustrations alike attest. I have also heard some say that if the triangle has a right angle it cannot be transformed, since a right angle has no remainder [complement]. But in truth their words are not correct, and they did not delve deeply in their studies, for who does not understand that the remainder of a right angle is 90°? This is true. I will not conceal that for triangles all of whose angles are acute, the transformation does not succeed by the method of the earlier authorities; and if you try to mark transformations like these by [means of] the intersection of circles, you will find the door shut and barred. Therefore I felt about with all the shepherds'

instruments and strove to reach the conclusion of the matter, and my efforts were not spent in vain: for I found in Ptolemy, in the second treatise, chapter ten, of his *Great Book* [*Almagest* II.10], and there it was demonstrated to me by proof. The words of the expounder of the Theonine [commentary] are of great profit in the lessons, and will bring us out into broad relief, God willing. And here it is for you:

§67. The poles of the sides of a right [angle] lie upon the sides themselves, and their distance [from the angle] is a quadrant, which is the measure of the right [angle]. The poles of the acute [angle] lie outside the sides, both on one side; and if the distance between the two poles is taken from the two sides outward, the distance between them will be the measure of the remainder of the acute angle. The poles of the obtuse [angle] either both lie between its sides, and their distance is the measure of its remainder; or one lies between them and one outside them, and the distance between them is the measure of the obtuse [angle]. You must extend the sides as long as a quadrant of great circles; then the arc of the great circle passing between the ends of the sides measures the magnitude of the angle, as I have already shown you. On the sphere, let there be right angle [נצבת] (figure 15 [ציור טו]) [איו] [A-Y-N] and its sides אי [A-Y] [and] ין [Y-N]. The pole of side אי lies on side ין [?], and likewise the pole of side ין…

on the side. According to this, the poles are א and ו, and their distance is a quadrant of a circle — which is the measure of angle אין, the right angle. And even if you were to extend it beyond a quadrant, you will still always find the poles on their sides.

Again, let angle עין be acute. Since arc ון, whose measure is less than a quadrant — as I showed you earlier — and the poles are distant from the arcs by a quadrant (per the tenth teaching), the pole of side ען will fall outside, at point ז; and likewise the pole of side ין from that same side, at point ב; and the arcs עת and [ת]ב [?] will be quadrants. When you take the shared arc עת, the arcs [ת]ע and בת will be equal, and each of them is the measure of angle עין.

If you take from point ו to point א, the other side, [it is] a quadrant. I mean: if you take from the pole-line, between the two sides, the arc את will be the measure of the remainder of angle עין up to the semicircle — that is, the measure of obtuse angle עיב [?].

Since arc ון [?], whose measure exceeds a quadrant (per the third teaching), and the poles are distant from the arcs by a quadrant (per the tenth teaching), the pole of side יב will fall at point ף, and the pole of side זי [?] at point תן [?] — both

between the sides. And if you extend side בי [?] up to the semicircle — I mean up to א — the arcs חע [?] and וא will be quadrants. When you take the shared arc, וע will equal the pole-distance ון. Now, אע is the remainder of angle ענוב [?]; so too זן [?] [taken] from the pole-distance is the measure of the remainder of the obtuse angle אים [?].

If you wish to mark side יב outside its side — I mean, that you make its pole at point ב — then arc ב, the pole-distance, will be the measure of the obtuse angle, since this is the remainder of [the side] whose measure was its remainder. Grasp this well.

After I have laid down these premises, incline your ear and hear the comprehensive teaching that applies to every spherical triangle — not as the earlier authorities ruled to the contrary:

When you place the three angles of the spherical triangle as poles, and draw upon them three great circles, you will find among the intersections a single triangle whose sides and angles correspond to the remainders of the sides and angles of the original triangle, and it will be of the very form already mentioned. That triangle is אבה [?], with its angles and sides known. Since angle א is obtuse and angles בג are acute, the points פמ [?] will be the poles of the sides of the obtuse angle — that is, אג [?] — falling between the sides (per the preceding teaching); and according to its indication their distance פמ [?] will be the measure of the remainder of the obtuse angle, 29.6 א [i.e. 29°6′].

But the poles of the sides of the acute angle ג will fall outside the sides, and they are [ת]מ [?], and that is the measure of the remainder of angle ך [= 120, i.e. the remainder] 26.120 [?]. And likewise the poles of the sides of angle [ב], outside them, at points פת, which is the measure of the remainder of angle 150 ,ב. If so, then triangle הפפ [?] is the one corresponding to triangle [ג]אב, and all is ready and prepared before you.

The following two-column table lists, on the right, the members of triangle אבג (angles in degrees and minutes), and on the left, the corresponding members of the polar triangle:

Member (אבג)	Value	Member (polar)	Value
א	31.53.102 [?]	פט	29.6.77
ב	0.30	פח	150
ג	34.59	טח	26.120
אב	50	פ	130

Member (אבג)	Value	Member (polar)	Value
אג	20.22.26	ט	40.37.153
בג	60	ח	120

The Second Corner (הפנה השנית)

[Source p. 184] An example: Let the form here — the second figure (ציור טז) — be triangle אבג, acute in all its angles, with its members known to us, namely:

Side	Value	Side	Value
אב	2.32	אג	50
בג	27.55	א	48.86
ב	10.66 [?]	ג	40.68 [?]

If one wishes to transform it into another triangle, we set upon its three angles three great circles: טרפל [?] upon pole א, טמק [?] upon pole ב, שפת upon pole ן. And you see that at the meeting of these circles are the poles of the sides of angle א — I mean, at point פ is the pole of side אג, and at point מ the pole of side אב; and at point [ת]ו מ [?] are the poles of side בג.

Since angle א is acute, the poles of its sides אב, אג will be outside them, on the two sides, per the teaching that has passed; they are פ and ט. According to this, פמ [?] will be the measure of the remainder of angle א. For the same reason, the poles of the sides of angle ג — its measure being [the angle] whose poles are מ,ה [?] — both fall on the side of side בג, outside it; and their distance פמ [?] [is its measure]. Likewise מ ,מ, the poles of angle ב [?], both fall outside side בג; and if so, their distance ממ [?] is its measure.

Just as side פט in triangle פמט corresponds to the remainder of angle א in triangle אבג, so too angle [פ]טת [?], which subtends side [ן]פט [?] in triangle פתט [?], corresponds to the remainder of side בג in triangle אבג — since side בג is less than a quadrant. For this reason, [from] מ, the side of angle מ [?], drawn upon center ב, holding within it point [ן] [?] — which is the pole of side פת — and פת encloses within it point ב; so then בג, the pole-distance, equals the remainder of מתזפ [?], or one says that angle מפ [?] equals the remainder of ב, which is all the same.

In just this same way you will establish that angle פטת equals the remainder of side אב, and angle טפת equals the remainder of side אג. You see that arc אך [?] is a quadrant, [as is] ונכש [?]; and when you take the shared arc בר [?], there remains

אג, the pole-distance, equal to רש [?] — which is the measure of angle פמ, as explained in the preceding teaching. The remainder of רש [?] — I mean רצ [?] — up to the semicircle שרצ [?], is the measure of angle טפ [?]. And since this equals the remainder of אג, which is 50, angle מפת will then be 130.

Likewise, the measure of angle פטמ [?] equals the distance אב between the poles of its sides, and its remainder בס [?] is the measure of [angle] פטת [?]. And likewise the measure of angle פמש [?] equals the remainder of the pole-distance of its sides, בג — I mean, [it equals] angle מש [?], which is the remainder of angle ממפ [?]; so angle ן [?] will equal the remainder of בג. All of these correspond, this to that. [These figures] answer for students and for the understanding:

Member (אבג)	Value	Member (polar)	Value
אבג	—	פטח	130
אג	0.50	מפת	—
אב	2.32	פטה	58.147
בג	27.55	טחפ	33.124
א	48.86	טפ	12.93
ב	10.68	טח	50.111
ג	40	פח	0.140

[Source p. 185] If the triangle were to have two of its [angles] obtuse and one acute — as is the case at the head of triangle בגם [?] in the second figure, where angle ם [?] is acute and בג obtuse — it is transformed into triangle ממפ [?]. For just as triangle בגם [?] corresponds to triangle [ג]אב — since its sides and angles are the remainders of triangle אבג except for the shared side ב, and angle ם [?] equal to angle א — so too triangle טפמ [?] corresponds to triangle טפ [?]; and therefore triangle מפת [?] is the one corresponding to triangle בגם [?] in the remainder of its angles and sides. And this is what was sought.

Likewise, if the triangle בג [?] were to have all three of its angles obtuse, then triangle ממם [?] would correspond to it, as in the second figure. Grasp this well.

As for the right-angled triangle, its transformation and substitution are easy. This goes by one of two ways: either by the way of the earlier authorities, in the first figure, or by my way — and behold, here is the palate that tastes. Since, after you transform it into an oblique-angled triangle by the way of the earlier authorities — where the remainder of the right angle becomes a side of 90 degrees — it is easy to turn the oblique [triangle] into another triangle by taking the remainder of its parts; there is no need to draw new figures, for the poles of the sides of the

right angle fall upon the sides, as if you were to say רן [?] in the second figure. It is impossible to direct the matter in this figure.

Now take up your vessels, write and record the matter for yourself according to understanding, for the teaching in its entirety is true and faithful. Many benefits issue from the transformation as a whole, to ease computation when one chooses among the triangles — as I showed you in the sixteen [?] rows of the first Corner, where, through the device of transforming one triangle into another, two or three teachings sufficed us to complete it, whereas others did not attain it [save] with great labors.

Taking the angles in place of the sides, or the sides in place of the angles, is greatly useful — especially with oblique-angled [triangles]. Beyond the satisfaction it affords in difficult and doubtful operations, [there is value] in altering and trying; and at times it is easier to find the unknown sides from the angles, and at times the reverse. There is a further great added benefit: that many teachings befit this [triangle] and not another, and a skilled craftsman is needed to fit the subject so that it is suited to its work.

I will inform you of this too: that every spherical triangle has three further triangles attached to it. For if you draw three great circles on a sphere, you will find among the intersections eight triangles — yet they are only four, since every two of them are equal to one another. Hence half the corporeal [?] sphere suffices you. And behold, if you look at the first figure of this teaching and consider triangle פתמן [?] in it, seeking its hypotenuse, you will find to its left triangle תקט [?], to its right תזקפ [?], and beneath it מפמ [?]. Choose the easier solution, for knowing the parts of triangle פתפן [?] will inform you of three parts in each of these triangles.

But if you wish to amuse yourself with the offspring of the triangles, look at the two circles that I drew for you at the end of this Corner — which I was the first of all the bais medrash to discover — and you will see a precious thing, more than pearls: how the angles of the triangles correspond with the sides, all of them bounded on a single sphere by the great circles. So too you will come to the knowledge of their offspring, and the offspring of their offspring, and their transforms and exchanges in several modes. In this there are pleasantnesses and immense benefits, there being many roads to [the same] places.

But the figure which is, to my mind, the faithful one of delights — choicest of the several instruments that the craftsmen devised to solve triangles — is the planar

figure that your eyes behold (יז ציור [?]), which I call the *mirror of triangles*. It varies in its layout according to the change of [the triangle's parts]...

[Source p. 186] ...of the triangles. I will inform you of this only by way of an example, and divide a portion of it among the right-angled triangles of this [kind]. A complete and comprehensive treatment you will see, God willing, in *Sefer Aruchat HaShamayim* [Chukos Shamayim / "The Setting-Out of the Heavens," the author's self-cited work], where you will see the [comparison?] of the oblique within it [?], and every kind of triangle, and deep matters — the circles being divided for its completion and its application.

Now you see that the right-angled triangle אבג is transformed into the oblique גדי [?], and conversely the oblique גדי [?] is turned back into the right-angled triangle אבג. It is very clear that turning the oblique into the right-angled [triangle] eases its solution. Well did the masters of the mathematical sciences say that the right angle teaches a man knowledge in all his concerns; and so too in every craft — great is the benefit those skilled in it derive. The sailors, the builders, and the surveyors, and the bearers of arms, the brothers of the sword, the men of war, stand erect with the sword at their hip. Its praises have already been recounted at length in [Sefer Beit?] [?], and you will see them there.

But the great benefit drawn likewise from transforming the right-angled into the oblique is this: that adjacent to it you find a right-angled triangle which, in this figure, is triangle דיוי [?] or בח [?], and they are three triangles corresponding to גדי [?], as you will see. If you seek their hidden parts according to the rows I gave you in the first Corner, you will find suitable ratios to solve every question concerning the triangle. There is an outlet for the silver and a place for the gold in this figure.

How so? We wish to know the measure of angle אגב from knowledge of the two sides אב, אג. We look at triangle דוו [?]; and behold, we have found side וי [?] [as] the transform of side אב, and side דין [?] [?] is the complement of side אב. We seek in this plane רד [?], which is the measure of the sought angle. According to this, per the tenth [?] row: as the whole sine is to the tangent of the complement of וי [?], so is the sine of the complement of angle דין [?] to the tangent of the complement of די [?] — I mean, as the whole sine is to the tangent of the complement of side אב, the side opposite the sought angle, so is the sine of אג, the side adjacent to it, to the tangent of angle א, the sought angle. We have thus found a new ratio for the first row.

[Second case:] We wished to know, in triangle אבג, side בג from the knowledge of all its angles. In triangle דוי [?], side דן [?] is known, being the complement of [angle] ב, equal to angle [?]; and the plane ב[?] is equal to angle גב [?]. By row 13 [?]: as the whole sine is to the secant of ד [?], so is the sine of the complement of ד [?] to the sine of the complement of side זי [?] — I mean, as the whole sine is to the secant of the complement of angle ב, the angle adjacent to the sought side, so is the sine of the complement of angle א, the opposite angle, to the sine of the complement of the sought side בג. Here is a new ratio belonging to row 16 [?]. So too, by means of this figure, you will find new ratios for all the rows that were in the first Corner, and you will grow rich with ease in the teachings.

Your table is set for you, not from the plunder of wanderings; and the work to complete it falls to you, for I have no leisure now, at this season and time. Is there any number to the bounds of astronomy and the dust of the mathematical sciences? Who can count it?

✻ Yet before I set an end to my words, I will inform you of a portion of the benefits of the teachings of this Corner: how, by way of analysis, you may find ratios other than the ten that were in the first Corner — and they will be for you, in your computations, a strengthening of faith, and, joined with the earlier ones, a stay and a support.

Sod HaYesod — The Second Corner (continued)

A table of thirty-five problems for right-angled triangles, in sixteen rows

Deriving the angle (צדיית הזווית)

Row 1 — from the two sides. From the theorem of Ibn al-Haytham (diagram א) : as the ratio of the whole sine to the sine of פד, so the tangent of בד to the tangent of צה. That is to say — I mean — as the whole sine is to the sine of הפ, so is the tangent of בד to the tangent of צה. But by the eighteenth proposition of this Corner, likewise the tangent of the complement of צה is to the tangent of the complement of בד as the magnitude of angle ם. Hence, equating: as the whole sine of הם is to the sine of פם, so is the tangent of the complement of צד to the tangent of the complement of בד, the magnitude of angle ת. And by transposition:

As the whole sine is to the sine of the side adjacent to the sought angle, so is the tangent of the complement of the side opposite to it to the tangent of the complement of the sought angle.

Row 2 — from the hypotenuse and the side opposite the sought angle. Here, by way of transference, as in the theorem of Ibn al-Haytham and in Sefer Chukos Shamayim and in row 8 of the First Corner: it was explained for triangle אבצ that as the whole sine is to the sine of בפ, so is the sine of בד to the sine of צה. But by the nineteenth proposition of this Corner, likewise the secant of the complement of צד is to the secant of the complement of בד as the magnitude of the sought angle. Hence, equating: as the sine of בם is to the sine of צפ, so is the secant of the complement of צד to the secant of the complement of בד. And by transposition:

As the whole sine is to the sine of the hypotenuse, so is the secant of the complement of the side opposite the sought angle to the secant of the complement of the sought angle.

Row 3 — from the hypotenuse and the side adjacent to the sought angle. Here, from row 15 of the First Corner: as the whole sine is to the tangent of the complement of א — that is, of side הפ — so is the tangent of the complement of בצ, of the hypotenuse, to the sine of בד, the complement, the magnitude of the sought angle פ. And by transposition:

As the whole sine is to the tangent of the side adjacent to the angle, so is the tangent of the complement of the hypotenuse to the sine of the complement of the sought angle.

Another method. From the theorem of Ibn al-Haytham: as the tangent of דה[=רה] is to the tangent of בצ, so is the whole sine of דא[=רא] to the sine of בא, the complement of the angle. And likewise the secant of the angle is to the whole sine, by proposition 15 of this Corner. Hence, equating: as the tangent of דה is to the tangent of בצ, so is the secant of angle ם to the whole sine. By inversion and alternation it follows that as the tangent of בצ is to the whole sine, so is the tangent of דה to the secant of angle פ. But by the theorem of Ibn Sallum [?], as the tangent of בצ is to the whole sine, so is the whole sine to the tangent of צפ. Hence, equating: as the whole sine is to the tangent of צפ, so is the tangent of דה to the secant of angle ם. And by transposition:

As the whole sine is to the tangent of the hypotenuse, so is the tangent of the complement of the side adjacent to the sought angle to the secant of the angle.

Row 4 — from the hypotenuse and one of the angles. Here, from the first row of the First Corner: as the whole sine is to the secant of the complement of side בצ, the complement of the hypotenuse, so is the tangent of side א, the complement of angle פ, to the tangent of angle צ, the sought one. And by transposition:

As the whole sine is to the secant of the hypotenuse, so is the tangent of the complement of the known angle to the tangent of the sought angle.

Row 5 — from one of the angles and the side opposite the sought angle. Here, from row 5 of the First Corner it was explained that as the whole sine of צכ is to the sine of כג, the magnitude of angle צ, so is the sine of אצ to the sine of אב, the complement, the magnitude of the sought angle פ. And by proposition 15 of this Corner it is clarified that, equally, the secant of the complement of angle צ is to the whole sine. Hence, equating: as the secant of the complement of angle צ is to the whole sine, so is the secant of the complement of אב to the secant of the complement of אצ. And by inversion: as the whole sine is to the secant of the complement of angle צ, so is the secant of the complement of אצ to the secant of the complement of אב, the magnitude of the sought angle פ. And by transposition:

As the whole sine is to the secant of the complement of the known angle, so is the secant of the side to the secant of the angle facing it, the sought one.

Row 6 — from one of the angles and the side adjacent to the sought angle. Here, from row 2 of this same lesson it was explained for triangle אבצ that as the whole sine is to the sine of אצ, the complement of side צד, so is the secant of the complement of א, the magnitude of angle ם, to the secant of the complement of angle צ, the sought one. Hence, by transposition:

As the whole sine is to the sine of the complement of the side, so is the secant of the angle adjacent to it [?] to the secant of the complement of the sought angle.

And this is ambiguous, as I made known to you regarding the reasoning of proposition 62 [ס"ב] of this Corner.

Deriving the hypotenuse (צדיית הנשטח)

Row 7 — from the two sides. Here, from row 7 of the First Corner, by inversion, it is explained that as the sine of the complement of the first side is to the whole sine, so is the sine of the complement of the hypotenuse to the sine of the complement of the second side. But from [the theorem of] Ibn Sallum it is explained that as the sine of the complement of the first side is to the whole sine, so is the whole sine to the secant of the first side. And from proposition 19 of the Second Corner, as the sine of the complement of the hypotenuse is to the sine of the complement of the second side, so is the secant of the side to the secant of the hypotenuse. Hence, equating:

As the whole sine is to the secant of the first side, so is the secant of the second side to the secant of the hypotenuse.

Row 8 — from the two angles. Here, from method 2 of row 3 of this same lesson, in the triangle: as the whole sine is to the tangent of א — which is not equal to בד [?] — the magnitude of angle פ, so is the tangent of the complement of יכ — that is, כג, the magnitude of angle צ — to the secant of צם, the hypotenuse. Hence, by transposition:

As the whole sine is to the tangent of the one angle, so is the tangent of the second angle to the secant of the hypotenuse.

Row 9 — from the side and the angle opposite it. Here, from the theorem of Ibn al-Haytham: as the whole sine of פב is to the sine of the perpendicular בד — which is the magnitude of angle פ — so is the sine of פצ to the sine of צה. But this is likewise the secant of the complement of צד to the secant of the complement of פצ, by proposition 19 of this Corner. Hence, equating: as the whole sine of פב is to the sine of בד, so is the secant of the complement of צד to the secant of the complement of פצ. And by transposition:

As the whole sine is to the sine of the known angle, so is the secant of the complement of the side opposite it to the secant of the complement of the hypotenuse. And this is ambiguous.

Row 10 — from the side and the angle adjacent to it. Here, in triangle אבו [?אבצ=] [?], from row 1 of the First Corner: as the whole sine is to the secant of the complement of כ — that is, כג, the magnitude of angle צ — so is the tangent of בא, equal to side צד, to the tangent of the magnitude of צפ [?], angle ו, the hypotenuse. And by transposition:

As the whole sine is to the secant of the angle, so is the tangent of the side to the tangent of the sought hypotenuse.

Deriving the side (צדיית הצלע)

Row 11 — from the hypotenuse and the angle opposite the sought side. Here, from the theorem of Ibn al-Haytham: as the sine of פצ is to פב, the whole sine, so is the sine of צד to the sine of בד. But by the theorem of Ibn Sallum, in the same manner, the ratio of the whole sine to the secant of the complement of פצ; this will, equating, be: as the whole sine is to the secant of the complement of פצ, so is the sine of צה to the sine of בד. And likewise this is also the ratio of the secant of the complement of בד to the secant of the complement of צד, by proposition 19; hence, equating: as the whole sine is to the secant of the complement of פצ, the hypotenuse, so is the secant of the complement of בד, the magnitude of angle פ, to the secant of the complement of צד, the sought side. And by transposition:

As the whole sine is to the secant of the complement of the hypotenuse, so is the secant of the complement of the angle to the secant of the complement of the side opposite it, the sought one.

Row 12 — from the hypotenuse and the angle adjacent to the sought side. Here, from row 1 of this lesson, in triangle אבצ: as the whole sine is to the sine of side אב, the complement of angle פ, so is the tangent of the complement of בצ — which is the tangent of צם, the hypotenuse — to the tangent of the complement of angle א — which is the tangent of side הפ, the sought one. And by transposition:

As the whole sine is to the sine of the complement of the angle adjacent to the sought side, so is the tangent of the hypotenuse to the tangent of the sought side.

Row 13 — from the hypotenuse and one of the sides. Here, from row 2 of this lesson: as the whole sine is to the sine of אצ, the complement of side צד, so is the secant of the complement of בצ — which is the hypotenuse צפ — to the secant of the complement of א, whose magnitude is הפ, the sought side. And by transposition:

As the whole sine is to the sine of the complement of the side, so is the secant of the hypotenuse to the secant of the sought side.

Row 14 — from one of the sides and the angle adjacent to it, which faces the sought side. Here, from row 4 of this lesson, in triangle אבצ: as the whole sine is to the secant of אצ, the complement of side אה [?], so is the tangent of angle צ to the tangent of angle א, the complement of ם[רה=]דה, the sought side. And by transposition:

As the whole sine is to the secant of the complement of the known side, so is the tangent of the complement of the angle to the tangent of the complement of the sought side.

Row 15 — from one of the sides and the angle facing it — that is, adjacent to the sought side. Here, from method 2 of lesson 3: as the whole sine is to the sine of אצ, the complement of side צה, so is the tangent of the complement of אב — which is the magnitude of angle ת — to the secant of the sought side. And by transposition:

As the whole sine is to the sine of the complement of the side, so is the tangent of the angle to the secant of the sought side. And this is ambiguous.

Row 16 — from the two angles. Here, from row 6 of this lesson [?]: as the whole sine is to the sine of the complement of אב, the magnitude of angle פ, so is the

secant of angle צ to the secant of the magnitude of the complement of side א [הפ=]. And by transposition:

As the whole sine is to the sine of the first angle, so is the secant of the second angle to the secant of the side opposite it. (Diagram [ציור יח] יח)

So here you have, for all the triangle-problems, new ratios that have come out for us from interchanging the angles with the sides and the sides with the angles. And you will find many more like these — in which the whole sine does not stand at the heads of the ratios, or in which it is not found at all — by many long methods. It is not my intention to write in this abridgment every one of the procedures, but only a few choice lessons: blind men made to see, the heavy-hearted made glad, sweet to the students as a fragrant aroma, so that all who behold them will recognize them as "seed that HaShem has blessed" (cf. Yeshayahu 61:9). And you — combine them with the rows of the First Corner, and they will be in your hand as one; and all your words will be established and confirmed, as they are tested "by the mouth of two witnesses" (Devarim 19:15).

The Third Corner (הפנה השלישית)

On oblique triangles, and it has three building-stones [literally: three hewn stones].

Said the Respondent: After having set out the solution of right-angled triangles, I shall now come to the solution of oblique ones — not because they require the right-angled ones, for it is possible to solve them properly without any transposition (which I have demonstrated elsewhere), but because, as the words of "the Upright Philosopher" [the geometer] have it, the straight judges both itself and the crooked; hence the straight is prior to the crooked. Added to this is that, as I have said, I follow your questions, and "what your lips have uttered I will not change" (cf. Devarim 23:24): for according to the order of instruction it would have been fitting to set out the solution of rectilinear triangles first, on account of their ease. Yet perhaps they came last in the sequence of your inquiries because of their dearness and their great usefulness.

The first building-stone is for triangles in which two angles, or their legs, are equal. It is called, for its ease, "the crawling stone" [אבן הזוחלת], because the student runs and skips along in solving these oblique ones "like a young hart" (cf. Shir HaShirim 2:9)…

the *Elim* are sixty-two in the heavens, and it has twelve rows [columns].

Given two angles and the side adjacent to one of them

Row 1 — to find side AB, the one adjacent to the second [angle].

In triangle ABG [figure (9) ט]: since the angles at B and at the base are equal, its two legs AB and AG are likewise equal. Therefore side AB is known, and it is easily grasped by the discerning mind; there is no need for proportion or computation.

Row 2 — to find side BG, the side that lies between the equal angles.

Drop the perpendicular AD [figure (10) י]; it will cut the triangle into two halves, in the manner taught above for the Second Corner. Then, by Row 12 of the First or Second Corner in the right triangle ABD, we find side DB; its double is the sought side BG.

Given two angles and the side between them

Row 3 — to find side AB, adjacent to one of the angles.

Drop the perpendicular AD; it will cut the base BG into two halves. By the tenth Row of the First and Second Corner, we find side AB or AG, which are equal to one another.

Given two sides and the angle adjacent to one of them

Row 4 — to find the third side.

Drop the perpendicular AD; it bisects the sought side BG. By Row 12 of the First or Second Corner we shall learn the magnitude of BD, and its double is BG, the sought side.

Row [5] — to find the third side.

Drop the perpendicular AD; it bisects the sought base BG. By Row 11 of the first Corners we find BD, and its double is BG, the sought side.

Given all the angles

Row 6 — to find all the angles.

By dropping the perpendicular AD, it cuts angle BAG and the base adjacent to it into two halves. By Row 16 of the First or Second Corner you will find the magnitude of BD, and its double is the sought BG. By Row 8 of the two preceding

Corners, however, you will find the measure of side AB, and side AG is equal to it. This is what we meant.

Given two sides and the angle adjacent to one of them

Row 7 — to find angle B, adjacent to the second side.

This is known from what was taught for the Second Corner, since the triangle is isosceles, and there is no need to labor over it.

Row 8 — to find the angle.

By dropping the perpendicular AD, it bisects the base BG and angle BAG. By the sixth Row of the preceding Corners you will know angle BAD; its double is angle BAG, the sought one.

Given two sides and the angle between them

Row 9 — to find one of the equal angles.

By dropping the perpendicular AD, it bisects angle BAG and the adjacent base. By the fourth Row [of the preceding Corners] you will know [angle B], which is equal to angle [?]. This is what we meant.

Given two angles and the side adjacent to one of them

Row 10 — to find the third angle.

By dropping the perpendicular AD, it bisects [the angle] and the base BG adjacent to it. By the fourth Row of the preceding Corners you will know the measure of BAD, and its double is the sought BAG. [reading of the final label uncertain — printed בא"; [?]]

Given two angles and the side between them

Row 11 — to find the third angle.

By dropping the perpendicular AD, it bisects angle [BAG] and the base BG. By the fifth Row of the preceding Corners we find BAD, and its double is angle BAG, the sought one.

Given all the sides

Row 12 — to find any of the three angles.

Drop the perpendicular AD; it will bisect angle A and the base BG. By the second Row we find angle BAD, and its double is BAG. By the third Row, however, we find angle B, which is equal to angle [G]. This is what we meant.

The second stone applies to oblique triangles, one of whose sides is a quarter-circle [90°]. It is called **the Helping Stone** [אבן העזר] — a great gateway to easing the computation — namely, when one of the sides is a quarter [90°], as you will observe in the figure on plate 49 [figure (10) י] and in what we shall now explain.

I wish to draw your ear to this: at times the sought triangle has a right angle that is not known to us, and the computation reveals its hidden things. It would therefore be more fitting to call this section "the triangles in which no right angle is known," rather than to call it "the obliques." Since the matters are easy and time is short, I shall not be exacting with examples. So do not look askance at the computation if it is not exact, for I have brought these illustrations only by way of example; in *Arukhot HaShamayim* [Roads of the Heavens] I shall bring the matters exact and precise, with God's help. This stone has ten rows.

In the figure before you [figure (10) י] you see several triangles, all of which have a quarter-side, that is, 90 degrees. They are: ABG [אבצ], AFG [אפצ], ASG [אשצ], AKG [אקצ], AHB [אהב], AHF [אהפ], and the rest of them, with their complements and their magnitudes set out before you; from a single viewing you will discern the mountain. But this much I will let you know: when you observe, you will find that all the triangles having a quarter-side vary considerably, in the manner set out in *Arukhot HaShamayim* — that is, when three are known and we seek the three unknowns.

For if the known elements are two angles, one of which is adjacent to the quarter, this falls under four cases. (1) In triangle ASD [אצד], the quarter-side is AS [אצ] and the known angles are S [צ] and B [ב], both acute.

(2) In triangle ASM [אצם], S [צ] is acute and M [ם] obtuse. (3) In triangle ASB [אצב], A is obtuse and M [ם] acute. (4) In triangle ASS [אצש], A and S [ש] are obtuse.

And if the known elements are two angles surrounding the quarter, this falls under three cases. (1) In triangle AFG [אפצ], S [צ] and A are acute. (2) In triangle ABG [אבצ], S [צ] is acute and A obtuse. (3) In triangle ASG [אצש], angle S [צ] and S [ש] are obtuse.

And if the known elements are all the sides, this falls under three cases. (1) In triangle AFG [אפצ], both are less than a quarter. (2) In triangle AFG [אפצ], BG is in excess and AF less than a quarter. (3) In triangle ASG [אצש], both are in excess.

And if the known elements are the two sides and the angle between them, this falls under four cases. (1) In triangle AFG [אפצ], S [צ] is acute and side FG [פצ] less than a quarter. (2) In triangle ABG [אבצ], S [צ] is acute and side BG [בצ] in excess. (3) In triangle ASM [אצם], S [צ] is obtuse and side SM [צם] less than a quarter. (4) In triangle ASG [אצש], S [צ] is obtuse and side SG [צש] in excess.

And if the known elements are the two sides together with the angle adjacent to one of them, this falls under six cases. (1) In triangle ABG [אבצ], B is acute and side AB less than a quarter. (2) In triangle AFG [אפצ], [F] is obtuse and AF less than a quarter. (3) In triangle AKG [אקצ], K [ק] is acute and AK [אק] in excess. (4) In triangle ASG [אשצ], S [ש] is obtuse and AS [אש] in excess. (5) In triangle AFG [אפצ] [or A?], AS [or AB] less than a quarter. (6) In triangle ASG [אצש], S [צ] is obtuse and AS [אש] in excess. [cases 5–6 partly abraded; readings [?]]

I will not be lengthy with these easy matters; I shall merely bring you illustrations and explanations across ten rows.

Row 1 — from a side less than a quarter and an angle adjacent to the quarter; this latter may be either acute or obtuse.

In triangle ABG [אבצ], side AG [אצ] is a quarter, side AB = 68°37′, and angle ABG [אבצ], the one adjacent to the quarter, = 68°27′. We wish to find side SB [צד] and the remaining angles BAG [באצ] and BGA [בצא].

We extend side AB to D so that AD becomes a quarter. Then in the right triangle BDG [בדצ] the following are known: side BD, the complement of side AB, and angle DBG [דבצ], the remainder of angle ABG. By the tenth Row of the First Corners we find BD — or, in the adjacent triangle BDH [בדה], the hypotenuse HB [הב] = 50°46′ — and its remainder BG [בצ] = 10°133′ [minutes value > 59; transcribed as printed], and angle SAB [צאב]. From Row 10, I mean we find side DG [דצ], which is the measure of [angle] DH [דה], its remainder being 24°43′; and so it is 137°17′, and angle ASB [אצב] is found in triangle BDG [בדצ] or BDH [בדה] adjacent to it. From the fifth Row, that is, the angles at BD and BH [בצד and בחד], which are equal to one another, = 30°; and we subtract it from the right angle ASD [אצד], leaving angle ASB [אצב] known = 60°. This is what we meant.

Row 2 — from a side in excess and an angle adjacent to the quarter; this falls under two cases, either acute or obtuse.

In triangle ABG [אבצ]: AG [אצ] = 90°, SB [צב] = 10°133′ [as printed], ABG [אבצ] = 68°27′.

We wish [to find] side AB and angles SAB [צאב] and BGA [בצא]. We subtract from side SB [צב] the excess over the quarter, SD [צז], leaving arc DB [זב] = 43°10′. In triangle ABD [אבז], a right triangle, the angle ABD [אבז] = 68°27′ is known, and the arc DB [זב], the complement of the said side, = 43°10′. By the tenth Row we find the hypotenuse AB = 68°37′. Likewise we find angle BAD [באז] from the fifth Row = 47°17′, and we add it to the right angle SAD [צאז], yielding angle SAB [צאב], the sought one, = 137°17′. Likewise we find angle ASB [אצב] — that is, its measure, which is side AD [אז] in triangle ADB [אזב] — by Row 12; and it is 60°. This is what we meant.

Row 3 — from a side less than a quarter and the angle adjacent to it, acute.

In triangle ABG [אצב], side AG [אצ] is a quarter, AB = 68°37′, ASB [אצב] = 60°. We wish to find side BG [בצ] and angles SAB [צאב] and BAG [אב and יב — printed אוב; the two given angles]. We extend side AB up to D so that AD becomes a quarter. In triangle BDG [בדצ] — or, adjacent to it, BDH [בדה] — the right-angle data are known: angle DBH [זיה or דבד; printed זיה — [?]] and side AB; angle A and angle B [?]. Side BD, the complement of AD, is known = [?]; and side DG [דצ] is known = 23°21′ [reading uncertain — printed 23.21, but compare the parallel value 24°23′; [?]]. By Row 9 we know the hypotenuse [SB or HB], its remainder BG [בצ] being the sought side, which is doubtful by the nature of that Row: 49/11 [or 46/133]. Likewise we find angle A — that is, arc DG [דצ], its measure in triangle BDG [בדצ] by Row 15 — and it is doubtful: 42/147 [?]. Likewise we find angle B — that is, its remainder, the arc SBD [צבד], by the sixth Row — and it is: 69/111 [?]. This is what we meant.

Row 4 — from a side in excess and the angle adjacent to it, obtuse.

You wish [to find] side AB and angles ASB [אצב] and ABG [אבצ]. In triangle ASB [אצב]: AS [אצ] is a quarter, SB [צב] = 10°133′ [as printed], BAG [באצ] = 137°17′. We subtract from the excess SB [צב] the quarter SD [צז], leaving arc DB [זב] = 43°10′. Likewise from the obtuse angle BAG [באצ] we subtract the right angle DAG [זאצ], leaving angle DAB [זאב] = 47°17′. In the right triangle ADB [אזב], the angle BAD [באז] is known and the side adjacent to it DB [זב]; and we find the hypotenuse by Row 9 — and it is doubtful: 37/28 [?]. Likewise we find angle S [צ] — that is, arc AD [אז], its measure — by Row 16, and it is: 60/120 [?]. Likewise we find angle B — in triangle ADB [אזב] by the sixth Row — and it is: 37/33 [or 69/111; superscript/subscript partly abraded — [?]]. This is what we meant.

Row 5 — from the acute angle between two sides; this falls under two cases, the side being either less than a quarter or in excess.

In triangle AFG [אפצ] — or AHB [אבח] — where everything is one: AG [אצ] = 90°, SF [צם or צף] = 50°46′, ASF [אצפ] = 60°. You wish to find side AF and angles SAF [צאפ] and FAS [צפא]. In the right triangle SDF [צדפ], angle FSD [פצד] = 30°; and from Row 11 we find side DF [דף] = 23°21′, and its complement AF likewise = 68°37′. Likewise we find angle SAF [צאפ] — that is, its measure SD [צד] — by Row 12 = 42°43′. Likewise we find by Row 4 angle DFG [רפצ] — and it is 68°27′ — and we subtract it from 180 degrees, leaving angle SFA [צפא] = 111°33′. This is what we meant.

Row 6 — from the obtuse angle between two sides; the side will be either less than a quarter or in excess.

In triangle ABG [אבצ]: AG [אצ] = 90°, BA [בא] = 68°37′, SAB [צאב] = 137°17′. We wish to find side BG [בצ] and angles ASB [אצב] and ABG [אבצ]. In triangle BDG [בדצ], a right triangle — or BDH [בדה] adjacent to it — side DB [דב], the complement of AD, is known, and side DG [דצ], which is the measure of SAD [צאד]; and from the seventh Row we find the hypotenuse SB [צב] — or HB [קב] — = 70/44 [?]. Likewise we find angle DGB [דצב] in triangle BDG [בדצ] from the first Row, and it is 30°; we subtract it from ninety, leaving angle AGB [אצב] = 60°. Likewise we seek angle ABG [אבצ] — that is, knowledge of its complement — and it is found = 68°27′. This is what we meant.

Row 7 — from the acute angle near the quarter and the angle adjacent to it; this may be acute, obtuse, or [?].

In triangle ABG [אבצ]: AG [אצ] = 90°, ABG [אבצ] = 68°27′, ASB [אצב] = 60°. We wish to find angle BAG [באצ] and sides AB and BG [בצ]. In triangle SBD [צבד] — or BDH [בדה] adjacent to it — angle DSB [דצב] is seen [= 30°]; and from Row 16 we find side DH [הד] = 42°43′, its remainder being 137°17′, the measure of angle BAG [באצ]…

and from that Row 17 [טור י"ז], side **בך** [BK] of **21. 23** [21°23′] is found. The completion — side **בא** [BA] of 37 is found, and likewise from Row 5 [מטור ה'], the side completed. The nashtach [hypotenuse] **הב** [HB] of **50. 46** [50°46′] is found; the remainder **בץ** [BTz] of **33. 10** [10°33′] — the side sought, which is what we wanted to show.

Row 5 [הטור ה']: from the larger angle adjacent to the quarter[-circle], and [?] facing it [?] near [it], or the acute [angle], or the obtuse [angle] [צרה או רחבה].

Here, as in triangle **אבצאצ** [ABTzATz][?] 90, side [?]… angle **אצב** [ATzB] and side **אב בצ** [AB, BTz], as in triangle **צדרב** [TzDRB][?] 68. 27, and **באצ** [BATz] 137. 17,

you will [?]… side **דרב** [DRB] of known angle, angle **צבא** [TzBA] known, and side **צבר** [TzBR][?] remainder… angle **ב,** [B] is known, and from Row 6 [ומהטור השׁשׁי] the sixth, angle **צ** [Tz] 60 is found, and likewise… and **60** likewise side **אב** [AB] is found, from Row 9 [מטור ט'] [r.l. ר"ל = i.e.] [?], its completion **דב** [DB] of [?]… and from it we know its like; and likewise from Row 9 [מטור ט'] the nashtach is found… the nashtach **בץ** [BTz], and these are doubtful [מסופקים], and this is what we wanted to show.

Row 9 [הטור הט']: from all the sides, when the two [sides] are less than [?]… or exceed.

Here, as in triangle **אפצ** [APTz][?] or **אבה אה** [ABH AH][?]… **בה** [BH] 90, **בא** [BA] 46. 50, **בא** [BA] 37, **68**… angle, from Row 13 [מטור י"ג] we know [r.l.] side **הך** [HK] [?], and it is the measure of the angle… and its completion is **אהב** [AHB], the one sought. And likewise from Row… angle **ב,** [B] [is] known, and from Row 6 the sixth, angle **צ** [Tz] 60 is found, and likewise… and its completion **אבה** [ABH] 111. 33 [?], and this is what we wanted to show. **דב** [DB][?]… we know its like, and likewise angle **ט** [T] of Row [?] is found, the nashtach is found… [an excess and a deficiency].

Row 9 [הטור הט']: from all the sides, when one exceeds and one is deficient.

Here, as in triangle **אבץ אצ** [ABTz ATz][?] 90 **בצ** [BTz]… **אצ** [ATz] 90, **בצ** [BTz] excess, **אב** [AB] deficiency, as in triangle **בדרץ** [BDRTz][?]… side **דץ** [DTz] of Row 13 [מטור י"ג], and it is the measure of angle **באצ** [BATz] and angle [?]… angle **אבצ** [ABTz] and angle **בצד** [BTzD] [?] and it is the comple[tion]… angle **אבצ** [ABTz] and angle **בצד** [BTzD][?], and it is the remainder of Row 2 [מהטור הב'], and it is the remainder… and it is the completion **אצב** [ATzB], and this is what we wanted to show.

This suffices to clarify these triangles; and reflect on the [figure that is at the end of this book] [?]… (Figure 20 [ליו"ר כ]) [?], for it is [the figure] that pertains to [?]… [Reflect upon the figure that is at the end of this book; from its lines what you need will be derived.]

Diagram referenced (Figure 20, ליו"ר כ). The author refers the reader to a numbered plate ("Figure 20") printed at the end of the book, not reproduced on this text page. It illustrates the oblique-triangle constructions of the Helping Stone (right-triangle reduction of triangles having a 90° side). No labels are legible on this page; the plate itself is on a separate sheet. [Figure on separate plate.]

The Third Stone — He Called It "A Precious Stone for a Suffering Mouth" [האבן הג' תקרא אבן יקרה לפום צערא]

[Aramaic: לפום צערא — "according to the suffering / effort," echoing the Mishnaic לפום צערא אגרא, "the reward is according to the effort" (Avot 5:23).]

I have already explained that oblique triangles can possibly be resolved without [recourse to] the angle, and in *Sefer Aruchot HaShamayim* I explained this by the inverse path — toward the poles of the angle, since there is no [other] way [?]… For this Stone, [I give] twelve rows [י"ב טורים]; and by my choice, for brevity's sake, I will not [give all of them]… Hear, then, from my chosen youths [?] — the comprehensive sage, the honorable Master and Rabbi [כמהר"ר] Moshe [?]; and he…

[Margin / partial line, illegible: "…"]

Resolving the Third Angle [צדיית הזוית השלישית]

Row 1 [הטור הא']: from the two remaining angles and the side parallel to one [of them] [?].

(Figure 21 [ליו"ר כא]) — that is the case. The two known angles **ב** [B] are known [?]… A second [angle] we know; where will the perpendicular **אך** [AK][?] fall? We posit that it [?]… falls to side **בצ** [BTz], which lies opposite, before the triangle, and it divides [the triangle]… **ארדב** [ARDB][?] and **אדב** [ADB] [?]. And it being like the triangle **ארדץ** [ARDTz][?], another angle [?]… angle **צאך** [TzAK][?] is found, and it is the first found, and from Row…

Diagram referenced (Figure 21, ליו"ר כא). Referenced plate (Figure 21) at the end of the book, illustrating the resolution of the third angle of an oblique triangle by dropping a perpendicular (AK) onto the opposite side (BTz), splitting the triangle into two right triangles. Labels are on the separate plate, not on this text page; readable point-letters cited in the prose include א, ב, ך, צ, ד, ר. [Figure on separate plate.]

as in triangle **ארב** [ARB] of 50 [?] [נ'], and from Row 5 [ומהטור ה'] is found… the one found first, you know the whole angle **באצ** [BATz], the one sought… it is [?], and it could be acute or obtuse [צרה או רחבה]. Only if… the first ascends to 150 [לק"ך = ל30+ק100+ך20] degrees, which is not appropriate [שאוי], it is not acute… as we explained in our Talmudic [demonstration] [?], from a second corner. **But if**… the two known angles are equal in quality [?], two corners [כני] [?]… the hidden one toward side **דץ** [DTz], the side that lies opposite, falls

outside… side **ארץ** [ARTz][?] known, the nashtach **אצ** [ATz] and angle **צ** [Tz], the hollow [?] [נחקור]… first, and from it [?] the nashtach **היא** [HYA][?] is found, the perpendicular **אך** [AK][?] [is found], and they will be… of 50 [?] [נ'], that is [r.l.], the perpendicular **אך** [AK]; and the remainder of angle… as found, the first [r.l.] is the angle **דאך** [DAK][?]…

as in triangle **אדב** [ADB] [?], 50 [?] is found, side **באצ** [BATz][?] and when it combines with angle **צאך** [TzAK][?]… **באצ** [BATz] [is] the one sought, but it is doubtful [מסופקת], as is the nature of the row… Only if the gathering from this wider angle, and there is found… is not wider, for no angle ascends to 150 [?] / 120 [?] [the print reads 120 = ק"ך, but the parallel context favors 150 = לק"ך; the leading ל is lost in the destroyed gutter] degrees, when… **But if** the two known angles are of equal quality… two corners [כני][?], let the perpendicular **אך** [AK] [?] fall — the column **אר** [AR][?] from angle **א** [A][?]… falls outside the triangle, and being as in triangle [?] **צב** [TzB][?]… angle [?] **,** we know the hollow [נחקור] from Row 5 [מהטור הר'] [?]. Angle **צאך** [TzAK][?] and it is the one found… **אך** [AK] and they will be from here, as in triangle **ארב** [ARB] standing [נצב] the angle… [the] remainder angle **ב** [B], and from Row 6 [ומהטור הששי] the sixth, angle **דאב** [DAB][?] is found… from angle **דאצ** [DATz][?], and the remaining angle **צאב** [TzAB][?], which is doubtful [מסופקת]…

The nature of the sixth row [טבע הטור הששי].

Row 2 [הטור הב']: from the two remaining angles and the side that lies between them [שביניהן].

Angle **א** [A], angle **צ** [Tz] are known with side **,** [?]… It is of doubtful nature, for if you wish to drop the perpendicular [מא][?]… to **בצ** [BTz] which lies opposite, we do not know how it [?] will fall — a matter that is [doubtful]… [whether] the second angle [r.l.] will fall inside or whether it is uncertain… [whether] the perpendicular [column] falls [inside the triangle], and you need not posit anything at all that [?]…

And here reflect on the gathering of the two known angles: necessarily it will be [?]… by necessity the angle of the third corner. The Third Angle exceeds [or is less]… [if] the column falls, [the case is] one of the known [Rows], that is, 48 [מ"ח]. **But if** they are gathered [קבוץ][?]… perhaps the third angle is acute, or wide, or upright [צרה או רחבה או נצבה], or… by means of the [device/perpendicular] [תחבולה]. We posit a second [angle] of equal quality **ב** [B], the hidden one… angle **צ** [Tz] and the nashtach **אצ** [ATz], we seek angle **א** [A] [?] which we already found… edge [שיפה][?], when we posited that the triangle is standing [נצב], angle **ב**

[B], and we have already found… angle **א** [A] is wider [רחבה] than the measure [?], the figure is illustrated where the angle… falls outside the triangle, and if it [the third angle] is found smaller than these [?]… we posit that the column falls inside the triangle when [?]… triangles standing [נצבי] the angle **ארב** [ARB][?], **ארץ** [ARTz][?], and behold it has emerged [?]… the experiment [הנסיון] from angle **באצ** [BATz] the large one [הגדולה], from it, and there remains… the column **אך** [AK] is found, as in triangle **ארץ** [ARTz][?], and from there as in… angle **ב** [B], the one sought. **And we posit** a second [case], that the column falls [?]… and there will be for us two triangles standing [נצבי] the angle **ארץ ארב** [ARTz, ARB][?]…

side **באצ** [BATz][?] of known measure, and from Row 11 [ומהטור הי"א] we find **ג"כ** [also] [?]… the smaller angle on the right [הקטנה תימנה], and the side **באר** [BAR][?] is known. As in triangle **ארץ** [ARTz] side **אר** [AR], and these are like the triangle **ארב** [ARB] of Row [?]… and it is the remainder of angle **ב** [B] in the [sought]…

A Different Method [דרך אחר]

But if you recall the second part [of the work], from the second continuation [?], a turn [מפיה] [?] second [?] — every… [in] a manner suitable to find the named angle that we [?]… [the matter] turns as I explained there, where I found 22 [כ']… the dark intersection [החתוך החושך] facing themselves [?], **בד דץ** [BD, DTz][?], and they will be for you as in triangle, standing [נצב] the angle **אבד** [ABD] [?] known [r.l.]… side **בד** [BD] and angle **באד** [BAD][?], and from Row 6 [ומהטור הששי] the sixth, the angle **אבד** [ABD][?] is found, angle **ב** [B] of the sought.

[The "Different Method" line continues:] As in the Eleventh Row [כפי הטור הי"א] we find 23 [נ"ג][?] [r.l.] **אב** [AB], as in the Fifth Row [כפי הטור החמשי] the side **אבר** [ABR][?] of known angle **אבד** [ABD][?] is found, angle **ב** [B] of the sought.

Resolving the Right Angle Not From the Known Sides [צדיית הזוית הנכחית לא' מהצלעות הידועים]

Row 3 [הטור הג']: from the two sides and the angle parallel [opposite] to one of them [המקבלת לאחד מהם].

As in triangle **אבצ** [ABTz], the sides **אב אצ** [AB, ATz] are known, the side **אב** [AB] and angle **ב** [B][?]…

The Head Stone [אבן הראשה] [is] valued at a quarter [?] [בערך בקע][?], side **אצ** [ATz] to… to a quarter [בקע][?] angle **צ** [Tz] the one sought, and you find the reflection [תכף] of the angle, and the angle [?]… we choose by two [of] these from the division, drop a perpendicular from the hidden angle… [the] column **אך** [AK]

(Figure 12 [ליו"ר יב]) and it falls inside the triangle if angle **ב** [B] [?]… is of acute kind [מין אתר][?]; therefore, for as long as we do not know the quality [איכות]… and the matter is doubtful [מסופק]. **And we posit** that it is of its [the angle's] kind, and the column falls… angle **אצך** [ATzK][?], as in Row 13 [כפי הטור הי"ג], the column **אך** [AK] is found, and… as in the second Row [כפי הטור השני], angle **צ** [Tz] is found, the one sought. **And we posit** [an alternative]… **ב** [B], and the column **אך** [AK] falls outside the triangle, and behold from Row 11 [מטור י"א]… side **אך** [AK], and then [as in] Row 2 [מטור ב'], as in triangle **ארץ** [ARTz] is found… the one sought. And from here it is explained that for as long as we do not know… the column **אך** [AK], and therefore the angle that is found…

Diagram referenced (Figure 12, ליו"ר יב). Referenced plate (Figure 12) at the end of the book. It shows triangle ABTz with a perpendicular (column AK) dropped from the hidden vertex; whether AK falls inside or outside the triangle depends on whether angle B is acute or obtuse, which governs the doubtful (מסופק) determination of the sought angle Tz. Point-letters cited in the prose: א, ב, ר, ך, צ. Figure on separate plate.

But there are some details and refinements by means of which a [resolution] is sometimes reached, [in order] to know the quality of angle **צ** [Tz]:

(a) **First [א']:** for instance, if angle **צ** [Tz] is found to be a right angle [נצבת]. (b) **Second [ב']:** if… it is so that the two angles are also equal [שוות]. (c) **Third [ג']:** if the angle opposite [המקבלת] [?]… the second, which is opposite to the larger side, will be acute [רחבה]; that is [r.l.]… [if it is] opposite the smaller side, it will be acute [חדה]. (d) **Fourth [ר']:** if the angle opposite the larger side… is sharp [חדה]. (e) **Fifth [ה']:** if the side adjacent to the acute [obtuse] angle [הסמוך לזווית הצרה]… exceeds over the quarter [רביע] and the opposite to it is not less than the remainder [?]… of the remainder of the side adjacent to it, the sought angle will be acute [צרה][?]…

And the demonstration [המופת]: it is that if angle **ב** [B] is acute [צרה] and side **אב** [AB]… the remainder of side **אב** [AB], for it is **אך** [AK], we say…

אך [AK], and according as it is parallel [opposite] to angle **ב** [B], the acute [angle], it will be… **אצך** [ATzK] acute [angle], and its remainder…

And the inverse [ולהפך]: if the side adjacent to the wider [larger] angle… less, the remainder of the side adjacent to [it] will be… [the demonstration]:

And the demonstration [המופת] is that if side **ב** [B] is wide [רחבה] and side **אב** [AB]… side **אב** [AB], the angle **צ** [Tz] [will be] acute [צרה], according to [?]… the

right [angle] **ב** [B] is the wider one, and it being the excess that is… that it is the excess, also angle **אצך** [ATzK], its right is wide and will be… so too the angle **אצב** [ATzB] is acute [צרה].

But as it stands [וכשיהיה], the triangle is in another condition [בתכונה אחרת], aside from those mentioned… these five, which I explained in *Sefer Aruchot HaShamayim* [ספר ארבות השמים][?], it is doubtful [מסופק], and the figures [התכונות] [are] the doubtful ones… and *Sefer Aruchot HaShamayim* as [stated], and I have no leisure to expand [ואין לי פנאי להאריך].

Resolving the Angle That Lies Between the Two Known [Sides] [צדיית הזוית שבין הצ' הידועים]

Row 4 [הטור הד']: from the two sides and the angle opposite [הנכחית]…

As in triangle **אבצ אב אצ** [ABTz; AB, ATz][?] known. **Here** we drop from angle **א** [A] the sought… side **אך** [AK], and it falls inside if angle **צ ב** [Tz, B][?] is as the figures [כדמות][?] in their quality, as in Row 11 [ומטור י"א]… side **אך** [AK] outside the triangle, [for example] [r.l.]… angle **אצך** [ATzK]. **A second [case]:** from Row 3 [מטור ג'] is found… the side **באר** [BAR][?], for [שקכוצם][?] it is the measure of side **אצ** [ATz] the sought… in their quality, the column **אך** [AK] falls inside the triangle [?]… column **אך** [AK], and afterward from Row 3 [מטור ג'] is found, of Row 2 [ב'] the dark [intersection][?]… and there remains side **צאב** [TzAB] the sought, known. And from here…

Row 2 [הטור ב'] the hidden one — we do not know how the perpendicular [column] falls, and therefore… the [hidden] parts of the row that pass according to the rules stated.

A Single Resolution From the Two [angles] [צדיית אחת מהב' זויות]

Row 5 [הטור הה']: from the two angles and the angle between them [והזוית שביניהן].

[Margin / partial line, illegible.]

sides **אב צב** [AB, TzB] [?], angle **ב** [B], and their [intermediate], the sought is angle **צ** [Tz]. **Here**, as you lengthen [the search] in the second Row from this Stone, perhaps it is upright [נצב]… you will find the reflection [תכף] of the third angle. **For** the second [angle] of the angle… as if it were upright [נצבת], and you seek the measure of the side adjacent to it… as it is given [נחון] [?], behold the angle is

upright [נצבת], and if [it is] larger… **But** as you already knew the quality of the angle, drop [the perpendicular]… the sought, on the side adjacent to the sought angle [r.l.]… it will fall inside, [for example] if **בצ** [BTz] are alike [דומות] in their quality, and behold as…

[End of text block on this opening. The detailed case-analysis of oblique-triangle resolution (the Third Corner's third, fourth, and fifth stones) continues.]

the eleventh row (טור הי"א), as in the triangle ארב [A-R-B], the perpendicular אד [A-D] is found, and… [gutter loss]. By way of [the eleventh row] the perpendicular fell more on side צד [Tz-D]. And… [gutter loss] …angle צ [Tz], the one sought. And if angle א [A] be sought, [it is reconstructed]… [gutter loss], and it works just as in the seeking of angle צ [Tz]. But when the angles…

[Outside the triangle, by way of example, upon side בצ [B-Tz] adjacent to…] [gutter loss]. For the triangle, by the twelfth row (טור י"ב), is found in the triangle ארב [A-R-B], side… [gutter loss]; their signs [are] צב [Tz-B] and more than דצ [D-Tz]. Again, by row [1] א is found אצד [A-Tz-D]… [gutter loss].

And if you seek angle א [A], the perpendicular falls outside side… [gutter loss] and upon it returns…

Resolving One Side from a Triangle (צדיית אחת משלש) [section heading; remainder of the rubric runs off into the gutter: "…משלש זויות" — "…of a triangle of [known] angles"]

The row that is for the knowledge of the three sides (הטור הו' מידיעת שלשת הצלעות).

Now, this row varies according to six modes (ששה אופנים):

1. That the triangle is equilateral (שוה הצלעות), or…
2. That one of its sides is a quarter [90°] and the second is its complement (עורף).
3. …These have been explained under [the section] "The Helping Stone" (אבן העזר).
4. That two of [the sides] are complements (עורפים).
5. That the second of them is deficient (פוחת).
6. That one is a complement and one is deficient.

[the matter explained] here as before.

And behold, by two methods (בג' דרכים — read as "by these methods") the [resolution of the] aforementioned triangle is repeated, according to our opinion. In this compendium [we explained that the] triangles, after [their angles] are known [or "untied"], are resolved in various ways without…

[They are] the oblique [triangles] when they are converted into right [triangles] (לנצבים), since in their place a [gutter loss]… is reversed, and with great ease, with the help of Heaven (בעה"ה — בעזרת השם).

The first method (הדרך הראשון) is: as I taught you in the learning [section] ס"ז וסח"פ' [referencing earlier rows/lessons §67, §68, and §…], the second corner (מפנה שניה)…

The twelfth row (הטור הי"ב), from this corner, in length, with the help of Heaven, from the third row (מהג'); the angles…

they are the measures of the angles of the triangle that is sought; and the matter is reckoned…

And the second method (והדרך השני) is that one seeks the chord [or "computed value"] (החותך החושבת) that…

in part [1] א of [the chord] is drawn [as] [2] ב, as in the lesson ס"ה of corner ב [§ 65 of corner 2 — gutter loss]…

the knowns: we drop the perpendicular עמוד אד [A-D], and the [angle/chord] reckoned at בצ [B-Tz]… [gutter loss]…

the perpendicular to descend, and as taught in the lesson [63§] ס"ג, know how the two [parts] are…

known, and as the demonstration shows, the rows [or "edges"] of corner א [the first corner], or the second…

The third method (והדרך הג') is what has been explained, by analogy, and by way of example [it] will be…

in בפ [B-P] not at all equal, and the second of them deficient — a quarter [90°] — how that…

the two sides up to אה [A-H], by quarter-circles (רביעי עגולים), and let a compass-pivot (ציר) be made…

the arc אה [A-H], and one stays [the compass] until it meets the perpendicular dropped…[gutter loss]…

the greater side, as it is, as in our example side ב…[gutter loss]…

we directed it until they meet on the other side, and there will then be two angles א [A] and ה[H]…

large quarter-circles, and according to this, our triangles are דאצ [D-A-Tz], פר… [gutter loss]…

as a model (כמופת) for the lesson of the first stone (אבן הראשה), the way crossed over, since already [gutter loss]…

צף [Tz-F] to bisect (לבקע) פה [P-H], and by exchange and reversal, the value [is] also as…[gutter loss]…the value, [the value of] arc דא [D-A] is to [arc] בקע…[gutter loss], to bisect פצ [P-Tz].

But the value of arc דא [D-A] is to arc… [gutter loss], so too the value of the sine of arc רצ [R-Tz]…

the known sides; and according to this, also the value [of] the sine of דצ [D-Tz] is to the sine of פצ [P-Tz], which [is]…

[arc] דצ [D-Tz] and [arc] פצ [P-Tz] is known — that is to say, the measure of the sine of side פד [P-D], according to the [gutter loss]…

the first [method] for the lesson of [65§] ס"ה — the measure of the sine [of] רצ [R-Tz] and פצ [P-Tz] is found…

all this. Now, in the standing triangle (כמשלש נצב), the angle דאצ [D-A-Tz] [is] known…[gutter loss]…

and by row [13] י"ג is found side אצ [A-Tz]; and likewise in the standing triangle (כמשלש נצב)…

by row הי"ג [the thirteenth] aforementioned, is found side חצ [Ch-Tz] [or הצ [H-Tz]], and we subtracted it…

the sought angle (הזוית המבוקשת).

But, [for] angle בדפ [B-D-P], it is found, as in the triangle ראצ [R-A-Tz]…[gutter loss]…

we know side דא [D-A], and we extend [the perpendicular?] דצ [D-Tz], and we found…

and the remaining angle בדפ [B-D-P] is known. The angle בפר [B-P-R], it is found… [gutter loss]…

that we know side פה [P-H], and we extend פצ [P-Tz], which [is] the angle…

And by way of an addition (ולתוספת), for clarity: let us posit side בד [B-D] [as] **66. 1** [recte:? minute field 66 >59; probably 6°6′ or 1°6′; confirmed as printed, under magnification] and… [gutter loss]…

and so the supplement [will be] דא [D-A] **59. 23** [23°59′], and…[gutter loss]…

we have explained: the greater arc is found [to be] רצ [R-Tz] **39. 41** [41°39′], and… [gutter loss]…

In the standing triangle דאצ [D-A-Tz], the [right] angle is found…[gutter loss]…

as **100000** to the cosine [of] דא [D-A] $^{23\cdot59}/_{109449}$ [sine 23°59′ / 109449], so… [gutter loss]…

the sine of the complement [of] אץ [A-Tz] $^{35\cdot8}/_{81781}$ [35°8′ / 81781]; and so in the triangle…

side הצ [H-Tz], so…

as **100000** to the cosine [of] פה [P-H] $^{2\cdot56}/_{100181}$ [sine 2°56′ / 100181], so…[gutter loss]…

the sine of the complement [of] הצ [H-Tz] $^{3\cdot49}/_{99779}$ [sine 3°49′ / 99779]; and when we subtracted…

the arc הא [H-A] **19 31** [read as printed — possibly 31°19′] — this is what was sought (שריל = שר"ל, "this is what was required"). And likewise for its resolution…

as **100000** to the contact [or "touch"/tangent] of פה [P-H] $^{2\cdot56}/_{5124}$ [2°56′ / 5124], so…[gutter loss]…

the sine of the complement [of] angle פ [P] $^{52\cdot24}/_{61020}$ [52°24′ / 61020]; and likewise…

row [3] ג, is found angle או [A-W / A-V — gutter loss]…

as **100000** to the contact [tangent] of אד [A-D] $^{23\cdot59}/_{44488}$ [23°59′ / 44488], so… [gutter loss]…

the sine of the complement [of] angle ראצ [R-A-Tz] $^{30\cdot0}/_{50019}$ [30°0′ / 50019], and the…

more than the measure of [arc] בדפ [B-D-P] **120** [= 180 − 60; the supplement remainder (180° = מק"ף), internally consistent — not an impossible]…[gutter loss]…

A second example (דמיון שני), in the triangle אנב [A-N-B] (figure 22 — ציור כב), whose side אנ [A-N]…[gutter loss]…

side אאב [A-A-B — likely אב, A-B] up to ד [D], a quarter [90°], and a compass-pivot is made [at] נק…[gutter loss]…

with the third side בג [B-G] at the point פ [P], and they shall be… [gutter loss].

[two] large quarter-circles אר [A-R] [and] אה [A-H], and [it is] demonstrated for the lesson of the first stone (אבן הראשה) which is…[gutter loss]…

בדפ גהפ [B-D-P, G-H-P], so that angle פ [P] is bisected, equal in the reckoning, the second of them…[gutter loss]…

the perpendicular הג [H-G]; thus the sine of פב [P-B] [is] to the sine of side עמוד [the perpendicular]…[gutter loss]…

פב [P-B], thus the sine of הג [H-G] [is] to the sine of בד [B-D], and reversed, as… [gutter loss]…

to the sine of פב [P-B]. But the value of the sine of הג [H-G] [is] to the sine of… [gutter loss]…

the knowns goes [proportionally]; also the value of the sine of גפ [G-P] [is] to the sine of פב [P-B]…[gutter loss]…

[which is] known, [namely] that it is the third side, the given [one]; therefore from what is drawn…[gutter loss]…

the arcs גפ [G-P], פב [P-B] each one by itself, and they shall be for you…[gutter loss]…

the knowns, and by row [13] י"ג is found, [in] the two right triangles (בשתי הכאות צלעות)…[gutter loss]…

angle א [A], the one sought; and this is the way of the ancients (דרך הקדמונים). And how you…[gutter loss]…

you must, for the two right triangles, until you know the first angle…[gutter loss]…

to know all three angles of the triangle that remain; so it will avail you…[gutter loss]…

the remaining ones.

And by way of an addition (ולתוספת), for clarity: let the triangle be אבג [A-B-G] (figure 23 — ציור כג), in which side אב [A-B] [is] **36 0** [36°0′; the apparent "360 0" is a merger of 36 and 0 at low magnification, corrected on the scan], and side…[gutter loss]…

[side אג [A-G]] **31. 120** [recte:? minute field 120 >59; possibly 31°20′ or 1°31′; confirmed as printed, under magnification] and side בג [B-G] **0. 64** [64°0′]. And behold, the value [of] פב [P-B]…[gutter loss]…

the value [of] פב [P-B] is to פב [P-B], as the value of the sines [of] the supplements of the sides…[gutter loss]…

that is to say, as the value of הג [H-G] $^{30\cdot31}/_{50778}$ [30°31′ / 50778] to בד [B-D] $^{27\cdot0}/_{45399}$ [27°0′ / 45399], and likewise this value is known, and also the bisection [is] known…[gutter loss]…

the two arcs גפ [G-P] [and] פב [P-B], that it is the side גב [G-B] given, and therefore [the] sought…[gutter loss]…

[it is] given; and therefore is found the arc of the greater [angle] פג [P-G] [as] **57 33** [33°57′ — read as printed], and the…[gutter loss]…

the arc פב [P-B] **3. 30** [30°3′], and from these the sought ones [are] known. As in the triangles גהפ [G-H-P] בדפ [B-D-P] from the row…[gutter loss]…

sides רפ [R-P] [and] פה [P-H], in this manner.

as **100000** to the cosine [of] גח [G-Ch] [or גה, G-H] $^{30\cdot31}/_{116079}$ [30°31′ / 116079], so…[gutter loss]…

the sine of the complement [of] גפ [G-P] $^{33\cdot57}/_{82952}$ [33°57′ / 82952], so…

the sine [of] arc הפ [H-P] **96289** — which is **39 15** [15°39′ — read as printed], and likewise in the triangle בדפ [B-D-P]…[gutter loss]…

as **100000** to the cosine [of] בד [B-D] $^{27\cdot0}/_{112232}$ [27°0′ / 112232], so…[gutter loss]…

[is] the sine of the complement [of] בפ [B-P] $^{30\cdot3}/_{86658}$ [30°3′ / 86658], so…

the sine of arc דפ [D-P] $^{13.44}/_{97145}$ [13°44′ / 97145]. And when we joined…[gutter loss]…

the two sides הפ [H-P] [and] דפ [D-P], the whole is found (נמצא כללות)…

the arc דה [D-H], which is the measure of angle א [A] **29. 23** [23°29′] — this is…[gutter loss]…

and likewise is found, in the triangles, the [points] standing at the angle גהפ בדפ [G-H-P, B-D-P], upon…[gutter loss]…

29 — this is what was required (זה מה שר"ל).

as **100000** to the cosine [of] גח [G-Ch / G-H] $^{30.31}/_{58943}$ [30°31′ / 58943], so…[gutter loss]…

[of] בד [B-D] $^{27.0}/_{50952}$ [27°0′ / 50952]…

to the sine of the complement of arc ג [G] [number trails into gutter: $_{50}$…]…

[A boxed/inset numeral block at the column foot, partly abraded:]

	value
sine of גפ [G-P]	$^{30.31}/_{116}$…
(complement / supplement)	$^{33.57}/_{82952}$ [33°57′ / 82952]
	39 15 [15°39′]
sine of בפ [B-P]	$^{27.0}/_{112}$… [27°0′]
sine of דפ [D-P]	$^{30.3}/_{86658}$ [30°3′]
tangent-compl. פג [P-G]	$^{30.81}/_{148535}$ [recte:? minute field 81 >59; confirmed as printed, on the scan]
sine of גח [G-Ch]	$^{30.31}/_{58}$…
	$^{59.57}/_{172856}$ [read as printed]
sine [of] בפ [B-P]	$^{27.0}/_{50}$…
sine [of] ג [G]	$^{28.54}/_{87550}$ [28°54′ / 87550]
פבד [P-B-D]	$^{28.16}/_{88073}$ [28°16′ / 88073]

But (ואולם), if you do not desire to know [all] of the aforementioned triangle…[gutter loss]…

and you do not go by the way that its bounds were [set] first…[gutter loss]…

the true side adjacent to the sought angle (ד"מ — "for example"): if you seek… [gutter loss]…

it will be a quarter [90°] and fifty (וחמשים — וחשים), [for] the construction (המחונה) upon the pivot ג [G], and you mark…[gutter loss]…

for the lesson of [§65] ס"ה know the arcs אר [A-R] [and] רב [R-B], each one by… [gutter loss]…

רקב [R-K-B], the points of the angle, [are known] from the [given] knowns, and is found, from the row…[gutter loss]…

and these the ancients deemed [worthy], were the [ones currently] available… [gutter loss]…

up to ד [D], and you mark the arc דה [D-H], and it is found, according to the demonstration…[gutter loss]…

פדהג [P-D-H-G] is found, from row הג' [the third]…[gutter loss]…

Resolving the Opposite Side (צדיית הצלע המקביל) [section heading; the heading continues: "…המקביל" — "the parallel/opposite (side)"]

The row [that is] for [knowing] (הטור חז') from the two given angles and the side between them (משתי זויות והצלע המקביל) to one [of them]… [gutter loss].

Now (הנה), in the figures (בתמונות) that are drawn [in] §27 (שבדף נ"ז) [or "page 57"] (figure 21 — ציור כא), as… [gutter loss]…

the true [side] adjacent to angle צ [Tz], and we desire side אץ [A-Tz], the side parallel/opposite (המקביל) to angle…[gutter loss]…

the first [side], with the division, we strike (הקיש — "we infer by proportion"), and we say: as the value of the sine of…[gutter loss]…

angle ב [B] is to a quarter [90°], so will be the fourth [proportional] — side אץ [A-Tz], the [side]…[gutter loss]…

the perpendicular אד [A-D], and there will be for you two [right triangles], and the triangles [are] the points of the angle…[gutter loss]…

as in the triangle אבד [A-B-D]; and afterward, by row [9] ט, it is found, as in the triangle…[gutter loss]…

a row according to its nature [is] doubtful (מסופק), it will be — that is to say, also the aforementioned side…[gutter loss]…

Also (גם), regarding the dropping of the perpendicular (הורדת העמוד): whether it falls outside or whether it falls within…[gutter loss]…

the first [case] of this corner, and there is no need to repeat. And to the…[gutter loss]…

the side found. (א) **One** (א'): if the given side is the…[gutter loss]…

a quarter [90°], or deficient. All the more so when the parallel/opposite (המקביל) to angle…[gutter loss]…

the true [side], to the smaller angle, of the known [angles], it is the quarter [90°]…[gutter loss]…

the larger one will be a complement (עורף). (נ) [§50] **(2)** [or ב]: if the given side [is] a complement (עורף)…[gutter loss]…

then the second true side, adjacent to it, will not be deficient…[gutter loss]…

the given [side is] deficient and parallel/opposite to a problem (לצרה — "to a difficulty"); and the second angle (והזווית השניה)…[gutter loss]…

deficient — its remainder [is] (פחות מ-שאריה ה-)…[gutter loss].

Resolving the Two Sides (צדיית ב' הצלעות) [section heading]

The row [is] (רטור הה' — read הטור הה', "the row, the fifth") from the two given angles and the side between them (משתי זויות והצלע שביניהן).

in the aforementioned figure, angle א [A] [and] צ [Tz] are known, and the side between them אצ [A-Tz]…[gutter loss]…

and we desire sides אב [A-B] [and] בצ [B-Tz]. Now, most of the authors (רוב המחברים)…[gutter loss]…

and they mixed the resolution of the angles with the sides, since they [are] truly…[gutter loss]…

they wrote in the resolution of the angles concerning the matter of the doubt that falls in…[gutter loss]…

aligned with the second row of this same stone (האבן), and afterward when…[gutter loss]…

there, with the help of row [10] י, of corner one (פנה ראשונה) and corner two (ושניה)…[gutter loss]…

But (ואולם), to know the measure of side בצ [B-Tz] — for this you need…[gutter loss]…

side רב [R-B], and as in the triangle ארצ [A-R-Tz], we search the side דצ [D-Tz]… [gutter loss]…

side בצ [B-Tz], the [one sought].

[At the foot, partly obscured by a stamp:]

[Censor stamp / library stamp, illegible — circular impression at the lower inner margin, no legible text]

[The authors explained the further resolution of the obliques (העקומים) in six lessons, [each] aligned [angle] against [side], six against six, and all…]

the doubt that falls in the dropping of the perpendicular — we have copied [it down]; behold, this row…[gutter loss]…

and afterward, when the measure of the perpendicular אד [A-D] is found, as I taught you…[gutter loss]…

and the second is found [as] side אב [A-B], adjacent to angle צ [Tz]… [gutter loss]…

[for] this you must, for the two right triangles that are in the triangle ארב [A-R-B], search…[gutter loss]…

side דצ [D-Tz], by row [14] י"ד, and gather them or subtract them, [for] it is the measure of…[gutter loss]…

side בצ [B-Tz], the one sought (המבוקש).

[End of folio 202.]

another method. Set side בצ [BTz] as known, together with the angles ב [B] and צ [Tz] adjacent to it.

But, as I taught you in the third part of Lesson 64 [ס"ד] of the Second Corner, for those figures shown on p. 42 [מ"ב] (figure 12 [ציור יב]), it was made clear to you that the ratio of the tangent of the complement of angle ב [B] to the tangent of the complement of angle צ [Tz] equals the ratio of segment דב [DB] to segment דצ [DTz]. And once we know the sum of the two cut-off pieces — that is, the whole side צב [TzB], or the difference between it and the cut-off piece falling outside the triangle — and the ratio between the cut-off pieces, we will arrive at knowledge of the cut-offs of the base, just as I showed you by an analogous case in the second

part of the second branch of Lesson 68 [ס"ח], where I expanded at length over three examples. Once you know the said cut-offs, you will have, in the right-angled triangles, the angles ארב [ARB] and ארצ [ARTz] [recte: אדב / אדצ — see anomaly a] known by the third [method], and by the tenth branch you will find the surfaces אצ [ATz] and אב [AB]. And this is easily understood.

Resolving the Side Between the Known Angles [צריית הצלע שבין הזויות הידועות]

Branch 9 [הטור הט']: from the two angles and the side parallel to one of them.

This branch corresponds exactly to the first branch of this Stone; its figures are transferred here from p. 57 [נ"ז] (figure 21 [ציור כא]), as are all its cautions. Now, ג''א [recte: גא — the perpendicular] אד [AD] falls within triangle אבצ [ABTz], in which the angles ב [B] and צ [Tz] are known together with side אצ [ATz]. In the right-angled triangle [ADTz] we find the angle אדצ [ADTz] [recte: angle at A in triangle ADTz] by Branch 11 [הי"א]; the perpendicular אד [AD]; and then, by Branch 13 [הי"ג], we find side צד [TzD]; and by Branch 15 [ט"ו], side [AD as needed]. Likewise, if the perpendicular falls upon צר [TzR — recte: צד / TzD], and the whole side צד [TzD] comes out as known [רב — recte], in triangle אדב [ADB] we add to it the sought צב [TzB]. Since Branch 15 [הט"ו], which we use here, is by its nature doubtful [מסופק], the side will likewise be doubtful.

And if the perpendicular falls outside the triangle, the perpendicular is found, and then the cut-offs of the base; we subtract the smaller from the greater, and side צב [TzB] comes out — falling outside the triangle. The cut-offs of the base are doubtful [מסופק], unless you know their character from another source, as I taught you in the Second Corner.

Resolving the Third Side [צריית הצלע השלישי]

Branch 10 [הטור הי']: from the two sides and the angle opposite one of them.

This branch corresponds exactly to the third branch of this Stone, and following the cautions stated there, search out the character of the angle opposite the second side, if you wish to know how the perpendicular falls. The figures are those of p. 42 [מ"ב] (figure 12 [ציור יב]): in triangle אצב [ATzB] sides אב [AB] and אצ [ATz] are known, and angle צ [Tz] is wanted, [as well as] side בצ [BTz]. We drop the perpendicular אד [AD] from the angle between the known sides, and in triangle אדצ [ADTz] — side צד [TzD] [and] angle אדצ [ADTz — recte: the cut-off and angle] — we find by Branch 11 [טור יא'] side אד [AD]; then by Branch 12 [טור

יב'] we find, in triangle אדצ [ADTz], side צד [TzD]; and in triangle אדב [ADB], by Branch 13 [טור יג'], we find side דב [DB]. We add the one to the other or subtract the one from the other, and side צב [TzB], the sought side, comes out.

Resolving the Third Side [צריית הצלע השלישי]

Branch 11 [הטור הי"א]: from the two sides and the angle between them.

[recte: this is the eleventh branch; the rubric repeats "Resolving the Third Side" — see anomaly b.] This branch corresponds to the fifth branch of this Stone, and with its figures, the triangle is tested where the perpendicular falls. In triangle אבצ [ABTz], whose two sides אב [AB] and צב [TzB] are known together with the angle ב [B] between them, once you have found triangle אדב [ADB] by Branch 7 [מטר ז'] [recte: מטור ז'] — [whether it falls] inward or outward — and after you know its measure by Branch 11 [טור י"א], you seek in triangle [ADTz the remaining parts], and you find side אב [AB], the sought side; the matter is easy for one who understands.

Branch 5 [הטור הה'] recalled, with its conditions and rules:

The modes [are]: (1) all three angles equal, or two; and you will find the remainder in the Stone of the Crawler [אבן הזוחלת — the First Stone]. (2) One of the angles [is a right angle]: then the parallel sides are quadrants of circles, and the magnitude of the third angle is the measure of side [opposite] the right angle. (3) It has two right angles, or all [of equal sides]. (4) All its angles, or two of them, are obtuse; and then make the triangle that is opposite to it, which has two right angles [recte: two acute angles]. (5) Two of its angles are acute and one obtuse. For example: triangle אבצ [ABTz] (figure 12 [ציור יב]), angle ב [B] = 8°50′ [recte: ח 50 — read as printed]; angle צ [Tz] = 16°18′; and angle א [A] = 111°54′. We want all three sides. We drop from the obtuse angle א [A] onto בצ [BTz] the perpendicular אד [AD], which divides angle א [A] into two angles. The ratio between them is known, being the ratio of the tangents of the complements of angles ב [B] and צ [Tz] of the base, as was made clear in the first part of Lesson 64 [ס"ד], that is:

as the tangent of the complement of angle צ [Tz] = 42°28′ [recte: 28 42 צ — angle צ's complement is read as printed], which is 48022, is to the tangent of the complement of angle ב [B] = 10°81′ [recte: 81 — minutes >59, transcribed as printed; see numerical impossibles], which is 98814, so is the tangent of צאד [TzAD] to the tangent of באד [BAD]. And the sum of these is also known, namely

angle באצ [BATz], the given 111°54′. Thus, by the first sequence of Lesson 65 [ס"ה], we know [angle] צאד [TzAD].

Each [angle] by itself we set in proportion thus:

98814		98814
48022		48022
55°57′	27′.6 [recte: 27.6]	

As the ratio of the tangents [בקעיו — recte: tangents], 146836, is to the difference, 50792, so is the tangent [נוגע] of half-angle א [A], 147977, to the tangent 51207. This angle is called the angle of addition and subtraction; for if you add it to the half of angle א [A], which is 55°57′ [recte: 55 57], you find the measure of the larger angle באד [BAD], 83°3′ [recte: 3 83 — read as printed; see numerical impossibles]; and if you subtract it from it, you find the smaller [angle] צאד [TzAD], 28°51′ [recte: 51 28]. After this has come into your hand, by Branch 5 [מטור ה'] you find, in the right-angled triangles: the base [cut-off] ב [B] = 35°0′ [recte: 0 38 — read as printed; see numerical impossibles] [recte uncertain]; side אב [AB] = 20°38′; and side אצ [ATz] = 20°6′; and by Branch 14 [מטור י"ד] you find side דצ [DTz] = 38°0′ [recte: 0 38] and דצ [recte: דר] = 4°3′ [recte: 4 3 ודצ]; their sum, 41°4′ [recte: 4 41], is side צב [TzB]. This is what was to be demonstrated.

And if the angles of the base differ in their character and you wish to drop from the third angle a perpendicular that falls outside the triangle, use the second part of the first sequence of Lesson 65 [ס"ה], and this is easily understood. But you can always drop the perpendicular within the triangle, and that is the more fitting — choose it.

Another Method [דרך אחר]

If you wish to convert the triangle into a triangle whose sides are known and to seek its angles, as I taught you in Lessons 66 [ס"ו], [ס"ח] 68, and 80 [פ'] of the Second Corner — then the measures of the angles found are the measures of the sides you seek, according to the conditions of the lessons. Use the sixth branch of this Stone. Whichever seems good in your eyes, do; for the two ways are both good, as one. Nor is this conversion valid only in a triangle of known angles, which you would convert into a triangle of known sides, and vice versa; rather, in all the triangles of the twelve branches of this Stone the conversion applies — by means of the two circles I drew at the end of the Second Corner. Examine closely and you will find various modes for the resolution of each of the twelve branches.

For example: let there be a triangle one of whose angles is 76° [recte: this example's first angle is read as printed] and the sides around it: the first 60°0′ and the second 110°0′; and we want the third side. We make a triangle one of whose sides is 11.21 [recte: 11°21′ — read as printed] and the angles around it: 110°0′ and the second 120°0′; and it is the remainder of one of the sides of the given triangle. We seek the third angle, which is 54°0′; and so the sought third side is 54°0′. The matter is easy for one who understands.

There is no need to expand. The time presses upon me — many and grievous misfortunes have surrounded me, and may God shorten the days of those who cause me to shorten and to "heap up the stack before its time" [cf. Iyov 5:26]. For here am I, [a burden] and my heart is for all my people, that I might set my purpose — let me set out my own portion. And my eyes [are upon you]: now I will counsel you — keep my eye [upon you]. Examine well my words; perhaps I erred in my haste. And the Holy One [recte/uncertain reading] will draw you near to Him, for I did not intend to acquire a name, but only for your benefit and your good. At another time I will write to you according to your heart, and I will fulfill all your need, with the help of God.

The Fourth Corner [הפנה הרביעית]

On Right-Lined Triangles, Having Two Stones [במשלשים ישרי הקוים ולה ב' אבנים]

The First Stone is called *sapir* [Sapphire], for it is like the white of the sapphire — clear and pure as the heavens [cf. Shemos 24:10], lucid in its purity — [serving] for the resolution of right-angled triangles. Its foundation rests entirely upon Lesson 31 [recte: כ"א, see anomaly c] [ל"א — printed] of the Second Corner, where I taught you by demonstration that the ratios of the sides of a right-angled triangle equal the ratios of the sines, tangents, and secants of the angles opposite them; and that you can, by three modes, seek each and every one of the three unknown parts of the triangle when you know the other three — except when you know the three angles, in which case you cannot know the sides with precision, but only the relation between them. Yet note: if you look at the second or fourth or sixth figure of the foundations, or at the figures opposite them (figure 24 [ציור כד]), you will see that the line parallel to one of the three sides makes angles equal to the angles adjacent to that side; and likewise, if there are several lines parallel to the sides, they make equal angles. Therefore you cannot know the measure of the sides from the angles alone, unless you also know the measure

of some one side. This Stone has eight branches, wrapped in sapphires [מעולפים בספירים — cf. Shir HaShirim 5:14].

Resolving the Ratios of the Sides [צריית ערכי הצלעות]

Branch 1 [הטור הא']: from any angle.

As the first of my words, let me remind you that in right-lined right-angled triangles, when you know one of the angles besides the right angle, you immediately know the third angle — by Lesson 32 [ל"ב] of the First [Corner] of the foundations — which is the complement to two right angles; and therefore there is no need to know more than the two angles alone.

Let there be triangle אבצ [ABTz] (figure 25 [ציור כה]), whose angle א [A] = 36°52′ and angle צ [Tz] = 53°8′, the golden ratios being their sines as known. If you set the surface אצ [ATz] [the hypotenuse] to half the diameter, 100000 — that is, אב [AB] will be the sine of angle צ [Tz] and צב [TzB] the sine of angle א [A] — then these will be the ratios of the sides: אצ [ATz] = 100000, אב [AB] = sine 8000 [recte: 80000 — read as printed; see numerical impossibles], צב [TzB] = sine 6000 [recte: 60000 — read as printed; see numerical impossibles].

But if you set the side as half the diameter, the ratios of the sides will be: thus אב [AB] = 100000, בצ [BTz] = tangent 7500, אצ [ATz] = secant 12500; and the second [set], of angle א [A] = 36°52′. But if you set בצ [BTz] as half the diameter, the ratios of the sides will be: thus בצ [BTz] = [100000], אב [AB] = tangent 13334, אצ [ATz] = secant 16667; and both [sets] are of angle צ [Tz] = 53°8′.

And just as you found in the right-angled triangle, so too you find in the oblique triangle — that is, אבצ [ABTz] — for if you seek the sines, tangents, and secants of its three angles, the ratios of the sides will be known to you, and these they are:

	א [A] = 80°	ב [B] = 60°	צ [Tz] = 40°
sine [בקעה]	9848	8660	6427
tangent [נוגעה]	56712 [recte: 57712 — see numerical impossibles]	17320	8390
secant [חותכה]	57587	20000	13054

Resolving the Side [צריית הצלע]

Branch 2 [הטור הב']: from the hypotenuse and one of the acute angles.

Since, by Lesson 32 [ל"ב] of the First [Corner], all its angles are known according to Lesson 21 [כ"א] of the Second Corner, we set up the proportion: as 100000 is to בצ [BTz], so 6000 is the sine of א [A] [recte/uncertain] to side צם [TzM — recte: צב / TzB] 3, and so to side אב [AB] 4. Or proceed by way of method 3: set the hypotenuse אצ [ATz] to half the diameter, and sides אב [AB] and בצ [BTz] will be the sines of the opposite angles — thus אצ [ATz] = 100000 is to 5, so אב [AB] = 8000 is to 4, and so צב [TzB] = 6000 is to 3. Or let the hypotenuse be the secant of angle א [A], or that is, צ [Tz] — that is, you set side אב [AB] or בצ [BTz] to half the diameter, and the ratios will be thus:

as אצ [ATz], the secant of angle צ [Tz] = 53°8′, which is 16667, is to 5, so the tangent of angle צ [Tz] — that is, אב [AB] — is to 4 [recte uncertain], and so the sine of צ [Tz] [recte: בקע צ] = 100000 is to 3. Or in this manner: as אצ [ATz], the secant of angle א [A] = 36°52′, which is 12500, is to 5, so the tangent of angle א [A] — צב [TzB] = 7500 — is to 3, and so the sine of אב [AB] is to 4.

But the choicest of ratios is the one whose foundation is entirely the sine, where you need not divide; yet it is good to know the long ways too, that they may serve you as a check in your calculations. Be that as it may, we choose the short way in this teaching. And note: at times the given surface [the hypotenuse] will be the whole sine, and then you need neither double nor apply [a divisor], for the sines of the angles are themselves the measures of [the sides] …

the sides that are adjacent to them. Likewise it will be easy for you, since whenever the sine [בקע] is one of [the sides], then the tangent [נוגע] and secant [חותך] of the angles at the center are themselves the measures of the surface [נשטח, hypotenuse-projection] and of the second side; understand this well.

Resolving the Side

Lesson 3 — From a side and one of the acute angles

In the said triangle, if you seek side **אב** [AB] as set out: just as sine **א** [A] 6000 is to **צב** [TzB] 3, so is sine **צ** [Tz] 8000 to **אב** [AB] 4. And if you seek side **צב** [TzB] as set out: just as sine **צ** [Tz] 8000 is to **אב** [AB] 4, so is sine **א** [A] 6000 to **צב** [TzB] 3. Or proceed by this route, and you will escape the [labor of] division: just as **צב** [TzB] 160000 [recte: 100000?] is to 3, so is tangent **אב** [AB] 4 to 4 [?]. And likewise, as the ratio of **אב** [AB] 100000 is to 4, so is tangent **צב** [TzB] 7500 to 3.

Resolving the Side

Lesson 4 — From a side and the hypotenuse-surface

In the said triangle, let side **אב** [AB] be 4 and the surface **אצ** [ATz] be 5. Subtract, then, the square of **אב** [AB], 16, from the square of **אצ** [ATz], 25; the remainder is 9, and the root of the square 9 is 3 — which is the measure of **צב** [TzB] sought, as taught in Lesson 7 of the First [Corner], "On Foundations."

Another route: just as **אצ** [ATz] 5 is to **ב** [B] 100000, so is **אב** [AB] 4 to **צ** [Tz] 8000; and in the table of sines you will find 8°53′ [literally "8.53"], whose complement is 36°52′ [transcribed as printed "52.36"], and its sine [בקעה] is 6000. Then set up a second proportion: just as **אצ** [ATz] 100000 is to 5, so is **צב** [TzB] 6000 to 3.

Another route: take the surface **אצ** [ATz] as the radius [half-diameter], and **אב**, **ב** [AB, B] will be the sines of the acute angles, and you will find as at first.

Another route: take **אב** [AB] as the radius, and **אצ** [ATz] will be the secant of angle **א** [A]; and it will be: just as **א** [A] 4 is to 100000, so is **אצ** [ATz] 5 to the secant of angle **א** [A], 12500, which is 52°36′ [transcribed as printed "52.36"], and its tangent is 7500. Then frame a second proportion: just as **אב** [AB] 100000 is to 4, so is 7500 to 3, the side **צב** [TzB] sought.

Resolving the Hypotenuse-Surface

Lesson 5 — From a side and one of the acute angles

In the said triangle, let side **בצ** [BTz] be known. We then frame the proportion: just as sine of angle **א** [A] 6000 is to **צב** [TzB] 3, so is sine of angle **ב** [B] 100000 to **אצ** [ATz] 5. Or, if side **אב** [AB] is known, we frame: just as sine **צ** [Tz] 8000 is to **אב** [AB] 4, so is sine **ב** [B] 100000 to **אצ** [ATz] 5.

Another route: take the given side as the radius. Then: just as **צב** [TzB] 100000 is to 3, so is **אצ** [ATz], secant of angle **צ** [Tz], 16667, to 5. And likewise: just as **אב** [AB] 100000 is to 4, so is **אצ** [ATz], secant of angle **א** [A], 12500, to 5.

Resolving the Hypotenuse-Surface

Lesson 6 — From the two sides

In the said triangle **אבצ** [ABTz], from the sum of the squares of the two sides **צב**, **בא** [TzB, BA] — which is **צ** [Tz]…

25 — extract the side of the square, which is 5, and that is the measure of **אצ** [ATz], the surface.

Another route: take one of the sides as the radius, and frame the proportion thus: just as **אב** [AB] 4 is to 100000, so is **צב** [TzB] 3 to the tangent of angle **א** [A] 7500, which is 36°52′ [transcribed "52.36"], and its secant is 12499 [recte: 12500?]. Then again frame: just as 100000 is to **אב** [AB] 4, so is 12499 to **אצ** [ATz] 5.

Resolving the Angle

Lesson 7 — From the two sides

In the said triangle: From the preceding Lesson you will find the surface **אצ** [ATz] 5; and so frame the proportion **אבצ** [ABTz] — just as **אצ** [ATz] 5 is to **ב** [B] 100000, so is **בצ** [BTz] 3 to the sine 6000 of angle **א** [A], which is 36°52′ [transcribed "52.36"]; and likewise sine **אב** [AB] 4 is to sine **צ** [Tz] 8000 of angle **צ** [Tz], which is 8°53′ [transcribed "8.53"] — and this is the route of the ancients, who did not know the tangents and secants and their use.

Another route: take one of the sides as the radius, say **א** [A], and **בצ** [BTz] will be the tangent of angle **א** [A] and **אצ** [ATz] its secant; and it will be: just as **אב** [AB] 4 is to 100000, so is **בצ** [BTz] 3 to the tangent of angle **א** [A] 7500, which is 36°52′ [transcribed "52.36"] [recte for "25.36" / "א"?], and its complement is angle **צ** [Tz] 8°53′ [transcribed "8.53"]. And likewise, were you to take **צב** [TzB] as the radius, it would be: just as **בצ** [BTz] 3 is to 100000, so is **אב** [AB] 4 to the tangent of angle **צ** [Tz] 13334, whose angle is 8°53′ [transcribed "8.53"], and its complement is angle **א** [A] 36°52′ [transcribed "52.36"].

Resolving the Angle

Lesson 8 — From a side and the hypotenuse-surface

So, in the said triangle: just as **אצ** [ATz] 5 is to **ב** [B] 100000, so is **אב** [AB] 4 to the sine of angle **צ** [Tz] 8000, which is 8°53′ [transcribed "8.53"], and its complement is 36°52′ [transcribed "52.36"] for angle **א** [A]. And likewise **צב** [TzB] 3 is to sine **א** [A] 6000 — 36°52′ [transcribed "52.36"] — and its complement, 8°53′ [transcribed "8.53"], is angle **צ** [Tz].

Another route: take one of the sides as the radius, and the proportion will be thus:

אב [AB] 4	to secant of angle **א** [A]	12500	which is 52°36′ [transcribed "52.36"]
as	to 100000	so is **צא** [TzA] 5	and its complement 8°53′ [transcribed "8.53"], which is angle **צ** [Tz]
צב [TzB] 3	to secant of angle **צ** [Tz]	16667	which is 8°53′ [transcribed "8.53"]
			and its complement is angle **א** [A] 52°36′ [transcribed "52.36"]

Route 3: take the hypotenuse-surface as the radius, and frame the proportion thus:

אב [AB] 4	to angle **צ** [Tz]	8000 — 8°53′ [transcribed "8.53"]
as **אצ** [ATz] 5	to 100000	
צב [TzB] 3	to angle **א** [A]	6000 — 52°36′ [transcribed "52.36"]

The Second Stone is called **odem** [ruby], and it has six lessons. They [the lessons] are loftier than pearls [literally: their substance is finer than pearls]. And although it was my intention to explain the matter in its diverse kinds, of many sorts, for now — for shortness of time — I will mention only the simple modes, which hold true [as they stand] upon the lessons of the preceding Stone.

Resolving One of the Sides

Lesson 9 — From the two angles, that is, all of them, and one side

Now, if the given angles are equal (see **figure כו** [Ko]), then the corresponding sides are also equal, according to the teaching of each from the Second Corner. And if you drop a single perpendicular onto the third side, **בצ** [BTz], it cuts it in two, and from Lesson 2 you find side **ב** [B] [half-base], whose double is the measure of side **בצ** [BTz]; and likewise the measure of **אב**, **אצ** [AB, ATz].

But if the angles are unequal, as in the second triangle (see **figure כז** [Ko"z]), whose angle **ב** [B] is 36°52′ [transcribed "52.36"] and angle **צ** [Tz] is 67°23′

[transcribed "67.23"] [recte: 23°67′ — minutes >59, see below], then angle **א** [A] will be — by the teaching of Lesson 9 [recte: א"ל?] of the First [Corner] — 75°45′ [transcribed "45.75"] [recte: 45°75′ — minutes >59], and side **אב** [AB] is 20; and you set up the proportion thus:

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as sine **צ** [Tz] 9231	to **אב** [AB] 20	so is
sine **ב** [B] 6000	to **אצ** [ATz]	9228/9231 12
sine **א** [A] 9692	to **בצ** [BTz]	9220/9231 20

Or proceed thus: drop a perpendicular **אד** [AD] onto the hidden side **בצ** [BTz], and it will fall either inside or outside [the triangle]; and by Lesson 2 the right-angled triangle **אדב** [ADB] becomes known, and so side **בד** [BD] becomes known. And by Lesson 3 you find side **דצ** [DTz] in triangle **אדצ** [ADTz], and you add it to **בד** [BD], or subtract it from **בד** [BD], and the result is the measure of side **בצ** [BTz] sought.

Now, for seeking side **אצ** [ATz] from the outset, you proceed in this manner: drop **אד** [AD], and it becomes known to you from Lesson 2; then again, in triangle **אצ** [ATz, scil. אדצ], from Lesson 5 you find side **אצ** [ATz] sought. By "from the outset" I mean that you should never need all three sides; so for seeking each one alone you must have a short route that does not require seeking another for the sake of the [first] one. For if, after you have found one, you wish to seek the others, there is no doubt that you can obtain the matter in several ways, once most of the parts of the triangle are known to you — that is, once you know the measure of side **בצ** [BTz] added to the first three known ones, you will find side **אצ** [ATz], whether from the tenth Lesson or from Lesson 11, which come, God willing, [later]. But the straight road is that you choose to know what is sought directly from the three known [parts]; and this you shall know in every "Resolving the Side." The Lessons, whether for right-lined [triangles] or for curved [spherical] ones — there is no need to prolong.

Resolving the Side

Lesson 10 — From the two sides and the angle corresponding to one of them

Now, in an isosceles triangle the angles at the base are also equal, from the Fifth [Corner] to the First. And all three angles will be known, from Lesson 12 to the First; and we then frame the proportion and say: as the ratio of sine **צ** [Tz] 9231 is

to side **אב** [AB] 20, so is sine **א** [A] 6000 to **בצ** [BTz] 9220/9231 20. Or we drop a perpendicular **אד** [AD], and in triangle **בדצ** [BDTz] we find side **דצ** [DTz] from Lesson 2, whose double is the side **בצ** [BTz] sought.

But when the triangle is one of all-differing sides — the one called *scalene* [scaliyno] — as in the form **אבצ** [ABTz] (see **figure כה** [Ko"h]), in which side **אצ** [ATz] together with side **אב** [AB] and the corresponding angle **צ** [Tz] are among the knowns, then frame the proportion thus: just as **אב** [AB] 13 is to sine of angle **צ** [Tz] 36°52′ [transcribed "52.36"], which is 6000, so is **אצ** [ATz] 20 to sine 9230 of angle **ב** [B], which is 67°23′ + 37°112′ [transcribed "37.112 / 67.23" as a paired/fraction form] [recte: minutes >59 — see anomaly note]. And after you know two of the angles, you know — from Lesson 12 to the First — also the third angle **א** [A].

[recte for the angle just found]. Then again frame a second proportion: just as the sine of angle **ב** [B] is to **אצ** [ATz] the side, so is the sine of angle **א** [A] to **בצ** [BTz], the side sought.

Another route: drop a perpendicular **אד** [AD] onto the hidden side **בצ** [BTz], and it will fall inside or outside, along the extension of the said side; and from Lesson 2 there are found both the perpendicular **אד** [AD] and side **רצ** [RTz, scil. דצ] in triangle **אדצ** [ADTz]; then again, from Lesson 4 in triangle **אדב** [ADB], side **בד** [BD] is found, and you add it to side **דצ** [DTz] or subtract it from it, and the whole **בצ** [BTz] sought comes to light.

But whether side **בד** [BD] ought to be added to **דצ** [DTz] or subtracted from it is in truth doubtful — just as the manner of dropping the perpendicular **אד** [AD] is doubtful: whether [it falls] inside or outside, as your eyes will see. In the figure before us [drawn], there are two triangles of thirty equal parts [?], yet one [is] larger than its fellow; for side **אצ** [ATz] is shared between the two, and angle **צ** [Tz] [is shared] between them, and side **אב** [AB] is equal in both — yet side **בצ** [BTz] of the one is larger than that of its fellow. The angles **אבצ** [ABTz] [differ]: in the one obtuse and in the other acute; and likewise angle **צאב** [TzAB] differs between the two. Therefore, one who frames the proportion and decides at once upon the measure of the unknowns will err without doubt, unless he first knows the character of this second angle — that is, whether angle **ב** [B] is obtuse, in which case the perpendicular falls outside the triangle, or whether it is acute, and falls inside the triangle.

To clarify the matter, let it be, for example, in the right-lined triangle **אבצ** [ABTz], **אצ** [ATz] 20, **אב** [AB] 13, angle **צ** [Tz] the corresponding [angle] 36°52′

[transcribed "52.36"]; and you seek the third side **בצ** [BTz]. We frame the proportion thus: just as **אב** [AB] is to sine **צ** [Tz], so is **אצ** [ATz] to the corresponding sine **ב** [B], and you find it 9230. Now if we assume it is the lesser, 23°67′ [transcribed "23.67" — minutes >59], as it would be in the larger triangle, then it would be — from Lesson 12 to the First — and it is shared with [the supplement], the angle 23°67′ [transcribed "67.23"] and its remainder 37°112′ [transcribed "112.37" — minutes >59]; and who shall test which of them ought to be taken? — angle **א** [A] the third [is] 75°45′ [transcribed "45.75" — minutes >59] and its sine 9692; and frame the proportion thus: just as sine **צ** [Tz] 6000 is to **אב** [AB] 13, so is **א** [A] 9692 to 5998/6000 20, the side **בצ** [BTz].

But if it is angle **ב** [B] that is obtuse — that is, 37°112′ [transcribed "112.37" — minutes >59] — as it is in the small triangle **אבצ** [ABTz], then the third angle **א** [A] will be 30°31′ [transcribed "31.30"], and the proportion will be thus: just as sine **צ** [Tz] 6000 is to **אב** [AB] 13, so is sine **א** [A] 5077 to **בצ** [BTz] 11 approximately.

You see, then, how side **בצ** [BTz] differs and is in doubt as to its measure. This [is] true [in part]; at times the matter will be revealed and the doubt cleared, and this [happens] in two places:

(**א**) When the side corresponding to the known angle is larger than the second known [side] — for then the angle corresponding to it [the second side] will be acute, whatever the case. If the known angle is acute, then since the [first] side is larger and corresponds to the larger angle, the second angle, corresponding to the smaller side, will be acute all the more so. And if the known [angle] is obtuse, it is impossible — by the straightness of the lines — for the angle to be more than obtuse, by [Lesson] 1 to the First [recte: מל"ם / "from L"M"]; and so we know how the perpendicular falls, inside or outside the triangle.

(**ב**) When the side corresponding to the known angle is smaller than its known fellow, and by means of the proportion the angle corresponding to the second known side is found to be a right angle — for then the leg of the right angle is itself the perpendicular. And there is no need to say [more] when the triangle is equilateral or isosceles and the two legs are known, for then the perpendicular falls inside; and all is clear as mentioned at the head of this Lesson.

But when the side corresponding to the known angle is smaller than the second known [side] and the known angle is acute, and the second known angle is acute…

uncertain — for we would not know how the perpendicular falls, or from which point. Once you have sought one of the remaining angles, you may, if you wish,

learn the other [angle]. Everything written here we will [later] transcribe with full justification in Lesson 13, which follows, God willing; there is no need to repeat it here.

Resolving the Third Side

Lesson 11 — from two sides and the angle between them.

When the known sides are equal, drop a perpendicular from the known angle to the unknown side; then by the procedure of Lesson 2, take its cuts בד and רצ [B-D and R-Tz], and their sum is the measure of the third side.

But if the triangle is one of unequal sides, drop a perpendicular from the unknown angle to the known side. If it proves necessary, extend it outside the triangle — and this is unavoidable when the known angle is obtuse. If, however, it is acute, then investigate its complementary sine up to ninety degrees: should you find it [the sine] greater than the known side, know that the perpendicular falls outside; if not, it falls inside, beyond doubt. The reason for the matter is set out by way of analogy in the Second Corner. Let the triangles before you (Figure 29) be אבצ [A-B-Tz], where side אב [A-B] is 100000 and angle אצב [A-Tz-B] is 0.50 [i.e. 0°50′ in the inverted minute·degree notation; [?]], and side בצ [B-Tz] in this one is 9000, and in that one 4000. You see in these triangles that angle צאד [Tz-A-D] equals the complement of angle צ [Tz]. Consider, then: if the sine of angle צאד [Tz-A-D] is greater than side בצ [B-Tz], there is no doubt that the side measured against its magnitude will be larger than בצ [B-Tz], and בצ [B-Tz] will be extended outward. But if the sine of angle צאד [Tz-A-D] is less than side בצ [B-Tz], then a portion of side בצ [B-Tz] suffices to lie opposite it, and the perpendicular falls inside. There is no need to elaborate.

Resolving the Angle

Lesson 12 — from the three sides.

If the triangle is equilateral, each of its angles is one-third of two right angles — that is, 60 degrees. And if it is isosceles, the perpendicular dropped inside bisects the third side, and by the procedure of Lesson 8 you will learn the angle.

But for a scalene triangle, the modes of resolution among the authors are numerous, while I now seek brevity, not the prolixity that comes of multiplying matters. So I will bring you two modes.

The first mode is founded on the pillars of Euclid's Lesson 13 of [Book] Two. Let triangle אבצ [A-B-Tz] (Figure 30) have side אב [A-B] = 5, אצ [A-Tz] = 7, בצ [B-Tz] =

8, and let its angles be sought. We always drop the perpendicular onto the largest side, so that it falls inside the triangle — even when the angle is obtuse. Thus the sum of the square of the smaller side אב [A-B], 25, together with the square of the larger צב [Tz-B], 64, is 89. When we subtract from this the square of the third side, אצ [A-Tz], 49, there remain 40; and the half of this, 20, if divided by the side made known as the larger, 8, gives the smaller cut בד [B-D] as 2½ in the result. By this, the larger cut רצ [R-Tz] will be 4½.

The proof: Euclid demonstrated it in Lesson 13 of [Book] Two for acute-angled triangles, and as we explained in our commentary upon it, the matter holds also for right-angled and obtuse-angled triangles. It is fitting that the perpendicular fall on the largest side, for the sum

of the squares of the sides enclosing the obtuse angle — that is, 89 in the diagram before us — equals the sum formed from the square of the side opposite that obtuse angle, which is 49, plus twice the area, 20, formed from the side בצ [B-Tz] = 8 onto which the perpendicular falls and the cut adjacent to the obtuse angle, namely בד [B-D] = 2½. For twice the area is 40, and together with the square of side אב [A-B], 49 [recte: 25; [?] — the text appears to repeat 49 in error], it is 89. It is therefore clear that if you subtract from the sum of the squares of the sides enclosing the angle — which is 89 — the square of side אצ [A-Tz], 49, there remain 40, which is twice the said area. Now since בצ [B-Tz] is known in number, divide 40 by it, and its half, 5, [is what] enters into it [?] — half of which gives the cut בד [B-D] = 2½, and the cut רצ [R-Tz] will be 4½. This is clear, as explained.

From here it is evident that whenever two sides of a triangle and the measures of the cuts of those two sides are known, you can readily find the third side. For [the remainder] will be the square of the third side: if you subtract from the number 89 the number 40, there remain 49, whose root, 7, is the measure of the square root of אצ [A-Tz] [i.e. the length אצ = 7].

The second mode — the one I use, which is peculiar to me, for I devised it for myself — is by way of *al-jabr* [Arabic: al-jabr, "algebra"]. We posit that the cut בד [B-D] is a "thing" [the unknown], and accordingly the other square will be רצ [R-Tz] = 8 minus the [first] "thing." And in accordance with Lesson 47 of [Book] One: if you subtract from the square of אב [A-B], 25, the square of the first thing, the perpendicular אד [A-D] comes out as the root of a square of 25 minus a square of [one] thing. Likewise, if you subtract the square of צ [Tz] — which is 64, less 16 "things" [from the binomial (8 − thing)²] — the other square, from the square of אצ [A-Tz], 49, the perpendicular אד [A-D] likewise comes out as the root of a

square of 16 things minus 15, less the other square. So the equality between these is:

> 1 square + 25 = 1 square + 15 things + 16. [as printed; "פ"=square, "ם"=thing/root, "א"=unit]

Add to each [side] 15 things [recte: subtract; [?]], so that the equality is between 16 [units] and 40. Then balance and say: if 16 units equal 40, how much equals 1 unit? You will find the equality, the root, [to be] 2½ — and that is the measure of רב [R-B; i.e. cut בד]. Accordingly, רצ [R-Tz] = 4½, and this is well understood.

And once the cuts have become known to you through Lesson 8, you will find the measures of the three angles. This is what was intended.

Resolving the Angle

Lesson 13 — from two sides and the angle opposite one of them.

If the sides are equal, the angles too are equal, by Lesson 5 of [Book] One; and when one of them is known, the rest too are known, by Lesson 32 of [Book] One. But if they are unequal, you can reason by proportion and say: as side אב [A-B] is to the sine of angle צ [Tz], so is אצ [A-Tz] to the sine of angle ב [B]. Or drop the perpendicular א [A], and you will have in the triangle אדצ [A-D-Tz] three knowns; and by the procedure of Lesson 2 find the perpendicular א [A]; then again, in triangle אדב [A-D-B], find by the procedure of Lesson 5 angle ראב [R-A-B] or ראב [R-B-A] [i.e. the angles A and B about the foot D], whose complement or remainder are the sought angles.

But as I instructed you in the tenth Lesson, the resolved angle is uncertain in its measure, since we would not know how the perpendicular falls; the figure there shows you the truth of the matter, and there is no need to elaborate — for in that Lesson the construction is set out in full.

Resolving the Angle

Lesson 14 — from two sides and the angle between them.

When the sides are equal, the angles too are equal, by Lesson 5 of [Book] One; and if you subtract the given angle from two right angles, the remainder is the measure of each of the [other] angles — or subtract half the given angle from 90 degrees, and the measure of the angle remains.

But if the sides are unequal, drop a perpendicular from one of the unknown angles, א [A], and in triangle אד [A-D], by the procedure of Lesson 2, find the sides אד [A-D] and רב [R-B]; and if you add בצ [B-Tz] to it, or subtract it from it, the measure of רצ [R-Tz] comes out. Then by the procedure of Lesson 7, find angle צ [Tz], and from there you will learn the second angle.

In my commentary on the *Almagest* I have explained these matters with various proofs. But today my supporters have sustained me with raisin-cakes and refreshed me with apples [cf. Shir HaShirim 2:5], and have asked of me reckonings and computations — men at ease and untroubled. I cannot do as my heart [would wish]; and when my mind takes pity upon me, I shall return, I shall look, I shall pasture among the lilies [cf. Shir HaShirim 6:2–3] — the counsel of the wise and discerning.

The Treatise *Sod HaYesod* is finished and complete.

Praise to God, Creator of the world.

Figure Plate I

This plate holds sixteen lettered figures, ציור א (Figure 1) through ציור טז (Figure 16): spherical and plane triangles, circle constructions, and one rectangular array of small reference triangles. The figures are arranged in several columns; numerals beside arcs are degree-and-minute values (written as `deg.min`, e.g. 168.16 = 168° 16′).

Diagram. A lune / two-arc figure (crescent shape formed by two intersecting arcs).

- ג [G] (top): vertex of upper arc, with adjacent ה [H] at top
- ט [T] (upper interior): intersection point
- א [A] (mid interior, below ט): interior point
- ד [D] (right): right base
- ה [H] (lower-right base, at right-angle tick)
- פ [P] (left): left vertex

Diagram. A semicircle/quadrant construction with inscribed chords and a central point; several radii and a triangle are drawn inside.

- ז [Z] (top): apex
- ב [B] (upper right): point on arc
- ה [H] (left): point on arc

- ג [G] (center): central point
- פ [P] (lower center): foot point
- ט [T] (lower right): right base point
- ד [D] (lower, interior): interior point [?]

Diagram. A circle with numerous degree-values inscribed around the circumference and along chords, evidently a worked spherical-triangle example.

- Inscribed/marginal numerals (reading around the figure): 11.44, 100.2, 168.16, 79.58, 50, 85.23, 48.34, 154, 37, 131.2, 79.58
- ד [D] (right): point on circle
- א [A] (lower right, near 131.2): point on circle

Diagram. A circle with an inscribed quadrilateral/triangle and internal perpendiculars; labels on a rectangular inset at upper left.

- א [A] (left): vertex
- פ [P] (upper left): point
- ה [H] (right): point on circle
- ז [Z] (lower right): point
- ד [D] (lower): base point [?]

Diagram. Partial circle with crossing diagonals (continuation of the ציור ד construction at left edge).

- א [A] (right): vertex
- (remaining labels run into the gutter and are not legible) [?]

Diagram. A lune (two intersecting arcs) with three interior labels.

- ז [Z] (top): upper vertex
- ה [H] (interior): upper interior point
- ב [B] (interior): mid interior point
- ד [D] (lower interior): lower interior point
- א [A] (left): left vertex

Diagram. A square/rectangle with inscribed quarter-arc and diagonals from a corner — a right-triangle-in-square construction.

- ז [Z], נ [N] (top edge, left to right): top corners
- ה [H], ד [D] (interior, on diagonal): interior division points
- ט [T], מ [M] (left edge): left-side points

- א [A] (lower left): corner / origin of radii
- ש [Sh], ד [D], ה [H], ב [B] (bottom edge): base points
- ל [L] (right edge): right-side point

Diagram. A large circle with an inscribed figure of many crossing chords and interior labels (a complex spherical-triangle / transversal construction).

- ט [T], ו [V], ז [Z] (top): upper points
- מ [M], ש [Sh] (upper left): points on circle
- ק [Q], ה [H] (interior left): interior points
- ז [Z] (center): central point [?]
- ל [L], ה [H], ב [B] (lower interior): lower points
- ה [H], ל [L] (bottom): base points

Diagram. A small spherical triangle (curved-sided triangle / "sail" shape) with two labeled lower vertices.

- א [A] (top): apex
- ב [B] (lower left): base vertex
- ג [G] (lower right): base vertex

Diagram. A rectangular array of small reference spherical triangles, arranged in a 3-column × 5-row grid, each a curved-sided triangle annotated with right-angle marks, small dotted/dashed marks (∴, ◊, △), and the vertex letters ב/ז/ה. The grid is headed by the catchword-style label line: מטורתשיע"י · מטורשושי … · מטורט"ז · מטורת… [a mnemonic / reference string keyed to the triangle types; abbreviated forms, read as printed].

- Each cell: a curved triangle with vertices marked ה (top right), ז (top left), ב (bottom); right-angle square at one corner; the interior symbols (∴, ◊, △, ::) distinguish the case (right angle / two angles / etc.). Individual cell symbols transcribed collectively rather than cell-by-cell.

Diagram. A lens / vesica figure (two arcs meeting at top and bottom) with a horizontal chord.

- א [A] (top): top vertex
- א [A], ב [B] (mid, on chord): chord endpoints
- ג [G] (interior): center point
- ד [D] (bottom): bottom vertex

Diagram. Two stacked small spherical triangles (curved-sided) with a brace; the upper one labeled at its vertices.

- א [A] (top): apex
- ג [G] (lower left): base vertex
- ד [D] (lower center): foot point
- ב [B] (right, with brace): right vertex

Diagram. A full circle divided by several diameters/chords radiating from the center, with labeled points around the rim — a great-circle / rhumb construction.

- ד [D] (upper right): rim point
- ה [H], ז [Z], ח [Ch], ט [T] (upper interior, clustered): division points [?]
- ב [B] (right): rim point
- א [A] (center): center
- ג [G], ה [H] (upper left): rim points
- ז [Z] (left): rim point
- ש [Sh] (lower): rim point

Diagram. A figure of several overlapping circles (a Venn-like / intersecting-circles construction) with many interior labels and small asterisk/star marks (✻) at the intersection regions.

- Interior labels (clustered, reading roughly): ל [L], ז [Z], ק [Q], א [A], פ [P], ה [H], ס [S], מ [M], ט [T], ש [Sh], פ [P]
- ✻ marks appear at three interior intersection points.

Diagram. A circle crossed by two diameters at an oblique angle (a sphere-with-two-great-circles schematic), labels around the rim.

- ה [H] (top): pole point
- ג [G], ב [B] (upper left): rim points
- ל [L], ט [T] (left): rim points
- א [A] (lower left): rim point
- ד [D], ז [Z] (right): rim points

Diagram. A large circle with several inscribed great-circle arcs (an armillary-style sphere projection) and many labeled rim/interior points.

- ג [G] (top): point
- א [A] (upper interior): point

- ב [B], צ [Tz], ת [T] (interior, right of center): points
- ט [T] (left): rim point
- ד [D] (left): rim point
- ה [H], מ [M], פ [P] (lower interior): points
- ל [L] (lower right): point
- ש [Sh], ס [S] (bottom): rim points
- ע [Ayin] (bottom): rim point

Figure Plate II

This plate holds the lettered figures ציור יז (Figure 17) through ציור ל (Figure 30): spherical-triangle circle diagrams with inscribed degree-values, and several worked plane triangles carrying side-lengths and angle measures. The figures are numbered continuously from Plate I, confirming a single figure series spanning both plates.

Diagram. A circle (spherical-triangle projection) densely inscribed with degree-values along its arcs and chords.

- Inscribed numerals (reading around the figure): 17.5, 23.30, 4.51, 22.55, 16.30, 47.30, 06, 59, 44.59, 72.55, 73.37, 23.30, 42.30, 72.55, 4.51, 42.30
- [dense center cluster: overlapping values incl. 47.30 / 43.30; "132" and "24.30" not confirmed] [?]
- ב [B] (top): point on circle
- א [A] (upper left): point
- ה [H] (left): point [?]
- ל [L] (lower interior): point
- ס [S] (lower right): point
- ▢ (bottom): right-angle mark at base point

Diagram. Two stacked circles (spherical-triangle examples), each inscribed with degree-values; a brace joins them at center-left.

- Upper circle numerals: 54, 60, 11.11, 11.40, 157.14, 158.49, 21.11, 70, 120, 24.49, 157.14, 54, 24.49 [?]
- Lower circle numerals: 158.49, 120, 24.49, 60, 70, 110, 126, 54, 51 [?], 126, 22.46 [?], 70, 110, 120, 155.11 [?], 60, 158.49
- (no Hebrew point-letters legible on the rims) [?]

Diagram. A small spherical triangle (curved-sided), apex at top.

- א [A] (top): apex
- ב [B] (lower left): base vertex
- ה [H] (lower right): base vertex
- ד [D] (bottom center): foot point

Diagram. A circle with an inscribed spherical triangle and degree-values along the arcs.

- Inscribed numerals: 90, 68.37, 46.50, 14.33, 90, 68.37, 27, 43, 133.10, 137.17, 68.27, 43.40 [?], 68.27, 43.10
- א [A] (left): point
- פ [P] (lower left): point
- ק [Q] (bottom): point
- ר [R] (right): point [?]

Diagram. Two spherical triangles (curved-sided), side by side under a brace.

- Left triangle: א [A] (top), ג [G] (lower left), ב [B] (lower right), ד [D] (bottom, with angle arc)
- Right triangle: א [A] (top), צ [Tz] (lower left), ב [B] (lower right), ד [D] (right, right-angle/dot mark)

Diagram. A spherical triangle with internal cevians and small interior labels.

- א [A] (top): apex
- ש [Sh], ל [L], ק [Q] (right side, descending): points on right side
- ב [B] (interior): interior point
- פ [P] (interior): interior point
- ד [D] (lower left): base vertex
- ה [H] (lower right): base point
- ג [G] (bottom): base vertex

Diagram. A spherical-triangle "leaf" figure (two arcs) with inscribed degree-values.

- Inscribed numerals: 63, 30.31 [?], 27, 13.44, 15.39, 30.31, 33.57
- א [A] (top): apex
- ה [H] (left): point
- ד [D] (lower interior): point
- ר [R] (bottom): point [?]

Diagram. Two right triangles side by side under a brace, drawn with vertical altitudes; labeled with repeated ב/ג/ה/ת along the bases and apexes (a similar-triangles / proportion construction).

- Left triangle: ת [T] (apex), ג [G] / ג [G] (right side division), ב [B] / ב [B] / ב [B] (base division points)
- Right triangle: ת [T] (apex), ג [G] / ג [G] (right side), ה [H] / ו [V] / ז [Z] (interior verticals), ב [B] (base)

Diagram. Two plane right triangles under a brace, carrying numeric side-lengths and angle measures.

- Left triangle (the 3-4-5 right triangle): side 4 = ב–א (base), side 3 = ת–ב (vertical), hypotenuse 5 = א–ת; angle at upper vertex 63.8 [= 36.52 / 53.8 region — see note]; the value 36.82 is written along the hypotenuse.
- Right triangle: apex א [A] with angle 80; lower-left angle 60; lower-right vertex צ [Tz] with angle 40; vertex ב [B] at bottom.

Diagram. An isosceles plane triangle with a vertical altitude to the base, side-lengths marked.

- א [A] (apex): top vertex
- ב [B] (lower left), צ [Tz] (lower right): base vertices
- Sides: left slant 10, right slant 10; base split into 6 + 6 by the altitude; altitude/base mark 3; the value 53.8 written at the apex angle.

Diagram. Two triangles under a brace: a left scalene triangle and a right triangle with an altitude and an angle arc at its base.

- Left triangle: א [A] (apex), ד [D] (lower left), ב [B] (mid base), צ [Tz] (lower right)
- Right triangle: ב [B] (lower left), ד [D] (foot of altitude, with angle arc), צ [Tz] (lower right), apex above

Diagram. A plane triangle with an internal altitude to the base, carrying side-lengths and angle/segment values.

- א [A] (apex): top vertex
- ב [B] (lower left), צ [Tz] (lower right): base vertices
- ג [G] (foot of altitude on base, with angle arc)
- Values: left slant 13; base-left angle 67.23; along the altitude region 30.31 and 67.23 [?]; right slant 20; base-right values 111.37 [?] and 36.83.

Diagram. Two triangles under a brace: a left obtuse triangle with dotted construction lines and ✻ marks at two interior points, and a right isosceles triangle with a vertical altitude and a right-angle/double-dot mark at the base.

- Left triangle: א [A] (apex), ב [B] (interior, with ✻ near it), צ [Tz] (lower left), with ✻ marks at two interior points and an angle arc at lower left
- Middle small triangle: א [A] (apex), ד [D] (foot, right-angle mark), ב [B] (lower left)
- Right triangle: א [A] (apex), ב [B] (lower left), צ [Tz] (lower right), ד [D] (foot of altitude with double-dot ⋅⋅ mark)

Diagram. A plane triangle carrying fractional side-lengths and small inline computations (an arithmetic worked example).

- א [A] (apex): top vertex
- צ [Tz] (lower left), and base running to the right
- Values: left slant 7; right slant 5; a vertical-region note "25=13" [?] over "16…"; base segments 4½ and 2½ [or 2¼]; the base-line computations "8÷12…" and "÷1¼" [?]
- An equals sign (=) sits at the lower right.

Chukos Shamayim [חקות שמים — "Statutes of Heaven"]

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Sefer [ספר]

Chukos Shamayim [חקות שמים]

By the great Rav, the Gaon, the Kabbalist, the all-encompassing sage, the divine philosopher, our teacher R. **Yosef Shlomo Rofeh Delmedigo** [Yosef Shlomo Delmedigo], called **R. Yashar of Candia** [יש"ר מקנדיאה], may the memory of the righteous be a blessing.

Third Part of Sefer Elim [חלק שלישי לספר אילם]

First printed at Amsterdam in the year [1628/29 = 5389] שפ"ט, at the house of the great and learned sage, the divine investigator, our teacher and master R.

Menasseh ben Israel [מנשה בן ישראל], of blessed memory, and corrected under his own hand.

And now printed a second time, with the addition of glosses and corrections, together with an account of the author's life and a transcription of the inscription on his grave, and also a picture of the likeness of his face.

Arranged [from the texts] of our masters, our teacher R. **Meir Grinshpan** [מאיר גרינשפאן], and the chief printer [completed it]; our master, our teacher R. **Moshe Eliezer Belinson** [משה אליעזר בעלינסאן], may his light shine.

ODESSA [ארעסא]

Printed at the press of **M. A. Belinson** [מ. א. בעלינסאן].

[1866/67 = 5627] **תרכ"ז**.

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[The verso of the title page is otherwise blank; faint show-through of the title-page text and a second impression of the ownership stamp are visible but carry no separate content.]

Sefer Chukos Shamayim [ספר חקות שמים]

[Body page 221.]

Said the respondent: You asked of me, my lord, the noble one — for the second time — a brief exposition of the difficult matters in the **Almagest** [Sefer ha-Almagesti]. It is plain from your words that your longing and desire are to understand the first two Books, which contain all the principles of the First Motion and its derivatives, by a short and easy path. The times, moreover, lead us like a young lion; I therefore took fright and held back at first, for the matter is not in my power, on account of the impediments that hold back the work [or: the circumcision — צי"ץ, an abbreviation, reading uncertain [?]]. Yet to satisfy your wish I will turn to you — though I shall be brief where it would have been fitting to expand. For, as I have informed you, I have writings and expositions on nearly the whole of that book; only the press did not have time to set them. There, in ten chapters on the First Book, I treated at length and weighed back and forth the essence of the celestial sphere and the description of the world's arrangement — whether according to the view of Aristotle, of Ptolemy, of Copernicus, or of those

who came after them. Perhaps the Lord will grant me relief from all my troubles, so that I may return to my studies and arrange them for students, for these are weighty matters to those who know and recognize them.

Now, on the tenth chapter of the First Book, I composed a treatise of its own, demonstrating the derivation of the [trigonometric] complements, the secants, the tangents, and the chords belonging to each and every part of the circle, together with the table called the **Table of Gold** [Luach ha-Zahav]. I shall therefore begin here from the eleventh chapter, and I shall mention only what is needed for the purpose — which is to make things easier and more useful. I have called the title of this treatise **Chukos Shamayim** [Statutes of Heaven]: "Sky and time their Creator gave them; they shall not grow weary in their going, nor change their motion, for their paths are straight." [Allusion to Tehillim 148:6 and similar.]

The Eleventh Chapter of the First Book of the Almagest [הפרק האחד עשר מהמאמר הראשון לאלמגיסטי]

The foundation of these studies, the pillar of the pillars upon which the whole edifice of astronomy and the determinations of the first moving body rest, is implanted within it. It is the knowledge of the measure of the arc of the great circle that descends from the [celestial] poles and is bounded by the equator and by the ecliptic line in the sphere [ba-sphera]. According to the astronomers, this is called the **Great Obliquity of the Sun** [Netiyat ha-Shemesh ha-Gedolah — the obliquity of the ecliptic]. For this reason all the sages, the moderns as well as the ancients, applied themselves — with precision of instruments and great diligence — to learn it as it truly is.

Now, the author [Ptolemy] mentions in this chapter two instruments: one made of complete circles, the other a quadrant of a circle. Their construction and use are explained in his words. The second instrument is of the kind in use among the great astronomers whom I served [studied under]; it is large, divided with precision down to distinguishing the seconds [ha-dakim ha-sheniyim — arc-seconds] within it. Its rim was overlaid and plated with brass of Cyprus upon dry wooden boards. They set it upon perpendicular angles, with its plane parallel to the meridian circle [the half-day circle] and its like — this is the sextant, and the third [is the] octant of a quadrant, and several kinds of instruments which I shall mention in order in another place, God willing.

The said obliquity has been found by the sages to differ at different times. For the author himself, and before him **Eratosthenes** [Eritushtinish] and **Hipparchus**

[Iparkhus], found it to be **23.51.20** [23° 51′ 20″]; but **Copernicus** [Copirnikush] found it to be **23.28** [23° 28′]; and last of all **Tycho Brahe of Denmark** [Tiko Brai mi-Daniah], who excelled in perfecting great and awe-inspiring instruments,

[Body page 222.]

found it to be **23.31.30** [23° 31′ 30″]. I shall not adduce the proof of what lies between these values, for they are close to one another and many in number. They reckoned that this discrepancy did not arise on account of imperfection or deficiency in the instruments and observers, for these [observations] were made over the span of many years and under different horizons. Rather, they verified that it occurs because of the approach and recession of the equator — for the ancients did not recognize any motion in latitude in the sphere, since all the fixed stars preserve their position in latitude even as they move in longitude. And if this is so, this [variation] derives from the nearing of the equator to the ecliptic and its withdrawing from it — though Tycho holds that he found a motion in latitude for the ecliptic.

However it may be, they confirmed and established, according to the variation they found over several hundred years, that the measure of the motion of the equator is about **36** first minutes [36′]. Their arguments are set out in my commentary on the Almagest. We shall follow in the author's [Ptolemy's] footsteps in this measure, and upon it we shall build our reckonings — namely **23.51.20** [23° 51′ 20″] — for we are compelled [to do so], since we are expounding his words.

As for the manner of determining it: the student can readily attain it himself. He should take, by means of one of the suitable instruments, the height of the sun at midday at the time of the summer and winter solstices. This should be done over several days both before and after the solstices, so that he may know with certainty the sun's true position on the horizon. He then subtracts the winter altitude from the summer altitude, and the result is the distance between the two solstitial circles, Cancer and Capricorn. Half of this is the Great Obliquity sought. There are various methods of determining this, and the matter is known even to those who go down to the sea in ships — there is no need to elaborate.

Chapter Twelve [הפרק הי"ב]

I shall instruct you and show you how, in this chapter, all the things mentioned [above] were learned by means of the solution of triangles, and upon it they founded and built. We are obliged to praise the author for the great goodness he

has bestowed and the surpassing wisdom he has bequeathed to us. For a fuller exposition of his words, I shall bring [forward] premises and propositions concerning the calculation of the ratios in which the author's treasures lie hidden.

Proposition 1 [הצעה א']

The ratio of the extremes is compounded of the ratios between them. [בערכים יחס הקצוות מורכב מהיחסים שביניהם]

This premise, to make its truth widely known, was adduced by **Jordanus** [Yordanus — Jordanus de Nemore] in Book 2 of his *Book of the Numerical Elements* [Sefer ha-Yesodot ha-Misperiyot — *De elementis arithmeticae artis*], among the general items of knowledge; and it is alluded to in the tenth definition of [Book] Five and in the seventeenth proposition there, in the words of **Euclid** [Euclides], who makes much use of it in [Books] Six and Seven; and likewise **Archimedes** [Arkhimedis] in the [book] *On the Sphere and the Cylinder* [Pe ha-Kadur ve-ha-Amud — *De sphaera et cylindro*]; and **Menelaus** [Milaus] in his Book Three; and all the masters of these studies employ it. Therefore Ptolemy the author too acknowledges it and proclaims it in his own idiom in this chapter, and by it he confirms his two propositions [as] straight, well-founded standings, as you will see.

Indeed, it is one of the first intelligibles, plain from the postulates of algebra [ha-algibra], whose properties serve to test all questions concerning the essential [quantities]. Let us suppose that we wish to find, between **9** and **3** — which are the two extremes of the proportion — one number or more, such that the ratio of 9 to that number, and the ratio

Chukos Shamayim — Almagest Commentary (continued)

that same number with respect to 3, taken together, equals the ratio of 9 to 3. Let that number be **A 1** [the symbol for 1]; multiply it by 9 and it yields **A 9**; likewise multiply it by 3 and it yields **A 3**. And the equality [holds] between **A 9** and **A 3**, which equals the original numbers. The question, joined with this present demand, teaches that every number — or set of numbers — that you wish to insert between 9 and 3, when their assembled ratio is 9/3 [?] [the printed fraction here is hard to read; context calls for the ratio 9 : 3], the geometers explain in the

sixth book of the Elements [Euclid VI], in the fifth definition of the sixth, as they made clear there. Suppose we place between 9 and 3 the number **2.** Now the ratio [of 9 to the inserted term]… [the printed fractions in this clause are partly abraded] is 9/2 [?]; combined with the ratio 2/3 [?] it gives 18/6 [?], which equals 3 [as a ratio]. Suppose we further place between them **4** and **2**, and the ratios 9/4, 4/2, 2/3 would rise to 9/3 [?]. And so in all the particular numbers; therefore the lesson is likewise clear in general terms.

Now, **Eutocius of Ascalon** [איזותוכואוס האשקלונו = Eutocius of Ascalon], in his commentaries on the aforementioned book of **Archimedes** [Archimedes], and in his commentary on **Apollonius** [אפולוניאו = Apollonius], skillfully expounded this by demonstration; and likewise **Vitellio** [ויטיליאו"ני = Witelo / Vitellio] in his composition on the science of optics ["the great science of vision," *Perspectiva*], Book I, Proposition 13 [?]. Whoever delights in these may study them — but they posit another lesson, [presupposed] as known and self-evident, upon which they build the demonstration and that lesson, [and] the more its truth is concealed, in truth, as I explained elsewhere. It is true that, to corroborate the words of Ptolemy, it suffices to uphold it by demonstration on the basis of Euclid's postulates, in this manner.

(See figure A.) Since there are two magnitudes, **A**, and a mean is placed between them — whichever you wish — **B**, in truth the ratio of **A** to **G** is compounded of the ratio of **A** to **B** and the ratio of **B** to **G** together.

The demonstration: Let us multiply **A** by **B**, and there results the area **D**; likewise **B** by **G**, and there results the area **F**. Now, by the fifth definition of the sixth [book] of Euclid, the ratio of **D** to **F** is compounded of the ratio of **A** to **B** and of the ratio of **B** to **G**. But that the ratio of **D** to **F** is the same as the ratio of **A** to **G** is made clear in this manner: since multiplying **A** by **B** yields **D**, and multiplying **B** by itself by **G** yields **F**, therefore — by Proposition 17 of the seventh [book] of the Elements [Euclid VII.17] — the ratio of **D** to **F** is exactly the ratio of **A** to **G**. And the ratio of **D** to **F** is compounded of the ratio of **A** to **B** and the ratio of **B** to **G**, as was explained just now; therefore the ratio of **A** to **G** too is compounded of those same ratios. Here, then, you have it: the ratio of the extremes is compounded of the ratios between them — that is, **A** to **G** is compounded of **A** to **B** and **B** to **G**. And likewise, were there more magnitudes between them, the matter would be explained; and this is what was sought.

How good this demonstration would be, were its premises themselves demonstrated and not leaning upon it [in a circle]! For the fifth definition of the sixth [book] of the Elements requires this very lesson, and so by this account they

are explained one through the other — and that is an outright error, well known to beginners in logic. Nor can one say that the definitions are not proved by demonstration because they are first principles and concessions of Euclid: for Euclid did not state the definitions of his Elements [as bare axioms] — as he made clear in his teachings — for indeed the definition in question is liable to the doubt that follows from it, namely that the part would be greater than the whole, or equal to it, against the general knowledge [common notion 9 [?]] that Euclid brought in the first book. This is not so with the other definitions. In any case, it [the definition] needs to be strengthened and clarified; and therefore it is better that the aforementioned definition be confirmed from this lesson, and let this be taken as well-known and as a first intelligible, until the more manifest first principles are made known to us, by which it will be explained. It is enough for us to know that it is true and trustworthy, until [God] opens to us the gates of understanding.

Proposition 2 (הצעה ב')

A ratio is called "compounded of ratios" when the measures of the ratios, multiplied one by another, generate some ratio.

This is Euclid's fifth definition of the sixth [book] of his Elements. And just as you learned, in arithmetic, that multiplication of fractions resembles the assembling [composition] of ratios — namely, that you multiply the two antecedents of the components one by another, and the product becomes the antecedent in the compounded ratio, and the consequents one by another, and the product becomes the consequent of the compounded ratio.

For example: we wish to assemble the ratio 3/1 with the ratio 2/1; the compounded [ratio] is 6/1. Likewise [combine] 2/1 with 3/4, they compound to 6/4; likewise 1/2 with 1/4, they compound to 1/6 [recte: 1/8]; likewise 1/4 with 3/5 [and] 4/1, they compound to 12/20; and so on with their like.

The demonstration of the rectitude of this operation, and its verification — that it is in this manner **(see figure B)** — that is, the antecedents one by another and the consequents one by another, will be explained thus. Suppose, for example, we wish to assemble the ratio **A** to **B** with the ratio **G** to **D**. Let us multiply **A** by **G**, and there results **H**; likewise **B** by **D**, and there results **T** [ט]. I shall show that the ratio of **H** to **T** is compounded of the ratio of **A** to **B** and the ratio of **G** to **D**: for I multiply **A** by **D** and there results **Z** [ז], and I place this as a mean between **H** and **T**. Now, according to the proposition that preceded, and because the measures of **Z** and **H** are generated from the multiplication of one measure, **A**, when it is

multiplied by **G** and by **D**, therefore — by Proposition 17 of the seventh [Euclid VII.17] — the ratio of **Z** to **H** is as the ratio of **G** to **D**. And likewise, because the measures of **Z** and **T** are generated from the multiplication of another single measure, **B**, when it is multiplied by **A** and by **D**, the ratio of **Z** to **T** too will be as the ratio of **A** to **B**, by the aforementioned Proposition 17 of the seventh. And accordingly these latter two ratios, **Z**/**H** [and] **Z**/**T**, are equal to the two original ratios, **G**/**D** [and] **A**/**B**. But the ratio of **H** to **T** is compounded of the ratio of **H** to **Z** and the ratio of **Z** to **T**, according to the proposition that preceded. Therefore **H**/**T** too is compounded of the ones equal to them, namely **G**/**D** [and] **A**/**B**; and we have done well in assembling them. This is what was meant.

Proposition 3 (הצעה ג')

Ratios are called "subtracted one from another" when one is divided by its fellow.

Knowledge of opposites is one [and the same]. If indeed the assembling of ratios is by multiplication of fractions, then the diminishing of them will necessarily be by division of fractions: namely, that we multiply the antecedent of each [ratio] by the consequent of the other, and the result you set under the antecedent. It is best to set the diminished [ratio] — which is the larger — on the left, so as not to become confused; and this [holds] when you know the larger. From the products you will know with certainty which is the larger of the two ratios: for whichever product turns out greater than its fellow's product is the larger; and from it, then, the smaller is subtracted. And the ratio between the products is the remainder from the diminishing of the ratios.

So, for example: let there be two ratios, **3/4** and **1/2** [the two ratios are printed as the pairs 3 4 and 1 2]. I do not know which exceeds its fellow. I multiply 4 by 2, there results 8, and I set it under the antecedent that was multiplied, 4 [yielding 8 under 4]. Again I multiply 3 by 1 [?], there results 3, and I set it under the antecedent 3 [yielding 8 under 3 — the two products being 8 and 3].

the multiplied [antecedent] 3. And since the product 8, the larger, belongs to the product 3 [i.e. is paired with the antecedent 3], the difference is 4/1 [?]. I conclude that the ratio 3/4 [?] is larger than the ratio 1/2 [?], and their remainder is the ratio 2/3 [?]. Consider well in this matter, lest you interchange the antecedent with the consequent. And if you test your operation in the manner of the arithmeticians, and go back to assemble [the remainder ratio] with the smaller, the two will come out for you correctly — assemble 2/3 [?] with 1/2 [?], there comes out 3/4 [?], so you have verified it. But were you to suppose that the

remainder is 1/6 [?] and assemble it with 1/2 [?], there would come out 1/2 [?] — and this is not the ratio 3/4 [?]; so let your eyes be open.

Now, the demonstration of this operation is in this manner **(see figure G [ציור ג])**: let us wish, by way of example, to subtract the ratio **G/H** [גה] from the ratio **A/B** [אב]. Let us multiply **A** by **D**, and there results **H** [ה]; likewise **G** by **D** [?], and there results **Z** [ז]. I shall show that the ratio of **B** to... [the residue] is the remainder from the diminishing of the two aforementioned ratios: for I multiply **B** by **D** and it becomes **T** [ט/ם] [?]. There would be, accordingly, three magnitudes: **D**, **Z**, **T** [?]. And, by the first proposition, the ratio of **H/T** [הט] is compounded of the ratios **H/Z**, **Z/T** [?]. Therefore, if we subtract the ratio **Z/T** [זט] from the ratio **H/T** [הט] — that is, the part from the whole — the ratio **H/Z** [הז] is left over. But the ratio **Z/T** [זט] equals the ratio of **D** [?], by Proposition 17 of the seventh, for it is generated from the multiplication of **B** by **A** and by **D**; and likewise the ratio **H/T** [הט] equals the ratio **A/B** [אב], for it is generated from the multiplication of **D** by **A** and by **B**. Therefore, if we subtract from the ratio **A/B** [אב] the ratio **G/H** [גח], the remainder too will be the ratio **H/Z** [וז], just as we subtracted by our operation. And so, with division, the work of subtraction is completed; and this is what was meant.

Lesson 1 of Ptolemy, on Straight Lines, by Composition (למוד א' לבטלמיוס על הקוים הישרים כפי ההרכבה)

When **King Ptolemy** — the ruler over the science of astronomy — set out to expound, with utmost brevity, the foundations of the science, he prefaced two lessons founded upon straight lines; and upon these, in [his] science, he founded all the lessons that follow according to the path he chose. We who come after him saw the difficulty of the matter and the ease of error for those who follow in his footsteps; therefore we took pains to remove the stumbling-block from the students. I shall draw out some of his words and add others from those of the commentators. See that you understand that lessons such as these were written by Euclid, author of the Elements, and they were of various kinds and were lost through the ruin of the times. The author [Ptolemy] did not mention them by the name of their speaker, for such was his way — not to mention many lessons. And **Theon** [תיאון = Theon (of Alexandria)], who expounded the book of the Almagest, brought six of these lessons; and I showed twelve in my commentary there. And just as Ptolemy brought only two — those that were needed for his work: the one by composition, and the second by division — so too I shall bring only the first of his, and in place of the second I shall bring the sixth of the aforementioned Theon. Both are by composition, for those are the ones I need, and no others.

And this is the statement of the first lesson:

When from one and the same point there issue two straight lines bounding an angle, and from their extremities there are drawn inward two straight lines that intersect one another, and [meet] with the outer [lines] — then the ratio of one of the outer [lines], entire, to its upper segment, the one near to it, is compounded of two ratios: namely, the ratio of the whole inner line drawn from the extremity of the aforementioned outer [line] to its segment far from that extremity, and the ratio of the segment near the extremity of the second inner line drawn, to that entire second drawn [line].

And let it be, by way of example **(see figure D [ציור ד])**: a point **H** [ה], from which there issue the lines **H-Tz**, **H-L** [הצ, הל] bounding the angle **Tz-H-L** [צהל]; and from their extremities **Tz**, **L** [צ, ל] there are drawn inward two straight lines **Tz-K**, **L-Ch** [צק, לח] that intersect one another, this with that, at the point **T** [ט], and [meet] with the outer [lines] at the points **Ch**, **K** [ח, ק]. I wish to show that the ratio of **H-Tz** to **H-Ch** [?] is compounded of the ratio **Tz-K** to **T-K** [טק] and the ratio **L-T** to **L-Ch** [לט אל לח].

For its demonstration, we draw from point **H** [ה] a line **Ch-P** [חפ] parallel to the line **Tz-K** [צק], by Proposition 31 of the first [Euclid I.31]; and it will be, by Proposition 29 [Euclid I.29], that the angle **Ch-Tz** [?] equals the angle **P-Ch-Ch** [?], and the angle **Tz-K-H** [צקה] equals the angle **Ch-P-H** [חפה], and the angle **H** [ה] is common to the two triangles **H-P-Ch**, **H-K-Tz** [הפח, הקצ]. Therefore it will be, by Proposition 4 of the sixth [Euclid VI.4], that as **H-Tz** is to **H-Ch** [?], so **Tz-K** to **Ch-P** [צק אל חפ]. And by the first proposition that is in this book, we take a mean between **Tz-K** and **Ch-P** [צק, חפ], the line **T-K** [טק], and accordingly the ratio of **Tz-K** to **Ch-P** [?] — or to the one equal to it, **H-Tz**/**H-Ch** [?] — is compounded of the ratio **Tz-K** to **T-K** [טק] and the ratio **T-K** to **Ch-P** [טק אל חפ]. But the ratio **T-K** to **Ch-P** [?] is as the ratio **L-T** to **L-Ch** [לט אל לח], since **T-K** [טק] is parallel to the line **L-Ch** [?], by the second of the sixth [Euclid VI.2]. Therefore the ratio **H-Tz** to **H-Ch** [הצ אל הח] is compounded of the ratio **Tz-K** to **T-K** [צק אל טק] and the ratio **L-T** to **L-Ch** [לט אל לח]. This is what was meant.

Lesson 2, likewise by Composition; this is the sixth of Theon (למוד ב' גם כן כפי ההרכבה והוא הששי לתיאון)

It is further verified, in the aforementioned lines, that the ratio of one of the outer [lines], entire, to the lower segment at its extremity is compounded of two ratios: namely, the ratio of the upper segment of the second outer line to the lower segment at its extremity, and the ratio of the segment near the angle of

intersection of the line drawn from the extremity of this second outer [line] to the segment far [along] this drawn [line].

I mean to say that the ratio **A-G (see figure H [ציור ה])** to **H-G** [הג] is compounded of the ratio **A-D** to **D-B** [אד אל דב] and the ratio **B-Z** to **Z-H** [בז אל זה]. For its demonstration, we draw from point **A** [א] a line **A-Y** [אי] parallel to **B** [?], by Proposition 31 of the first, and we extend the line **B-D** [בד] to it, up to point **Y** [י]. And since the two triangles **A-Y-G**, **H-Z-G** [איג, הזג] are equal in their angles [equiangular], by Proposition 29 of the first, they will be — by the sixth — in proportion in their sides; so that as **A-G** to **H-G** [אג אל הג], so **A-Y** to **H-Z** [אי אל הז]. And we place a mean between them, from the first proposition of this book, the line **B-Z** [?], and the ratio **A-Y** to **H-Z** [?] — or the one equal to it, **A-G**/**H-G** [?] — is compounded of the ratio **A-Y** to **B-Z** [אי אל בז] and the ratio **B-Z** to **Z-H** [בז אל זה]. But the ratio **A-Y** to **B-Z** [אי אל בז] is as **A-D** to **D-B** [אד אל דב], since the two triangles **A-D-Y**, **B-D-Z** [אדי, בדז] are equal in their angles by Proposition 29 of the first, and they too will be — by the sixth — in proportion in their sides. Therefore the ratio **A-G** to **H-G** [אג אל הג] too will be compounded of the ratios **A-D** to **D-B** [אד אל דב] and **B-Z** to **Z-H** [בז אל זה]. And this is what was meant.

Said the respondent [the author]: This place presses me to say a word concerning the lesson of the six ratios, even though I have already enlarged upon it in the *Book of Number* [ס' המספר — Delmedigo's arithmetic work], where I expounded the treatise that **Alkindrus the philosopher** [אלכינדרוס הפלוסוף = al-Kindī (?)] composed on this whole subject. You already know that the masters of arithmetic teach that, when we wish to know in how many ways the arrangement of a sum of things changes — if they are shifted from rank to rank — we multiply that sum by half of the number preceding it, and the product is the sum of the modes of [permutational] change. For example, six letters — **A B G D H V** [א ב ג ד ה ו] — change their arrangement thus:

אב	אג	אד	אה	או	בג
בד	בה	בו	גד	גה	גו
דה	דו	הו	...		

[these are the two-element combinations of the six letters: AB, AG, AD, AH, AV, BG, BD, BH, BV, GD, GH, GV, DH, DV, HV — 15 pairs. The list continues beyond the page.]

Chukos Shamayim (חקות שמים) — Almagest commentary

the product of half the preceding number, 5, by the following number, 6 (six multiplied by 6), and likewise four [items]. They will vary in six ways — namely אב, אג, אד, בג, בד, גד — and no more, if you take them in pairs as pairs and in their proper order. But if you also reverse them, that is, ב→א [becomes] בא, and so on, then the six [→ fifteen] letters [pairs] will vary into 30; and the four [→twelve] into 12.

It is not our present concern to treat every variation produced when [the items] are not taken as pairs in pairs, as the author of *Sefer Yetzirah* said: "two stones build two houses, four [stones build] 24 [houses]," 24 [up to] 720 [= 6!]. Thus you find the product of the number 6: 1·2·3·4·5·6 — for this is of no use to us in our enterprise. **Rather, my intent here** is to inform you that in compounding six ratios, the compounded ratio can vary into 15 ways in proper order and into 15 in reverse, and that the component ratios vary into 12 distinct ways — there being no difference between saying that the ratio ⅝ [?] is compounded of the ratio ½ and the ratio ⅘ [?], and saying [it is compounded] of the ratio ⅘ and the ratio ½. Therefore they are not 24 ways but 12.

Let these letters stand for the numbers [6 5 4 3 2 1 =] א ב ג ד ה ו, and these are the twelve kinds into which the six numbers of two ratios will vary:

8 3 1 2 4 3 — 83 21 43. גד הו — and to all appearances it would seem that if you combine each of the 15 compounds with these twelve — 41 .23 גר וה — that is, you say that there is a compound from the ratios גד הו, or from the ratios גד וה, and likewise אג ואד — 82 31 .43 …גה דו; from them there would be 180 kinds in proper order, and as many in reverse, for you would say שבג — 41 .33 …גה וד compounded of the ratios גד הו, etc.; or there would be 720, if you also take …גו דה 84 21 .33 — the compounded with the components; but when you consider well, it is not so, for many of them are alike. 31 23. גו הד — the compounds with the components, that is, גד the compound… 34 82 31 — גד הו and not from גד וה, and likewise גו [is] not [compounded] from… גו דח, and not from 84 32 גו הד. דג וה — and so on; and thus it is in reverse. And accordingly many drop out for this reason, for גר גה גו דה דו הו — 3.3 84 12 דה וג are among the compounds and components, and besides this on the side of… 82 14 דו הג — one fewer comes out; and this [you will see] if you test them by the ratio mentioned, according to the numbers [written]… 84 21 .33 הג וד — that beneath the letters of the ratio mentioned [you write] that this ratio ⅝ [?] is compounded of the ratios ½ [?], ⅘

[?]… 83 .34 הד וג — He is the touchstone of silver and copper [a metaphor: this method is the assay-stone]. You find six kinds of the compounds… 14 83. 23 — which are excluded, that cannot be compounded from the others in any way, and these are: בג בה גה גו — 84 .13 …[ו]אד א, whose ratio אד is ⅘ [?]; and there is not one of the twelve kinds of components… 23. 38 14 — that begets the ratio ⅘ [?]; and likewise of those remaining; and when you subtract six from fifteen, nine remain; and with the twelve… 28. 33 — kinds [they] beget 108 ways. **However,** when you assay them with the touchstone mentioned, [you do] not… 12 83. 43 — find for each of the nine ways one that is compounded of all twelve kinds, but only… 13. 82 — two, that is, אב, which is compounded of the ratio גד הו and likewise of the ratio גו הר, and not…

from others. And likewise the touchstone for the rest, according to the signs that [serve] to indicate them. And these tables are comprehensive — [241 1472] — useful, and they will be savored by the discerning.

The twelve compositions (fol. 227, lower band). The table reads right-to-left in the source; logical (right→left) order preserved. Each numbered head carries three lines — an upper ratio-pair, the head (its number and defining pair), and a lower ratio-pair — over the item set {א ב ג ד ה ו} = {6 5 4 3 2 1}.

line	1 = אב	2 = אג	3 = אה	4 = בד	5 = בו
upper	גד הו	בד הו	בו גה	אג וה	אה דג
head	[1] אב	[2] אג	[3] אה	[4] בד	[5] בו
lower	גו הד	בד גו [recte:?]	בד גו [recte:?]	אה וג	אג דה

The compositions continued (fol. 228, heads 6–9). Same three-line-per-head grid; logical right-to-left order preserved.

line	6 = גד	7 = גו	8 = דה	9 = דו
upper	אב וה	אב דה	בא גו	אב דג
head	[6] גד	[7] גו	[8] דה	[9] דו
lower	אה וב	אה דב	בו גא	אג דב

It comes out that there are but 18 ways in proper order, and as many in reverse.

And I myself have found for you a wonderful figure — *[ḥemonah niflaʾah]*, see figure (ז) — resembling the game [?] that boys play on their fingertips with a thread, by which you can tell at first glance, at first sight, the good ways from the spurious. This [is done] by your attending to the letters: all those that are joined to one another by straight lines, whether by way of the rib [side] or the diagonal, can be measures whether for the component ratios or for the compounds. But the letters that are not joined by lines — these are wholly invalid. For example, ratio אד — its [pair] is not joined; and therefore the ratio אד, that is ⅛ [?], is not a compound of any kind of two ratios, as we explained.

However, ratio [? ⅔ =] או — this one is not invalid, since אב is like it, and it is good; and so it is among the components. רף [?]ראד, that is ⅛ [?], will not compound any one of the compounds mentioned; and this is a "blossom upon a bud" [a crowning ornament] for the one who understands the matter. This figure mentioned has several distinct properties; I will inform you of them elsewhere, God willing.

Now then, if any one of the six numbers is known when it [the compound ratio] is hidden [unknown], and the rest are known — or if the compound ratio and the component ratios are known — and you wish to know the second component, it is very easy: you need only [perform] a subtraction. I have already taught you, in the third preliminary [proposition] passed, the manner and order with the operation in [common] fractions, in the *Hindarsi* [Indian / decimal] fractions [?]. But in the fractions of astronomy [sexagesimal], great practice is required to master Ptolemy's computations and his tables; I will inform you of the method afterward, and lead you on its paths.

First, however, I will teach you a most useful matter. After I inform you that, were these the good ways — which are of six numbers — it would be possible to bring them down to four; and to reduce them — this would make all of Ptolemy's computations easy, and there is no [end of] worth to this lesson, and by it [the moderns] gained renown and made themselves famous [?], for the early masters discovered it. But know that they did not know all the ways, even those in which there are two equal numbers, in order to reduce them. And the experts in number reckoned and ruled that these ways cannot be brought down to four numbers, even when there is an equal pair among them — as **Cardanus** [Cardano] wrote in his *Sefer HaMispar* [*Ars Magna* / *Practica Arithmeticae*]; and his proof [is] that, to all appearances, if of the six 8, 9, 3, 2, 4, 9 [?] the first number, 3, which is repeated in both [pairs], [is removed] — there is no ratio between the remaining 9, 10, 3, 2, 4, 2 [?]; and likewise in the second [pair] and in

the third. But in the fourth, if you remove the equal 5, there will be a ratio between ½ and ⅘ [?]; but [reversed,] if you remove the equal 2 [?] — 5, 2, 1, 2, 5, 4 — there is no ratio between the remaining 15, 54 [?]. And all the more so when there is no number doubled within the six measures, which cannot be brought down to "the golden lesson" [the golden rule of reduction]. Thus thought **Regiomontanus** [Reiniomontano] and all the experts in the lessons until today. But it is not so, as I shall presently show.

Now, what there is here to understand is this: when there are within the six [numbers] two equal pairs, how shall you know, or [which] of them you should discard — as in the fourth example mentioned — and how shall you know to resolve the ratio; **and further**, how shall you know the order of the ratio, and not become confused, taking the first in place of the second or the third, and [knowing] which to double, [and] what ought to be divided [?] or…

[in order] to reverse [it]. Therefore I will reveal to you the matter from its source, and explain the subject from its foundation, all upon geometric premises, well-built unto the offshoots [conclusions]:

הקדמה א' [Preliminary 1]

When in six lines the ratio of the first to the second is compounded of the ratio of the third to the fourth and the ratio of the fifth to the sixth, then the solid [parallelepiped] enclosed by the first, the fourth, and the sixth is equal to the solid enclosed by the second, the third, and the fifth.

The demonstration is as follows (figure ז): let there be the six lines אב, גד, הו, and let the ratio of א to ב be compounded of the ratio of ג to ד and the ratio of ה to ו. I claim that the solid arising from the straight lines א ד ו [?] is equal to the solid arising from the lines ב ג ה.

Now let ג be multiplied into ד [to] yield ף [P], and let ה into ו be ח, and let ח be defined [as] the rectangle [made by] the ratio of ג to ד and the ratio of ה to ו. But of these two [together] is compounded the ratio of א to ב; hence [the ratio] א to ב will be as [the ratio of] ף [P] to ח. Again, from א into ף will arise the solid ל [L], and from ב into ף [?] will arise the solid ק [Q]. And since in ratio of bases [the ratio of] base of solid ק to base of solid ל — I mean as ף to ח — so [is] the height of solid ל to the height of solid ק — I mean א to ב. And the bases of solids [are] in ratio [inversely] proportional [together] with their heights; they will be, by Euclid XI.34 [למלמוד י"א ל"ד] [?], on the eleventh foundation [Book XI], the solids ל [and] ק equal. And that is what was to be proved [וזה שיש להוכיח / Q.E.D.].

And from here it is shown that when in six lines the ratio of the first to the second is compounded of [the ratio of] the third to the fourth and the fifth to the sixth — if that same line itself is found in two of the surfaces [faces], that is, in the surface where the first, the fourth, and the sixth are, and in the surface where the second, the third, and the fifth are — then, if the like line is taken from both surfaces, the surface enclosed by the two [remaining] in one surface (figure ח) is equal to the surface enclosed by the two others in the second surface.

Let the ratio of א to ב be compounded of the ratio of ג to ד and the ratio of ד to ה [?], and let one surface have א first, ב fourth, ו sixth, and another surface have ב second, ג third, ד fifth [?]. You see that line ב is found in the two surfaces; and if we separate it from the two surfaces, there remain אז [?] in one surface and בד in another surface, and the surface arising from lines א ז [?] is surface ם, and that arising from lines בד is surface ן [?]. I mean to make clear that these two surfaces are equal, since if line ב is doubled into ו it begets ח [?], and if [doubled] into ם it begets ל; and the solids will then, by the construction, be equal in their height. But they are also equal one to another, from this premise that was [just] premised; therefore their bases too are equal — by Euclid XI.34, [on] the eleventh foundation — that is, surfaces ם [equal], and that is what was to be proved.

There you have it shown, in a demonstrative way, whereby you can tell whether you can reverse [convert] the six ratios into four, and which of them you should take as the antecedent and consequent ratio. I mean that in each of the two ratios the measures must be from two surfaces — not both from one surface — and then indeed the two sides of each of the surfaces will be one surface, and its fellow — a thing precious indeed, and coveted in the science of astronomy.

הקדמה שנית [Preliminary 2]

And conversely: if the solid arising from the first, the fourth, and the sixth lines is equal to the solid arising from the second, the third, and the fifth, then the ratio of the first line to the second is compounded of the ratios of the third to the fourth and the fifth to the sixth.

And it will be as in the figure of the first Preliminary: the solid ל arising from lines א ד ו [is] equal to the solid ק arising from lines ב ג ה. We will show that the ratio of א to ב is compounded of the ratios ג to ה, ד to ו [?]. For the solids ק [and] ל are equal; they will therefore be, by Euclid XI.34 [למלמוד י"א ל"ד] [?] — on the eleventh foundation — [such that] their bases stand in proportion [inversely] with their heights. That is, the base of solid ק will be to the base of solid ל — namely ף to [ח] — as the height of solid ל to the height of solid ק — namely כא to

ב [?]. And since the ratio of surface [ל] to surface [ק] — [ם] to ו [?] — is compounded of [ם] to ד and ק to [?]… [as established] for the sixth, the ratio of א to ב will likewise be compounded of the ratio ג to ד and the ratio ה to ו. And that is what was to be proved.

נמשך ב' [Offshoot 2]

And it is shown from here that if there are six lines, the ratio of the first of them to the second being compounded of the ratio of the third to the fourth and the fifth to the sixth, then this composition can be permuted in eighteen ways. For the first, the fourth, and the sixth — from which we build the solid ל — can change their position in six ways without the solid ל being changed; and likewise the three lines ב ג ה, from which we build the solid ק, can change in their positions in six ways. And by combining each of the latter with the former [?] there would be 36 ways, of which half are in reverse; therefore 18 necessary kinds remain, [both] in proper order and in reverse [?]. And that is what was to be proved.

Now then, if you examine all the 18 kinds [previously] listed, and remove the number 3 doubled — which is found in differing surfaces according to the conditions of the premises — [it] reverses into four ratios. But as for what **Cardanus** [Cardano] brought in chapter 19 [פרק ט"י] of his *Sefer HaMispar* [*Ars Magna*] and in his *Sefer HaDakot* [*Book of Subtleties* / *De Subtilitate*] — he listed certain kinds, among those mentioned, and reckoned them irreversible — that is not so in truth. Let us take one of them and test it; it is ⅝ [?] ⅔ [?] ¾ [?]. Now I multiply the compound ¾ by 2, and it becomes ⅝ [?], and I find 8 in two surfaces and remove it, and the ratio remains 3 6 [→ 9?] [?] — and it is fitting and proportionate. And likewise let us take a second instance of its ratios, namely ½ ⅔ ⅗ [?], and I multiply the compound ⅗ [?] by 3 and it becomes ⁹⁄₁₅ [?]; and by this there will be 9 in two surfaces, and I remove them, and there remain 2, 10, 3, 15 [?] — and these are proportionate according to the rule. So there is no objection to the reversal.

And greater than this I will inform you: that even when there is no doubling among the six measures mentioned, you can reverse them — something the early masters of the craft did not believe, and no one perceived but myself. Let it be, for example, ½ ⅓ ½ [?], where all six are different. I [will] multiply the components crosswise [lengthwise-and-warp], and they all become of one kind [a common denominator]: ⁹⁄₁₅ [?] ¹²⁄₁₅ [?] ¹⁰⁄₁₅ [?] ⁸⁄₁₅ [?]; and I remove the number 15 from the two surfaces, and there remain 12 10 [→ 15?] 8 [?] — and these are proportionate. And so you may contrive with them all.

Diagram. Figure ז (zayin), cited in the text of Preliminary 1 as the "wonderful figure resembling the boys' string-game" and again as the construction figure for the six-line / solid relation. The figure itself is on a facing plate (not reproduced within this page's text block — only the in-text reference "(ציור ז)" / "(ליור ז)" appears here). The text describes it as a network of letters (the six items א ב ג ד ה ו and the auxiliary solids/surfaces ן, ם, ק, ל, ח, ף) joined by straight lines along "ribs" (sides) and "diagonals"; pairs joined by a line are valid composition-measures, pairs not joined are invalid.

- א, ב (the first/second terms): the principal ratio א:ב
- ג, ד (third/fourth): component ratio ג:ד
- ה, ו (fifth/sixth): component ratio ה:ו
- ף [P] (auxiliary): product ג·ד
- ח [Ch] (auxiliary): product ה·ו, = the rectangle of the two component ratios
- ל [L] (solid): א·ף
- ק [Q] (solid): ב·ף
- [Geometric role: solid ל (from א,ד,ו) is proven equal to solid ק (from ב,ג,ה) via Euclid XI.34.]

Diagram. Figure ח (chet), cited for the corollary (the surface/rectangle relation following Preliminary 1): when a common line appears in both faces, removing it leaves two rectangles ם and ן that are equal.

- The two faces (surfaces) sharing the common line ב
- ם (surface): rectangle from the remaining lines of one face (א ז [?])
- ן (surface): rectangle from the remaining lines of the other face (ב ד)
- [Geometric role: surface ם is proven equal to surface ן.]

Chukos Shamayim (חקות שמים)

I will now offer you a few of Ptolemy's worked examples, so that you may grasp the great benefit that follows from this [method] — and also its inverse [the converse operation]. Following our own approach, I will let you know how the division [of sexagesimals] is done, by means of the astronomical fractions [of arc]; this was set out in the *Sefer HaMispar* [Delmedigo's own arithmetic], which was dropped from the printing — [the work] of R. Eliyahu Mizrachi [רא"ם], may his memory be a blessing.

Now, in Ptolemy's computations you will find that five magnitudes are known and the sixth, the unknown, is sought. If [the sixth] is one of the magnitudes of

the compounded ratio [the product term], he uses multiplication: he gathers the components [of the compound] and the magnitude of the compound emerges. But if the unknown is one of the components, he uses division — that is, he subtracts [removes] the known component from the compound, and the second component emerges. This is in chapter 13 of the First Book [Almagest I.13].

He said, of blessed memory: when one subtracts from the ratio 120 : 48 [or: 120^{a} to 48^{b} — written above the line] the ratio [?: subscript figures here read approximately 120 : 24 with fractions, but the small glyphs are abraded — [?]], there remains the ratio [?]. And in accordance with the rule for the subtraction of ratios — if you multiply them crosswise [numerator by denominator] — there are produced 48 sixtieths of degrees, 31 degrees, 55 firsts [minutes] [transcribed as printed: 55.31.48 in the running sexagesimal order]. For there is no need to multiply the number by 60 — for [otherwise] you would of necessity have to divide it back by 60 afterward — rather it is enough that you raise the numbers up by one rank from their rank. Thus 120 with 120 rise up to 14400 degrees, which are 240 sixtieths of degrees. And the remainder will be thus [?: subscript abraded — approximately 55.31.48 [?]].

And since the fifth number, besides the [other] four, is known in Ptolemy's computation — namely 120 — we know that this remainder equals [?: subscript ~57.15.24 [?]]. This, together with the rule of the Golden [rule of three / *regula aurea*]: if 240 equals 180, how much does 120 equal? — and you find the result easily, since [120] is half [of 240]: 55.31.48. But at times it is difficult, when [the divisor] is smaller than the known [dividend] or larger, and it carries fractions and must be divided down [reduced] or up [raised], and you would need a long labor. It is therefore better to shorten the matter from the outset and to invert the five known [quantities] into three.

You find, by the rule of three [Golden rule], the unknown at once — and this is that you discard twice 120, that is, the fourth and the fifth [terms], and two ratios remain, and the parts [denominators] of each ratio are of twice [60] firsts [i.e. expressed in the second sexagesimal rank], and the three [terms] are known. We set the proportion and say: if 120 yields 60, what do 55.31.48 [yield]? And there come out 57.15.24, and this is the unknown. ✻ Likewise in that same chapter you find Ptolemy saying that if one subtracts from the ratio 55 : 48 [with subscript ~55.31.120 [?]] the ratio [?: subscript ~103.55.120 [?]], there remains the ratio [?].

Now if you multiply these in the manner indicated for the subtraction of ratios — the two crosswise — and divide them by sixty, you find the remainder [?: subscript ~35.24.120 [?]]. And we set a second proportion and say: if 240 yields

120, 35′ 3′ 84′ [as printed], how much do they yield? You find 42.1.48 — and this is what was sought. But in inverting the six [terms] back to the [first] four, from the outset you discard the 120 [that occurs] in the third and the sixth [positions]; and it will be as the ratio of 120 to 48.31.55, so 23.55.103 [transcribed as printed: 23.55.103] to what [the unknown]? And you find 42.1.48, which is the sought quantity.

And likewise in chapter 14 of the First Book [Almagest I.14] Ptolemy says that if one subtracts from the ratio [?: dense subscript ~55.15.120 with several fraction-figures, abraded — [?]] the ratio [?: subscript ~57.15.24 / 31.111… abraded — [?]], that is [?: ~120.1… [?]] — and here the components are equal and the compound is the doubling [square] of each one, and it is not felt [does not register a difference].

So I will let you know that even by Ptolemy's own way, with six ratios [terms], there is no need to multiply all four numbers crosswise, but only two of them by themselves. I will give you an example in the geometric fractions [sexagesimal fractions]. We wished to subtract [from] the ratio 12 : 4 [as printed, subscript "מ ¹²/₄"] the ratio 2 : 6 [subscript "²/₆"]. I multiply 2 : 6 [?] — two of them crosswise — and the result 72 I divide by the second consequent term, 4; and the remaining ratio will be 18. Beneath it I place the first antecedent, 2; and the remaining ratio is 18 : 2, that is 9 [subscript "¹⁸/₂", i.e. 9]. Or, conversely, I divide 72 by the antecedent 2, and there come out 36; and beneath them I place the consequent 4; and the ratio is 36 : 4 [subscript "³⁶/₄"], that is 9. Or I multiply 2 by 4, and the result 8 I divide by 6, and there come out 1⅓.

And I place upon it [beside it] 4 : 3 [subscript "¹²/₄ → ¹/₃" as printed, i.e. the residual 1⅓; [?] on the exact fraction-glyph], that is 1⅓ — or that I multiply 8 by 1 [as printed, subscript "⁹/₁"], and there comes out [the result]; and upon it I place 6 [subscript], and they become 6 : ? [subscript "⁶/₃", reading uncertain — [?]].

The reason for the matter is as follows. After I have multiplied even four [terms] crosswise, there came out numbers whose ratio between them and between the aforementioned [original] terms is known; and we need to divide them by means of one of the numbers of the subtracted ratio — that is, the subtracted [term] — for then by necessity one of the multiplied [products] reverts. For it is established that if you multiply one number by another and divide the resulting [product] by the [first] one, the first [number] comes out. And if I multiply 2 crosswise, there come out 8 and 72 [as printed], and I divide them by 4 — they become [a result] — and by 6 they become 1⅓ [as printed, with subscript figures]; and you see that there come out the first [original numbers, 12 and 2]. Hence there is no need to

multiply both of them crosswise, but only two of them; and the result you divide [thus], and the third ratio will be whole [an integer] or with one of the modes I have described.

This division [you do] with the second antecedent, and there comes out the antecedent of the third ratio; and beneath it you place the consequent that you have not [yet used]. Thus the work is somewhat shortened in these computations. See now how, in Ptolemy's example, this third [ratio] comes to your sought quantity by the two modes; and if you wish to multiply them crosswise, it will be thus:

[Worked computation table — Ptolemy's example] Two parallel column-blocks of cross-multiplication. The table reads right-to-left in the source; the logical right block is given first below. Each block has a header row of three sexagesimal triples, then a column of products, with a sum and a final divided result at the foot.

Right block:

117	31	15
48	31	55
5616	3627	6435 · 1705 · 825 [row as printed, five figures]
1488	961	465
	720	
87	135	36 · 13 [as printed]
5693	5250	8152 · 2183 [as printed]
94 · 54	30 · 52	23 · 45 [as printed]

Left block:

109	44	53
24	15	57
2616	1635	6213 · 2508 · 3021 [as printed]
1056	660	795
	1272	
47	136	55 · 50 [as printed]

2663	2827	8200 · 3353 [as printed]
44 · 23	7 · 40	53 · 24 [as printed]

Here it is set out plainly for you: after the multiplication and the division by sixty, there comes out the remaining ratio, and it equals 26.52.54 / 15.31.117 [as printed in the marginal sexagesimal fractions — [?] on the small figures], as Ptolemy wrote — for every thing reverts to the parts out of which it was compounded. Indeed, after the parts 48.31.55, if we divide by this each one of the aforementioned products — that is, the number 2663 and so on, and likewise the number 5693 and so on — there comes out of this the second [factor] by necessity, 15.31.117 [as printed]; and therefore it is in vain that he troubled to multiply it. But from dividing 2663 and so on — which is the product of the second multiplication — by 48.31.55 aforementioned, there comes out 54.52.26 [as printed], and this is as Ptolemy wrote.

And so that the rule of division may be fluent in your mouth, take this illustration. Place the dividend above and the divisor beneath it. Now I investigate, to know how many times 48 fits into 266 [as printed], and it appears that it fits in 5 times, with 263 left over [as printed: 5 times, remainder 263]. Then again I place the divisor 48 beneath, and at first sight it appears to fit into the remainder 5 times, leaving thereby 23 [as printed]. But if we multiply 31, the divisor that follows, by the number 55, they make 1705 [as printed], and these are more than was intended above — for 23.7 [as printed] are only 1387 — and therefore the divisor 48 fits into the number 2663 only 54 times by itself, for its product with 48 amounts to 2592; and we subtract it from 3662 [recte: 2663] and there remain 71 [as printed]. And this we convert to firsts [first-order minutes] — that is, we multiply by 60 — and they become 4260, and upon it we add 7, and they become 4267; and from this I subtract the product of 31 of the divisor with 54, which is —

1674, and there remain 2593. And again I subtract the product of 55 by the divisor 54, which is 2970 [as printed] — from 40, two-secondths [?: "from 40 seconds" — small figures uncertain] — and these seconds make 3010, and there remain attached to them 2593 [?] firsts, which are 49, which together with the 40 [?: the running tallies here are dense and partly abraded — [?]]. Then again I place the divisor beneath the firsts of the remainder, which is 44.40.53.21.25 [as printed]. And at first sight it appears to fit in 53 times — for if we subtract the product of 53 by 48, which is 2544, there remain 40 [as printed]. But the product of 31.55 by 53 cannot be subtracted from 40 [as printed]; therefore [it fits] 52 times, and we

multiply it by 48, and the product 2496 we subtract from 2544, and there remain 48; and likewise we multiply 31 by 52, and the result 1612 we subtract from 40 [as printed], after we have appended to it [borrowed] the remaining 27, which together with the 40 are all 1660 [as printed], and there remain 8.53.21 [as printed].

And likewise we multiply 55 by the divisor at 52, and the result 2860 — after [the remaining] 7.40.53.21.2663 [as printed: a string of running remainders, transcribed verbatim] it is impossible to subtract 590 [as printed], so we append 47 to them, and they all become 2873 — 43.31.55.55.55 [as printed], and there remain 13. So the remainder is then 21.1.13.21 [as printed]. Again [21.1.13.21 →] 31.31.48 [as printed].

I place the divisor beneath the seconds, and to shorten the matter I multiply 21 by sixty [subscript margin: 26.52.54 // 48 // 31.31.48], and upon them I add 1, and they are 1261, and they fit in 26 times, and there remain 19; and these, when they are carried over to thirds and we add upon them 13, become 793. And in truth they will not suffice for 31.55 to be subtracted from them when these are multiplied by 26 — which make 829 [as printed] — but the deficiency is 36, and Ptolemy reckons these fractions as nothing; therefore he ruled that they are 26.52.54. And when we place beneath them 117.31.15, the remaining ratio will be thus [subscript: 26.52.54 / 15.31.117], as in Ptolemy's words.

Here, then, is the division you wanted. And if you wished to divide in the reverse — that is, 2663.40.53.7.21 [as printed] by the divisor 117.31.15 — you find 10.35.39.22 [as printed]; and when you place them with the consequent it will be [subscript dense — [?]], and it equals the first remainder.

But as for what Ptolemy said — that the remaining ratio 55.55.55.31.43 [as printed] equals the ratio [subscript ~26.52.54/117.31.15] — they found it by the rule of three [Golden rule], thus: if 117.31.15 yields 54.52.26, how much do 120 yield? And by the multiplication and the division they found 56.1.53; and he chose this rather than the first two, even though all three are equal, because the number 120 is the diameter of the circle and is the guide [base] in astronomical operations, and the computation is easier when the consequent of a ratio is whole [an integer] rather than fractional.

And further, it is possible to find this last ratio when you divide the antecedent of the compound ratio, 109.44.53, by the antecedent of the component, which is 117.31.15, as I have shown you; and you will find that there fits in it 56.1.53, and this is the antecedent that remains. And likewise divide their consequents, one by

the other — that is, 48.31.55 by 24.15.57 — and the remaining consequent 120 is produced. And it is shown that the ratio 56.1.53 [subscript marginal] equals the ratio [subscript ~54.52.26/57.15.24 — [?]].

You are able to test all these operations one against another with the fixed balances [check-sums] and casting-out [of the deficiency] as is customary among the masters of computation [the arithmeticians]. He further said, in that same last chapter of the First Book, that when one subtracts from the ratio [subscript ~109.44.53/55.31.48] the ratio [subscript ~112.23.56/56.23.42 and a "more" mark — [?]], that is [subscript ~95.2.41/56.23.112], the [marginal: r"l ~20.28.120 // 101 …] — and it is easy to grasp that if you double the two [terms] of which they consist — that is 109.44.53 with 42.1.48 — you find 24.47.18.39.12.46 [as printed], and you divide them by 55.31.48, and there come out 41.2.95 [as printed], which is the antecedent of the remainder, and beneath it you place the consequent, and it will be 41.2.95.1.4 [as printed — [?] on the last figures], and this is the antecedent of the remainders. And he said: if 56.23.112 —

— how much do 120 yield? You find 101.28.20 [as printed]. And with one of the modes I have shown you, you reach your sought quantity. There is no need to expand on plain and clear matters, suited even for children.

In another place I will let you know the manner of extracting square roots and cube roots from these fractions [sexagesimals] by means of a table, demonstrated without any toil or labor at all; and I have many ways stored up, but it is not my intention to bring all the modes here in this abridgment.

הקדמה ראשונה עגולית לבטלמיוס — First Geometric Preliminary, of Ptolemy

When the diameter of a circle cuts some chord, the segments of the chord [into which it is cut] stand in the ratio of the [chord-segments] *bekaim* of the arcs at the ends of the [two] chords. This is the fruit of Ptolemy's lesson, and the gist of his words. But so that you may see that the *bekaim* [chord-segments / "sines" in the segment sense] which the moderns discovered were not hidden from us, I will bring his words just as they are in his great book [the Almagest]. He said, of blessed memory:

Let there be a circle (ציור ט / figure ṭ [9]) [labeled] א ב ג, whose center is ד [dalet], and let three points be taken on its circumference, as you please, א ב ג — only that each of the arcs אב ,בג be less than a semicircle; and this you will keep in

mind every time we use such arcs in what follows. Let the lines אן [aleph-nun] and דה [dalet-he] be drawn.

He says that the ratio of the chord of double the arc אב to the chord of double the arc בג is as the ratio of line אה to line הג. For we draw from point א a perpendicular אן upon the line דב [aleph-nun upon dalet-bet]. And since these lines are parallel to one another, and line אן falls between them, the angle גנא [gimel-nun-aleph] is equal; therefore [by Euclid] from [proposition] four to [proposition] six, the ratio of אז to גי will be as the ratio of אה to הג. But the ratio of אז to גי [aleph-zayin to gimel-yod] is as the ratio of the chord of double the arc אב to the chord of double the arc בג — for [by] from [proposition] three to [proposition] three [an Euclidean reference], these are the halves of the aforementioned chords, which are the *bekaim* [chord-segments] of the aforementioned arcs: that is, [אז] is the *beka* [chord-segment] of arc אב, and גי the *beka* of arc בג [as printed]. Therefore [by] from [proposition] five to [proposition] five [Euclidean ref], the ratio of אה to הג will likewise be as the ratio of the chord of double the arc אב to the chord of double the arc בג. This is what was meant.

You see that Ptolemy understood well the nature of the *bekaim* [chord-segments / sines], except that, according to his approach — of using the cut [whole] arcs [chords] — they did not come into his work [as a separate function], and just as he understood the matter, so did Albategni [?: אלבחין = al-Battānī — [?]]. But we, according to our method, will henceforth use the *bekaim* [sines], and we will explain the author's lessons with them — for there is to them another rule and law together with the [whole] chords, as the author himself explained, with his reasoning and grounds with him.

הקדמה שנית עגולית והיא שלישית לבטלמיוס — Second Geometric Preliminary, which is the Third of Ptolemy['s]

When the chord of some arc is extended outside the circle and meets the diameter extended outside on that same side, the ratio of the whole extended chord to the cut [external segment] outside the circle is as the ratio of the perpendicular dropped onto the diameter from the [far] end of the one chord, to the perpendicular dropped from the end of the [other] chord [where it] meets the second diameter [?: phrasing dense — [?]].

Let there be a circle (ציור י / figure yod [10]) [labeled] **היום** [the figure's circle-label string], chord יו, which when extended meets the diameter at point א —

he says that the ratio of יא to וא is as [the ratio of] יד, the *beka* [chord-segment] of arc יום, to וע [vav-ayin], the *beka* of arc וס [as printed]. For the triangles אעו [aleph-ayin-vav] of our triangle are right-angled and similar [equiangular]; and they are proportional [by] from [proposition] four to [proposition] six. Therefore the ratio of יא to יד is as וע to וא; and alternando [by interchange], the ratio of יא to וא is as יד to וע [as printed] — and this is the sought quantity.

Chukos Shamayim

(Almagest commentary, Book I — chord-segment theorems and the construction of the chord table)

Now understand this: from any one of these Preliminaries, Ptolemy drew a fine theorem. From the first Preliminary one consequence follows — and that is the second proposition [Almagest I.13]; and from the same Preliminary, which is the third in his book, a fourth proposition follows [Almagest I.14]. In truth, from each one of these Preliminaries both consequences emerge: from the first Preliminary one follows, as does the second consequence in his book; and from the said Preliminary, the two [...] follow. One must not wonder at a man as great as he was failing to notice this, given that his way was to be brief in his mathematical demonstrations — as Mamilios [?] and Hipparchus [איפרכוס] are praised in the words of the experts who clarify his words. He needed both of these Preliminaries to prove his theorems on the cut arcs: in the first proof, which proceeds by *division*, he uses the first Preliminary; and in the second proof, which proceeds by *composition*, he uses the said second Preliminary. And because I have seen the great benefit that follows from these Preliminaries — beyond the need that compels their mention for proving his theorems — I have drawn from each one a single consequence, even though both consequences emerge from each, as I have explained in my commentary on him. We too shall need both of them to prove our matters, as He shall show with God's help; therefore I have brought both of them in as well.

The first consequence [Almagest I.13] is what becomes clear from this: that when you know some arc subtending half the circle, divided into two parts, and you also know the ratio of the chord-segments (*bekaim*, i.e. sines) of those parts to one another, you can easily determine the measure of each of them on its own. To prove this, he has here transcribed the figure of proposition 65 [ס"ה] of Book II [literally "the second face"] of *Sefer Sod HaYesod* [the Book of the Foundation's Secret].

Let there be a circle בג on center א, and let there be in it a known arc רדק — or, the angle דא — divided into two parts, ראפ פאה; and let the ratio of the chord-segment דך [sine of דך] to the chord-segment יך [sine of יך] be known. I say that each of the angles ראפ, פאה will also be known. Draw the chord דג [?] beneath arc דג — the chord דה. Since the ratio of sine דך to sine יה [?] equals the ratio of side דמ (drawn to angle ראפ) to side מה (drawn to angle פאה), by the first Preliminary mentioned; and the whole chord דה is known, since the arc constructed upon it is given — therefore, by what Euclid demonstrated in [Proposition] 18 of Book V, on proportion, the conjoined being proportional, we know also the sides דמ, טח [recte: מה] each on its own. Draw from center א to chord דה the perpendicular אלג; it bisects the said chord and the arc upon it, and the angles ראג, גאה will be equal, third to third, and known. By this, קי לן [?], which is the surplus over the half, is known. And since in triangle לאך the angle is right, angle לאד is known, and its complement too, [the angle] from 90° [literally "the second to 90"]. Then, in the manner R. Yitzchak [Israeli? — see note] demonstrated: if we conceive the triangle inscribed within a circle, the sides will themselves be the chords of the arcs corresponding to the angles, and the arcs are double the angles — for the whole circumference is 360° and the two right angles of the triangle are 180°. Therefore, when you know the angles you know the sides; for the ratio of the right angle to 180° equals the ratio of the [...] arc, [and] thus the ratio of every arc-angle to the [chord] constructed upon the side opposite it, whose arc is double. And once you know the chords — if the measure of some side is known — you can infer and determine the measure of every chord. And the chord דמ, if half the diameter, 120, equals ten, how much will a given chord be? And so forth. This was Ptolemy's way of determining the sides from the angles.

In this manner he determined the measure of line אל in the said triangle. Or, proceed by this route: from אך [?] קי לר, and from proposition 47 of Book I [Euclid I.47, the Pythagorean theorem] determine לא; then, going back, in right-angled triangle אלמ, from side אל and from [Euclid] I.47 determine אמ; and from the sides you come to know the chords and the arcs, which are double the corresponding angles. So find angle לאמ — which is the *added-and-subtracted* angle: for if you add it to angle אל, half the sum, you obtain the larger angle, דאפ [recte: ראפ], known; and if [you subtract it]...

(continues at the top of the next page)

the subtracted from the half, the smaller angle, פא, comes out, and the arcs are known as they are.

I know that Ptolemy has another figure for this, but the route I have shown you brings you into his inner chambers — and this [his own] is in truth a tortuous path. I wished to make known to you how many advantages and how good an addition the calculations of the moderns have over the ancients: for by the teaching of proposition 60 [?] of *Sefer Sod HaYesod*, it is easy to resolve these triangles and others as well, especially if you recall what I taught you in proposition 21 [כ"א] of Book II of *Sefer Sod HaYesod* — namely, that the ratio of angle to angle equals the ratio of tangent (*noga*) to tangent — you will run swiftly through your calculations. That is: inscribe upon center א, in the said figure, according to the distance, an arc of circle קלנ; let line דל be the tangent of angle ראל, and קלמ the tangent of angle לאט; and set the proportion: if דל gives tangent דאל, how much will מ give? — since all these are known. And you will find at one stroke the tangent of the sought angle לאמ. By proposition 1 [...], a consequence of proposition 60, Book II of *Sefer Sod HaYesod*, I have brought you an illustration. And here it is:

Let there be two known [given values] for the ratio, and let the angle be ראה [?]; let angle פאד be to angle פאה as 50778 is to 45399 [?]; and the inference will be thus:

[The following are paired worked tables, transcribed as printed. They read right-to-left in the source; the logical right-hand block is given first. Sexagesimal entries are written with the printed `degrees.minutes.seconds` dot notation as in the source.]

Right-hand table:

	50778
	45399
the tangent of	50778
half the sum	45399
32 דאה	

Left-hand worked block (with the conditional headings printed above):

If the sum	it gives the difference	how much will the tangent give
96177	5379	3409
	62486	this angle

(right side)		(left side)	
add to it	0. 32.	subtract from	0. 32.
half the sum דאה	1. 57.	half the sum קבוץ דאה	1. 57.
and angle דאפ comes out	33. 57.	and angle פאה comes out	3. 30.

The second consequence of Ptolemy [Almagest I.14] is that, if the ratio of the chord-segments (sines) of the two divisions and the difference between them be known, you can also determine each of the divisions on its own. And although by his proofs this second consequence follows from the second Preliminary, by our method it is clear from the first Preliminary. For just as I showed you in the consequence drawn from the first — by proposition 65 of Book II of *Sefer Sod HaYesod* — you need only exchange the ratios in his statement.

Suppose, for example, that we know the ratio of the chord-segments of the divisions ראפ to פאה — or [equivalently] side דמ to מז — to be as sine 40647 is to sine 5117 [recte: 5,117], and the difference of the angles to be ראם [recte: angle ראם] 36 .51. We wish to find each angle on its own. Then first I subtract from half the sum of the chord-segments דל the smaller chord-segment מה — or I subtract the smaller מה from the larger:

[Paired worked tables, read right-to-left in source; logical right-hand block first.]

Right-hand block (half-sum of the chord-segments):

step	value	value
the larger דמ	40647	40647
(the first in the rule of three) 17765	5117	5117
of angle לאפ, i.e. half [the difference], 45764	22882	45764
the tangent found for it is 33312	22882	33312
half the sum of chord-segments דל	5117	(blank)
	17765	(first)

122882, which is the third in the ratio; and infer thus:

if 17765... 17765 (first)

33312... 22882... tangent 42907, and its angle is 23. 13 ½

Left-hand block:

	value	value
the larger דמ	40647	40647
17765, which is the first in the rule of three	5117	5117
half the difference of the angle, i.e. angle לאפ	45764	35530
and it is 18. 25 ½; and from the table of tangents I find its tangent	33312	17765

So I take half the difference, 18. 25 ½, and angle ראם [recte: ראם / the relevant angle] comes out 41. 39, and angle פאד [recte: פאה?] 4 .48 [recte: 4. 48] — and that is what was sought.

half the difference is 18. 25 ½, and angle דאפ comes out 41. 39, and angle .4 פאה 48 — and that is what was sought.

And if you wish, you may draw both of these from the second Preliminary as well, and with much ease; there is no need to expand, for I have already explained these matters at length [in the commentary], and this is not their place — our purpose being only to take the benefit that issues from his theorems.

Proposition 1 [Almagest I, on the sphere] — and it is the second in Ptolemy:

When, on the surface of a sphere, there proceed from a point two arcs of great circles, each less than a semicircle, and from their endpoints two other arcs of great circles are drawn doubled, cutting one another and the first arcs — then the ratio of the chord-segment (sine) of one of the first, outer arcs to the chord-segment of the upper cut nearest it is compounded of the ratio of the sine of the inner arc, doubled from the endpoint of that outer arc, to the sine of the cut of this inner arc farthest from the said endpoint; and of the ratio of the sine of the cut nearest the endpoint of the second inner arc, doubled, to the sine of that second doubled arc. (See figure יא [fig. 11].)

That is to say: from point א on the surface of the sphere there proceed two arcs אב, אצ of great circles, each less than a semicircle; and from their endpoints ב, צ there are doubled toward them two arcs צפר, מפ, which cut one another at point פ and cut the first, outer arcs at points ד, ה. I say that the ratio of sine אצ to sine אק [?] is compounded of the ratio of sine צך to sine פך, and of the ratio of sine בפ to sine בה.

Their proof. Let point ג be the center of the sphere, and from it let three diameters proceed to the circumference of the [great] circles: that is, גא to circle צהא [?], which meets the chord צדק (drawn outside the circle) at point ך; likewise the diameter בד to circle צפל, which meets the chord צם (drawn outside the circle) at point ק; and likewise the diameter גב to circle בפח, which meets the chord דה (drawn outside the circle) at point ל. Now since the three points ח, ק, ל [?] are in the same plane that contains triangle צפה — for they lie on its drawn sides — and they are also in the same plane that contains circle אד [recte: אדב] — for they lie on the lines proceeding from its center — they must therefore lie on the common section of the said planes, צפה, אדב. And by [Euclid] XI.3 the planes are cut along a single straight line; and if this line — which is חקל — be drawn, it will be exactly the model of proposition 1 of *Sefer Yesharei HaKavim* [the Book of the Right Lines] — that is, two lines חל, חצ containing angle ה [?], whose endpoints לצ are doubled by two others לק, צק, which cut one another at point פ, and so on. And as was demonstrated in the said proposition, the ratio of הצ to חה [?] is compounded of the ratio of צק to פק, and of the ratio of לפ to לה.

Now the ratio of חצ to חה [?], by the second, spherical Preliminary, is as the ratio of sine [...]; and the ratio of arc אצ to arc אד [?]; and the ratio צק to פק as the ratio of sine דצ to sine דפ; and the ratio לפ to לה as the ratio of sine בפ to sine בה. Therefore likewise the ratio of sine arc אצ to sine arc אד is compounded of the ratio of sine דצ to sine דפ, and of the ratio of sine בם to sine בה — and that is what was to be proved.

[The middle lines of this proof are dense with point-letters that are abraded in the gutter; several ratios (חצ/חה, the spherical-Preliminary correspondences) are reconstructed from the parallel structure of the standard Menelaus configuration and flagged [?] where the ink is not legible.]

To make the said figure intelligible to your imagination, cast your eye on a sphere and its circles, and imagine that arc אב is a quarter of the meridian circle and arc אצ a quarter of the equator; that point צ is the intersection of the equator with the ecliptic; that arc צך, doubled, is a quarter of the ecliptic; that arc בך, doubled, is a quarter of the [great] circle from the pole of the meridian to the equator; and that arc כ [?] is the sought arc — that is, the measure of the inclination of point [...] which is on the ecliptic. Arc צה [?] will indicate which *right ascension* or *oblique ascension* it is; and arc א [?] will be the great declination of the sun. At point א the meridian circle and the equator are cut at right angles, and the chords of the arcs drawn in this figure are nothing other than [proposition] 10 of the first proposition of *Sefer Yesharei HaKavim*. The figure of these four cut arcs is the

corner-stone of the [spherical] triangles, and is continually found in the hands of the masters of computation, as you shall see with God's help.

Proposition 2 [Almagest I, spherical]:

Further, I wish to explain a proposition that I needed, which Ptolemy did not bring among the said cut arcs on the sphere — namely, that the ratio of the sine of one of the outer arcs to the sine of its lower cut is compounded of the ratio of the sine of the upper cut of the second outer arc to the sine of its lower cut, and of the ratio of the sine of the cut of the doubled arc from the endpoint of the second outer arc — I mean the cut beginning from the doubling — to the sine of the cut of the arc farthest from it. That is: the ratio of sine אצ to sine הצ is compounded of the ratio of sine אך [?] to sine דב, and of the ratio of sine בפ to sine פה. (See figure יב [fig. 12].)

To prove it, we draw from point ג, the center of the sphere, to the circumference of circle צפד, at the place where it is cut by the others, three diameters — that is, גצ, גפ [recte: צ, פ, ד] — and the chord אדק is drawn outside the circle until it meets diameter גצ, which is also drawn out, at point ז [?]; and we draw the two chords בא, אב [recte: בא, בה], which are cut by the said diameters זד [recte: גד], זפ [recte: גפ] at points ך, ץ [?]. So by this there will be three points ב, ז, ר [recte: ך, ץ, ז?] on a single straight line, since they lie together in two planes — that is, in the plane of triangle אבה, for they are on its drawn sides; and in the plane of circle צפד, for they are on the lines proceeding from its center. Therefore, if upon these three points the line זך [?] be drawn, that line will be the common section of the two planes; and the straight line will be, by [Euclid] XI.3, exactly the model of the second proposition of *Sefer Yesharei HaKavim*, which is the sixth of the said Theonius [Theon] — that is, that from a single point on the sphere there proceed lines אב, אג making an angle at point א, whose endpoints ב, ג are doubled by two lines בה, גד, which cut one another at point ץ, and so on. Then, as we explained there, the ratio of אג to דקב [?] is compounded of the ratio of אך to רב [recte: דב], and of the ratio of בז to זה.

Now from the second, spherical Preliminary, the ratio of אב to הג equals the ratio of sine אצ to sine הצ; and from the first, spherical Preliminary, the ratio of אד to דר [recte: דב?] equals the ratio of sine אד to sine חם [recte: דב]; likewise the ratio of בץ to זה equals the ratio of sine בפ to sine פה. Therefore also the ratio of sine אצ to sine הצ is compounded of the ratio of sine אך to sine דב, and of the ratio of sine בפ to sine פה — and that is what was to be proved.

Chukos Shamayim (חוקת שמים)

R. Yashar [Delmedigo] said: It has not escaped me what Vieta [תיביט, = François Viète] wrote, that something occurred to him against Ptolemy or against Menelaus — from which it followed (that is, he sought to refute the claim) that the chords drawn outside the circle in the figures just mentioned must always, of necessity, meet the diametral lines issuing from the center of the sphere, whereas in fact these lines are sometimes parallel and sometimes diverge from one another. He therefore found it necessary to complete Ptolemy's words. And in truth the attentive reader can readily grasp toward which side the chord must be drawn, and which arc he should take first — whether to the right or to the left — and so find what he seeks. In my commentary on the Almagest I have brought all the figures and the variations that arise, so there is no cause to doubt Ptolemy's demonstrations and the reckoning of triangles I build upon them. For according to the first proposition (*hatzaʿah*) of *Sefer Sod HaYesod*, everything that follows from these cut arcs has been demonstrated by another proof, and all his words are true, and they are right ways to the discerning, and straight to those who find knowledge. And if I delay in completing my commentary on the Almagest, wait for me, for it is a great work; and meanwhile ponder Vieta's words above, together with the *Sefer Menelaus* — for it is in the Holy Tongue [Hebrew], and there he will instruct you to go left or right; and you will reach the end, and rest, and stand for your lot at the end of days.

And now I shall present the roots [foundational propositions] from which the solution of triangles branches out.

Root 1 (שרש א׳)

When there are, on the surface of the sphere, two right triangles sharing one acute angle in their base, then the ratio of the sine [*beka*] of the surface [*nishtach*] of the first to the sine of the surface of the second equals the ratio of the sine of the perpendicular side of the first to the sine of the perpendicular side of the second.

By "surface" [*nishtach*] I mean the side opposite the right angle [the hypotenuse]; and of the [other] sides, one is the base and the second stands perpendicular upon it.

Transcribe here the figure on folio 60 [שברף ס׳], which is the model of the cut arcs, or [use] this figure. I say that the ratio of the sine of בה — the hypotenuse of the spherical triangle באה — to the sine of בפ — the hypotenuse of the spherical

triangle בדפ — is as the ratio of the sine of side אד, the perpendicular, to the sine of side דפ, the perpendicular. (Figure 13 [ציור יג])

The demonstration is what was clarified in the first spherical Proposition: that the ratio of the sine of צא to the sine of הא [?] is compounded of the ratio of the sine of צד to דפ and of the ratio of the sine of בפ to the sine of בה. Now the sines of צד [recte: צד and בה] are equal, since both are quarters of circles, and they lie on different surfaces [planes]. Therefore, by the first consequence of our first Preliminary [*Hakdamah*] in this book, it also follows — if the equals are cast out, namely the sine of צא (or its equal ב [?], both being quarters) against the sine of אד — that as the sine of בפ is to the sine of [the corresponding term], so the sine of אד is to the sine of דפ. And by exchange of the fourth-to-fifth [ratio terms], as the sine of בה is to the sine of בפ, so the ratio of the sine of אד to the sine of דפ — and this is what was required.

Another way: cast out צד ,צא, which are equal, and the ratio of the sine of בה — the hypotenuse of triangle האב — to the sine of בפ — the hypotenuse of triangle פדב — will be as the ratio of the sine of the perpendicular אד to the sine of the perpendicular ד [recte: דפ]. And this is what was required.

[Caution against an error:] Beware of erring and supposing that the ratios of the sines of the hypotenuses [*nishtachim*] are likewise as the ratios of the sines

of the bases — for were that so, then [it would also be that] the sines of the bases would stand to the sines of the perpendiculars by exchange [inversely], which is not the case, since these bases of ours are not [so] proportioned, as I explained in my commentary on the Almagest, with God's help. Examine carefully the matter of the cut arcs, and you will find that they are not straightforward, because their chords do not lie on a single surface [plane].

And from the aforementioned root there branches a branch of the tree of knowledge — to which the unnamed [scholars] could not attain, nor reach its lofty foliage —

Section 1 (סעיף א׳)

of [the solution of] all triangles. This is the first proposition (*hatzaʿah*) of *Sefer Sod HaYesod* — that is, that in all spherical triangles the sines of the angles are proportional to the sines of the sides opposite them. This is evident at once from what is stated in this root, for in right triangles the perpendicular [side] is itself the whole sine and is equal to the arc of a quarter-circle. And it will be: as the sine of בה — that is, the sine of angle ד, the right angle — is to the sine of side בפ

opposite it, so the ratio of the sine of אד — that is, the angle that is [a right angle, of] its measure — is to the sine of side דפ opposite it. And if you draw out the sides פב ,פד [?] to quarter-circles, ק ,ד [?] [reading uncertain], you will find by this same reasoning the ratio of the sine of ד — that is, the sine of פד — to the sine of side פב, as the ratio of the sine of חב [?] — the measure of angle פד [?] — to the sine of בד, the side opposite it. And this is what was required. (Figure 14 [ציור יד])

But in the oblique spherical triangles, the [followers of] al-Ashqilī [?, the geometer al-Shaklī / perhaps "the geometer"] explained it in this manner. Let the figure of triangle אבצ be the locus of angles; and let, first, angles ב and צ [be] acute (or obtuse). According to Proposition 48 [מ״ח] of the second part of *Sefer Sod HaYesod*, the perpendicular אד falls inside the triangle. And as was clarified just above, in the right triangle אדב it will be: the ratio of the sine of אב to the sine of ד, so the sine of אד to the sine of ב. And by exchange of fourth-to-fifth, as the sine of אד [recte] to the sine of אד, so the sine of ד [?] to the sine of ב. Likewise in the right triangle אדצ: as the sine of אד to the sine of צ, so the sine of אצ to the sine of ד. And exchanging fourth-to-fifth, as the sine of אד to the sine of אצ, so the sine of צ to the sine of ד.

It will therefore be, from the join of the fifth [terms] by the mixed ratio: as the sine of אב to the sine of אצ, so the sine of צ to the sine of ב. And by exchange: as the sine of side אב to the sine of angle צ opposite it, so the sine of side אצ to the sine of angle ב opposite it. And this is what was required.

The figure of the syllogism is before you. It is like the analytical syllogism, with the middle limit doubled in this figure; therefore cast out the two middle [terms] of the first ratio and the two extremes of the second — that is, the equals in the two ratios — and the result will be as in the bottom row. It is easy for the discerning when you set your eyes upon the configuration, and it does not confuse the mind, for it is clear as the moon.

Diagram. A schematic of the "compound ratio" syllogism for Section 1 — a two-row Menelaus-style proportion array with numbered terms (1–4) keyed above each cell, and a bottom row giving the reduced result. The terms are sine-labels (point-pairs); the numerals 0–4 above each cell index the term's position in the compound ratio (a 0 marks a term cancelled/cast out). Read right-to-left in the source.

Top ratio (term-indices above, sine-labels below; read right→left as 1, 2, 3, 4):

index	1	0	0	4
label	אב	אר [?]	ד״ [?]	ב

Lower ratio:

index	0	2	3	0
label	אד	אצ	צ״ [?]	ד

Bottom (reduced) row reads: "as the sine of אב is to the sine of אצ, so the sine of צ to the sine of ב" — i.e. the surviving terms after the two zero-indexed pairs are cast out.

- Term 1 (rightmost): אב
- Term 2: אצ
- Term 3: צ [angle]
- Term 4 (leftmost): ב [angle]

Now let angles צ ,ב be of different kinds [one acute, one obtuse]; then, according to Proposition 48 [מ״ח] of the second part of *Sefer Sod HaYesod*, the perpendicular אד falls outside the triangle. Let אד be in [the larger right] triangle, with the right angle at ד. [It will be] as the ratio of the sine of אב to the sine of ד, so the sine of אד to the sine of ב; and by exchange, as the ratio of the sine of אב to the sine of אד, so [the corresponding] —

— that is, as the sine of א to the sine of אצ, so the sine of צ to the sine of ב [reading partly reconstructed; the marginal note here reads " כערך א אל אצ כן צ אל ב"]. And likewise in triangle [ד]אצ: as the ratio of the sine of אד to the sine of צ, so the sine of אצ to the sine of ד. And by exchange: as the ratio of the sine of אד to the sine of אצ, so the sine of צ to the sine of ד.

It will therefore be, by equality and by mixing [the ratios], as the sine of אב to the sine of אצ, so the sine of צ to the sine of ב. And by exchange: as the ratio of the sine of side אב to the sine of angle צ corresponding to it, so the sine of side אצ to the sine of angle ב corresponding to it. And this is what was required.

And know that the same sine serves an angle and its supplement — that is, the sine of angle אצב together with the sine of angle אצד [are the same], as I taught you in *Sefer Sod HaYesod*; therefore I took angle צ without distinction. Examine [it]. And in the first proposition of *Sefer Sod HaYesod* I explained all this in brief — better, in my opinion, than the proposition of Geminus [כיינ, = Geminus?] in the sixth [book] on foundations; see there.

Section 2 (סעיף ב׳)

And from it [Root 1] there branches a second branch, useful for the solution of oblique triangles. Namely: if a perpendicular is dropped from the [apex] angle of an oblique triangle to the base opposite it — or to [the base] extended outside the triangle — then the ratio of the sines of the angles lying between the perpendicular and the sides equals the ratio of the sines of the supplements of the angles at the base, each [taken] with the angle opposite it.

And it will be (Figure 16 [ציור טז]): let the spherical triangle אבצ be oblique-angled. In right triangles [this] is not verified by the method [just] taught, since a right angle has no supplement except at the limit of the points [degenerate cases]; in those, however, it [the rule] is [trivially] verified. From angle א let the perpendicular אד be dropped to the base בצ. I shall show that the ratio of the sine of באד [angle] to the sine of צאד [angle] equals the ratio of the sine of the supplement אבד to the sine of the supplement [אצ]ד.

Suppose the angles at the base are of different kinds [one acute, one obtuse]; then, by Proposition 48 of the second part of *Sefer Sod HaYesod*, the perpendicular falls outside the triangle. Extend the said perpendicular [arc] to a quarter; let דק be a quarter, and let ק be the pole of circle בצ, by Proposition 13 of Theodosius, Book I [לחיאודוזיאוס]. Place points on the circle [arcs] as poles according to the distances דק, [and from] צ, ה, ז [reading uncertain] we draw quarter-circles בה, צה, and extend the sides of the triangle אצ, בא to ב, ה, פ [?]. It is evident from Proposition 15, Book I of Theodosius that the arcs בה, הצ, which descend from the pole ד [recte ק] of circle בצ to it — being quarters and standing perpendicularly upon it — and the angles הבצ, הצב, are right [angles]. For this same reason, the arcs בג, צפ, drawn from the poles ב, צ to the circles צגה, בפה, are quarters, and the angles פג [at the meeting] are right; and accordingly, arc פב will be the measure of angle אצד, and פף [recte] the measure of its supplement; and arc בצ [recte בג] the measure of angle אבד, and גה the measure of its supplement.

By Section 1 and by exchange of fourth-to-fifth, in the right triangle [צ]פא [?]: as the ratio of the sine of פה to the sine of הא, so the ratio of the sine of פאה to the sine of פ, the right [side]. Likewise in the right triangle גהא: as the ratio of the sine of גה to the sine of הא, so the sine of [ג]אה to the sine of [ג], the right [side]. It will also be — by equality and mixing [the ratios], by [the proposition] 23 [?] to the fifth: as the ratio of the sine of פה to the sine of גה, so the sine of פאה to the sine of גאה.

But פאה is equal to צ[א]ד [?], and הא is equal to באד [reading uncertain]. It will therefore be, by equality, [the proposition] 11 to the fifth: as the sine of גה [and] —

Diagram. A schematic compound-ratio array for Section 2 (the same syllogism form as on p. 240), with numbered term-indices (1–4) above each cell and the base radius 100000 / 10000 written at the row ends, and the surviving-term row beneath. Read right-to-left.

Top ratio (term-indices above; sine-labels below):

(radius)	index 3	index 0	index 1	label-row tail
100000	פ	פה	הא	פאה
label	פ	—	—	—

Lower ratio:

(radius)	index 4	index 0	index 2	label-row tail
10000	ג	גה	הא	גאה
label	ג	—	—	—

Transcribed cell content, top row (right→left): label פאה (index 1) · הא (index 0) · פה (index 3); end-radius 100000; with פ beneath. Lower row: label גאה (index 2) · הא (index 0) · גה (index 4); end-radius 10000; with ג beneath.

The bottom (result) line reads: "as the sine of פה is to the sine of גה, so the sine of פאה to the sine of גאה."

— that is, as [the sine of] פה is to גה, so פאה to גאה: the ratio of the sine of באד to the ratio of the sine of צאד; and likewise by exchange — and understand it. This is what was required.

But when the angles at the base are of the same kind [in their nature], and the perpendicular falls outside, the matter is evident from the configuration itself, provided only that you consider it as if point צ were in the place of point ד, and ק [recte פ] in the place of ה — and then it will be correctly and easily understood. And in *Sefer Sod HaYesod*, Proposition 64 [ס״ד], Part 1, I explained all this in a brief way.

Root 2 (שרש שני)

It is also demonstrated, in the aforementioned figure, that in spherical right triangles upon their bases — which share one angle — the ratio of the sines of the bases equals the ratio of the tangents [*nogʿim*] of the perpendiculars.

This is Proposition *Even Bochan* [אבן בוחן] of *Sefer Sod HaYesod*, and it is demonstrated for the cut arcs in another way, apart from the manner explained there. For it was made clear in the second spherical Proposition that the ratio of the sine of אצ to הצ is compounded of the ratio of the sine of אד to דב [recte] and of the ratio of the sine of בפ to פה. But the ratio of the sine of אד to דב is as [the ratio of] the tangent אד to the whole sine; and the ratio of the sine of בפ to the sine of פה is as the whole sine to the tangent פה — by Propositions 16 and 17 of the second part of *Sefer Sod HaYesod*. It will therefore also be: the ratio of the sine of אצ to the sine of הצ is compounded of the ratio of the tangent אד to the whole sine, and of the ratio of the whole sine to the tangent פה.

And by the indication of the first consequence of our first Preliminary in this book, if we cast out the whole sine — which is doubled, [appearing] on different surfaces — there remains: the ratio of the sine of the base אצ to the sine of the base הצ as the ratio of the tangent of the perpendicular אד to the tangent of the perpendicular פה. And this is what was required.

Diagram. A small compound-ratio array for Root 2, set in the right column: a single line with the tangent (*nogeʿa*) terms and the base radius 100000 appearing three times (R for each factor), arranged as the compound ratio that, after cancelling the doubled whole-sine R, reduces the base-sine ratio to the tangent ratio. Read right-to-left.

Reconstructed term layout (as printed, right→left):

- Right factor: בקע אצ [sine אצ] — radius 100000
- Middle factor: נוגע אד [tangent אד] — radius 100000
- Left factor: הצ בקע [sine הצ] — radius 100000, with נוגע פה [tangent פה] beneath
- Marginal label between: מרכב מ… וט… [= "compounded of … and …"]

The line resolves to: sine אצ : sine הצ = tangent אד : tangent פה (R = 100000 cancelled).

Another way: cast out אצ and the whole sine — which is the fifth [term] — in the six ratios, and the whole [resulting term], and as I taught you in *Sefer Sod*

HaYesod compounded of [the appropriate terms], it is at your discretion to reckon each one of the triangle's sides as the perpendicular and the other as the base — which is not the case in Root 1. Understand this.

And behold, your eyes see how, from Ptolemy's teachings, every spherical triangle is resolved with great ease, and you have no need of other teachings or books; for these two roots, together with the sections, suffice you and satisfy you, and his words will water you. And this is what you do: sketch the triangle you seek to resolve — that is, [knowing] three of its known [parts], to determine the three remaining unknown ones. Mark its given parts and examine it: if it is right-angled, [use] something like Section 1, by which half of the problems pertaining to triangles are resolved — as you saw in *Sefer Sod HaYesod*, whose power is truly great. And if not, draw out its sides to quarters, and examine the supplements of the sides and the arcs that are the measures of the angles; mark in them too the knowns, and perhaps by their means you will find a method. And if they do not suffice you, draw out further the supplements of the sides to quarters — as is the first figure in *Sefer Sod HaYesod*, which I did not for nothing call "Im LaBinah" [אם לבינה, "Mother of Understanding" / "If, for understanding"]. For there is no doubt that by means of the two roots and Section 1 you will find the ratio that guides you to knowledge of the unknown part, and you need no other book at all — for these teachings are easy.

to remember them. You need only the table of sines, together with the tangents and the cotangents. Suppose now that in the triangle [it appears as] **נצכ** [N-Tz-K], where the right angle is at **ב** [B] in triangle **בורפ** [?], we wish to find angle **צ** [Tz], the hypotenuse **אצ** [A-Tz], and the side **פצ** [P-Tz], apart from the perpendicular [?]. Then, drawing on Section 1, we form a proportion and say: as the sine of the hypotenuse **אצ** [A-Tz] is to the whole sine — that is, to the perpendicular opposite it — so is the sine of side **בצ** [B-Tz] to the sine of the sought angle at **אצ** [?].

Now if, from these known quantities, we wish to find a side and no suitable proportion presents itself for our purpose, then we must extend its sides to their complements [?]. R. Gershom [?] found this difficult. Yet here we have found help from Section 1 by way of the complements: that as the sine of the angle at **י** [Y], the complement of the known **צ** [Tz], is to the sine of **כא** [K-A?], the complement of the hypotenuse **אצ** [A-Tz], so is the sine of the right angle at **ב** [B] — which is the whole sine — to **אי** [A-Y?], the complement of the sought side **אב** [A-B].

Likewise, if you know the three angles of triangle **צרק** [Tz-K-Q?] and you wish to find the hypotenuse **צפ** [Tz-P], it will not suffice to bring the sides only as far as their quadrants; rather, you must extend the complements to quadrants as well.

Then you will find help from Root 2: that as the sine of **בצ** [B-Tz], the whole sine, is to the tangent of **כי** [K-Y?], the complement of angle **צ** [Tz], so is the tangent of **אב** [A-B], the complement of angle **פ** [P], to the sine of **בצ** [B-Tz], the complement of the sought hypotenuse **צפ** [Tz-P]. [?]

And so for the remaining cases, as they appear in *Sefer Sod HaYesod*, clear and well ordered. I have already cautioned you there, in [the figure called] *Even HaRosha* [the cornerstone], that if the triangle has obtuse angles or sides exceeding a quadrant — whether right-angled or oblique — you should take another triangle whose sides [are within bounds], so as not to grow confused; for the sines, tangents, and cotangents are equal [for an arc and its supplement]. And as I taught you in *Sod HaYesod*, Figure 22, Section 2, how to deal with the conversion [?] of the triangle's sides: once you know how to solve every right-angled triangle, you will recognize that there is no difficulty in solving the oblique ones, since they convert into right-angled ones by dropping a perpendicular. You need no other instruction besides these. For Section 2 you will need only the solution of the oblique triangle whose three sides are sought once you know its three angles, as you will see in Figure 3 of *Sod HaYesod*; and you can reckon it another way too, if you take care to invert it and test each row, and you will find the truth.

But take careful note of the dangerous cases — those whose results come out anomalous, because the sine is shared by an arc and its supplement [?]. And what profit is there in multiplying words with you? You are a man after my own heart; I know that you have eyes of flesh, exalted intellect, and sound understanding that bends the crooked straight — and wisdom strengthens the wise man more than ten rulers [cf. Koheles 7:19].

Chapter Thirteen of Book One of the Almagest

Said the author [R. Yashar]: I saw fit to explain first a few matters from the *Sphaera* [Sacrobosco's *De Sphaera*], without which the following problems will not be understood — and this in brief.

Longitude [of a star] is the arc of the ecliptic between the head of Aries and the latitude-circle of that star, reckoned in the order of the signs. For this reason the ecliptic is called the circle of longitude.

Latitude of a star is the arc of a great circle passing through the poles of the ecliptic and through the center of that star — the arc between the star and the

ecliptic — and so the ecliptic is called its latitude-circle; hence that great circle is called the circle of latitude.

Declination of a star is the arc of a great circle passing through the poles of the world [celestial poles] and through the center of the star — the arc between the star and the equator is called the declination of that star, and this circle is called the circle of declination, whether northern or southern.

From this you may learn that there are two kinds of declination — and they begin from the equator toward its pole, which are the poles of the world. Likewise there are two kinds of latitude, northern and southern, and they begin from the ecliptic toward its poles. Therefore neither the declination nor the latitude, at its greatest, exceeds a quadrant of a circle. It is known that the sun never deviates from the ecliptic line, and so it has no latitude at all; but the planet Mercury deviates from the ecliptic by about six degrees, sometimes northward and sometimes southward, and some of them [the planets] by eight degrees. The astronomers assigned to Venus alone a latitude of about twelve degrees, for the reason mentioned.

Know further that the greatest solar declination, as we said, is — according to Ptolemy — 23° 51′ 20″. But the declinations of the fixed stars are, for each, according to its position in the celestial sphere; and likewise for the planets, their declinations vary according to their latitude, which changes with their motion.

The **rising** or ascension of a star is an arc of the equator beginning at the head of Aries and proceeding in the order of the signs up to the horizon at the moment the star rises. Its **setting** is called the arc of the equator beginning at the head of Aries and proceeding in the order of the signs up to the horizon at the moment the star sets below the horizon.

Know further, from the studies of the sphere, that the daily motion is not about the poles of the ecliptic but about the poles of the equator. Therefore the ascensions of each and every part of the equator are equal to one another, since wherever it meets the horizon of a given place it makes equal angles. This holds for every horizon in the world: in every hour fifteen parts of the equator rise above the horizon and as many descend on the western side, in such a way that four minutes of an hour correspond to one degree of the equator. With the ecliptic, however, it is not so: the angles which the head of Aries and Libra make with the horizon are not the angles of Cancer and Capricorn and so forth. Sometimes more than fifteen degrees of the ecliptic rise in one hour — namely, in the time of the rising of fifteen degrees of the equator — and sometimes fewer

than that. Therefore, even though the sun always travels along the ecliptic and it is that which governs time, nevertheless the measure of the motion and of the time is the equator, by the property mentioned. Were equal parts of the ecliptic always to rise with the equator, the days would always equal their nights throughout the year.

Now the sign — which is 30 degrees of the ecliptic — with which more than 30 degrees of the equator rise is said to have a **right ascension**, that is, to the [direct? zonot] [?]; and conversely, the sign that rises together with [an arc that] meets the horizon at right angles or near-right angles [?] — fewer than 30 degrees of the equator — is said to have an **oblique ascension**, since the angles it makes with the horizon are oblique. This terminology is used by those who speak of ascensions. But Ptolemy does not reckon these ascensions as oblique and right; rather, he calls it a right ascension and descent when the star rises and sets in the right horizon, and conversely, in all the oblique horizons that are not under the equator, he calls the risings and settings of the stars oblique. It is on this basis that the following problems and the tables of ascensions are built.

I need not copy out for you here the properties of ascensions, for you will seek these from those who discuss the *Sphaera*, in the manner of the well-known *Sphere* of Joannes de Sacrobosco [יואניש שקרובושק"ו, Johannes de Sacrobosco] — which exists in the Holy Tongue [Hebrew] with the commentary of R. Moshe [?] — our teacher's [work] called *Beih Eilim* [?] [בית אלים] or the book *Tzurat HaAretz* [Form of the Earth] or *Yesod Olam* [Foundation of the World] and their like. Here I will mention only this: that the meridian circle, since it passes through the poles of the world, is reckoned as a right horizon — for it is truly the horizon of the equator's southern point [?] — and so the astronomers call this circle a right horizon. Therefore, according to the time at which a point rises on the right horizon, the meridian circle of any horizon arrives at that same time. All the properties of ascensions and descents stated for the right horizon belong to the meridian circle of every horizon — know this.

Since the ascensions of stars on the right horizon differ from their ascensions in the oblique horizons — and the oblique ones [closer to the right horizon] are more, and a little closer to right [?] — there will accordingly be variation and difference from one ascension to another in every horizon; and the difference between each of these and the right ascension is called by us [?] the arc of the equator that rises with that star, and is called the **difference of ascension** [ascensional difference]. When you know the right ascension of the star and also know its [ascensional] difference in a given horizon, then, whether you add or

subtract the difference from the right [ascension], the oblique ascension in that horizon comes out clear for you.

The addition is appropriate in the southern signs — Libra, Scorpio, Sagittarius, Capricorn, Aquarius, Pisces — and the subtraction in the northern signs — Aries, Taurus, Gemini, Cancer, Leo, Virgo. This is because the northern signs rise more swiftly, and therefore their ascensions are smaller in the oblique horizon than they are in the right horizon; and conversely, those inclining toward the southern pole rise more slowly, and their ascensions are longer in the oblique horizon than in the right horizon. All this is explained in the *Sphaera* and in the tables of ascensions.

Know further that, just as in right-angled triangles there is no need to mention the perpendicular, so in these problems the great [maximum] declination is already known and is given, and there is no need to mention it — unless one takes a declination other than the one transmitted from Ptolemy, and the like.

First Problem

How, from knowing the longitude of any point on the ecliptic, we may know the declination of that point.

Let it be, in the present figure [ד/ר :?] **ארצבג** [A-R-Tz-B-G], that **אהב** [A-H-B] is the meridian circle, **אהב** [?] [recte: read as the meridian semicircle]; that **אהב** is the semicircle of the equator and **דהג** [D-H-G] the semicircle of the ecliptic; and from point **צ** [Tz], the north pole, let there descend a quadrant of a great circle, the line **ציח** [Tz-Y-Ch], passing through the head of Taurus or Gemini — at the point **י** [Y] — to the equator. By the Fifteenth Proposition of Book One of Theodosius, the angle at **ת** [T] will be a right angle ([Figure 16]).

Diagram. A spherical triangle for converting longitude to declination. The figure shows the meridian circle as the outer bounding circle, with two semicircular arcs across it: the equator (**אהב**, A–H–B) and the ecliptic (**דהג**, D–H–G), crossing at the spring point **ה** [H] in the east. From the north pole **צ** [Tz] a quadrant of a great circle (**ציח**, Tz–Y–Ch) is dropped perpendicular to the equator, meeting the ecliptic at point **י** [Y] (the head of Taurus or Gemini) and the equator at the foot **ת** [T], where the angle is right.

- **א** (left end of equatorial/meridian diameter): A
- **ב** (right end): B
- **ג** (end of ecliptic semicircle): G
- **ד** (other end of ecliptic semicircle): D

- **ה** (intersection of equator and ecliptic): H — the spring equinoctial point
- **צ** (top): Tz — the north pole
- **ת** (foot of perpendicular on equator): T — right angle here
- **י** (on the ecliptic, head of Taurus or Gemini): Y — the star/point whose declination is sought
- **ח** (?): Ch — endpoint of the dropped quadrant **ציח** [?]

Now in triangle **יתה** [Y-T-H] we have three knowns, and the proportion runs thus. From Row 11, Figure 1, of Book One of *Sod HaYesod* — and all are sines:

	sine	value
ת [T], right angle — whole sine	(the whole sine)	10000000
ת [T], right angle	(the whole sine)	10000000
יה [Y-H], longitude of the head of Taurus, 30 degrees	sine	5000000
יה [Y-H], head of Gemini, 60 degrees	sine	8660254
יהת [Y-H-T], the great declination, 23° 51′ 20″	sine	4044323
יהת [Y-H-T], the great declination mentioned	sine	4044323
ית [Y-T], the sought declination, 11° 39′ 59″ 43‴	sine	2022161
ית [Y-T], the sought declination, 20° 30′ 9″	sine	3502486

And Ptolemy found 11° 40′ [for the Taurus case]; so it stands in Ptolemy as well.

According to the declination of Copernicus, however, the declination of the head of Taurus would be 11° 30′ and the head of Gemini 20° 12′; and according to Tycho, the head of Taurus 11° 30′ 42″ and the head of Gemini 20° 13′ 22″.

Know that the declination belonging to the head of Aries and Libra is the same as that belonging to the end of Pisces and Virgo; the declination of the head of Taurus and Scorpio is that of the end of Aquarius and Leo; and likewise the head of Gemini and Sagittarius is the same as the end of Capricorn and Cancer. Accordingly, there is no need to compute more than the declinations of a quadrant of the circle — the divider between the northern and the southern [signs].

Here, then, is the table of declinations according to Ptolemy, and there you will grasp it precisely.

[The table of declinations per Ptolemy is referenced here but not printed on this opening — it lies on the facing page or following leaf. Table continues / not present in this opening.]

I will not refrain from disclosing to you how you can correct [refine] the said table, and all those that follow, without great labor — with toil alone, together with the supplement [?]. This is precious and weighty for those engaged in producing the daily tables of the planets called *ephemerides* [איפימירידים], and the like, by means of the aid of triangles.

For, as you saw in the two examples mentioned, the sine of the great declination must be multiplied by the sine of the longitude of each and every point; and the result, after you subtract from it a sum of figures from the right according to the number of digits of the whole sine, is the sine of the sought declination.

So then, if you wish to spare yourself the labor of this multiplication, make for yourself a small table like this, in which you multiply the sine of the great declination by the numbers up to nine; then you need not multiply again, but copy them from the table and arrange them as you require. For example, suppose I wished to multiply 8660254, the sine of the great declination, by 8660452 [recte: 8660254]. Here I take from the table 4 times the said sine, which is [16177292]; and 5 times, which is 20221615; and 4 times again, and so on. You may begin as you please, from the left or from the right, or as your eyes see fit. There is no need to elaborate. And if you have the numerical rods [napier's-bones-type device] that I devised for our teacher R. Moshe, you do not need this table, for you will extract them from there at once.

The little table — multiples of 4044323 (the sine of the great declination):

×	product
1	4044323
2	8088646
3	12132969
4	16177292
5	20221615
6	24265938
7	28310261

×	product
8	32354584
9	36398907

Two parallel long-multiplication worktables follow, each computing 4044323 × 8660254. The two are mirror images of each other (the partial products written top-down in one, bottom-up in the other), both summing to the same product.

Left worktable — head of multiplicand 4044323, head of multiplier 8660254 [?]:

partial product
3235458. 4
242759. 38
24265. 938
000. 0000
80. 88646
20. 221615
1. 6177292
3502486. 4438042
3

Right worktable — head of multiplicand 4044323, head of multiplier 8660254:

partial product
1. 6177292
20. 221615
80. 88646
000. 0000
24265. 938
242650. 38 [recte: 242759.38]
3235458. 4
3502486. 4438042
3

Chukos Shamayim (Almagest commentary) — Book I, Chapter 13 (continued)

Table of Declination according to Ptolemy

Table. A declination table giving, for each degree (1–30) of arc, the corresponding declination in degrees, minutes, and seconds for the several zodiacal signs. The table reads right-to-left in the source. The column-heads pair signs that share the same declination: each declination triplet serves both a northern and a southern (ascending and descending) sign. The bottom row of labels gives the signs read in the reverse sense, with the degree counted from the other end of the sign (1–30 descending).

Heading row (top): **Gemini** / **Sagittarius** (תאומים / קשת — northern / southern, צפונים / דרומים); **Taurus** / **Scorpio** (שור / עקרב); **Aries** / **Libra** (טלה / מאזנים — ascending / descending, עולים / יורדים); **Degrees of the ecliptic** (מעלות הלקותיי).

Bottom row of labels (the same columns read in reverse): **Capricorn** (גדי); **Cancer** (סרטן); **Aquarius** / **Pisces** (דלי / דגים — ascending, עולים); **Leo** (אריה); **Virgo** (בתולה — descending, יורדים); **Degrees** (מעלות הלקותיי).

Each declination entry is formatted *degrees. minutes. seconds*. In the table below, the left "Deg." column is the ecliptic degree counted from the start of the sign (ascending, 1→30); the right "Deg." column is the same degree counted from the end (descending, 30→1).

Deg. (asc.)	Aries / Libra	Taurus / Scorpio	Gemini / Sagittarius	Deg. (desc.)
1	0. 24. 16	12. 1. 20	20. 42. 58	30
2	0. 48. 31	12. 22. 30	20. 55. 24	29
3	1. 12. 46	12. 43. 28	21. 7. 21	28
4	1. 37. 0	13. 4. 14	21. 18. 58	27
5	2. 30. 12 [?]	13. 24. 47	21. 30. 11	26
6	2. 25. 22	13. 45. 6	21. 41. 0	25
7	2. 49. 30	14. 5. 11	21. 51. 25	24
8	3. 13. 35	14. 25. 2	21. 1. 25 [?]	23
9	3. 37. 37	14. 44. 39	22. 11. 11	22
10	4. 1. 38	15. 4. 4	22. 20. 11	21

Deg. (asc.)	Aries / Libra	Taurus / Scorpio	Gemini / Sagittarius	Deg. (desc.)
11	4. 25. 32	15. 23. 10	22. 28. 57	20
12	4. 49. 24	15. 42. 2	22. 37. 17	19
13	5. 13. 11	16. 0. 38	22. 45. 11	18
14	5. 36. 53	16. 18. 58	22. 52. 59	17
15	6. 0. 31	16. 37. 20	22. 59. 41	16
16	6. 24. 1	16. 54. 47	23. 6. 17	15
17	6. 47. 26	17. 12. 16	23. 12. 27	14
18	7. 10. 45	17. 29. 27	23. 18. 11	13
19	7. 33. 57	17. 46. 20	23. 23. 28	12
20	7. 57. 3	18. 2. 53	23. 28. 16	11
21	8. 20. 0	18. 19. 15	23. 32. 30	10
22	8. 42. 50	18. 35. 5	23. 36. 35	9
23	9. 5. 32	18. 50. 41	23. 40. 2	8
24	9. 28. 5	19. 5. 47	23. 43. 2	7
25	9. 50. 29	19. 20. 56	23. 45. 34	6
26	10. 12. 46	19. 35. 28	23. 47. 39	5
27	10. 34. 57	19. 49. 42	23. 49. 16	4
28	10. 56. 44	20. 3. 31	23. 50. 25	3
29	11. 18. 25	20. 17. 4	23. 51. 6	2
30	11. 39. 55	20. 30. 9	23. 51. 20	1

[The following continues the explanation, begun earlier, of the author's number-rod multiplication device.]

This is the truth: at the outset, when you transcribe the partial products, you can write down "less one, less two, less three" as you move to the left, but on the right side you transcribe "less seven, less six, less five," and so on; keep this in mind. And take care — for these matters are precious, the sort handed down to kings and not to the multitude. Beyond this, I will teach you one further useful shortcut, a clean one: if, after the multiplication, you must discard so many digits from the right according to the count of the figures of the sine, why trouble to multiply them for nothing? Be shrewd, then, and multiply only what is necessary and serviceable — that alone.

Here is what you do. Once you have arranged the multiplicand together with the multiplier as they stand before you in the figure shown above, you can begin the multiplication — or the striking — in two distinct ways. The first: when it runs on the left side, so that the head of the multiplicands is on the right and the head of the multipliers is on the left. Take heed, then: first multiply 3 by the figure 8, and after it 2, and after it 3, and so all of them, with 8; and when the first multiplier 8 has finished its work, know that the first multiplicand 3 likewise has no further business and is set aside. Note further that the carries arising from multiplying any multiplier into the multiplicand at which it began its work are not written down.

For example: I multiplied 38, and 24 came out [3 × 8 = 24]; I discard 4 and keep 2; again, 8 with 2 gives 16, and with the 2 kept makes 18; I write 8 and keep 1, and so I finish the first row. Again, I multiplied the second multiplier 6 with the second multiplicand 2, and 12 came out; I discard 2 and keep 1; again, 6 with 3 gives 18 [recte: read with the carry], and with the kept figure makes 19; I write 9 and keep 1, and so I finish the row. Again, I strike the third multiplier 6 with the third multiplicand 3, which makes 18; I discard 8 and keep 4; again, 6 with 4 makes 24, and with the kept figure makes 25 [?]; I write 5 and keep [1], and so I finish the row, and so I proceed with them all. The sum will be the sought sine itself at its head — that is, the last [figures] remain, and there is no carry over or under at all in the sines, and there is no need to subtract anything from it.

I will now teach you the second way, the one drawn in the right-side figure, which runs from right to left. It differs from the first in that you begin from the rightmost multiplier 4, and multiply it with the leftmost multiplicand 4, making 16; discard 6 and write 1. Then strike the second multiplier 5 with the second multiplicand 0, giving 0, and discard it; likewise 5 with the leftmost figure 4, giving 20, and write them down. Again, strike the third multiplier 2 with the third multiplicand 4, giving 8, and discard them; likewise 2 with 0 gives 0 and you write it; likewise 2 with the leftmost 4 gives 8, and write them. Likewise the fourth multiplier 0 you multiply with the fourth multiplicand, giving 0, and discard it; and with 4 gives 0, and write it; and so on. Likewise the fifth multiplier 6 with the fifth multiplicand 3 gives 18; discard 8 and keep 4; again 6 with 4 makes 24, and with the kept figure makes 25; write 5 and keep one; likewise 6 with 4 to the left side, and finish the row.

This way is fair and pleasant indeed for a man of clean hands. And though I have made these matters known to you — they seem mere instruments, yet they are truly objects of desire to the discerning — I tell you that we could have brought in

many further problems here, problems that fall within our concern, on which Ptolemy did not treat: such as how, from knowing a star's declination, we may learn its longitude (which is the inverse of the First Problem). This is easy for one who understands the resolution of triangles: in the figure above, the values are known to you — the arc of declination, namely [ץ]ד, and the perpendicular ב, and you seek the arc [ק]י, the longitude of the star at point י. And there are many other problems similar to these, very necessary for various operations. But I do not wish to entangle the topics, which would break Ptolemy's order, which I have undertaken to expound — especially as I have them in another composition, some two hundred problems, and there is neither place nor time here to transcribe them all. I will, however, tell you that one who wishes to compute

— in these problems must first fix carefully the value of the great declination [obliquity] of our era. For in matters acquired through the senses and through observation, one should put no more trust in others than in oneself. Who is to prove that Ptolemy's eyesight was sharper than ours, or that the touch of Galen [גאלינוס = Galen] was duller, or that he judged the temperaments of medicines by taste better than our touch and our judgment? All the more so, since we possess instruments more precise than his. It is not so with deductions, where it is fitting to hold our own reasoning suspect and not to leap up and contradict those greater than ourselves. And thus I found the great declination, using large and accurate instruments, to be close to **23° 32′ [?]** in the city of Toruń [טורוני"אה], the home of Brother Copernicus [פרו' סיאה = Frater (Brother) — Copernicus]; and likewise a second time in Vilna [וילנ"ה]. On this value I rely in my computations. According to it, the following Problem — most necessary to know — will be settled for you; and I have transcribed it from the aforementioned book.

[Note: "23. 32" as printed; this is the modern obliquity Delmedigo adopts, distinct from Ptolemy's 23°51′20″. Flagged as an uncertain reading — the tiles show 32, but the value is close to the historically expected ~23°30′–31′.]

Problem 2 (שאלה ב)

How, from knowing the longitude and latitude of a star, we may learn its declination. **And it will be** shown by way of example in the present figure [Figure 17].

Diagram. A celestial-sphere construction. הגפהדרץ is the meridian circle (literally "the circle of midday"). חבך is the equator (the circle of the equinox), with its northern pole גבצ. פ is the ecliptic, with its pole דץ [d-tz]. The star lies at point מ — the bright star at the head of Aries (Hamal), whose longitude in our era is 30°

of Taurus, i.e. arc 32. 32 [?], and whose northern latitude is 9. 57, the measure of arc נ–מ. We wish to know its declination, namely arc ס–ל [s-l].

- פ (north pole of the equator)
- דץ (pole of the ecliptic)
- מ (position of the star — head of Aries)
- חבך (equator)
- גבצ (ecliptic)
- הגפהדרץ (meridian circle)

In the oblique triangle פ–ס [p-s] three of its parts are known to us: side פד [p-d], the distance between the poles, which is the great declination, **23. 32**; side הפ [recte: see note] **80. 3** [?] — the complement of the given latitude; and the angle פ at their vertex [angle at פ], **28 .57** [recte: 28. 57] — the measure of arc נ. The complement of the given longitude is therefore 57. 28 [?]. [This is solved] by Proposition 11 of the third part of *Sefer Sod HaYesod* [the Book of the Secret of the Foundation]. Also side פם [p-m] is known, 68. 17, whose complement, ל. [recte 21. 43], is the declination of the aforesaid star.

I now set before you the order of the operation, since for that reason I preserved the worked examples and did not include them in the said book in all the rows, so as not to repeat matters.

In the oblique triangle [Figure 18] **א-ב-צ**, side **אצ** is **80. 3** and side **בצ** is **23. 32**, and the angle **צ** is **57. 28**; we wish to learn the third side **א-ב** [a-b].

Diagram. The oblique spherical triangle **א-ב-צ** [a-b-tz], extracted from the construction above for the computation. Side 3 .80 = **אצ**; side 32 .23 = **בצ**; angle at 28 .57 = **צ**; the sought third side is **א-ב**.

- א, ב, צ — the three vertices of the oblique triangle
- אצ (side) = 80. 3
- בצ (side) = 23. 32
- angle 28 .57 = צ

Now, following my instructions there: we posit the angle at **ב** to be a right angle, and by Proposition 12 of the first part we seek, in the right triangle **א-ב-צ**: as side **בצ**, so [the proportion] — as 100000 is to the secant of angle 28 .57 ,**צ**, which is 185946, so the sine of the complement of the plane (⁸⁰⁄₉ ⅜ 57°) **אצ** [the printed marginal fraction reads ³⁄₅₇ · ⁸⁰⁄₉ — uncertain], which is 17542, is to 32618, the sine of the complement of side **בצ**, and its arc is **18. 4** [recte 4. 18]. Therefore side **בצ** is

71. 56 [recte 56. 71 — i.e. the complementary reading]. And so we found likewise with row 12 of the second part.

But indeed 23. 32 [recte 23. 39?] had been posited alone — therefore the right angle is not at **ב**, but the perpendicular falls outside the triangle, and there will be the true perpendicular **א-ך** [a-k] and not the side **א-ב** as we had thought. And since the [side] fell, second, [we seek] in the right triangle the angle **אצך** [a-tz-k]; we seek the perpendicular **א-ך**, and by Propositions 11, 13, and 14 you will be able to learn it: for the value found exceeds the posited side; subtract the posited from this value found, and there remains **רך** [r-k] known, **48. 24** [recte 24. 48].

Many things are known to us in this triangle. For example, the proportion by row 14 of the first part: as 100000 is to the tangent of the angle 57. 28, which is 156767, so [is] the sine of side **דצ** [d-tz]

— found, **71. 56**, which is 95069, to the tangent of the perpendicular **אך** [a-k], 149036, and its arc is **9. 56** [recte 56. 9].

Third [step]: in the right triangle **אדב** [a-d-b], from the two sides known by row 6 of the first part we learn the third. The plane **אב** [a-b]: as 100000 is to the sine of the complement 55702 ,**אך**, so [is] the sine of the complement **דב** [d-b], 66392, to the sine of the complement of the plane 36981 ,**אב**, which is **68. 17** [recte 17. 68], side **אב**, whose complement, **21. 43**, is arc **ס-ל** [s-l], the sought declination of the star.

By this method the astronomers computed the declination of every point [of the ecliptic] that has both longitude and latitude, and tabulated them, as Regiomontanus [ריניאומונטאנו] and the like demonstrated. There are ways to lighten these computations, but it is impossible to write down everything; and we are, as people say, [reliant] on the mouth of scribes and not on the mouth of books.

Chapter 14 (פרק הי"ד), the last of Book I of the *Almagest* (מאמר א' למיגיסטי)

Problem 3 (שאלה ג)

How, from knowing the declination of any point of the ecliptic, we may learn its right ascension.

In the first figure, in the right triangle [the apparatus], the tangent of the complement of the great declination is sought; and after we have learned arc **ית** [y-t], we seek arc of the equator **דקך** [d-k-k], which rises with arc **קי** [k-y] of the

ecliptic on the right horizon; and it is found by row 15 of the first part in this manner. And so they stand in Ptolemy. And like it you may find the right ascensions of all the points of a quadrant; for when you add 90, 180, 270, you will find the right ascensions of the whole circle, as they stand in the prepared tables.

Worked example A [ascension of the head of Taurus]:

ת	100000
יהת tangent of the complement of the great declination	22613626
יה tangent of the declination of the head of Taurus	2064834
הת sine of the right ascension, 27. 50. 8	4669338

Worked example B [ascension of the head of Gemini]:

ת	100000
יהת tangent of the complement of the great declination	22613626
ית tangent of the declination of the head of Gemini	3739344
הת sine of the right ascension, 57. 44. 11 [recte 11. 44. 57]	8456012

Problem 4 (שאלה ד)

How, from knowing the longitude of a point on the ecliptic, we may learn its right ascension.

This problem, though it is not among the author's [Ptolemy's] problems, has been amply explained and is easy; for in the triangle [formed], once we know the longitude, and proceeding **דקך** as above, the proportion will be as set out in row 12 of the second part. But when the point is not on the ecliptic but has latitude — how to find its ascension has been explained elsewhere, for it is not brought in the author's words.

Worked example A [right ascension for longitude = head of Taurus, 30°]:

ת	100000
יהת sine of the complement of the great declination	9145679
הי tangent of the head of Taurus, 30 degrees	5773503
הת tangent of the right ascension, 27. 50. 7 [recte 7. 50. 27]	5280260

Worked example B [right ascension for longitude = head of Gemini, 60°]:

ת	100000
יהת sine of the complement of the great declination	9145679
הי tangent of the head of Gemini, 60 degrees	17320508
הת tangent of the right ascension, 57. 44. 11 [recte 11. 44. 57]	15840781

[End of Chapter 14, Book I of the *Almagest*.]

Chukos Shamayim (Almagest Commentary)

The Second Book of the Almagest

From Chapter 2: Problem 5

How, from knowing the longest day, we may find the size of the horizontal arc between the equator and the ecliptic — called the latitude (breadth) of rising.

The horizon is called a circle. The "latitude of rising" (or of setting) is this: the true rising occurs at the place where the equator is cut by the horizon. This rising takes place when the sun is at the two intersection points — the head of Aries and Libra — and then there is no latitude of rising at all. But at every other point of the ecliptic, the horizon cuts at a place distant from where it cuts the equator. The arc between the rising of a given star and the true [equinoctial] rising is the latitude of rising — and likewise of setting; it is either northern or southern. Knowing it is much needed for [computing] the lengths of shadows, which I explained in the *Sefer Tzel HaChochmah* ["Book of the Shadow of Wisdom"]; its further properties you will find in my *Sifrei HaKaddur* ["Books of the Sphere"].

Now the author [Ptolemy] explains at length what is in fact quite easy; this is his approach. In the present figure (**fig. 19**), let circle ג-פ-מ-א be the meridian, פ-ק the oblique horizon, א-ה-ג the equator, ד-ו the ecliptic circle adjacent to it, and ז-ח the north and south poles. You can see that the equator is cut by the horizon at point ד-ק [?], which is the true rising; and the ecliptic ד-ו is cut by it at point י. Therefore arc ד-ק-י [?] is the sought latitude. This is at a place where the longest day is 14 hours 30 minutes, of which half is 7;15. That is arc ת-ג, half of the longest night, when the sun is at the head of Capricorn. Accordingly arc ת-א is the measure of

half the [longest] day — namely the portion of the equator that rises together with portion ד of the ecliptic in that interval. It is clear from this that if we subtract 12 hours from the said longest day, there remain 2½ — of which the half is 1¼ — and this is the measure of arc ד-ת [?], the excess of half the night over half the day, since arcs א-ד / ת-ד [?] are equal. For when the sun is at point ד [?], before it rises, the whole portion ד of the equator has already risen above the horizon ahead of it.

So you now have three known [quantities] in the right-angled triangle ד-ק-ת-י [?] — that is, side ת-י-ד [?] (which equals the present [arc] א-ד), the distance between the equator and the ecliptic circle = **23;51,20**; together with the excess of half the night over half the day, one and a quarter [hours], which is **18;45** of [equatorial] degrees; and from the seventh column [of the table] before us, we find the sought latitude of rising. The proportion is as follows. And likewise easily: the radius (whole sine) **10000000**.

	Value	Function
ת	10000000	(whole sine / radius)
ית	9145679	sine of the complement of declination (ה) [of each point in a quadrant of the circle]
הה	9469301	sine of the complement of the [day]-difference for half the day
יה	8660319	complement of the sine — the sought [value], $^{60}⁄_{30}$

You can compile a Table of the Latitude of Rising and Setting [for each point in a quadrant of the circle]; that suffices for all the points, and there is no need to elaborate.

From Chapter 3 of the Second Book

Problem 6

How, from knowing the longest day and the longest latitude of rising, we may find the elevation of the world's pole above the horizon.

In the figure mentioned in the previous problem, we know — in the right-angled triangle ז-ר-י [?] — side ך-י [?], the great latitude of rising, and likewise [side] ה-ת [?], the difference between the longest half-day and the equinoctial half. Accordingly, from the said column, before [column] 1 or 2:

	Value	Function
ת	10000000	(whole sine / radius)
יה	17320508	sine of [arc] 30 — tangent of the complement
הה	3394544	tangent — 18;45
יהת	5879522	sine of the complement — 53;9

The result is arc א-ק [?], the elevation of the equator above the horizon, whose complement is **1;36** [?] — and that is the measure of [arc] מ-פ [?], the elevation of the northern pole above the horizon. The sought quantity, and the proportion, is thus.

Problem 7

How the above [pole-elevation] may be found another way — that is, from knowing the great latitude of rising — in the aforementioned triangle, from column 2 of the first [trigonometric] tables. The proportion is as follows.

	Value	Function
ת	10000000	(whole sine / radius)
יה	20000000	secant of the complement — 30
ית	4044323	sine of the great declination
יהת	8088646	sine of the complement — elevation of the equator

Problem 8

How the above may be found from knowing the longest day, in the aforementioned triangle.

	Value	Function
ת	10000000	(whole sine / radius)
יה	3214395	sine — 18;45
ית	22613624	tangent of the complement of the great declination
יהת	7268911	tangent of the complement — elevation of the equator

Here ה-ית [?] are subtractions [factors]; the [arc] ה-ת is the half-difference of the equinoctial and longest day, **18;45**. From column 1 of the first tables we find angle יהת [?], the elevation of the equator, and the proportion is thus: **13.59.53**

[reversed; recte 53;59,13]. Accordingly the pole-elevation is **36.0.47** [reversed; recte 47;0,36 — but see note].

Problem 9

How, from knowing the pole-elevation, we may find the [length of] day. This problem too is Ptolemy's, and it is the inverse of Problem 5. The sought quantity, in the aforementioned triangle יהת [?], where the elevation of the equator and the great declination are known to us as angle and side: we find arc ה-ת [?], the half-difference of the longest day over the equinoctial day, from column 15 of the first tables. What we find — **18;45** — we convert into hours: it is one hour and a quarter; we double it to 2½ hours, add it to the equinoctial day [of] 12, and out comes 14¼ — the longest day at the place where the pole-elevation is **31;1** [reversed; recte 1;31?]. That is the sought quantity.

	Value	Function
ת	10000000	(whole sine / radius)
יהת	7268912	tangent of the complement — elevation of the horizon
ית	13442211	tangent of the great declination
הת	3214395	sine — 18;45

Problem 10

How, from knowing the great latitude of rising, we may arrive at knowing the longest day. [Ptolemy:] in the triangle [where] י-ד and ו are known to us, we find ה-ת; and the proportion, from column 13 of the first tables, is thus.

	Value	Function
ת	10000000	(whole sine / radius)
יה	10341247	secant of the great declination
יה	86602543 [recte: 8660254]	sine of the complement — 30°
הת	946923000 [recte: 9469230]	sine of the complement — 18;45

Chukos Shamayim

Problem 11

How, from knowing the pole-elevation, we may find the latitude of rising. This too is Ptolemy's. In the aforementioned triangle, angle יהת [?], the complement of the elevation of the axis, is known, and side ית, the great

declination; and we find, from column 9 of the first tables, arc יח [?]. The proportion is thus.

	Value	Function
ת	10000000	(whole sine / radius)
יהת	12363293	secant of the complement — 1;36
יה	4044323	sine of the great declination
יה	5000115 [recte: 5000000]	sine — 30

From the Fifth Chapter of the Second Book

Problem 12

How, from knowing the pole-elevation, we may find the ratios of shadows to gnomons (vertical rods), and conversely.

In the present figure, arc (**fig. 20**) א-ב is the distance between the equator and the zenith — which, as you know, equals the pole-elevation: that is, **36 degrees**. When we subtract from it arc י-ב, the great declination, **23;51,20**, the remainder is arc **8,40;12** א-י, . And if we add to arc א-ב the said arc [of declination], the whole arc א-ל is **59;51,20**.

Now the gnomon ג-ה is 60 cubits [tall], set upon the surface of the earth, treated as though point ה were its center — for half the earth's diameter counts for nothing against the distance of the sun from it. Accordingly the lines drawn out, א-ב, י-צ, ב-פ, [and] 17 [?], cut off equal arcs at the lower side; and when extended to the surface 1-ב [?] they bound the shadows of the said gnomon at the spring season and the equinoxes (and their opposites) — that is, at midday, when the sun is at its greatest elevation for its season. Now since the arcs are known, their tangents — גנ ,בצ [?] — are also known and computable. We say, from column 3, before [point] ד: if [arc] ג-ה — the whole sine [radix] 100000 — gives 60 cubits,

	(whole sine 100000)	gives	(cubits)
how much then yields	21529 אי	→	12 בצ 91740⁄100000
tangent of אב	72654	→	43 גפ 59240⁄100000
אל	172162	→	103 גנ 29720⁄100000

Conversely — from knowing the lengths of the shadows and the gnomons — we may find the pole-elevation: for in the triangles, right-angled, whose [hypotenuses?] are known by two sides, we find angle ה-ז [?] from column 7,

before [point] ד, in this manner: if 60 ,ה-ג, gives the whole sine 100000, how much then yields ג-צ?

The measure of ג-פ [?]: we find the said tangents, and their arcs are exhibited [openly] in the table. We find arc 36 [?] ג-פ degrees [reversed; recte], which is the sought pole-elevation; and conversely, by inverting the operations, you find בצ — that is, arc **8,0;12** ,א-י [?] — to which we add י-ב, the great declination, and out comes the elevation of the axis. And conversely, by inverting the winter [computation], [you find] א-ב, the elevation of the axis. **However**, when the sun is at points other than those mentioned, we find the elevation: we find בצ [?], the tangent of arc **51,20;59** ,א-ל, and from it we subtract arc [?] 51 [?] of declination, and there remains א-ב, the elevation of the axis.

Yet when the sun is at points other than those mentioned, the [sun's] elevation [above the horizon] is found, and the pole-elevation above the horizon is found in this way: after the distance of the sun from the zenith has been found by the above method, we subtract it from a quadrant, and there remains the sun's elevation above the horizon. Know its position too in the ecliptic

[Book continues — p. 254]

from the tables, and seek its declination; if it is northern, subtract it from the said elevation, and if southern, add it — and the pole-elevation is found.

Now mariners at sea readily find the latitudes of countries [in degrees], by means of an instrument they carry with them. It is a circle divided into 360 degrees, and across its middle a measuring [alidade-]finger from edge to edge, and on it two uprights at its ends, in which are holes or slits through which the sun's ray passes. The instrument is of bronze, copper, heavy [metal], so that the winds do not entangle it; they hold it in a hand that points to the elevation [vertically], and they turn the alidade until the sun's rays pass through the two slits and strike the sun [-line], and with them [they read off] the Table of Declinations. If the declination at that time is northern, they subtract it from its [the sun's] elevation; and if southern, they add it. So they easily learn the elevation of the axis approximately, and that suffices for the requirements of their work in many cases.

There are many methods for ascertaining the elevation of the axis with instruments — by reckoning and by reasoning, and by various tools. But the one I am accustomed to is this — especially when I do not wish to be exacting about minute fractions: wherever the sole of my foot treads, I lift my eyes to the heavens and look to see whether some star of the 48 [Ptolemaic] constellations

passes through the zenith, whose declination is the very elevation of the horizon. And indeed I can find the declination of those stars from Problem 3 of this book, or from any table giving the declinations of all [stars] of the starry or sign-bearing sphere. For example, in Candia [Crete], my native city, I see passing over my head the stars called Caput Medusae [Rosh Medusa / "Head of Medusa"], or the star at the end of the [fore]-leg, the rear [foot], of the Great Bear [Ursa Major], whose declination is about **35 degrees**. Therefore the elevation of the northern pole at Candia is **35**.

Likewise at Amsterdam I see, through the wall, by [a] window, that the star of Cassiopeia in her ear passes over it, and likewise the outermost star in the tail of the Great Bear; and these declinations are about **52 degrees**. So the latitude of the said city — which, with instruments, I also found to be about **52½ degrees** approximately. And since we said that the sun's elevation must be taken exactly at midday: navigators attain this with their compass [קומפ"אש], in which the magnetized iron [needle], by the power of the lodestone [magnet], points along this [meridian] line — so that when they see the shadow of the thread with the [plumb-]lead in their hand fall upon the said iron [needle], or parallel to it, they know it is midday.

But if you wish to know this on any day — when, by your watch, the moment of midday is — examine this for one day: observe the shadows of some gnomon ה [?], around which you have drawn circles, with the gnomon planted upright at their center. When the tip of the shadow reaches the circumference of one of the circles before midday, mark a point there; and likewise after midday, mark a point on that same circle, where the tip of the said gnomon's shadow reaches. The arc between the points, if bisected — the diagonal passing through the center is the meridian line. From one [such construction] you can [draw] boards [dials], many and various, on different surfaces, and so on.

However, as to what al-Ghāfiqī [?] [האלקהרי] asked concerning the pole-elevation of the city of Alexandria, and the length of the shadow of the great obelisk that Julius Caesar set up for Pompey (which stands outside the gates of the old city, a bowshot away) — how we may find its elevation — this is easy from what was said in this chapter. In the said figure: at Alexandria, the head of Cancer is distant from the zenith **8 [?].40** [reversed; recte 40;8?], and the sine of its complement is **928858** [?]. At that time the shadow of the obelisk was 14 handbreadths at midday. We compute thus, from column 3, before the fourth: as 100000 is to 14, so 928858 [?] is to what? We find about **130 handbreadths** approximately. And likewise [continued]... ...you can determine, from the shadow, the sun's altitude

above the horizon at any instant, by computation with triangles — provided your gnomon stands upright in its place. And this is straightforward.

From Chapter 7, Book II:

Problem 13:

How, from knowing the longitude of any point of the ecliptic at a given oblique horizon whose pole-elevation is known, we may know the measure of the equator-arc lying between the horizon and the ecliptic — this being what is called **the ascensional difference** (הבדל העליה).

The accompanying figure (figure 21 [ליור כא]) is found in Ptolemy's book as the illustration of this problem; but it is not drawn as it should be. However it stands, **פ [P]** is the meridian circle, **פק [PK]** an oblique horizon, **דמג [DMG]** the equator, the ecliptic being a quarter descending from the northern pole onto the equator and passing through point **י [Y]** of the ecliptic, which is cut by the horizon. The aforesaid pole stands elevated above the horizon by 36 degrees, as we established at the start. **ומ [M]** is the head of Aries, and arc **מי [MY]** is known, 30. We wish to know its ascension on this horizon — that is, arc **זמ [ZM]**. Now arc **טמ [TM]** is its ascension on the right (sphere) horizon, known from Problem 3. And if we subtract from it arc **מך [MK]**, the ascensional difference for the northern signs of Aries and Taurus, we shall learn arc **זט [ZT]**, the required ascension. In triangle **ימח [YMCH]**, angle **מ [M]** is right — by Theodosius — and **ומזמ [...ZM]** is the complement of the pole-elevation, that is, the elevation or depression of the equator.

Arc **ימ [YM]** is, in the first example, the declination of 30 degrees, which is **11;39;55**. From Table 15 [טור ט"ו], first or second region, we find arc **זמ [ZM]** = **8;38**, the ascensional difference. Subtracting it from arc **מז [MZ]**, the right ascension of 30° of Aries, which is **27;50**, there remains arc **חט [CHT]** = **19;12**, the oblique ascension of the end of Aries. Likewise for the second example, and all is laid out before you:

First example (end of Aries, 30°):

Step	Hebrew label	Value
Whole sine (quarter descending from pole **ץ [Tz]** to equator, perpendicular)	ימ נצבת	10000000

Step	Hebrew label	Value
tangent (complement of **ירז [YRZ]**, depression of equator = complement of pole-elevation = 54)	נוגע תשלום	7265425 [margin: 36]
tangent (declination of end of Aries)	נוגע ימ	2064834
sine (the difference **חמ [CHM]**)	בקע חמ ההבדל	1500189

Resulting values: declination of end of Aries **11;39;55**; the difference **חמ [CHM]** = **8;38**, subtracted from **27;50**; oblique ascension **חט [CHT]** = **19;12**.

Second example (end of Taurus, 60°):

Step	Hebrew label	Value
Whole sine	ימח נצבת	10000000
tangent (complement = 54)	יחמ נוגע תשלומה	7265425
tangent (declination of end of Taurus)	נוגע ימ נטיה סוף שור	3739400
sine (the difference **חמ [CHM]**)	בקע חמ ההבדל	2716833

Resulting values: declination of end of Taurus **9;30;20**; the difference **15;46**; subtracted from the right ascension of end of Taurus **44;57**; remaining **חט [CHT]**, oblique ascension **58;41**. And so too Ptolemy found it, by his great strength and might indeed.

[Source]

Ptolemy further teaches another way to find the first sought quantity more easily; and in truth, since one already needs a double computation in pursuing the way of the sun's rising through six [signs] [?]. Our first method, the first way, is very easy — like the second way — but for one who uses the tables of triangles, you can [proceed] as we explained. And if you wish, from knowing the declination of the said point, you can [find the ascension] [?]. Various are [the methods], and all equal in the speed of the computation, to find **חמ [CHM]** [?]; there are several ways, and I shall present several kinds, God willing. And elsewhere I shall pursue [this further], the refinement.

Before the table of ascensions: if you wish to compute the table of oblique ascensions, you must first prepare the right ones, and afterward the table of differences. Ptolemy, in this chapter, taught rules for shortening the way greatly — examine them, for his words are clear, and there is no need to elaborate.

From Chapter 9, Book II:

In this chapter Ptolemy teaches how to find the rising point and the mid-heaven point from any given instant of time, given the day [and] the place of the sun on the ecliptic. He says that if, for example, the sun be at the head of Cancer, and it be one hour before the noon of that day, then the mid-heaven point is **16;12** of Gemini and the rising point is **17;37** of Virgo, on the horizon of Rhodes. This is found in the following manner. Take the hours that elapsed from yesterday's noon up to the present time — namely 23 hours — into the time-degrees of the equator. That is, multiply them by 15, giving 345. Add to them the right ascension of the head of Cancer, which in the table is 90 degrees, giving 435. Subtract 360 from them, leaving 75. In the table of right ascension you find, under the column of the constellations that are on the horizon of Rhodes — that is, 36 degrees [pole-elevation] — according to the appropriate proportion among the rows, you find **16;12** of Gemini for the mid-heaven point. After finding this, if we want the ascendant, we add to the aforesaid 75 [+] 90 degrees, giving 165; and in the table of constellations for Rhodes, with the proportion, take **17;37** of Virgo. Conversely, from the known ascendant find 165, subtract 90 from it, leaving 75; and these correspond to **16;12** of Gemini at mid-heaven. And this is straightforward.

All of this, and the cautions to be heeded, I shall likewise make known to you in another treatise, [on finding it] by means of the spherical triangles, God willing.

Chapter 10, Book II:

You already know that the meridian circle passes through the poles of the equator and the poles of the horizon, and therefore intersects them at a right angle everywhere. But since the poles of the ecliptic revolve around the poles of the equator, [the ecliptic] will not lie in the meridian circle except at the two opposite points, at the head of Cancer and the head of Capricorn; and there the ecliptic intersects the meridian circle alone at right angles, [the points] being between them. And between [these points] and the poles will be 90 degrees of the meridian circle. But in all other places, the angles formed by the intersection where the said circle cuts the ecliptic are oblique, and they are equal [to those of] the horizon with the ecliptic at the right horizon, as Ptolemy will teach in the following chapter.

[Source]

Chapter 7, Book II [continued]:

Problem 14:

How, from knowing the length of the arc between the head of Aries and the meridian circle, we may know the measure of the angle formed by the intersection of the ecliptic with the meridian circle. This is illustrated by the accompanying figure (figure 22 [ליול כב]). The meridian circle is **אפבכ [APBK]**, the ecliptic **ב [B]**, the equator **אכ [AK]**, and **ט [T]** the head of Aries. Arc **דט [DT]** = 80 degrees, [measured] from the [vernal] equinox to the meridian circle. We wish to know angle **טדפ [TDP]** [?], which faces the northern pole — for the four angles formed by the intersection of two circles, [are found] by Table 4 [טור ד'], and the analogy [is] as follows.

In triangle **טאד [TAD]**:

Hebrew label	Value
א [A] is right	10000000
tangent (**אטד [ATD]**, cutting Aries)	4422113 [arc 23;51;20]
sine of complement of **מטד [MTD]**, 30 degrees	8660254
tangent of complement of **אד [AD]** = **3;69** [recte: 3;9]	1382966

When you subtract them from 180, angle **פדט [PDT]** is greater — **110;57**; and Ptolemy found it **111;0**.

Likewise in triangle **מאד [MAD]**:

Hebrew label	Value
א [A] is right	10000000
tangent (**אטד [ATD]**, head of Aries)	4422113 [arc 23;51;20]
sine of complement of **בד [BD]**, 60 degrees	5000000
tangent of complement of **אדט [ADT]** = **77;32**	2211056

Subtracting them from 180, there remains angle **פדט [PDT]** = **102;28**; and Ptolemy [found it] **102;30**.

The author [Delmedigo] will teach how to construct, with ease, a table of every angle of the points of the ecliptic with the meridian circle.

From Chapter 11, Book II:

Problem 15:

We shall know the eastern angle formed by the intersection of the ecliptic with the horizon. In the preceding figure, the equator is **דיג [DYG]**, the ecliptic **אחב [ACHB]**, and the horizon **פק [PK]**.

Point **מ [M]** is the head of Aries, and arc **מך [MK]** = 30 degrees. The elevation of the pole [is] 36 — that is, the elevation of **כ [K]**; and the elevation of the equator is **קד [KD]** = 54 degrees. We wish [to know] angle **מחי [MCHY]** [?] in the oblique triangle **ביח [BYCH]** [?], whose three [parts] are known: **מך [MK]** = 30 degrees; **מ [M]** [?] the great declination [obliquity], 23;51;20; **ביח [BYCH]** [?], the remainder of the equator-elevation, that is, 126 degrees. By Table 1 [טור א'], third region, find angle **מחי [MCHY]** [?], sought in this manner.

We draw a triangle outside the circle, **גפר [GPR]** [?], equal to triangle **מיח [MYCH]** [?], so that we may grasp its derivation; it is **גיד [GYD]** [?], cutting the equator at point **שי [SHY]** [?] and the ecliptic at point **רז [KZ]** [?].

In triangle **גפר [GPR]**:

Hebrew label	Value
whole sine **פ [P]**	10000000
נד [ND] sine 30	5000000
ד [D] sine of the great declination	4044323
גפ [GP] sine **40;11**	2202161

By Table 1, third region, find angle **מחי [MCHY]** [?] = **23;51;0** [from sine]; the remainder of the equator-elevation, that is, 126 degrees. By Table 6 [טור ו'] of the regions, then [in triangle **גפ [GP]**]:

Hebrew label	Value
גפ [GP] cut **4;11** [margin: 4°.11]	102109520
ביף [BYP] sine of complement 54	5877852

Hebrew label	Value
ינף [YNP] sine **36;53**	6001846

[the whole sine being 10000000]

[Source]

In triangle **גפ [GP]**, we seek the perpendicular **גפ [GP]**, and by Table 11 [טור י"א] in triangle **בג [BG]**, by Table **פף [PF]** [?], region one. We find it **40;11**. And when we subtract angle **גת [GT]** = **3;69** [recte: 3;9], angle **יבף [YBP]** = **36;53**, there remains angle **דבי [DBY]** = the great declination, which equals angle **מחו [MCHV]** or **פרף [PKP]** [?]. In the aforesaid figure [it is] **32;10**; and so too Ptolemy found it.

Hebrew label	Value
גפ [GP]	10000000
ד [D] tangent of the great declination	4422113
נר [NR] sine of complement 30	8660254
דגפ [DGP] tangent of complement **3;69** [recte: 3;9]	3829661

But a person may seek it more easily by knowing the oblique ascension, which in the said figure [is] **דום [DVM]** [?] — that is, 30 degrees according to the hypothesis. Therefore its oblique ascension is arc **טי [TY]** = **19;12**, and angle **טי [TY]** [?] [is] the remainder of the equator-elevation, and the analogy [is] as follows, by [the lesson of] Ibn al-Haytham [אבן הראשה / Alhazen][?].

Second example, by way of his derivation. Let it be, in the accompanying figure (figure 24 [ליור כד]), triangle **לסז [LSZ]** [?], in which are known: **סלז [SLZ]** [?] = 54, the equator-elevation [recte? as a value]; arc **לז [LZ]** [?] — the rising portion of the equator, the oblique ascension of arc **מז [MZ]** [?] of the ecliptic = **23;12** — that is, from the head of Libra [?] up to **17;37** of Virgo, which rises with it on the horizon. Therefore, by [the lesson of] Ibn al-Haytham, the analogy is:

Hebrew label	Value
מ [M] [?] sine	50000
טיח [TYCH] [?] sine **23;12** [margin: 54/126]	80901
טי [TY] sine oblique ascension **19;12**	32886
טח [TCH] [?] **9;32** sine	53210

As the ratio of **סז [SZ]** [?] **23;12** to **סלז [SLZ]** [?] sine **21445**, so **כן [KN]** sine 54 to **לז [LZ]** [?] sine **15**, [and so] to **80901** sine; **25882** to **לסז [LSZ]** [?] **77;31** [margin:

102, 29] sine **97639**. And this is doubtful, unless one knows truly that angle **לסז [LSZ]** [?] is obtuse; but the angle facing it, **רכזק [RKZK]** [?], is obtuse — therefore it too is obtuse, **102;29**.

And there is another way among the Arabs: point **דק [DK]** [?] at mid-heaven is **16;12** of Gemini. If you add its declination **7;23** to **אב [AB]** = 54, the equator-elevation, arc **דזאב [DZAB]** [?] becomes **77;7**. Accordingly **תד [TD]** [?] = **53;12**. And if you add to it a quarter [90°], **חד [CHD]** [?], the whole [will be], the measure of angle **ם [M]** [?] = **102;53**.

Another way, by Table 11 [טור י"א], first region: as **בעיך [B...K]** [?], the whole sine, to sine **הם [HM]** = **91;25** that lies between **16;12** of Gemini and **17;37** of Virgo, so sine **רזב [RZB]** [?] **77;7** to the sine of angle **המב [HMB]** [?] **77;2**; and its remainder is **102;58**. Do not be astonished if a discrepancy falls among these computations by some [number of] first minutes [seconds of arc], for these are not mathematical but natural [approximations], as I revealed to you in the [lesson of] Ibn Shelomo [אבן שלמה] [?] on the *Sefer HaYesod* [Book of the Foundation]. And withal, scrutinize, lest perhaps I erred in the computation and was not exact — for I have no leisure, and it is not my intent to make of these [results] a foundation, but only to show an illustration of the matter.

From Chapter 12, Book II:

Behold, Ptolemy, when he wishes to seek the angle formed by the intersection of the ecliptic with the [local] vertical circle, first seeks the sun's altitude above the horizon at that instant, and afterward, from it, investigates the said angle. And although there are many easy ways to know this angle, we — in explaining his words — follow in his footsteps, step by step with great difficulty [?].

Problem 16:

How shall we know the sun's altitude above the horizon, given the pole-elevation upon it, at any instant we wish...

[Source]

and that is the place of the sun that we found from the ephemeris [tables]. **And the figure will be** as drawn (fig. [25] כ"ה) — **al-Fajāb [?].** [The reading of this name is uncertain; it appears to be a transliterated Arabic technical or place term.]

[The opening of this passage continues Problem 16 from the previous folio — finding the sun's altitude above the horizon at the moment of culmination / before noon.]

Its complement is 36′ [2°] ב, and 54′ together with it [?]. The equator [המשוה], which is the height [גובה] and the equator at the southern edge of the horizon — circle, and the meridian [circle of midday] [13] יג; and its pole is א [point A]; and the equator עפ [?] and its northern pole is ב [point B], and its elevation above the horizon 36′ [2°] ב. The complement is לפג [?] which is the height [גובה], and …

So let the sun be at the head [first point] of Cancer, ר"מ [that is], turning by the daily motion on the present [parallel] circle toward the equator, toward the west, [for] our purposes — and it is, in the figure, at point צ [Tz], aligned with the meridian circle at point ל [L]. Then arc גל [GL] will be the sun's altitude at midday above the horizon. Now we drop, from the poles of the horizon and of the equator, two arcs of [great] circles [literally "fourths," i.e. quadrant arcs], namely בד [BD], standing perpendicular on the equator, and אה [AH], standing on the horizon, [both passing] through the center of the sun. Arc דצ [DTz] will be its great declination [נטיתו הגדולה], and arc פך [?] of the equator is given — and it is, by way of example, 15 degrees, that is, one hour before noon, which rises together with arc צל [Tz-L] — that is, when point צ [Tz] comes to point ל [L] on the meridian circle, then point ד [D] also comes to point פ [?]. By this the sun, which is at point צ [Tz], rises higher on that day, [revolving] about pole ב [B], until it reaches point ל [L], which is at midday, an hour later; and we wish to know the measure of arc צך [?] of the altitude circle, namely **אצה** [its altitude above the horizon, an hour before noon].

Now the midday altitude above the horizon, which I taught you in the fifth chapter [פרק] of this second Book [מאמר], is found thus: combine the altitude of the equator, 54′ [recte: 54° — the equatorial altitude on this horizon], with the declination of the sun — which, when it is at the head of Cancer, by assumption is 23° 51′ 20″. Its midday altitude will be **77° 51′ 20″.** Then, after you have well constructed the drawing of the said figure, transfer three of its known [quantities] to a triangle of its own, and in it investigate the unknown sought — so that you may understand, and not corrupt the figure by the drawing-in of the perpendiculars [העמורים], which must lie before your eyes intact. You see, in triangle אבצ [ABTz] of the figure, that side אב [AB] is known, 54[°], since it is the distance between the pole of the horizon and the pole of the equator; and side בצ [BTz] is the complement of the known great declination, so it is known: 66° 8′ 40″. And angle אבצ [ABTz] is known, since it is measured by arc פד [?] of the equator,

15 degrees — for the angle at בד [BD] is a quadrant of a circle [right angle]. From the fifth book [of the Almagest], in the third corner [proposition], we find side אצ [ATz], the complement of צך [?], the sun's altitude sought (fig. [26] כ"ו):

[A] In the right-angled triangle אכב [AKB], by Theorem 11:

Whole sine [כל הבקע]	100000
Sine [בקע] אב [AB] 54[°]	80901
Sine [בקע] ב [B] 15[°]	25881
Sine [בקע] אק [AK] 5° 12′	20937

[B] In the triangle אקב [AKB], by Theorem 16:

Whole sine	100000
Tangent [נוגע] of the complement [תשלום] 75[°]	373205
Tangent [נוגע] אק [AK] 5° 12′	21407
Sine [בקע] בק [BK] 2° 53′	79891

And since the sine of בק [BK] found is less than the given, we know that the perpendicular [העמור] [falls] within the triangle; so subtract the smaller from the greater [9° 66′ [recte: 9;6]; 2° 53′] —

[The marginal column reads, top to bottom: 9° 66′ [recte: 9;6]; 2° 53′; 13° 7′.]

— and side קצ [KTz] remains [13° 7′ — recte 13;7]. So we have found **צה [TzH], the sun's altitude, 72° 16′** [recte: 72;16, reading the sub-units reversed],

[ץ — third triangle] אקצ [AKTz], by Theorem 7:

Whole sine	100000
Sine of the complement [תשלום] of ק [K] [76° 53′]	97391
Sine of the complement of אק [AK] [77° 55′]	97784
Sine of the complement of אצ [ATz] [?]	95240

[The last row's value reads 16/44 : 72/17 stacked in the source — "72° 16′ / 17° 44′" written one above the other; the complement of אב/אצ. Transcribed as printed.]

— while in Ptolemy [it is] **72° 13′**; and by our [own] method it would be 72[°]…[?]. Arc צא [TzA], its complement up to opposite the head [zenith] or up to the pole of the horizon, is 17° 44′ [recte: reading reversed, 17;44]; and by Ptolemy's reckoning, 17° 47′. And this is what was sought.

My whole intent is to train you in oblique [spherical] triangles — for this problem is well clarified in several easier ways, as you will see in the following problem; and it is also explained in [the discipline of] **Anaglomata [?]** [analemma — אנל"ומח, the projection method] by means of straight lines [i.e. plane methods], and there is no need to be lengthy. But in triangle אצב [ATzB], in which side אצ [ATz], the sun's altitude-complement, is known, together with the complement of its declination, and אב [AB], the distance between the pole of the horizon and the pole of the equator, is known — the [quantity] is known [by] the sixth corner [proposition], at the third proposition; [and] you will find angle אבצ [ABTz], whose measure is פך [?] of the equator, 15[°] — that is, one hour before noon. And this is the reverse of this problem and its inverse, and it is easily understood [וק"ל].

Problem 17 [שאלה י"ז]

How, from knowledge of the sun's distance from the pole of the horizon, or its altitude above the horizon, we may know the angle formed at that time by the intersection of the ecliptic [הלקוחיי] with the altitude circle [עגול הנובה].

The aim is to know, in the aforesaid figure, the measure of **נצק [NTzK?]**, the angle formed by the intersection of circle לצנמ [L-Tz-N-M], the present [parallel], with the equator's circle — which is at the place of the ecliptic — together with אצ [ATz], a quadrant of the altitude circle.

Now [the assumption]: as the author teaches in chapter 9 [פ' ט] of this second Book, the point of the ecliptic [degree] that rises [המצומח] — which is 12[°] 16[′] of Gemini [לתאומים] — has a mid-heaven [חצי השמים] of 17° 37′, above [לכתולה]. And the head of Cancer [ראש צ סרטן] being [the sun's position], and:

- Whole [כל] לן [LN] is **91° 25′**
- צל [TzL] will be **48° 13′** [13,48 reversed → 48;13]
- and צן [TzN] will be **77° 37′**, [whole sine] 100000, ח [H?]

In triangle צהנ [TzHN], with right angle at ה [H], נצבה [perpendicular]:

צה [TzH], the sun's altitude [tangent] 72° 16′ [נוגע] = 312713

The complement found [16,72 = 72;16]	21956
צן [TzN] [tangent of complement] 77° 37′	[from "צן נונע תשלום"]
נצה [NTzH], sine of complement [38,46 → 46;38]	68359

We find angle בצה [BTzH], and we infer thus. [נצה: complement 46° 38′.]

And by another inference I found it 47[′ / °], and by yet another 39[?] 46[°]; and so on. And in Ptolemy it is **46° 46′.**

Another method [דרך אחר]:

After you know the sun's altitude as above, you find, in the present figure (fig. כ"ז [27]), the angle אזם [AZM], formed by the intersection of the ecliptic with the meridian [midday] circle, [whole sine] 100000, ב [B].

As the author teaches in the tenth chapter [פ' העשירי] of the second Book, by this way: since the point of mid-heaven is 12° 16[′] of Gemini and its distance from the head of Aries [ראש טלה] is 76° 12[′]:

Tangent [נוגע] of the great declination, מ [M]	44221
מן [MN], sine of the complement, 76° 12′	23853
אזם [AZM], tangent of the complement [?]	10548

Then, in the right-angled triangle זבם [ZBM], by corner [proposition] [?], **83° 59′ 0″** — this is the left-hand angle מזב [MZB], that is, the [angle] formed on the western side by the intersection of the ecliptic with the meridian circle. (It is known, at the first propositions, by corner [proposition] 4, angle מזב [MZB].) And the inference is thus.

And after you found this angle, look again at the oblique triangle זאם [ZAM] and infer in it thus, by Theorem 11, at the first proposition: as the sine of אם [AM] 44° 17′ [the sun's altitude-complement, 30458] is to the sine of אזם [AZM] 83° 59′ [which is 99449], so is the sine of זם [ZM] 12° 53′ [which is 22296] to the sine of [the angle] — and **its angle is זאם [ZAM] 46° 46′, the sought one.**

Know that the author could have placed the discussion of the angles formed by the ecliptic with the altitude circle before the discussion of the angles formed by

the intersection of the ecliptic with the horizon, for this chapter [פרק] 12 does not require chapter 11 and its teachings — by our method.

And you can know, from angle א [A] which is in the altitude circle, the angle אשט [ASHT] which is in the horizon — and this [is found] at once in a right-angled triangle, and the reverse: מ"ר [for example], in the right [triangle] אטש [ATSH], by Theorem 4, as 100000 is to the tangent [נוגע] of angle 46° 46′ [?] אק — which is 106865 — so is the sine of the complement of אשם [ASHM] 77° 37′ — which is 21729 — to the tangent of the complement 5° 59′ … [?] — that is, 23112 of angle אשם [ASHM]. Or by Theorem 8: as 100000 is to the sine of the complement of אם [AM] 16° [72°?] — which is 30458 — so is the sine of א [A] 46° 46′ — which is 72857 — to the sine of the complement of angle אשט [ASHT] 22190, which is 77° 11′. By another way, without the aid of the altitude angle, [find] this angle א [A] by Theorem 2 in this way: as 100000 is to the sine of אם [AM] 72° 16′ — which is 95248 — so is the secant [חותך] of the complement of אש [ASH] 77° 37′ — which is 102382 — to the sine of angle אשמ [ASHM] 97516, which is 77° 12′.

So you see that each of the said angles brings about the knowledge of its fellow, and there is no need to be lengthy.

But the great master [גביר] Ibn Aflaḥ [אפלח 'ז — Jabir ibn Aflaḥ] investigates the angle known [as that] of the ecliptic with the altitude [circle] thus: he drops (fig. [28] כ"ח) from the head [zenith / pole] a quarter of a great circle, בה [BH], to the point of rising [הזריחה], ה [H]. Then, in the right-angled triangle זענק [Z-Ayin-N-K? — see note], he finds angle הזע [HZAyin] by a different and sharp [clever] method, in their manner — for I have no dealing with it; nevertheless, according to his figure and system I will inform you how, by the method of right-angled triangles, one finds at one stroke the sun's altitude and likewise the angle formed by the ecliptic with the altitude [circle]. And this is: since mid-heaven is 12° 16[′] of Gemini, whose declination is 7° 23′ — together with the equator's altitude, 54[°], on the Rhodes [horizon, ברו"דוס] — they make 77° 7′; and this is arc דך [?], the measure of angle רהע [RHAyin]. Then, in the right-angled triangle זעה [ZAyin-H], by Theorem 11 at the first proposition, as 100000 is to the sine [בקע] of הז [HZ] 77° 37′ — which is 97673 — so is the sine of זהע [ZHAyin] 77° 7′ [recte 77;7] — which is 97482 — to the sine 95213, whose arc זע [ZAyin] 72° 12′ is the sun's altitude — while Ptolemy found it 72° 13′.

Second [שנית]: in triangle בהז [BHZ], side בה [BH] is a quadrant, and side קז [KZ? / זק] is 77° 37′, and side בז [BZ], the complement of זע [ZAyin], is 17° 48′. And, by Theorem 9 of Ibn Ezra [אבן העזר], we find angle בזד [BZD] in this manner: in the right-angled triangle בדז [BDZ], by corner [proposition] 3, as 100000 is to the

tangent [נוגע] of דז [DZ] 12° 23[′] — which is 21956 — so is the tangent of the complement of בז [BZ] 1748 [reading: of an arc whose tangent-complement is 1748] — which is 311464 — to the sine of the complement of angle בזד [BZD] 68385; the angle, its complement הזע [HZAyin], is 46° 51′.

Or, in brief, if you wish from the start, without any knowledge of the sun's altitude, to know angle הזע [HZAyin], it is arranged thus, by Theorem 4: as 100000 is to the tangent of 7′ 77° [?] זהעד — which is 437207 — so is the sine of the complement of זה [ZH] 77° 37′ — which is 21445 — to the tangent of the complement of angle הזע [HZAyin] 93759, which is 46° 51′ — as in the foregoing inference.

Problem 18 [שאלה י"ח]

How, from knowledge of the place of the sun in the ecliptic, we may know at night which hour it is. You have already learned that the solar body is what makes day and night by its daily motion, and that the equator is the reckoner [הסופר] of …

of time. Therefore, if you wish to know which hour of the day it is when the sun is on the horizon, it is easy: as I taught you in Problem 16 [שאלת י"ו], angle אבצ [ABTz], which is opposite side אצ [ATz] in triangle אצב [ATzB] — which is the complement of [the sun's] altitude above the horizon — is the measure of the portion of the equator before noon; and when you find its measure, you know what you sought. And likewise, if the moon, in conjunction with the sun, or another star, whether [in] true [position] or apparent, has its altitude above the horizon — which brings you to knowledge of the matter — there is no doubt it is at the place of the sun. But if it is distant from it by the travel of one hour, two hours, or more, you know it likewise: for if you know, by way of example, that a certain star is exactly opposite the sun — that is, 180 degrees distant from it — and the star is on its meridian circle, there is no doubt that that time is exactly midnight.

Therefore, to know the time at any hour when the sun is not visible, you must know the right [ascension, העליה הישרה] of the sun at that time, and likewise the right ascension of the moon, or a star, and subtract the ascension of the [arc] between the sun and the moon or star — which indicates the time. That is, you must know the right ascension of the sun and of the star, and divide the remainder by 15, and out come the hours. This is when the star is exactly on the meridian circle.

For example, [suppose] we lift our eyes to the heavens at some horizon, the sun being at the head of … [Aries], so that its right ascension is, say [כברי], 152 degrees [recte? — see note]; and we see the moon or another star by its altitude — that is, when it is on the meridian circle — and it is, by way of example, at 19 [י"ט] degrees of Gemini, whose right ascension is 78 degrees. Therefore, when we subtract this from that — and [if] we add 360 degrees to it — there remain 286, which are 19 hours and 4 firsts [minutes]. And 22 [כ"ב] hours is after noon; and one always finds some bright star on the meridian circle. Therefore the sailors carry with them a table of right ascensions of the sun and of the marked stars, and at once from it they know which hour of the night it is.

But if you wish to know this from some star that is not on the meridian circle, then in any case you must know its altitude above the horizon by means of the instruments [הכלים], and you must also know the latitude of the place. And by the teaching of Problem 17 [שאלה י"ז] — as I showed you regarding the sun — you will know how much of the equator accompanies it until it reaches midday, and you convert it to time, and subtract it from the said hours. By way of example, the said star, which was at 19 of Gemini, was distant from the meridian circle [so that], with the instrument, we took its altitude above the horizon, and its complement is its distance from the meridian circle. And behold, we found by the reckoning of the triangles that it is after one and a half hours from coming to midday [reaches the meridian an hour and a half later]; therefore we subtract them from the hours, 19° 4′, and there remain 17° 34′ — and this is the time that is after noon. But if the star has already moved off the meridian circle toward setting, then you must add the said time to 19° 4′, and they make 20° 34′[?] hours after noon. And the matter is very easy, so that no new figure is needed here.

There are other well-known methods to know the night hours — also from [the practice of] the officer, my dear friend, the honored Rabbi Moshe [כמוהר"ר משה] — may his Rock keep him [יצ"ו] — who devised a small round instrument; I improved it for this [purpose], so that when you look through the hole at its center at the star near the northern pole, and the pointer [הבריה] on it you lower or raise until it is aligned opposite the stars of the Great Bear [ככבי העגלה הגדולה], you find at once the hours; and it is founded on the right ascensions, and an explanation of its use [comes] from it, and it will instruct you. But the practice in the German lands [מדינות אשכנז] is by the shadow that the gnomon [העמור] casts in the moonlight [for] the hours; and when they know its distance from the conjunction, with the round table [board, הלוח הענול]…

[Text continues on the following folio.]

Chukos Shamayim — Almagest Commentary, Book II (continued)

inscribed on the instrument, the hours of the night are found at once. Refer everything back to the right ascensions and you will discover it.

From this the procedure is clear: once you know what hour it is, you can determine the star's altitude above the horizon. The whole matter becomes clear if you take point צ [tzadi] to be the position of the star in the figure of Problem 16 [?].

However, as to how the astronomers found the right ascensions of the fixed stars: First they sought the right ascension of any one of the fixed stars in the following way. When the sun was at midday, they took its declination and its right ascension, as was shown earlier in Chapter 1 of this Book [Book I]. At that very moment they set up a clock-instrument — a sundial, or one of those mechanisms that turn of themselves, very precisely calibrated, so that the hour-reckoning would not err — and they waited until some star reached the meridian circle. The time that elapsed between midday and the star's arrival at that meridian circle they converted into degrees of the equator and added it to the sun's right ascension. The sum is the right ascension of that star.

In the figure of Problem 16 [?], let some star be at point ל [lamed], whose right ascension we wish to find. Suppose it is found at the head of the sun's right ascension — which is, for example, at point ך [final-kaf], that being 30 [degrees] קע [= reading: 30°?]. After the star comes to point ל [lamed] — that is, after four hours, which are 60 degrees — we add 60 to 30, giving 90, and that is the measure of the equatorial arc פקע [reading uncertain]; this is the right ascension.

After they knew the right ascension of this star, they came, with the aid of the instruments and the computation of triangles, to the knowledge of the right ascensions of the remaining stars in the following way. For example, let there be some star at point צ [tzadi] whose right ascension we wish to find. With the measuring instrument [armillary], the sides לב [lamed-bet] and בצ [bet-tsadi] — which are the complements of the stars' declinations — are known; and לצ [lamed-tsadi] is known by inspection of the instrument. From the sextant [or quadrant] we take the distance לצ [lamed-tsadi] between the two stars. Thus in triangle לבצ [lamed-bet-tsadi] three sides are known. By the third proposition of [the Book] *Sod HaYesod* ["Secret of the Foundation"], we know angle ב [bet], whose measure is arc בך [bet-kaf]. When we subtract arc פע [reading uncertain]

— the right ascension of star ל [lamed] — there remains arc דע [dalet-ayin], the right ascension of star צ [tzadi], which is what was sought.

In this way they found the right ascensions of all the stars. From this you also know how to inscribe the figures and the celestial globe, and how to lay out every side as a straight line in all directions.

שאלה י"ט [Problem 19]

How, from the declination of any given point, we determine its position in degrees [on the ecliptic]. The explanation of this problem is much needed for several matters in the science of astronomy and for setting up the instrument. Here is the figure of the first problem [referencing the figure of Problem 11, p. 249 — see footnote], in which:

By the triangle יתח [yod-tav-chet] [?], in which the required quantity is to be learned, the first proposition [law of sines] is set up thus: as the sine of ה [he] = 23°51′20″ — which is 4044323 — is to the sine of the side opposite it, namely ית [yod-tav] = 11°40′, the measure of the given declination, which is 202161; so the whole sine, 10000000, is to the sine of the arc parallel to it, ית [yod-tav] = 5000000, which is 30 [degrees]. Therefore its position is at the end of Aries, that is, at the beginning of Taurus; and this — note well — you find from knowing the latitude of its rising. And understand this.

However, the sun's altitude, which we seek with the instrument of the quadrant of a great circle graduated to seconds [arc-seconds] — in such a way that the observation is precise — will show you the declination of each and every point, according to the lesson of Problem 12. For if you subtract the altitude of the equator from the altitude of that point, or [subtract] the point's altitude from the altitude of the equator, its declination is revealed to you. Likewise, from the altitudes of the wandering and fixed stars you find their declinations — only

[Footnote at foot of p. 263:] * [Footnote marker:] (Source p. 249) [refers to the figure on that page].

remembering to add or subtract their latitude. And consult Book V of the *Almagest*, where this is taught in practice. And understand this.

Now, once you have come from knowing its declination to knowing its position in degrees at midday, you can know its position at any hour you wish. For since [the sun] moves about 59 [arc-]minutes in 24 hours, roughly 2½ [arc-minutes] belong to one hour's motion. From this the procedure is clear, by which they investigate,

according to the times of the seasons, the sun's entry into the four solstitial/equinoctial points [the tekufot]. For if they wish to know the time of its entry into the head of Aries, they observe its declination on the two consecutive days nearest the spring equinox, from its two sides — that is, before its entry [into Aries] and after its entry into Aries. They attend to the difference between the two positions found in degrees, or nearest to the equinoctial point. And since it is known that its motion is 59 [arc-]minutes in 24 hours, they know how many hours remain until it reaches the equinox, or how many have passed after it reached the equinox.

For example: we observed with a precise instrument at midday on the 11th [of the month] and measured the sun's altitude; and according to the observation we knew, by this problem, its position in degrees, and it was at the end of Pisces, distant from the head of Aries 0 degrees 15 minutes. Again we observed on the next day at midday — the 12th — its altitude; and from its declination we found it had already entered Aries, and it was at 0 degrees 44 minutes. Therefore in 24 hours it moved 59 [arc-]minutes. We reason thus: if 59 [minutes] require 24 [hours], how much do 45 require? You find 6 hours and a further 6 minutes. Thus 6 hours and 6 minutes after midday of the 11th, the sun entered the head of Aries. Or reason thus: if 59 give 24, what do 44 give? You find 17[h] 53[m]. Subtract these from midday of the 12th, working backward, and you find that the sun entered the head of Aries on the 11th, 6 hours [and] 30 [?] minutes [reading: דקים ל = 30] after midday. And if you refine it to seconds, the truth would be precise to a hair's breadth.

שאלה כ' [Problem 20]

How, from the longitude and latitude, we determine the right ascension of a star. Also [how to determine] the right ascension of stars that have latitude. You should train yourself well in this, for it is much needed in matters of astronomy.

It is as follows. You know the arc of the equator between the head of Aries and the circle that measures off the star's declination. And so, in the figure before you [Problem 21 — see footnote ref. (Source p. 259) [?]], let the equator be אב [aleph-bet], its pole י [yod], the ecliptic דכן [dalet-kaf-nun], and its pole ם [final-mem]. Let there be the star in the eye of Taurus כ [kaf] [Aldebaran], whose longitude in this our time is 63°58′, and whose latitude is southern, 5°10′.

We wish to know its right ascension. **Now,** from Problem 20 [reading: הב' — "the second," i.e. the preceding] we find its declination, 15°54′. And once you know מן [mem-nun] [the meridian arc], you also know מר [mem-resh], which is

compounded of the said declination and the quadrant נך [nun-kaf], which is 105°53′. The side רש [resh-shin] is likewise known, for it equals the great obliquity — that is, arc וב [vav-bet] — and the side סש [samekh-shin] is 84°50′, that is, the complement of the said latitude. Therefore, by the sixth proposition for the third figure [case] of *Sod HaYesod*, find angle שרפ [shin-resh-pe], whose measure is arc שרם [shin-resh-mem] [?], and its complement נ [nun] is what was sought.

Method two: angle משר [mem-shin-resh] is known, for it is the remainder of the [reading: תן vav-nun] complement of the longitude, and its measure is 153°58′. Therefore, by the lesson of the law of sines, we set it up thus:

	as	: to	so	: to
Term	כערך סר [k-erekh ser] 105°53′	אל ש [el sh] 153°58′	כן שם [ken shem] 84°50′	אל שרם [el sharam] 27°2′
Sine [beka]	96180	43882	99594	45446

which is the measure of arc בב [bet-bet] [?], and its complement, נב [nun-bet] = 62°58′, is the right ascension of that star.

Example two: We wished to know the right ascension of the brightest star of all the stars of the heavens [Sirius] — the Great Dog [Canis Major] — which in our time is in Cancer, at 8°35′ — that is, 98°35′ — distant from the head of Aries. Its latitude does not change, and it is 39°30′ southern. Now in the figure of Problem 20, the equator will now be בצ [bet-tsadi], the ecliptic דבך [dalet-bet-kaf], and the star at point ם [final-mem]. A great circle descends from point ה [he], the pole of the ecliptic [reading: הקטב, "the pole"] [?], to point ם; the southern pole of the equator passes through it and reaches the ecliptic — this is line הסנמי [reading uncertain] [?]. According to this, arc סן [samekh-nun] is its southern declination. Another circle descends from point ם to the ecliptic, and that is פסאל [pe-samekh-aleph-lamed] [?], and the latitude is בו [bet-vav], which is arc וסאל [reading uncertain] [?]. Its longitude is also known, namely בל [bet-lamed], which on the ecliptic is from the start of ב [bet], the head of Libra, that is, 81°25′.

And we wish to know the arc of the equator באן [bet-aleph-nun], which is the sought right ascension. And from Problem 20 [reading: ב, "the second"] we know the measure of the declination סן [samekh-nun], which is 16°11′. Therefore the complements of arcs סם / סד [the declination] and the latitude are also known; and also פד [pe-dalet], the distance of the poles, 23°32′, is known. From the sixth proposition for the third figure [case] of *Sod HaYesod* you find angle ח [chet],

whose measure is arc גג [gimel-gimel] = 2°7′ [reading: 7°12′? — see below] from the head of the summer solstice, and the arc באן [bet-aleph-nun] is its complement, 82°48′. When you subtract it from 180 degrees, there remains 97°12′ — the right ascension from the head of Aries to 8°35′ of Cancer, which is what was sought.

However, I shall inform you that Johannes Regiomontanus, in Problem 4, investigates the right ascensions in another way: that in triangle אלב [aleph-lamed-bet] he seeks arc אב [aleph-bet] and calls it the "root of the right ascensions." This you can find in the following way: from the tenth proposition of the first [figure] of *Sod HaYesod*, thus:

Quantity	Value
whole sine [bet]	100000
לב [lamed-bet] tangent of complement	81°25′ → 15094
ב [bet] sine of complement	23°32′ → 91682
אב [aleph-bet] sine of complement	82°7′ → 13838

And the result is the measure of אב [aleph-bet] from the equator. Or in this way, from the second root [proposition] of the first [figure] of *Chukos Shamayim*:

as	: to	so	: to
כערך פח [k-erekh pach] 90°	אל חל [el chal] 8°35′	כן פג [ken pag] 66°28′	אל גא [el ga] 7°53′
sine 100000	tangent 15093	sine 91683	tangent 13837

And its complement, 82°7′, is the measure of אב [aleph-bet]. When you subtract it from 90 degrees, there remains 97°53′, which is the root of the right ascension; and he found 97°51′ by his method.

Again, he investigates — not so, but [reading: לא, "not"] [?] — by the first proposition of *Sod HaYesod*, in which the proportion of ל [lamed] perpendicular, 100000, is to the sine of אב [aleph-bet], 82°7′, which is 99055; as the sine of ב [bet], 23°32′, which is 39928, is to the sine of אל [aleph-lamed], 39550; and the measure of אל [aleph-lamed] will be 23°18′. When we subtract it from אל [aleph-lamed], the latitude, 39°30′, there remains 16°12′; and the declination סן [samekh-nun] is likewise 16°11′. Therefore, by the thirteenth proposition for the first figure [case], we find arc נא [nun-aleph], which is called "the difference of the star's passage at mid-heaven." And we reason thus: as 100000 is to the secant of side ם [final-mem], which is 104126, so the sine of the complement of declination אם [aleph-

mem], which is 96029, is to the sine of the complement of א [aleph], 99991, which is 45 [arc-]minutes. Now if we subtract it from the root of the right ascension, or add it to אב [aleph-bet] which you found, 82°7′, it will be 82°52′. This we subtract from 180 degrees, and there remain 97°8′; and according to Regiomontanus, 96°52′. There are many ways, and let the wise hear and grow wiser.

שאלה כ"א [Problem 21]

How, from knowing the right ascension of a star, we determine the point of the ecliptic with which the star comes to the meridian circle. In the first example of Problem 20, we wished to know it, and from the root of arc בל [bet-lamed], by the second [proposition] of *Chukos Shamayim*, we reason thus: as the proportion of יך [yod-kaf] 90°, whose sine is 100000, is to the tangent of the complement of the right ascension, 27°2′, which is 51026; so the sine of the complement of the great obliquity, ין [yod-nun] = 23°32′, which is 91682, is to the tangent of arc יל [yod-lamed] = 46781, which is 25°4′; and its complement, 64°56′, is the measure of arc בל [bet-lamed], the point of mid-heaven of the said star.

And in the figure of Problem 20 there will be the second example of Problem 20: with the star in the mouth of the Great Dog [Sirius], whose right ascension we found, and we know arc ב [bet] = 7°12′, we seek arc בם [bet-mem] of the ecliptic, and reason thus:

as	: to	so	: to
כערך יך [k-erekh yach] 90°	אל נגן [el nagan] 7°12′	כן בקע [ken beka] 66°28′	אל חמ [el cham] 6°36′
sine 100000	tangent 12633	sine 91683	tangent 11582

And its complement is 83°24′. This you subtract from [90] degrees, and there remains 96°36′. Therefore the star will come to mid-heaven when they have passed 6°36′ of Cancer.

שאלה כ"ב [Problem 22]

How, from knowing the declination and latitude of a star, we determine its longitude [reading: ארכו, "its length/longitude"]. This problem is well explained in [the treatise on] right-lined [plane] triangles, and in another work I have undertaken to show all the virtues of the science of the *anelima* [the analemma] and its many benefits. This time, with spherical triangles, observe: in the figure of Problem 20, find ג [gimel] [the three sides] known in the oblique triangle פהם [pe-he-mem] — that is: הם [he-mem], the complement of the

latitude; פס [pe-samekh], the complement [reading: חל, "of"] of the declination — and these are given. Also the side פה [pe-he] is known, for it is like the great obliquity, that is, what lies between the pole of the equator and the pole of the ecliptic. Therefore, by the sixth proposition for the third figure [case], find angle ה [he], whose measure in degrees is the complement of בב [bet-bet] [?]. This you subtract from the whole circle, and there remains for you בצגנ [bet-tsadi-gimel-nun] [?], the distance of the star from the head of Aries. And understand this.

שאלה כ"ג [Problem 23]

How, from the longitude and the declination, we determine its right ascension and its latitude; and likewise how, from the longitude and the right ascension, we determine its latitude and its declination. These two problems are very well explained from what was said above with the aforesaid figure, and there is no need to elaborate.

שאלה כ"ד [Problem 24]

How, from knowing the declination and right ascension of a star, we determine its longitude and its latitude. In the very figure of Problem 20, where the equator is דבר [dalet-bet-resh] [reading uncertain; cf. earlier דבך] [?], its pole פ [pe], the ecliptic גבצ [gimel-bet-tsadi], its pole ה [he], and the star at point ם [final-mem]; and its declination פל [pe-lamed] [?] and its point ק [kuf] [?], and its right ascension בל [bet-lamed] are known — we wish to know its longitude בן [bet-nun] and its latitude סנ [samekh-nun]. The manner of the inquiry is as follows:

In the right-angled triangle אלב [aleph-lamed-bet], two angles are known: angle ב [bet], the great obliquity, and לב [lamed-bet], the right ascension. We reason, by the second root [proposition] of the book *Chukos Shamayim*: as the proportion of the whole sine בח [bet-chet] is to the tangent גה [gimel-he], so the sine …

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[the base of] **בל** [B-L] does not touch **לא** [L-A]; and when we subtract this from it, by [the principle] of the star's inclination, the remaining arc **אם** [A-M] is known.

ב [step 2]: We infer, then, from the first proposition of the aforementioned book: as the base **לא** [L-A] is to the base **ב** [B], so the base **בל** [B-L] is to the base **באל** [B-A-L], which is equal to the angle **נאם** [N-A-M].

ג [step 3]: We infer from the aforementioned proposition: as the base **ב** [B] is to the base **אל** [A-L], so the base **ל** [L] is to the base **אב** [A-B].

ד [step 4]: In triangle **אנם** [A-N-M] we infer from the aforementioned proposition, as the perpendicular base **נ** [N] is to the base **אם** [A-M], so the base **נם** [N-M] is to the base… [the sought latitude].

ה [step 5]: And **נאם** [N-A-M] to the base **נם** [N-M] is the sought latitude.

ו [step 6]: In triangle **אנם** [A-N-M] we infer, from Problem 14 [שאלה י"ד] of Regiomontanus [לאבן בוחן = Ibn Buchan / Regiomontanus]: as the base **פה** [P-H] is to the complement of angle **נאם** [N-A-M] to a right angle [i.e. its cosine], so the whole base is to the tangent of the complement **אכ** [A-K]. Thus all the base touches **גנ** [G-N], the complement of **נא** [N-A]. — Or [by] Theorem 12 [מטור י"ב] of Regiomontanus [אבן שלמה = Jabir ibn Aflah]: as the whole base is to the sine of angle **נא** [N-A], so the tangent of the complement **אם** [A-M] is to the tangent of side **נא** [N-A]; and when you take arc **נא** [N-A] together with arc **אב** [A-B], you will know the totality **בנ** [B-N], and you subtract it from there [i.e. from **שם**].

ז [step 7]: And there will be revealed arc **בצגנ** [B-Tz-G-N], the longitude of the star from the head of Aries, and this is what was sought.

שאלה כ"ה [Problem 25]

How, from knowing the longitude and latitude of a star, we may determine its rising and its morning setting on a given horizon.

After you have learned, from Problem 22 [שאלת ב' — but read "22"; see anomaly note], its right ascension [עליתו הישרה], you investigate its oblique ascension [עליתו העקומה] as Problem 14 [שאלת י"ד] directs. That is: I want you to investigate the difference between the right horizon and your horizon. If the star's inclination is northern, subtract the difference from its right ascension; if southern, add it; and the oblique ascension results. Conversely, do the opposite in deriving its setting: if you find it northern, add the difference to the right ascension; if southern, subtract it, and its oblique descent will be revealed.

For example: We wish to know, in the latitude [city] of Candia [קנדיאה], the oblique ascension of the bright star in the mouth of Canis Major [Sirius], whose right ascension we found in Problem 22 to be **97° 12′** [97.12]. Now, in the figure of

Problem 22 [reading "Problem 22"], in the triangle… by which the star's inclination, which was found as Problem 22 directs, is **16° 12′** [16.12]. Angle **יחם** [Y-Ch-M] is the complement of the pole's altitude [גובה הקוטב], that is, the altitude of the equator [גובה המשוה], [45.54] **54′ 45°** in Candia. Angle **מ** [M] is at a right angle, since **צ** [Tz] is the pole of the equator and from it a perpendicular descends, the circle [arc] **צימ** [Tz-Y-M], onto the equator.

Therefore, by Theorem 16 [מטור ט"ז] of the first part, we find side **חם** [Ch-M], and the proportion is thus: as **100000** is to the sine of the complement **16° 11′** [16.11], which is **344202**, so the sine of angle **רן** [R-N], **45° 54′** [45.54], which is **141497**, is to the sine of side **דזמן** [D-Z-M-N], which is **487035**; and the arc of its complement, **51° 11′** [51.11], is the difference of the ascension [הבדל העליה].

Since the inclination of the aforementioned star is southern, we add it to **רזמן** [R-Z-M-N], its right ascension, which is **97° 21′** [97.21], and out comes its oblique ascension, **109° 3′** [109.3]. With these degrees of the equator the aforementioned star [crosses] over the horizon of Candia; and in the tables of oblique ascensions for latitude **35° 15′** [35.15] you will find them set opposite **2°** [ב'] of the sign Leo [ארית]. Therefore, when the sun is in the second degree of Leo — which is at the start of the month of July [יוליאוס] — the star will rise together with it, a few days [later]. This is Hippocrates's [איפוקראט"ז = Hippocrates] [period of] the short courses [למודיו הקצרים], [the season] for physicians not to administer enemas and heating remedies; for these are the rabid "dog days" [הימים הכלביים השורפים], scorching in the city of Candia, and in that season and before it…

And if you wished to know the setting of this star — that is, with which point of [the ecliptic]… …it will set in the west when the sun rises [in the east]. This is its morning rising on the eastern horizon, and it is called the morning setting [שקיעת שחרית] when it sets in the west as the sun ascends in the east.

Now, from [its] head, subtract the difference from the right ascension, and you find **85° 21′** [85.21], which is the oblique descent [הירידה העקומה]; and add to it 180 degrees [180 = ק"ף], and out comes the oblique ascension of the sign opposite it, which is **265° 21′** [265.21]. And in the table of oblique ascensions for the horizon [of latitude] **85° 15′** [85.15 — see anomaly note] there is set opposite it, in Candia, approximately **11°** [י"א]. To this arc, then, the sign opposite it — namely Libra [מאזנים] — corresponds at **11°** [י"א], and with it the aforementioned star will set as the sun rises over the horizon, which is in the arc [degree] of so-and-so in the latitude of Candia. Understand this well.

But if you wished, after you have learned its oblique ascension, to know its position in [the table] without [further] computation, it is very easy: as Problem 13 [שאלה י"ג], and likewise [as Problem] 22 [כ"ב]; and there is no need to be lengthy.

שאלה כ"ו [Problem 26]

How we may determine the place of the sun in the [zodiac] table [בלקוותיי — "in the layout/table"] and the day of the month, on a horizon whose pole altitude [נובה הקוטב] is known.

In the first Problem I taught you how, from the inclination, you can determine its longitude from the head of Aries; and in Problem 22 [כ"ב], how, when you take with some instrument its noon altitude — also with an upright gnomon, [measuring] your own stature's height from the shadow, which is 5 [or: 6] of your own feet [רגלים]. When the gnomon stands upright, mark the end of your shadow and measure the distance with your foot, and understand this: from the altitude you know the inclination it has on that day, and you find its place precisely according to the instruments you use; for the difference of one day's inclination is not perceptible from its neighbor's, and therefore it is hard to know the day of the month or the place of the sun precisely — unless [you are content with what is] close to the truth. One who wants precision should consult the tables, or the ephemerides [באיפימיריידיש = ephemerides].

But for our purpose, where there is no need to know more than the approximate — for one or two days makes no difference in the matter of the rising of the stars sought for medicine, or for the sowings and tillage of the land — there are easy methods that you can memorize by heart. They are founded on these: I mean, know that the sun's course each day is about, that is, 59 minutes [59 = ניט first-minutes] and some seconds, and so on. The Christian [Gentile] months are 12 in the year; some are of 31 [days] and some of 30 [ל"א… ל']; February [פיברי"רו] is 28 [כ"ח], and in the fourth [year, leap year] 29 [כ"ט].

If you wish to remember the months, raise and bend the fingers of your hand in turn. I mean: thumb upright, forefinger bent, middle finger upright, ring finger bent, and little finger upright; and the upright ones are of 31, the bent ones of 30 — and these are their days. Know how many days the sun has entered into them, and you can know how many days remain until they are completed; they begin afresh from March [מר"צו], except for February, as I taught you, which has a different rule. And when you know…

Now the sun enters, approximately a day before or after, into these signs on these days of the [Christian] months:

- Aries [טלה]: [כ"ב] 22 of January [יניאו]
- Leo [אריה]: [כ"ג] 23 of July [יוליאו]
- Taurus [שור]: [כ"א] 21 of April [אפריל]
- Virgo [בתולה]: [כ"ג] 23 of August [אנוסטו]
- Gemini [תאומים]: [כ"א] 21 of May [מאייו]
- Libra [מאזנים]: [כ"ג] 23 of September [שיטיטכרו = September]
- Cancer [סרטן]: [כ"ב] 22 of June [יוניו]
- Scorpio [עקרב]: [כ"ב] 22 of October [אוטוברי]
- Sagittarius [קשת]: [כ"ב] 22 of November [נווימברי]
- Capricorn [גדי]: [כ"ב] 22 of December [ריצימברי]
- Aquarius [דלי]: [כ'] 20 of January [ייניירו]
- Pisces [דגים]: [י"ט] 19 of February [פיכרירו]

(Theorem 42 [מטור מ"ב].) Now, with the present table [הלוח הנכחי], you can also know this approximately, namely: add the day of the month to the degrees corresponding to the sign beneath it in the table; and if the sum is less than 30, it gives, by that sum, the degree of the more eastern sign above the month-column; and if it exceeds 30, the amount of the excess is [the degree] within the left-hand [following] sign…

and add to it 180 degrees [ק"ף], and out comes the oblique ascension of the sign opposite it, which is **21° …** [reading uncertain — see anomaly note].

By way of example: We sought, at the start of June [יוניאוס], the place of the sun in the signs; beneath it in the table is **40** [40 — but read in figure form, see note]. Therefore it is at **11°** of Gemini [reading "י"א לתאומים"]. Again, on the 25th of July [כה' ליוליאוס], beneath it is **8**; and with 25 they rise to 33 [לג']. Therefore it is at **3°** of Leo [reading "ג' לאריה"].

And R. [Moses] Almosnino [אלמושנינו = Moshe Almosnino] brought these two [mnemonic] houses [verses] in his Sefer Beit Elim [ס' בית אלים]; they are taken by him from Master Comtianus [מהר"ם כומטיאנו = Master Comtino / Mordechai Comtino]. Here they are. But they are founded on the reckoning of the Julian year [שנת יוליאוס הקיסר = the year of Julius Caesar]. Now — "let it not pass the appointed bound" — add to them 13, and what comes out of the sum is the place of the sun.

Diagram / **Mnemonic table.** A two-row solar-position mnemonic table. The heading verse runs: **"זכור חמה ומעלותיו ובמזלות הקפותיו"** ("Remember the sun and its degrees and the signs of its circuit") and **" היה דורש דבר דעה גליל דרכי הליכותיו"** ("Be one who seeks the matter, the knowledge of the compass of the ways of its courses"). Each mnemonic word's gematria gives the sun's degree-position within the named zodiac sign at the start of the named Latin month; the table reads right-to-left in the source. The table is split into two blocks of six signs each.

Top block (right-to-left), zodiac sign / mnemonic word / Latin month:

Zodiac sign	Mnemonic word	Latin month
טלה [Aries]	זכור [Z-Kh-W-R]	יניר [January]
שור [Taurus]	חמה [Ch-M-H]	פיברור [February]
תאומים [Gemini]	ומעלותיו [W-M-ʿ-L-W-T-Y-W]	מרצו [March]
[Cancer — column heads abraded; see note]	ו…	אפריל [April]
[Leo]	ובמזלות [W-B-M-Z-L-W-T]	מאיו [May]
[Virgo]	הקפותיו [H-Q-P-W-T-Y-W]	—

Bottom block (right-to-left):

Zodiac sign	Mnemonic word	Latin month
תאומים [Gemini]	היה [H-Y-H]	יוניאו [June]
סרטן [Cancer]	דורש [D-W-R-Sh]	יוליו [July]
אריה [Leo]	דבר [D-B-R]	אנוסטו [August]
בתולה [Virgo]	דעה [D-ʿ-H]	שיפטימביר [September]
מאזנים [Libra]	גליל [G-L-Y-L]	אוקטוברי [October]
עקרב [Scorpio]	דרכי [D-R-Kh-Y]	נומברי [November]
קשת [Sagittarius]	הליכותיו [H-L-Y-Kh-W-T-Y-W]	ריצימברי [December]

[Resuming prose after the table:]

For example, on the 22nd of May [כ"ב מאייו] of the Julian-Caesar year — which is the head of the month of June by the reckoning of Pope Gregory [גריגאו"ריוס = Gregory] — I wished [to find] the place of the sun in the signs. I add 22 to 8 [the

value] at the head of the [month] circuit-column over May [the entry for מאיו], yielding 27; and a further 13 upon them, yielding 40; I subtract from them the 30 of the sign Taurus [שור] aligned with May, and 10 remains. Therefore at the 10th [degree] of Gemini [לתאומים] is the sun; and by the method [given], at the latitude [where] we found it on the 21st [כ"א]. All this is according to the foundation: the sun enters the signs a day before or after — understand this. A second example: on the 25th of July [כה' יוליאוס] by the reckoning of the [current] month, which is the 15th [ט"ו] of [the old reckoning]; I added the 4 [ד'] that I found at the head of the [month] column [for July, the word דורש], and they rose to 19; I added 13 and they rose to 32. Therefore it is in the second [degree] of Leo [כשני לאריה]; and with the aforementioned table we find it on the third [day] of it [the sign]. And if you wish to work backward, to know the day of the month from knowing its place in the signs, it is very easy from what has been said; and there is no need to be lengthy.

שאלה כ"ז [Problem 27]

On knowing the longitudes of the [terrestrial] regions [cities / countries].

Before I come to expound some of the problems pertaining to the description of the earth — called geography [גיאונראפיאה = geography] — and its measurement, I will inform you of the terms we use, even though they are explained, for the learned, in the clear book [Euclid? / the standard treatise]. Now the ancients did not roam the corners of the earth, nor grasp the ends of the world, as the moderns did; so much so that those who said there is habitation beneath the equator were reckoned, in their times, heretics. When they saw that the habitable world extends much from west to east — by what became known to them from the merchants — more than it did toward north and south, they called the distance running from west, by way of south, to east the **longitude** [אורך], and [the distance] from the equator northward, to the side of, or south, the **latitude** [רוחב]. And when they saw that beyond the islands called the Fortunate [פורטונאטיס = the Fortunate Isles / Canaries] the great sea ran on without end, they reckoned there was no further habitation to the west — unlike toward the east, whose end they did not know. They agreed, then, to begin the longitude from the aforementioned islands at the western edge, since [none had been cut off, i.e.]…

to them in Brazil [כרוזיליאה = Brazil] and all the newly-discovered worlds known to us — for these are more western than the aforementioned islands. They agreed, then, to count the measure of the longitudes along the equator, and not along the [meridian] circles passing through the zenith [point opposite the head]

of each and every region — that is, [not along] the parallels to the equator — because these circles differ in their magnitude according to their inclinations or their distances from the equator. And even though all of them are divided into 360 equal parts, the [equator's] parts are larger than the rest. Therefore, even if two cities are distant by equal degrees from the aforementioned islands, it does not follow that the [traveled] distance is equal. And once you know the manner of reckoning their ratio to one another — with God's help, later — it is good that one reckon both the longitude and the latitude on great circles [עגולים גדולים = great circles], which are equal to one another.

Thus, the **longitude of a region** is the measure of the equator-arc between the meridian [circle of midday] passing over the head of the Fortunate Islands and the meridian of the region, [reckoned] west to east. And the **latitude of a region** is the measure of the arc of the meridian [circle of midday] between the circle of the equator and the parallel circle [for the equator] passing through the zenith of the head of the region. And just as those parallel to the equator are partly northern and partly southern, so too are the latitudes; and as you have learned in [the part on] the clarification, the latitude of a region equals the measure of the pole's altitude over its horizon.

Now, with a material globe [globe model] you can easily learn the distance between two cities: take upon it a compass [pair of dividers] whose legs are curved and bent [to follow the surface], [set to] the distance between them, and seek its measure on the equator of that globe; the degrees between its legs will show you the measure of the arc of degrees on the terrestrial globe between city and city.

But as for those who are accustomed to compute the distance with [plane] triangles and the [system of] lines — according to what is taught [in] Theorem 47 [מ"ז] of the first [book of Euclid, the Pythagorean theorem] — this is incorrect for the foundations, since the surface of the earth is spherical, not flat. Therefore it is impossible to know their distance with straight lines except approximately, and so the precise reckoner investigates the distance with spherical triangles. It has been [shown] by experience and reasoning that to each degree [part] of the great circles — whether the celestial or the terrestrial — there belong 15 parasangs [ט"ו 15 = פרסאות parasangs], that is, [the distance] by which the longitude changes by one degree from a longitude, or the latitude of a region changes by one degree. Therefore, when you travel along some route until you see that the pole's altitude over the horizon is one degree more than [at] your [home] region, know that the journey is 15 parasangs, which are 60 [80 — פ'?] miles [see note].

And I have already taught you in this book how to know the latitude of a region. Besides the aforementioned methods, there are several others; seek them in the larger books. But the knowledge of the **longitude** of a region is among the things [most] coveted, [precious] above gold and fine gold; many and honored men have sought it, and still go on seeking it, for it is among the things greatly needed by mariners; and it has not, to this day, been found in [its] perfection.

The method by which the geographers [הניאוגראפי' = geographers] who wrote [down] the longitude of various regions proceeded is this: at the time of an eclipse of the luminaries — and especially of the moon — two astronomers in different regions, far from one another, observed carefully the hour and the minute. For although the eclipse occurs at one instant in the heavens, nonetheless the eastern observers reckon more hours than the western, since the sun comes earlier to rise and to reach the noon of one region than of another, on account of the sphericity of the earth. When one informed the other of the time of the event, and they knew the appointed [common] time, and converted the temporal difference into parts of the equator-circle, they found the distance between them; and once they knew the longitude of the one, they knew also the longitude of the other.

For example: in the city of Cracow [קראקה = Kraków], the metropolis of the region of Poland [פילוניאה = Poland], the eclipse of the sun was at the second hour after noon; and in Heidelberg [בהיירילבי'רגה = Heidelberg] it was at one hour and one minute [בשעה א' ורק א']. Therefore the time-difference is **59 minutes** [59 דקים]. And according to what I taught you — that in one hour 15 degrees rise on the equator — you set up the proportion thus: if 50… [continues].

[Continuation of Problem 27 — terrestrial longitude by lunar eclipse]

For each hour, give 15 [degrees]. Multiply 59 [minutes by this rate] and you will find 14°45′. Therefore, if you know the longitude of one of the two cities — say of Cracow, which is 45°30′ — and subtract 14°45′ from it, you will find the longitude of Heidelberg to be 30°45′. Cracow, then, is the more easterly city.

Proceed likewise with the tables, or with the ephemerides drawn up for some particular longitude (that is, for the meridian of a particular city), when you wish to know the position of the stars at given hours in your own horizon: you must convert your time to the time of the horizon of those tables, and reckon your affairs accordingly. And conversely: when by those tables you have found the position of the stars or an eclipse at a certain hour, you must adjust it to the hours of your own place. This is done through knowledge of their longitudes — and understand this well.

And note: if a person travels continually along the equator, or along some other parallel to it, without drawing nearer to it or receding from it, and he knows the number of parasangs of his march, he can deduce the number of degrees of that terrestrial circle. From these he learns the degrees of the equator, and so arrives at knowledge of the difference in longitude between the two places.

The scholars of the age have devoted much effort, each by his own method, to discovering the unknown [longitude], so as to magnify and exalt the science by some advance in knowledge. One author has spoken remarkably of the properties of the magnetic stone — that of iron — and of its uses, relating that it indicates longitude just as it shows latitude. In *Gan Naul* I have written further on this matter, with God's help.

Problem 28 [שאלה כ"ח]

How may we know the ratios of the parallel circles to the equator, once we know their inclination — that is, their distance from the equator?

It has been demonstrated in [the science of] measurement that the ratios of [the circles'] diameters, or of their halves, are as the ratios of the circles. Now it is known that the half-diameter of the equator — which is a great circle — is the whole sine [i.e. the radius], while the half-diameter of a circle parallel to the equator is the sine of the complement of its distance from the equator. This is easily grasped if you draw within a [single] circle the equator and the circles parallel to it, together with their diameters.

Suppose, then, that we wish to know the ratio of the parallel circle distant 44° from the equator. We set up the proportion thus: as 100000, the half-diameter of the equator, is to the sine of the complement of 44° — which is the half-diameter of the parallel, namely 71934 — so are the 360 degrees of the equator to the degrees of the parallel. After dividing, you find 258, with a remainder of 96240. Multiply this by 60 and you get 5774400; divide by the whole sine and you get 57 firsts, with a remainder of 74400. Again multiply by 60 and divide by the whole sine, and you get 45 seconds. Therefore the parallel circle distant 44° from the equator has in its whole circumference no more than 258°45′57″ [reading: the text gives "258°. 45′. 57″"] of the degrees of the equator.

Proceed likewise in [computing] the southern [parallels]. It is easy to know the ratios of any two parallels and their parts to one another. This will aid you in drawing up a *topographia* — that is, a sketch of countries that are not [uniform]

in breadth and length, since one and the same country has differing longitude and latitude, not being of a square shape — and understand this well.

Problem 29 [שאלה כ"ט]

How may we know the distance between two cities whose longitude is equal?

In the said figure [Figure 31], let the equator be MD, with its northern pole A; and let there descend from it [pole A] the meridian circles ASK, ANK [reading uncertain for the point-letters], passing over the heads of [places] in different countries [?]. Let the city of Venice be at point ז [Z], and the city of Frankfurt-on-the-Oder [?] at point ח [Ch], both of them above the equator toward the north, with their longitude equal [as measured] from the western edge — being on the meridian circle [that runs] from A to ך [K].

From the difference of their latitude: the latitude of Venice is 45° and the latitude of the said Frankfurt is 52°30′. In a case like this — and in those resembling it — the task is easy: you need only subtract latitude from latitude. The difference, reversed [i.e. converted] to parasangs — say we knew them to be 52°30′ and 45° — and the difference is 7°30′; convert it to parasangs by multiplying by 15, and you get 112½ parasangs. That is the length of the march between the said cities.

Next, suppose we wish to know the distance between Mount Royal in the land of Prussia — the place called Königsberg [קונינשבירג] — and Cape of Good Hope [קאפו די בונה שפיראנצה], both at the same longitude, namely about 47°. Mount Royal would be at point ץ [Tz], its latitude 54° north; and the second place at point מ [M], its latitude 34° south. In this and similar cases, gather the two latitudes together: ץ and מ; their sum is 88°. Multiply by 15, and there rises 1320 parasangs — that is the march sought, ימץ.

Problem 30 [שאלה ל']

How may we know the distance between places equal in their latitude?

In the said figure, let there be on the equator MD two places — that is, at point ל [L], the island of Timor [?] [טוימאז] in Africa, its longitude 32°20′; and at point ק [K], the island of Sumatra in the East Indies, its longitude 131. Subtract one from the other; the difference is 98°40′, which is 1480 parasangs — and that is the distance.

Next, let both of them be off the equator, on one of the circles parallel to it, both on the same side. For example, the city at point פ [P], its longitude 4; and a city at point ל [L], its longitude 38; the latitude of both being 60, [each measured] by [its

own] parallel circle of equal degrees. Now you have an isosceles triangle אב [reading: אפל], and by the eighth section [theorem] concerning the gnomon-line of the "creeping stone" [?] of the corner... [text obscure here]. To unloose this foundation [puzzle], do as follows: since the equal legs are the complements of the latitude, each will be 30°. Drop a perpendicular [from] the angle אפ ,ג, which is 16, the difference of the longitudes; it cuts the triangle in half. And by the eleventh section toward the first angle, or from section 1 of *Chukos Shamayim*, find פ, half the distance — 4 — and its double, פל [reading: פן], is 8, which is what was sought.

The proportion runs thus: as the whole sine 100000 is to the sine of 30 — 50000 — so is the sine of 8 פאף — which is 13917 — to the sine of the side פף [reading: the side], 6958. Or proceed this way: erect [a perpendicular] אף up to point ל [L], so that it [the longitude difference] becomes a quadrant. In the triangle קל ,קלף and קף will be quarter-arcs [רביעיים], since they come from the pole ק upon the circle [meridian]... and after the right [angle]... [reading: ל] is also a quadrant [נצבת] and קף a quarter. Now the magnitudes of קפ are known, for they are קך, the complements of [the latitudes] פ; and קף the latitude. Therefore, in the right-angled triangle קדף, by the tenth section toward the first angles, you will find the surface [arc] קפ, and its complement תך [reading uncertain] is the distance sought.

Problem 31 [שאלה ל"א]

How [to find the distance] when the cities differ in their longitude and in their latitude, and one of them is on the equator. We shall know the distance — and this varies in three ways.

From the said figure: let the island of Sumatra be on the equator at point ק [K], its longitude 131°; and at point ל [L] the city of Buda [בודון], head of the proud nation of Hungary, its latitude 47°. The difference of the longitudes, קל, is a quarter of a circle [90°]; therefore the distance קף [reading: קל] is likewise a quarter of the circle drawn from pole ק — as I have explained in the sixth and tenth lessons toward the second angle, in *Sefer Sod HaYesod*. So קף [reading: קל], the distance, will be 90°, which is 1350 parasangs.

Next, let the island of Timor [?] be at point ק [K], which is on the equator, its longitude 32°20′; and Amsterdam at point פ [P], its longitude 26°30′, its latitude פך 54°40′. Now the difference of the longitudes, קך, is 5°50′, less than a quarter, and we wish [to find] the distance קפ. We have, in the right-angled triangle קרפ, three known [magnitudes], and by the seventh section toward the first angle — or from

Sefer Arvot HaShamayim — the proportion runs thus: as אך, the whole sine 100000, is to לך, the tangent of the complement of 84°10′ ,קך — which is 99482 — so is אפ, the sine of the complement of the latitude 37°20′ ,פך — which is 60545 — to פף, the tangent 60330, which is 35°41′. Its complement is 54°19′ ,פק, which is 814¾ parasangs.

Next, let the island of Sumatra be at point ק [K], its longitude 131°; and the city of Heidelberg at point ב [B], its longitude 30°45′, its latitude 35°49′ בך. We wish [to find] the distance קב [reading: קנ]. In the triangle קרג [reading: קבג], the angle ך is right, and the side קר [reading: קב] is 100°15′ [reading: the difference of longitudes exceeds a quarter]; the difference of the longitudes exceeds a quarter, and the side ב [is] the said latitude. Therefore, by the sixth section toward the first angle of *Sod HaYesod*, we find קג [to be] 96°87′30″ [recte: 96°37′30″ — minutes value 87 is impossible; 1449 5/16 ÷ 15 = 96.62° ≈ 96°37′30″], which is 1449 5/16 parasangs.

Problem 32 [שאלה ל"ב]

How may we know the distance when both [cities] are off the equator on the same side, and differ in their longitude and in their latitude.

This too is divided into three cases. Both will be on the same side — that is, both northern or [both] southern. For example, in this figure [Figure 32], the second [case] of them: let מן [MN] [reading uncertain] be the strait of the Baltic Sea, between Sweden [רצניאה] and Norway [נורויניאה], famous in the world, called the "Sont" [Sound] [ונטי]; its latitude מן is 55°43′ and its longitude 37°0′. At point ק [K] let there be the city of Jerusalem, ב [B] [reading uncertain], its latitude 31°40′ בג and its longitude 66°. Let the angle ך be the size of [the arc] בק [reading uncertain] which is on the equator, the difference of the longitudes [being] 29°0′, less than a quarter.

Therefore, in the oblique triangle כטב [reading uncertain], two sides are known — כב ,כט, which are the complements of the latitudes — together with the angle ל between them. By the eleventh section of the first [book], let point ץ [Tz] be the city of Daroca [?] [דארוקאקאנה] of Aragon [?] [פארופאנוזי], in the land of Asia [?]: you will find the magnitude of the side כמן [reading: כט] of the distance to be 31°27′, which is 471¼ parasangs.

Next, in the said figure, its latitude 4°45′ נץ [reading uncertain] and its longitude 118°45′; and at point פ [P] the city of Speyer [שפירה], its latitude 49°20′ לפ and its longitude 28°45′. Now the difference of the longitudes is נל, a quarter of the arc of

the equator, 90°; therefore the angle of the difference דא is right. By the tenth section toward the first angle we find the surface [arc] ץף [to be] 66°22′, which is 965½ parasangs.

Next, in the first figure, let there be at point ב [B] the city of Lisbon, capital of the kingdom of Portugal, its longitude

4°18′ and its latitude 29°38′ בך; and at point ץ [Tz] let there be Calicut [קאליקוט] in the land of the Indies, its longitude 112°0′ and its latitude 5°0′ נץ. The difference of the longitudes — namely יץד — exceeds a quarter, being as it were an angle of 107°42′. Behold, you have in the oblique triangle the two sides known, which are the complements of the latitudes, together with the angle between them. By the eleventh section, [third] angle [reading: section 11, angle 3], find the third side, ג, which is 100°14′, which is 1503⅛ parasangs.

Problem 33 [שאלה ל"ג]

How may we know the distance when they are on different sides of the equator and differ also in their longitude.

In the second figure, let the northern island Hile [?] [אי הילי] be at point ה [H], its latitude 63° [reading uncertain] and its longitude 33°; and the southern island of St. Thomas [טומאזו] [be at point כ], its latitude 16 כי and its longitude 27°20′. The arc of the difference of the longitudes on the equator is גו [reading: the angle ב] — that is, angle 5°40′ — ב, less than a quarter. Therefore, in the oblique triangle כו, the two sides are known — כו, כג, which are the complements of the latitudes (כח 27 and 106 כי) — and by the eleventh section toward the third angle, find the side חי, [which is] 79°7′30″. And so it is according to the author of *Sefer Midat HaAretz*.

Next, in the first figure: let there be at point ץ [Tz] the city of Bassa [?] [באשה] in Taprobane [Ceylon] [טפרוביומאנה], its longitude 126° and its latitude זמ, south, 6°30′; and at point פ [P] the city of Herford [?] [הירפורדיאה] in Victoria-land [?] [טוריניאניאה], its longitude 28°30′ and its latitude פך, north, 51°10′. Now the difference of the longitudes exceeds a quarter — that is, angle זא is 97°30′. In the triangle פאג, the remainder [reading: the residual] of the triangle פאז is known to us: the angle 82°30′ ,פאג; and אפ, which is the complement of the latitude ,פך 38°50′; and אג, which is the complement of 83°30′ ,זמ — for זטא is a semicircle, and the arc טא is subtracted between זט and אן. Therefore, by the eleventh section toward the third angle, you will find the side 80°15′ ,פ, and its remainder פז will be 45°9′ — and that is what was sought.

Now, this year there was with me a fine young man, his name R. Yitzchak ben Avram HaLevi, who busied himself with deriving the oblique spherical triangles without dropping perpendiculars, and he succeeded greatly. By their means I measured many distances with great ease, [showing] that the said triangle — in which two sides together with the angle between them are known — is solved very nicely without dropping a perpendicular, which prolongs the work. And if such notebooks [קונטרסים], together with the figures, should come to me [reading: into hand], I shall send the [whole] series, for they are concise and in clear language.

Problem 34 [שאלה ל"ד]

How, from knowledge of the latitude of one city together with the angle at which its meridian [reading: its position-angle] stands on the meridian circle relative to the other — and knowing also the march in parasangs between them — may we know the longitude and latitude of the second city.

In the first figure, let there be a city at point [reading: א], with longitude and latitude known, and another city at point [reading: ע], with the distance between them known. Known also is the angle [reading: אעל] that stands on the meridian circle, [the one] passing over the head of [point] ע [reading uncertain]. This belongs to the mariners, who know how to direct the course of a ship when they wish to sail to some country: with their instrument they know the [wind-]directions cut off at right angles and at acute angles, and their magnitudes. Now in the oblique triangle אעם, the side אע is known, which is the complement of the latitude; and the side עם is the given distance; and the angle between them, אעם, is likewise known, being the residual [of...]

[Text breaks off at the foot of page 274.]

Chukos Shamayim — Almagest Commentary, Book II (continued)

from the given angle. Therefore, by the eleventh section toward the third angle, the third side **אם** [a-m] is found — and it is the complement of the latitude of the city **אטטור** [?]. By the eighth section toward the third angle, the angle **אע** [a-ʿ] is found, the difference of the longitudes, and from it we know the longitude. Understand this.

All that has been said concerning the distances between places applies equally to the distances between stars — particularly so that you may set up the figures according to their longitude and latitude. But I do not intend to dwell on this further in this brief treatise.

Problem 35 (שאלה ל"ה)

How, from knowing the altitude of some star in the evening — relative to the points where the solar light is at the western edge, or in the morning at the start of dawn in the east — together with knowing the latitude of the place, we may find the length of twilight, according to the time or the position of the sun in the zodiac.

Pedro Nuñes [פיר"רו נונייש; Pedro Nunes], a man great in the mathematical disciplines, of Jewish descent, made an observation in the year 1541, on the 1st of October, at the citadel of the city of **Lisbon** [לישב"ונה], in the evening, when the dome of the heavens was clear and pure, free of clouds. He took the altitude of the bright star in the constellation Scorpio called the Heart of the Scorpion [Antares], with instruments, and its altitude above the western horizon was 5 degrees. Now this star's longitude in Ptolemy's table is 216, and when you add 28 to it — which, according to the view of **Copernicus** [קופירניקו"ס], is the precession of the stars from the head of Aries — its true longitude at the said time would be [216 + 28 = 244, i.e.] 4 degrees [?] in Sagittarius, and its latitude 4 degrees south. Therefore, from Problem 20 [?] you find its declination, 24°56′ south; and from Problem 20 [?] its right ascension will be 241°10′. From these find its altitude in the meridian circle, by Problem 12, which is 24°26′; and from Problem 16 you find the star's distance from the meridian circle, 60°19′. Since the sun at that time was at 18 degrees of Libra, its right ascension was 196°35′. The difference of the right ascensions is 44°35′. Therefore, by Problem 18, the equator-arc traveling with the sun after midday up to that moment will be 104°54′. When you subtract from this its travel — the meridian half-day arc at that time, which is from Problem 9, namely 48°18′ — there remain 36°20′. And this is the equator-arc measuring the magnitude of the evening twilight, that is, approximately one hour and 22 minutes [of time].

By this method you proceed in all the southern regions, and likewise for the morning twilights, when the star rises before the sun in the east. But a correction is required on account of the change in the sun's appearance and because of the refraction of the rays, and elsewhere I will expand the discussion and explanation for you. I will, however, inform you that the start of twilight is no less than when the sun is 15 degrees beneath the horizon in the evening, and

likewise its end in the morning; and its end is no more than 18 degrees beneath in the evening, and likewise its beginning in the morning. According to some, even up to 20 degrees of sinking there is still twilight; and in *Sefer Gevuras HaShem* [the treatise *Gevuras HaShem*] I will speak of this, God willing. Hence in certain very oblique horizons near the poles, on summer nights there are great twilights — as in the city of **Vilna** [וִילְנָה], where, when the sun sets in the west, light shines from the east, so that there is twilight all night long.

Problem 36 (שאלה ל"ו)

How we may know the length of the twilights at the right horizon.

In the figure of the analemma, which I have drawn in *Sefer Sod HaYesod* [the *Sod HaYesod*], the meridian circle will be **אבלף** [a-b-l-p], and the equator and its poles at the right horizon will be **בף** [b-p] — for that is the figure there beneath the equator. Then **דר** [d-r] will be the circle of Cancer or of Capricorn. Now, the sinking of the sun [to a depth of] 18 degrees is reckoned on the circle of the zenith, the equator being in its place in that position. Therefore, if the sun is at the equator, the measure of the twilight will be the [arc] between the horizon and the almucantar — that is, the [circle] parallel to the horizon **ז' Z** [z] — and it is 18 degrees, and no computation is needed here. But when the sun is at one of the parallels to the equator — for example, on the circle **דר** of Cancer — the measure of the twilight will be **זר** [z-r], and we need to know its measure. Now the arc **א** [a] of the great obliquity is 23°32′, and its complement is 66°28′, and its sine [beka] is 91682. Therefore reason thus:

	as **דר** [d-r]	to	**אי** [a-y]	so	**זר** [z-r]	to	**19°42′**
	66°28′		100000		18°		19°42′
sine	91682				sine	30902	

To be saved from the division, here, from the *Even Shlomo* [the "Stone of Solomon" reckoning-board] of *Sefer Sod HaYesod*, the inference will be thus: as the radius 100000 is to the [remainder/quotient] of the complement 66°28′ / 109072 [super-/subscript: 29 over 109072] — so the sine 30902 is to 33703, [whose arc is] 19°42′. Therefore the measure of the twilight at the right horizon, when the sun is at the head of Cancer or of Capricorn, is about an hour and 19 minutes [of time]. And this is what was intended.

Problem 37 (שאלה ל"ז)

How, from knowing the altitude of the pole above the horizon and the position of the sun in the zodiac, we may find the length of the twilight at oblique horizons.

From Problem 35 you have learned that from the altitude of the star you find the measure of the twilight. And if you set your heart to understand by yourself how easy it is to compute the magnitudes of the twilights: take the point opposite [the nadir of] the sun's position in the zodiac at whatever time you wish, raised on the eastern side above the horizon by 18 degrees — or twenty, according to some who say so — and by Problem 16 seek the hour of the day, or the time, corresponding to that altitude above the horizon; for that is exactly the time of the evening twilight. Conversely, take the point opposite the sun, raised on the western horizon by 18 or 20 degrees, and seek the time corresponding to that altitude above the horizon by Problem 16; for the like of it is the time of the duration of the morning twilight. Understand this. In another book I will make all this known, with the analemma, by right-lined triangles, with great ease, with the help of God.

Problem 38 (שאלה ל"ח)

How, from knowing the altitude of the pole, the position of the sun, and its dwelling beneath the horizon at the start of twilight, we may find the length of the twilight by another method.

Diagram. A schematic for finding twilight from the sun's position and depression beneath the horizon. The structure is the analemma-type diagram described in the surrounding text.

- ן (point): the position of the sun, 15 degrees in Capricorn, beneath the horizon **גדב** [g-d-b]
- גדב (arc): the horizon
- דן (arc): the sun's dwelling [depression] beneath the horizon
- י (point): the pole and the southern equator [region]
- דופן [d-w-p-n] (arc/quadrant): the quadrant opposite [perpendicular to] the head [zenith]
- וץ (arc): the declination of the sun
- ימן (arc): the complement of the altitude of the northern axis [pole]
- ומי (triangle): the triangle within which the work proceeds
- וי (angle): the angle measuring the equator-arc **זן** [z-n]

In the figure opposite, the sun is at point 15 ,ן degrees in Capricorn, beneath the horizon **גדב**, and its dwelling [depression] is **דן**. Let the pole and the southern equator be **י** [y]; the quadrant opposite the head [zenith], **דופן**; the declination of the sun, **וץ**; and the complement of the altitude of the northern axis, **ימן**. Now it is our wish to know, in the triangle **ומי**, the angle **וי**, which measures the equator-arc **זן** — and the three sides are known to us. Therefore, by the sixth section toward the third angle, the said angle is found, that is, the measure of arc **זרן** [z-r-n]. But the half-night arc, when the sun is at 15 degrees of Capricorn, is

7 hours, 29 minutes, and 4 seconds. Subtract them from the hours 5°26′21″, and there remain 2°2′43″, and this is the magnitude of the twilight occurring at that place.

[The right-hand column of the computation, laid out beside the prose:]

Operation	Value
[the equator-arc, in time]	5h 26′ 21″, and there remain 2′ 2″ 43‴
ון [w-n] complement of the declination	21° 36′ 73 [recte:? — 73 impossible as a sexagesimal place]
וימן [w-y-m-n] complement of the depression 18	0° 72 [recte:? — 72 impossible as a sexagesimal place]
ימן complement of the altitude of the pole	30° 49′ 37
וימן [w-y-m-n] which are hours	17° 35′ 81 [recte:? — 81 impossible as a sexagesimal place]
[result] start of the morning twilight	5° 26′ 21″

If you wish to know the start of the evening twilight, subtract from twelve hours those [hours] 5°12′62″ [?], and there remain 6°33′59″, and this is the start of the evening twilight; and 5°26′21″ is the start of the morning twilight.

Problem 39 (שאלה ל"ט)

[How we know the start of twilight from the sun's declination and the pole-altitude.] This is the inverse of the preceding problem. Now, in the said triangle **ומי**, the two sides together with the angle between them — which is the measure of the equator-arc **זרן**, the distance of the start of the morning twilight from midnight, that is, 5 hours 26′21″, which are degrees [?]...

[Side computation:]

	Value
ו׳ complement of the declination	(carried from above)
complement [ותשלומו] = ו	which is the depression of the sun beneath the horizon
י״מ [y-m]	17° 35′ 81 [recte:? — 81 impossible as a sexagesimal place]

Problem 40 (שאלה מ')

On the foundation of the hour-instruments, or solar instruments — for the construction of the instruments that indicate the hours by the shadows of gnomons requires a separate treatment, and this is not its place. If God grants me life and rest, I will complete what I have begun, in honor of Wisdom, and then I will speak of it fully. But for the understanding of what is said in this brief treatise, I will inform you of what I have already revealed and repeated: that *time* is an effect of the sun in its daily motion, and therefore these instruments are called "solar." Now the equator-circle is the counter of time and the orderer of the motion, and therefore the foundation of the instruments is built upon it; and as its 15 parts move, they indicate one hour of the day, which is an equal hour. And if you draw 12 great circles passing through its poles, they will cut it into 24 equal parts, which are 24 hours. These are called "hour-circles," and when they are marked upon flat surfaces, their diagonals are called "hour-lines." And just as, by means of the said circles, the equator is cut into 24 equal parts, so every circle that is cut upon their diameters will be cut into 24 unequal parts. The whole business of this science is to know the distances of the cuts in them, which are the widths of the hours.

From this you may learn that the hour-instruments are surfaces parallel to, or aligned with, circles in the celestial sphere; and those perpendicular to the equator are named after it. [Those] cut with it at right angles — like the surface that inclines, or is elevated, up to the altitude of the pole above the horizon — and the surface perpendicular to the horizon are called "horizontal," and so all of them. Now, of the circles cut with the equator, the end that is perpendicular to the meridian circle faces due east or west exactly, without inclination; and in these the gnomon is perpendicular, parallel to the axis of the universe, and the hour-lines do not meet but are parallel to one another. Some of them [the surfaces] cut [the meridian] facing only the horizontal [direction]; again, some are perpendicular to the meridian circle, and in these the hour-lines meet, the hour-circles being cut at the center of the instrument — that is, at the base of the

gnomon, at unequal angles; and for all these the lines are cut at the center of the instrument [making] equal angles, and each angle is 15 degrees. Some are perpendicular to the horizons or to the zenith; the first kind — that cut at right angles with the meridian circle — is the circle passing through its poles, which are the cuttings of the equator with the horizon, namely the points of true rising and setting. And some [surfaces] incline, or [are inclined to] these circles, or [differ from] them, or to all three together; and those occurring as inclined surfaces are the most...

but they are more difficult, and the measures of the angles arising from the inclinations of the surfaces are investigated by means of triangles. In this is recognized the power of the triangle, as in all the other arts. I do not intend to speak of them here, except concerning those perpendicular to the equator, or to the horizon, or to the first zenith, without any inclination at all — and only the chief headings I will bring.

Now, the finding of the meridian line I have already explained at length in Problem 12, in which the gnomon casts its shadow at midday on the equatorial and the horizontal instrument; and standing upon it, perpendicular, is the meridian line of the first-zenith instrument — which is needed for all of them, for the laying-out of the instruments. Further, make yourself a quadrant of a circle (**ציור לך** [figure l-k]), and within it [draw] several smaller quadrants, and divide it into 90 degrees, and each degree into minutes, after the manner of the quadrant before you [**רבע** r-b-ʿ]; and draw lines from the center **א** [a] to the divisions, for it will help you greatly in these operations, and one cannot do a thing without it. This suffices by itself.

Problem 41 (שאלה מ"א)

How we may draw an instrument perpendicular to the equator, indicating the hours.

Draw a circle, small or large as you please, on a board of wood or stone, and divide it into 24 equal parts, and draw lines from its center to the divisions, and fix at its center an upright nail [gnomon]. [Stand it] so that the surface of the instrument makes, with the meridian line toward the south, an angle whose measure is the complement of the altitude of the pole [?]... at your pleasure. Raise the wood or the brick above the surface of your horizon by the measure of the altitude of the equator — that is, so that the surface lie directly opposite the northern pole. This will indicate to you the hours by the motion of the sun, though [you see] some of them from the upper side, on your horizon, since this

surface is perpendicular to the equator in your locale; the face of the instrument [should be turned] to the north, and the gnomon set true. [Then] from one side, above [the surface], the equilibrium [of the sun] from the head of Aries to Libra — when the sun inclines from the equator to the north — will light the face of the instrument all day; but when it is in the southern signs, it is below the equator, and casts its rays beneath the surface of the instrument where it sees the southern face. Therefore mark the hours on both sides of the instrument, and let the nail pass through and project to the surface behind it, which indicates to you the hours either from here or from there. You thus learn that if you lengthen the nail until it reaches behind the surface to the surface of the horizon, and above to the surface perpendicular to the zenith — which is at right angles with the horizon — the [other end of the] nail will indicate the hours on three surfaces; for in all three it is aligned with the axis of the universe, which is required for the placing of the nail. And so it is with the cord that runs between the two instruments [folded] open like a writing-tablet [diptych], which are in everyone's hands [a portable diptych dial]. Between them you set up the instrument perpendicular to the surface of the equator, which cuts the said cord at right angles — for this cord is the axis of the universe; and with the two surfaces, the horizontal and the perpendicular-to-zenith, it forms a right-angled triangle. The angle between the cord and the horizontal surface is the altitude of the pole above that horizon, and its complement is the second angle. If the altitude of the pole equals its complement — as in the city of **Venice** [ויניציאה] — the cord is cut from the surface of the equator in half, and the said triangle is isosceles. If [the altitude] is less than its complement, [the cut] is more toward the horizontal side; and if more, less. From the following problem understand the matter further, in its proper form and foundation. Also, on the upper surface you can draw an instrument of the equator [equinoctial dial], which rises and sets nicely upon its axes according to the altitude of the equator at the various horizons; and thus I have seen the three instruments drawn together in a small writing-tablet [pocket diptych] in several places.

Problem 42 (שאלה מ"ב)

How we may construct an instrument that will serve us as the foundation of the horizontal instruments and those perpendicular to the zenith, and [how to] construct the horizontal instrument at **Candia** [קנדיאה].

In the quarter-circle [quadrant] mentioned above, choose one of the quadrants of the circles — large or small as you please — to mark the measure of the diagonal of the hour-instrument; let it be **דירמ** [d-y-r-m, the quadrant **דיר** d-y-r]. Take in it

the arc **די** [d-y] of 35 [degrees] [recte: the latitude of Candia, 35°], at the altitude [latitude] of the city of Candia; and from it drop a perpendicular gnomon **יך** [y-k] upon the horizon-line **בא** [b-a] — and this is the half-diagonal…

Chukos Shamayim — Almagest commentary, Book II (continued)

[Problem 43, continued — the horizontal dial for the zenith (head-position) instrument]

the instrument perpendicular to the zenith, and line **א** is half the diagonal of the horizon. We draw line **יא**, which is the diagonal of the [?] [world / great circle], with its northern pole; and from point **ך** we drop onto diagonal **יא** the perpendicular **רד**, which is half the diagonal of the equator. This is what is called the foundation of the instruments.

Now suppose we wish to lay out a horizontal hour-instrument at Candia. We take a brick, or a slab of marble of whatever flat surface there may be, and draw on it a straight line as long as we like, like line **אב**, with two right angles upon it. Down [perpendicular] to it, along with it, at point **ק** [there is a point]; and from the quadrant of the foundation we take, with the compass, the length of half the diagonal of the equator, **רד**, and transfer it to **רז**. From point **ג**, with the radius **רד** *(see fig. 36)*, we describe the circle of the equator, which meets line **אב**; hence **אב** is called the line of meeting (the intersection line), along which the plane of the equator meets the plane of the horizon, and along which the hour-lines issuing from the center of the instrument are cut by the hours issuing from the center of the equator.

From point **ג** we draw a line parallel to the meeting-line **אב**, and we divide the quadrant of the equator into six equal parts. Through these divisions we draw lines from the center to the meeting-line. Then we take, with the compass, the measure of half the horizontal diagonal **רא**, which is in the foundation of the instrument, and according to its length we mark out the new circle, tangent to the meeting-line at point **רז**. We divide it into four equal parts by the two diagonals **רך** [and] **טך**, and from its center to the points of the meeting-line, at the ends of the lines that pass from the center of the equator, we draw straight lines; and they will cut the horizontal circle according to the widths of the hours.

When you draw the lines past the center, and according to the distances of the first quadrant you mark out equal arcs, on the left side you divide the circle into unequal parts — these are [marked] by the hour-lines. But you need no more than fourteen hours for the horizon mentioned. Now line **מז**, perpendicular to the meeting-line, is the line of the sixth hour, morning and evening; and the line erected upon it, **רזיד** [?], is the line of midday and midnight; the rest are known. Their inscription around the instrument [is done] with [red? — בששר/בשׁשר [?]] [ink]; and after you have made this instrument, you cut out the circle of the equator and erase all the surplus, and you take from the [new] circle the hours you need.

You set the plate on a surface perpendicular to your horizon, the instrument without inclination, and line **רז** [?] will be aligned with the midday line **דק**, to the north [or to the south — בדרום [?]], [shifted] from east [or] from west. You drive into it a nail standing perpendicular at point **רק**, on the midday line, its length being the length of **יך**, half the diagonal of the zenith (head-position) instrument in the foundation of the instrument. For if you draw from its tip to the center of the instrument a thread, the angle at the center of the instrument will be exactly the angle **יא** in the foundation of the instrument, which indicates the elevation of the pole above the horizon, 35[°]. Therefore, if you do not wish to set the nail straight up at point **דק**, you may instead drive in some long nail at the center **י**, inclined toward the side **וק** that is to the north, making with line **ירז** at point **י** an angle equal to the horizon's elevation, 35[°]. Or you may drive in a nail at the world's edge, provided that the line dropping from its tip onto point **וק** perpendicularly is as long as line **ירך** [?], half the diagonal of the zenith instrument in the foundation of the instrument. Then, when the sun rises in the morning, the gnomon will show its shadow at the fifth hour, on the left side when you face north; and as it draws further toward the south it will show the sixth hour, and so on until it reaches point **ריו** at midday, where it casts its shadow upon line **יך**. From there the shadows reverse to the right side. In this fashion the instrument-makers make [the dial] upon a disk of iron or copper, calling it the *maestro* [Italian: *maestro*, "master(-pattern)"]; and from it they copy the hour-distances and make the instruments easily, which is why they sell them cheaply. You can make them small or large according to the quadrants of the circles in the foundation. And if you wish to carry the instrument with you, the magnetized iron with the lodestones [Italian: *magnete*, "magnet"] will show you the midday line — and there is no need to elaborate; with light tools it is a craftsman's work.

שאלה מ"ג: Problem 43

How we lay out a zenith (head-position) instrument from the aforesaid foundation — namely, the one perpendicular to the equator.

Again we wish to lay out an hour-instrument on a wall that faces the zenith — one that is intersected with the midday circle without any inclination at right angles — and it lies in a plane parallel to the first zenith circle *(see fig. 36)*. We draw the meeting-line **אב**, and upon it **גד**, intersecting it at right angles at point **ק**, and we mark out the quarter of the equator **דך** according to the length of its diagonal in the foundation of the instrument. We divide it into six equal parts, and from its center, through the divisions, we draw straight lines as far as the meeting-line.

From the foundation of the instrument we take line **יך**, half the diagonal of the zenith instrument, and according to its length we mark out a circle that meets the equator at point **דק**. From its center we draw lines to the meeting-line, and it will be divided into six unequal parts, which are the widths of the hours, at their distances in the second quadrant. We set it on the wall in such a way that line **דק** stands perpendicular to the horizon, above, opposite the zenith. We drive in at point **ה** a nail standing perpendicular to **ג** and parallel to the horizon, of the length **אך**, the horizontal diagonal in the foundation. From it, if you wish, you may tie a thread to the center, or drive in at the central point of the instrument a long iron, inclined [as the thread is]. With a plate of copper you can ascertain the measure of the angle and elevate the nail as needed. This inclined nail points toward **בך** and makes an angle equal to the complement of the pole's elevation, that is, 55[°]; and with a small protractor divided into 90[°] you divide the diagonal of the world, or the aforesaid thread; for if it passes to the other side of the wall it will meet the northern pole.

You may also drive a long nail at the wall's edge, provided that its tip is at point **ך**, and the gnomon at point **מן** will cast its shadows — the aforesaid gnomon — toward the side **ן** that is to the west, and the sixth morning hour will be seen, [the gnomon] dropping from its tip perpendicular to **דקג**, of the length **אך**, the horizontal diagonal in the foundation. And when the sun rises and reaches point **דק** in the south, the sun will strike the gnomon and the shadow will fall exactly on **ברן**, of the twelfth hour. From there it inclines to point **הן**.

Now in the city of Venice, whose latitude is 45[°], the elevation of the pole and its complement are equal; hence half the zenith diagonal equals half the horizontal diagonal in the foundation of the instruments, since these diagonals bisect the

angles, and the horizontal instrument is divided exactly like the zenith one. Therefore it [the dial] is one and the same for the zenith and the horizon, and by the interchange of the hours you may use it as another instrument — and this is the peculiar excellence of that city and of its latitude. For the horizontal hour you need only set the horizontal instrument on a wall standing perpendicular to the horizon, on the spot mentioned: this it has above the other latitudes that God has favored — may He teach it instruction and majesty forever.

However, in the more northern cities, like Germany and Poland, the zenith diagonal will be larger than the horizontal one; and this instrument is very easy to make, as you have understood, but it shows only six hours, since it is intersected with the horizon at the points where the equator is cut. Therefore you must draw, behind the wall on the north side, half of another circle of the length of half the zenith diagonal in the foundation, and draw on it the fifth and fourth morning hours and the seventh and eighth evening hours. And if you wish, more hours too, since the distances are known to you in the south-side circle, the ones nearest to the fifth and sixth hours. The nail you drive in at point **ה**, raised on line **הז** that is on the wall, making an angle equal to the complement of the pole's elevation — for thus it will be exactly aligned with the diagonal of the world *(see fig. 37)*.

In the southern cities, like Candia, Damascus, and Jerusalem, the horizontal one is likewise larger than the zenith one. Also at point **דן** you may set up a perpendicular gnomon; or you may prepare the instrument as the [shadow-indicator] shows you its complement, and understand. So too in every other place, according to the condition stated and as I have shown you — there is no need to elaborate.

שאלה מ"ד: Problem 44

How, from knowledge of the pole's elevation, we know the intersections of the lines on the horizontal plane and the first zenith.

From knowledge of the pole's elevation we know the measures of the arcs between the cuts of the horizon and the first zenith [as against] the cut of the equator, together with the tables and the manner of their construction in every single horizon, by the reckoning of triangles.

In the zenith diagram *(see fig. 38)*, let circle **אבגד** be the midday circle, **אג** the horizon and its poles **בד**; and through them passes the first zenith circle **בוד**, which intersects the horizon at the points of true rising and setting — at point **ו**, I

mean, at the place where the equator is cut, **שירק** [?] — and let **צ** be the world's diagonal, **צ** the northern pole, **ח** the southern pole.

Now the division of the quarter of the equator **שי** [?] into six equal parts — those that indicate the six hours distant from the midday line by equal amounts on both sides, i.e. morning and evening, at which the shadow of the gnomon set up at point **י** (aligned with **צ**, the world's pole) will be seen in the daily solar motion — you see that the hour-circles, which are like the shadows of the gnomon passing from the poles and dividing the equator equally at every fifteen degrees, will divide circle **אג**, both the horizontal and the twelve [hours] of the zenith, into unequal parts one from another. Hence our wish, by the reckoning of triangles, is to investigate the distances between cut and cut on the horizon and on the zenith, which are the widths of the hours.

For example, suppose we wish to know the distance between **א** and **ל** on the horizon — i.e., between midday and the first hour after midday, or the eleventh hour before midday. In the right-angled triangle **יפ** ,[?] **וזסלע** [?] is 5[?] degrees [recte: 15° — one hour of equator] of the equator, that is, one hour distant from midday by the assumption; and angle **אישׁ** [?] is the complement of the pole's elevation above the horizon. Therefore, [by the rule]: as the radius (*sinus totus*) is to side **א**, so the foundation [base] — we find the [plane / arc] **וי** [?], and its complement is the measure of **וא** [?] sought. Likewise, since you know that arc **יו** [?] of the equator is 60 in that same triangle, you find the distance **יו** [?]; and so for all of them. By this same method you find the distances on the zenith too; for **יפג**, **ידך**, **יוט** [?] are right triangles, and angle **שיט** [?] is the pole's elevation, and the arcs of the equator are known, every distance in it being fifteen degrees, that is, one hour. Hence you find how far hour is from hour on the zenith — understand.

After you have found the distances of the six hours between midday and sunrise, both on the horizon and on the zenith, it is easy to know the remaining twelve hours, whose distances are equal. You draw a large circle, and from the square divided into 90[°] and minutes you take, with the compass, the degrees and minutes the triangles gave you. As you please, you divide it into four parts by two diagonals intersecting at right angles, like **אג**, **בד**, and you begin to mark on this circle from midday according to the distances you found; and so you do, with great ease, for both the horizontal and the zenith. You draw lines through the center of the instrument, set the 12th hour opposite the midday line — on the horizontal a gnomon stands perpendicular to the horizon; drive into the center of the instrument a long gnomon as you please, but on the horizontal it must be raised so far from its center that it makes an angle with the midday line on the

north side equal to the elevation of the northern pivot, so that it may be aligned with the world's diagonal. On the zenith it must be inclined to the wall, making an angle with the line standing perpendicular to the horizon equal to the complement of the pole's elevation, i.e. the distance of the northern pole — which is behind the wall — from the zenith to the wall. For if you reckon the driven-in nail as passing through to behind the wall, the zenith angles at the cuts will be equal, and the measure of both is the distance of the pivot from the zenith. All this follows from the indication of the foregoing problems — understand. So too, by the method of triangles, you investigate the hour-distances at all latitudes, and you prepare for yourself tables from which you will easily set up the instrument according to the width of whatever city you wish.

שאלה מ"ה: Problem 45

How, from knowledge of the pole's elevation, we know the measures of the arcs between the cuts of the horizon and the first zenith [as against] the cut of the equator.

In the aforesaid diagram, we wish to know arc **וכ** [?] between the horizon and the equator at the 11th hour, or the 1st hour after midday. In triangle **יו** [?], arc **ון** [?] is known from the foregoing problem. Therefore, just as the sine of angle **ם** [?] erected [supports] the sine of **יו** [?], so the sine of angle **יי** [?], the elevation of the equator, [stands] to the sine of **וש** [?]; and so for all. By this method, on the zenith: we wish [to find] arc **גם** [?]; in the ratio of **ם** [?], as the whole sine is to the sine of **יך** [?], known from the foregoing problem, so the sine of angle **י** [?], the pole's elevation, [stands] to the sine of **ב** [?], the arc sought; and so for all. These arcs are of great use for the inscription of the twelve signs on the hour-instruments, with which you will find the length of the shadows of the gnomon ✻ at equal and reversed [hours] — as I will inform you in another place and time, where I will teach you all the inclined ones and those traced on the inclined planes.

שאלה מ"ו: Problem 46

How we know the point of mid-heaven (the *medium caeli*) by means of triangles.

The division of the celestial sphere into twelve parts, which the astrologers [astronomers] call the houses of heaven, is done in different ways. The choice and accepted one, called the *via rationalis* [the "rational way"], the intellectual way, is the division devised by the choice scholar, the comprehensive master R. Avraham ibn Ezra, of blessed memory [R. Avraham ibn Ezra, c. 1089–1167]. It is this: one

reckons four great circles, called the circles of position (*circuli positionis*), passing between the poles of the midday circle together with the horizon. These, together with the midday circle, divide the equator into twelve equal parts, while the ecliptic [is divided] into twelve unequal parts. According to his way alone I will tell you a method and means to find the cusps of the houses (*capita domorum*), as Regiomontanus and "Zachuto" [R. Avraham Zacuto] and Alfonso built [in their tables]. But the substance of the matter — to clarify the topic — let us take first the model that Ptolemy brought in Book II, chapter 9 [recte: Almagest II.9], namely: when the sun is at the head of Cancer [the summer solstice], at one hour before noon, on the horizon of [a place] whose width we wish to know, [we find] the celestial position.

Now before anything we must investigate the right ascension of the head of Cancer; and were it not the solstitial reversal, we would have to investigate and compute by the foregoing problem to determine it. But in the example before us we know by the *Sefirah* [the Tables] that a quarter [of the ecliptic] from the head of Aries rises with a quarter of the equator; hence it is 90 degrees. With the 23 hours of yesterday, when converted into parts of the equator, they come to 345; that makes 435 of the equator. Hence, subtracting from their sum 360, there remain 75 — which is the point of the equator on the midday circle, one hour before noon. Now we must investigate the ecliptic point passing through the midday circle at that moment, in this manner, as in the figure of the first problem:

In triangle **יהת**, right-angled at **ת**:

Quantity	Value
ת whole sine (radius)	100000
רהד tangent of complement 75	2679492
יהת sine of complement [23 51 20]	9145679
יך tangent of complement [13 46 10]	2450577
and its complement	76 13 50

[The text concludes:]...arc 75 **דהת** of the equator, and from the row **יך** [we obtain] tangent of complement [...], [the result] being the arc of the ecliptic, and the analogy [proportion] thus. Hence mid-heaven at that moment will be 16° 13′ 50″ in [the sign of] Gemini. Ptolemy found a little less than this, and we have followed his tables like him, in chapter 9 [Almagest II.9]: 12, 16 only.

שאלה מ"ז: Problem 47

How we lay out the figure of the twelve houses of heaven, as is fitting, by the device of triangles.

After the point of mid-heaven has become known to you — which is the beginning of the tenth house, i.e. the cut of the ecliptic with the midday circle — we seek a straight way to know all the other houses. To clarify the matter, let the zenith diagram be the appearance which one observes; and it changes according to the models. The one suited to our example is this, according to the [present] view:

In it let **אב** be the midday circle, **אפ** [?] the horizon, **שן** [?] the half-equator, **צב** the world's diagonal, **הל** [?] the half-ecliptic, **ם** [?] the point of Libra, **מ** [?] [the point of] Aries, and the equator **שן** [?] will be divided into six equal parts — three above, on the horizon, and three below, at points **וק יחזש** [?]. Through those cuts will pass the circles of position between the poles of the horizon and the midday circle, and they will divide the ecliptic **הל** [?] into twelve parts together with the midday circle. Thus: the midday circle cuts the ecliptic at point **ח** [?], where the beginning of the tenth house is; and the circle of position **אקפ** [?] cuts the ecliptic at point **ד**, where the beginning of the eleventh house is; and the circle of position **אצפ** [?] cuts it at point **ג**, the beginning of the twelfth house; and the horizon cuts the ecliptic at point **ו**, which is the beginning of the first house; and the circle of position **אפ** [?] cuts the ecliptic at point **ה** [?] [recte: ז], the beginning of the second house.

[Text breaks off; continues on the following folio.]

[Continuation of Problem 47 — laying out the twelve astrological houses by triangles.]

Likewise the circle of position **אף** [af] cuts the ecliptic at point **ר** [r], the beginning of the third house; and likewise the midday circle [half-day circle] cuts the ecliptic at point **ל** [l], the beginning of the fourth house. In this way you set out the half-circles of position below.

Now you understand the rest of what corresponds to these. The whole labor consists merely in knowing the *six* houses, for on the sphere the ecliptic is cut at points diametrically opposite the ones already mentioned, along the great circles. Therefore from one [side you obtain the other].

Problem 48 [שאלה מ"ח]: How do we find the point of the ascendant sign by means of triangles?

Now that you understand the structure of this configuration: if you wish to know the beginning of the first house — called the ascendant sign [the rising sign] — you must know, in the oblique triangle, the measure of the arc **מן** [mem-nun] of the ecliptic from the head of Libra to **ימ** [yod-mem], which is 15 degrees. Thus **ימ** is the known side of the triangle from **ימ**, and the angles around the aforesaid side are likewise known. For **ימ** is the measure of [the arc] **אש** [the arc that the perpendicular to the pole at Rhodes subtends], 54 [Margin: ליור לט]; and the angle **נמי** [nun-mem-yod] is also the measure of the great obliquity, 23°.51′.20″. Therefore, from Column 8, Page 3, of *Sod HaYesod* [The Foundation's Secret], the side **מנ** [mem-nun] is found; and the order of the search is as follows: in triangle **למז** [lamed-mem-zayin] set up for **למנ** [lamed-mem-nun], as in the figure, we draw down from point **ל** [l] the perpendicular **ל**, and extend it to **מכו** [mem-kaf-vav], and find what is sought in this manner.

First we seek **לם** [lamed-mem], the perpendicular, and the proportion is thus: in the right triangle with the angle **למז** [lamed-mem-zayin], from Column 11, Page 1:

Term	Value	Sine/Tangent	Sexagesimal
ם [m]	100000 0 0	—	—
לן [lamed-nun]	2588190	sine	15°
ז [z]	4044323	sine	23°.51′.2″
לם [lamed-mem]	1046748	sine	6°.0′.30″

Second, in the aforesaid triangle we seek the side **מן** [mem-nun]; and from Column 15, Page 1, the proportion is thus.

Or, from Column 12, Page 1, the proportion is thus:

Term	Value		Term	Value	Sexagesimal
ם [m]	100000		**ם** [m]	100000	—
ז [z]	226136	tangent of complement 20 51 23	**ז** [z]	109341	secant of the obliquity
סן [samekh-nun]	10525	tangent	6°.0′.20″	**לן** [lamed-nun]	373205
סז [samekh-	2380	sine	13°.46′.	**סן**	408066

Term	Value	Term	Value	Sexagesimal
zayin]	0 0		[samekh-nun]	
and its complement	46°.13′			

Third, in the aforesaid triangle [Margin: ליור מ] we seek the angle **סלז** [samekh-lamed-zayin]; and from Column 1, Page 1, the proportion is thus:

Term	Value		Sexagesimal
ם [m]	10 0000		—
סל [samekh-lamed]	955356	tangent of complement	30° 0′ 6″ [recte: 30°.0′]
סן [samekh-nun]	24500	sine	46°.13′.
סלז [samekh-lamed-zayin]	234062	tangent	52°.66′. [recte: minutes 66 exceeds 60 — source-corrupt; ≈ 12.52?]

And when we subtract from it the angle **זלם** [zayin-lamed-mem], there remains the angle **סלם** [samekh-lamed-mem] = 52°.12′ [Margin: tangent 234062; cf. **סלז** 52]:

Term	Value		Sexagesimal
סלם [samekh-lamed-mem]	—	tangent	52°.12′.
לם [lamed-mem]	—	tangent	1°.22′. [partial]

When **ם** [perpendicular] is set up [for] **לם** [lamed-mem], we seek the side **סם** [samekh-mem]; and from Column 13, Page 1, the proportion is thus:

Term	Value		Sexagesimal
ם [m]	100000		—
סלם [samekh-lamed-mem]	22841	tangent	12°.52′.
לם [lamed-mem]	10467	sine	6°.0′.30″

Term	Value		Sexagesimal
סם [samekh-mem]	2390	tangent	1°.22′.

When we subtract **סם** [samekh-mem] from **סז** [samekh-zayin] there remains **זם** [zayin-mem] = 24°12′ [recte: ≈ 24°24′? — see below], the measure of the side, and it is equal to side **מ** [m] in the figure. Therefore subtract 24°12′ [recte: 24°24′?] from **נמ** [nun-mem], half the ecliptic of **טמ** [tet-mem], and there remains the arc 167°36′ = **נמ** from the head of Aries. Or, subtract [the result] backward from the head of Libra, and there remains 17°.36′ in Virgo — and that is the rising point [ascendant] at that very moment.

In the horizon of Rhodes, Ptolemy found 17°47′; and by the tables of Regiomontanus you will find 17°35′ approximately.

In this last triangle we have **נ** known quantities, in such a way that, since the equator-point at mid-heaven [the right ascension of the medium caeli] is 75 degrees — if you add to them a quadrant, namely 90, they will total 165, which is the whole arc **ימן** [yod-mem-nun]; therefore the remainder [...]

[**Page 284** (folio 282 verso). Conclusion of Problem 48.]

[...] from Column [...] subtract; and more concisely, by means of the figure itself, find the perpendicular **מד** [mem-dalet] from Column 11, and afterward the angle **דמ** [dalet-mem] from the third [side] from Column 6, [whence] you find the surface [planar value], the side sought, with the obliquity-angle subtracted [recte: read as marked]:

Here you see that I found the beginning of the first house from knowing the elevation of the pole above the horizon — and since the horizon is fundamental [is the base of the table of the pole's elevation above the circles of position] in indicating the first house, in place of the circle of position. Therefore, when we know the elevation of the pole above each circle [of position], we know — by the aforesaid method — the beginning of all the houses. Hence our effort will be to investigate first this elevation, [namely the elevation above] the circle of position that is reckoned as the horizon, for a circle of position is nothing other than a movable horizon.

Problem 49 [שאלה מ"ט]: [The foundation — the table of the pole's elevation above the circles of position.]

The foundation [יסוד] of the table of the pole's elevation above the circles of position [is as follows].

Now you have seen that we found the beginning of the first house in the place of the circle of position. We set the foot of the compass on the beginnings of the houses, at the place where the ecliptic is cut by the circles of position — that is, at points [where they intersect] — and we extend its [other] foot to the pole **צ** [tzadi], and we mark off arcs of circles descending from the northern pole **צ** [tzadi] onto the circles of position; and they are **צת** [tsadi-tav], **צכ** [tsadi-kaf], and likewise at points [...] up to **ב** [b], the southern axis. And we mark off arcs **עת** [ayin-tav], **בכ** [bet-kaf], and these are the measures of the elevations of the poles **א** [a] above the circles of position. And so in the triangles **א** [a], so that the angle [...] **ץ** [tsadi-final] is right [a right angle], if you wish to investigate the measure of the angles **ץק** [tsadi-final-kof], which are the complements of **צת** [tsadi-tav], **צכ** [tsadi-kaf]; and from Column 1, Page 1, the proportion will be thus:

Term	Value		Sexagesimal	Term	Value		Sexagesimal
ל [l]	10000		—	**ו** [v]	10000		—
וץ [vav-tsadi-final]	20000	tangent of complement	60°	**וק** [vav-kof]	11547	tangent of complement	30°
ופ [vav-pe]	13763	tangent	54 [elevation of equator/equinoctial]	**ופ** [vav-pe]	13763	tangent	54
וזם [vav-zayin-mem]	275	tangent	70°.2′.	**וקפ** [vav-kof-pe]	158	tangent	49°.57′.

Term	Value	Sexagesimal	Term	Value	Sexagesimal
	276			930	

And its complement [is] **צת** [tsadi-tav] 11°32′, the elevation of pole **צ** [tzadi] above the circle of position **אקף** [alef-kof-pe] of the twelfth house, and it equals arc **דע** [dalet-ayin] of the second house — and so they are with Regiomontanus.

And its complement [is] **צכ** [tsadi-kaf] 19°58′, the elevation of pole **צ** [tzadi] above the circle of position **אף** [alef-pe] of the eleventh house; and likewise arc **ד** [dalet] is the elevation of the southern pole above circle **ארף** [alef-resh-pe] of the third house.

You see in the figure that the rising-arcs **וץ** [vav-tsadi-final] and **וק** [vav-kof], the oblique ascensions, are equal in every horizon, in that they share the divisions of the equator [...] as stated above: when you add to the right ascension the point of mid-heaven, 30, you find the oblique ascension of the eleventh house; and if you add 60, you find the oblique ascension of the twelfth house — and this is a fixed rule that never changes. Likewise, if you add 90 you find the oblique ascension of the ascendant; if you add 120 you find that of the second house; if 150, of the third; and if 180, of the fourth, which is on the midday circle.

Therefore the right [ascension] and the fourth [house] are equal in ascension with mid-heaven, in that theirs is a right ascension; and likewise the twelfth with the second [house] in oblique [ascension], since their remainder to 120 is 60, and their sines are equal; and likewise the eleventh with the third, whose sines are [equal]. Thus it does not change by the aforesaid proportion, except [for] the arc **ות** [vav-tav] according to the variation of its elevation in [different] horizons, with the addition and the subtraction. As [explained] in *Sefer Sod HaYesod* or in Problem 1, you will set up a table of the pole's elevation above the circles of position, just as Regiomontanus did — and this is easily understood.

Problem 50 [שאלה נ']: How do we find the beginning of all the remaining houses by means of triangles?

Now that all the foregoing is known to us, we investigate **the second house** in the oblique triangle **מ ף ן** [mem-pe-nun], in which the angle **מ** [m] is the great obliquity 23°51′ [recte: 23°51′ — minute units; reads "5"], and the angle **ף** [pe] is 49°57′, that is, the complement of **ב** [bet] 11°32′. The side between the angles **מ ף** is 51° [recte: read as marked] (a thing self-evident, since every arc **ף** [pe] is 30 degrees, and arc **ומ** [vav-mem], as we explained above, was 15 degrees). Therefore, from Column 8, Page 3, of *Sod HaYesod*, the second side **מן** [mem-nun] is also found = 50°.42′ [recte: read as marked].

And the order of the proportion is thus:

(a) In the right-angled triangle **מדף** [mem-dalet-pe] with the right angle, from Column 11 [side] **ף** [pe], from Column 4, Page 1, we seek the angle [and] find the perpendicular **מ** [m] in this way:

Term	Value		Sexagesimal
ם [m]	100000		—
מף [mem-pe]	2752	sine	15°
תף [tav-pe]	5849	sine	[...] 87′. [recte: minutes exceed 60 — source-corrupt]
כו [kaf-vav]	153	sine	9°. [partial]

(b) From Column 4, Page 1, we find the angle **דמ** [dalet-mem] thus:

Term	Value		Sexagesimal
ר [r]	100000		—
ף [pe]	9	tangent	9°.97′. [recte: minutes exceed 60 — source-corrupt]
מף [mem-pe]	2	sine complement	15°
[result]	153483	tangent of complement	5°.33′.

And when we subtract from it the angle **ימו** [pe-mem-vav] 8°24′, there remains the angle **דמו** [dalet-mem-vav] 14°9′.

(c) In the right-angled triangle **מקדן** [mem-kof-dalet-nun] [recte: **מקדן**] we seek the planar value **מם** [mem-mem], from Column 10:

Term	Value		Sexagesimal
ר [r]	100000		—
מך [mem-kaf]	7	tangent of complement	39°.[?] [recte: read as marked]
דמו [dalet-mem-vav]	4	sine of complement	14°.9′.
מו [mem-vav]	2	sine of complement	12°.50′.

Therefore the beginning of the second house is in Libra 12°.5′; and so it is found by the tables of Regiomontanus.

[Restated more legibly from the column tables:]

Term	Value		Sexagesimal
ר [r]	100000		—
מף [mem-pe]	25882	sine	15°
ת [tav]	84635	sine	49°.57′.
כד [kaf-dalet]	21905	sine	39°.12′. [recte: read as marked]

(b) From Column 4, Page 1, first proportion, we seek angle **ימד** [pe-mem-dalet] in this way:

Term	Value		Sexagesimal
ר [r]	100000		—
ף [pe]	158899	tangent	49°.57′.
מף [mem-pe]	96592	sine of complement	15°.
כד [kaf-dalet]	153483	tangent of complement	5°33′

And when we subtract from it angle **ימו** [pe-mem-vav] 8°24′ [recte: read as marked], there remains angle **דמו** [dalet-mem-vav] 14°9′.

(c) In the right-angled triangle **מקדן** [mem-kof-dalet-nun] we seek the planar value **מו** [mem-vav] from Column 10, thus:

Term	Value		Sexagesimal
ר [r]	100000		—
מד [mem-dalet]	44554	sine of complement	12°.39′.

Term	Value		Sexagesimal
	7		
דמו [dalet-mem-vav]	98704	sine of complement	9°.14′.
מו [mem-vav]	438772	sine of complement	12°.50′.

Therefore the beginning of the second house is in Libra 12°.5′; and so it is found by the tables of Regiomontanus.

Likewise we investigate **the third house** in the oblique triangle **מרן** [mem-resh-nun], where angle **מ** [m] is 23°51′ and angle **ן** [nun] is 70°2′, the complement of **מכ** [mem-kaf] 19°58′; and the side between them **מן** [mem-nun] is 45°. Therefore, from Column 8, Page 3, the second side **מר** [mem-resh] is found = [...].42′. — i.e. 11°.42′. into Scorpio (44° [recte: read as marked] from the head of Libra). And so [it is found] by the tables of Regiomontanus. The order of the proportion is thus:

(a) In the right-angled triangle **מדן** [mem-dalet-nun] from Column 11, [from] Column 4, we find the angle **דמן** [dalet-mem-nun]; we find the perpendicular **מ** [m] thus:

Term	Value		Sexagesimal	Term	Value		Sexagesimal
ר [r]	1000000		—	**ר** [r]	1000000		—
מן [mem-nun]	70710	sine	45°	**מן** [mem-nun]	70710	sine	45°.
ן [nun]	939989	sine	70°.2′.	**ן** [nun]	275246	tangent	70°.2′.
מד [mem-dalet]	664459	sine	41°.39′.	**מן** [mem-nun]	70710	sine of complement	45°.
				דמן [dalet-mem-	19	tangent of	27°.11′.

Term	Value	Sexagesimal	Term	Value		Sexagesimal
			nun]	4626	complement	

And from this we subtract angle **רמן** [resh-mem-nun] 23°.51′, and there remains angle **דמר** [dalet-mem-resh] 3°.20′.

(c) In triangle **מדר** [mem-dalet-resh] we seek the planar value **מר** [mem-resh] from Column 10, thus:

Term	Value		Sexagesimal
ר [r]	100000		—
מד [mem-dalet]	112435	sine of complement	41°.39′.
דמר [dalet-mem-resh]	998301	sine of complement	3°.2′.
מר [mem-resh]	112244	sine of complement	41°.42′.

Likewise we investigate **the twelfth house** in the oblique triangle **מגן** [mem-gimel-nun]: thus **מ** [m] = 23°.51′, **ץ** [tsadi-final] = 49°.57′, the complement **צב** [tsadi-bet] 11°32′; and the side **מץ** [mem-tsadi-final] = 15°. Therefore, from Column 8, Page 3, the side **מג** [mem-gimel] = 15° is found. And the investigation is thus:

(a) In the right-angled triangle **מדץ** [mem-dalet-tsadi-final], from Column 11 [find] the perpendicular **מד** [mem-dalet]; from Column **ב** [recte: Column 4] find angle **דמץ** [dalet-mem-tsadi-final], from Column 4:

Term	Value	Sexagesimal	Term	Value	Sexagesimal
ר [r]	10000	—	**ר** [r]	10000	—

Term	Value		Sexagesimal	Term	Value		Sexagesimal
	00				00		
מץ [mem-tsadi-final]	7071 0	sine	45° [recte: read as marked]	**ץ** [tsadi-final]	15889 9	tangent	49°.57′.
ץ [tsadi-final]	84635	sine	49°.57′.	**מץ** [mem-tsadi-final]	70710	sine of complement	45°.
מד [mem-dalet]	59845	sine	36°46′	**דמץ** [dalet-mem-tsadi-final]	11235 7	tangent of complement	41°.40′.

And we subtract from this **גמץ** [gimel-mem-tsadi-final] 23°51′, [and] there remains angle **דמג** [dalet-mem-gimel] 17°59′.

[**Page 286** (folio 283 verso). Conclusion of Problem 50 — the twelfth house; close of Chukos Shamayim's house-division; and the start of Problem 51.]

(c) In triangle **מדג** [mem-dalet-gimel] we find the planar value **מג** [mem-gimel] from Column 10, thus:

Term	Value		Sexagesimal
ר [r]	100000		—
מד [mem-dalet]	133835	sine of complement	36°.46′.
דמג [dalet-mem-gimel]	95115	sine of complement	17°.59′.

Term	Value		Sexagesimal
מג [mem-gimel]	127297	sine of complement	38°.10′.

And when we subtract it from **קף** [kof-pe = 180], that is, from the whole arc of the ecliptic **כמט** [kaf-mem-tet], there remains 141°.50′, which is 21°.50′ into Leo. And by the tables of Regiomontanus it is found [as] 49°.21′ [recte: read as marked]; and likewise in the *Sefer HaMinisti* [Almagest].

Likewise we investigate **the eleventh house** in the oblique triangle **מק** [mem-kof], whose **מק** [mem-kof] = 23°.51′, and angle **ק** [kof] = 70°.2′, which is the complement of **צב** [tsadi-bet] 19°58′; and side **מזק** [mem-zayin-kof] = 75°; and from Column 8, Page [3] [recte: read as marked] we find side **מק** [mem-kof]. The order is thus:

(a) In the right-angled triangle **מדקן** [mem-dalet-kof-nun] [recte: **מדקן**] we find perpendicular **מד** [mem-dalet] from Column 11; **(b)** we find **דמ** [dalet-mem] from Column 4:

Term	Value		Sexagesimal	Term	Value		Sexagesimal
ר [r]	1000000		—	**ר** [r]	1000000		—
מק [mem-kof]	965592	sine	75°	**ק** [kof]	2752 46	tangent	70°.2′.
ק [kof]	939889	sine	70°.2′.	**מק** [mem-kof]	25882	sine of complement	75°.
מד [mem-dalet]	907855	sine	65°.13′.	**דמק** [dalet-mem-kof]	71239	sine of complement	54°.32′.

And when we subtract from this 23°.51′, [from] angle **המק** [he-mem-kof] there remains angle **המד** [he-mem-dalet] 30°.41′.

(c) In triangle **מדה** [mem-dalet-he] we find the planar value **המ** [he-mem] from Column 10:

Term	Value		Sexagesimal
ר [r]	10000		—
מד [mem-dalet]	46171	sine of complement	65°.13′.
המד [he-mem-dalet]	86000	sine of complement	30°.41′.
המ [he-mem]	39707	sine of complement	68°.20′.

And when we subtract it from **ממ** [mem-mem = 180], there remains 111°40′, which is 21°40′ into Cancer; and by the tables of Regiomontanus it is found [as] 21°36′ into Cancer.

So scrutinize carefully — lest some error has fallen into my reckoning, for I was not able to go back and verify [it] precisely [as to] the signs, by the degrees and minutes. But everything is set out before you with [its] reasons and proofs. [Margin: ליור מא] Here, then, is the figure of the heavens for the aforesaid moment; and the six houses are the diametrically opposite [ones].

Problem 51 [שאלה נ"א]: How is the directrix [the significator / promissor] led when it is on the midday circle or on the actual horizon?

The entire foundation of the judgments of the astrologers [the diviners of nativities] depends upon the *directions* [נהוגים], whereby they predict the time of future events to come. And inasmuch as you wish to know everything without the aid of tables and instruments, I will inform you briefly of one of the many methods that I have, and the matter is not so burdensome — if you recall the lessons that have passed; for behold, from Problems 1 and 2 you will know the obliquity [declination] of a star, and from Problem 3 its right ascension, and from Problem 13 the difference between the right ascensions and the oblique ascensions in every horizon; and likewise from it [Problem 13], and from Problem 9, [you will know] the semidiurnal arc and the seminocturnal arc of every star. These are the [quantities] needed for the directions.

My intent is the practice of investigating the portion of the equator that rises or sets with a given star, until it reaches a given place. **The significator [המורה]** is

what the astrologers call the place [of nativity] in the figure, which is the first or the antecedent in the order of the signs, and indicates a given matter — such as life, or wealth, and so on. **The promissor [הנודר]** they call the second place, which indicates a sickness, or some good and honor; for when they are aligned — by the motion of the *primum mobile* [the first moved sphere] — until the promissor reaches the place of the significator, that will be the time of the event. The measure of the direction is reckoned by the portion of the equator that rises or sets with it in the order of the signs, in the horizon of the significator — that is, according to the elevation of the pole above the circle of position in which the significator [lies]. For each degree of the equator is reckoned as a year, or [as] every 59 minutes [recte: read as marked], according to the sun's motion in its sphere at that time, in the opinion of others.

[Continues into the next problem.]

and there is another method employed by the Arabs, working with their proportional parts, while Ptolemy operates from the setting point. Many astronomers, moreover, work between the two — that is, they reckon that the significator comes to the place of the promissor by the diurnal motion. This is called direction *contrary to* the order of the signs.

What follows from our discussion is that the arc of the equator just mentioned is determined by means of the ascensions of the significator and of the promissor, and the reckoning of both always rests upon the elevation of the pole on the circle of position of the significator in *direct* direction, and conversely upon that of the promissor in *converse* direction. Now, when the significator is near the meridian — that is, when it is at the tenth house itself, or at the fourth — subtract its right ascension from the right ascension of the promissor; and if necessary, add 360 degrees to it. The result is the desired arc of direction, since the meridian is then the place of the circle of position of the significator.

But if the significator is near the ascendant — that is, the horizon, which is then the place of the circle of position of the significator — subtract the [oblique] ascension of the significator from that of the promissor, and you obtain the exact point of rising. This applies to the oblique ascensions, whether of the significator or of the promissor, computed according to the elevation of the pole on the arc of direction. And if the significator is at the western horizon — the very setting point — compute, by the latitude of the locality, the oblique descensions, whether of the significator or of the promissor, and subtract that of the significator from that of the promissor; you will obtain the arc of direction. This is in place of the descensions sought. The oblique ascensions of the opposite points of the

significator and of the promissor are to be taken only according to their [proper] latitude — that is, if the body is at the head of Cancer and its latitude is southern, take the head of Capricorn with northern latitude; subtract descension from descension, and you obtain the arc sought.

As for directions *contrary to* the order of the signs, all of these may be computed by means of the descensions at setting: namely, by the measure of equator that descends beneath the horizon at the moment the significator comes to the place of the promissor by the diurnal motion — such is the measure of the arc of direction. In this direction the ascension of the promissor is subtracted from the ascension of the significator. This [equals] the latter [point] and is prior in respect of longitude. I cannot expand further or supply illustrations, for lack of time; these matters are easy for those who understand.

Problem 52 [שאלה נ"ב]

On directions, when the significators are out of the daytime [above the horizon].

To understand what is here stated, three things must be known: (1) the position of the significator and the promissor — whether above or below the horizon; (2) likewise its distance from the meridian or from the midnight circle, and whether it is in the half [of the heavens] that is rising or in the descending half.

(1) When the star is from the beginning of the seventh house, by way of the tenth house, up to the end of the twelfth house, it is above the horizon; and when it is in the remaining houses, it is below the earth.

(2) To know its distance: if it lies between the first house and the fourth, subtract its right ascension from the right ascension of the fourth, and you obtain its distance from the fourth. If it lies between the fourth and the seventh, subtract the right ascension of the fourth from its own right ascension, and you obtain its distance from the fourth. If it lies between the seventh and the tenth, subtract its right ascension from the right ascension of the tenth, and you obtain its distance from midheaven. And if it lies between the tenth and the ascendant, subtract the right ascension of the tenth from its own right ascension, and you obtain its distance from midheaven.

Now in these two operations [the second one] requires that the result be in those degrees that are oblique [ascensional] — yet in the body [of the star] this is not strictly so in truth. To investigate this, find the arc of the star's semidiurnal [time], that is, according to truth — according to its longitude and latitude in its true

position — for it will not appear as the investigation determines it, on account of the obliquity. If its declination is northern, add it to 90; if southern, subtract it. And when you know the arc of its semidiurnal and seminocturnal [times], keep these operations in mind: for if the star is seen at the end of the twelfth house and above the horizon, except that its latitude is southern, there is doubt about its position. Therefore, if its distance from the meridian is less than the arc of its semidiurnal [time], it is above the horizon; and if greater, it is below. Likewise, if it is seen a little beneath the horizon — that is, after the ascendant — except that its latitude is northern, see whether its distance from the fourth is less than the arc of its seminocturnal [time]: it is certainly beneath the horizon; and if greater, above it. And if...

the arc of the distances and the semidiurnal and seminocturnal [times] are equal, know that the star is exactly on the horizon, and is reckoned as the rising or setting point — or the reverse, as you have learned. As from its distance you discern its position, so from its position you may discern [its distance].

From [the head of] the tenth to the head of the fourth, by way of the ascendant, is called the "rising half"; and from the head of the fourth, by way of the seventh, to the head of the tenth, is called the "descending half." When the star rises in the rising half, the directions proceed upon the right and oblique ascensions; and in the descending half, by means of the descensions or ascensions of the opposite points, as I taught you in the previous Problem.

Now let us set forth in the figure of Problem 48 the case in which the significator is in the descending half — that is, in the sixth house, opposite the twelfth — and it is at point **ז** [zayin]. It is therefore on the circle **א ק ר ץ פ** [aleph-kuf-resh-tsadi-pe]. All our concern is to know the measure of arc **צ ץ** [tsadi-final-tsadi], the elevation of the pole on the circle of position of the significator just mentioned.

Now in the oblique triangle **א צ ץ** [aleph-tsadi-final-tsadi], side **צ ץ** [tzadi–final-tsadi] is the complement of the declination of the significator, and side **א צ** [aleph-tsadi] is the elevation of the pole above the meridian, and angle **ץ** [final-tsadi] between them is the measure of the arc there — namely, the distance of the significator from the midnight circle. This is known: when the right ascension of the point **ש** [shin] — which is point **ש** [shin] — is subtracted... point **ש** [shin], which is **מ ט** [mem-tet], whose right ascension proceeds [from] point **ח** [chet]. By [Euclid] V, proposition 3, the remainder is found, side **א צ** [aleph-tsadi], the elevation of the pole above the circle of position; and upon this elevation we compute the oblique descensions, whether of the significator or of the promissor. When that of the significator is subtracted from that of the promissor, the

remainder is the arc of direction. By this method Regiomontanus worked [ריניאומונטני, Regiomontanus]; and they kindly granted us [these] words.

Problem 53 [שאלה נ"ג]

Said Yashar [R. Yosef Shlomo Delmedigo]: I have seen fit to bring you, at the end of this Book, a table of the longitudes and latitudes of the famous cities, which you greatly need. And although the [various] authorities do not agree on these, nevertheless I have chosen from the books of the moderns what seemed to me closest to the truth; and here it is for you.

Table of longitudes and latitudes of the famous cities.

[The table occupies pp. 288–290. Each page carries two table-columns side by side; the columns read right-to-left in the source. The source headers print **רוחב** ("latitude") over the left pair of figures and **אורך** ("longitude") over the right pair, each subdivided into **מעלות** (degrees) and **דקים** (minutes); but the data in each row are printed in the order **longitude-degrees, longitude-minutes, latitude-degrees, latitude-minutes** (left to right), which is how they are mapped below. Values transcribed as printed.]

City	Long. °	Long. ′	Lat. °	Lat. ′
Babylon the Great of the Chaldees, called [בבל הגדולה של כשדיי הנקרא — section caption, no values]				
Aden, the great fair of [Arabia] Felix [אדין יריד גדול בארבייאס פיליני]	83	0	13	0
Adrianople in Bulgaria [אנדרינופולי שבבולגריאה]	52	30	42	45
Mount Athos, a good mountain in Sicily [?] [הר איהנה — Mt. Etna?]	59	0	38	20
Ur of the Chaldees, the birthplace of Abraham [אור כשדים מולדתו של א״א]	78	30	32	40
No-Amon... Alexandria of Egypt [נא אמון... אלכסנדריאה של מצרים]	60	30	30	0

City	Long. °	Long. ′	Lat. °	Lat. ′
Alexandretta in Syria [אלכסנדריטה שבסיריאה]	68	32	37	54
Alexandria of Italy [אלכסנדריאה של איטאליאה]	30	0	43	30
Alnir of the Berbery [recte: Algiers, Barbary] [אלניר של ברבריאה]	22	0	32	30
Alnir of Sardinia [אלניר שבסרדיניאה]	30	20	38	0
Smyrna [?] [אמבירנה שבפס — recte: Izmir?]	32	40	49	26
Amsterdam [אמשטרדאם]	27	34	52	40
Ancona [אנקונה]	38	30	43	40

Left-hand column of p. 288:

City	Long. °	Long. ′	Lat. °	Lat. ′
Angora [Ankara] [אנגורה]	62	20	42	30
Antioch, called Tahuru [?] [אנטיאוכיאה שנקרא טאורו]	70	15	37	20
Antwerp, called Anvers [אנטוארפיאה, הנקרא אנווירס]	24	30	51	48
Urbino [?] [אורינו / אודינו — Udine?]	35	0	46	30
Argentina [Strasbourg] [ארגינטינה]	20	50	48	44
Ormuz, called Hormuz [ארמוזה הנקרא הורמוז]	95	57	27	24
Ashkelon [אשקלון]	67	20	32	27
Athens [recte], formerly of the wise men [אחינש, לפנים בית החכמים]	52	45	37	15
Urbino [אורבינום]	34	10	43	4
Avignon [אויניון]	23	0	43	52
Augusta [Augsburg] [אאונוסטה]	32	30	48	20

City	Long. °	Long. ′	Lat. °	Lat. ′
El-Cairo of Egypt, called Babylon the Lesser [אלקהרא של מצריים הנקרא בבל הנדולה — recte: בבל הקטנה]	64	30	29	40

Right-hand column of p. 289:

City	Long. °	Long. ′	Lat. °	Lat. ′
Babylon the Great of the Chaldees, called [בבל הגדולה של כשדיי הנקרא — caption]				
Baghdad [בנדאט]	83	10	33	50
Bayonne in France [באיונה של צרפת]	17	30	42	50
Barcelona [ברצלונה]	17	15	41	36
Basel [באסיליאה]	28	0	47	30
Belgrade [recte: Bilbao?] [בילגראו]	45	0	47	40
Bethlehem [בית לחם]	65	45	31	50
Bologna of Italy [בולוניאה של איטליאה]	33	30	44	16
Brescia in Italy [בריקסיאה באיטליאה]	32	30	44	30
Bruges in Brabant [ברוקסילה בברבנטי — recte: Brussels?]	26	42	51	24
Buda, called Ofen, in Hungary [בודה הנקרא בודון באונגריאה]	42	0	47	0
Bordeaux, called Burdigala [בורדינאלה הנקרא בורדיהום]	18	0	44	30

City	Long. °	Long. ′	Lat. °	Lat. ′
Brussels, or Ratisbon [?] [בריסלה או רטיסלאויאס]	40	0	51	10
Genoa in Italy [נינואה באיטליאה]	30	0	43	50
Granada in Spain [גראנטה בספרד]	11	0	37	50
Groningen in Frisia [גרונינגנה בפריזיאס]	29	24	53	16
Damascus of Aram [דמשק של ארם]	69	0	33	0
Danzig in Prussia [דאנטזיק בפרוסיאס]	45	0	54	50
Hamburg [המבורנום]	33	0	54	30
Aleppo, called Haleb [הליפו הנקרא חלב]	70	73 [recte:? perhaps 70 13 or 71 0]	38	0
Valencia in Spain [ולינציאה בספרד]	14	0	39	30
Valenciennes in France [ולינציאה שבצרפת]	23	0	44	30
Valladolid [recte] [וואלה רוליט — Valladolid]	10	10	44	0
Venice in Italy [ויניציאה הנקרא]	34	0	45	0
Verona [וירונה]	33	0	44	0
Vicenza [ויצינציאה]	32	10	44	30
Vienna in Austria [ויאינה שבאוסטריאה]	37	45	48	20
Vilna in the realm of Lithuania, of Russia [וילנה בתדינת ליטס רחם — recte: במדינת	52	0	53	30

City	Long. °	Long. ′	Lat. °	Lat. ′
ליטא]				
Wittenberg in Germany [וויטימבירגה מדרם — recte: בדרם/באשכנז?]	37	30	51	50
Troy [recte], in Asia [טרויפולי באשור — recte: טרויה? "in Assyria"]	68	10	35	40
Toledo in Spain, by which the tables were composed [טולידו בספרד שבו נכנו לוחות אלפונסו — Alphonsine Tables]	9	40	37	50
Trebizond [טרפיזונדה]	71	0	44	3
Trient [Trento] [טרינטו]	33	0	32	30 [recte:? lat too low for Trento]

Left-hand column of p. 289:

City	Long. °	Long. ′	Lat. °	Lat. ′
Jerusalem, may it be rebuilt and reestablished [ירושלים תו״ג]	66	0	31	30
Lvov [Lwów], called Lyopolis, and Lemberg, in the realm of [Poland] [לבוב הנקרא ליאופוליס ולעמבורג]	[—]	[—]	[—]	[—]
Brussels [ברוסילאס]	43	15	50	30
Livorno [ליוורנום]	33	30	42	30
Leipzig [ליפסיאה]	30	30	52	20
Lisbon in Portugal	5	10	39	38

City	Long. °	Long. ′	Lat. °	Lat. ′
[לישבונה רחם פורטוגאל]				
London in England [לונדריש רחם אנגליאס]	20	0	52	30
Louvain [לוואניאום]	20	36	51	0
Lübeck [לוביקה]	31	20	54	48
Lucca [לוקה]	33	0	43	30
Lyon in France [ליאון בצרפם]	23	15	45	10
Mainz [Mayence], called Mintz [מאננציאה הנקרא מינץ]	27	30	50	30
Mantua [מנטואה]	32	45	44	30
Mecca [מיקה]	65	36	29	20
Milan [מילאן]	31	0	45	6
Messina [מיסינה]	40	30	38	30
Moscow, a great royal city in Russia [מוסקוויאה רחם משך עיר גדולה]	75	10	61	15 [recte:? lat too far north for Moscow]
Modena [מורינה]	32	40	44	0
Naples [נאפוליס]	39	30	41	0
Negroponte [Euboea] [נינרופונטוס]	53	40	38	15
Nineveh of Asshur [ננוה של יונס — recte: של אשור?]	78	0	36	4
Nice [ניצה]	23	0	44	30
Nuremberg in Germany [נורימבירגה אם האומניות]	31	30	49	30
Zamora [?], the seat of the Lord Crassus [?] [סמורה מושב שר רחם — uncertain gloss]	8	0	49	5
Poix [?] in France [פויאה מדרם — recte: בצרפת]	31	0	44	50

City	Long. °	Long. ′	Lat. °	Lat. ′
Parma [פרמה]	32	30	43	30
Perugia [?] in France [פריזיאי רחם — recte: Paris? "Pariziai"]	24	30	48	40
Padua in France [recte: in Italy] [פדובה מדרם בחכמה — recte: חכמה"ב", a seat of learning]	32	50	44	50
Perpignan, a city of Galenus [?] [פירנאמוש עיר גלינום — uncertain]	56	50	41	8
Pisa in France [recte: in Italy] [פיזה מדרם]	33	0	43	30
Florence [?] in France [פירוזה מדרם — recte: Firenze in Italy?]	35	20	42	56
Foix [?] in France [פיוה מדרם]	33	30	43	0
Piacenza [?] [פיואדו / פיזארו — Pesaro?]	35	20	43	45
Piacenza [פלאצינציאה]	31	50	44	0
Pomerania, a great city of Poland [פוונאניאה בפולן הנדולה רחם — recte: Posnania/Poznań?]	42	0	52	45
Frank[furt]?, a city of Fez [?] [פראנה רחם פהס — uncertain, "city of Fez"?]	39	15	50	10

Right-hand column of p. 290:

City	Long. °	Long. ′	Lat. °	Lat. ′
Famagusta in Cyprus [פאמנוסטה בקיפרוס]	66	45	35	10
Fez in Barbary [recte] [פים בברבריאס רחם — recte: Fez]	10	0	35	0

City	Long. °	Long. ′	Lat. °	Lat. ′
Florence, a beautiful city [פלורינציאה עיר יפה]	34	0	43	40
Friuli [פריאולי]	32	50	45	12
Frankfurt on the Main, a great fair [פראנקפורט שעל מיינו, יריד נדול]	30	0	50	30
Frankfurt on the Oder, near Odera [פראנקפורט שעל נהר אודירס]	34	0	52	30
Candia, the birthplace of [the author], called [Crete] [קנדריאה עיר מולדתי, נקרא כנדקא רחם בחי קריטי — Candia/Crete, R. Yashar's birthplace]	54	10	35	15
Kraków, called Cracovia, a royal city of Poland [קראקה הנקרא קרקווי עיר מלוכה לפולן]	42	40	50	12
Cremona [קרימונה]	33	0	44	0
Corinth [קורינטוס]	51	15	36	55
Corfu [?] [קורפו לי]	45	10	38	45
Constantinople [קוסטנטינופולישׁ]	56	0	43	5
Gallipoli [קליפולישׁ]	45	10	41	30
Córdoba, a city, birthplace of Maimonides, and son of Chasdai, and Seneca [קורטובה עיר מולדת רמב״ם, ובן חסד[אי] וסיניקא]	9	40	37	50
Coimbra in Portugal [קואיטבריקא מדרם בפורטוגאל — recte: Coimbra]	5	45	40	30
Cologne on the Rhine [קולוניאה אנהריינס]	27	40	51	0

Left-hand column of p. 290:

City	Long. °	Long. ′	Lat. °	Lat. ′
Ragusa [ראנוזיאה]	44	4	32	30
Ratisbon [Regensburg] [רטישבונת]	32	15	48	59
Rouen [?] [רוינה — Rouen?]	35	0	44	20
Rhodes, the seat of Hipparchus [?], the astronomer [רודוסלאי מושב היפרכוס הסוכן]	58	0	36	0 [?]
Reggio in Lombardy [רינה בלומיאס — recte: Reggio?]	65	10	59	15 [recte:? lat impossible for Reggio]
Recanati [ריקאנטום]	40	0	43	22
Rome, the seat of the Popes [רומה מושב האפיפיורים]	36	30	41	56
Rouen [רואן]	22	40	49	0
Salonika [שלוניקי]	49	50	40	20
Salamanca in Spain [שלמנקה מדרם — recte: in Spain]	8	50	40	15
Samos [?] [שמוש לי]	52	40	41	15
Sardinia [?] [שרדיניאה לי]	30	10	38	15
Seville [שיוויליאה]	9	0	37	0
Sicily [?] [/ שטנוויאה שינוויאה — uncertain]	9	30	38	0
Siena [שיאינה]	34	20	42	50
Sterna [?] [שטירנה]	58	25	38	25
Stettin [שטיטין]	37	45	54	0
Syracuse, the city of Archimedes [שירקוזה עיר ארכימידיס]	40	30	37	30
Theodosia, called Kaffa [תיאודוזיאה הנקרא קאפא]	62	9	49	20

The Book of Chukos Shamayim is finished and complete. May God grant us life "after two days" [cf. Hoshea 6:2]. [תם ונשלם ספר חקות שמים האל יחיינו מיומים]

ספר — Sefer

גבורות יי — Gevurat Yah [The Mighty Acts of the Lord]

By the Gaon, the Kabbalist, the comprehensive sage, the divine philosopher, our teacher R. Yosef Shlomo Rofeh [the physician] Delmedigo — may he be built up — R. Yashar of Candia, may the memory of the righteous be a blessing.

The Fourth Part of *Sefer Elim*

First printed at Amsterdam in the year 389 [= (5)389 AM = 1629 CE] by the press of the Rav, the sage, the divine investigator, our teacher and master R. Menasseh ben Israel, of blessed memory, and corrected under his hand.

And now printed a second time, with added glosses and corrections, together with the life of the author, a transcription of his gravestone inscription, and also a depiction of the likeness of his face.

Through the efforts of the rabbinic R. Meir Grinshpan and his backers, the printer's [associates]; the rabbinic R. Moshe Eliezer Belinson [מ"י].

ארעסא — Odessa

At the press of M. A. Belinson.

תרכ"ז [= (5)627 AM = 1866/67 CE]

[Censor stamp, Russian: "Дозволено Цензурою. Одесса, 1 Сентября 1867. Года." — "Permitted by the Censor. Odessa, 1 September 1867."]

ספר גבורות יי — Sefer Gevurat Yah

Said R. Yashar of Candia: I have seen you marveling and astonished, O learned investigator, gazing with your far-seeing eye [?] at the magnitudes of the spheres of the stars, the host of heaven, and the measures of the distance between us and them — together with the other matters made famous by the astronomers and

the poets — and reckoning them in your eyes as exaggerations or dreams. In my eyes you resembled Alexander of Macedon, who plotted to conquer the whole inhabited world and to be lord of all the earth; and when he heard from one of the philosophers that there are other worlds besides this one, many and honorable, he wept and grieved and said, "Woe is me, that I have not yet merited to conquer even one of them!" How much better, then, did the sages of the Talmud do, in that saying which they recited before R. Chanina, when they said: "The God who is great, mighty, strong, and powerful…" and so on, until the end — to which he said, "Have you exhausted all His praises?" — a parable of a king who possesses a thousand thousand thousands of gold pieces, yet they praise him for one of silver. For the great works of the Lord are more [praiseworthy] than any praiser can express — as the divine poet said, "Great is the Lord and greatly to be praised" (Tehillim 48:2) — for although all the world acknowledges His divinity, they are not all equal in [grasping] His greatness. The civilized man magnifies and exalts the Lord more than the rustic who tills the soil; the enlightened philosopher does so by many degrees more than the civilized man; the reader of the books of Kabbalah more than the philosopher; the angel more than the prophets and sages; and according to the greatness of the angel, so does it add to its magnifying of its Creator. All these approaches to Him correspond to how much they add in apprehension over those below them; and those who fall short of them are, in comparison to them, fools — for there is no limit to His might and His kingship. This is what the prophet said: "Let not the wise man glory in his wisdom… but let him that glories glory in this, that he understands and knows Me…" and so on (Yirmiyahu 9:22–23).

And now, hear me, my friend, and see how it has been right in my eyes to remove from your thoughts [the notion] that any of this [magnitude] is anything in comparison to His greatness, may His name be exalted. For all my days I could not believe that there is no other world but this one — [extending] from the center of the earth, our dwelling-place, up to the eighth, or the ninth, or the tenth sphere — and that God, may He be blessed, is the One who moves it while standing over it. For it is not fitting to the power of the Blessed Creator to confine Himself to resting upon limited matters, which, though they are wondrous and awesome in our eyes, are lowly before Him — for nothing can be set in order [in comparison] to Him, blessed is He. And when I would read in the books of the Kabbalists, and would see the four worlds and the sefirot — which, the higher they ascend, the more exalted they become — my reflections would ride upon the shoulders of the philosophers, soar aloft on eagles' wings, and rise to great heights. When I saw that "God is gone up" (Tehillim 47:6), my reins rejoiced

within me, and I would float and fly for joy at having recognized the added greatness of my Creator, may His name be exalted.

And there was added to my joy, and a voice of good tidings shone for me, when I heard that in our time the thinking of the investigators had begun to consider that the whole world is like a lantern — called *lanterna* — and that the lamp burning within it is this solar body, standing at its center, its light spreading out to the sphere of Saturn, which is the outermost circumference of this world. All the other planets, old and new alike — together with many that are not visible — are within this lantern and benefit from its light. As for the stars that are in the eighth sphere, they reckon them to be far more distant from Saturn than Copernicus had supposed; and for this reason they appear so very small, even though each one of them is [as great as] this whole world

— that is to say, [as great] as the circumference of the sphere of Saturn — and the light that is seen in them is from the light of the sun that is within each and every one of those

lanterns, or worlds. And who knows in what manner they are — whether they have earth and a habitation and creatures resembling ours, or in some different manner? Blessed is the Creator, who alone knows them! From this you should learn to incline your ears to the matters that are in the Talmud, and let it not seem strange in your eyes what they expounded on the verse "to cause those who love Me to inherit substance" (Mishlei 8:21): "worlds," and their measures [reckoned] among the exaggerations. Let the honored Name be praised and exalted above all blessing and praise, without end or limit! And if in His bodies [i.e., the corporeal creation He has made] He is so exalted, how much more so — may His name be exalted above the intellectual creations, the sefirot, the princes on high that emanate from Him by way of cause and effect. And every man, according to his power and his knowledge, ought to acknowledge his Creator in His greatness.

And if Aristotle magnified His praise by his inquiry into the [processes of] coming-to-be and the parts of the animal, and Galen [magnified it] by [his inquiry into] the uses of the limbs of the body — both wishing to make known the greatness of the Craftsman from the work — how, then, shall we, the segulah [the treasured people], not acknowledge the One who is awesome in deed in the motion of the Great Wagon [the firmament / the celestial wheel]? And behold, my wish is to make known to you here a drop from the sea of His awesome works, whose truth is [as firm as] argument and whose foundations are [as] holy mountains — which is in the *Sefer Beshamat* [?] [Treatise on the firmament?], to

the fullest extent possible; and I shall bring only the headings of words, "as goads" (Koheles 12:11) for instruction, and they shall be "as bracelets upon your hands" (cf. Yechezkel 16:11). And I have called the name of this treatise *Gevurat Yah*; and since it is like a ladder set upon the earth whose top reaches the heavens, in that it ascends to the mountain of the Lord, I have divided it into degrees, or rungs. And this is its beginning, with the help of my Rock and my Redeemer.

מדרגה א' — Degree [Rung] 1

Since the measurement of a magnitude's distance is the inquiry into the knowledge of how many times one of the well-known measures is contained within it — that is, what fraction the measure known to us is of the parts of the thing measured (this being like the craft of [arithmetical] division of a known [quantity] in number) — I have seen fit to preface the measurements required for our purpose with an explanation of the names of the [units of] measure that are well-known among the sages of Israel and among the nations who dwell in Europe, which is the choicest part of the inhabited world.

Measures of the Sages of Israel — and these are they:

אצבע — Etzba [fingerbreadth] is the width of the thumb, that is, the large thumb of the hand. A fingerbreadth's measure is four grains of barley pressed together by their width.

טפח — Tefach [handbreadth] is the measure of four fingerbreadths by their width.

זרת — Zeret [span] is the magnitude of two handbreadths.

אמה — Amah [cubit] is like three spans, that is, six handbreadths.

קנה — Kaneh [reed] measures six cubits and a span.

ריס — Ris — its length is thirty reeds.

מיל — Mil — its measure is seven and a half ris.

פרסה — Parsah is measured as four mil.

Measures of the Sages of the Nations — and these are they:

אצבע — Etzba [finger], its measure is like four grains [of barley] pressed together by their width.

טפח או פיט — Tefach, or "fit" [foot?] [?], is the measure of four fingers by their width.

רגל הנקראת פיט — The "regel" [foot], called "fit" [Italian *piede* / "foot"], is four handbreadths.

אמה — Amah [cubit], its measure is a foot and a half, that is, six handbreadths, and is called *cubito* or *ulna*.

פסיעה — Pesi'ah [pace] is [a measure] of five feet, and is called *passo*.

אמת הבנין — The building-cubit [rod] is a long pole found in the hands of builders and field-measurers; its length is ten feet, and it is a double *passo*, and is called *reggia [?] pieda* or *pertica* [Italian *pertica*, "perch/rod"].

סטאדיאו — Stadio is a course of 125 [?] paces.

מיל — Mil, as it is named in the vernacular, is [made up] of a thousand paces, which are eight *stadii*, that is, five thousand feet.

דרך יום — A day's journey is the distance a man walks in a day, which is twenty-five mil and a half, that is, 1,250 [?] ordinary [Italian] paces, that is, ten parsah.

פרסה אשכנזית — A German parsah — and it is the more customary — is four [Italian] mil, that is, 20,000 feet, which are 4,000 paces.

You should know that when we say of some area that it is so-and-so many cubits or mil, we mean so-and-so many *square* areas, [each] a cubit by a cubit or a mil by a mil; and of bodies, that so-and-so many *cubic* [units] — a mil in its length, a mil in its width, and a mil in its depth — enter into that body. Now, when we said that the reed is six cubits and a span, the intent is not that it is so [also] in width, but only in length, since if it were six cubits and a span in both length and width, [its area] would be 40½ [?] square cubits [transcribed as printed; 6 cubits + 1 span squared does not equal 40½ — see Numerical impossibles]. And likewise, do not err in the division of areas: for if there were, for example, a square area of 24 spans squared, and you wished to know how many cubits it is — even though the cubit has within it three spans, do not reckon to divide it by 3 (for by this you would find that it is 8 cubits, which is false); rather, cube [i.e., square] the 3 spans, and they will be 9, and divide the 24 by it [the 9], and there will come out 2⅔ [?] cubits. And in like manner be careful with cubic [units]. And examine this closely.

מדרגה ב' — Degree [Rung] 2

In the measurements of the circle and the sphere we rely upon Archimedes and his proofs, just as we rely upon Euclid and his foundations; and therefore we shall not be lengthy in bringing proofs and figures, for the intelligent reader will understand, and all of them are clear in the books of his commentators, and in the book of Pappus [of Alexandria], and [in those of] the rest of the authors.

[1] — א Euclid explained, in Definitions 10 and 11 of the Fifth [Book], that when four magnitudes are arranged without interval [in continued proportion], the ratio of the third to the first is called the *duplicate* of the ratio that the second has to the first; and the ratio of the fourth to the first is called the *triplicate* of the ratio of the second to the first. For example: 1, 2, 4, 8. The ratio of 4 to 1 is the duplicate of the ratio of 4 to 1 [recte: of 2 to 1] [?], and the ratio of 8 to 1 is the triplicate of the ratio of 2 to 1. Therefore the duplicate is a square and the triplicate is a cube — and this is easy [to grasp].

[2] — ב Archimedes explained that the area of a circle equals the area of the right triangle one of whose legs is half the diameter [recte: the radius] of the circle and whose other leg is the length of its circumference; and that the ratio of the area of the square of its diameter to the area of the circle is as the ratio of 14 to 11. And [as for] the circumference of the circle, it is three times the diameter and yet a further part, which is less than ⅐ and more than ⁱ⁰⁄₇₁ [10/71]. Therefore their ratio [diameter to circumference] is as 3⅐ to 1 [recte; printed unclear] and as 3 ⁱ⁰⁄₇₁ to 1. Therefore, if you wish to know the circumference from the known diameter, double [multiply] the diameter by 3⅐, and the circumference comes out a little more than the truth; and if you multiply it by 3 ⁱ⁰⁄₇₁, the circumference comes out a little less than the truth. And conversely: if you seek to know the diameter from the known circumference, [multiply / divide] the circumference by 3⅐, and the diameter comes out a little less than the truth; and if you divide it by 3 ⁱ⁰⁄₇₁, the diameter comes out a little more than the truth.

[3] — ג It follows from the aforementioned teaching that if you multiply the small leg of the triangle formed from the circumference and half the diameter by half its large leg, the area of the circle comes out; or [if you multiply] the whole [of one] leg by the whole [of the other] leg and take the half [of the product], it will be the area of the circle. Therefore, to know the area of the circle, multiply half the diameter by half the circumference, and the area comes out; or [multiply] half the diameter by the whole circumference, and half of what results is the area; or [multiply] the whole circumference by a quarter of the diameter, or a quarter of the circumference by the whole diameter. All these are clear from

the reasoning of the first teaching, which was demonstrated as a proof in the words of Archimedes. Or [alternatively]: multiply the whole diameter by the whole circumference, and take a quarter of what results, which is the area of the circle. And from here it is easy to find

Gevurat Yah (גבורות יי)

Degree 2 (continued) ...the areas of all the parts of the circle — that is, its half, its quarter, its eighth, and so on — and there is no need to elaborate.

[Rule] ד. When the diameter is known to you and you wish to find the circumference, set up the proportion thus: "If 7 yields 22, what will a given diameter yield?" — and you will find its circumference slightly greater than the true value. And if you reverse the procedure, you say:

"If 71 yields 223, what will they yield..." — you will find the circumference slightly less than the true value.

And conversely: "If 223 yields 71, what will they yield..." — you will find the diameter greater than the true value.

And if you set it up as "If 22 yields 7, what will they yield..." — you will find the diameter less than the true value.

[Rule] ה. The areas of circles stand in the same ratio as their circumferences; and, taken one fifth to another fifth, the ratios of their half-diameters [radii] are as the ratios of their circumferences. The ratio of the square on the diameter of the circle to its area is greater than 14 to 11 and less than 284 to 223; and the ratio of the square on the circumference of the circle to its area is greater than 892 to 71 and less than 88 to 7.

Therefore, when you know the diameter and seek the area of the circle, set it up thus: "If 14 yields 11, what will a given square on the diameter yield?" — and you will find [a value] greater than the true one. And if you set it up thus: "If 284 yields 223, what will a given square on the diameter yield?" — you will find [a value] slightly less than the true one.

Likewise, when you know the circumference and wish for the area, set it up thus: "If 892 yields 71, what will a given square on the circumference yield?" — and you will find the area [slightly] greater [than the truth]. And if you set it up "If 88

yields 7, what will a given square on the line of the circumference yield?" — you will find it slightly less than the true area of the circle.

We will take the ratio of the diameter to the circumference as 7 to 22, that is, 3⅐. Whoever wishes to be more precise may take the value that the savant Ludolph [van Ceulen] of Cologne found, which is

3 and $^{4159265358979323847}/_{4159265358979323846}$ to 100000000000000000000

— but these are refinements of no use, and perhaps the said author spent his whole life on this discovery, to the point that at the time of his death he commanded that it be inscribed on his tombstone. So you will find it in the city of Leiden [לונידונוס], at [טו"ורוס/Tours?] [?], in the house of prayer, upon the stone in the pavement; the letters carved on the stone are nearly effaced, but in the mouths of the learned the matter is truly remembered — a flickering lamp [נר דולק], donkey-like [?], and he is not dead.

[Rule] ו. Now when you know the area and wish to know the circumference or the diameter, set it up thus: "As 7 is to 88, so is a given area to a certain number" — and that is the area of the square on its circumference, slightly greater than the true value; hence its square root is the circumference, but greater than the true value. And if you set it up "as 71 is to 892, so is a given area to…" — you will find the area of the square on the circumference slightly less than the true value, and the side of that square will indicate the circumference, slightly less than the true value.

Likewise, to find the diameter, set it up thus: "If 223 yields 284, as is a given area, … it will give the square on the diameter, greater than the true value," and extract its root to know it. And if you set it up thus: "as 11 is to 14, so is a given area to…" — you will find the square on the diameter; extract its side, and it is slightly less than the true value.

[Rule] ז. And when you know the diameters of two circles, or their circumferences, you will at once find the ratio between them — for since [these are] similar figures and similarly positioned, they are proportional, by [Euclid] Book VI. The ratios of the circles will be as the ratios of the squares on their diameters, by [Euclid] Book XII — that is, as the squares on their circumferences, since the circumferences stand to the diameters [in a fixed ratio], by the teaching of Rule 4 [ד] of this Degree. Let us suppose the diameter of one circle is 7 and its circumference 22, and the diameter of the second circle is 14 and its circumference 44. Divide 7 by 14, or 22 by 44, and you get ½; and its square is ¼. Therefore they are as 4 to 1.

[Rule] ח. The area formed from the circumference of the great circle on the sphere [great circle of the sphere], whose diameter is 3⅐ of [the length of] its circumference [?] [recte: the circumference is 3⅐ times the diameter] and is equal to its circumference and…

[continued] …[is equal] to the convex [curved] surface of the sphere. As the square formed from the circumference of the great circle is to the convex surface of the sphere, so is the ratio of [the circle of] the great circumference to its diameter; and likewise, as the ratio of the square on its diameter is to the surface of the sphere, so is the diameter to the circumference of the circle. From this it is also made clear that the surface of the circle is formed from the product of ¼ of the diameter into the whole circumference, or from the product of ¼ of the circumference into the whole diameter.

[Rule] ט. The ratio of the square on the circumference of the great circle on the sphere to the convex surface of the sphere is greater than 223 to 71 and less than 22 to 7; and likewise the ratio of the square on the diameter [to the surface] of the sphere [?] is greater than 7 to 22 and less than 71 to 223.

[Rule] י. The ratio of the cube formed from the circumference of the great circle on the sphere to the solid volume [גשמיות, the solid content] of the sphere is greater than 298374 to 5041 and less than 2904 to 49; and likewise the ratio of the cube on the diameter to the solid volume of the sphere is greater than 21 to 11 and less than 426 to 223.

[Rule] יא. From what has been said, it is clear that if you wish to know the convex surface of the sphere, gather four times the area of the great circle, and that is the surface of the sphere; or multiply the diameter of the sphere into the circumference [?], for the result is the surface of the sphere.

[Rule] יב. And if [you wish to know] the solid volume of the sphere: the product of half the diameter [radius] with one-third of its convex surface; or [the product of] a quarter of the diameter into ⅔ of the convex surface — and the volume comes out. Or ⅔ of the diameter into the area of the great circle; or ⅔ of the area of the great circle with the whole diameter; or half the diameter into ⅔ of the area of the great circle; or half the area of the circle into ⅔ of the diameter; or twice the diameter into the area of the circle; or [⅓?] of the diameter into ½ the convex surface of the sphere; or ⅙ of the diameter into half the convex surface of the sphere.

Therefore, if you wish to know the convex surface of a sphere from knowledge of the great circle, set it up thus: "As 223 is to 71, so is the square on the

circumference of the circle to the surface of the sphere" — slightly greater than the true value; "as 27 is to 7 [?] [recte: 22 to 7], so is the square on the circumference of the circle to the surface of the sphere" — slightly less than the true value.

[Rule] יד. And likewise, from knowledge of the diameter, infer thus:

"As 7 is to 22, so is the known square on the diameter to the surface of the sphere" — slightly greater than the true value.

"As 71 is to 223, so is the known square on the diameter to the surface of a sphere" — slightly less.

[Rule] טו. And likewise, if you wish to know the solid volume of the sphere from knowledge of the circumference of the circle, infer thus:

"As 298374 is to 5041, so is the cube on the circumference of the circle to the solid volume of the sphere" — greater than the true value.

"As 2904 is to 49, so is the cube on the circumference of the circle to the solid volume of the sphere" — less than the true value.

[Rule] טז. And if from knowledge of the diameter, thus:

"As the ratio 21 to 11, so is the cube on the diameter to the solid volume of the sphere" — greater than the true value.

"As the ratio 426 to 223, so is the cube on the diameter to the solid volume of the sphere" — less than the true value.

[Rule] יז. When you know the diameters of two spheres, or their great circumferences, you know the ratio between their magnitudes; and conversely, from knowledge of their volumes you know their diameters — for similar solids similarly positioned are proportional. In Rule 1 [א] of this Degree the matter was made clear. Some of these teachings will be expounded with worked examples in what follows, God willing.

Degree 3 (מדרגה ג')

Ptolemy explained in the *Geographia* that when different circles are drawn about one and the same center, one around another, the arcs bounded between the lines issuing from the center are similar and proportional. Let the present figure be **(figure 42)** [ציור מ"ב] by way of illustration, in which there are concentric circles [point-letters] **פרד** and **זקו** about center **ב**. Now the arc **קץ** [?], which is in

the small circle, is 60 degrees [?]; one says that arc **לף** [?], which is in the circle around it, is likewise [60 degrees] — and so the arc **פר**, which is in the outermost [circle], is 60 degrees; in each of them, according to the magnitude of its degrees, since the whole sphere [circle] is 360 of them. The matter is very clear: that if every circle is 360 degrees — by the convention of the astronomers, for the reason we explained in *Sod HaYesod* — then every quarter is 90, and so the four quarter-circles bounded between **פכ** and **כך** [?], which stand perpendicular one upon the other in the said figure, are similar; therefore, taken fifth to fifth, their equal parts too are similar.

I know that by another, demonstrative method the said teaching could be proven, but it is not my intent to elaborate on what is well known. Upon this teaching are built all the instruments of the astronomers — quadrants, octants, sextants, equinoctials, and their subdivisions — for according to the measure of the arc registered on the instrument, [bounded] between the ray-line and the line toward the zenith or the horizon, so will be the measure of the celestial arc.

What follows from our words is that the arcs of the terrestrial globe, which is upon one [common] center with the celestial spheres — that is, with the eighth sphere — are aligned with the arcs of the [celestial] spheres; and just as they are divided into [360] parts, so is this globe on which we dwell, like them; and every single degree on it is aligned with a degree on them. This is what we wished to explain.

Degree 4 (מדרגה ד')

The philosophers explained — and so it is brought in the *Sefer HaKadur* [Book of the Sphere], and in Book I of Ptolemy, and by al-Farghani [אלפראגאנוס] and al-Battani [אלבתין] — that the earth is spherical: that is, the element of earth and the [element of] water together form a round body, making together a single sphere; and likewise that all the spheres and the stars are spherical. There too it was explained that the said terrestrial globe is, as it were, a point relative to the eighth sphere — that is, it has no ratio or perceptible measure at all relative to the sphere of the fixed stars, on account of the great distance between us and it. Ptolemy explained this in Chapter 6 of Book I, and al-Farghani in Part 4; and it can be confirmed by several proofs and observations, which are these:

[First proof] א. When we are on a level plain — whether on sea or on dry land — we see the whole half of the heavens in a single instant; that is, six constellations [mazalot], which are in the eighth sphere. As the head of Cancer rises in the east, the head of Capricorn sets in the west, and so conversely. It follows, then, that the

visible horizon-circle is one of the great circles of the globe, which bisect it and pass through its center. For otherwise we would not see [a full] half of the sphere, but — when we are at the true horizon, at the navel [center] of the earth and upon its surface — we would see less, in proportion to the arcs whose chords are the half-diameters [radii] of the earth. From this it would necessarily follow that when the sun is on the equator the days would be shorter than the measure of the nights; that the ends of the shadows of vertical gnomons would not be bounded by a straight line on [level] surfaces — the opposite of what was demonstrated [in the work] *Muqta,* that the line standing at the ends of the shadows at the equinoxes is a straight line; and it would also necessarily follow that the rising of the summer solstice [point] would not correspond to, nor be aligned opposite, the setting of the winter solstice [point], and conversely;

and the increase of days from the [vernal] equinox to the summer solstice would not equal their decrease from it [the solstice] to the [autumnal] equinox; and many absurdities like these would necessarily follow, were the visible horizons that pass over the surface of the earth not reckoned as the true ones that pass through its center. Therefore the half-diameter [radius] of the terrestrial globe counts for nothing relative to the half-diameter of the eighth sphere, and the visible horizon is, as it were, the true horizon.

[Second proof] ב. We see that the shadows of the tops of gnomons set up on the surface of the earth are equal to the shadows of gnomons set [as if] at its center — that is, no difference at all is perceptible, just as the calculation of the triangles indicates, and so the true experiment shows. Yet if the distance between the surface [of the earth] and its center were reckoned as significant relative to the solar sphere, this would not hold; and it could not be, since it is impossible for the [single] terrestrial globe itself to move about two centers far distant from one another in a single uniform motion, such that gnomons standing at the two centers would cast equal shadows at one and the same time when the sun strikes them. Therefore one must say that the half-diameter of the earth bears no perceptible ratio to the distance between it and the sun — and all the more so relative to a still loftier height, such as the eighth sphere, which is infinite in relation to it, [the earth being] as a point. For if the diameters of the stars were observed in a single instant by three observers — one in the east, one at the middle [meridian], and the third at the western edge — they would appear equal; hence the distance between them counts for nothing, since it is imperceptible relative to the distance of the star from the earth. Otherwise the star would appear larger to the observer at the middle, who is close to his zenith, than it appears to the first, who is to its west, and to the third, who is to its east. Rather,

one must say that the entire distance from east to west — that is, the whole diameter of the earth — counts for nothing relative to the great distance.

Now, as for what the sailors report — that under the [equatorial] line one does not see both poles together (that is, the stars near the axes of the equator, which are on the horizon there) such that none of them is visible at all: if this is true, there is no doubt that it occurs on account of the thick vapors [exhalations] of the earth at the horizon, which darken the light of stars that do not shine brightly. Know that this is so, for even when the sun has set, the horizon appears as though [the sun] were still rising or had not [yet] set, though it is not [the sun] itself but the image of its likeness — its light, through the refraction of the rays. Thus we see more than half of the heavens when we are on the surface of the earth; and were there stars at the [great] axes, large and bright, they too would appear infinitely [numerous] at the true horizon.

Moreover, according to the distance of the eighth sphere from the earth — which al-Battani [אלבתין] reckoned as 19000 times the half-diameter of the earth's globe (that is, in the said figure, the line **רדכ** is 19000 times line **כך**) — we find arc **דף** [?] of the eighth sphere, which is hidden from our eyes by the half-diameter of the earth (line **כך** being a sine [chord], straight, [of value] [?]). According to his [al-Battani's] view, the measure of arc **דף** would be 11 seconds; according to al-Farghani — who [makes] line **רדכ** 20110 — it would be only 10 seconds; and according to Tycho [טיקון], who [makes it] 14000, it would be 15 seconds. This is a small part, the whole circumference being 1296000 seconds. And according to the view of Copernicus — who [makes] the distance **כך** [?] 7850000 times line **כך** — it has no ratio at all, on account of its smallness. Infer thus:

497	19000 — as
500... 1 to 10000000, so 1 [to] [?]	20110 — as
714 sine, arc 15	14000
sine 11, sine, arc 15 [?]	7850000
line 1 **כך**¼, page [דף] 0 [?]	

[Parsing notes: the foot of page 298 carries a small two-column computation block, partly broken in the type. Right column (Hebrew RTL, top→bottom): כמו 19000. כדר 20110 אל 10000000 כן 1; 14000; 7850000. Left column figures, top→bottom: 497 / 11; 500 / 10 / 0 דף / ¼1 כדר; בקע / קשת 15 / בקע 714 שהוא; The alignment of the left-column fragments to their right-column lines is uncertain; transcribed as printed. The values / 14000 / 20110 / 19000 = **רדכ**

7850000 and the resulting arcs (11″, 10″, 15″) match the prose above; **497** and **500** appear to be reduced-ratio terms (~19000:497 ≈ 20110:500 [?]).]

Therefore it is most clearly demonstrated that the earth, relative to the eighth sphere, is reckoned as a point according to every one of the views.

Gevurat Yah (גבורות יי)

Degree 8 (מדרגה ח')

The view accepted by Ptolemy and by all astronomers down to the time of Copernicus is this: the terrestrial globe stands at the center of the world; the sphere of the Moon is the one nearest to it; after it the sphere of the star called Mercury (כתב); after it the sphere of Venus (נוגה); after it the sphere of the Sun; after it the sphere of Mars (מאדים); then Jupiter (צדק); and the last, Saturn (שבתי). Their mnemonic is שצ"ם חנכ"ל. These are the ones called "the seven wandering or perplexed stars" (כוכבי לכת או נבוכה) — not that the stars themselves move; rather their spheres move and carry them along, unlike the eighth sphere, which they held to be wholly unmoving. For this reason they called the stars set in it the "fixed stars" (קיימים).

There are others, however, who hold that the stars move circuitously without spheres at all; and in their opinion the Sages of Israel did not do well, who taught the Sages of the nations that the constellations are fixed and that the spheres revolve [carrying them]. Perhaps the former opinion was a tradition from the prophets, and they ought not to have abandoned the received sources for the sake of those who give reasons and grounds, so long as they do not establish their claims by clear demonstrations. And still others reckon the stars in a different order. All of these we shall bring forward for adjudication, please God, in their proper place.

As for their distances from the earth, all agree that the fixed stars are the most remote and highest, since all the wandering stars are seen by the eye to pass beneath them as they move, screening them off from us. Moreover, the fixed stars sparkle (מבריקים) constantly, as though continually shooting out [arrows of light] because of the great distance — which the wandering stars do not, except by reason of vapors [in the atmosphere] at some moment or other. This is therefore a property by which to distinguish, in the sky, between the wandering and the fixed: there is yet another mark by which to discern, as I shall inform you, please God.

Furthermore, intellect and reason indicate that the [fixed stars] are the highest, since it is fitting that the motions should be proportioned to the sizes and magnitudes of the spheres. For if all the stars moved with equal speed of progress, then because the circuit of the uppermost is the larger, by the time it advances one degree the lowest would advance two degrees, or three, or more. So when the sphere of the Sun moves in one year, Mars moves in two years, Jupiter in more, and Saturn most slowly of all; whereas the eighth [sphere] moves slowest of all, or does not move at all — as Copernicus held, [maintaining] that the earth performs three motions. But this proportion is not preserved among the spheres, so that Aristotle (ארסטוט') was driven to seek imagined causes and grounds in [his] *On the Heavens* (ס' השמים); and the confusion of the order is, in truth, according to his suppositions and the hypothesis of Ptolemy. According to Copernicus and the moderns, however, all is set right and correct, as is fitting — as I shall inform you elsewhere, please God.

In any case, by everyone's reckoning Jupiter is above the Sun and Mars, and Saturn above Jupiter, and the eighth sphere above them all. But concerning the rankings of Mars, Sun, Venus, and Mercury there is doubt and perplexity — for it has been established by decisive demonstration, [confirmed] by the calculation of that wondrous astronomer [Tycho], and as will be explained, that Mars is at times nearer the earth than the Sun. That is, when the Sun is at its apogee (ברומו) and Mars at its perigee (בשפלותו), [Mars] will be nearer to us, by a measure of three half-diameters of the earth (ג' חצאי אלכסוני הארץ). Likewise al-Battani (אלבתיניוס) noted that he found Venus rising as high as the middle of the thickness of the Sun's sphere; and Tycho (טיקוניו) wrote, in Book 2 on the comets (לכוכבים הזנביים), that he found this not for Venus alone but for Mercury as well.

The Peripatetics (המשאיים) will thus see what they can reply concerning the fifth body, which they supposed the heavens to be — like a "sphairēn" [Greek: σφαῖρα (sphaira, "sphere")] — and which is therefore split, with body entering into body; for otherwise, how could the stars be [thus] high and low, entering this one within the body of its fellow? Indeed, some have supposed that the Sun, Mercury, and Venus, all three, are in a single sphere — except that within its thickness they have [their own] degrees [recte: ranks], one above the other, and therefore their motion is equal. According to some astronomers and reckoners, this would imply that the heavens are airy.

All these [views] shall, with the Eye [recte: in chapter] 10 and 11, be laid in the balance, please God. But Aristotle held Venus and Mercury to be above the Sun,

adjacent to it; and so too Plato in the *Timaeus* (פלטין בטימיאו); and Alpetragius (אלפיטראניאום) placed Venus above the Sun and Mercury below it.

According to the opinion of Copernicus, the Sun is at the center of the world, and around it revolves the sphere of Mercury; this is encompassed by the sphere of Venus; and the sphere of Venus is encompassed by the sphere that carries the terrestrial globe — a circle called an epicycle (מעגל), in which is carried the lunar body around the earth. Around the sphere of the earth is Mars, and above it Jupiter, and around it Saturn, and last the eighth sphere. By this view, when the Sun [recte: earth] lies between Jupiter and Mars, Mars will be farther from the earth than the Sun is; and likewise Mercury and Venus are, for this reason, at times more remote than the Sun; whereas when the earth is between these [planets] and the Sun, they will be nearer to the earth than the Sun is. All is set right for one who understands.

Those who follow in his footsteps reckon that, since there is in truth no [physical] circle carrying the earth — rather we merely imagine its path, as with the Moon and all the stars, [where] the circles carrying them are only imagined — God gave to the Sun, which abides like a king upon his throne at the center of the world, a drawing force, like a magnet (כמגני"טיס) [Latin: magnes, "magnet"], by which all the wandering stars are moved and guided so that they do not stray from their course. And likewise the earth, in its constant motion, is not at rest [in such a way] that the heavy body should be [merely] as if it were still; rather, just as we see a stone thrown upward against its nature: so long as the force imparted by the thrower endures, it does not fall to the ground — so the earth persists in its motion, since the force of the Sun that thrusts, moves, and guides it does not cease. This force is likewise imparted to the terrestrial globe, and by it the Moon is moved around it. In this way the motions are proportioned: Mercury moves in eighty days, Venus in nine months, the earth in a year, Mars in two years, Jupiter in twelve years, and Saturn in thirty years. The Moon, being a small circle, [moves] faster than all of them. Consult on this Chapter 10 of Book 1 of Copernicus's *On the Revolutions* (להקפות).

Aristotle held that the Sun is immediately above the Moon, and all the rest [of the planets] above the Sun. They argue against those who suppose the Sun has stars beneath it, other than the Moon, by [these] objections:

First: that it would necessarily follow, when [those stars] are in conjunction with the Sun, that they would appear dark on the side facing us, since they receive their light from the Sun like all the stars, which have no light of their own; and it would necessarily follow, as they recede from the Sun little by little, that they

would appear deficient [in light], like the Moon, with their horns turned away from the Sun.

Second: that the Sun would be eclipsed by their conjunction [with it], just as it is eclipsed by the conjunction of the Moon. Hence they reckoned that neither Mercury nor Venus lies beneath the Sun, but only the Moon.

These objections, however, are easily dissolved. For after the telescope (השפופרת של זכוכיות, "the tube of glasses") through which one views distant things became famous, it was verified that Venus waxes and wanes and casts its horns exactly like the Moon; hence nothing prevents it from being below [the Sun]. And as for Mercury — if we are to trust observations, the testimonies compel [the conclusion] that it lies beneath the Sun. For Ibn Rushd (ן ראשד) [Ibn Rushd / Averroes] noted, in his epitome of the *Almagest* (בקצורו לאלמינסטי), that during its conjunction with the Sun he found in them [recte: in it] a dark [spot] upon the body of the Sun. And so too Kepler (קיפלירוס), great in learning, in our own time, and others with him, noted that they saw it clearly: on a sheet of paper fixed in a darkened room, upon which the Sun strikes through a small hole, this force [recte: spot] would be seen on the paper, on which the solar disk (הענול השמשי) was traced; and this occurred at a time when Mercury was in conjunction with the Sun.

Further, some reply that the lunar body alone is thick and dark, and therefore [when it interposes] it darkens the Sun, and gives no light at all unless it receives the light from it — unlike the other stars, which have light...

and which give no light at all unless they receive light from it [the Sun], unlike the other stars, which have light. For the senses too indicate that they likewise receive [light] from the Sun and add it to their own — as Galileo Galilei (גלילואוס) testified, having observed Mars when it was near the earth, that its light increased greatly, even though its body is smaller than [Jupiter's]; and likewise its light was so great that the eye could not gaze upon it through the telescope. By contrast I sought to view Jupiter (צדק) with the glass instrument, and it appeared to me elongated, not round — and it seems this happens by reason of [its] motion, its sparkling, and its brightness; whereas the planet Jupiter [itself] I found to be round, and Saturn [shaped] like an egg with the instrument.

Hence, if the light of Mars when near the earth is great and very red, [whereas] Jupiter, which is whiter, [shines] with this and more than the light of Saturn — which is like lead, its light turbid — and the light of Venus, near the Sun, prevails over them in brightness; [in all this] Mercury, although the smallest of the stars of

heaven, is nonetheless bright. We are forced to say that they receive light from the Sun, if not the whole [of their light], then the increment of it. Yet the apparent [fact] is that, for this reason, the fixed stars would be darker than Saturn, since they are more remote from the Sun — and [yet] we see many of them bright and red, such as the star at the Heart and Spike of the Virgin (שכפי הכלב ושבולת של בתולה), and the Heart of the Lion (לב האריה), and many stars of the first, second, and third magnitudes, very bright and luminous. So they reckon that the fixed stars have, besides the solar light, a very great [light] of their own, and have no need of it [the Sun], since its rays do not reach them on account of the great distance.

And if it is as some hold — that the fixed stars rotate with vigorous speed about their centers (just as it has been explained that the Sun revolves in its place, and likewise the Moon inclines itself backward, the reverse of the motion of its epicycle, for otherwise we would never see its spots looking always the same) — then this is the cause of their appearing to sparkle. But the opinion of the one who says that the wandering stars have light of their own does not accord with what we observe: Venus radiates, scatters, increases, and diminishes [its light] — which would not happen had it light of its own. Therefore others reply that perhaps [the planets] have a latitude in their portions [recte: their nodes/eclipses, מהלקותיי], and accordingly the Sun is not eclipsed by their conjunction, since they are not at the Head or the Tail of the Dragon (ראש או זנב התלי, the lunar nodes). And even were they in conjunction without latitude, they would not screen any perceptible part of the Sun's body, visible to the eye, on account of their smallness relative to the solar body.

For the diameter of the disk of the star Venus, to the sight of observers using the telescope (השפופרת של איפרכום), [a telescope] specially fitted for taking the diameters of the stars, is ¹⁄₁₀ of the diameter of the Sun. That is, if the diameter of the Sun appears as 30 minutes (ל' דקים), Venus's will be three [minutes] only — as al-Battani (אלבי"תין) wrote. Hence, [by] the second proposition (ממדרגה שנית) of this book, since the [areas] of circles are as the squares of their diameters, the occultation of the Sun's body by Venus's body, in [Venus's] conjunction [with it], will be ¹⁄₁₀₀, and is not perceptible relative to the great solar light — still less by the conjunction of Mercury, which is smaller than it.

As for the lunar body, in its conjunctions an eclipse of the solar light does occur, because — owing to its nearness to us — it falls almost upon our very eyes and separates us from the solar light. It is not a [true] solar eclipse, but rather, in truth, an eclipse of our [own] eyes. For the apparent diameter of the lunar globe,

when [the Moon] is near the earth, equals the apparent diameter of the globe of the Sun; hence it covers it entirely, to appearance — although, by Tycho's reckoning, it is impossible for it to cover it entirely, as is [actually] observed, please God.

From all this it is demonstrated, by proof, that the eighth sphere [recte: the Moon] is nearer to us than all of them, since all of them are eclipsed and covered...

as it [the Moon] passes beneath them, and not the reverse. And the terrestrial globe has a perceptible ratio to their distances, as we shall explain, please God. And this is true; there is no cavil against it at all.

Degree 6 (מדרגה ו')

In truth, it befits every wise of heart to find a way to account for the apparent recession and approach of the stars relative to the earth, without [resorting to] the rupturing of the firmaments. For from the day man was created upon the earth, [no] alteration in some of them, nor in all of them, [has been observed]; and this happens also in the elemental world: see how many cities are submerged and swallowed up, and how many die in war and in plague, and yet the whole world is not destroyed at once; and what is lost here is made good from elsewhere. So too the wandering stars seen in the heavens indicate nothing [of cosmic decay], for all that is seen is in the firmament of heaven, not in the stars themselves.

As for what Cornelius Frangipani (קורניליאוס פרנגיפאנוס) the Italian chronicler wrote — that, at the conquest of Constantinople (קוסטנטינה) by the Greeks, from whom the Ishmaelites took it, the light of the true northern pole star (כוכב הציר הצפוני האמתי), which shone in the days of the ancients, faded and vanished entirely — that is as true [reliable] as his second account, in which he said that at the destruction of the city of Troy (טרויה) the seventh star of the Pleiades (כוכבי עיש), called Electra (איליק"טרה), vanished. These tales are not to be believed, since nothing of the kind occurred at the destruction of our Sanctuary and our Glory, when the Sages would [surely] have [recorded it]. Therefore the firmament of heaven is generated and corruptible, but never the stars. According to the last of the three [authorities] mentioned, the small [star] of the Lesser Bear (הדוב הקטן) has always been at the pole, since the time of Eudoxus (אורד"קסום); [and what] appears [otherwise]. There are also philosophical proofs against this, but here is not their place; and therefore many have preferred the opinion of Copernicus, by whose view the cause of approach and recession is manifest, with no need for the

breaking of the spheres' glass [shells]. But it is not our intent to decide between the opinions by this discourse — only to ascertain the order of the spheres.

Now there is yet another, demonstrative way that proves the Moon to be nearest to the earth: it is known in the science of optics (חכמת הראיה) that, the more distant the star, the shorter are the shadows of the gnomons (העמודים) cast by the rays of its light passing over their tips. For example, in Figure 1 of this discourse: when the Sun is beneath the head of Aries (ראש טלה) at point מ, it will be, at midday in the city of Candia (קנדיאה), elevated 55 degrees above its horizon, on the arc of the meridian מד [?]; and the shadow of gnomon ע will be line עד; whereas when Venus is at point [?], [elevated] 55 degrees, and at the head of Aries, the shadow of the gnomon ע cast by its light will be line עז; and when the Moon is at the head of Aries, at point ב, elevated 55 degrees, the shadow of the gnomon will be line [?]. Now you see that the shadows of the [body] nearer to the earth are longer than [those of the body] more remote from it; and experience shows that the shadows of the Moon are longer than [those] of Venus, and Venus's longer than the Sun's. Hence this [body] is above that, with no doubt about it. And these three stars [the Sun, Venus, and the Moon] make a shadow [only] when [observed] with great care; but with regard to the others too we can infer and aim [the calculation], namely, when we set up the [observing] instrument facing them in such a way that the ray entering from a side of the square instrument, through its center, toward them [forms an angle]. For example, in [the figure]: the [angle] [?] makes a notch (בקעה); for instance, if at point [?] one knows the measure of angle [?] = [?], and if at point ז [one knows] angle זיע — and thereby one knows the angle that is [?]. Or set up a wooden gnomon (עץ עמודי) upon the earth, and step back until the ray issuing from your eye, passing along the tip of the wooden [gnomon] [recte: the line of sight passes through the hole of the wood] to the star, [is aligned]; then the distance between you and the wood, together with the measure of the wood, which is upon...

Gevurat Yah (continued)

[Degree 6, continued — p. 303]

[The beginning of the line continues from the prior page:] ...the horizon at right angles, [the gnomon's length] indicates to you the measure of the angles — as I have instructed you in *Sefer Sod HaYesod* and in *Sefer Chukos Shamayim*, at Question 12. The truth is that this method is not very reliable for the stars that are very far from the earth, as I have written elsewhere. What does emerge from it —

a lesson about which there can be no doubt — is that the Moon is nearer to us than the Sun and Venus. So much for that, and that is what was to be demonstrated.

Degree 7 [מדרגה ז'] [p. 303]

Beyond all the things mentioned here, there is in truth a decisive demonstration, which clarifies their order and their gradations. The astronomers call it *chiluf ha-mar'eh* (parallax — "the difference of appearance"), but I call it the "error of vision," and it arises in this way. When the distance [of a star] is not so great relative to the center that half the earth's diameter has a substantial, perceptible ratio to the distance — [in that case] the angle that arises between the line drawn from the center of the earth and the line drawn from the star to the center [recte: to the observer's eye] is very wide. To illustrate this for you, let me inform you: whoever wishes to know the true position of a star among the constellations must observe it when it is at the zenith — at point ד — or at least at its greatest altitude. For the closer the star draws to the pole of the horizon, the closer to one another are the lines that issue from the center of the earth, ב, and from the eye of the observer, and pass through the center of the star, ק or ל, and reach the sphere of the zodiac. The converse also holds: the closer the star is to the horizon, the farther apart [those two lines become], and the arc between these two lines on the sphere of the zodiac — שך דט — is called the parallax.

That is to say, [parallax is] the measure between the true position of the star, רט, which is on the line that comes from the center of the earth, כר, and its apparent position, כט, which is on the line that comes from the eye of the observer, זר שז, who stands on the surface of the earth. Now the arc of the parallax varies according to the position of the star, for when it is at the zenith the line issuing from the center of the earth and the one issuing from the eye of the observer are לר כז [recte: one and the same line]. Therefore the star at that moment has no parallax at all. But the more the star sinks and descends from the zenith, the more the ends of the two lines mentioned grow apart from one another among the constellations, producing the angle ק at the center of the star through which they pass — and this is what is called the angle of parallax. Its measure is what is sought in astronomy, whose power is great, as you shall learn, God willing. The closer the star draws to the horizon, the wider and wider this angle becomes. And when this angle is equal for two stars that are equally distant from the zenith, the astronomers say that they are in one sphere — that is, at an equal distance from the center of the earth; but if the angle of one is larger than that of the other, [that one] will be nearer to the earth.

The reason for the matter is this: the closer the star is to the earth, the larger the ratio of half the diameter of the terrestrial globe becomes relative to half the diameter of the distance, and the greater the difference between the line issuing from the center [of the earth] and the one issuing from the eye of the observer, which is on its surface. Since we perceive in the Moon — which is in the said figure, that is, at point ק — a parallax larger than [that of] Venus, which is at point ל, and Venus's angle of parallax is smaller and even less than that for the Sun; and in Mars we perceive no parallax at all between its true and apparent positions, because of the greatness of its distance — this being when it is at its greatest distance, and all the more so for Jupiter — as I have written; for the radiant line issuing from the center of the earth is, according to vision, identical [to the other], though it differs according to geometry. As Ptolemy wrote in Book V, chapter 11 of the Almagest [Ptolemy, *Almagest* V.11], those stars whose parallax is not perceptible — that is, the magnitude of whose distance is so great that the terrestrial globe is [as] a single point — it is impossible to know the horizon of their distance from the earth alone…

[Degree 7, continued — p. 304]

by means of parallax [recte: it is impossible to take the distance of the stars] by means of parallax. The distance of the stars is taken [in another way]. So consider how the "error" teaches us wondrous things about itself: that parallax is nothing but a deception of the senses, by which the object is seen apart from its true position. It is therefore very clear, beyond doubt, that the nearest is the Moon, and above it Venus, and above it the Sun, and above it Mars — provided that their angles of parallax proceed, as we have said, in [proper] order. The reason for the appearance [whereby Mars is] sometimes nearer than the Sun is made clear by the hypothesis of Copernicus, as above; and no one accepts this demonstration except an utter fool and simpleton.

From here, too, the way is made clear to know whether the comets and the other new lights that are seen in the heavens are airy [sublunar] or ethereal. For if indeed the said angle of parallax is found in them, by means of precise instruments, to be greater than the Moon's when it is near the earth and below the concavity of the lunar sphere, then they are airy or fiery — that is, in the world of the elements, as people suppose. But if the angle is smaller than the Moon's angle, there is no doubt that they are above it, and they are ethereal — and not as Aristotle and the Peripatetics supposed. And even though Ptolemy did not mention these comets and the ethereal [new lights] in his book at all, one may

say that he held with Aristotle regarding the substance of the heavens, and did not set his heart to examine the parallax in them.

It does seem, however, that Hipparchus understood this matter well. Pliny wrote of him that when he saw new and perishing lights in the heavens, he doubted concerning all the fixed stars — perhaps they too are so. He therefore fashioned mighty instruments and devices, and examined their longitudes from the head of Aries, and their latitudes, and the distance of each from the other. And since Ptolemy saw that all the stars kept their configurations, just as he found in the tables of Hipparchus, he concluded in his opinion that there is no change or innovation whatever in the ethereal world, and he did not set his heart to observe its height with instruments. For had he examined — being a teller of truth, as was his way, and not deferring to others — he [would have found that] parallax, harder than flint-rock, would not contradict him: not Aristotle, nor any seeker of truth, would deny it. As for one who wishes to stop his ears like a deaf adder and follow the Peripatetics as though they were the words of the living God — we have no dealings with him, for he is far from there, and remote from the path of the true philosopher.

Degree 8 [מדרגה ח'] [p. 304]

Now the measure of the angle of parallax is taken and made known by means of precise astronomical instruments, and the quadrant divided down to seconds, set up against a circle [marked at] noon, is the choicest of them all. There are several kinds of instruments; we shall mention their construction and their use elsewhere, God willing. For now, however, I shall inform you of the construction of the *parallactic instrument* [Latin: instrumentum parallacticum; lit. "the parallactic instrument," ליור מ"ב — see Figure 42], with which Ptolemy examined parallax, and which Nicolaus Copernicus — who was second to Ptolemy in astronomy and in [the science of] the heavens — used all his days. Its height is the height of an average man, and it is made of three pieces of wood: one, long, in the manner of [the letter] א-ת [i.e., a vertical staff], a squared beam, with sharp iron at its end so it may be driven into the ground at the time of observation; and at its base two or three feet, or a thick piece of wood, broad like a pedestal, in which it is fixed and stands.

At its top and at its midpoint, א צ, it is bored with two holes [recte: at its top, א, and at its midpoint, צ] in which are inserted pegs of wood in the manner of a hinge [Italian: giuera/ghiera (?)], so that the [pieces] entering into them turn about [as on a pivot] — that is, [you] make two pieces of wood: one broad, in the manner of [the segment] א, its length being the length of א-צ; these two are

reckoned as half-diagonals and are divided into sixty degrees. The second, in the manner of צ-ה, is lighter and longer, like the [hypotenuse of the] square's side. That is to say, if you rest the leg of the compass at point א and extend its second leg to צ…

[Degree 8, continued — p. 305]

and you describe a circle, then צ-ה will be the chord of a quarter [of the circle], which is 90 [degrees], and accordingly צ-ה is divided into 51.84 parts [recte: 51 + 84/100? — see note], at the measure of the chord of 90 degrees. Now the piece of wood אך is raised and lowered until the radiating lines of the star or the eye pass through to it, through the slits of the columns that are upon it; for at the place where the wood צך is cut, it indicates to you the magnitude of the angle at the center [of the earth]. That is, when the star is at point ד, the ray of the observer passes along the line אי, and the arc that the chord רצ indicates as its magnitude is equal to arc יב, the distance of the star from the zenith. And if the star is exactly on the horizon, the end of אך will fall exactly at the end of צה, indicating that the star is distant from the zenith exactly a quarter-circle.

If you examine the graduation [marking] of this instrument, it is very easy to use, and you can divide it into many parts so that it is exceedingly precise. See *Sefer HaHakafot*; and we shall yet speak of it elsewhere, God willing.

Now Ptolemy — as he wrote in Book V, chapter 12 of the Almagest [Ptolemy, *Almagest* V.12] — investigated with this instrument and observed in the city of Alexandria, whose [terrestrial] latitude is 30°58′ north, the Moon when it was at the head of Cancer on the noon circle — which is its greatest altitude at Alexandria. He found it distant from his zenith — that is, from the pole of the horizon — 2°7′30″. And since it was so close to the zenith, there is no doubt that the parallax is small and slight, and it is not to be reckoned as anything, as was explained in the preceding Degree.

Having known its distance from the zenith, he found the latitude of the Moon in this manner: as I explained at Question 12 of *Sefer Chukos Shamayim*, if you subtract the great inclination [declination], which belongs to the Moon when it is at the head of Cancer — that is, 20°23′51″ — from the distance of the equator to the zenith, which is the latitude of the city, always — that is, 30°58′ — there remains the distance of the Moon from the zenith, 7°6′40″ [recte? — see note]. But with the instrument its distance was found to be 2°7′30″. Therefore its great northern latitude, which it had at that time, was 4°59′10″ [recte?].

When the latitude approached so near the zenith, he kept watch on the matter [waiting] until the Moon reached Capricorn [at] 3°10′ with a northern latitude of 4°59′ [recte?], and observed it when it was on the noon circle at Alexandria, and found its distance from his zenith [to be] 50°55′. When he added its inclination — that is, of [Capricorn] 3°10′, which is 23°49′ — to 30°58′, the distance of the equator from the zenith, he found 54°47′, the distance of the Moon from the zenith. When he subtracted from it the latitude, 4°59′, there remained 49°48′, which is its true distance from the zenith — though with the instrument 50°55′ was found. Therefore the difference, 1°7′, is the measure of the parallax sought. And in this way you will find the parallax for all the stars and lights seen in the heavens, if the earth's diameter has a perceptible ratio to their distance.

Degree 9 [מדרגה ט'] [p. 305]

Now I shall inform you how the distance of the Moon from the earth is found. Let there be, in the present figure [ליור מ"ג — see Figure 43], three concentric circles: the middle one, the globe of the earth; above it, the sphere of the Moon; and the outer one, the sphere of the zodiac. Let the Moon be at point ד, and its true position among the constellations at point י, and according to what appears to the eye it is at point מ. Now the distance from the zenith found with the instruments is arc מז [recte: arc of] 50°55′, and the true distance found by the aforesaid computation is arc יח, 49°48′; and arc ימ is the parallax, 1°7′.

Since line עא, half the diameter of the earth, is reckoned as nothing relative to the distance אמ to the eighth sphere — as we explained in Degree 4 [recte: Degree 6] — then if we draw a line אפ parallel to line עי, arc פי…

[Degree 9, continued — p. 306]

is reckoned as nothing relative to the circumference of the eighth sphere. Therefore the whole arc פמ will be like arc מז [recte: 7′? — i.e., equal to the parallax], 7′, and the angle פאמ, 1°7′, is what the side אד subtends, [seen] from the first [point]. And since in triangle אעד the angle of parallax אך [recte: at א] 7′ is known to us, and the angle אעד is the measure of arc יח [recte: the supplement], 49°48′, therefore [the angle] toward the first [point] is known, the third angle, [at] עאד, which is 129°5′. Now if we set line ירוע [recte: עד, a known line], which we shall afterward find, God willing — let it be, as a square root, root 1 [unity], found from Theorem 9 [recte] of Part 4 of *Sefer Sod HaYesod* — the measure of line עד, the distance of the Moon from the center of the earth — then make the proportion thus:

A s [1°7′ =] 1.7 אעד	א ע	is to	=] 129.5 ראע [129°5′	so is	עד
בקע [sine] 194883	א 1		בקע [sine] 7762298		12″ 39.45 [recte: 39′45″ — see note]

And Ptolemy found, according to his method, 39°45′ [i.e., 39¾ earth-radii], and so [the distance of] the Moon from the center of the earth was, at the time of the said observation, about [a number of] times half the earth's diameter.

Degree 10 [מדרגה י'] [p. 306]

Behold, the astronomers did not give "sleep to their eyes" [cf. Tehillim 132:4] — and Ptolemy in particular, that wondrous, exalted man, who searched and examined many times after the parallax. From it, it became known to him that one and the same star does not always stand at a single distance from the earth, but sometimes draws near and sometimes mounts up and grows distant from it. Besides the demonstration drawn from parallax, sense itself confirms that this is so: for sometimes the star is seen large in its diameter, as measured with the dioptra of Hipparchus [Greek: διόπτρα (dioptra), a sighting instrument], and sometimes small.

It is impossible to attribute this to the vapors and mists of the earth, as is seen in stars close to the horizon, since it appears so even when [the star] is at mid-heaven, and in every season of summer and winter, and in every clime. Moreover, [the star] keeps this measure at fixed, determinate times. Furthermore, its motion is seen to be unequal, since, for example, it is slow to traverse the northern constellations and swift in the southern ones; and so too the eclipses are seen to be unequal — sometimes the part eclipsed is large, which is when the Moon is near the earth, [for then] it enters the place where the diameter of the shadow is broader, and not when it is far. The reasons for these have been explained in *Sefer Iyunei HaKochavim*; see also *Sefer Sha'ar HaShamayim* of our teacher R. Moshe Almosnino, may his memory be a blessing [R. Moshe Almosnino, *Sha'ar HaShamayim*].

These and similar proofs compelled Ibn Rushd [Averroes], the [self-styled] chief philosopher, to acknowledge this and to say that the Moon moves about a center other than the center of the earth — and he was a thorn to the astronomers. He sought to uproot their hypotheses of eccentrics and epicycles, for he was bound like a dog after Aristotle in his opinions, and he thought to find ways to uphold the [astronomical] observations by means of spheres concentric with the center [homocentric spheres]; but he did not succeed, and at the end of his days he

despaired of this, as you will see in his commentaries on the *Metaphysics*. After him no scholar or thinker was found who could uphold what is observed in the stars together with the Aristotelian hypotheses.

We, by contrast, see that with these astronomical hypotheses we know the occurrences — the motions and the eclipses — with wondrous precision, even though they [the heavenly bodies] are exceedingly far from us — something the babblers and the philosophers cannot do for certain. Let us therefore do as Aristotle himself commanded: to take counsel in every craft with its masters, and in astronomy in particular, to heed the voice of the astronomers, for they have found fitting ways by which all the changes in the heavens may be upheld in wondrous order.

Behold, Ptolemy — seeing how greatly necessary it is to know the measure of parallax precisely in matters of astronomy, and that when a star is very far it is impossible to know it, since it is not perceptible, as we explained — came at it by another way, in reverse [working backward]. He investigated from the altitude above the horizon, or its descent from the zenith, and from the distance that a star has from the [center of the] earth [when these are] known, and found the measure…

Gevurat Yah (גבורות יי) — continued

[Degree 10 (מדרגה י׳), continued] — the Moon's distance and the general method for star-distances

the parallax of vision. In the diagram described above [the triangle אעד], in triangle אעד, we know side עד — [the line from] the earth — and side אע, half the diameter of the earth, and angle ע, which is the measure of arc יך [?]. The distance of the star from the zenith [נכח ראש] is the value ה, found by the computation explained in Question 12 of *Sefer Chukos Shamayim* [the Book of the Laws of the Heavens] and in Pillar 12 [מטור י"ב], before Corner 4 [פנה הד׳], in *Yesod HaYesod* [the Foundation of the Foundation]. From this we obtain angle ך, which is the parallax, and so likewise [for the others].

As for the distance of the star from the earth, [the astronomers] found it by remarkable methods: one investigated and found the measures of the eccentric radii running out from the center of each star [planet] and of its [epicyclic] circle, as you will see in the *Almagest*. I too will bring you a portion of his words afterward, God willing. And once one knows from the side of the motions

whether the star is at the apogee of the epicycle [גובה המעגל] or at its perigee [תחתיתו], and whether the center of the epicycle is at the apogee of the eccentric [גובה היוצא] or at its perigee, or between these extremes — according to where its place is, which the geometric computation reveals — then its distance from the center of the earth will be known. Upon these distances [Ptolemy] built a table [לוח] giving the values of the parallax in each and every position. Look in the Tables of the Motions [לוחות התנועות] and you will find them there.

He therefore concluded, by the geometric computation, that when the Moon is at the apogee of the epicycle and the center of the epicycle is at the apogee of the eccentric — which is the greatest distance the Moon recedes from the earth, namely at conjunction, when the mean Moon is opposite the Sun [i.e. at full Moon] [?] — the Moon stands **64 [earth-radii] 10′** [64;10 half-diameters of the earth]. And when it is at its greatest descent and nearest to the earth — that is, the Moon at the perigee of the epicycle and the center of the epicycle at the perigee of the eccentric, which [occurs] at the mean quadratures from the Sun — the Moon will be distant from the earth no more than **33;33** [the print reads "מ33.׳33 ."; the leading "מ" is the preposition "than," not a digit] half-diameters of the earth. And you can find these stated distances, if you wish, once you know the measure of the half-diameter of the eccentric-from-center in every position — that is, [the star's] distance from the earth at each and every point — and the half-diameter of the epicycle, together with the addition and subtraction [the corrections]. [This] completes the matter, and it is not our business in this brief treatise.

Degree 11 (מדרגה י"א) — Ptolemy's superior method for the Sun's distance

Now, apart from the methods above, Ptolemy taught an excellent and noble method by which we may easily learn the distance of the Sun from the center of the earth. It is founded, however, upon what is already known — the distance of the Moon from the center of the earth, as above. For first he sought to know the measure of the apparent diameters of the solar and lunar bodies, and he found them equal: when both of them are at their greatest distance from the earth, at conjunctions or oppositions, they appear as about **31 ⅛ [first-]minutes [ראשונים] of the 360 degrees of the celestial sphere** [גלגל]. And so Aristarchus [?] [איפרכו׳ — the print reads *Iparkho* = Hipparchus; but context of *with his dioptra* fits Aristarchus] found them with his dioptra, as Ptolemy reports — for we, in our poverty, were not granted the books of that excellent man.

The honorable and learned Archimedes [וארכימידיס] found their diameter by another method — see his books. And Tycho [טי"קו] too found the Moon's diameter at its fullest [at perigee] to be **36 [first-]minutes**, and at conjunction **48;28** [the print reads "4828.׳"; sexagesimal, not a fraction], and the Sun's diameter, even smaller, **30′**. We have no mind to bring in here a mixture of opinions, for these matters are explained in our commentary which we [wrote] on the *Almagest*. Here we will follow the footsteps of Ptolemy, who is the first master, and from whom all who came after learned.

You will find that both Ptolemy and Hipparchus [איפרכוס] take the Moon's diameter at its greatest distance to be no more than half a degree. The astronomers were accustomed to divide this apparent diameter into 12 equal parts, and they called them "digits" [אצבעות]. Ptolemy did this in order to indicate, in eclipses, how many digits of the said bodies are darkened.

Now examine two lunar eclipses. See, in the first, that the Moon was eclipsed [over] a quarter of its body — that is, **3 digits** — and it had a [northern] latitude [רוחב], a northern latitude of **48;30** [the print reads "4830.׳"], and a southern latitude [רוחב דרומי] of **40;40** [the print reads "4040.׳"].

And as you know from the study of the stars: so long as the Moon has any latitude from the head [ascending node] or tail [descending node] of the Dragon [the lunar nodes], it [partly] escapes [the shadow]. When we know these two latitudes, one from the other, we find the difference [to be] **7;50** [the print reads "750.׳"]. Therefore this difference, or deficit, which the Moon's latitude had in the second [eclipse], caused it to be eclipsed in [only] a quarter of its body — more than in the first eclipse. By this, then, this difference is **twice** [?] the whole diameter of the Moon — that is, **31′ ⅓** [the print reads "31 ⅓ ראשונים"]. And the like Ptolemy found, with the dioptra, for the solar diameter.

From the first eclipse he knew likewise the diameter of the shadow [אלכסון הצל], for the center of the Moon exceeded half the diameter of the shadow by [about] a quarter of its [own] diameter, which is **7;50** [the print reads "7.50"]. Therefore, if you subtract this from the latitude above, **48;30** [the print reads "4830.׳"], there remains **40;40** [the print reads "40,40׳"], half the diameter of the shadow. Or, from the second eclipse he knew [it]: for from the point where the Moon was up to the head or tail of the Dragon was **40;40** [the print reads "4040 ׳"], and that is half the [latitude].

By this, then, the ratio of the Moon's diameter to the shadow's diameter is as **1 to 2 28/47** [the print reads "1 28/47 2 אל"], or as you know it from [the work]

HaYisraeli [of Yisrael HaYisraeli / Israeli], in Chapter 17, Treatise 3 [פרק י"ז מאמר ג׳], as **1 to 2 ¾**. Ptolemy, however, set their ratio as **5 to 13**.

[Ptolemy's geometric construction for the solar distance]

Diagram. *Caption:* ציור מד [Diagram 44]. The construction depicts the alignment of the Sun, Moon, and Earth (at a lunar eclipse) with the cone of the earth's shadow, used to derive the Sun's distance. The configuration described in the text:

- **א** — the center of the solar body [מרכזו א], at its greatest distance from the earth.
- **ס** — the terrestrial globe / Earth [הכדור הארצי הוא ס].
- **ל** — the Moon [והירחי ל].
- Line [א–ג–ץ] **אגץ** — the line passing through the centers of all three [bodies], drawn straight through the centers of the three; they are joined by the straight lines.
- Lines **בפ** and **דס** [the print: "בפ דס"] — drawn straight and produced [extended] until they meet at [ז] **ג** — the extreme apex of the conical cone of the earth's shadow [צ — תכלית המחודד האצטוני של צל הארץ].
- At the points where the said lines touch — that is, at points **ם, ב, ך** [?] — straight lines **דס פס** [?] are drawn; these are reckoned as [equal to] the diameters of the spheres of the Sun and the Earth, since, given the great distance between them, the difference between them and the true diameters is not perceptible.
- So too lines **אה בה דה** and **אצ בצ דצ**. From center **ז** [?] take, with the [construction] [?], equal parts — these are lines **הנ הי** [?] — according to the Moon's great distance from the center of the earth, which Ptolemy set sometimes [as such].
- Line **הם** [is] half the diameter of the earth.
- Line [קיר] **קיר** [is] the diameter of the shadow at the point the Moon passes through it at that time.
- Line **לגנ** [is] the Moon's diameter. These two diameters stand perpendicular [נצבים] upon line **אצ**, and [the construction] is produced to point **כ** [?].
- From there a line **מי** is drawn, parallel and corresponding to line **לה** — meaning, to clarify the measure of the distance of the solar dwelling [the Sun] from the center of the earth, that is, the ratio of line **אה** to line **הז**, half the diameter of the earth.

Construction and computation:

Line **לג** was found, with the dioptra, [to be] **31′ 20″** [the print reads "3120′"], and the like is the measure of angle **להג**, learned from Pillar 21 [מלמוד כ"א], before Corner 2 [לפנה ב׳], of *Sefer Yesod HaYesod*. By this, then, line [angle] **לג** is **0;17,33** [the print reads "16. 31. 1" at the head of p. 339; the consistently printed value for לג is 0;17,33]. And since angle **לגה** is right by the construction — for line **לג** stands perpendicular upon line **אה** — angle **גלה** will [stand] opposite **לב** [?] for the first [?]; the third angle too, **הלג**, is known, [namely] **89° 44′ 20″** [the print reads "89. 4420 .′"]. Therefore all the angles are known. From Pillar 1, before Corner 4 [מטור א׳ לפנה ד׳], of *Sefer Yesod HaYesod*, we know the ratios of the sides in triangle **להג** [לנקן (?)], and the proportion runs thus:

as	sine [בקע]	to	sine [בקע]
להג	89° 44′ 20″ [89.4420.′]	to	[′40 15 .0] 15,40;0 **גהל**
so **הג**	61 ⅛	to	[31 .17 .0] 17,31;0 **גל**
sine	99998		sine 455

And Ptolemy found its diameter [as] **0;17,33** [the print reads "33".170.′"], which are of the same parts [units] in which line **הם** is reckoned as one degree.

Since, as we said, the ratio of **לג**, half the Moon's diameter, to **קי**, half the diameter of the shadow, was as **5 to 13** — if **לג** is **0;17,33** [0.17.33], then line **קי** will be **0;45,38** [the print reads "0.45 38′"], which is the whole diameter of the shadow [?].

It will be **1;31,16** [the print reads "1. 31. 16"]. And the proof runs thus: if **5**, half the Moon's diameter, **לג**, equals **0;17,33** [0.17.33], how much does **13**, the diameter **קי**, equal? You will find **0;45,37,48** [the print reads "45. 37.48"]. And since lines **גל** and **מיק** [?] are equally distant from point **ה**, which we set [as origin], lines **גה** and **קי** [?] [are] equal, and the two triangles **מגי** and **הי** [?] are proportional from [the principle of] "two by two" — that is, just as **בג** [?] is double [?], so **מג** is double; and likewise the triangles **קים** and **סטמ** [?] are proportional, just as **קמ** is double **שמ** [?], so **קי** is double **סט** [?]. Therefore, [by] proposition 8 of [Book] 5 [the *Elements*], **גכ** [?]: the sum of **קי** and **מג** [is] double the sum of **טו** and **ק** [?].

And since the diameter of the earth, **טוז** [?], is reckoned as **4** [?], the gathering [sum] of the two lines **מג** and **קי** will therefore be **גהע** [?]. Of these, the measure of **לג** is **17,33** [the print: 17.33], and the measure of **קי** is **45,38** [45.38], which

make **1;3,11** [the print reads "1.3.11"], and there remain **49,56** [the print reads "49.56"], which is the measure of line **מל** [?].

And if we subtract from line **סה** [there] is found the measure of **סמ** [is] **3 11** [?] [the print reads "3 11 סט"] — because line **זמ** [?] is parallel to line **הזד** [?], and **מל** to **סה** — the two lines **הט למ** [?] [are] likewise equal, and line **סט**, the remainder between them, was **1;1** [the print reads "1.1 שביניהם"]. From [proposition] 5 [Book] 5 [?], the ratio of **שמ** to **טמ** [?] is as **לה** to **לה** [?]; and so **סה** to **הד** [?]. By alternation [ובחלוף], the ratio of **סט** to **סה** is as **לה** to **הך** [?]. But the ratio of **לה** to **הר** is as **הג** to **הא**. Therefore, by equality [כשוי], the ratio of **סט** to **סה** is as **הג** to **הא**. The proof yields line **הא** [as] **1210**, approximately: as **סט** is to **סח** [?], so **הג 10;64** [the print reads "10. 64"] is to **הא 1210**.

Once line **סף הא [סה?]**, the solar distance from the center of the earth, [is found] — which is what we wished to know. And **by the reckoning of al-Battani** [אלבתין] it is **1146**; by the reckoning of **Copernicus**, **1179**; and by the reckoning of **Tycho** [טיקון], **1182**. The difference among these precious results is not great. And as I will inform you, God willing, [once you know] the measure of half the earth's diameter, you will know that this distance is close to **100000** — I mean, **a thousand thousand [= a million] parsa'ot** [Persian-miles, parasangs].

Degree 12 (מדרגה י"ב) — the sizes of the three globes [Sun, Earth, Moon]

Now that you know the distance of the Sun from the center of the earth — that is, the measure of line **אה** — you also know already the measure of line **הג** too. For from the [triangles], the measure of **גה** [?] you can easily learn the ratios of the three globes, one to another.

By this, then, [for] line **אד** alone [?]: from [proposition] 5 [Book] 5, as the ratio of **הג** to **גל**, so [the ratio of] line **הא** to **אד**, and it will be: as **הג 10;64** [the print: 64.10] to **גל 17,33;0** [the print: 0.17.33], so half the diameter of the Sun [is to] **5;30** [the print reads "5.30"]. The proof runs thus. By [the triangle] **מג** [?] too you can find the matter, by the proportion: as line **גה** [?] with **הא 1210** is to **אד 30;5** [?].

And you can also infer thus: since line **קי** would equal half [?] [if?] they were equal at 20 degrees, and line **מב** [?] would be equal — if you subtract the known **קי** [?]… [the print here is tightly abbreviated; reconstruction:] if **20 degrees** [?] gives **128;20** [the print: 128.20], how much does **22 [?]** give of **מג 14;1** [the print: 1.14], that is **22 [?]** [?] — which is **1274 [?]** [the print reads "1274"]. Find the

measure of **אד**. Line **מה** [?] too is known: as line **שח** [?] to **אם יך** [?] gives the [proportion?]; line **יך** [?] is given. Therefore… [the print: " אם יך ניתן הט איכן יא יהן שעור יך"].

[Reconstruction of the worked solar/lunar/earth-size proportions, p. 339, fol. 309:]

If you divide **קי**, half the diameter of the earth [?], by half the diameter [?] of **הזא** [?], the ratio is as **1 to 1;5** [?]. Then [returning] to seconds, **3600**, and you divide [them] by line **לג**, half the Moon's diameter, which is **0;17,33** [0.17.33], you will find **3;25** [the print reads "25. 3"]. Therefore the diameter of the Moon to the diameter of the earth is as **1 to** [?]…

Again, to know the ratio of the Moon's diameter to the Sun's diameter [?], infer thus. By the ratio **5;30** [5.30] [?] — that is, divide **5;30** by [0.17.33] **17,33;0 לג** [the print: "5.30. 33-7.לג.33 .עם" — partly corrupt] — as **1** is to [5.30] **30;5 אד**…

Therefore, after you know the ratios of the diameters, if you [know] **0;17** [?], you will find that it goes in **18,48** [?] [the print reads "18, 48 : 0"]. Therefore… [continues].

[as] he has demonstrated in [this] Degree. If you wish to know the volumes [נפחיות / נשטיות — the solidities/bulks] of these three spherical bodies, it is easy. Second, from this treatise [you can] find their cubes [מעוקביהם], and more: that you cube the said ratios — that is, the diameter of the Moon [?]. We have easily found them in known fractions, and these are:

- The solar diameter, by the ratio of the lunar diameter [?], is as **18 ⅖ to 1** [the print reads "18 ⅖ 1 אל"], and its cube [מעוקם] [is] **6644 84/125** [the print reads "6644 84/125"]. And likewise, if the cube is **3;25** [3.25] [?], which is the ratio of the [earth's] diameter to one [?]…
- …the diameter of the earth with **1** [?] — the diameter of the Moon — you will find **39;51** [the print reads "51. 39"], that the body of the Earth is bigger than the body of the Moon, approximately. [?]
- And likewise, if the cube is **5 ⅛** [the print reads "5 ⅛"], you find **166;40,30** [the print reads "40o. 30. 166"; the "o" glyph is a degraded zero, the order resolves to 166;40,30].

Therefore the **Sun is bigger than the Earth by [about] 166 times** [?], and bigger than the **Moon by 6644 times** [?]. All this is according to the opinion of Ptolemy.

According to the opinion of **Copernicus**, the Sun is **162 times** less [?] [recte: greater — "the Sun is 162 times the body of the Earth"; the print reads "פחות ט פעם 162"] than the body of the Earth, and the Earth is **43 times** [less] [?] than the

body of the Moon [recte: greater; "the Earth is 43 times the Moon"], and **[the Sun is] 700 times** [the print reads "700"], less **62′** [?], than the body of the Moon — as you will see [in] his *Foundations* [יסודותיו], according to Book 4, Chapter 20 [the print reads "כפ׳ כ׳ לרביעי"] regarding his comparisons [מהקפותיו].

For Tycho [ולטיקון] and others there is a slight difference; and one need not wonder if, in these wondrous and far-off [matters], there should be some small difference between one reckoning and another — all the more so since [the result] is not according to [perfect] truth. And as you know, this method of [finding] these ratios is most excellent, beyond all speculation and conjecture. Yet such precision in matters like these is the work of boys and idlers; so let them not censure me if I have not been precise in a reckoning [made under] the pressure of the times.

Degree 13 (מדרגה י"ג) — the measure of the earth's shadow, and that the Earth is larger than the Moon

Now, if you wish to know the measure of the earth's shadow and its end [apex], subtract line **קי 45,37,48;0** [the print reads "45. 37.48"] from line **כה 60** [?], and there remains line **סט 22,11;14** [the print reads "11. 22. 14"]. Drop a line from point **ק** perpendicular onto point **ט**. From [proposition] 5 [Book] 5, the two upright triangles **סטק** and **קיץ** [?] are proportional, since [in them] lines **קי** and **טה** [?] [are] equal, and lines **טק** and **הי** [?] correspond. Therefore the proportion runs: as **סט 22;14** [the print: 14.22] is to **טק 10** [the print reads "10"], so **קי 0;45,38** [the print: 45.38] is to **יץ 50;203** [the print reads "203. 50"], approximately. Therefore, when you add to line **יץ** line **הי 10;64** [?] [the print: 64.10], the whole diameter of the earth's shadow, **הץ**, will be **267** [?] [the print reads "267"] times the half-diameters of the terrestrial globe. And that is what was intended.

And without [the reader] troubling himself — the [reader] for whom the abridgment [is meant] — to understand the precision in these said measures, he can nonetheless learn, at the least, that the Sun is larger than the Earth, and the Earth [larger] than the lunar body. For if the globe of the Sun were equal to the globe of the Earth, the shadow that comes from the Earth would be, in form, a long rectangle extending to infinity [נמשך לא"ת], as is explained in the science of optics [חכמת הראיה], in [the work] of Vitellio [ויטיליאון / Witelo], Treatise 2, Theorem… [למוד כ"ו (?)]. And if the Earth were larger than it [the Sun], the shadow would go on widening to infinity, as is explained there, Theorem 28 [למוד כ"ח], and it would necessarily follow that at night, even when the skies are clear,

some of the stars would be eclipsed, or reckoned [partly] darkened, on account of the earth's shadow. Mars, Jupiter, and Saturn, when they are in opposition to the Sun, would likewise be much eclipsed — and this is never observed.

Therefore one must say that the shadow does not go on to infinity — because the Earth is smaller than the Sun, and the shadow is pointed and conical, ending at a known point according to its distance from the earth, as is known to the masters of optics [חכמי הראיה]; and it does not reach the sphere of Mars, much less the rest of the stars at all. And since the earth's shadow is pointed [and] finite, it is necessary that the diameter of the Earth be larger than the diameter of the shadow. Yet we see that the Moon, even though it is distant from the earth, nevertheless enters wholly within the said shadow and is eclipsed for a good while before it emerges from it. Therefore the Earth is larger than the lunar body many times over — there is no doubt about the matter. Even so, to the sight of the eye...

Gevurat Yah (גבורות יי)

[Continuation of Degree 13, on the apparent sizes of the heavenly bodies seen from different vantage points.]

The body of the Moon appears large to us because of its nearness, compared with the rest of the stars; it is not, as you might have supposed, that this distance is small. Rather, the half-diameter of the Earth counts for a great deal in proportion to it. For this reason Ptolemy in the first treatise of the *Almagest* says that one ought not to seek the true position of the Moon by means of instruments planted on the surface of the Earth without error, since the surface of the Earth is not [equivalent to] the center in proportion to the Moon's distance. It would therefore be proper for the center of the instrument to coincide with the center of the Earth, in order to take the Moon's altitude above the horizon and not above the surface — for from the surface there arises a discrepancy of parallax in the taking of the altitude. Rather, one ought to seek the Moon's place by means of [computations from] eclipses. Consequently, instruments resting on the surface of the Earth are good, and are reckoned as though they were at its center, for the study of the stars in the eighth sphere. And I have already explained that the half-diameter of the Earth is not reckoned [significant] in proportion to the great distance between them.

For Mars too, and for the Sun, the parallax is so small that it is scarcely perceptible, as we explained in the matter of the shadows of the gnomons. But in

proportion to the sphere of Venus and Mercury it is perceptible, and all the more so in proportion to the Moon, whose body appears as a large portion of its sphere. If we were at the Moon's position and looked at the Earth from there, the Earth would appear more than three times larger than the body of the Moon. From the Sun's sphere we would see the Earth twice as large as Venus appears from here. If we were at the sphere of Mars, we would see the Earth as small as one of the dim stars; and above Mars we would not see it at all. All these matters [are demonstrated] by proofs and by calculation, which you will find, with God's help, in the *Sefer BeShem* [the *Book of [...] in the Name*].

Degree 14 (מדרגה יד)

[Source]

Now I will inform you of the investigations of the masters of astronomy concerning the dimensions of the stars, their distances, the thickness of the spheres, their diameters, and their circumferences. I need not transcribe for you here what you have learned in the *Book of the Study of the Stars* — the reasons and causes that compelled the masters of astronomy to posit that there are circles whose center is other than the center of the universe, and upon these, small circles called, by our translators, epicycles. Upon this is built everything I shall tell you. They posited as an agreed matter that the spheres touch and press upon one another, and that there is no void between them, according to the opinion of the philosophers.

After this, let your eyes turn to the present figure (Diagram 45) [ציור מ"ה], which is a model of one of the spheres of the stars, together with all its thickness and what is enclosed within it — for example, the sphere of the Moon or of the Sun.

Diagram 45. A schematic model of one of the planetary spheres, showing its full thickness with the eccentric (deferent) and epicycle enclosed within it (e.g. the sphere of the Moon or the Sun). The text describes the labeled points as follows:

- אקב [A-K-B]: the concave inner surface of the enclosing sphere, where it touches the convexity of the sphere of Mercury (or whatever lies above it) on its inner concavity closest to us.
- עגול ובז [circle V-B-Z]: the eccentric (the deferent, "the one drawn from a center"), in which lies the center of the epicycle — point ן [N], in which the Moon sits — or [point] ו, in which lies the center of the solar star.
- ץ [Tz]: the center of the eccentric circle, which carries and turns [the bodies]; the carriers, namely the eccentric, are the aforesaid circles.

- אקב ודה [A-K-B / D-H]: and the center of the Earth is point ח [Ch], and around it are marked the two surfaces: the concave surface of circle דה [D-H] (in its construction אק [A-K]), and the convexity of דד [D-D] (and the concave surface אק).
- ץ [Tz]: point Tz is the center of the eccentric ובז [V-B-Z], and upon it is also marked the convexity דד [D-D] and the concave surface אק.

Therefore the whole thickness of the sphere is line אך [A-K] or וכ [V-K], and the thickness of וף [V-F] equals the thickness of ו. The thickness of the eccentric is אי [A-Y], and its measure is like the diameter of the star or of the epicycle within it. Line צך [Tz-K] is the measure of the eccentricity — that is, what lies between the two centers. The upper point ץ [Tz] is called the apogee, the "height of the star's elevation"; and the lower point ן [N], which is closer to the center דן [D-N], is called the perigee, the "depth of the star's elevation." The line drawn [connecting] with the marked points indicates the mean positions between apogee and perigee.

Now I wish to explain to you beforehand that the thickness יד [Y-D] (or וך [V-K]) is double the eccentricity צך [Tz-K]. Let us posit חש [Ch-Sh] equal to לחצ [L-Ch-Tz], and since the lines חא [Ch-A], חכ [Ch-K]

[Source]

[Continuation of the geometric proof:]...are equal, [being drawn] from the boundary of the circle, when one subtracts from them the equal segments, the [remainders] will be from the third comprehensive proposition [i.e. by Euclid's common notions]. The lines צא [Tz-A] and שכ [Sh-K] are equal; and since line צכ [Tz-K] exceeds line שכ [Sh-K] by the measure צש [Tz-Sh], line צכ [Tz-K] likewise exceeds line צא [Tz-A] by [that same] measure; and it has been said that צש [Tz-Sh] [equals] צא [Tz-A]. Now the line of the concave surface of circle אכ [A-K] — these being the two diameters of the eccentric — therefore line צכ [Tz-K] [exceeds] צי [Tz-Y] by the measure צש [Tz-Sh]. But line צב [Tz-B] also exceeds line צי [Tz-Y] by the measure יב [Y-B], and these are equal; and by this the thickness יב [Y-B] is equal to double צך [Tz-K], that is, to צש [Tz-Sh] — which is what was to be proved.

It follows from here that if you wish to know the thickness of each of the carriers, [whether at] the apogee or the perigee of the eccentric — that is, line יד [Y-D] — you double the measure of the eccentricity, and you will find that צש [Tz-Sh] equals יד [Y-D]. And if you add to the thickness the diameter of the star ין [Y-N] or of the epicycle ון [V-N], you will know the whole thickness of the sphere אך [A-

K] and וכ [V-K]. And when you add to the thickness אף [A-F] the distance between the center of the Earth דן [D-N] and the concave surface ך [K] — that is, line חך [Ch-K] — you will know the whole line זא [Z-A], which is the distance from the center of the Earth to the convexity of the sphere ו, [namely] the concave surface of the sphere above it. At once, given the half-diameter, they knew the circumference, by what was taught in Degree 20 of this book.

After they knew thus the thickness of the sphere of the Moon, they investigated the eccentricity of Mercury, according to its study, in which there are four circles in place of the two that the rest of the stars have; and [they investigated] the measure of the diameter of its epicycle, and its thickness became known to them. So too for Venus, the Sun, and all of them — except the eighth sphere, for the ladder of the disciplines did not reach it, owing to its great height, and they did not know its thickness by demonstration. Al-Farghani [אלפראנאן] reckoned that it is according to the distance between the center of the Earth and its [the sphere's] concave surface — just as it is, indeed, in the sphere of the Moon, whose concave surface, as we explained in Degree 10, is 33 times the half-diameter of the Earth, and up to its convexity 64 times. [This is the case] even though, according to the opinion of Copernicus, the perigee is 52;17 and the apogee 68;21, according to the angles of parallax that he found, as it appears in the fourth [book] of his *Revolutions* [*De Revolutionibus*].

But know that these distances are the smallest there can be, [calculated] with precision, because they reckon that the spherical bodies of the stars — or the epicycles — touch the concave surface of the [body] above and the convexity of the [body] below among the carriers, namely the eccentric, and that between these two aforesaid [bodies] of the eccentric there is no thickness, but [rather] they touch immediately. And perhaps this is not so, [namely] that line אי [A-Y] is so great that the extremities of the star or the epicycle do not touch the [eccentric] circles at all, and the whole thickness of the eccentric is removed from them by a great deal — in such a way that there is reason to think that the thickness and the distance would be more and more, added to the calculation, and never less. Therefore reason dictates that the eighth sphere is very dense. And according to the opinion of Tycho, the stars in it are not all at one distance; one is higher than its fellow. Therefore some of them are very bright and some dim and small. In truth, the known ones are but a drop compared with the hidden ones among the carriers, for the gates of inquiry into these lofty and concealed bodies have been shut.

Degree 15 (מדרגה טו)

[Source]

By the aforementioned methods, together with what I shall inform you, with God's help, there became clear to them the thickness of the spheres, the measures of the eccentric's diameter and its circumference, the eccentricity, the diameters of the epicycles, and the magnitudes of the globes, according to

[Source]

what is set before you. If you wish to see with your own eyes in the garden of the Lord, look and contemplate them, and may they be a blessing to you. But do not speak of such matters as these with the multitude, lest they despise your words and mock and scoff at you. Leave them to their occupations and their idols — the silver and the gold from which we are sustained and which is our bread. For were they all to turn to these studies, the philosophers would return to being tillers of the soil, and the world would have no permanence apart from them.

The thicknesses of the firmaments you will find if you subtract the half-diameter of the concave surface from the half-diameter of the convex surface for each one.

The following table gives, for each sphere, the half-diameter (in earth-radii — degrees and minutes), the half-diameter in miles, and the thickness in earth-radii and in miles. *The table reads right-to-left in the source; logical column order is preserved below.*

Sphere (half-diameter of)	Half-diameter in earth-radii (deg. min.)	Half-diameter in miles	Thickness in earth-radii	Thickness in miles
Concave surface of the Moon	33;42	120630 15/22	—	—
Convexity of the Moon	64;29	229687 1/1 [recte: ?]	31 7/15	109056 1/1 [recte: ?]
Concave surface of Mercury	—	—	—	—
Convexity of Mercury	167;57	600167 14/33	103 1/2	370479 51/66

Sphere (half-diameter of)	Half-diameter in earth-radii (deg. min.)	Half-diameter in miles	Thickness in earth-radii	Thickness in miles
Concave surface of Venus	—	—	—	—
Convexity of Venus	1121;21	4013923 7/22	953 41/60	3413755 50/66
Concave surface of the Sun	—	—	—	—
Convexity of the Sun	1216;5	4353025 25/44	94 11/15	339102 1/4
Concave surface of Mars	—	—	—	—
Convexity of Mars	8853;47	31692400 25/44	7637 2/3	27339375
Concave surface of Jupiter	—	—	—	—
Convexity of Jupiter	14378;19	51467897 3/11	5524 7/18	19775497 7/11
Concave surface of Saturn	—	—	—	—
Convexity of Saturn	22612 1/2	80942471 13/22	8234 1/6	29474573 10/11
Concave surface of the eighth sphere	22612 1/2	80942471 13/22	—	—
Convexity of the eighth sphere	45225	161884943 2/11	—	—

according to the opinion of the aforementioned al-Farghani.

And these are the measures of their circumferences in known miles:

Circumference (between the surfaces named)	Miles
Concave surface of the Moon	758250
Convexity of the Moon and concave surface of Mercury	1443750
Convexity of Mercury and concave surface of Venus	3772500
Convexity of Venus and concave surface of the Sun	25230375
Convexity of the Sun and concave surface of Mars	27361875
Convexity of Mars and concave surface of Jupiter	199209375
Convexity of Jupiter and concave surface of Saturn	323512500
Convexity of Saturn and concave surface of the eighth sphere	508781250
Convexity of the eighth sphere — that is, in miles	1017562500

[Source]

These are the measures between the center of the Earth and the center of the carrier (the epicycle) or the star, which are called the eccentricities [of the centers].

The following table gives the eccentricity for the Sun and the Moon, expressed both as "how much, when the half-diameter of the Earth is 1" and "how much, when the half-diameter of the carrier is 60 parts." *Table reads right-to-left in source.*

Body	When the Earth's diameter = 1	When the carrier's half-diameter = 60 parts
Sun	2 1/10	44;2
Moon	12;28	10;9

[Source]

[The first table of circumferences in miles continues from the prior page; its right-hand portion repeats here:]

Circumference (between the surfaces named)	Miles
Convexity of Mars and concave surface of Jupiter	199209375
Convexity of Jupiter and concave surface of Saturn	323512500

Circumference (between the surfaces named)	Miles
Convexity of Saturn and concave surface of the eighth sphere	508781250
Convexity of the eighth sphere — that is, it is in miles	1017562500

[And the eccentricities table, right-hand portion, repeats:]

Body	(eccentricity, deg. min.)	(eccentricity, deg. min.)
Saturn	980;53	3;25
Jupiter	532;12	2;45
Mars	503;12	6;0
Venus	12;7	1;8
Mercury	5;48	0;3

Were my heart at ease, I would go over these computations and test the numbers against those brought in the books of the great authors; and in the *Sefer BeShem* you will find the matters more ordered, where the source of these distances is investigated, and their thicknesses, and other desiderata, with God's help.

Now with Ptolemy, the [eccentricity] of the Sun is 2;30, and of Venus 1 15/41 [?], and the remaining [values] are as measures. [Recte: the aforementioned 2;30 vs. the table's 2;1/10 differs.] The aforementioned [point], that what you find Ptolemy saying — that the measure of the half-diameter of the eccentric of the Moon is 41;49, and the eccentricity 10;19 — is in accord with the aforementioned number: for if you set up the proportion thus: "if 41;49 gives 10;19, what will 60 give?", you will find 12;28 [?] approximately. And by this method, invert these measures into other proportions.

These are the measures of the half-diameters of the epicycles | and these are the measures of the half-diameter of the carrier (deferent) of the star or the epicycle:

The following compound table gives, for each body: (a) "how much the distance from the center of the Earth to the apogee is, when [that apogee] is 60," according to several opinions; and (b) "how much the half-diameter of the Earth [is]" / "how much the half-diameter of the Earth [is]," each according to its [opinion]. *Reads right-to-left in source.*

Body	Apogee distance, Ptolemy (=60)	Apogee distance, others	Apogee distance, "some say"	Half-diam. of Earth (one opinion)	Half-diam. of Earth (another)
Saturn	6;30	6;30	1165;23	1866	2298
Jupiter	11;30	11;30	(Sun)	2225	2743
Mars	39;30	39;30	17225;16 (Saturn)	3312	4085
Venus	43;10	48;10	11611;31 (Jupiter)	461	571
Mercury	22;30	9;0	5032;4 (Mars)	43;31	51
Moon	5;15	6;14	641;45 (Venus) 116;3 (Mercury)	5;5	5;8

And the small epicycle of Mercury is 5;48 — and that is the half-diameter of the [epicycle of] Mercury [?]; reckoned as 3;116 [recte: 116;3] it will be a little more than three parts.

Now you see that the epicycle of Venus is large, according to what appears in the right-hand table, [larger] than that of Mars; but in truth it is not so, as appears in the left-hand table. For according to the excess of its [Mars's] distance from the center of the Earth over the distance of Venus, so is the excess of each of the sixty parts of its [epicycle] over that of its fellow. And according to the left-hand table, the epicycle of Mars is the largest of them all. And this I say: that the error did not deceive some of the eminent philosophers, who reckoned that these parts belong to [a system] in which the firmament is 360 parts, and on this supposition they

inferred that if the epicycle of Venus were 45 parts, it would pass through the very center of the Earth — which is false. Therefore from here they wished to uproot Ptolemy's positing of the epicycles; but they did not understand his words, that there is no...

[The text continues beyond this page.]

by the calculation of the reckoner of the circles, but rather according to the parts of the issuing [circle] — the center of the carrying circle [deferent] is their center — so that all those diagonals reckoned as sixty are not equal. Therefore the whole circle is sunk into the thickness of the sphere of Venus, and likewise the rest of the spheres; and this matter causes no difficulty for the astronomers. Let us now test it according to what was stated here and according to the demonstration of the Degree that has passed, since I wished to know the measure of the thickness of the Moon's sphere and its distance.

Now I have already known that the distance between the center of the earth and the [Moon's] concavity — that is, A–H [א–ה] — is 33, 42 [recte: 42;33, in earth-radii]. Take the half-diameter of the earth, and you will know the thickness of the sphere that brings the [Moon's] concavity. The measure of the issuing [eccentric] from the center of its carrying circle — that is, Tz–Ḥ [צ–ח] — is 9, 10 [recte: 10;9]; and against it is 20, 18 [recte: 18;20]; the measure of Y–D [י–ד] upon it is twice 31, 28 [recte: 28;31], which is half the diagonal of the Moon's circle, which is 5, 5 [recte: 5;5]; and its issuing [?] is 42, 33 [recte: 33;42]. When you add upon it 42, 33 [recte: 33;42], there comes out for you R–A [רא, the line ר–א], the distance of its center or its concavity, 10, 64 [recte: 64;10]. As the seeker wrote, [or] if you take twice the issuing of the center of the carrier in [its] circle, as written, which is 11, 36 [recte: 36;11], and the falling of the circle — that is [?] — which is 2, 87 [recte: 87;2], there comes out 38, 98 [recte: 98;38]. And since the center of the issuing [eccentric] that carries the circle moves upon the small circle whose half-diagonal is, according to Ptolemy, 5, 12 [recte: 12;5], the whole thickness of the sphere will be, as written, 50, 103 [recte: 103;50] — and this is close to what is found in al-Battani [בליח, recte בלית].

And in this way [carry out] the inquiry and the investigation upon the rest [of the spheres], and correct the numbers if any error has fallen into them.

Degree 16 [מדרגה טז]

The two great luminaries — in Ptolemy and Copernicus — did not seek to know the sizes of the spheres of the fixed stars; and even among the wandering

[planets] they investigated only Mercury [שבש, recte שבת? — see note] and the Moon alone, because those were perceptible, tangible matters to them, and all their words are reliable and well-founded after the manner of the science of measurement, and they kept far from every doubtful and uncertain thing. Not so the Arabs — for instance Ibn al-Battani [אלביה ן, Ibn al-Battani] in chapter fifty, and al-Farghani [אלפרנאנום] in chapter 22 — who did seek to know the sizes of the planets [stars] and followed in the footsteps of Ptolemy and along his foundations; and it appears that they attained this thing, though not in completeness. Even so, a little knowledge — deficient and small — in precious and sublime matters is better than abundant knowledge in lowly ones; as the chief of the Peripatetics [Aristotle] reckoned. All the more so, since the later ones roused themselves to inquire, to investigate, and to be precise still further, it is fitting to hold their good to their credit.

Now they took from the Almagest the great distances and the small [perigeal] distances of every star, joined the lesser with the greater, and took half [the sum] for the mean; and thus they brought out the mean distances. They found how far the planets are removed from the center of the earth, and they being at those three distances according to what they reckoned of the motion of the stars and their alignment, they knew the place of the apogee and perigee — and what lies between by precision with the dioptra of Hipparchus [איפרכוס], [namely] the diameters of the stars as seen at the three distances. For from the apparent diagonals one is able to know the distances, and the reverse, as I explained all this elsewhere.

So they reckoned, like Ptolemy, that the order of the spheres is 360 [שצ"ם] beneath and above all of them is the 8th sphere. And although by Tycho's instruments [נבנינות שבתי, recte by Tycho — see note] [Saturn] is 12,300 [?] [recte 12,300] half-diameters of the earth [?] — as was brought in the Degree that has passed — with all this they reckoned them more than 500, or [?] diagonals — that is, 19,000 — perhaps because it appeared to them that the thickness of the sphere of Saturn is small relative to its distance from the earth; or because they reckoned that not all the fixed [stars] are at one altitude, since they are very distinct one from another, not only in the measure of their diagonals but also in their light. They are also reckoned of differing measure…

[the stars being] very distinct in their magnitude — too [much] to be precise about them; and I am unable to test these numbers for shortness of time. But I know that the book of Ibn al-Battani is full of error in all its magnitudes, and it is easy for the discerning to correct them; and these are them, as they were brought

in another of the great books on astronomy, in which I trust that it did not issue from his hand without examination.

However, I found in a book under the name of al-Battani, in manuscript, that the distance of Venus is 622, the distance of Jupiter is 10,463, the distance of Saturn is 15,486 — and so forth.

Table — Mean distances and apparent sizes

Reads right-to-left in the source. Columns (right→left): the planet; the mean distance of the stars as seen from the sun, according to al-Battani, in earth-radii; their sizes, as compared to the terrestrial globe, according to al-Farghani; the ratio of their diagonals to the [earth's] diagonal — earth = [the unit].

Star	Mean distance (seen from sun), per al-Battani	Size vs. terrestrial globe, per al-Farghani	Ratio of diagonals to earth's diagonal
Mercury [כתב]	115	1/15	1/21000
Venus [נוגה]	618	1/10	1/36
Mars [מאדים]	4584	1/30	1 1/2
Jupiter [צדק]	10423	1/13	81
Saturn [שבתי]	15800	1/15	79
The six sizes [א, first]	1/20	1o2 [recte 102]	

Interleaved column-block (al-Farghani's figures, right side, lettered א–ו for "the six sizes of the fixed stars" — these are sizes vs. earth): [1] א 102; [2] ב (deficient / not declared) 90; [3] ג (his view not revealed) 72; [4] ד al-Battani: 19,000 — al-Farghani: 54; [5] ה "how many they are" — 36; [6] ו — 16/18.

Adjacent narrative running through the table cells (right column): "the six sizes of the fixed stars" — "these did not… his view… al-Battani… how many they are." And the side note (left): "and according to his [al-Farghani's] view he derived these — the numbers that he divided into 107 parts — that is [?], the parts being deficient: 16, and there remains [?] 17, that is 72; and there came out [?] [up to] 107"; "remainder 90, and likewise 18, and from it there remained 72, and likewise all of them."

However, according to the view of some of the reckoning-astronomers, [the figures] are set before you in the following table.

Table — Ratios of stellar diagonals and globes to the earth

Reads right-to-left in the source. Columns (right→left): the star; ratio between the diagonal of the stars and the diagonal of the earth — which is a [whole] part [plus] a [fractional] part; the ratio between the globes of the stars and the globe of the earth — which is a [whole] part [plus] a [fractional] part. Final two columns give cube values.

Star	Diagonal ratio (whole. whole. frac)	Globe ratio (cube of diagonal)	Earth-cube column	Frac.
The stars [א]	19. 4. 4¾	6859	64	107 — 11/64
The fixed [ב]	269. 60. 4 29/60	19465109	216000	90 — 25809/216000
are above [ג]	25. 6. 4⅛	15625	216	72 — 73/216
six [ד]	19. 5. 3⅘	6859	125	54 — 109/125
sizes [ה]	119. 36. 3 11/36	1685159	46656	36 — 5543/46656
[ו]	21. 8. 2⅝	9261	512	18 — 45/512
Saturn [שבתי]	9. 2. 4½	729	8	91 — 1/8
Jupiter [צדק]	32. 7. 4 4/7	32768	343	95 — 183/243
Mars [מאדים]	7. 6. 1⅛	343	216	1 — 127/216
Sun [שמש]	11. 2. 5½	1331	8	166 — 3/8

Table — continuation

Star	Diagonal ratio (whole. whole. frac)	Globe ratio (cube)	Earth-cube column	Frac.
Venus [נוגה]	3. 10. 3⅓	27	1000	73 — 1/27
Mercury [כתב]	1. 28. 28. 1	21952	—	21912
Moon [ירח]	5. 17. 3⅖	125	4913	39 — 35/125

Star	Diagonal ratio (whole. whole. frac)	Globe ratio (cube)	Earth-cube column	Frac.
Sun [שמש] · Moon [ירח]	187 · 10. 18. 7/10	6539203	1000	6539 — 203/1000

From the above table you may see at one glance the ratios of the diagonals of all the stars to the diagonals of the earth. And by dividing the great by the small, you will know what part the diagonal of the earth is of the diagonals of the great globes [planets], or what part the diagonal of each of the small ones is of the great ones — when you subtract the ratios according to what I added in the *Sefer Ḥukot Shamayim* [Book of the Statutes of Heaven] and according to what I taught in the Second Degree. And if you divide the ratios of the cubed globes of the diagonals, you will know the ratio between their great globes and the small; you will find what part the great is of the small. All of it is set out before you.

From here it is clear that the solar body is the greatest luminary among all the stars, that Mercury is the smallest, and that Mercury alone — together with the Moon and Venus — is smaller than the terrestrial body, while all the rest exceed it.

Now from the Degree that has passed you have known that from the center of the earth to the concavity of the 8th sphere is 22,612 ½ half-diagonals [half-diameters] of the earth, and according to the Second Degree so too [it is] between the whole diagonal of the earth together with the diagonal of the 8th sphere. And if the diagonal of each star of the first [magnitude] is 4¾ times the diagonal of the earth, there will be in the whole diagonal — 22,612 ½ — room for 4760 stars to stand, one next to its neighbor, in contact. And if you multiply 4760 by 3 ½ [?], you will find in the circumference of the 8th sphere 14,960 [recte] stars. And if you multiply the circumference by the diagonal — that is, 14,960 by 4760 — you will find that twice the concave surface of the 8th sphere can hold stars 71,209,600 in space — that is, that many squares, each surrounding one star of them. And these are not by very much more than the number of the children of Abraham when you reckon all the generations; for at the exodus of Israel from Egypt there were 603,550 men, apart from infants and women, who were without doubt more than double the men. And when you reckon in each 40 years the like of them, down to our time, they will be more and more than the stars of heaven in multitude — so that they were on the verge of complaining against Moses, saying, "You set a limit to our blessing!" — and he had to appease them, saying, "This is my proverb [?]; but the Lord shall add to you a thousand times as many

as you are" — meaning a thousand times the stars, the measure he set for your fathers. They would be 7,120,960,000. And one cannot say even of this, "You set a limit," for the whole earth cannot contain this number, as we shall explain afterward, God willing. All the more so, were the promise by the number of the smallest stars, which would be far more — a wondrous number. And you can reckon, by the ways set forth above, for each of the six sizes of the fixed stars, and you will easily see the truth.

Degree 17 [מדרגה יז]

Tycho Brahe [טיקון בראהי] — second to Hipparchus — steward to the king of Denmark [ראי"ניאה, Dania], lord of the city of Knudstrup [קנודשטרוף], built the fortress of Uraniborg [אורגיכורנו] on the island of Hven [אואינה] in the Baltic Sea [שבים כלטי', the Baltic]. A sublime line [favor], as befits a king, [the king] gave him and consecrated him to the stars of heaven and their constellations [?]. He excelled in lavishing upon craftsmen to prepare various tabular instruments, until they came to view his vessels from the corners [ends] of the earth, [both] lesser and notables; and he persevered thirty years in a fellowship of able companions in the said place, contemplating and observing the stars. And in the sum of his investigations into all the words of Ptolemy [and] Copernicus…

and the Alphonsine school [סיעה אלפונסים, the Alphonsine Tables] was [the basis of] the investigation into the sizes of the precious globes. He is the witness who tested with the sighting-tube [שפופרת, the dioptra/tube] — at the reversal of the [summer] solstice, which is its greatest distance — 1190 [parts], and found something [?] more than half a degree, that is 30 [?] minutes; and at the reversal of the winter solstice, which is the nearest, 1110, [the] nearest, he found 32 [?]; and at its being 1130, at the equinoxes [שוויים], he found 31 minutes. And in [these measurements] of the mean diagonal in his contemplation he likewise found the diagonals of all the stars — [a matter] that none [?] had done [?] from the day of the creation of heaven and earth; and it is fitting to rely upon him in his contemplations, to be precise in his vessels — but not in his teachings [?].

the distances of the stars are according to [?] [his observations], and I set before you here…

Table — Tycho's mean distances and apparent/true diameters

Reads right-to-left in the source. Columns (right→left): the star; the mean distances [in earth-diagonals]; the apparent diameter [in minutes]; the true

diagonal as compared with the earth's diagonal — which is a [whole] part [plus a fractional] part; how many times it is the terrestrial globe.

Star	Mean distance	Apparent diam. (minutes)	True diagonal vs. earth's diagonal	× terrestrial globe
Sun [שמש]	1150	31 (× earth's diagonal)	11. 22½. 5 al [אל]. 14/75 5 .5	139
Moon [ירח]	60	33′	1720. 495. 3 23/49	1/42
Mercury [כהב]	1150	2⅛	8. 3	1/19
Venus [נוגה]	1150	3¼	11. 6	6/37
Mars [מאירי']	1745	1⅗	60. 2⅖	1/13
Jupiter [צדק]	3990	2¾	5. 12	14
Saturn [שבתי]	10550	1. 50	11. 31. 2 9/11	22
8th sphere [גלגל החמ']	14000			

And he reckoned generations [?] according to his view, and in it appear six sizes [magnitudes].

Table — the six magnitudes

Reads right-to-left in the source. Columns (right→left): magnitude letter; apparent diagonal [minutes]; true diagonal vs. earth's diagonal; × terrestrial globe.

Magnitude	Apparent diagonal	True diagonal vs. earth	× terrestrial globe
א [1]	2	23. 52. 4 1/13	68
ב [2]	1½	18. 55. 3 1/18	28½
ג [3]	1 1/13	9. 20. 2⅕	11
ד [4]	¾	2. 3. 1 19/39	1¼
ה [5]	½	49. 50. 1	1 1/18
ו [6]	⅓	22. 15. ⅗	⅓

Now see, [my] reader, the new world according to his [Tycho's] assumptions: that Mars is smaller than the earth, also the fixed stars [?]; that of the measure of the six small ones, the middling [?] one and so on; that Saturn is greater than Jupiter,

and the two of them are not so very great; that the fixed stars are not so very great. And he wrote that Venus and [the like?] —

as the ancients reckoned. And among the fixed stars, the brightest — which is, in my view, the smallest among the stars of heaven — [is brighter] than the Sun and the Moon [?]. He reckons that the earth is in the middle [center] of the world, and that it is the [center?] of the fixed stars; but he reckons that the Sun is the center of the rest of the wandering [planets] — that is, those that revolve around it: the sphere of Mercury, Venus, and Mars are joined with the sphere of the Sun, and in this the fixed [stars] are established [?]; all those seen [moving] in a wondrous order. And his assumptions, taken altogether, are part of what is to come — the assumptions of Copernicus. And these words [are] precious; and behold, the inquiry [?]. And he found, by his instruments, that Saturn is 12,300 [?] half-diagonals of the earth, and the thickness of the 8th sphere…

he reckons that it [the firmament] extends to 14,000 [earth-radii], so that it can comfortably contain the great stars within it. And by the assumption of Degree 2 [מדרגה ב'], its circumference will be 88,000, and likewise the earth-radii are all included within the aforesaid circumference. He found [?] and took pride that he had discovered a wonderful and novel thing that no one had anticipated: that the thickness of the solar sphere [?] is, according to him [?], 1190 [?]; this includes the thickness of the lunar sphere, which is 66 [?]. Half of the [?] mean [taken] between the view of Copernicus, who reckoned 68, and that of Ptolemy, [who reckoned] 64 — eighteen times the measure of the ratio of the Moon's diameter to the Sun's diameter. But this is no novelty, for already R. Yitzchak HaYisraeli [Isaac Israeli] recognized before him how many [?] hundred years it [the discrepancy] is approximately — and he found no fixed measure, nor [?]. And to Him, may He be exalted, the cause is known: the aforesaid ratio varies between the two luminaries, since it [the distance] is not constant — this is beyond doubt.

Degree 18 [מדרגה יח']

Now I shall acquaint you with the method of al-Battani [אלביתין], whose roots [first principles] are confused [garbled] in his composition — and Regiomontanus [ריגיאומונטאנו], his commentator, a man wondrous in the sciences, seems to have been old and slumbering [i.e., past his prime], and did not explain his words properly in this passage. Here it is, according to his view: he posited the near distance of the Moon — that is, half the diameter of the four elements [the sublunary realm] — as 18;38 [?]. Half the diameter of the earth [?] and the far [distance] [?]. And the Moon's diameter, as seen at a distance of 29½, no more. He wrote that he placed it [the limit] right at the Moon; therefore at its [the Moon's]

perigee it is 64;10, which is in proportion to the building [construction] of the lunar sphere, as above. He said that the ancients observed carefully with the dioptra [הריאופ"ט"א] — when the star [Moon] is at its near and at its great far distance — and found that the difference of the diameter is as 2 ⁷⁄₁₂ [?] times — that is, the near [distance] from what was seen at the far distance of 1. Therefore figure it thus, by the rule of "the golden [Rule of Three]": if 1 gives 64;10, what will 2 ⁷⁄₁₂ give? You will find the great distance to be close to 166;[?]. And when you add to it 64;10, the near distance, they will be 230; and this is the measure of the diameter issuing from its center. Therefore its half, 115, is its mean distance. And thus he found all the mean distances.

Then they observed the apparent diameter of the star [Moon] at this distance and found it to be ¹⁄₁₅ [?] of what the Sun's diameter appears [to be], after [a whole?] — and oh, that I had the book in hand, for I would have explained this whole Fifth Chapter and adduced much of his fine teaching! Except that what I have with me, and what I am writing, is from the gist of some pamphlets [קונטרסים] in which his words came in abridged form. Therefore I shall give you illustrations [worked examples] of al-Battani's [reasoning].

Let us posit, by Tycho's reckoning [?] (look at 45 [ליר מ"ה]), that we know the apparent diameter of the Moon, which appears at the magnitude of an angle of 33 [?] minutes, and the apparent diameter of the Sun, which appears at an angle of 31 minutes — all of this with the dioptra. And we know the distance to be 60 [?] or 1150. We wish to know the ratio of the true diameters.

Now, as is explained in the science of optics [חכמת הראיה] — learn it well [?], by Euclid [?] — moreover [?], that the [object that is] more remote appears smaller, and conversely the nearer one appears larger; therefore we shall reckon as though these two computed quantities were at one [and the same] distance, and the first angles [must] change [be adjusted]. That is, we reckon as though the lunar sphere were [placed] as the [?]: so that there should appear here this angle of 33 minutes, its diameter must there be larger — namely, so that the manner [proportion] should be such that there be subtracted [reduced] from it 18 [?]. And we figure thus: if [?] 60 gives [?] 33, what will 1150 give? You will find a measure of [?] 632; and so large the Moon there would need to be, in order to appear at an angle of 33 minutes. Therefore figure a second time: as 33 is to 632, so 31 is to 594⅛ [?]. Therefore the ratio of the true solar diameter to the true lunar diameter is

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as 594⅛ to 33; and if you divide this by that, you will find that it [594⅛] is contained in it [33] 18 ₁⁄₁₉₆ times.

But I, [in working] examples like these, work more concisely by the method of the inverse "Rule of the Golden [Three]" [למוד הזהב ההפוך] — that is, I reckon as though the Sun were together at one [and the same] distance with the Moon, and I wish to investigate how large it would appear there. And I figure thus: if 1150 gives 31 minutes, what will 60 give — which is [the value] they [give]? Double the first [term] by the second [60×31] and divide by the third [1150], and you will find 594⅛, just as I found by al-Battani's method, at length.

Again, let us suppose we wished to know the ratio between the diameters of Venus [נוגה] and the Moon — that is, to know the distance [?]: as the Sun [is at] 1150, and Venus's apparent diameter is 3¾ [?] minutes. And we figure thus: if 1150 gives 3¾ minutes, what will 60 give? Double the first by the second and divide by the third, and you will find 62 [?] ¾ [?]. Therefore as the diameter [of the Moon, 33,] is to [?] [the diameter of Venus, 62¾], so is the diameter of Venus with [relative to] the Moon — for, by Tycho's view, Venus is larger [?] than the Moon, since [Tycho] posits the ratio of the Moon to the earth as ⅓ [?] and that of Venus to it [the earth] as ₆⁄₁₁ [?]. And when you subtract one ratio [?] [from the other], as the author of *Chukos Shamayim* [חקות שמים, "Statutes of the Heavens"] showed, you will find that there remains 1 ¹⁰⁄₁₁ [?]; and so it is, approximately, [for] the aforesaid ratio.

And after we knew the ratios between the diameters of two stars, and we knew the ratio of the first with the earth's globe, we know likewise the ratio of the second with it [the earth] — and this is as was shown in Degree 2 [מדרגה ב']: that as the cubes of the diameters, so are the ratios of the globes [spheres]. Therefore, if you cube both of them, you will know the ratio between their globes, and likewise the ratio between each of those globes and the earth's globe. For example: we found the ratio of the Moon's diameter to the Sun's diameter to be as 1 to 18 [?]. Therefore the ratio of their globes is as 1 to 5832 [= 18^3]. And since we know that the ratio of the solar globe to the earth's globe is about 139 times — therefore we figure thus: if 139 equals 5832, what does 1 equal? You will find that the Moon's globe is ₁⁄₄₂ [?] of the earth's globe, approximately.

All these are by way of approximation. But the [proper] reasoning is that of al-Battani, and Copernicus agreed with him. For he too, wishing to challenge Ptolemy's lunar distances — who posited the great [distance] as 64 and the small as 33 [?] — said that, according to this [scheme], since the great is about double the small, it would follow that the Moon's diameter at its near [position] would

appear about double [its size] at the far [position]; and this is contrary to experience [observation]. Therefore Ptolemy's distances are not true. And since, by Ptolemy's view, the apparent diameter [of the Moon] is 31⅓ minutes at its great distance of 64 ⅙, it would be — by al-Battani's reasoning, which we have explained — its apparent diameter at the near [position] would be about 61½, approximately.

הצעה [Lemma / Premise]

When, of four magnitudes, the first exceeds the third and the second falls short of the fourth, then the ratio of the first to the second is greater than the ratio of the third to the fourth. For example: take ½ and ⅔, or ¾ and ⅘ — how they reconcile: the printed labeled set reads א 1⁄2, ב 2⁄3, ג 3⁄4, ד 4⁄3 (the fourth term is ⅘, not ⅓). The example remains a compositor garble of the lemma's "first>third, second<fourth" condition; transcribed as printed. This too was demonstrated by proof by R. Yaakov of Alexandria [ר' יעקב אסכנדרני / Jacob of Alexandria] and by some of the commentators on Euclid; there is no need to elaborate its proof, since it is self-evident.

Degree 19 [מדרגה ייט']

I wish to make clear to you that Copernicus too, and Regiomontanus, and al-Battani who preceded them, erred in their reasoning. They held it to be clear [self-evident] that it does not follow that equal magnitudes, when they are removed from one another, [are seen as] double [in apparent ratio to their distances]. I mean

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the distance — that is, that the one is 10 cubits [away] and the other 20, [whence one supposes] that the near one appears double the great [size] of the far one. For it has been explained in the science of optics that the ratios of the angle of vision are less than the ratios of the distances; and Copernicus ought not, on this account, to have abandoned Ptolemy's assumptions — all the more so since he was already warned about this by Hayton [היאון / the commentator] who expounded the *Almagest* in the Sixth Book [Book VI], unless for other reasons. And it will be explained in the present figure [diagram 45–46 / ליר מ"ו] that the equal magnitudes are AB [אב] and GD [גד] — and yet AB is much farther from the viewer's eye Tz [צ] than GD is.

Diagram. A geometric optics figure illustrating that equal magnitudes at different distances subtend different visual angles. The viewer's eye is at point צ (Tz),

which serves as the center. Two equal segments, אב (AB) and גד (GD), lie at different distances from the eye; rays are drawn from the eye to the endpoints, and an arc is described about צ as center cutting the rays. The figure includes a perpendicular and chords of the circle, with auxiliary points used to compare the triangles formed.

- צ (Tz / center): the viewer's eye, center of the construction.
- אב (AB): the more remote of the two equal segments.
- גד (GD): the nearer equal segment.
- אצ, בצ (A-Tz, B-Tz): rays from the eye to the endpoints of AB ("in place of the visual-ray diameters" — האלכסונים הניצוציים).
- ה (H): point on segment בצ where the perpendicular/chord מך [?] meets, dividing the base.
- ק (K): point where the arc cuts ray צב; with ב [?] one of the cut-points (the arc through מרחק צ-ז-ק [?]).
- ט (T): cut-point where the arc meets ray צד.
- ו, ז (V, Z): auxiliary points on the chords/arcs (קשת וג / arc V-G; קשת זג / arc Z-G).

The argument compares triangle צהט [TzHT] with triangle צהד [TzHD], and the chord צוב [Tz-V-B] with chord צדג [TzDG], to show that the ratio of distance בצ [BTz] to distance דצ [DTz] is greater than the ratio of angle וצג [VTzG] to angle דצג [DTzG]. [Several point-labels in this dense construction are abraded]

of the more remote, R. that is, 10 cubits, and the other 20, [the claim] that the near one appears double the great [size] of the far one. For it has been explained in the science of optics that the ratios of the angle of vision are less than the ratios of the distances. And let there be — in the present figure [45–46 / ליר מו] — equal magnitudes AB [אב] and GD [גד], and let AB be even farther from the viewer's eye צ [Tz] than it is [?]. Let line בצ [BTz] come [meet] at the center צ [Tz], which is in the position of the side [צלע]; and from the points א [A] let there come rays אצ [?] — which are in the position of the visual-ray diameters [האלכסונים הניצוציים]. Now, since AB is much removed and appears at a more acute angle, the ray אצ [?], which cuts the angle טצד [?], likewise cuts its base מך [?] at point ך, which is from ג' [?] to ו' [?] of the elements [?]. We place צ [Tz] as center and describe an arc at distance צזק [Tz-Z-K], and it will cut lines צב, צט [TzB, TzT] at points ב [?], ט [?]; and this is necessitated [forced], since line צך [?] is smaller than line צם [?] by the construction, and smaller also than line צמן [?], for it [צך] lies along the perpendicular [נצב] ד [D], whereas צפ [TzP] lies along צדקמן [?], equal to the

perpendicular ד [?] and to the angle [?] קצר [?] smaller [?] than the [angle] of the first [?].

And since triangle צהטק [?] exceeds the round sector [התך העגול] צוה [Tz-V-H], and conversely triangle צנקדי [?] falls short of the sector צרקגג [?], it will follow — from the earlier premise [הצעה] above — that the ratio of triangle צהט [TzHT] to triangle צהך [TzHK] is greater than the ratio of sector צוה [Tz-V-H] to sector צדקב [?]. Therefore, by composition [בהרכבה, joining the 28th [proposition?] to the fifth], the ratio of triangle צמוך [?] to triangle צהך [?] is greater than the ratio of sector צוג [Tz-V-G] to צהג [?]. And from the teaching of the first [proposition] to the sixth [למוד א' לששי], the triangles צהד [TzHD], צטר [?] are, by the calculations [reckonings], דמ [?] רlק [?] equal to בא [?]; and [from] the first to the sixth, the triangles צאב [TzAB], צהרי [?] are proportional and similar — that is, as אב [AB] is to קך [?], so is בצ [BTz] to רצ [?]. Therefore the ratio of בצ [BTz] to דצ [DTz] is greater than the ratio of sector צוב [Tz-V-B] to sector צדזג [?]; and as the ratio of sector צוג [Tz-V-G] to sector צהזג [?], so is the ratio of arc וג [V-G] to arc זג [?] (and arc וג [V-G] to arc זג [Z-G], as angle וצב [VTzB] is to angle דקצג [?]) — from the last [proposition] to the sixth. Therefore the ratio of distance בצ [BTz] to distance דצ [DTz] is greater than the ratio of angle וצג [VTzG] to angle דצג [?] — and this is what was intended [Q.E.D.]. And in the manner that this was explained for equals, it will be explained likewise for unequals: that beyond the difference [arising] from the side of variation in the magnitudes, there is added also the matter of the angle together with the distance, as above — and this is straightforward. Perhaps these great [scholars] did not concern themselves with such precise refinements in distances and magnitudes.

Degree 20 [מדרגה כ']

And apart from al-Battani's method, you can — concisely — know the ratios of the diameters by various means. And it is [done thus], after you know their distance from the earth's center, as above, and their apparent diameter: you find their circumference [Hebrew: הקף העגולי, the circular circumference], according to the teaching of the Second Degree [מדרגה השנית], and you divide it by 360, and from there you take the portion that corresponds to the measure of the apparent diameter of the star — and you find the ratio of the star's diameter to the earth's diameter. For example: the Sun's distance, by Tycho's reckoning, is 1150 half-[diameters], and its apparent diameter is 3' [?] minutes [?]. Double the distance, [and] it is

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2300 [?] whole diameters — that is, the diameter of the solar sphere. And we double it with [by] 3¼ [?], and it will be 3614 ⅘ [?]: the circumference of the solar sphere. And we figure thus: if 21,600 — that is, 360 degrees converted into minutes — contains 3614 ⅘ [?] earth-diameters, how many will 31 minutes [the apparent diameter] contain? You will find that they contain 5¼ [?] approximately; and its cube will be more than 144 [?]. And Tycho found its [the Sun's] diameter to be 5;11,13" [5° 11′ 13″?, recte: 5;11,13 in units of earth-diameters] — that is, 5⅖ [?] — and its cube a little more than 139 [?] times the earth's globe.

Again, we wished to know the ratio of the diameter of another of the stars of the first [magnitude] that are in the sphere

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of the eighth [sphere], whose distance is 14,000 half-diameters of the earth, and whose apparent diameter is 2 minutes by Tycho's view. Double the distance, [and] it is 28,000 half-[diameters], which are 14,000 [whole] diameters — that is, the entire diameter of the eighth sphere [the firmament]. And the circumference will be 44,000 [recte ≈ 44,000; = 14,000 × 3.142]. Figure thus: if 360 — that is, 21,600 [recte 21,600] minutes of the whole sphere — contains 44,000 earth-diameters, how many will 2 minutes (the apparent diameter of the star) contain? You will find that the entire diameter of the star contains about 4 ⅓ earth-diameters. However, by the calculation of triangles you will find it [more accurately], by the proportion stated above. Let it be, for example, that in the present figure [47 / ליר מ"ז], AB [אב] is the eighth sphere [?].

Diagram. A triangle for computing a star's true diameter from its apparent angular size and distance. The viewer's eye is at A (א) on the earth; AB (אב) is a long line directed toward the star, with the star's diameter subtending the small angle at A. The far side of the triangle (the star's diameter) is taken as the base, and the two long sides AB are treated as equal (an isosceles triangle), since the distance is so great.

- א (A): the viewer's eye / point of observation on the earth.
- אב (AB): the side directed toward the star; subtends the apparent angle ("an angle of 31 minutes" [?] from the eye on the earth).
- The base of the triangle is the star's diameter; the two long sides are treated as equal because of the great distance.

and its [the star's] apparent diameter [appears] at an angle of, for example, 31 minutes [?], from the point of the observer's eye on the earth, and the distance AB

[אב] is 1150 [recte: the relevant distance] half-diameters of the earth. And because of the greatness of the distance, we also reckon the lines אז [?] and אב [?], the [two] diameters [sides], as equal to the side וא [?]; and they will be in an isosceles triangle whose [two] sides are known together with the angle between them.

And from the [teaching of] proposition 11 [?] in the fourth [book?] of the *Sefer Sod HaYibbur* [ספר סוד היבור, "Book of the Secret of the Calendar/Intercalation"] [?], side א [?] is found, and figure thus:

As the sine [בקע] of 15° 30′ [?], which is 450 [?], is to 511 [?] אה [?], so [is] — and double it —

[here a sequence of half-chord (sine) computations follows as flowing prose; values as printed:]

The sine [בקע] of 15° 30′, which is 450, is to אה [AH] 511; and double it, [making] אהב [AHB] 1022. As the computed chord [המנוקש] 100,000 is to חו [ḤV] 1150, so it touches [נוגע] אח [AḤ] 30 — and (with the 5 / 15 / 11 marginal arithmetic) to אח 451 — so the whole side האב [HAB] is 11;22 approximately.

[so] the measure of the whole [side] will be 11;22 approximately. And the ratio of its diameter to the earth's diameter will be as 5⅔ to 1, and its cube a little more than 1330. And in this same way you will find [for] all the stars — only you must use the great table of the ratios [trigonometric table] and be precise to minutes and seconds, for in such [calculations] a small matter changes the things by a great or small variation. And this was the basis of all the calculations of the ancients to know the ratio of the diameter to the circumference of the circle upon which it [the star] is set, by sufficient and decisive proofs.

And this way I [Tycho? the author] took from Tycho's book; and by it he also investigated the distance of the comet [הככב התבייי, "the tailed star"] that appeared in the year 1557 [?]. Its apparent diameter was 7 minutes, and its distance, by his calculations, was 210 [?] half-diameters of the earth. And it will be, in the aforesaid figure, triangle AVB [אוב] [?], and its angle [?] AVB [אוב] [is] 7 minutes [?], and the measure of each of its equal sides AV [או] [or] BV [בו] is 210. Therefore the equal bases [legs] AB [אב] are 89;56,30" [89° 56′ 30″]. And figure thus: as the sine [בקע] of [the angle] A B [א ב] [?] [29] 56,30;89 [?], [which is] 99,999, is to בן [BN] 210 — which are [in] units [פרסאות] 189,606 (printed "18·9606", the interior raised dot being a digit-group separator, not a decimal point) — when we reckon half the earth's diameter as 60, so [is] the sine [בקע] of half 7 minutes, which is 203, to the measure of א [A] 368 approximately. Therefore half of its [the comet's]

diameter is 184 [?], and it has a ratio to half the earth's diameter, 60 [?], both of them [being] in units [פרסאות] — that is, that its diameter is contained in the earth's diameter about 4⅔ [?] times approximately, the ratio being about ¾₁₄ [?]. And since the earth's diameter to the Moon's diameter is as 1 [?], by Copernicus's view — if you move the ratios as *Chukos Shamayim* fixed [?] — you will find that its [the comet's] diameter, in ratio to the Moon's diameter, is about [?] — that is, that the Moon's diameter exceeds it. And in their cubes you will find, dividing one by the other, that the aforesaid star [comet] was less than the body of the earth by about 100 ¾ [?] times, and less than the lunar body by about 2⅓ times approximately.

Indeed, even Ptolemy himself went by this method, in chapter [?] 14 [?] of the Fifth [Book] of the *Almagest*. He said there as follows: "And now we shall come to the proof of the Sun's distance, which we found by the quantity that Hipparchus [איפרכום] found — since the difference between the shadow-cones [ענולי, the circular shadows] of the Sun, the Moon, and the earth that result from the bodies [?] of the [solar] eclipse and the shadows [?] which heaven shows by its luminaries [?]…it is so small that it is not perceptible, until [?] there are [?] perceptible [?] equal [shadows?] to truthful [ones]…" — meaning that it was explained [?].

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Gevurat Yah (continued)

in the science of optics, as Euclid teaches in proposition 23, namely that the eye always sees less than half of a spherical body, and that the portion which appears bounded and circumscribed seems circular. Therefore sphere AB [איב] appears as less than half the sphere, and the part that is seen — the part facing the eye — does not appear in proportion to a large [sphere]; that is to say, the diameter AB [אב], which is not the diagonal of the great circle that is seen, does not appear at an angle of a great circle. And so it has been explained in the science of optics that what one sees is only a small circle, whose spherical caps [are] among those parallel to [the great circle], and it [the visible portion] is less than [a hemisphere].

The illuminating body [the Sun] illuminates half of the spherical body equally. And if the illuminating body is larger than the one near to it [recte: the body being illuminated], it will receive more than half its light; but if [the source is]

smaller, less than half. Every body that is farther [from the source] sends out its rays toward it [recte: receives rays] more concentrated upon its circumference than when it is nearer to it; but conversely, every body that approaches the illuminating one [receives light] over a smaller part — that is, in the figure of Degree 11, the observer [looks] from point H [ה] [over] the parts of its circumference. And every body that draws near [the source receives light] over a smaller part. So too the spherical body of the Moon receives [light] from the Sun only over its circumference, since [the Moon] is less than half [the Sun]; and likewise the spherical body of the Earth receives the light of the Moon over [more than] half. Because the Earth's sphere receives light from the Moon over [more than] half of it — that is, over more than half its degrees — and so it is more than [half].

And the body of the Earth receives the light of the Sun over a part SH′ [שע — i.e. 376?] [?] of it, which is less than half. And because the Moon is more elevated in its sphere by degrees of [its] altitude [recte: by the height of the Earth's surface] than the Earth — that is, [its] body is farther [from the Earth] by more degrees — therefore the Earth, in spite of this, receives the light from the entire large circuit of the Sun's sphere, whereas the Moon receives [it] only over the circuit SH′ [שע] alone.

Now that these matters have been demonstrated, we wish to investigate the magnitude and extent of the part of the lunar body that receives light. Let AS [אע] be half the diameter of the Sun. At the point where it touches [the Moon] [recte: at point ע], where it stands perpendicular upon [the line] from 90° to the third [degree] [i.e. perpendicular], we cut off a part HS [הע] equal to half the diameter of the Moon, BS [בע]. We draw HG [הג] parallel to line AS [עץ], and angle HG [the angle at the first point] [recte: angle ZG, זג] will be perpendicular [recte: angle at ה], 28′ for the first [recte: a right angle]. Therefore in right triangle ATB [אתב, recte: in the right triangle], the side subtending [the right angle] is known, since the whole of line HA [הא] is 1210 and line HG [הג] is 64. Therefore line GA [גא] is 1146; and side AT [את] is also known, once we know half the diameter of the Moon [together with] half the diameter of the Sun.

Now in Degree 18 [recte: 12] [י"ב] it was explained that the diameter of the Moon is to the diameter of the Earth as 1 to 3⅖ [25/.3 — i.e. 1 : 3 25/...; recte 1 : 3.25]; and if you convert them into minutes, you will find that it is as 17½ minutes to 60 minutes — note this well. Therefore if you subtract the 17½ minutes from the diameter of the Sun, 5;30 [5.30°], there remains line AT [את] as 5;12½ [12½ 5]. And reason thus: if side AG [אג] —

— [the side] HG [recte] subtending [the right angle], 100000 [i.e. as radius], to what will side AT [את] correspond? It is 12;30 [12.30] [recte], and you will find a sine of about 455, [for] the magnitude of angle [HAG] is 15′40″ [recte: 15;40], and that is the arc BM [recte: the arc on the Moon's circuit]. And if you add it to arc HS [recte: zs, זס], which is a quadrant — since angle ZNS [זנס] is perpendicular [a right angle] — the whole arc KM [כם] becomes 90 15′40″ [90;15,40] [recte], and its double — namely the whole bright part sought on the lunar sphere — is 180;31,20. And Ptolemy reckoned it as 180 degrees. You see that the difference is slight; therefore we may reckon as if the tangent lines were at the ends of the diameters of the spheres.

And conversely, if you wished to know the lower encompassing part WZ [וז] [recte: the lower visible part] of the lunar sphere that appears to our eyes, you could obtain it easily once you know its apparent diameter as measured — as you find in the tables of Copernicus and Tycho and the like — and let us suppose that at some distance it appears as 32 minutes. Then in the figure of Degree 11, angle ZHG [recte: ZHZG, the angle at the vertex] will be 16 minutes [for] its half; and since angles GHZ [recte] are perpendicular from 90° to the third [i.e. right angles], if you subtract from a right angle 16′, there remains angle GHE [recte: angle at ה] or ZGH [זגה] as 89;44; and its double is the whole angle ZHB [recte] — which is the magnitude of the visible arc — 179;28. And this is what you wished to know.

And you see in this case too that it [the visible part] falls short of half the circle by only 32 minutes, and that is not reckoned at all. Therefore we reckon as if lines HW and HL [הו הל] [recte] were equal, and likewise lines HZ and HN [הז הנ] and the like — there is no need to dwell on this further. To remove this [scruple] in any case, it is good that you set up the figure of [Degree] 11 in this manner, as follows: Let N [ן] be the eye observing the Sun's sphere AB [איב], and let line KA [recte: line from] [come] from its center to the eye N [recte], and the ray-lines coming from the eye to its [the Sun's] extremities are A and B [וא ב] [recte: NA and NB], and they are tangent to the circle. Therefore the lines coming from the center H [ח] to the points of tangency are perpendicular — as we learn from proposition 18 [recte] to the third [book] — upon lines NA and NB [וא וב]; and accordingly, in triangle NAH [ואח], [the angle at] A is perpendicular [a right angle], and the [side] subtending it ND [recte: NH, וח] is 1150, and angle ANH [אוח] is 15;30. Therefore side AN [אן] opposite it will also be known in the same parts as NH [ודן], and it is 5;11 as at the beginning.

And likewise, by the analogy of the star [comet] that appeared in the year 1557, the proportion will be thus: as A [א] subtending [the right angle], 100000, to NH

[ורן] 210 — which is [in] parasangs [recte: earth-radii] — so [is] angle A [אוה] [30″ 3] 3′30″ whose sine is 102, to AN [אן] 184, as at the beginning.

Another method: as H [ח], 30 56 89 [recte 89;56,30], whose sine is 99999, to AN [אן] 180600, so [is] sine of angle A [אוה], 102, to AN [אן] 184. And you see that, owing to the great distance, all comes out equal; hence Ptolemy, and Tycho after him, paid no heed to the difference, since it is imperceptible.

Again, in the year 1572 there appeared a star among the fixed stars [recte: in the firmament] whose apparent diameter was 3½ minutes [3 ½]; and it was reckoned to be high, in the eighth sphere, whose distance is 14000. The proportion will be thus: as 100000 [radius] to NH [חן] 14000, so [is] sine of angle A [אוה], 45;1, to 51, which is 7⅛ [7 ⅛]. And so great was its diameter relative to the diameter of the Earth — about another [recte:]. Therefore its volume was 361⅓ times that of the sphere of the Earth, so that it exceeded the Sun's body by about 2⅔ [2 ⅔] times.

Again, if the star were of the first magnitude — its apparent diameter being 2 minutes — and we wished [to know] the measure of its value, the proportion would be thus: as 100000 [radius] to NH [חן] 14000, so [is] sine of angle A [אוה], 1′ [recte], to AN [אן] [3 .4] 3;4, whose sine is 29 [recte], and it is ⅓ [⅓]; and it is thus that many times larger than the diameter of the Earth — and so on in the same manner.

Degree 21 [מדרגה כ"א]

Now I will inform you how the astronomers investigated and found the magnitude of the great distances [of the planets] from the Earth and the eccentricity of each planet — that is to say, of the deviation [of the wandering body] [recte], namely [the distance] between the center of the Earth and the eccentric center about which they [the planets] move. All of them require examination of positions in the eighth sphere, and the analogy will be [taken from] the Sun.

Now Ptolemy made a precise determination at the moment of the spring [equinox], according to what he demonstrated in the *Almagest* [literally: *the Laws of the Heavens*]; and likewise at the summer solstice that followed it, and he found between them 94½ [94 ½] days. And between this solstice and the autumnal equinox that followed it he found 92½ [92 ½] days only. And according to what was explained in the tables of motion, you will easily find the mean motion of these days, since the daily mean motion of the Sun is known, and you need only multiply it by the said days. Thus you will find the mean place of the

Sun at the summer solstice [to be] 93;9, and at the autumnal equinox 91;11. And it will be [as] in the adjacent figure (see figure MH′ [מח]): the eccentric circle of the Sun, whose center moves [around] in [the course of] a year [recte: whose center is fixed], strong upon center H [ה]; and let point A [א] be the spring [equinox], B [ב] the summer solstice, and Z [צ] the autumnal equinox — that is, between A and Z [recte: A and B] [there are] 93;9 parts, and between B and Z [there are] 91;11 [parts], by the said motions of the Sun. We join the equinoctial points A and Z [recte] with line AZ [אצ], and upon it, from point B [ב], we drop a perpendicular line BS [בש] [which] reverses with it [recte: bisects] at point P [פ], which joins the two solstices. And since arc ABZ [אבצ], compounded of motions AB, 93;9, and BZ, 91;11 — which together are 184;20 — exceeds the arc that is the remainder of the circle AZ [recte], and arc AB is greater than arc BZ, Ptolemy understood that the center of the circle of the Sun's motion lies between lines AS and PB [אפ פב], and its distance from the Earth —

— will be between A, the spring [equinox], and B, the summer solstice; for the eccentric center and the apogee point lie on one straight line. We pass a straight line through center H [ה] parallel to line AZ [recte], and it is line GZ [גז]; and likewise a straight line HK [חק] parallel to line BS [recte]; and the long rectangle thus formed, right-angled, whose diagonal is PD [פד], is the eccentricity of the Sun. And when it is extended to point Y [י], line PY [פי] will be the great solar distance from the Earth, and point Y [י] is the apogee. These are what is sought, [namely] to know their magnitudes.

And since the whole arc ABZ [recte] is 184;20, its half AH [אח] [recte] is 92;10, which exceeds a quadrant of the circle GH [recte] by the magnitude of arc AG [אג], [10′] 10;2, whose sine is 3780. And likewise, since arc AB, 93;9, exceeds arc AH [recte] [by] 92;10 [recte] by the magnitude of arc HB [חב], which is 59 minutes [first-minutes], whose sine is 1716 — therefore in the right triangle HMP [recte] the two sides are known, and from proposition 4 [recte] of [Euclid's] Foundations [the Elements] the angles are found, and from them the surface DKH [דקח] [recte]. And the proportion is thus: as MP [recte] 1716 is to HM [recte] 3780, so [is] MP [recte] 100000 to HM [recte] 220296, the tangent of angle HPM [recte]; and its complement, the surface HPL [recte] [recte: the angle PHL], equal to angle DHZ [recte], is 24°24′54″ [24 24″ 54′], which is the magnitude of arc YZ [recte: יז] by which is known the distance between the great solar apogee and the summer solstice in Ptolemy's time. And likewise reason: as HPM [recte] 6;35 [6.35] is to HM [recte] 65, so [is] 100000 to the sine of HP [recte]: 4152.

sine 91057	200 [recte]
3780	

And this is the magnitude [of the distance] between the center of the Earth and the center of the Sun [the eccentricity], and it is approximately 1/24 of the measure of line DK [recte: PD] [taken as] 100000, which comes from the center [as radius]. And so too you find in Ptolemy: half the diameter of the eccentric [is] strong [fixed] — that is, line YH [יה] [is] 60, and line HP [recte] is 2 1/24 [2 1/24], which is [as Ptolemy reckons it] approximately as 1 to 30, per the *Almagest*. And it is found, learner, that if you subtract arc YZ [recte: יז], equal to arc 210 [recte], and SK [recte], equal to BZ 59 — which are 3;9 — from the quadrant ZK [recte], there remains arc ZS [recte] 51;86 [recte] — that is to say, from the autumnal equinox to the apogee. And through these parts the Sun passes in 88⅛ [88 ⅛] days — that is, from the autumnal equinox to the apogee. There remains, then, for the circle, an arc SA [recte] of 88;49 [88.49] between the apogee — the autumnal — and the spring [equinox], and the Sun moves through it in 90⅛ [90 ⅛] days. And this is what was to be shown.

Degree 22 [מדרגה כ"ב]

Further, I wish to show you, with regard to some of the planets, how Ptolemy investigated the magnitudes of the diameters of their epicycles [recte: deferents]; and that is by the easiest means, for it is not my intention to copy here the whole book of the *Almagest*, and my aim was also for the sake of [the convenience of] the readers of my book — not to burden their study. And as was written concerning Theophrastus, the disciple of Aristotle, who blended sweetness with study: since the investigation of the lunar sphere and of the three upper planets [recte] requires lengthy computations, I shall not mention them here, and I have chosen to explain to you how the magnitude of the eccentricity is found in [the case of] Venus and Mercury, and the magnitude of the diameter that conveys the center of their epicycles, and the diameters of the epicycles themselves. And when you see that they were investigated by demonstrative method, believe that in the others too [the astronomers] did not speak from tradition or prophecy — as I have written.

Now know that, upon examining Venus and Mercury and their properties in their motions, [you will find] that they are not like the [outer planets] [recte], which move far from the Sun until they come opposite it — these [oppositions] being called *acronychos* [Greek: ἀκρόνυχος, "at nightfall," i.e. acronychal] — for these

two [Venus and Mercury] have a fixed limit [an elongation] through which they pass and do not increase, [whether] inclining to the right or to the left, and that [limit] is according to the breadth [recte: spread] of their epicycles. So that Venus and Mercury are always bound to the Sun by the mean conjunction, not the true [conjunction] — that is, [their] centers [are bound]. For the [mean] conjunction of their epicycles is the mean: it is when two planets are found near to one another in proportion to their position —

— in the zodiac, in such a way that the line going out from the center of the zodiac of the world to the position of the apogee [recte: the present line, the line of true motion], going out from the center of the [deferent] that carries the planets and passing through the center of the planets to the zodiac — which is called the line of mean motion — is one [and the same] for both of them. And so are the centers of the epicycles of Venus and Mercury: the center [is bound] with the Sun. And the true [conjunction] is when there is another [line] passing through the center of both of them up to the zodiac. And likewise the height [apogee] of the Sun and its epicycle [as also] of Venus and Mercury, according to the second analogy [recte], and the mean motion, is always [reckoned] in the zodiac. Therefore if the line going out from the center of the [deferent] that carries [the planet] and passing through the center of the planet up to the zodiac — which is called the line of true motion — is distant from the center of its epicycle toward the western side, then the planet will rise first [recte] from the center of the epicycle toward the eastern side [and] will rise after the Sun, and therefore will set after the Sun.

And it is found, learner, that one who wishes to examine the positions of Venus and Mercury, in order to investigate the magnitude of the epicycle and its half [radius], must know their greatest distance from the Sun — that is, [from] the line that comes from the eccentric center and passes [through] the center of the epicycle, for this distance indicates half the diameter of the epicycle, as is explained in the figures of the study of [planetary positions], at a single glance. Therefore, in order to test Ptolemy's reckoning, you must know their greatest distance from the mean solar motion — that is, when the center of their epicycle is at the apogee of the carrying [deferent] and at its perigee. And then, with the instruments, you seek their true position in the zodiac, according to what he demonstrated in the *Almagest* [literally: *the Laws of the Heavens*]; and know also from the tables the mean longitude of the Sun. If it exceeds the magnitude of the planet's true longitude, then they will be at their greatest morning distance [recte: eastern]; and if it is less, they will be at their western distance.

And I will bring you an example from Ptolemy, which he adduced in chapter 2 of the tenth [book] of the *Almagest* concerning Venus; and from the sequence of his words you will understand the matter fully. He said there that he received from Theon the astronomer, who was in his time, who observed and reckoned the planet Venus in the year 129 of the Christians [CE], on the 19th of the month June; and according to the distance and the position it had relative to the fixed stars that he mentions there, he learned that it was at degree 36, [and] minutes 10, of Aries, and its southern latitude [was] 30 [recte: 1°/...] [?]. And since the mean motion of the Sun at that time was 24 45 [recte: 24;45] of Taurus, whose longitude exceeded the longitude of Venus, it was distant from the Sun by a distance of 48 44 [recte: 48;44] [recte: the difference], and the center of its epicycle was at the apogee of the carrier [deferent].

Again, Ptolemy himself observed Venus in the year 136 of the Christians [CE], on the 18th of the month November, and investigated its distance [from] the fixed stars, until it became known to him that its true place was 12;50 [12 50] of Capricorn, and the mean motion of the Sun was less than it, being at 25;30 [25 30] of Scorpio. Therefore its distance was western, 20;47 [20 47] [recte], and it [the center of the epicycle] was at the perigee of the carrier. And Ptolemy decreed that the greatest distance of the center of the epicycle — that is, the apogee of the carrier — is when the Sun is at 25 of Taurus, and the nearest [perigee] when it is at 25 of Scorpio; for he found no distance greater than this, nor any smaller than this, when [comparing] place with place, [these planets] being at all the other points in the sphere of the zodiac.

Now turn your eye to this figure, in which is the circle of the zodiac, MTHG [מתהג], described upon the center of the Earth B [ב], and the circle of the carrier [deferent] — [whose] center [is] the center of the epicycle of Venus — is MTHG [מתהג] [recte], described upon center A [א]; and let them be divided in half by the diameter [recte]; and let the center of the epicycle [carrier] be at the apogee of the carrier at point H [ה], which is 25 of the sign of Taurus, according to the observation of Theon mentioned above; and likewise let the center of the epicycle be at the perigee of the carrier at point D [ד], 25 of Scorpio, according to the observation —

Diagram. A schematic of Ptolemy's Venus model introduced at the foot of the page. The accompanying prose names the circle of the zodiac drawn on the Earth's center, the carrying deferent drawn on its own center, and the apogee/perigee of the deferent at opposite points of a diameter. Only the labels

named in the running text are legible; the figure itself continues onto the following page.

- ב (center of the zodiac circle): the Earth's center
- א (center of the carrier/deferent): center about which the deferent carrying Venus's epicycle turns
- מתהג (circumference): points of the zodiac circle MThHG
- ה (point on diameter): apogee of the carrier, at 25° of Taurus
- ד (opposite point on diameter): perigee of the carrier, at 25° of Scorpio

Gevurat Yah (גבורות יי) — Fourth Part of *Sefer Elim*

Degree 22 (מדרגה כ"ב), continued — Venus eccentricity, concluded; Mercury

as Ptolemy [did], so that it will be at the eastern side, with [point] ש in the west. Now from the centers of the [two] circles, in the two positions, lines are drawn to the true places of the star, namely ד and די; and to these are drawn lines tangent to the circles from the center of the world, ב, namely בפ and בי. These are the lines of the true motions as they appear to the two observers mentioned above at the said times. At point פ the star was in the morning, and at point י in the evening.

Now, since the lines connecting the third [center] — the lines הפ and די — stand perpendicular to the lines פב and בי, therefore, by the right [triangles] standing at פה, the angles at פה are known by you: the angles בפה, namely [angle] 44;48, and ידב, namely [angle] 47;20 [recte: 47;20 — read as printed], which are the measures of the sides of the distances הפ and די, as will be explained. This [is] in accord with Proposition 22 of Book 2 [of Euclid] for the foundation of the foundation; and what follows, with Proposition 1 of the First Premise of the *Book of the Heavenly Configurations* [the *Almagest*]; and Proposition 32 of the First [book]. The third angle, too, namely [angle] ידב, will be known — return [to] Proposition 1 of the Fourth Premise for the foundation of the foundation — and we shall know the measures of all the sides, and the demonstration is as follows:

As 100,000 is to הב, so [is] the chord of [angle] 44;48 — which is 70,463 — to הפ. Thus.

As 100,000 is to בד, so [is] the chord of [angle] 47;20 — which is 73,531 — to די. And to the dispute… [the discussion continues]: since הפ should be equal to די — how can they be measured as unequal quantities? This was explained in the

science of optics, and sense-experience attests that whatever the perceived [object] is nearer, its angle appears the larger; and so it is with the circle in its lower [position] — which brings [the explanation], and understand it.

Second, reason [argue] thus: As 73,531 is to הב, [namely] 100,000, so [is] [?] to הב [namely] 100,000, [and] to בד, [namely] 95,828. [Their] joining together is the measure of the whole diagonal הדבד and its halves; [namely] בד is 95,828, [and] this is the measure of line אה, or אד. And when we subtract [it] from line הב, [namely] 100,000, there remains line אב, [namely] הבד 195,828 [recte: 195,828 — read as printed; this is the sum 95,828 + 100,000 = בד + הב], the measure between the centers, 2,086.

And if you set half the diagonal which brings [i.e. half of אד], its half אה [is] 97,914, [in] one part [where] the radius is 60, the value [of which] is: as 97,914 yields 60, the eccentricity 2,086 [yields] how much? It yields 2,086. You find the eccentricity 1;16 [recte: 1;16]. And likewise, as 97,914 yields 60, how much does 70,463 yield? You find the measure of הפ, half the diagonal of the circle, 43;10.

And this is all that we sought to know.

Now, Nicolaus Copernicus, who reckons that the terrestrial globe is carried on a circle that revolves [about] the sphere of Venus, would find the measures of the diagonals of the sphere of Venus and the sphere of the Earth, by Ptolemy's assumptions, with great ease. He reckons, by the said figure (figure 49), that circle מתהג is the sphere of Venus, and נשק is the sphere of the Earth. Let ש be the point of elevation [apogee] and ן the [point] of depression [perigee]; and from them draw lines שט and ות tangent to [the sphere of] Venus at the place where the star was found in the said observations; and let perpendiculars be raised, in accord with Proposition 3, from line אט, namely את, upon the tangents. So that the right triangles standing [are] שא and תא. Now angle ש [is] 44;48, and angle ן [is] 47;20. Hence, by the right triangles standing אטש [and] אתן, all the angles are known, and from them the measures of the sides — that is, אט [is] 7,046 in what אש is reckoned [as] 100,000, and likewise את is 7,353 in what אש is reckoned [as] 100,000; and in what את equals אש, which is 7,046, [then] שבן will be [?] 9,582. Hence the whole [of] שבן will be 19,582; its half שב [is] 9,791, and the remainder אב [is] 209. And in what the whole שב — the diagonal of the sphere of the Earth — is reckoned [as] one [unit], אט will be 43;10 and אב approximately 1[;0]. And in what שט is reckoned [as] 100,000, אט (or את) will be 7,193, and אב 208 approximately. Thus far [Copernicus] in chapter 21 of the Fifth [book] of his *Revolutions*.

Once more my hand is stretched out to show you how Ptolemy found the measure of the diagonal of the circle of Mercury and the eccentricity…

[of Mercury]. He has already informed you of its paths in the preceding Degree, which are precisely like the paths of Venus. Now, Ptolemy, in chapter 7 of the Ninth [book] of the *Almagest*, investigated and sought out fully the far and near distances of Mercury from the Sun. And in chapter 8 he verifies that he took its distance from the fixed stars and ascertained its true place once in the morning: it was at 12;20 [recte] of Virgo, and the mean place of the Sun was at 9;15 of Libra; hence its distance in the morning was 3;19 [reckoned as the difference 9;15 − 12;20 of the sign before]. Then a second time, in that same year, he found the true place of Mercury in the evening at 4;20 of Taurus, and the mean place of the Sun was at 11;5 of Aries, so that its distance was 23;15 morning [recte: evening]. These are the two extreme [limiting] distances of Mercury, namely when the Sun is at the elevation [apogee] and at the depression [perigee].

Let it be, by the said figure: ד the [point of] elevation, and so on, 9;15 of Libra; and ו the [point of] depression, which brings [it], and ב'5 [recte: 11;5] of Aries. And [let] Mercury be at point צ in the morning and at point כ in the evening; and Ptolemy's eye was at the center of the world, ב. From it there go out lines קי, namely בצ and בכ, tangent to the circle at the place where the star was. And from the center of the circle let there be lines כ ,הצ and בס perpendicular to the tangent lines, in accord with Proposition 3. And since angle בצ of the great distance is 3;19, by Proposition 32 of the First [book], in the right triangle [whose] angle is at בצדק [recte: בצק], all the angles are known, and Proposition 1 of the Fourth Premise for the *Book of the Foundation of the Foundation* makes known the measures of the sides. And likewise in the right triangle [whose] angle is at בכר, the angle of the distance 15;23 ,רבכ, is known. Hence all the angles and the measures of the sides are known as above, and the demonstration is as follows:

Statement	Term	Value
As the chord of [angle] 90 is to הב, [namely] 100,000, so [is] the chord of [angle] 3;19	אל צה	32,639
And as the chord of [angle] 90 is to דב, [namely] 100,000, so [is] the chord of [angle] 23;15	אל כך [recte: כר]	39,474

[Table reads right-to-left in source; the left column gives the resulting line and its computed value.]

Hence, by [the rule of] equality, and by composition, casting off the 90 of equals as was shown in the *Book of the Heavenly Configurations*: it will be, by the proportion — as 4כ [recte: כ, the chord of 90] is to צך [recte: צה], [namely] 32,639, so [is] הב, [namely] 100,000, to רב' [recte: דב']. Hence one finds that דב' equals 82,685. And when you add to it הב, [namely] 100,000, the whole line הבד — the diagonal that brings [it] — will be 182,685, and its half אה (or אך) [is] 91,342. And if you subtract from this line דב, [namely] 82,685, there remains line אב, [namely] 8,657, the measure between the two centers.

And after we have known the diagonal of the circle, the eccentricity, and the half-diagonal that brings [it], all three [being] in numbers of one kind, we shall reckon the half-diagonal that brings [it] [as] one part and shall set up a proportion thus: As אך, [namely] 91,342, equals 60', how much does אב, [namely] 8,657, equal? You find the eccentricity 5;41. And likewise set up the proportion: as אך, [namely] 91,342, equals 60 minutes, how much does צה, [namely] 32,639, equal? You find 26;21, the measure of the half-diagonal of the circle.

And Ptolemy found, afterward, the measure of the small circle on which the center that brings [it] moves; but consult [it], for it is not my desire to lengthen further.

But Nicolaus Copernicus, who admits no [eccentric] circles, would reckon, by the said figure — and נשק the sphere that brings the Earth, upon center ב, and on טוזג [recte: וטהג, after fig. 49] the center [?]; the sphere that brings the star of Mercury [is] drawn upon center א. And from it he draws out lines אמ [recte: אט], and upon them perpendiculars את. And lines שט and ות; and behold אט [and] את are the chords of the angles, or the extreme distances of Mercury from the Sun, that is, אשט [is] 3;19, and אות [is] 23;15. And one finds the measure of אב, the eccentricity, 5;41; and אט (or את), the half-diagonal of the sphere of Mercury, 26;21 [recte: 26;21] — in what the half-diagonal of the Earth, ב, is reckoned [as] one, with that same reckoning that we too found for the [eccentric] circle. And the rest of his pleasant words, look [for them] in chapter 27 and following, from the Fifth Treatise of his *Revolutions*.

Degree 24 (מדרגה כ"ד) — The thicknesses of the spheres and their distances from the Earth

After I have informed you of all the above, let me return to what was sought — what you asked to know: the way, by a demonstrative method, to know the thicknesses of the spheres and their distances from the Earth.

I say that first you must settle and agree that their order is according to [the sequence of] the seven planets, for so the ancient astronomers agreed. Afterward you shall investigate, and learn, according to Ptolemy's indications in his wondrous book the *Almagest*, the measures of the eccentricities of the centers that bring the stars [i.e. the deferents], or the [epicyclic] circles, and likewise the measures of the diagonals of the circles — all of them in [the unit] where the half-diagonal that brings [each] one is reckoned [as] sixty parts.

And as I have shown you, the sixtieth part of one [body] is not equal to the sixtieth of its fellow. Therefore you need a stratagem to convert them all to one [common] measure, so that you may know their proportion. And this is very easy, once you already know the distance from the center of the Earth to the concavity of the sphere of the Moon, which is, according to Ptolemy, 33;33 of the half-diagonals of the Earth; and according to others, 33;42 — as I have arranged for you in all the [tables] that have passed [above].

Upon this known [value] you shall set up your ladder and ascend to the Eighth Sphere. For clarification of the matter, set your eyes on the figure of Degree 14, by which you will understand the thickness of the sphere of the Moon. And you already know, from the investigation, that in what line צן, half the diagonal of the bringing circle [deferent], is reckoned [as] 60, the eccentricity צך is reckoned [as] 12;28½, and line ין, half the diagonal of the lunar [epicyclic] circle, [is] 5;15. And when you subtract ין from צן, there remains line צי [namely] 54;45. Subtract from this the thickness — which, as we explained in Degree 14, equals twice the eccentricity, namely the measure of line צש, [namely] 57;24 [recte] — there remains line צך [namely] 29;48. Add to it half the eccentricity, [namely] 12;28½ [recte], there remains הזד [recte: דזך] [namely] 16;2½, the distance from the center of the Earth to the concavity of the Moon. And since דזך equals 33;33 of the half-diagonals of the Earth, therefore set up the proportion thus:

Statement	Result
If 16;2½ equals 33;33 [and] 12;28½ [is] the eccentricity, you find:	9;55 for [line] צח [recte: צי]
How much do 5;15 [for] the half-diagonal of the circle equal?	4;10 for ין

Hence you add to דזך, [namely] 33;33, the thickness די, which is twice the eccentricity, [namely] 19;50, and upon them twice ין, [namely] 8;20 — that is, line אי. You find the measure אדן [recte: אזן], which is from the center of the Earth to

the convexity of the Moon — which is the concavity of Mercury — [namely] 61;43 of the half-diagonals of the Earth.

And likewise, if you wish to know the thickness of the sphere אך [recte: זך], subtract from אזך [the value] 61;43 — that is, the measure of דז, which is 33;33 — and you find 28;10, the said thickness. And likewise, if [you wish] the measure of the half-diagonal that brings [it], add to דז, [namely] 33;33, [the value] צח [recte: צי] [namely] 9;55 and ין [namely] 4;10, and there comes out צז [namely] 38;47 [recte: 47;38?]. And so you enter into the inner chambers of the [respective] sphere.

But, since we took the measures by the distance of the concavity of the Moon, and by the diagonal of the [epicyclic] circle and the eccentricity, [which are] less than what was assumed in the table of Degrees 15 and 16, you will find all the [values] found by [this] demonstration to be less than the reckonings of the said table. However, according to the opinion of the sage who composed the said table — that the eccentricity is 10;9 [recte: 9;10?], and the half-diagonal of the circle [is] 5;5, and the distance from the concavity of the Moon to the center of the Earth [is] 33;42 — you will find them [the values] exact and close to his reckonings.

And here, by his way, is their arrangement; and as you found in the sphere of the Moon, you find הזא [recte: דזא] [namely] 64;10, so too in the sphere of Mercury, when you reckon that line דז [is] 64;10 of the half-diagonals of the Earth, אך [is] 30;28 [recte: see table]. And convert the measure of its eccentricity and the diagonal of its circle, which is known by [the fact that] half [the]…

Line	Value (col. 1)	Value (col. 2)
חד [recte: דז?]	42	33
בי	18	20
אי	10	10
חא	10	64
אד	28	30
צו	56	48

[Table reads right-to-left in source; the rightmost column holds the line-labels, then two numeric columns (degrees, then minutes/sub-units). Several letter-labels and the column roles are abraded.]

the diagonal that brings [it] [is] 60, to the half-diagonals of the Earth. And when you know its convexity, you will know the concavity of Venus, which is above it,

immediately. And so you ascend, by [these] degrees, on the mountain of the Lord. Yet I have already become weary of my work, and the season has set its scent upon me; and I have set in my vessel the balm of my joy. If the Lord grant me to complete the *Book of Joy* [Sefer Simchah], your soul will delight as I desire, and then the nations will say, "The Lord has done great things" (Tehillim 126:2). And now, behold, I have come down from the lofty heavens, lest "they break forth against us" (cf. Shemos 19:24) — the angels of God.

Degree 25 (מדרגה כ"ה) — The size of the Earth

Behold, you have seen that all the measures and distances of the spheres are built upon the measure of the diagonal of the Earth, and are proportioned to it. And if we do not know its measure, all is vanity, and the spirit shall touch all the said [values]. And the chief of poets [the Psalmist / or R. Avraham ibn Ezra] said:

> A man who would give his wealth to strangers, and not to his kinsman, Is like an eye that sees what is far from it, but does not look and does not see what is around it.

[Two lines of verse; rendered as a couplet. The first hemistich of each line precedes the second in the source layout.]

Therefore it is fitting for us to investigate it greatly, and to be precise about it — all the more so, since the matter is near to us, more than the far and hidden heavens. And I have heard many say, "Who has measured the earth?" — for although in this time the deeds of the sailors, the men of England and Holland, are wondrous (they enter by way of the Strait of Magellan, and the other ways are known to them, and they go round and come by way of the south-east, and return after one or two years to their place, as I have spoken mouth to mouth with several of them) — nevertheless the understanding one will say: how do they estimate the [length of] travel, when they go in a slant, sometimes to the north and sometimes to the south, according to the blowing of the changing winds? And who knows the way of a ship in the heart of the sea? The paths of the dry land are not known: their ways are crooked, and between them are seas and high mountains and forests and dangerous roads. And if they do not travel always beneath the equator, or beneath some [other] circle running from north to south without deviation, they will err — and this is impossible to keep over a great journey in such [conditions].

Yet [astronomical] science is [the means] for sound men to escape all the hindrances mentioned, and to know its [the Earth's] measure while they are quiet

in their homes, or strolling secure and tranquil. And this [is] by various methods; and I shall mention some of them, the easiest and soundest.

The first method belongs to the ancients, according to what their words have brought in the books of Strabo and others of the…

[the geographers]: that in the city of Syene, which is precisely beneath the Tropic of Cancer, there is a well that has been dug, very deep — and it is a wonder to many who see it and marvel — that the Sun illuminates it entirely, down to its depth, as Pliny also wrote. And this [happens] when [the Sun] is over its head [the zenith], at the head of Cancer, at midday. And at that very time they observed and saw, as Eratosthenes reported, that the columns [gnomons] cast no shadow at all to any of the sides of this city, [for] 150 stadia around. And accordingly, the diagonal [path] of the Sun [over the meridian] [is] 300 stadia [for the noon shadow-difference]; in the manner that they intended, that as the proportion of [those] stadia [is] to the whole circuit of the terrestrial globe, so [is] the diameter of the Sun to the circumference of its sphere. For the whole globe of the Earth is commensurate with the whole solar sphere, as we explained in Degree 3. Therefore all the determination depends on knowing what part of the 360 degrees is the diameter of the Sun. Now the sages of Egypt, with the hourly water-clocks, found it [to be] 1/750; and when you multiply 750 by 300 stadia, you find the whole circumference of the globe of the Earth [to be] 225,000 stadia. And according to Cleomedes [חיאוניס], the expositor of the *Almagest*…

[The discussion of the Earth's circumference continues.]

in Treatise V, and Proclus in chapter 3 of his abridgment of that same book. There you will find the construction of these instruments. And in truth the *Kalifsidris* [Greek: κλεψύδρα (klepsydra, "water-clock")] of sand, or the self-moving machines that exist among us, are better than the timepieces of the ancients; for that reason Ptolemy had no use for them. But Aristarchus too wrote, in the seventh and eighth books on the measures of the Sun and Moon, that it [the solar diameter] is 28′49″ — that is, $\frac{1}{750}$ of the circle — and this appears equal to the diameter as the Moon is seen. Now Archimedes computed it according to his own investigation, with a large *Diopra* [διόπτρα (dioptra, sighting instrument)], as less than $\frac{1}{164}$ of the right angle [of the quadrant], that is, between 27′ and 32′55″ [as printed]. But he himself said that one should not rely on these investigations, since they are not demonstrative.

And Ptolemy, by what I have already mentioned, examined it both with the dioptra and by means of eclipses, and confirmed that the solar diameter is

constant without variation — whether the Sun is at its apogee, taking it at, or at its perigee, 31′20″ [as printed; recte: 31′20″ — i.e. 0°31′20″] — and that it is equal to the diameter of the Moon at the Moon's apogee. But Proclus, in his chapter 3 mentioned above, according to what he was prompted to by the words of Sosigenes — the astronomer of Julius Caesar, who saw what is written in his [Sosigenes'] book on circumferences (he is named therein) — examine and see that the solar eclipse is sometimes more [complete] [?]: the round disc of the Sun all around it [the Moon] is not covered by the Moon. So too many of the moderns have testified, who saw this in their own time. Therefore it is impossible that the lunar diameter should be equal to the solar one in the gnomons and the *barbukim* [?: instrument-term, perhaps "graduated arcs"] as Ptolemy supposed.

Now Albitin [Al-Battānī] [?] found the solar diameter at its apogee 20″ [as printed], and at its perigee 33′40″ [as printed; the minutes appear sub-unit-reversed — recte: 33′ 40″]. And Copernicus: 31′48″, and at its perigee 33′45″. And Tycho: at apogee 30′ [some minutes], and at its perigee 32′ [as printed]. And as I have written about this, it is proper to bow the head to Tycho's investigation, to verify his instruments. Therefore the solar diameter, when it is at the head of Cancer [the summer solstice], will be 30′ [arc-minutes]; and the circumference of the Earth will be 21600 stadia [as printed; this figure is corrupt for the Earth's circumference — recte the 225,000-stadia result of the preceding Degree; flagged]. This is what he meant by this method.

מדרגה כ"ו [Degree 26]

The second method, likewise belonging to the ancients and attributed to Eratosthenes, works by means of the shadow cast by an upright gnomon standing on the surface of the Earth. He found that the city of Alexandria in Egypt lies on the noon-circle [meridian] of the city of Syene — that is, that their longitude is the same, measured from the western edge — and that the distance between them is 6183⅓ stadia [as printed]. And at the city of Syene, as I wrote, when the Sun is at the summer solstice at midday, the gnomons cast no shadow, for the shadow runs straight to the center of the Earth. This is not so at Alexandria, which is more northerly, where the gnomon casts its shadow toward the north.

Let it be as in the figure before you (figure [51] נא):

Diagram. A geometric construction for Eratosthenes' shadow method of measuring the Earth. At point **א** [alef] stands an upright gnomon in the city of Alexandria; its height is known, and its shadow, cast from a known ray-tip, is likewise known (whether measured directly, or estimated, or computed from its

[the Sun's] altitude as taught in chapter 5 of Treatise II of *Sefer Chukos Shamayim* [the author's astronomy]). Because the arc subtended is so small in proportion to the whole circumference of the Earth, no error results if we treat it as a straight line. Point **ה** [he] is the city of Syene, where the Sun strikes with its ray **פבך** and reaches the navel [center] of the Earth — this is the well **ה**. The gnomon at point **א** is likewise pictured as extending [its line] to the center **ק** [kuf].

- **א** (Alexandria): the upright gnomon
- **ה** (Syene): the deep well, the point where the solar ray reaches the Earth's center
- **ק** (center): the navel [center] of the Earth
- **צ** (ray-tip): the tip of the gnomon's shadow at Alexandria
- **ד** (shadow-tip): foot of the shadow, in right triangle **דאג**
- **ג** (gnomon foot / shadow line)
- Ray **פבך / צבך**: the solar ray to Syene reaching the Earth's center

In the right triangle **דאג** (right-angled), two sides are known — namely the gnomon and the shadow. And from column 1 of corner [section] 4 [?] of *Sefer Sod HaYesod* [the author's trigonometric tables, "the Book of the Foundation's Secret"] one finds the measure of angle **ד**, and likewise [its complement] angle **ד** [at the apex], by Proposition 29 of Book I, since the rays of the Sun here [צג]…

are reckoned as if they were parallel to one another, on account of the smallness of arc **בא** lying between the two cities in proportion to the solar body — as I shall explain in *Sefer Ma'ayan Chatum* [the author's promised work, "the Sealed Spring"], God willing. Therefore arc **בא** too — the measure of angle **ה**, which is known — is the distance between the two cities **ה** and **ב**. And according to what the author of the book on the circumference of the Earth, Eratosthenes, reported, he found arc **א** to be ⁴⁄₅₀ [as printed] of a degree [recte: ¹⁄₅₀ of the great circle, i.e. 7°12′]. Therefore, if you set up the proportion and say: ⁴⁄₅₀ [as printed] gives 6183⅓, what will 360 give? — you find the circumference of the Earth to be 252000 stadia.

However, in Strabo, Treatise I, you will find that Hipparchus criticized Eratosthenes for reckoning the distance between Alexandria and Carthage [קארטאגינוס] as 13 thousand stadia, when it is not even 9 thousand — so that none of his measurements is exact. And Posidonius found by Eratosthenes' method — that is, by the shadow — less than this, namely 2000 stadia less, except that he reckoned the distance between the two aforementioned cities to be only 5000 stadia [as printed; recte: he reckoned a shorter base], not so great; and according

to others, no more than 4560 stadia. Consult Cleomedes on this in his book, and Pliny and Strabo, regarding what they reported and what they set down there, and Vitruvius [ווויטרוויאום] in Treatise II [?], chapter [?]. Hipparchus too reckoned the circumference at 25000 stadia [as printed], and the difference between them is great.

מדרגה כ"ז [Degree 27]

The third method is the one peculiar to Posidonius, according to what Cleomedes (mentioned above) wrote — namely, that he observed the fixed stars and saw the star Canopus [קנו"בוס], which is at the stern of the ship called *Argo* [ארג"יש]. It is a very bright star, but in the lands of Greece it is not visible, and for that reason Aratus [אראטוס] did not list it among the stars. At Rhodes it passes along the line of the horizon and immediately sets, and even there it is visible only from a high place. But on the noon-circle [meridian] of Alexandria it stands high above the horizon — according to what Proclus reported in *Sefer HaKadur* [his book "On the Sphere"], more than a quarter of a [zodiacal] sign, that is, 7½° [as printed]. And Rhodes is on the same meridian as Alexandria, as Ptolemy too wrote in chapter 3 of Treatise V of the *Almagest* [לטיניסטיו]. Therefore, if you multiply the distance between Rhodes and Alexandria — which is 5000 stadia — by 360 and divide it by 7½, you find 24000 stadia.

By this method you can seek the altitude of the meridian, that of some known [star], when it rises [?] on another horizon, with very precise instruments, down to the second [of arc]; and you will see how high it stands above the horizon. Then again seek, on a second horizon lying along another longitude with the first, the altitude of the same star mentioned, subtract one altitude from the other, retain the difference, and survey the distance traveled between these two cities. See how many miles it is, and reason as follows: "if such-and-such degrees of difference give such-and-such miles, how many miles will 360 degrees give?" — you find the circumference of the whole Earth. This is a higher path [a superior method], except that it requires great precision in measuring [the distance].

And you must choose level ground over which you can sight some marked landmark and walk toward it without the path bending; and when you come to the first marker and know the distance between them, you proceed from there to a second marker. If the second is in a straight line with the first and the place you set out from, all three — this you can determine easily and well. But if not, take the angle of position of the two with the third (and I have already taught its nature in *Sefer Chukos Shamayim*, or with the *Magnitis* [magnetic / compass] instrument), and write down the lengths according to the measure of the angle on

paper. So you continue your path, for afterward the crooked sides will yield the measure of the straight path. And all this depends on measuring the distance exactly; and [it is best] when the Sun is at the solstices, for then no difference in its altitude is sensed...

from day to day, and you can take the Sun's altitude over one city and proceed from there until you see that the difference in altitude is, say, ⅙ of a degree [as printed], from which you reason about the whole circumference. For the longer the path you travel, the greater the error in measurement; therefore it is good to keep it short and to be precise. Now know this not only about the distance covered along a great circle [the meridian], but about any meridian whatever, provided you know your distance from it. For example, two cities under the equator: in one an eclipse occurs at midday, and in the other [it occurs] one hour past midday — we then know that the difference between them is 15 degrees [as printed]. And if we know exactly the distance between them, from that we easily know the whole circumference. But if both are at a latitude of, say, 30 degrees toward one side, we know the value of the parallel circle that is distant 30 degrees from the equator, from what I taught you in *Sefer Chukos Shamayim*; and from that you can likewise know the circumference, just as from the great circles above — and you will know it correctly and easily.

מדרגה כ"ח [Degree 28]

The fourth method belongs to one of the later mathematicians, called *Maroliqus* [Maurolico] [?], who was learned and acute. His method is this: that one climbs a high and steep mountain — such as Mount Ida [הר אירא] in Candia [Crete], or Etna [איתנה] in Sicily [שיציליאה] and the like — and by the method of determining heights, expounded in geometry with the instruments, we know its altitude. Let it be as in the figure before you (figure נא [recte: figure for this Degree]):

Diagram. A geometric construction for Maurolico's mountain-height method of measuring the Earth's radius. The globe of the Earth is drawn about center **פ** [pe]. The altitude of the mountain — Mount Etna, which the author chose because it was in the city of his birth — is line **אב** [alef-bet]; its height we determine with the instruments. Upon it is set the measuring instrument, so that one of its sides is perpendicular, upright over the mountain like line **אב**, while the other side, **אק** [alef-kuf] [recte: **אצ**], along which the dioptra is inclined, is tilted until we sight along the line of the solar ray, or some rising or setting star on the horizon, at point **צ** [tsade]. Line **פצ** [pe-tsade] is drawn to its [the Earth's] far edge from the Earth's center; it is perpendicular, upright on the tangent **אצ** [alef-tsade] by

[Euclid] III.18 [Book III, Prop. 18]. From the tilt of the dioptra we know angle **א**, and by [Euclid] I.32 [Book I, Prop. 32] angle **אפצ** too will be known.

- **פ** (center): center of the Earth's globe
- **א** (mountain summit): top of Mount Etna, apex of the gnomon
- **ב** (mountain base): point on the Earth's surface at the mountain's foot; line **אב** = mountain height
- **צ** (sighting point): the point on the Earth's far horizon where the line of sight (solar ray / rising or setting star) meets the surface; tangent point
- **ק** (auxiliary point on the radius / radius **פב**)
- Line **פצ**: radius (tangent-perpendicular) to point **צ**
- Line **אצ**: line of sight (tangent) from summit to horizon point

And according to what was taught in column 1, corner [section] 4 of *Sefer HaYesod* [Foundation], you find the values of sides **אפ אצ צם** in *bekaim* [sexagesimal sine-parts] [?]; or you reckon **פצ** [pe-tsade] as the whole *beka* [the whole sine, the radius], and know from the *Luach HaZahav* [the "Golden Table" — the trigonometric tables] the measure of the segment **אפ**; and when you subtract from it **פב** [pe-bet], the whole *beka*, there remains **אף** [recte: **אב**] known in the numbers of the Golden Table. Then again set up the proportion and say: "if **אף** of the table gives **אב** known from measurement, how much will **פצ**, the whole *beka*, give?" — and you find the measure of **פצ** in the numbers of the altitude of **א**; and that is the half-diameter [radius] of the Earth sought.

Or you may go by this path: after you have estimated angle **א** and know its complement **אצ** — the elevation of the tangent line at **ד** [recte] meeting the ray **צ** [recte **צד**] at point **ק** [kuf] [as printed], and line **חן** [recte] reaching it — then in the triangles **פבה פצה** [pe-bet-he, pe-tsade-he] the angle at the center **פ** is equal, since their sides **פב פצ** are equal and the angles at **ב** and **צ** are right angles. Also the angles at **ההצ** [recte] are equal, by [Euclid] I.26 [Book I, Prop. 26]; hence, by [Euclid] I.4, the angles at the center **פ** too are equal. And since we will know the whole angle **אפצ**, we know also its half **בפ** [recte], and its complement **בדפ** [recte]. Therefore first, in the right triangle **אבה** [right-angled], from knowing the angles and side **אב**, by [Euclid] column 3 of corner 4 of *Sefer HaYesod*, we know side **בה** [recte] according to the measure of **אב**. Then afterward, from all [knowing] the angle and side **בה**, in the right triangle **אבה** [recte **פבה**], whose angle **פבה** [pe-bet-he] is right, side **פב** [pe-bet] is the half-diameter of the Earth sought.

Or, by this method: after you know **בה** [bet-he] in the right triangle **אבה** [recte], that is, knowing…

to you, from the instrument, angle **א** and its complement, by [Euclid] I.32 [Book I, Prop. 32], you know **אדק** [recte], and you also know the altitude from which [you sighted] **א** — know **בה** [bet-he] as above — and then you also know, by [Euclid] I.47 [Book I, Prop. 47, Pythagoras], line **אך** [alef-kaf]. When you add to it **קצ** [recte], the line equal to line **בק**, as is understood from [Euclid] I.47 of Book III [recte: Book I, Prop. 47], the square arising from the whole tangent line **אצ** equals the square arising from the whole line from **א** to **אב** [recte]. Therefore, if you divide the square of **אקצ** [recte] by **אב**, the mountain's altitude — for line **אך** is greater than line **אב** by the measure of **בך** [recte], and you subtract from it line **אב**, and there remains more than **ך** [?] — [you obtain] the measure of the whole diameter of the Earth's globe. And from the diameter known by what was taught in Degree 2 [recte: Degree 22, or the cited Degree], you find the circumference. This is what we wished to explain.

מדרגה כ"ט [Degree 29]

Just as the methods for measuring the circumference of the Earth are many and varied, so the opinions [the resulting values] differ greatly, some of them far from the truth — as is the opinion of the ancients whom Aristotle mentioned in Book II [of *De Caelo*], the *Lishtim* [?: those who computed], who reckoned its circumference at 400000 stadia, that is, 50000 miles; and the opinion of Cleomedes in his Treatise I, that it is 300000 stadia, that is, 37500 miles. And Hipparchus, according to one opinion, reckoned, like Eratosthenes, that it is 252000 stadia [as printed; the manuscript shows "20" truncated — recte 252000], that is, 31500 miles; and others said that Hipparchus, in his own opinion, [reckoned] 277000 stadia, that is, 34625 miles. The ancients among the Arabs reckoned it [blank/missing figure] miles. But Alfarghani [אלפ-נאן, al-Farghānī], in his book, and Alāṣīn [אלאצין, al-?] in the book of the [?] reckoned it 20400 miles. And some of the moderns [reckoned] no more than 19080 miles, and their words seem closer to the truth.

But most of the moderns tested it, whether by sea-route or by land-route, and found 15 [as printed] *parsaot* [parasangs] [recte: per the total below, 15 parsaot per degree] for each degree, so that the whole circumference is 5400 parsaot, that is, 21600 miles — and this is close to Alfarghani's measure, which the Arabs uphold. They say that King Al-Ma'mun [אלמאנון] was very exacting in the matter and sent surveyors to the plains of Shinar [שינגר, Sinjār], near the seacoast, where the observer can sight far in a straight line; and they investigated very carefully

and found for each degree 56⅓ miles only. Now many think to reconcile all the opinions, saying that the measures are not equal [the units differ] and that they all agree or are close; and it seems that everything depends on the precision of measuring the distance from place to place, on which this whole reckoning is built — and a small thing greatly multiplied [throws it off], and the error is not sensed.

And although my own opinion is with the moderns, nevertheless I have agreed for now to follow another path: the opinion of Ptolemy, who in his *Geography* agreed with the opinion of Marinus [מרי"נוס], his predecessor, in the survey of the Earth, who set it at 180000 stadia, that is, 22500 miles. And according to what I taught in the lessons of Degree 2 [recte: cited Degree], you find its diameter to be 7158⅓ [as printed], and its half [radius] 3579 miles. And within this great thickness there would be some [hollow] — for metals are not generated except by [blank] — therefore they place within it the chambers of Gehinnom of which the Kabbalists speak.

And conversely, if you double the diameter to ⅓ [recte], you find the aforementioned circumference, that is, 22496[⅓] [as printed; recte 22500]; and when you double its half — which is 11248[⅔], half of the aforementioned diameter — you find the area of the great circle to be 40257614[¾] square miles. And if you multiply it by 4, you find the circumference of the surface [area] of the Earth's globe to be 461030458[⅓] [as printed] square miles. And when you reckon, over four thousand years, a hundred generations — that is, every forty years another generation, so there are 100 generations — and in each generation there are [as many as] those who came out of Egypt, who were within 3 parsaot, that is, 144 square miles, then they would need 14400 square miles only. And even if you reckon the rest of mankind [non-Israelites] with them, half the surface area of the Earth's globe — which is dry land, as we shall explain — would suffice for them. But the Gaon…

Gevurat Yah (continued)

[Degree 29, continued — Earth's resurrection-population aside, sphericity of land and water]

Our master Saadia [Gaon] wrote in his *Sefer HaTechiyah veHaPedut* (Book of Resurrection and Redemption), of blessed memory — the seventh question: *Will the world have room enough for all those who will live, be fruitful, and multiply at the Resurrection of the Dead?* The answer worth knowing is this: those who rise at

the Resurrection of the Dead resemble the rest of created beings in their conduct, their needs, and their sustenance; they too require place and time like the rest of mankind. Therefore He will renew for them a different place and a different time — namely, the World to Come — and they will enter it not through toil and labor, but in ease and repose.

Moreover, from the time Israel went out and the Torah was given to mankind, two thousand two hundred years had passed, which were thirty-two generations; and each generation numbered a hundred and twenty myriads of men and women. Even if we were to say that all of them do penance, they would not fill much of the world — only one part in a hundred and fifty of the world, when we assign to each and every one more than two hundred cubits to dwell in, he and his beast, with whatever space they sow. For the men of [all] the generations were 120 myriads [= 1,200,000], thirty-two times over [reading 32 ,ל"ב], which in [precise] reckoning comes to **3,840,000**.

When we assign them two hundred parsa'ot by two hundred parsa'ot of land — which is one part in a hundred and fifty of the earth — and reckon them in cubits, with three miles to every parsa, and four thousand cubits to every mile, by the Mosaic cubit, where each cubit is two and a half [cubits and a third? — see note] of these cubits of the [resurrected] generation, then there falls to each one, in his place, a breadth of two hundred and eighty-eight cubits. What difficulty is there in this, that the commentators should be confounded by it? — thus far [his words].

And if you double the area of the circle by two-thirds of the diameter, you will find the [volume of the] body of the sphere to be **192,109,336,734** cubic miles.

Now it is agreed among all the perfect philosophers that these two elements — earth and water together — form a single spherical shape. Beyond the shadow that arises at lunar eclipses, which shows the sphericity of the earth, there are several well-known and famous proofs. I mean: spherical according to appearance, for the great mountains and the valleys are like small blisters or fissures, on the order of a grain of barley, which we know is not [a defect] in the precisely-determined geometrical sphere. The body [of earth and water together] is called "the sphere" after the dry land, [even] though the dry land within it is of little and slight proportion. As Aristotle wrote in the First [Book] of the *Meteorology* (*De Caelo*? — *Meteorologica* I), he said there, regarding the quantity of the earth — which contains fifty-two [reading 52 = נ"ב] times the volume of the water near it — that [the water] bears no perceptible ratio to the encompassing [sphere]; thus far [his words].

You already know that the heavy earth among the elements is, accordingly, prior to the water in taking the center of the world; and conversely, for the element water, lighter than earth, the upper place is fitting — just as we see that quicksilver sinks below lead, and iron, which is the heaviest of the metals [sinks lowest]. There is no doubt that, were the dry land a perfect sphere by itself, without mountains and valleys, the waters — even if they were few — would cover it entirely, and no creature could survive.

And so it was before the mountains were born, until God decreed that mountains should rise and valleys sink within it. The waters, which by their nature flow downward, entered the recesses within the body of the earth, filled the hollows and the fissures, and leveled the depressions until they covered it; and the sphere was made even out of the two of them, and half the sphere was exposed — as you see in the figure of the earth, that more than half its surface, by its structure, is dry land. This is the meaning of "Let the waters be gathered" [Bereishis 1:9]. And as the Psalmist said: "Your righteousness is like the mighty mountains, Your judgments are a great deep; man and beast You save, O Lord" (Tehillim 36:7), and he says: "You set a boundary they may not pass, that they return not to cover the earth" (Tehillim 104:9) [reading הלכ' = an abbreviation marking the citation continues].

So long as the dry land has heights and depths in a single place — mountains here and fissures and depressions there — there is no way at all for the sea to cover all the dry land, for the portion of water is a small thing in proportion [to the land], and is, as it were, like a drop in a bucket. The waters do not stand except in depressions. So Aristotle wrote, and as will be explained, God willing. Were the waters ten times the earth — as certain Peripatetics suppose — they would of necessity prevail over the mountaintops and cover it entirely by their nature, which is impossible.

[It is impossible] that there should be two spheres upon a single center, the one larger than its fellow, such that the smaller is not entirely encompassed by the water — as is explained in the *Elements of Spheres* (*Sphaerica*). Some think that [the earth] is held [in place] in the manner of [bodies] that persist [by some sustaining cause]; and this is the meaning of "He suspends the earth upon nothingness" (Iyov 26:7). And they tremble and worry over this, lest the sea return to its [former] strength.

But it is among Aristotle's grave errors — among the things his disciples ascribed to him, though it is not so — to hold that the quantity [of bulk among] the elements is in the ratio of ten to one; for if you melt a handful

מדרגה ל' [Degree 30]

But as for the saying of Aristotle, far removed from what his disciples ascribed to him — though it is not so — that the proportion of the elements [in bulk] is ten to one: rather, if you dissolve a handful of earth into water, it becomes ten handfuls, for the spongy [substance] fills a larger place than the solid. So we see in fats and tallows, which are several quarter-measures while liquid, and when they congeal become [the size of] an olive. So too the powder of gunpowder stands compressed within a tube in one place, and when the fire ignites it, the cavity does not contain it, and it bursts out, uproots mountains, and levels valleys, and the earth quakes at its sound. The like happens with the bolt of fire that forms in a cloud, and in thunder like these.

And Copernicus, in chapter 1 of the first treatise of his *Revolutions* (*De Revolutionibus*, Book I, ch. 1), demonstrates by proof that it is impossible for the water to be even seven times the earth in the terrestrial sphere — for one of two absurdities would then follow: either that all the dry land would be covered by the water, or that the water would occupy the center of the world and the earth would float upon it, so as to be exposed on some side. I will now explain his words to you in the present figure.

Diagram. Two concentric spheres on a common center, illustrating Copernicus's reductio that the water cannot be seven times the earth. The text reads: "(figure 51) — let אהדן [aleph-heh-dalet-nun] [reading the lettering as in text] be the [whole] sphere comprising the two elements upon center **ב** [bet], and אהג [aleph-heh-gimel] be the terrestrial, earthy sphere upon center… and center **ב**."

- ב (bet): center of the whole sphere comprising both elements (earth + water)
- אהדן (aleph–heh–dalet–nun): the whole sphere comprising the two elements [point-set label as printed]
- אהג (aleph–heh–gimel): the terrestrial, earthy sphere
- אד (aleph–dalet): a diameter of the whole sphere = 2 [units]
- אג (aleph–gimel): a diameter of the earthy sphere = 1 [unit]
- א (aleph): the small exposed portion of the earthy sphere

If the ratio of the diameters of the spheres is [as] one-third the ratio of the spheres themselves — I mean by their cubic roots — then the diameter **אד** [aleph-dalet] = 2 and the diameter **אג** [aleph-gimel] = 1. Consequently the whole earthy sphere would lie outside the center, and only a small part of it — namely **א**

[aleph] — would be exposed; the rest would be submerged in water, the light waters being at the center and the heavy earth floating. This is false. Such is the upshot of his words.

And there is no need for this [supposition], for those who sail the Ocean Sea — in east and west, north and south — testify that they have never found a depth greater than three miles, and in a few places two miles or one mile only. Hence even if we reckon all the depths as equal — that half the surface of the said sphere, by its structure, should be of uniform depth, one parsa [≈ 3 miles] — even though they are not equal, since the mountain-roots of the earth [are uneven], the whole quantity of the water would be **80,515,229** cubic miles.

And when we divide the two [thirds of the] body of the earth's sphere — **192,109,336,734** — [by it], there enter into it about **2,385** times [over]. And according to those who reckon the depth at half a parsa or less, the water would be about **⅛₀₀₀** [reading 1/8000 — see note] of the dry land. But if you reckon the circumference of the earth at **5,400** parsa'ot and the depth at one parsa, you will find the water to be **⅛** [reading 1/8 — see note] of the terrestrial sphere; and if the depth is half a parsa, the water would be one part in a thousand of the dry land. And this is clear, that one need not stammer over it at all.

And if you wish to know how many drops are all the waters: double [i.e. cube] their whole bulk — the aforesaid **80,515,229** — taking its cube; and they will accordingly be all of them **80,515,229,000,000,000** cubic paces. Multiply that by **1,000,000,000** paces — for, from Degree 1, the length of a mile is a thousand paces, and one must [include the cube]. By the factor **125** they will be feet: **10,064,403,625,000,000,000**. And these multiply by **64**, and they will be handbreadths: **644,421,832,000,000,000,000**. And these double [multiply] by **64**, and they will be fingerbreadths:

Gevurat Yah (continued)

4,122,379,724,800,000,000,000 [fingerbreadths]; and these, by **64**, will be barley-grains: **1,688,526,735,278,080,000,000,000** [reading 1688526735278080… with trailing zeros].

And when you reckon every barley-grain as formed of a hundred drops — even though it is not really so large — you will find **16,885,267,352,780,800,000,000,000,000** drops.

And if you multiply the number of grains by the number **2,385**, you will find how many barley-grains is the whole terrestrial sphere. And for greater brevity in the

reckoning: the number **2,385** barley-grains weigh a *litra*; and the weight, the heaviness, of the whole terrestrial sphere will be as many *litra* as the number of barley-grains stated above.

And if your heart misgives you, that in the recesses of the earth's interior there are metals and quicksilver that weigh more and more than water — reckon that **2,385** [grains?] weigh three or four *litra*, and the whole matter is revealed to the discerning, and is known to him who orders his words; for the waters too are not equal in weight. For you will see that Pliny writes, that in his country, in the district of Cara [Caria? — see note], there are two springs, the one unlike its fellow: everything cast into the one sinks, and in the other everything floats. [Such is] the Dead Sea [*Yam Sedom*], in which men float like grass, owing to the abundance of its salt and its thickness.

מדרגה לא [Degree 31]

All the philosophers maintain that there is no [empty] place at all in the world, but that all of it is filled with one of the four elements; and by what stratagems are [things] brought about in the rising of water, so that no void be left — not because the void itself acts (for it is sheer privation), nor because the water fears and worries over the void, but because that place is fitting for water; for its way is to enter into hollows, if no other body bars it. And [the elements] are continually thrust off by the air, so they are compelled to rise as far as they can; and likewise the air does, and the light [bodies] by their ways.

You will test this and see that no void exists [in this way]: make a vessel of lead or of leather, fill it with water through a small hole, stop the hole well, and make a small hole in its bottom — and you will see that the water will not come out by any means. The reason is that the air below holds [it] back, since it fills the place; and so long as the water is not thrust from [the bottom by] something else, [the waters] will not descend. But if you open it a little above, then the air above thrusts the water, and at once it flows, being [drawn] down by its nature. In casks of wine and beer one sees this matter at all times.

But know that the earth is not compressed and does not yield, whereas the water yields — for [though] the thruster shows that they shift place, they are not [truly] compressed so as to enter a smaller place, even if you press them: the vessels will break and they will go out [first]. But the air is compressed, and this too has a limit. And if you test it with a vessel of leather that has two holes, and insert into it a tube or funnel that does not reach the base, with its mouth narrow and the second hole stopped, there is no doubt that it will not fill its whole cavity, for the

air is compressed but not wholly destroyed. And if you press to force more water into it with a syringe [*siringa*], you will not get in more than a certain measure, until the remaining cavity is left for the air, which is compressed within. And if you open the second hole, you will see in practice the air going out from there, the water thrusting it; and then you can fill it.

And if you fill a leather vessel with water, stop it well, and bring it near a fire until it heats, you will see it tear with a loud report, because the water turns to air and has no room to expand. Thus there is a limit to its compression. And although the waters are not [themselves] compressed, withal the force of the air passes through, to the other side, and raises the water with it and gives forth a sound as it goes out — as those skilled in such devices know. [But] the over-subtle [reasoners?] will not understand how

the air is thrust, if there be no void; for thrusting and resistance mean that a large body enters a small place, and this is impossible to imagine — unless you say that there were in their place void hollows for [the body's] spongy[ness], so that the place is thrust [together] but the place itself is not compressed. Perhaps there is some fixed measure of void in nature's view, but not that part should be thrust far into part; and this is not the place for these matters.

But our wish is to know the quantity of the [heavenly] bodies, or the distances between the heavens and the earth. Now you already know that there is no simple element such that they are [mutually] proportioned, one to another [by reduction]; for in the art of alchemy they extract from a single matter all four elements and the quintessence [reading הנשמה החמישי = "the fifth essence/spirit"]. And if you test [this]: fill a vessel and stop its mouth, seal it with the seal of Hermes [*ch* ותם הירמיס — Hermetic seal], boil it much, then take it from the fire and let it cool much — [the matter] that turns to air [will reveal itself]. So you will see in quicksilver, which turns to air and enters the body of gilded objects.

You will find all the water you set in their measure within the vessel deficient [in amount]; but if you open its mouth while it is still boiling, we find a great wind comes forth, [the water] having turned to air. And you will see, as with quicksilver, which turns to air and is absorbed into the body of gilded objects, [the vessel] much harmed thereby — or it rises upward by the force of the fire and clings to [the vessel]; and being hot, it is rarefied, and afterward it cools and returns to its nature, becoming quicksilver as at first.

Hence the air near us is filled by the force of the solar rays, which are doubled [reflected] at the earth and return through it; though they reach only as far as the

region of the vapors — for the solar heat continually mixes the elements. And the air, though by its nature cold, grows warm; and at the mountaintops too one sees the nature of the air, that the snows are perpetually there.

And if you wish to know the warmth of the air and the difference from day to day, prepare for yourself one of these vessels —

Diagram. A thermoscope (air-thermometer): a glass tube partly filled with a strong liquid (vinegar or the like), sealed where the tube enters the vessel so that no air enters and the strong liquids — wine vinegar or a like fluid — do not escape, with graduated marks for reading the daily change of temperature. The text describes its use rather than labeling points; no point-letters are given for this figure (only "figure 52" / ציור נב is cited).

— (figure 52) — and they shall be sealed where the tube enters the vessel, so that no air enters and the strong liquids do not escape, which are wine-vinegar or the like. Now in the one [vessel], if you apply heat above, the air warms and seeks more room, and the waters descend; so it happens that when the air is hot they descend much, and when it is cold they rise. And you make marks [graduations] on the tube to know the increase and the difference from day to day.

But in the second [vessel] it is the reverse: if you heat it from below, the waters rise upward; and this is better than the first, for you can open the hole of the lower tube and draw off some of the water, so that when you heat it much there is room for the water to rise higher. And the longer the glass tube, the more wondrous the matter will be.

Now if you wish to know the height of the clouds, you can attain it — without much error, owing to their nearness to us — with the [same] instruments. And since [the clouds] are always in motion, the observation must be made by two men at a single instant. Suppose, for example, the eye of the one at point **ג** [gimel], and the second at point **ד** [dalet], and they wish to know the height **יה** [yod-heh] of a cloud — that is, the measure of the perpendicular line standing erect upon the horizon **אב** [aleph-bet].

Diagram. A cloud-height triangulation by two observers using a dioptra. Two observers stand at **ג** and **ד** on the horizontal base; sight-lines run from each through the dioptra's sights up to the cloud at **י**, the cloud's height being the perpendicular **יה** dropped to the horizon **אב**. The angles at the two stations and the known baseline **גד** yield the height by similar triangles.

- אב (aleph–bet): the horizon line (horizontal base direction)

- ג (gimel): the position of the first observer's eye
- ד (dalet): the position of the second observer's eye
- י (yod): the cloud (apex of the perpendicular)
- יה (yod–heh): the vertical (perpendicular) line — the height of the cloud sought
- זוית ירדן (angle yod–dalet–nun? [reading as printed]): the angle and its complement [*she'arito*] at the first observer
- זוית ינד (angle yod–nun–dalet): the angle at the second observer, who stands at point ג
- גד (gimel–dalet): the known distance (baseline) between the two observers

Now from the instruments the measure of the angle **ירדן** [yod-resh-dalet-nun, as printed] is known, and its complement, by the observation of the viewer sighting its height through the holes of the dioptra; and [likewise] the angle **ינד** [yod-nun-dalet] by the observation of the one standing at point **ג**; and the side **גד** between them is known. Therefore turn to side **ד**, to settle the foundation by the triangle **גדי** [gimel-dalet-yod], from the two angles between which is known the perpendicular [reading mapping the line]. Erect the perpendicular of the angle **יהג** [yod-heh-gimel] [as printed]; thus draw the proportion: as the whole base is to **יג** [yod-gimel], so is **ג** to **יה** [yod-heh]; and [this is] easily understood [וקל"ל = ודי קלהבין, "and it is readily grasped"].

The experiment shows that the clouds are not raised to the same height in every clime and in every season: in the northern places, and when the sun is in the southern signs of the zodiac and its heat is weak, the vapors do not finely [coalesce], nor do they rise more than three miles; but in summer they rise a parsa and a half. According to the nature of the places and the seasons the matter varies.

Another method: Know the altitude of the sun [taken] with the instrument.

Diagram. A second method for cloud-height by solar altitude: the sun's altitude is the arc **אק** [aleph-qof] in the circle on **אקב** [aleph-qof-bet], measured for half the day [*chatzi hayom*]; an observer sights the sun and the cloud and derives the height from the altitude angle. (Figure 53 is cited; the surrounding text breaks off at the page foot, so the full point-set is not given here.)

- אק (aleph–qof): the arc of the sun's altitude
- אקב (aleph–qof–bet): the circle on which the altitude-arc is measured
- חצי היום (chatzi hayom): "half the day" — the measured value [reading מו"ג? / "from…" — see note]

— and let, for example, its altitude be (figure 53) the arc **אק** [aleph-qof] in the circle on **אקב** [aleph-qof-bet], of half the day, measured [from?] **מו"ג** [reading uncertain], in which the eye…

[The text breaks off at the foot of page 338 and continues beyond this opening.]

the eye, and when the sun illuminates the cloud from behind it, its shadow falls perpendicular onto the horizon like a line [reading קו, "line"]. Its length, that segment **הו** [he–vav], is also perpendicular, as is the segment between the eye and the beginning of the shadow — that is, side **גה** [gimel–he] — known. Therefore, for the first [observer], angle **גיה** [gimel–yod–he] is also known, and the upright **יה** [yod–he], which is what is sought, becomes known.

But if the eye is on the cloud's shadow — that is, at point **ז** [zayin] — and you preserve angle **ך** [kaf], and since **ה** [he] is perpendicular at the place where the shadow descends, the pillars rising upon the horizon will be known, as will angle **יד** [yod–dalet]; and side **יה** [yod–he], the sought quantity, will likewise be known. By these methods you can determine the height of the bow [rainbow] within the cloud, the size of its arc, and the distance of its center from your eye — which is half the diagonal [reading אלכסון] of the [?] [final word abraded].

Degree 32 (מדרגה לב')

I shall further inform you of the upper limit to which vapors rise above the earth — that is, up to the convexity of the air and the concavity of the elemental fire, as the multitude of philosophers suppose — which will be as in the present diagram. Let the earth's sphere [be drawn] about center **ז** [zayin], the sun's sphere **כסה** [kaf–samekh–he] about center **ה** [he], and according to the distance **חין** [chet–yod–nun] let us draw the sun's orbit **אבפח** [aleph–bet–pe–chet] about center **ין** [yod–nun]; let the true horizon be **אן פ** [aleph–nun pe], and the apparent one parallel to it **גור** [gimel–vav–reish]; and from the pole of the horizon **ב** [bet] let us drop a perpendicular pillar **בוין** [bet–vav–yod–nun]. Suppose the sun to be beneath the horizon at the onset of the morning twilight, which, according to most astronomers, begins when the sun is sunk **18** degrees beneath the horizon — that is, by the measure of arc **אך** [aleph–kaf], the arc subtended from angle **אשך** [aleph–shin–kaf].

Since the rays of the sun touch the earth's sphere, they will be [tangent] from point **מ** [mem] to point **ג** [gimel] [reading מי"ח לשלישי = "from M to G(?)"; the small letters here are abraded — [?]] thus, side **יכך** [yod–kaf] being perpendicular, the whole arc **יצע** [yod–tzadi–ayin] beneath the horizon will be illuminated, while its

remainder **יומע** [yod–vav–mem–ayin] will be dark, within the shadow **ילע** [yod–lamed–ayin]. Let us suppose that the height of the vapors surrounding the earth reaches the concavity of the round [boundary] **עטמצ** [ayin–tet–mem–tzadi] — there is the thick air, and from there upward [the air is] clear and pure, in which no clouds form, close to the nature of fire — or, according to some, that there is the beginning of the [Greek: αἰθήρ, aithēr, "ether"] [reading האית"רי], who do not grasp the use of so much air there.

Now you see that in the morning twilight, although the whole arc **צמ** [tzadi–mem] benefits from the sun's light, nonetheless the sun's rays cannot reach straight to the eye of the observer standing at point **ך** [kaf], because the convexity of the earth blocks them. But when they are at point **מ** [mem] — thick vapors — they redouble the ray striking them at **כ** [kaf] [?] and send out line **מך** [mem–kaf]; this is a secondary light, called twilight-light, which does not shine like the sun's. Now if we reckon that point **מ** [mem] is the upper limit of the rising vapors, our wish is to know the measure of line **מך** [mem–kaf], which is their height.

And now I shall instruct you how to know it. In Degree 20, concerning lunar light, I showed [you] the measure of the portion that receives light from the sun; and by that same method seek to know arc **וצע** [vav–tzadi–ayin], the half-diagonal of the earth, which is **72600** [for] the earth's illuminated sphere [reading "the half-diagonal of the earth, which is 72600"]. You already know that line **חז** [chet–zayin = the radius] is **1210** [?] minutes [reading דקים], and that half the diagonal **רב** [reish–bet] is **5½** times the half-diagonal of the earth, **ויץ** [vav–yod–tzadi]; and all this is to be known from Ptolemy. Subtract **יץ** [yod–tzadi] from **חך** [chet–kaf], and there remains **הך** [he–kaf], **4½**, which is **270** minutes. And if you draw line **זך** [tzadi–kaf] parallel to **ליב** [lamed–bet] and equal to it, then [angle] **מך** [mem–kaf] at the first [observer] [is **88°** — reading מכ"ח לראשון as "88 for the first"], and since the [...] **יבך** [yod–bet–kaf] is perpendicular, then likewise [69 =] **סכ"ט** for the first [observer], angle **רד** [reish–dalet] being perpendicular [reading זוית רד נצבת].

Therefore, in the right triangle **דח** [dalet–chet] the ratio will be: as **רזץ** [reish–zayin–tzadi] **72600** is to **דך** [dalet–kaf] **100000**, so is **חך** [chet–kaf] to segment **חן** [chet–nun] **372**, which is **13** minutes. Therefore the whole angle **ירן** [yod–kaf–nun] is **13.90°** [reading 13. 90; transcribed as printed — minutes-component appears as 90, sexagesimally impossible — [recte: 13°90′ is out of range; likely 13° plus a fraction, source-corrupt]], and double of it is arc **יצע** [yod–tzadi–ayin], **180.26°**, the illuminated [portion] of the earth. After you know this, if you subtract from angle **יזח** [yod–zayin–chet] the angle of the sun's setting noted above, **18** degrees,

the remainder is angle **יזע** [yod–zayin–ayin], **13.72°** [reading 13.72; transcribed as printed]. And when you subtract it from the perpendicular **וצ** [vav–tzadi], the remainder — angle **ויר** [vav–yod–reish] — is known, **47.17°** [reading 47.17; transcribed as printed]. Its half is **וצר** [vav–tzadi–reish], since **וץ** [vav–tzadi] are equal [reading שווים]. And if you subtract the square of each of them from the square…

of the common [hypotenuse], there remain the squares of **וכ** [vav–kaf] and **ים** [yod–mem], the two [sides], equal; and their sides are equal. Therefore, [subtracting] from **מח** [mem–chet] [reading "from 88"] the common **רץ** [reish–tzadi], there remain the squares [reading מרובעי] of **וכ** [vav–kaf] [and] **ים** [yod–mem], equal — hence **וץ** [vav–tzadi] **8.53** [reading 8. 53; transcribed as printed]. According to this, in [the figure] there will be, in the equal triangle, in the two triangles **זם ים** [zayin–mem, yod–mem], and there will be a half [reading חצי]; in the third [triangle], all the angles are known, and side **וי** [vav–yod] — the measure of the perpendicular — [makes] angle **רוץ** [reish–vav–tzadi] [right], for **יו** [yod–vav] is perpendicular from **88** [reading מי"ח, here "from 88"]. So, as the ratio of angle **ורץ** [vav–reish–tzadi] **1.846** [reading 1 846; transcribed as printed], whose subtended arc is **98800**, is to the earth's diagonal in miles, **9** [reading 9 — the last figure is abraded; transcribed as printed, [?]]. Therefore deduce [reading חקיש].

And when you subtract from it line **רן** [reish–nun] **3579**, there remains line **רם** [reish–mem] [reading רם] **3623** [recte: 3579 + line-segment; the subtraction here gives the residual segment — values transcribed as printed]; as **רן** [reish–nun], the whole subtense, is to **מן** [mem–nun] [reading "to MN"], **44** miles [is] the height of the vapors. And I have learned that al-Lāṣīn the Arab [reading שאלאצין הערבי — likely a garbled transmitted name, [?]], in the *Book of Twilights* [Sefer HaNeshafim], and the Rambam [Maimonides] in his Commentary on the Mishnah, reckoned this height [at] **52** miles [reading נ"ב מיל] — it seems they reckoned that the twilight begins even when the sun is **19 or 20** degrees beneath the horizon. But the method I have written is demonstrative and true, and there is no cause for the discrepancy except in accordance with the premises. Examine and you will find it as we have said.

Degree 33 (מדרגה ל"ג)

After it has been made clear to you by demonstration that there is no doubt as to the quantity of the element water and the element air and their ratio with the dust [earth], and it has become further evident to you that the elements are not multiplied tenfold according to the [successive] powers of ten [reading "according to the tens" — i.e., the supposed 1, 10, 100, 1000 progression] except [in] the

matters stated falsely in the [pseudo-]demonstrations [reading המשאיי' = "of the Peripatetics(?)"], by [supposedly] firm proofs the view of the Peripatetics is uprooted — for if it were as they say, [the elements stacked] in this ratio:

Earth (ארץ)	Water (מים)	Air (אויר)	Fire (אש)
1	100	1900 [recte: 1000 — the progression 1 : 10 : 100 : 1000 is intended; printed "1900" under air and "10" under fire are mis-set]	10

]

— then, according to this, the quantity of all four elements up to the concavity of the moon would be **1111** times the dust [earth] alone. And this is false, for it was made clear in Degree 10 that from the earth's center to the moon's concavity is **33.33** [reading 33.33] halves [half-diameters / radii]; so the whole thickness of the sphere of the elements is **33.33** whole diagonals [diameters].

According to what I taught in Degree 2, the ratio of the earth's sphere to the bulk of the four elements is as **1** [reading 1 ערך = "the ratio of 1"] [the earth] to the bulk of the four — that is, as their cubes: **35937** with [plus] **1** [reading 35937 עם 1], and all the more so since some of the reckoned [figure] is greater [than] **33.33** [reading 33.33מ יותר]; whereas if the whole spherical quantity were only **1111**, the diagonal would be a little more than **10**, so that the cube would be **1111** [reading 1111 כמעוקב כדי שיהיה]. And who would believe that the moon's height is not more than nine half-diagonals of the earth from the earth's surface, contrary to what has been demonstrated?

And if [you object] that the [aforesaid] intent was about the ratio above [the **33.33**] in terms of the diagonals of the elements and not their spheres, then according to this **1111** half-diagonals would be the whole diagonal of the four elements, and the [ratio] between their sphere and the earth's sphere would be **1000000000** [one billion] to **1** [reading 1 אל 1000000000 ערך], and the waters would be **1000** times the dust — and all this is contrary to the truth and the demonstrations stated above.

It is demonstrated that there is no [other] ratio among these elements except the well-known tenth [reading העשירי הנדחה — "the rejected tenth"] — that is, that they are not graded by adding one over its fellow. For if you add to the earth's half-diagonal, **3579** miles, **52** [reading 52] miles — the thickness of the [air-]belt —

the half-diagonal of the three elements is **3631** miles, and the whole diagonal is **7262.** When you multiply it by **3⅐** you find the circumference of the air, **22823** miles. When you multiply half of it, **11411⅝**, by its half-diagonal, **3631**, you find the area of the circular [surface] of the circumference, **41435934** square miles. When you multiply this by ⅔ of the whole diagonal — that is, by **4841⅓** — you find the corporeality [volume] of the sphere comprising the three elements: **171238200605^{10}⁄$_{21}$** cubic miles. When you subtract from this the corporeality of the earthly sphere stated above, there remains the corporeality of the air alone: **8495834503^{13}⁄$_{21}$** cubic miles. Therefore the ratio of the sphere of earth and water is more than **22** times the quantity of air.

But the quantity of the element fire is also revealed. For you know that up to the moon's concavity is **120630** miles approximately. Thus the whole diagonal is **241261⅕** [reading 241261, fraction ⅕]; multiply it by **3⅐** and you find the circumference of the four elements, **758250^{10}⁄$_{49}$.** When you multiply half of it, **379125^{5}⁄$_{49}$**, by **120630⅝**, half the diagonal of the four elements, you find the area of the circumference: **4573413186^{284}⁄$_{343}$** square miles. When you multiply it by ⅔ of the whole diagonal — that is, with **160840^{20}⁄$_{21}$** — you find the corporeality of all four elements in cubic miles: **7355921325133^{31}⅓** approximately [reading 7355921325133, fraction ⅓; transcribed as printed]. When you subtract from this the corporeality of the three elements stated above, you find the quantity of the element fire alone, in cubic miles: **7355720719962075¼₃** [reading fraction ¼₃]. Therefore the ratio of fire to the earth's sphere is more than **38289** to **1**, and the ratio of fire to air is about **865803** to **1**, and a little more. This is what was meant by what was said in the preceding Degree and in Degree 15. Know, then, the measures of the elements and of all the spheres.

If you [wish] to know how many miles the moon runs in its sphere in one day, divide **758250** by **24** hours, and you find **31593** miles each hour, approximately. If you [wish] to know how much the sun runs in an hour, divide **25230375** by **24** and you find **1051265** [reading 1051265; transcribed — note 25230375 ÷ 24 = 1051265.6, so the figure is the integer part]. Therefore the sun runs more swiftly in an hour, all day long. Likewise you find that every star in the eighth sphere runs in one hour **42398437½** miles, if you divide its [daily course] by **24** [hours] of its convexity, as you know from the aforesaid Degree. And a man does not walk that many miles in **2904** years, at the average daily pace, continuously.

And if you set your heart to it, know that in a quarter of an hour you cannot recite the Shema more than sixty times; therefore in a whole hour, **240** times. And if you divide the hourly course of the firmament stated above by **240**, you find

that in the time it takes you to recite the Shema once, the star travels **176660** miles. Likewise, if you divide the said hourly course by the earth's circumference, **22500**, you find **1884** approximately — and that many times would [the star] circle the earth's circumference, were it moving at that swiftness with which the firmament moves in an hour. This is most wondrous and astonishing, and hard to believe — yet [it follows] from Copernicus.

Degree 34 (מדרגה לד')

I have no doubt that many will open their mouths against us and reckon these computations of ours as audacity and arrogance. Yet I have already stated beforehand that my intent was to make known among the peoples the wonders of the Lord. I also know full well that it is better for a man to seek to know himself — the chambers of his belly, the structure of his house, and what is near to him — and not to set his heart on lofty things, "and the eyes of the fool are at the ends of the earth" [Mishlei 17:24], all the more so since these matters are of no use for a man's life and his hard-won sustenance and the food of his house, which is what most of the world seeks in our times. But what shall I do for this guest [my soul], since for its sake it came beneath the shelter of my roof to inquire and explore in wisdom — and being like a bird trapped in the fowlers' snare, it does not cease to lift its eyes to the host of heaven that stand over it, by its nature, as before? "Cast a staff into the air and it returns to its root" [Aramaic: זרוק חוטרא לאוירא, אעיקריה קאי — "throw a stick in the air, it stands on its root," Talmudic proverb: nature reverts to its origin]. For from my youth I considered my ways, knowing that "I shall not live forever" [reading לא לעולם אחיה], and I saw that one who passes on and departs should leave behind some trace and sign that he was in this world — that his going should not be like the flight of shadows, forgotten like them, his name going down with him to the pit of destruction.

And [as] the Lord God knows my nature, that I am lowly in my own eyes and recognize the smallness of my knowledge, and I have not "walked in great matters" [Tehillim 131:1] — yet this is my view: that when I am in assemblies and in the session of elders, in every matter and word of wisdom they speak, I understand their discourse and take pleasure in their company, and am not [left] desolate or [merely] elevated; for whether in Torah or in external [secular] wisdom, I am among their company [reading מנגינתם]. And if the deficient ones

mock us over the calculation [reading הפלגת המספרים — "the exaggeration of the numbers"], that they [the verses] rely on the plain sense of Scripture, which shows that there is no number to the dust of the earth and to the drops of the sea — I too concede to their words that these have no fixed and bounded number,

[such] that there should not be more or less, since the earthly body is not arranged in [exact] sides and angles, nor are the seas equal in their depth. There is no [precise] knowledge for things that are not [precisely] ordered; yet we, in all the aforesaid measures — whether celestial or earthly — said that they are so by reckoning, [though only] approximately, since in such matters one cannot be exact to the limit.

Now you will find that R. Shlomo Yitzchaki [Rashi], in his commentary on the verse "and your seed shall be as the dust of the earth," etc. [Bereishis 28:14], wrote, may his memory be a blessing: "Just as it is impossible for the dust to be counted, so your seed shall not be numbered." And concerning him, the perfect and wondrous man, our master R. Eliyahu Mizrachi [Re'em], of blessed memory, wrote: He said this because the intent of this verse is the negation that follows necessarily from the affirmation. And although [in general] the contradiction of the antecedent does not yield the contradiction of the consequent — this [holds] when the antecedent is more particular than the consequent; but when they are equal, as in "your seed shall be as the dust of the earth," or when one of them is the differentia or the property of the other — there the contradiction of the antecedent does yield the contradiction of the consequent, necessarily. End of his words.

[There follows a logician's gloss expounding Mizrachi:] The interpretation of his interpretation: Scripture was compelled, in any case, to say "and your seed shall be as the dust," to convey that they are convertible [terms]; and it made the analogy [reading הקיש] conditionally and connectedly [reading מהרבק], and did not say "if [you can count the dust]," etc., since the conjoint [sense] is understood of itself — that the analogy holds, and one cannot err, whether it yields an affirmation or yields a refutation, whether in the antecedent or in the consequent. For otherwise, had Scripture wished to abbreviate, it would not say "and your seed shall be as the dust"; rather it would state the whole analogy: "If the dust can be counted, [your seed] can be counted; but the dust cannot be counted, therefore your seed too cannot be counted."

But this [full syllogism] is not obligatory when the consequent is more inclusive than the antecedent. For example: if it is a man, it is alive; but it is not a man — therefore it is not alive — does not follow, for perhaps it is alive while being an ox or a donkey. Likewise, from the truth of the consequent the truth of the antecedent does not follow — that is, "however, it is alive, therefore it is a man" is not necessary. But in convertibles it is verified in every case. For example: if it is a man, it is rational, [capable of] laughter; or indeed if it is not a man, then it is not

rational or [capable of] laughter; and likewise: indeed it is rational or [capable of] laughter, therefore it is a man — [this is] necessary. Therefore, by choosing brevity, Scripture chose this [form] to reveal that they are convertibles, and did not complete the analogy, [from] which it is understood that we mean to yield the contradiction of the consequent from the contradiction of the antecedent. This is straightforward [וק"ל].

It might say "the property" and "the intent," and I would grasp the beginning of the analogy; and it seems to me that "He is not a man, that He should lie" [Bamidbar 23:19] — that Scripture did not intend to say that it is impossible for the dust to be counted, but rather that it is hard to count, for the reason we stated: that the dust [particles] are not [all] equal and that its quantity is not ordered. Likewise you find that [Scripture] was exact regarding the stars, saying "if you are able to count them" [Bereishis 15:5]. And I have already shown by demonstration above that they have a number — even if the whole eighth sphere were full of them — for indeed all the stars visible in the heavens, comprised within the **48** [classical] constellations, together with what the discoverers [explorers] have lately found toward the south, are not even **1200**, except for a few that appear with the tube of glass [the telescope] — just as the whole Milky Way [galactic ring] also appears to be small stars; but without the aid of the instrument, an eye, [however] keen and good, finer than the aforesaid, cannot in all the heavens behold them.

Only, when one gazes at them on the bright winter nights, very many more will appear to him, owing to the doubling of the rays in the thick air. And you have already seen, in the glass that shows the [single] dinar as if it were a thousand dinars, and in the mirrors that [show] several images — so too in the heavens there will appear three images of the sun [parhelia]; but one who is expert in them and in their forms sees that they are only the ones recorded in the tables of [Greek: Ἵππαρχος, Hipparchus] [reading איפרכוס], together with what the moderns have found — such as Tycho [Brahe], the Re'em [R. Eliyahu Mizrachi? — reading כרא"הי, abbreviation uncertain, [?]], and the sailors.

Yet that there is a power [capable] of numbering things even greater than the above is made clear in the book of Archimedes [ארכימידיס] titled *The Sand-Reckoner* [literally "the number of the sand"]. For in his own day, too, people refused to believe that there could be a number for the sand — even if the whole terrestrial globe, that is, the sea and the dry land, were reckoned as sand. For although in the opinion of some it was not finite in number, even so they held

that no human had it in his power to give it a number that would not fall short of the said [quantity of] sand.

To demonstrate their error, Archimedes in the said book first determined, with wonderful skill, the distance of the sun from the earth, and afterward proved by demonstration a number such that even if the entire world up to its outermost sphere were filled with sand, [that sand] would not equal it. Now, on the basis of the measures of the diameters that we have set out, we can see the matter by another, easier route. I give it to you according to our reckoning, posing the difficulty in brief. And just as he assumed the parts to be far more than they actually are, so as to silence the deniers, so we too shall assume more than he did, in order to make the matters more extreme and so better suited to the computation:

[Premise] א. Let us reckon that a grain of sesame [שומשמין] contains 10000 grains of the finest sand.

[Premise] ב. Let us reckon that across a fingerbreadth there are 40 sesame grains set side by side — even though Archimedes found only 35 [Archimedes' figure 35; the text states the deliberately inflated 40].

[Premise] ג. Let us reckon the mile larger than we assumed it in Degree 1, where it was 80000 fingerbreadths; now let us reckon it 100000.

[Premise] ד. Let us reckon the diameter of the earth larger than Ptolemy [טלמיום] assumed it — 7159 [?] — and let it be 10000 miles.

[Premise] ה. Let us reckon larger, too, the diameter of the concavity of the eighth sphere — which, as we explained, comprises 45225 earth-diameters according to al-Farghani [אלפראנאן] — and now let us reckon it 100000 diameters; and it will then be as many times the miles [in an earth-diameter] as there are miles in the earth's diameter, that is, 1,000,000,000 miles. Therefore, if a small sphere whose diameter is one fingerbreadth contains 40 sesame grains [along its diameter], its cube [volume] will be 64000 sesame grains — and the cube [volume] of one sesame grain is 1, since (as was shown in Degree 20) the ratio of spheres is as the cubes of their diameters. And since one [sesame] grain contains 10000 [grains] of sand, a sphere whose diameter is one fingerbreadth will then contain 640,000,000 grains of sand; and although they are not so many, we will reckon it still more, and let them be 4,000,000,000 [the inflated figure; cf. 640,000,000 just computed]. All this is reckoned relative to the sphere whose diameter is one mile: since the mile, by our assumption, is 100000 fingerbreadths, its cube [volume] will be 1,000,000,000,000,000 fingerbreadths. We resolve this into grains of sand, that is,

we multiply it by the grains of sand of the [one-]fingerbreadth sphere above, and they will be 1,000,000,000,000,000,000,000,000 [10^{24}] grains of sand. And all these are reckoned a single part [a unit] relative to the eighth sphere: for since its diameter was assumed to be 1,000,000,000 miles, its cube [volume] will be 1,000,000,000,000,000,000,000,000,000 [?] [a 28-digit string in the source; transcribed as printed — digit-count uncertain]. We multiply it by the grains of sand of the sphere whose diameter is one mile, [as] above. The whole corporeality of the world, then — from the center of the earth to the concavity of the eighth sphere — will contain fewer than this many grains of sand:

1,000,000,000,000,000,000,000,000,000,000,000,000,000,000 [?] [a very long zero-string in the source; transcribed as printed — the exact digit-count is not securely countable at this resolution].

And so it is set out plainly for you. When Scripture said "until they could not be numbered" [Bereishis 32:13 / cf. 16:10] — "for there is no number" — the Torah spoke in the language of human beings, who are astonished and exaggerate the matter beyond what it is. So too the Ibn Ezra [הראב"ע] wrote that [a multitude] is called *revava* [a myriad] on account of the multitude of the number; and places of *guzma* [rhetorical hyperbole] cited in Scripture have already been noted in the Talmud.

strengthen them. So Scripture translated, "for none [other] does the deed lay any obligation upon the craftsman" — but [the meaning is otherwise]. [The closing of the preceding passage is cryptic at the page-head; rendered as the letters give.]

מדרגה לה [Degree 35]

Said the author [ישר", Yashar]: The benefit that flows from contemplating the work of God is very clear to every wise of heart. For He is awesome in the worlds He created out of absolute nothingness, and in the wondrous formations within them. One who is no expert in their design is like a man who, should a stone fall from a wall, or a beam crack, would not know its history nor understand its defects; or like a deaf-mute who builds a ship but neither sees nor feels for her when she is in the heart of the seas, in a storm-wind, with pouring rain, and the hailstones have loosed all her tackle. But this our God will guide us; He Himself, in His own person and glory, and the officer set over the great and holy House, is the One who holds the rigging firm, who builds up her breaches and her walls; and He, the great pilot, spreads her sails for our ship and raises her banner until He brings her to her desired harbor. So too He does in the heavens, by means of the angels, who are like the sailors steering the celestial ships, doing His word,

attentive to hear the voice of His utterance — for by this they liken their workings to the will of their Maker and the desire of their Former. And we — we do not know what to do in this cage, for the heavens are as high above the earth, and it is not in our hand to attain [it]; rather, [it is enough] that He has granted us to subdue it [our nature] and habituate it to His service and His fear. For as our body stands to the bright celestial bodies, so our soul stands to their souls — or their inclinations [natures]. From this we learn our lowliness relative to God, may His name be exalted, and we fear and dread His commands; for from heaven He made us hear His voice through His servants the prophets, and He — for all His exalted occupations among the retinue on high — turns toward us, watches over us, and sends our help from the sanctuary whenever we call upon Him. This truly teaches that He made us, and we are His.

Perhaps it was in order to liken [his art] to God and His angels, in [their] moving the precious bodies, that the wondrous Archimedes [ארכמידיס] — whose line goes out through all the earth, and his words to the end of the world — intended [his boast], when he wished to show an easy means by which the whole terrestrial globe might be moved, as he prided himself in saying: "Give me, then, a place to stand outside the terrestrial globe, and I will move the earth." One versed in the mechanical sciences knows that there are several ways by which what he proposed to do would not be beyond him. And in reflecting that, through the science of flesh-and-blood, blessed be the faithful God who created and apportioned thus my intent in this whole treatise, I will set before you a single, paved and easy road — for I have not found, in the writings of the said sage (whose works are all rare in this age), this matter [so treated] — after I first lay down some premises [literally "after the premising of premises"].

הצעה א [Premise / Hypothesis 1]

The **center of gravity** [מרכז הכובד] is called, among the masters of mechanics [מיכאניקא], the point such that, when any body whatever is suspended from it, [the body] will not incline to any side; and in the book *Maʿyan Chatum* [מעין חתום, "Sealed Fountain"] you will find further explanation of the definition of this center. As for the place at which it stands and does not tilt, the matter is plain: if you take a knife or a spoon and lay it across your finger, the place where it does not fall is its center of gravity. And in the pans of a true [accurate] balance, the midpoint of the beam — from which they hang in equilibrium, equidistant from the center — is the center of gravity. It is quite clear that if two equal weights are equidistant [from the center], so that they stand on a straight line, neither will

outweigh its fellow. But if one is more distant, it will outweigh the other. (See figure [53] נג.)

הצעה ב [Premise / Hypothesis 2]

If the beam of a balance is divided into 10 equal parts, then, by what was stated, there will be on each side of its center five parts. If at the end of the first part on one side — say, the right — there is a weight of 12 litra, and on the left side, at the end of the second part, a weight of 6 litra, they will weigh equally. Likewise at the end of the third part, 4 litra will suffice — which is a third of 12; at the end of the fourth part, 3 litra — which is a quarter of 12; and at the end of the beam, 2⅖ [2 and a fraction] — which is a fifth of [12] — will weigh the same as the 12 litra. For the farther a weight stands from the center of gravity, the more it weighs, so that a small weight, [placed] far [from the center], suffices to stand in equal balance with a great weight near the center. All this is well known in the [class of] balances called *statera* [שטטיריש, the steelyard], as in the figure before you (see figure [53] נג). And behold, you see that a small weight, not far from the center, is the one from which the instrument is suspended [the fulcrum-weight].

And if the second weight is at point א, [a distance] of one-fifth of the first [arm], and the distance of [point] א from [point] ס equals the distance of ס [from the center], even so they will stand in equilibrium and the beam זא will not incline to any side. But if the weight is greater, the beam will incline until [point] א comes to [point] ג. Likewise, at point ב — which is nearer the center by one-fifth — at the distance of ס[?], if the weight is more than 1/10 of [the weight at] ם[?], [the beam] descends [at] ה, [and point] ב descends to ד.

הצעה ג' [Premise / Hypothesis 3]

When one end of the beam descends, the other end rises — as in the descent of point ב to ד, [point] ה rises (as above). Likewise, if we add weight at end ה, the falling beam הך rises until it comes into equilibrium [at] דזאך, or rises higher; and the arcs [described] will be as the distances from the center, that is: the arc אן is five times the arc produced by the rising of דקם[?], and likewise בך[?] is ten times the arc ם[?], according to the distance from the center. So will the arcs be — as is explained in the book of Jordanus [יורדאנוס] on weights [*De ponderibus*].

הצעה ד' [Premise / Hypothesis 4]

Among the general principles in mechanics is this: he who adds force adds time — that is, he who lightens the weight prolongs the time of its motion; for the time of the motion is proportioned to the motion of the [moving] weight. That is: since

the diameter ם is one-fifth of the diameter א, or one-tenth of [diameter] ב, it follows that if we wish to raise point [ם] one pace, point א must be lowered 5 paces [to come] to [ג], or point 10 ב paces [to come] to ד; and in this [same] time we [can] raise the weight ד. For if we suppose it [the weight at ם] to be 50 litra, then a weight of 40 litra at point א [error in source — see below], or a weight of 5 litra at point ב, will raise it [in equal measure]. But know this: the farther the weight is from the center, the longer the time it takes to raise [the load] than it would were the weight sufficient to raise it nearer the center — that is, if at point 50 ם litra are equal to 10 at point א or 5 at point ב, and you place a further litra at point ם, it will raise [the load at] א, or [at] ד, with more speed than they [10 or 5] would when raising point ם; and so the moving body [at] א will travel in shorter time than the travel of [point] ד.

Do not mistake the special property of [rotating] bodies that approach the center [the axle], [supposing] that the large [outer parts] move in a different time from the small [inner parts]. Rather, reckon them as moving in equal motion, except that the large is slower than the small, for the circumference of the large [wheel] is many miles [greater]. For this reason Tycho [טיקו"ן] was compelled to posit, in his *Astronomiae instauratae mechanica* [literally "the book," i.e., his mechanical work], that Venus [נוגה] —

— [Venus] is in one sphere together with the sun, their motions being equal; whereas he ought to have sought to show that they are not in one sphere, and that their motions are not equal, as Copernicus [קופירניקוס holds]. But this is not the place to elaborate.

הצעה ה [Premise / Hypothesis 5]

All instruments of mechanics are built and founded upon the balance, as you may see: a weight of stone of 200 litra requires four men to lift, [or] one man with a beam set under it to raise it. For in such a beam there are three points: one is the place where the beam rests — which is called *cuneus* [קוניאום, the fulcrum / wedge] — and it is the center of gravity; the second point — that is, the [near] end of the beam — is the place of the weight of the stone; and the third point, at which the two hundred litra are lifted. So too, with the *antenna* [האנטינא, the yardarm/spar] of ships, they raise great loads of merchandise — for the weight is not far from the center, while the other end of the antenna, where they pull, is far from the center.

By [these] means it is clear that any weight placed on the line passing through their center weighs nothing at all. But if it is placed on the line perpendicular to

the diameter, far from the center, at a quarter of the diameter it will weigh half the [full] weight on the [whole] arm; and if at the end of the diameter, it will weigh as much as its [full] weight; and at the points between them, according to the proportion above. These matters I will treat fully in the *Sefer Besamim* [בשמת, "Spices"], with many benefits beyond these in human affairs — more than in any other of the disciplines of the sciences.

מדרגה ליו [Degree 36] [printed לי"ו; for "Degree 36," following Degree 35 — transcribed as printed]

In wheels made round with teeth [gears]: if, by the time the wheel turns once, the [crank-]beam turns 8 times, and the thickness of the wheel's diameter equals the thickness of the beam's diameter, then in truth the power of the diameter in traction will be 8 times the power of the beam. This will be [shown] in the figure before you (see figure [55] נה): the beam is א, and upon it is [a handle] ן by which one of the laboring workmen turns it, and the toothed wheel is ב, and its diameter דק is equal in thickness to the thickness of [the beam]; and let wheel ב have 48 teeth, and beam א [have] 6 [teeth], equal to the teeth of the wheel in their measure. I say that the worker must turn beam א eight times if he wishes to revolve wheel ב one complete revolution; and that the weight suspended on the diameter [of the wheel], of mass one litra, balances against a weight of 8 litra suspended on diameter דק; and that he raises it, or the worker draws [it], whose strength is 50 litra, like five men — that is, [as] 400 litra — which, by his turning the handle of beam א, he does not feel as a weight greater than 50 litra. Only, what five men would draw in an hour, were they turning beam א, this one [man] will draw in eight hours. All this is clear from the premises above.

Likewise, if wheel ב had 60 teeth and beam א [had] 6, the beam would revolve 40 [recte: 10] times until the wheel turned once [60 ÷ 6 = 10; the source reads "40" — flagged]; and the weight on the diameter of the wheel דק, [were it] heavy [at] 600 litra, would be, to him who turns beam א, no more than 60 litra. And conversely: he will move in ten hours what ten men would move were they revolving beam א in one hour. In this way you will understand all [cases] analogous to these — and this is what we wished to say.

And it is clarified from this that force may be multiplied without limit, so that there is no weight in the world that is beyond the power of God to [move]…

the apparatus of mechanics, to draw it toward her [the Earth] and shake it — and it will be as in the figure before you.

Diagram. A schematic of the imagined Earth-moving engine described in the text: the globe of the Earth, fastened with sapphire chains and bound by the diagonal axle of the first geared wheel, with the train of gears that the operator turns. The caption reads ציור נ"ו [Figure 56]. The text supplies the labels that follow.

- Globe of the Earth (כדור הארץ): the great weight to be lifted.
- Sapphire chains (שלשלאות של סנפירינון): the bonds — strong enough not to snap, by which the Earth is gripped.
- א (aleph): the first wheel, of 100 teeth, turned by its crank-beam.
- ב (bet): the first crank-beam (בריח), of 10 teeth.
- ג (gimel): the second wheel, on the diagonal axle of the first crank-beam.
- ד (dalet): the second crank-beam, of 10 teeth.
- Then wheels ה, צ, י, ל, ך and further crank-beams ק, ך of the gear train, ending at the handle (בית יד) the man turns.

We considered her [the Earth] to be at rest. The chains — they are sapphire — and the bonds will not be untied; nor will the cables of craftsmen [be cut], nor will the mighty beams snap. Now grant us, please, a place outside her — a rock, a mountain, or a hill, a fixed standing-place that will never give way — on which we may take our stand and set up the apparatus of mechanics, ready to rouse the Earth from her place, as your own eyes will see, every detail laid out as in the figure.

[The Earth] will be bound to the diagonal axle of the first wheel א, which has 100 teeth, and it [the wheel] is turned by its crank-beam ב, which has 10 teeth. From what was learned earlier, by the time wheel א has revolved once, the crank-beam ב will have revolved 10 times. But the load on crank-beam ב is one-tenth of the load on the diagonal axle א. Now suppose another wheel ג passes through diagonal axle ה, like the former one, having 100 teeth, and turned by its crank-beam ד, which likewise has 10 teeth. By the same proportion as before, crank-beam ד will revolve 10 times for wheel ג to revolve once, and now the Earth weighs at notch ד only 1/100 of what she weighed at notch [ב]; and so on in order. If wheel מ is then turned by crank-beam ק, and wheel צ by crank-beam ל, and wheel י by crank-beam ך, and likewise wheel מן by crank-beam ך [?], you will find that at the handle ך of the man who turns it the whole Earth weighs only 1/[?] of what she weighed at crank-beam א — that is, [Lacuna: figure not legibly inked].

Degree 36 [continued]. The reduction is sequential. Therefore, by means of six [?] wheels, one must turn the said load 10,000,000 times — that is, the operator must turn 10,000,000 times. And if he adds still more wheels, up to the thirtieth and

beyond, the whole Earth would weigh, at the operator's handle, a single litra; and a small lad could manage it, swaying her at once, in a moment, until she trembled and tottered and her pillars shuddered.

But the years of one's making would not suffice to roll a single tooth of wheel א one turn — let alone the whole wheel itself. For suppose a man turns the operator's handle 10,000 times a day: then in a thousand years of unbroken turning he would have turned it only 3,650,000,250 times, and in six thousand years 24,900,001,500 times only. And even if 1,000,000,000 men turned together for six thousand years, they would not suffice to roll wheel א one single turn. So you see that the more the wheels are multiplied, the lighter the load grows — until it softens away to nothing — yet it would still require an age upon an age.

And besides the matter of the load, it appears that the nearer one approaches the center of the Earth, the more the weight increases — so much so that there is no doubt in my mind that no instrument of my devising could move the Earth at all. So too those who voyage on the sea of Norway have observed that the water there is so very deep that a litra of lead is so heavy at that depth that no man has the strength to draw it up from the sea. For this reason Aristotle wrote, in the fourth book of *On the Heavens* [De Caelo IV], that the nearer the weight draws to the Earth, the faster it moves — meaning that the more a weight acquires of its proper place, the more it acquires of the weight's own form. And in flowing waters the matter is most plainly seen: if you bore a hole at the bottom of a barrel full of water, the barrel empties — but not in a uniform time. At first the water flows out of it faster, then more slowly; and in the middle [it flows] faster than at its end. For the higher the water stands — that is, the thicker and heavier it is — the more it presses and urges and is distant from the spout, and all that is beneath it hastens to come out. So too, if you fit a spout into the hole, it empties faster than a vessel they call a *pimkush* [siphon] [?]; the longer the spout is, the faster it empties — as with hollow reeds.

the conduits, the springs, or the ships, the reason for the difficulty of drawing a weight up from the depths is made plain. For if the *pimkush* [siphon] is 10 feet long — even if its bore were made very wide — it raises the water with more weight, yet more easily; for it is not the abundance of water entering into the cavity that is the cause, but rather the depth, or height, is what does it. As I said, the device is no narrow matter if it is 20 feet long.

[The mechanical lifting of great weights] is a great thing for the Earth, and useful to the builder, for raising up great and precious stones, and for floating sunken ships from the sea, and for moving columns from place to place — as they did in

Rome in the time of Pope Sixtus the Fifth in the year 1585, when they raised the obelisk: a column — that is, a pointed shaft — of speckled marble, all of one piece, whose height was 85 feet, its greater thickness 9 [feet] and its lesser 6½. It was made at Heliopolis [On / Aelia] by command of Pharaoh king of Egypt before the destruction [of the Temple]; it was [a pillar] of the sun that they used to worship. Caius Caligula transported it to Rome, and there it became a holy image — woe to the man! — to the honor of Augustus and Tiberius the Caesars; and it was cast down to the ground in the time of the aforesaid pope. The excellent master, the mechanician Domenico Fontana, raised it with the aid of six hundred men and a hundred and forty horses and forty-six mechanical engines called *trochleas* [pulley-blocks], and with wondrous device they transported and set up the columns at the labor [court] or threshing-floor of the Vatican — and it is seen there to this day, as a sign and a wonder to the many.

And if these words of mine reach the ears of God-fearing men, men of truth, upright of heart, they will be pleased, and will recognize that my intent is acceptable and my soul clean of every blemish. The great ones will be advocates pleading for me, running before the taskmasters; nor will I be ashamed before the scoffers, nor afraid of the hairy he-goats and the painted, dissolute, zealous, cunning riders, who, though they envy [me], have seized hold of my righteousness, which I have grasped and will not let go — undisturbed by the voice of the mockers, who [drive men] like sheep. *"Yosef, a lion's whelp"* [?], and a rod for the horse and a whip for the donkey will shepherd them and chastise them with whips and with scorpions, [as it is said] over the gate of "the multitude." Blessed is the Lord [775]; [977 — numeral as printed] the Lord.

*

Said *Yashar* [the upright one — the author], at the close of this honorable treatise, which is a ladder of degrees, and whose rungs the wise will understand: I have seen fit to bring you an abridged *Table of Sines* [לוח הבקעים]. For the whole long *Golden Table* — in which are the sines, the tangents, and the secants belonging to each and every minute of every degree up to 90 degrees — I shall set out separately on its own (it is at hand), and also after the manner of our own construction and inquiry, in *Sefer Elim*; for since all the calculations in the three treatises mentioned above are built upon this table, I bring you here a table in which the sines belonging to each whole degree will suffice you, as I shall explain.

Now suppose we wished to know the sine of 30 degrees 30 minutes. In the column of degrees we seek the sine of 31 degrees, which is 51503; likewise we find the sine of 30 degrees, which is 50000. The difference between them is 1503. We then

reason thus: if the 60 minutes between 31 and 30 give a difference of 1503, how much will 30 minutes give? You find [it]; and this you add to the sine of 30 degrees, so that the sine of 30°30′ is 50752. And in the long table of sines you find 50753 — so you see that there is no difference between them except in the last digit of the sine, and that does no harm to what is sought.

Again, we wished [to know] the sine of 30 seconds. So we entered the table and found the sine of half a degree, which is 872; and the sine of one degree

is 1745, and the difference between them is 873. We therefore reason thus: if 1800 seconds give 873, how much will 30 seconds give? You find 15 approximately, which is the sine of 30 seconds. Again we wished [to know] the sine of 84°12′. Now the sine of 85° is 99619 and that of 84° is 99452, and their difference is 167. Therefore, as 60 is to 167, so is 12 to 34, approximately; we added this to the sine of 84°, giving 99486. (Always take a little more if you wish to be exact, since the sines are less than the truth.) And so the sine of 84°11′ is 99485 and that of 84°12′ is 99486.

And conversely, if you wish to know the arcs from the given sines, you find what you seek by the same proportion. We wished to know the arc of the sine 50752, which is not in the said table; but the larger one next to it, 51503, has the arc 31°, and the smaller, 50000, has the arc 30°. Their difference is 1503, and the difference between our sine and the sine smaller than it is 752. Therefore we reason thus: if 1503 give 60 minutes, how much will 752 give? You find 30 minutes, and joining them with the degrees they make 30°30′; and so for like cases.

But if you wish to know the tangent or the secant of any arc, you must first know its sine. Then we found, by the teachings I showed in the second Article of *Sefer HaYesod* [Book of the Foundation] — its figure is copied here, and it is explained in the fifteenth proposition of the said Article — that as the sine of the complement of arc [ז"ב] [?] is to the sine [of the arc], so is the whole sine [דב] to the tangent of the sought arc [קב]. For example, in the case of 30°30′ we reason thus: as the sine of the complement of [30°30′], which is 59°30′, namely 86168 [recte: 86163], is to its sine 50353, so is the whole sine 100000 to the tangent 58904. And likewise you find its secant from the fifteenth proposition of the said Article, where it was explained that the whole sine is the mean proportional between the sine of the complement of the arc and the secant of the arc — that is, as in the said figure, as the sine of the complement of the arc [הפ], whose secant is sought, is to the whole sine [דה], so is the whole sine [ד-דב] to the secant [דך].

For example, in the said case, as the sine of the complement of 30°30′, which is [the sine of] 59°30′, namely 86163, is to the sine [of 30°30′] 50353, so is 100000 to 116059, which is the secant of the sought 30°30′; and likewise in similar cases. And conversely: if you knew the secant and wished [to know] its arc, the proportion is: as the secant is to the whole sine, so is the whole sine to the sine of the complement of the sought [arc]. And likewise, from knowing the tangent you may come to know its arc: as the tangent is to the whole sine, so is the whole sine to the tangent of the complement of the sought [arc] — since the whole sine is the mean proportional between the complements of the secants and the tangents. And to the discerning, knowledge comes easily.

[Footnote, p. 349:] *) *Ma'ayan Ganim*, p. 163, [figure] [31] ל"א.

**Table of Sines (לוח הבקעים) — Source: **

The table reads right-to-left in the source. It is arranged in four pairs of columns; each pair gives a degree (arc, מעלות קשת) and its sine (בקע) on a radius of 100000. Alternate pairs are headed "complement degrees / complement sine" (מעלות תשלומה / בקע תשלומה) — i.e. the sine of the complement (= cosine). The four logical column-pairs are presented below in source order (rightmost pair first), then I give the consolidated reading.

Logical column order (right → left in source):

1. מעלות קשת (arc-degrees) | בקע (sine) — degrees ½–22
2. מעלות תשלומה (complement degrees) | בקע תשלומה (complement sine) — degrees 89½–68
3. מעלות קשת (arc-degrees) | בקע (sine) — degrees 23–45
4. מעלות תשלומה (complement degrees) | בקע תשלומה (complement sine) — degrees 67–45

Arc (deg)	Sine	Compl. deg	Compl. sine	Arc (deg)	Sine	Compl. deg	Compl. sine
½	872	89½	99996	23	39073	67	92050
1	1745	89	99985	24	40673	66	91354
2	3490	88	99939	25	42262	65	90631
3	5233	87	99863	26	43837	64	89879

Arc (deg)	Sine	Compl. deg	Compl. sine	Arc (deg)	Sine	Compl. deg	Compl. sine
4	6975	86	99556	27	45399	63	89100
5	8715	85	99619	28	46947	62	88295
6	10453	84	99452	29	48481	61	87462
7	12187	83	99254	30	50000	60	86602
8	13917	82	99027	31	51504	59	85717
9	15643	81	98769	32	52992	58	84805
10	17365	80	98481	33	54464	57	83867
11	10081	79	98153	34	55919	56	82904
12	20791	78	97815	35	57357	55	81915
13	22495	77	97437	36	58778	54	80902
14	24192	76	97029	37	60181	53	79863
15	25882	75	96592	38	61566	52	78801
16	27564	74	96126	39	62932	51	77714
17	29237	73	95630	40	64279	50	76604
18	30902	72	95105	41	65606	49	75471
19	32557	71	94552	42	66913	48	74314
20	342	70	93969	43	681	47	73135

Arc (deg)	Sine	Compl. deg	Compl. sine	Arc (deg)	Sine	Compl. deg	Compl. sine
	02				99		
21	35837	69	93358	44	69466	46	71939
22	37461	68	92718	45	70710	45	70710

עד הנה עזרנו י׳ ונשלם ספר גבורות י׳ — "Thus far the Lord has helped us, and the Book of *Gevurat Yah* is completed."

Sefer Ma'ayan Chatum [מעין חתום — "The Sealed Spring"]

[Title page — decorative border, full-page; the fifth treatise of Sefer Elim opens here.]

Sefer

Ma'ayan Chatum [מעין חתום — "The Sealed Spring"]

By the Rav, the Gaon, the Kabbalist, the all-encompassing sage, the divine philosopher and physician — our teacher R. Yosef Shlomo, physician, Delmedigo — may his Maker preserve and sustain him — **the Yashar [ישר] of Candia** [יש"ר מקנדיאה — i.e. R. Yosef Shlomo Rofei, acrostic "Yashar"], may the memory of the righteous holy one be a blessing.

The Fifth Part of Sefer Elim [חלק חמישי לספר אילם]

First printed at Amsterdam in the year [פ"ט]ש [= 389, i.e. (5)389 AM = 1629 CE] according to the abbreviated reckoning, in the house of the Rav, the sage, the inquirer, the divine — our teacher R. **Menasseh ben Israel** [מנשה בן ישראל], of blessed memory, who proofread it himself.

And now printed a second time, with added glosses and corrections, together with a history of the author's life, a transcription of his tombstone epitaph, and also a likeness of his portrait.

Through the agency of our teacher R. Meir Grünschpahn [גרינשפאן], and added by the printers; our teacher R. Moshe Eliezer Belinson [בעלינסאן], may his light shine.

Odessa [ארעסא]

By the press of M. A. Belinson [בדפום מ. א. בעלינסאן].

תרכ"ח [= (5)628 AM = 1867/68 CE]

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[Sefer Ma'ayan Chatum — text begins]

[Section] [1] א. How can the part be greater than the whole — which runs against sense, against reason, and against the ninth common notion [i.e. axiom], on which Euclid built his foundations, namely that the whole is greater than each of its parts?

The answer. A part can indeed be greater than the whole, as I explained to you in the demonstration in chapter 8 of [Book] V of the *Elements*, where, when [quantities] are set in proportion in unequal measures [i.e. in proportions], it can happen that the greater [part] equals the whole or is greater than it. The proof is as I demonstrated in *Sefer Chukos Shamayim* [ספר חקות שמים, "The Statute of the Heavens"], and following Euclid's definition. For this reason, turn to the sum of proportions that I explained to you — for example, the sum of ratios. The proportions are these: ¾ added together with ³⁄₁₂ — and that equals ⅔ — and so the whole comes to be equal to each of its parts. Likewise the sum ¾ + ⅔ rises to 1¼ [?]; thus the whole here is less than each of its parts [?]. And so, too, the sum 1½ is ¼ [?], and the like of these by the thousands. [The fractional arithmetic in this passage is partly inconsistent as printed; values transcribed as legible. See QA.] One must not say that seven [?]. This is as in the lessons of geometry, where the product of [two factors] multiplied is less than each of the multiplicands that was explained, just as is demonstrated in proposition 8 of [Book] V of the *Elements*: when [quantities] are set in proportion in measures that are not equal, the ratio of the larger measure to the third is greater than the ratio of the smaller to the third, and conversely the ratio of the third to the smaller is greater than its ratio to the larger. Hence the ratio of 1 to 2 is greater than the ratio of 1 to 4 — and so, by this very route, the part [comes to be] greater than the whole.

Nor may one object that the summing of ratios is invalid, for it has been demonstrated and proven, and Euclid, Theon the commentator, Archimedes, Menelaus, and Ptolemy all employ it; its usefulness is great, as is set out in the great books, and the learned do not reject it.

As for what one of the great Sephardi scholars of Jewish stock objected — namely, that the very essence of ratios is nothing existing in itself outside the thinking mind, but is entirely dependent upon the operation of the comparing intellect, comparing one number to another or one magnitude to another: for you cannot say that the relation and ratio between them is itself a thing that is *not*, since it is not a continuous quantity having extension, like a line, a surface, or a body, and likewise not a discrete one, since it is not a number — and if so, there is nothing to wonder at if the first intelligible, which states that the whole is greater than the part, does not hold true of it; for that [axiom] is true absolutely only of things existing outside the mind, not of mental and imaginary things — thus far his words. In *Sefer Bechinat* [ספר בחינת — likely *Bechinat Olam* or a "Book of Examination"] I shall pursue him further [?] and join his words [to the discussion].

Others have said that this is like two men, the ratio of whose health, in each case, to his sickness is as the ratio of 1 to 2; so that if these two equals are combined into a third man, his constitution will be in the ratio of 1 to 4 [?].

But what seems more reasonable, and more settling to the mind, is what I now tell you: a ratio is called large or small according to its distance from the mean and its nearness to it, and the mean is the ratio of equality, 1[:1]. Now the ratio 1:4 is more distant from the [ratio of] equality than [the ratio] more distant... [?]; from it [comes] the ratio 1:2. Therefore, whoever's constitution is ratio 1 — that is, whose health is one and the corruption of his humors is [?] 3 [?] — comes more ill than one whose constitution is 1. So you have it: the ratio 1, the whole that arises from the two ratios, the 1, is greater than its parts — and understand this well.

[Section] [2] ב. How can two differing magnitudes have one and the same ratio to a third magnitude, and conversely?

The answer. Euclid's proposition 16 of [Book] V confounded all the geometers who studied it, when he says there (of blessed memory) that the straight line set perpendicular upon the end of the diameter falls outside the circle, and that between this straight line and the circumference of the circle no straight line falls at all; and that the angle of the semicircle — that is, the inner [angle] formed from the diameter and the half of the circumference — is greater than any acute angle

made of straight lines; and the outer [angle], formed from the line of tangency and the circumference, is the smallest of the angles made of straight lines. Now there is no doubt [?]; [the figure here is what is] called among them [the geometers]... [?] which is drawn upon the figure of the said angle **(ציור נז [Figure 57])**. But many of Euclid's commentators [hold that] it is more acute than the [angle] made of two straight lines, and therefore the physicians, the craftsmen of the surgeon's knife [or "skilled craftsmen"], chose for their instrument that the angle made of the curved and the straight line — when it is **(ציור נז [Figure 57])** the angle of tangency just mentioned... [?]. They denied it and refused to grant that the said angle was a measure and a quantity, because they reckoned that the inner angle formed

from the circumference and the diameter is a right angle [literally, "stands at a cubit/perpendicular"]; and on this view the angle of tangency is no measure at all, since the angle formed from the tangent line א-ב [aleph-bet]... [?] is a right angle and is equal to the [angle] formed from the line of the circumference ז-ב [zayin-bet] and the diameter ב-ז [bet-zayin] — that is, the [angle] ח-ב-ז [chet-bet-zayin]. And if it is as they say, then there is here no occasion for this puzzle at all. Many strange matters, too, would fall away from the books of the geometers, were proposition 16 just mentioned uprooted from the body of foundations.

But I am wary of sin, for the consensus among Euclid and the commentators and the ancients is that the said angle is a measure and a quantity; and it is known that Euclid brought into his foundations nothing but propositions famous and accepted among the ancients — the trustworthy Pythagoreans, faithful in their teachings — such that Archimedes, Apollonius, and all who came after them upheld and received his words. No one cast doubt or murmured against him — for everyone declares him firstborn [foremost] in foundations — except for Pietro Ramo [פיט"רו רא"מו — Petrus Ramus / Pierre de la Ramée], a sharp-witted man called a craftsman, a new hand among the geometers, who took exception to the order of the foundations (or to his presenting the demonstrated [propositions] even to youths). So too he mocks him over the twenty-first proposition of the first [book], where he shows that the two sides of any triangle together are greater than the third — saying that the dogs chasing the hare know this, that they go by way of the longer side and not by way of the two shorter sides. His words are savory and correct; but in everything he argued against Aristotle and the ancients, he will yet render account before [me] in *Sefer Bechinat*. And since I have there expanded at length upon the nature of the said angle, I shall keep my words brief here.

Still, it does seem that the said angle is a measure — for it exceeds and falls short in its magnitude and admits of extension, which is the property peculiar to quantity. It grows and widens the smaller the circle becomes, and the larger the circle grows, [the angle of tangency] shrinks and narrows. For you see that the angle ב-א [bet-aleph] is greater than the angle ד-ב-א [dalet-bet-aleph], and this one greater than the angle ה-ב-א [heh-bet-aleph], and this one greater than the angle ו-ב-א [vav-bet-aleph], and this one greater than the angle ז-ב-א [zayin-bet-aleph]. And if it is so, then it is a measure.

Here there is room for inquiry, a deep matter — and it is that, by proposition 8 of [Book] V, when two unequal magnitudes are set in ratio to a third magnitude, the larger magnitude will have a ratio greater than the ratio of the smaller to the third. To prove it, the commentators on his words infer that we double the difference between the magnitudes until it exceeds the third magnitude — and this is the foundation of that proof. Let us posit that the greater of the two magnitudes is a right angle, א-ב-ן [aleph-bet-nun], and the smaller is the angle of the semicircle together with the diameter — that is, the angle ז-ב-ז [?] — and the third magnitude is the angle ר-ז-מ-ז [?] itself. Now it is clear that the angle of tangency ר-ז-ב-א [resh-zayin-bet-aleph?], which is the difference between the two magnitudes א-ב-ז and ז-ב-ה [?], even if doubled a thousand thousand times, will never equal the third magnitude ר-ז-ב-ז [?]. I mean that we should double them — yet not draw them out of their nature, which is to be an angle of tangency; and the doubling will be by shrinking the circle more and more, as we said, since [the tangency angle] widens accordingly, by the nature of continuous quantity. Yet that it never equals the magnitude ר-ז-ב-ץ [?] is plain: for if you say it does, it would follow that the angle ז-א [?] equals — if doubled a thousand times — some acute angle made of straight lines; and it would follow that one part in a thousand of an acute angle made of straight lines equals the angle ק-א [?]. And this runs against Euclid in this proposition 16, which intends that the angle ב-א [bet-aleph] is smaller than any acute angle made of straight lines. Therefore, if [the angle] ה-ב-ץ [?] is greater than any acute angle made of straight lines, and the angle ד-ז-ב-א [?] is smaller than any acute angle made of straight lines — and even if doubled a thousand times it does not equal an acute angle made of straight lines — all the more does it never exceed the third magnitude ר-ז-ב-ז [?]. If so: since the difference ד-ז-ב-א [?], between the two unequal magnitudes א-ב-ן and ז-ב-ך [?], does not exceed [it], one must say that the ratio of א-ב [aleph-bet], the larger magnitude, to ז-ב-ך [?], the third magnitude, is as the ratio of ז-ב-ך [?], the smaller magnitude, to ז-ב-ד [?], the third — and so you have it that two unequal magnitudes have one and the same ratio to a third magnitude, and

conversely, the third has another ratio to two unequal [magnitudes]. And this is a true marvel.

And if one wishes to push away the said proposition 16 of Euclid, and to say that the right angle א-ב-ן [aleph-bet-nun] is not greater than the angle ח-ב-ן [?] — my son, beware, lest the whole edifice of the wisdom of geometry collapse to the ground, for all the foundations are bound to one another like a chain, and the light of reason will be confounded, throwing all the books into confusion. So do not be hasty to raise [objections]. "Know well the face of your flock; set your heart to the herds" [Mishlei 27:23].

[Section] [4 ,3] ג ד. How can there be a ratio of equality between two unequal magnitudes, and how can the sine [בקע] of the smaller arc be greater than [that of the larger]? [The heading runs the two questions together as printed; the marker reads "ג ד".]

The answer. It is a labor in my eyes to think of replying to this [?] — that the officer, the chief, our teacher R. **Moshe** [משה — the editor/inquirer], intended to be clever; and one is not to suppose this in earnest, for his knowledge [?] was upright and faithful in the [mathematical] disciplines. Now it never entered the mind of any learned scholar, engineer, or astronomer to say that the ratios of arcs are as the ratios of their sines [בקעים, "sines"], or as the ratios of their chords [יתרים] which are their doubles. Nor did the astronomers say that the sine — that is, [the sine] of 90 degrees — [is as the arc]. Were it so, then, by the lesson of the [Table of] Gold [i.e. the golden table / table of sines], we would find R [the radius] = **100000**, so that, for example, the ratio of an arc of **30** degrees to a sine [would equal a fourth — that is, ¼ of the whole]; and the sine [בקע] of 90 degrees would be — but in the Table of Gold we find that the sine of 60[°] = **86602**. Therefore it is plain that the ratio of the arcs is not as the ratio of the sines; and so, too, of the chords which are their doubles. [86602 is the printed value; the standard sine of 60° at R = 100000 is 86603 — flagged. See QA.]

[The astronomers] did not say that [arcs and sines] are proportional, but rather that they are *matched* and *correlated* to one another. Therefore, whoever says that between an arc and its complement there is a ratio of equality is not exact; for, as the truth is, between their sines there is [a ratio of equality] only when it happens that the [arcs] are equal — namely when each is a quarter [of the circle] — just as [there is equality] between their sines [in that case]. But all the [trigonometric] computations are built upon the sines or the chords of the arcs, not upon the arcs themselves, as you saw in *Sefer Sod HaYesod* [ספר סוד היסוד] and in *Sefer Chukos Shamayim* [ספר חקות שמים]. By their means we find the arcs

and the angles, once we know their qualities — for if we do not know whether they are larger or smaller than a quadrant, or [whether] one is a right angle, the matter remains in doubt; since one and the same sine, and one and the same chord, is shared by an arc and its complement [literally, "its remainder"], as is explained in the second figure [pinah] of *Sod HaYesod*.

Greater than this puzzle, I say: were the arcs proportional to their sines, it would follow that between arc and arc there is a ratio compounded of the larger and smaller differences together. This is proven thus. Let there be arc א-ב [aleph-bet] with its sine ב-ך [bet-kaf], and arc צ-פ [tsadi-peh] with its sine פ-ק [peh-kuf]. Then the ratio of א-ף [aleph-feh] to צ-פ [tsadi-peh] is the ratio of the larger difference; whereas the ratio of the sine of arc א-ף [aleph-feh] — which is ב [bet] — to the sine of arc צ-פ [tsadi-peh], which is פ-ה [?], is the ratio of the smaller difference together — and that is ב-ז-ב [?], and is correlated. But this [identity] is true absolutely only in the sum ק-[?] [?], which is פ-ד-ה [?]: this is the ratio of the smaller difference; therefore the ratio of א-ב [aleph-bet] to צ-פ [tsadi-peh] is the ratio of the difference [?] which adds to or subtracts from the semicircle — if up to their sines [there is] a ratio of equality, [the arcs] will be equal.

But when the sum equals the semicircle — [to say so] is a notorious falsehood, and it would not be believed unless [the arcs] were quadrants. [This holds] also for their sines [literally, "that they too"], since in every place where you cut the semicircle into two parts, there will be between the sines of the [two] arcs a ratio of equality, [in] that one and the same sine is shared by an arc and its complement; and on this view all the arcs would be equal — and this is nonsense [literally, "trivial"]. But it is not nonsensical [absurd] that the sine of the smaller arc be greater than the sine of the larger arc, when one [arc] exceeds and the other falls short of a quadrant, or when both exceed [it]; for the larger arc has the smaller complement, and the smaller [arc] the reverse — a larger supplement or complement [remainder]; and the sines follow their supplements.

As appears in the figure before you **(ציור נט [Figure 59])**: although for the larger arc א [aleph] the sine ב-ך [bet-kaf] is smaller than the sine ו-ז [vav-zayin] of the smaller arc צ-ן [?] — nevertheless the remainder [complement-arc] ב-צ [bet-tsadi] is smaller than the remainder א-ן [aleph-nun], and the sines are matched to them. On this view the exact reckoner will see that the larger sine belongs always to the larger [complement] and the smaller to the smaller. Only one must not build an edifice upon this and set [arcs] in proportion, saying that according to the size

of the arc the sine increases over the [other] sine — for that is false, as I showed you.

And behold, I shall inform you of a matter that puzzles — that it does not follow [merely] because the larger ratio is denominated by a larger number, the smaller by a smaller, and the equal by an equal, that, according to the size of the number naming the larger denominator over its fellow, [the larger] will be greater than it. For although this is true...

Ma'ayan Chatum (continued)

in the ratio of the double and the square. That is, the ratio of 4 to 1 is greater than the ratio of 2 to 4 [recte: of 4 to 2], inasmuch as the multiplier of 4 over the multiplier of 2 is itself a doubling. Even so, in the case of other ratios of the "greater difference" kind this does not hold true, since the ratio of 8 to 1 is compounded of [several] ratios — that is, of [the ratios] they make, ⅔ ⅔ ⅔ ⅔ — and so it is a triple [ratio] of the ratio ⅔; whereas if you were to judge it by their multipliers, the ratio of 8 to 2 would be the quadruple ratio, and yet it is not a quadruple but a triple, as we said: for if you gather ⅔ ⅔ ⅔, you will find 16/9 [?]. Therefore these [steps] make it, and it is a triple [ratio]; hence you must not judge from the multipliers. Investigate the other ratios likewise, and you will see the truth upon inquiry.

In the same way let me make known to you that Euclid did not err — lest his foundations [Euclid's *Elements*] mislead you — for he explained, in [propositions] 6 and 8 [recte: 4 and 24] of the First Book, and 24 and 25 [of it], that when in two triangles the sides are equal and the angles contained between them are equal, then the base of the one is equal to the base of its fellow; and if the contained angle is larger, the base too is larger, and if smaller, smaller. So you might think to say that the base is larger or smaller in proportion to the excess or deficiency of the angle — but this is not so. Rather, the ratio of the [one] angle to the [other] angle is much greater than the ratio of the [one] base to the [other] base, as I have explained elsewhere.

So it is too with sines accompanying the differing arcs: the ratio of arc to arc is greater than the ratio of sine to sine — whether the larger sine be greater than the smaller sine, or whether it be smaller than it. For the ratio of arc **א** [A] to arc **ה** [E] is a ratio of the "greater difference" kind, whereas the ratio of **בד** [BD], the sine of **אב** [AB], to **פה** [PE], the sine of **צפ** [TsP] [?], is of the "smaller [difference]" kind. There is no need to prolong [the discussion] further on this.

§[5] ה

How is the chord [sagitta/versine] greater than its arc? [?] In Ptolemy's division, the semicircle is 180 times one [unit] [recte: the half-circumference is 180 parts], and the half its diameter is 120. According to this, the difficulty falls back upon the division of the diameter into 120 parts; for by the Archimedean ratio of the circumference to the diameter, 22 to 7, it ought to be that the diameter is 114 6/11 if the circumference is 360.

But R. Yitzchak HaYisraeli [R. Isaac Israeli] explained the matter in his book: that Ptolemy did not divide the diameter into parts equal to the parts of the circumference, but rather into [parts] smaller than them. For had he divided it into 114 6/11, the number would be hard to divide into whole parts; therefore he chose the number 120. And although a part of the diameter is not equal to a part of the circumference, this does us no harm in the computations of astronomy, since we do not wish to know the areas of the circles but [only] the measures of the ratios, from which we proceed to knowledge of the arcs — as is clarified for one who studies the book of the *Almagest*. We have already explained the merit of the number 120 in our commentary upon it, as well as from what I wrote in section [פנה] 20 [?] of *Sefer Sod HaYesod* concerning the merits of the number 60, which is the half lying between [?] its degree; and the astronomers chose the number 12 for the zodiacal signs and the months, since there is none smaller than it [yet] richer in parts, having [the divisors] 1/2, 1/3, 1/4 [+1/6 +1/12]. So too each month, dividing it into 30, has more parts than the number [60/30] — that is, [the divisors] 1/2 1/3 1/4 1/5 1/6 1/10; and because this is deficient by 1/4, they doubled it, making it 60. And so every degree is divided into its many divisors, for it is of full measure and the number 360 is deficient by only 1/4 [?]. Therefore there is no difficulty in saying that the arc is 24 and the chord 25; and the two arcs are [as it were] gold coins, and the chord one of copper. There is no need to prolong [the discussion].

§[8–7–6] וזח

How is it that one number has two [different] roots, and two equal measures [yield] differing roots; and how can being [*yesh*] be explained as coming from nothing [*me-ayin*]? These three numbers [problems] have astonished you. **Answer:** [These are] explained beforehand and clarified in the *Sefer HaBe'ur* [Book of Explanation]. Let me teach you the rule [*algorithm*] of numbers together with the "minuses" [negatives]:

The "minuses" [negatives] generate a "plus" [positive], and if a minus [is taken] with a plus [it yields] a minus. This is to say: if you multiply a plus by a plus, or a minus by a minus, [it yields a plus]; and the sign of the plus shall be [×] and the sign of the minus [−]. [As I explained in *Algebra* and in *Olam* (?).] Take, for example: 5 plus 3 [of] 5 plus 4 = 5 plus 35 [?]. Let us illustrate with: 6 plus 5, [times] 5 plus 35 [?] generate a minus. [And the proof:] if you multiply five minus two — which is 3 — by seven minus three — which is 4 [?]: to multiply 5 minus 2 (which is 3) by 7 minus 3 (which is 4) yields, [a plus]. And so: 15 plus 5 with 6 minus 2, that is 6 plus [?] 29, [times] 35 [?] generate [the product] 35 minus 29 plus 6, that is, 12.

[The worked terms here are corrupt in the printing; transcribe as printed:]

- 6 plus 5 … 5 plus 35
- 15 plus 5 … 6 minus 2 (i.e. 6 [+] 29 35) → 35 minus 29 plus 6 = 12 …20 with 4 generate 90 minus 10, that is, 80. And so test all their numbers, [whether] positive or with-minus [negative], and you will find the pupil's truth: 5 with 15 [?]. So set forth in all their numbers [the rules of] positive and product. Now let me make known to you, after this premise, a wondrous thing — by which you will know how to refute one who denies the possibility of the creation of being from nothing, as we, the children of [our] covenant, believe: that God created the world from absolute nothing. And it is this:

If you multiply numbers smaller than nothing [negatives], they generate positive numbers — and one cannot deny the operation, which is already clear and has been verified with the numbers of [the] product. So behold, [several] numbers of nothing generate being; and the like of them [are] many. [The worked examples:]

(right operand)	(left operand)
10 minus 4	7 minus 3
9 minus 8	7 minus 3
90 minus 16 1 [?]	49 minus 42 [?]
and they are 6 minus	and they are 16 minus

So too let this be explained to you: that one number has two [square] roots. Let the number be 4 quartic [fourth-power terms, "squares of squares"] minus 12 squares plus 9 squares [recte: plus 9 (units)]; and behold, its root is "3 roots [×] minus two squares" [?], and likewise its root is "1 square [?] minus 3 roots." Who does not know that these two roots are different? For if the root [variable] equals 2, the first root will be six minus eight, and the second root eight minus six; and

the difference between them is like that between being and nothing. And such [cases] are many beyond limit — one cannot deny them, unless he be a man who has transgressed his bounds.

From here it is clear that for these two squared numbers — ב 19 ד 4 ר [..] [×]... which are equal, there are two different roots, namely those mentioned: ד 4 ת. ג 12 פ. א 9. [The matters are well explained in *Sefer Ir Giborim*, with God's help.] Upon this principle even the wise man and master of stratagems gave no heed to these properties, and therefore was not able to answer my fourth question: that its intent was to give me a number other than two, such that when its square is subtracted from it, two remain. He proceeded by the formula of algebra and posited the number as 1 ["the thing," *x*], and its square equal to "2 plus 1 thing" [$x^2 = x + 2$]; and he took half of 1 [the coefficient] and squared it — and that is 1 [¼] — and with the number 2 it becomes [2¼], and its root is 2 [recte: 1½], that is, 1½, [?] and with the half of 1 [½] it becomes 2; and this is the sought root. From here he concluded that it is impossible for this property to belong to any number other than two. And when I showed him 4 1/5 [?] or [other cases], he acknowledged [it] and was astounded. [And there is yet more to say upon these matters; in *Sefer Bechinas Olam* (?) I shall expand, with God's help.] [..] 4 [?] [..] — [recte: the worked terms ב 19 ת. ג 12 פ. ד 4 / ד 4 ת. ג 12 פ. א 9]

§[9] ט

How is it that in the mathematical sciences they explain the explained and the evident things by means of proofs drawn from premises [that are] more hidden? Answer: One who knows how to demonstrate by proof that the sun shines at noon is reckoned by me as one who knows how to demonstrate by proof that there is a God in the world — even though the sun is among the clear sense-objects, while God is exalted, hidden from the eyes of every living thing. Therefore they labored greatly to explain the first intelligibles; and this is the work of the wise [author of] the *Physics* [HaTeva] — see Aristotle, there. And that great and exacting man, the emperor [Julius Caesar] Scaliger, boasted that he had a proof of the existence of motion, which Dionysius, for all his philosophizing, could not [achieve] except by his pacing back and forth, long and short. From here you may understand that Ramus seized upon Euclid unjustly when he explained the self-evident things — for the minds of those who go deep are not at ease with what appears to the senses, since the senses all err as one if the [proper] condition is not preserved in them, as the commentators on the *De Anima* [*Sefer HaNefesh*] explained. By the science of optics [*chochmas ha-habatah*] the errors of sight are explained; and likewise by the crossing of the fingers the

touch errs, and one will think the one thing to be two; and the masters of stratagem make sound reach the ear and the well, when in truth there is no rattling, [as] one who strikes with a hammer… …So too the preparers of dishes deceive those who taste. In sum: in all matters one must not rely on the senses unless the intellect bears and examines them, agreeing with the sense-objects together with them. Now perhaps the student will object: that what I said about the first [proposition] is not absolutely true, since it may be that the figure is not properly drawn — and on this account he was forced to explain it by [a] proof. True it is that on the face of it, one who explains the explained things seems to be furnishing proof for the more hidden [from the more evident], as I explained in *Sefer Chukos Shamayim*.

And behold, you will find the like of this in Euclid's foundations and in [those of] Jordanus on numbers. For example, in [proposition] 16 of the Seventh Book, R. [Euclid], may his memory be a blessing, says: if two numbers are multiplied one by the other, they generate equal numbers — meaning, you multiply 1 [recte: 2] and it generates 1 [the product], and likewise you multiply 3 by 1 and it generates ד [4 — the product 4 (?)], saying that 3 equals 4 [?]; even the ox and the ass would not refuse this. Also for fractions it is not so very clear — for example, that the half of a fourth is the fourth of a half. Nevertheless, in the multiplication of many numbers the proof is useful: for if you change their position, it serves you as a touchstone to test your operation. And up to the end of the operation, are not the two ways alike? There is nothing [worth saying], except in the [theoretical] studies, where there is no place for [practical] utility.

So [these values] appear to me, [as in] the proof of [proposition] 6, in chapter 8 [?]:

(right column)	(left column)
253	348
348	253
2024	1044
1012	1740
759	696
88044	88044

In [proposition] 11 of the Fifth [Book] he said, in [the manner of] the Seventh Book: that the ratio of two equal measures to a third measure is equal, and the ratio of the third to the [two] equal [measures] is equal. And in the eighth

[proposition] of it [the Fifth Book] he said that if they are unequal, the ratio of the larger to the third is greater than the ratio of the smaller to it — and so forth. These matters are very well explained; and he explains them from the sixth definition, which he set down [first], where he explained that the measures of which he speaks — [namely] that the ratio of the first to the second is one [the same as] that of the third to the fourth — are [those] of which the [magnitudes] equally multiplied [as multiples] of the first and the third are equal to the [magnitudes] equally multiplied of the second and the fourth, whether they be equal, or [the one] exceed [the other], or one add an equal measure. But in [the case of] ratios it is not so: for the ratio of 2 to 4 is not as the ratio of 4 to 2.

[This last is] more confused and more remote from reason than the first. But to one who goes deep it will not be reckoned so — for had he not informed us in the definitions what is intended by saying that "this ratio is identical in itself with such-and-such a ratio," and what is "greater" and "smaller" than it, we would never know that for [two] equal [magnitudes] there is one [same] ratio to the third, and for unequal ones a greater and a smaller [ratio]. Therefore the sixth definition of that book is, as it were, [a thing established] by [logical] inference; and it is thus:

> The measures whose multiples are equal[ly related] are equal — they are [those] of one ratio; and equal [measures], whose multiples are equal, are equal[ly related]; therefore the equal measures have one ratio between them.

The rest of the proof is explained according to our method there.

So [it is] for all of them, to those who understand. And since the studies of the Fifth Treatise are of much utility in all branches of knowledge — in measures, in melodies, in motions, in times, and in every work of arrangement [music/harmony] — the masters of the [mathematical] studies were compelled to establish them by faithful proofs. And behold, **Chamad** [Ḥāmid] ben Yosef the Ishmaelite [the Muslim] returned to explain by proofs three definitions of this treatise, and nothing worthy of note came of it, as appears in the *Epistle on Ratio and Proportionality* [*Iggeret HaYachas ve-haHityachasus*] which he composed in his Arabic; and in *Sefer Bechinas Olam* (?) I shall expand, with God's help.

§[10]ˈ

A wonder: how is it possible to bring near and to draw the extremities together without limit, and yet [the one] does not reach the middle, nor do

the two equalize with one another, even though they are forever drawing nearer? Answer: I shall make the difficulty greater, and say that there are unequal measures such that, even if you add to the smaller without limit and subtract from the larger without limit, even so [the smaller] will not become any closer to the great measure than it was before the addition and the subtraction.

This will be explained from the picture of the second [figure] [תמונת התמר הב — fig. 2]. Let these two measures be [the] angle **צזך** [TsZK?], a rectilinear acute angle [recte: an acute angle bounded by straight lines], and the angle of contact **אבך** [ABK?], which is from [proposition] 16 of the Third Book, [an angle] smaller than every rectilinear acute angle. ...then if you subtract from the angle **צזך** [TsZK?], for example, its half **בזך** [BZK?], and to the angle of contact **אבך** [ABK?] you add — provided you do not draw it out [too far] from the angle of contact, that is, that the angle **אב** [AB] or **אבג** [ABG] [remains]; if you subtract from the angle **צזך** [?] [its half], and [add to] the contact-angle **צזב** [TsZB?] — [yet] so as not to exceed... So too with however many divisions of the angles you make without limit, in order that the angle of contact may draw near and diminish from the angle **צזב** [TsZB?] without limit — even so they will never equalize, nor will [the one] draw nearer than it was at first; even if we do not say that "without limit, even so they are drawing near."

You have informed [me] of how much the city [?] would say [recte: corrupt]. **צזח** [TsZCh?] — [if] you subtract 1/100000000 [one hundred-millionth] [?] of the small [angle] times, and add upon the small one as many times — they would be equal; and this is against what is explained in [proposition] 16 of the Third Book, that no angle of contact is equal to a rectilinear [angle]. Therefore the two of them [these] measures do not draw near at all. Apart from this [argument], which is founded upon the aforesaid proposition — [a proposition] doubted by some of the moderns — as I explained in section 22, the matter is clear from several [other] measures: as if you should say, [it is] like the ratio of the circumference together with the intellectual [conception of the] circle [?]; it is of great use, as I explained in *Perek Gevuros HaShem* [?]; and likewise [for] the sides of the figures [polygons] inscribed [in a circle] and approaching the two ends [of the diameter, or the curve] without ever equalizing, as Archimedes and Vieta [ויאיטה — Vieta?] did, [for] the polygon. Now the diameter being a fixed [quantity], the chord and the deficiency are found from the true [value] of the circumference, in the ratio of a fixed and exact diameter — [yet] of an inexact circumference; and the deficiency [is] exact, while the truth [the precise value] is always hidden between the extremities and is not revealed.

But [proposition] 1 of the Tenth Book of Euclid has a way of its own, beside what I intended, and concerning it I shall speak in another place, with God's help. After them [come] many more [polygons]; and it is truly a divine matter, which God gave to the sons of man to be occupied with. So too [for] those magnitudes that are not known [as commensurable] — as if you should say the side [diagonal] of a 7- and a 9-[gon] — and some say the chord [excess] is found.

But concerning the roots [square roots] of deaf [irrational] or mute numbers, whose root the mouth cannot utter — the matter is very clear. And for further clarification, let me make known to you the extraction of square roots in general. For example, let the number be **1. 51. 29** [recte: 1 51 29, i.e. 15129] whose side or square root you seek. You begin from the right, and every [pair of] 2 figures separate off with a line [or stroke] according to the [number of] separations; so many will be the figures of the sought side. And by our example there will be 3 figures — that is, its root will rise up to the rank of hundreds. For the first separation, which is on the right side, indicates the units; the second, the tens; and the third, the hundreds. Therefore in the aforesaid square, which is **אבגד** [ABGD] [fig. 16 (?) — לוד ם (?)], make 3 separations — that is, 3 squares: **אצ** [Ats], **יצ** [Tsi], **יג** [YG] — and they are formed from the three sides **אל** [AL — recte] equal to **לכ** [LK], **כב** [KB]; and the large square **אבגד** [ABGD] is compounded of the 3 small squares and, further, of 2 areas [taken] twice — that is, of the area **מצ** [MTs] and **צך** [Tsk], the equal ones — that is, of the [area] formed from side **לצ** [LTs] equal to side **אל** [AL], and from side **לך** [LK?] — that is, the double of the first side by the second side, twice — and likewise of the area **בי** [Bi] and **וד** [VD], the equal ones — that is, of [the area] formed from the double of the two sides **אל** [AL] **לם** [LM] joined together. It is as if you said: each side **אם** [AM] to the third side **מ** [M]... twice; that is, the 3 squares together with the two areas, [taken] twice, as if you said with the four areas, complete the whole large square **אבגד** [ABGD], as aforesaid, and so forth.

Now we began from the leftmost separation and found the number 1, and its side [is] 1; and since, when it is [multiplied] by itself, it is in the rank of tens-of-thousands [10000], therefore the square **א** [A] is like 10000, and its root 1 will be like 100. Secondly, [we take] the double of the side by 1, and it will be 2 — that is, let us posit the area **של** [Shel/of] equal to the area **צך** [Tsk] of our number [recte: corrupt]. **צר** [Tsr] is the double of **קי** [Ki?] [?]; and had the area **צש** [Tsesh] been by itself [equal to] 51, that would be the number of the second separation. We divide that side by **צר** [Tsr], which is the double of the root, or twice the first side **אל** [AL], and the side **של** [Shel?] equal to **לם** [LM] would come out — except that

the area, [namely] the chord [excess] with the square **צי** [Tsi], is 51; therefore we divide 51 by **צר** [Tsr], the double of the side, [and so forth].

The right-margin number column (top→bottom), with its row-labels (read RTL in source):

Label	Value
אצ [Ats]	10000
מצ [Mts]	2000
צה [Tsh]	2000
צי [Tsi]	400
בי [Bi]	360
יד [YD]	360
יג [Yn] [the prose names this square יג]	9
אבגד [ABGD]	25129 [recte: 15129]
צלעו [its side]	1. 2. 3.

Ma'ayan Chatum (The Sealed Spring) — continued

so that we can subtract from it the square צי [ṣ-y]. Now the figure 2 [in the second digit position] is double the first side. א"כ [Thus:] the digit 1 enters into the number 51 [recte: read from context as the remainder being worked] twenty times, with 11 remaining; and since לס [the carried value] is in the place of the tens, then the digit 2 enters in 2 times, and that is the second side, and its square צי [ṣ-y] gives 4. We subtract that from the number 11, and 7 remain. Therefore the standing surfaces — the remaining vertical columns at מהדג [m-h-d-g] — equal 729.

Again I set surface מ [m] equal to surface ידי [y-d-y], so that the whole surface laid out in a straight line, together with square נף [n-f], equals 729. Line יף [y-f] is double אלם [a-l-m] — that is, its length is 24. We divide by it this third difference, 729, and it enters 3 times. Therefore מב [m-b], the third side, is 3; and in the rank of the others, when they are doubled with 240 [?], they come to 720, and its square יג [y-g] is 9, which gives 729. So then the product 15129 is a square, and its side is 3, and that is what was sought.

And I must further explain to you the extraction of these surd square roots [שרשים אלמים, "mute/irrational roots"], so that you grasp the matter. Let it be, for example *(see figure [61] סא)*, that square אבגד [a-b-g-d] is 12, and we want its root, which is irrational. We set the nearest square below it, the one arising from a whole root — namely [9] ט, I mean the smaller surface הזזגן [h-z-z-g-n] [?], whose root is 3, which is side דזן [d-z-n] or הז [h-z]. Accordingly the excess — I mean the head [remainder] of the surface that stands as a right-angled gnomon around square הוזג [h-w-z-g] [?], which for brevity we shall henceforth call the "standing surface" [vertical gnomon] — is 3.

Now if we transpose surface ב [b], or its equal, and set it in a straight line with surface אן [a-n], the whole elongated surface אנ [a-n] equals its aforesaid standing surface, and therefore equals 3 like it; and line דקוע [d-q-w-ʿ] [?] will be 6 long; and surface וז [w-z] equals 3 minus the square of אזק [a-z-q] [?]. Had surface דקצ [d-q-ṣ] [?] alone equaled three, we would have divided it by the length of side דזע [d-z-ʿ] [?], and its other side שזק [sh-z-q] [?] — equal to it, אש [a-sh], the sides of the small square אדן [a-d-n] [?] — would have become evident at once. Therefore the whole of קי ידקן [q-y / y-d-q-n] [?] would equal ½ 3 [3½], which is the root — except that, as we said, surface וקצ [w-q-ṣ] is ½ 3 [3½] less the square of אזק [a-z-q]. Therefore the root of square אבגד [a-b-g-d] is less than ½ 3 [3½], since line אש [a-sh] or ידק [y-d-q] is less than one half. Hence if you try to square 3½ by itself you get ¼ 12 [12¼] — so it overshoots. And if you set line יק [y-q] at 1 alone, so that the whole side אד [a-d] is ⅔ 3 [3⅔], then its square is 11 $^{259}/_{343}$ [11 and 259/343], which is less than 12. You are at liberty to refine the side of the small square אד [?], and you can find roots ever closer — whether exceeding the squares or falling short of them, ad infinitum [לא"ת], and this is clear.

From this you will understand the reason for the practice of doubling the nearest side and adding one, which becomes the denominator for the remaining fraction. For example, we sought the root of the number 6. The nearest side is 2, whose square is 4, and 2 remain. Double the side is 4, and with one more it is five, and that is the remaining denominator, 2 [the remainder]; so the nearest side for the number 6 is ⅖ 2 [2 and 2/5], since it lacked 2 of the completion of the square — hence one was doubled [folded in as] 4, as your eyes behold in the figure.

(See figure with the labels 13, 12 [?]) — The square is 4; and when you add to it ⅖ 21 of the surfaces, which are twice in its side, and further דקן [d-q-n] [?], which is one, these come to [?7] ז, and with the square 4 they rise to 9, which is square זג [z-g]. Therefore ⅔ [?] of the excess over square 4 is ⅔ of the deficiency that the number 6 has from completing square 9 — and grasp this. But this yields a square

less than the true one; and if you reckon the side as ⅜ 2 [2⅜] it will be in excess. There are many ways to approximate, whether by excess or by deficiency, ad infinitum.

However, in the fractional [sexagesimal] computation of astronomy, the extraction of the closest roots is beautifully laid out. Let it be, by way of example *(see figure [62] סב)*, that square אהיל [a-h-y-l] is 54 degrees, and we want its root.

First we find the nearest root — that is, line קודרה [q-w-d-r-h] [?] — and it is 7 degrees, and its square 49 is square דהוז [d-h-w-z] [?]. We subtract it from the whole, and the standing surface אדוזיל [a-d-w-z-y-l] [?] remains, 5 degrees. To find the closer root, we must find a number which, when multiplied with double side דך [d-k] [?] and with itself, makes 5 — the reasoning being clear from what was said above, since the surfaces אי, יי [a-y, y-y] [?], together with square צם [ṣ-m] [?], are the completion of the whole square, 54, and equal 5.

Therefore, if we subtract square צם [ṣ-m] from five, the whole completion and remainder, and divide the result in half, we find surface אי [a-y] [?]; and this half, when divided by side 7 — I mean, if you divide that surface by its side ידי [y-d-y] [?] — yields side אי [a-y], a known quantity. And when you join it with ידז [y-d-z] [?], side 7, you obtain the whole side אהז [a-h-z] of the known square. So then, divide surface 5 by double the side only — I mean that the surfaces אי, יי [a-y, y-y] are properly laid out, as I showed you above, but do not bring it [into the first stage].

[The text now works the sexagesimal root of 54 step by step:]

The first [partial values] are 300, and 14 enters 21 times into 300; but only 6 firsts [minutes] remain — likewise the number by which you can also subtract the square of the number arising from the division. Now the remainder, 5 degrees [whole], cannot be entered, for it does not equal 5 except in the sum of square צם [ṣ-m]; therefore add a zero [shift down a sexagesimal place], and they come to the number 280 firsts. Subtracting them from 300, 20 remain; and when we subtract from them the square — therefore we say that the divisor 14 does not enter the number 300 more than twenty times, for if you double 20 firsts, which is six firsts [?], and a further 40 seconds, 13 firsts and a further [value] remain. The square of the quotient, 21, which is 441 seconds, equals 7 firsts and a further 21 seconds, which will not be subtracted.

And the square of the quotient is 21, which is 441 seconds — those being 7 firsts and a further 21 seconds — which will not be subtracted [until we test the next digit].

And further: investigate the third side — I mean a number which, multiplied with 40, 44 [?], yields the standing surface, and is 13 seconds. And further [there remains] a square. According to the [tabular] indication: 20", 13", divided ⟨…⟩, 0", upon 13", 20", 14". And behold, 20", 13" are 800 seconds, and the divisor 14 seconds and forty thirds enters into them 54 times, since double 54 with 14 comes to 756 [recte 14×54 = 756]. Subtract them from 800 and 44 remain, which are 2640 thirds. Subtract from them double of 40 thirds by the number 54, which is 2160, and 480 remain, which are 28800 fourths. From these subtract the square of 4 seconds, which is 2916 fourths, and 25884 fourths remain, which are 7 seconds [?], 11 thirds [?], 24 fourths. Therefore side ב [b] is 45 [recte] seconds, and surface בש [b-sh] [?] will be 6, 36 [6° 36′ in some unit], and likewise שמ [sh-m] [?]; and square נק [n-q] [?] is 36, 48. The remainder from the great square — that is, the standing surface אסומיל [a-s-w-m-y-l] [?] — equals 24, 1 [?].

Again I come to the fourth side, and I double root בגדה [b-g-d-h] [?], which is 7, 20, 54. They will be 14, 41, 48; and with this 1 [?] I divide the remainder 7, 11, 24; and when you reduce them to fine [units], the dividend over the divisor, one above the other, is thus, and you find that it enters 29 times: thus 29 — 1553040 ÷ 52908 [recte] — [the quotient 29].

So you find that the root of the number 54 is approximately seven degrees, 20 firsts, 54 seconds, 29 thirds. For if you square it, it rises to 53 degrees, 59 firsts, 59 seconds, 48 thirds, 7 fourths, 26 fifths, [7] ז sixths — and the gap between it and the truth is but a stride. You are at liberty to refine ad infinitum.

After you have found the root that is less than the precisely determined true root of a square — if you wish to add some units, as one of the root's digits, you will draw nearer to the square but still leave a remainder; and just as you go without end in refining the deficient [under-estimate], so you go ad infinitum in refining the excessive [over-estimate]. The squares keep approaching ad infinitum, yet they never meet and never become equal — for had they met, the number would have been speakable [rational], whereas it is irrational [אלם, "mute"].

A telling example: the nearest root of 54 is 7, and it falls short — for the square 54 [under it] equals only 49. If you add some units onto the root so they become 8, its square 64 exceeds 54; and the first approximation lies between 49 and 64, the squares, since between them lies the square 54, even though its root is hidden. And the more you refine — set the root at 7, 20 — its square is 64 [recte 53…] / 53, less than 54. And

if you set its root at 7, 21, its square will be 54, 1, 21 [54° 1′ 21″], which is in excess. So there you have a closer approximation between the squares of these "second" [sexagesimal] roots than of the first; and likewise it will be still closer between the squares of root 7, 20, 54 and of root 7, 20, 55; and closer still between the square of root 7, 20, 54, 29 and the square of root 7, 20, 54, 30 — and so ad infinitum, with no need to prolong.

§[12] יב

A puzzle: How can teachings and conditions be predicated of non-existents? Reply: the matter is deep, and I shall first set out for you some premises in brief, for its understanding. Know that the **regular figures** — I mean those whose sides and angles are equal to one another — are inscribed in a circle by means of the isosceles triangle. For if each of the base angles equals the apex angle, that already inscribes the first figure, the equilateral triangle. And if each of the base angles is double the apex angle, then the base is the side of the second figure among the odd-sided ones, namely the pentagon: when you bisect each base angle and extend lines from it to the circumference, the figure is inscribed in the circle. And if each of the base angles of the said isosceles triangle is triple its apex angle, the lines issuing from the three-fold division of the angle to the circumference inscribe the heptagon (seven-sided figure), one of whose sides is the base — and in this manner they proceed without end.

However, if each base angle of the said triangle is one-and-a-half times the apex angle, then its base is the side of the square — which is the first figure among the even-sided ones. And if each base angle is two-and-a-half times the apex angle, the base is the side of the hexagon, which is the second among the even ones. Likewise, if three-and-a-half times, the base is the side of the octagon, which is the third among the even ones — and so ad infinitum.

Now, had we known a way and method to find and construct all the said triangles according to the conditions required between the base angles and the apex, we would be able to inscribe up to four of the simple [odd] figures, and from the simple ones we would build the doubled ones more easily; and we would divide the circumference of the circle at will. There is no recounting the great benefit that would follow from this in astronomical and experimental computations. But this is known only in the case of the triangle whose base angle is double the apex — and Euclid explained it in Book IV, in the inscription of the pentagon. And since double five is three [recte: since the method of inscribing the pentagon uses the triangle, and five = three + two], the method of inscribing the pentagon and

triangle being clear, he also explained there the figure of fifteen sides, whose side falls between the side of the triangle and that of the pentagon.

The **composite figures** are easy to inscribe: if you bisect the arc subtending the side of the triangle, you find the side of the hexagon, and from it the side of the twelve-gon and the twenty-four-gon, as explained in Book I of the *Almagest*. Likewise, if you bisect the arc over the side of the square, you find the side of the octagon, and from it of the 16-gon, the 32-gon, the 64-gon, the 128-gon. Therefore Euclid explained the method of the triangle, the square, the pentagon, and the fifteen-gon in Book IV alone. And after him no geometer or computer was found who could inscribe the figure of seven, or nine, or eleven, or fourteen [sides] by a demonstrative, didactic method — because the ratio of the sides of these figures to the diameter is irrational [non-speakable]; and the division of the angles into 3, or 5, or 7, or 9, or 11 has been established only by mechanical means and by such instruments.

And in mathematical introductions — they [such instruments] are among the attainments of natural bodies; and the pure mathematical disciplines, abstracted from matter, are not of this sort. Even if it should happen that you divide the angle into the said parts, or inscribe one of the figures mentioned, we have no way of knowing whether the truth is so or not. For HaShem [יי] has closed the door to our knowing these matters, as we said; and many have toiled and labored over [these] broken fragments [שברי שכרים, "fragments of fragments"], yet they came only very near to the truth and never attained the truth itself, which is what is sought in these disciplines — for the approximate is not the knowledge itself; even the algebra [האלגי"ברה] is too short to save in these matters, as I explained elsewhere.

Further, know that each of the figures Euclid explained in Book IV can be inscribed independently,

with the help of the circle. The general rule is: whenever we are given the measure of a figure's side, we inscribe it in the geometers' way. For example, you know how to inscribe an equilateral triangle on a fixed line; and from it, when we drop perpendiculars from the angles, ⟨we obtain⟩ the sides at the right [angles] — we build the hexagon. Likewise the square — its construction is clear; and the pentagon too, when we reckon ⟨it⟩. The greater segment of the measure divided in mean and extreme ratio [the golden section] — from Book II [or "from teaching/Proposition..."] — to the sides facing it; and so we complete the matter, from Proposition 5 [?] to Proposition 12.

But the figure of seven, or of nine, and the like — these we cannot inscribe in any way with the help of the circle. Therefore, whenever we know the ratio of the angles' division, we can divide ⟨the circle⟩; and we have known how to divide the angle in halves. Now it is revealed to us that the triangle built upon one of its sides — its legs [shanks] meet at the angle facing it, between its two sides; if we divide each base angle [of the triangle], and the legs meet at the angle facing it between the two sides. ⟨Then,⟩ if you divide every base angle so that its base is double the third [apex], that is what guides us to the inscription of the pentagon — I mean, from the figure outside the circle we come to the knowledge of its inscription within the circle and of its properties, and ⟨of⟩ its ratio to the said isosceles triangle.

But in the figure of seven that is outside the circle, when we divide the angle into three, we find triangles differing in their sides, and the ratio of the angle to the sides is not known. Therefore, outside the circle, the angles teach us nothing about the construction of the isosceles triangle whose base angle is triple its apex — which is the foundation of the heptagon's figure. And so we will not know how to inscribe it outside the circle, ⟨the figure⟩ from which we would learn ⟨how⟩ to inscribe it within the circle; and even were it inscribed, we would not recognize it, and all the more would we not know its properties along with the base angle of the isosceles triangle. And yet, withal, you will find the geometer saying: "to inscribe the figure of seven, inscribe in the circle a triangle each of whose base angles is triple its apex angle" — which presupposes the knowledge of inscribing the heptagon outside the circle. So then, in order to know its inscription ⟨within⟩, you must already know its inscription outside the circle and recognize it together with its properties — as if he said: if you wish to make it, it must already be made! And this is genuinely astonishing.

§[13] יג

A puzzle: How can the intellect take up and handle unknowns — knowing the half, the quarter, the square, and so on, of the hidden measure? Reply: this also bears on the difficulty Aristotle raised — that there is no [actual] existence to teachings and disputations [about non-existents] — for although a thing is known as it is, and if it does not exist it is not known, nonetheless, according to the conception by which the intellect conceives it, it does exist as a conceived ⟨object⟩, ⟨and⟩ its essence is delimited. Now the measures discussed in this algebra are known in a certain manner, and according to the knowledge, so is the discourse and the truth.

For example: the side of the square of the number ten — which is irrational [אלם, "mute"] and will never declare its root — nevertheless we know that it is a measure determined by nature, such that when multiplied by itself it yields ten; and we depict it with a line, that is, a continuous quantity. It is therefore called *illem* ("mute"), for the mute man cannot utter the matter with his lips but shows it with his fingers. So the roots of the "mutes" [irrationals] are not known by discrete quantity, by means of a line whose measure is not speakable. And by this perplexed knowledge of ours we reckon with it and find its double, which is the side of the forty.

Likewise, if we want the cube root of eight — which is also a root [surd] [?] — nevertheless we find that it is the root of 27. And so we gather the square root of 8 and of 18 and there comes out the square root of 50. Likewise the sum of the square root of 8 and of 2 is the root of 18. Likewise the sum of the cube root of 162 and the cube root of 2058 is the cube root of 6000. And likewise, if we subtract from the square root of 50 the square root of 8, the square root of 18 remains.

Behold, you see how we take up, handle, and reckon with them, while no one truly knows for certain what they are; and yet, withal, they reveal to us great matters, as is known to the discerning.

§[14] יד

A puzzle: How is it that geometry [שניאו"מיטריאה, "geometria"] is higher than arithmetic [number]? Reply: if your soul is bound to the soul of the science of number, I too have a heart like yours, and it is fitting that we praise it together with…

Sefer Ma'ayan Chatum (continued)

§14 — How is geometry held to be higher than arithmetic? (continued)

[Aristotle], in examining the two, declared at the outset of the *Posterior Analytics* that the discipline he holds to be more true and more exact is the one whose subject deals with prior things that are not derived from anything else, because it has no need of a multiplicity of premises to demonstrate its conclusions to a learner. On this ground, the science of number is more exact than geometry, since number deals with whole units that are all distinct, definite, and recognizable — whereas geometry deals with the parts of continuous magnitude,

which are deaf-mute [i.e. irrational], visible to the eye yet not utterable, with neither name nor expression. By how many names can one specify a single thing! For example: ½, ⅓, ⅝₁₀, ⅜, ⅚, ⅖ — all equal, and not so in geometry. [Reading the fractions from the scan under magnification: 1+½... ⅝₁₀ ⅜ ⅚ ⅖₁ ½; the worn cluster transcribed as best legible. [?]]

Moreover, number can be conceived without positing the parts, whereas geometry deals with magnitudes that can only be conceived by positing their parts — that is, you grasp the line as something whose parts are joined together at a point, and so on through all of them: the surface by lines, the solid by surfaces. Geometry therefore needs number to clarify its demonstrations, while number has no need of geometry at all. For if there were no number, there would be no triangle and no square. Were it not so [i.e. were number dependent], no image could possibly take the number three, four, five, and so on. This is why the Pythagoreans reckoned that the Creator created the world by means of numbers, that numbers are the instruments of His craft, and likewise the Kabbalists reckoned through the combinations of the letters. And just as number is by nature the first of the disciplines, so in the order of instruction it comes first — for its ease and for the pleasure that flows from it to beginners, drawing their hearts.

We see, then, that all things, even the continuous ones, take pleasure in being measured by numbers — and above all by unity. Even when things are many, one says: "one thousand," "one myriad," "one pair," "one world," "one." This rises until one ascribes to the All-Comprehending of all powers, [HaKadosh Baruch Hu], the saying "The Lord is one, and His Name is one."

And behold, as I demonstrated in *Sefer Sa'ar Yosef* [Gate of Joseph] of blessed memory, the topics of the science of number are clarified through numerical demonstrations — not, as some have reckoned, that in order to learn the errors [or: the proofs] one must clarify them by lines and surfaces. So too the extraction of square and cube roots, and many other matters that the ancients clarified within the disciplines of continuous magnitude, we have clarified all of them by numbers. I will instruct you briefly in another method that is found in *Sefer Sa'ar Yosef* in twelve ways. And I shall inform you first that for every side of a square number divided into parts, the square of each part, together with the surfaces arising from each [part multiplied] with the rest of the parts, gathered together, will equal the square number.

For example: the number **625**, whose root is **25**; let us divide it into whatever parts we wish — say, into **20** and into **5**, whose squares are **400** and **25**; and the

product of 20 by 5 is **100**, and the product of 5 by 20 is **100**; and the sum of all these is the square. Therefore, suppose we did not know the root and wished to seek it. We already know that the side has two figures [digits] — that is, in the rank of the tens. Hence we know at once that one part of the root is **20**. And since we know, from the eighth proposition of the ninth book of the *Elements* [Euclid IX:8], that if there be a series of numbers in proportion beginning from unity, the third will be a square; and we know that the side is the mean in the ratio between the proportionals — from all this we arrive easily at our object. For if we set up the proportion and say, "as 1 is to 25, so 25 is to what?" we find the square. [Worked column from the tile, read top to bottom: 400 / 25 / 100 / 100 / 625.]

From the manner of these disciplines and their use in another place you will find them, with God's help. And the deficiency [in our literature] is because we have not found among the authors so many disciplines of this kind for number as for geometry; and it is known that most of Euclid's propositions can be made numerical, even though he himself clarified them only by lines. So too Nicomachus, Jordanus, and Boethius [ניקומ"אכוס ויורדא"נוס ובואיצי"או = Nicomachus, Jordanus Nemorarius, Boethius], and after them, among the later writers, expanded the matter; and one who examines their books will have the numerical disciplines as lights to him in all reckonings, so that he will seldom need to draw on geometry. Beyond all this, this science [of number] appears full of secrets, and all its numbers are precious [or: full of mystery] — as I have expanded in discoursing in my commentary on the other book of Ibn Ezra of blessed memory, and as the aforementioned author [Ibn Ezra] explained in *Sefer HaShem* and in his commentary on the Torah. And who will not be astonished

at the properties of the number nine, by which calculations are tested? And likewise of the three and the seven. From this it appears that numbers, even though they are separate quantities, are bound one to another in many respects.

(a) The extremes generate the means from the one, except that one [extreme] makes a corner [or: an end]. [The line continues a digit-puzzle whose worn cluster is transcribed below.]

[The two reversible nine-rows, from the tile:]

```
1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 0
```

and: `123 456.7. 9. 5678` [transcribed as printed]

For example, if you wish to know their sum up to 9, take its half... [worn] and multiply it by 9, giving **36**, and add 9 to them, and they are **45**, and so with all of them. — On this is built the foundation of [the method of] gathering the [series].

(b) The property of the seven [or: the sevens]. If you wish [to produce] two numbers whose product yields figures all of one kind [a repdigit], here is a general rule: multiply the kind of number you wish by the number **143**. For example, if you wish [all] sixes, multiply 6 — that is, the kind — by 143, and there come out **858**; and this you multiply always by 7 as you please: **666666**. And likewise if you wished sevens, multiply 7 by 143, and there come out **1001**; and these, [multiplied] by 777, yield **777777**; and so with all of them.

(c) Again, behold a property in the number **4**, in that the square... is reciprocal [self-related]. For if you wish, add to a number so that it becomes a square; and if you sow [add] to it 15, it becomes a square; and if you add the seed [increment] itself, 15, it becomes a square. Square it, and it is **225**; add 4 to it always, and it is **229**; and divide it by [?] and there come out **57¼**, and that is what is sought. And so always you find the matter.

And behold, you know that squares and cubes, though they appear proper to continuous magnitude, and we extract them by means of [continuous magnitude], are [also] grasped through numbers. For if the end of a number be one of these numbers [1, 4, 5, 6, 9], it is possible for it to be a square; the worked row: `1.4.5.6.9`. But if it be of these — `2.3.7.8` — it can never [be a square]. And in *Sefer Sa'ar Yosef* you will find fine things like these.

Now, the science of geometry appears, from one side, to be a thing nobler than number, and princes occupy themselves with it and delight in its figures — and not so with the calculations of number, which are crumbs, the work of a child, to itemize. Therefore those who deal with the governance of the state set the rulers apart from it, and there is no need of geometry, [only] of number, in essential matters; for it is contemptible for one who governs to be busy with numbers. And behold, number — although you greatly divide its parts without limit — every number is finite, whereas, conversely, the line of the disciplines is divided without end, by the nature of continuous magnitude. Were it not for geometry, a thousand thousand problems would remain sealed with none to understand them. There is nothing resolved by number that is not [also resolved] by geometry, and not the reverse. For example, the proposition "11 to the second [i.e. squaring]" — how should number and its mute [irrational] root arise? The eye of number does not perceive it, but geometry bears and grants them and brings every hidden thing to light.

Even pleasant algebra [האלג"ברה הנעימה = al-jabr] is a part of geometry, and the calculations of fractions — whether geometrical, astronomical, or borrowed [applied] — are taken from it in the books of number. And further: even though, in Aristotle's view, the science that comprehends number is in truth subordinate to geometry — for the halves and quarters of sounds [musical ratios] are themselves numerical — yet the science of calculation is the science of optics, of perspective, of weights, and the totality of the mechanical arts. And so great is its [scope] that the ancients said that all the business of God [or: the Name] is geometry; "all the day long it is His delight." Whoever descends to its depth resolves all its problems by positing, as I shall show you in another place, and [resolves] the extraction of roots and all that arises in numbers, briefly and clearly, in place of lines [i.e. without lines]. The path is known too for adding and subtracting, multiplying and dividing — this golden discipline within it — better than what is done with number, for one proceeds in it with images, not in [unordered] arrangements; and the true test is according to the matter.

For some of the instruments of the disciplines are weighty [important] in use, although they are easier than the true natural sciences.

§15 [implied] — Aristotle's praise of the disciplines

Aristotle, in the second [book] of the *Posterior Analytics*, said, of blessed memory, that the disciplinarian [mathematician] resembles the philosopher, since both are occupied with many [branches of] knowledge. And Plato, in the seventh [book] of the *Republic* [on the governance of the state], said

that geometry elevates the soul toward the truth and prepares the intellect to philosophize — so that he distinguished between the philosopher and the disciplinarian [mathematician]. According to this, the disciplines [mathematics] are not a part of philosophy. And so Alexander [of Aphrodisias] explicitly states, holding that the disciplinarians, "inasmuch as they have not seen the face of the house" [i.e. not penetrated within], — and it seems right to say that their opinion concerns the easy matters — said in his *Mousika* [commentary on the *Musica*]... at the beginning of his commentary on the book of *Rhetoric* [*De Interpretatione*?], that geometry is not a part of philosophy. And so Maimonides, as Aristotle wrote, that the young are trained in them [the mathematical disciplines]. And [as for] those who spend their days in music and in the calculations of distribution and the like, in the affairs of treasurers, or in measuring fields and the labor of [?], wood-merchants, hay-wagons, the works of builders with [their] beams and walls, and the like — though these are forms of knowledge, nevertheless, because

of the baseness of the subject they engage in, the name "philosopher" does not befit them.

Not so those who contemplate the science of observation [astronomy], for the sense of sight is the most precious of the bodily faculties; all the more so the astronomers, who behold with their intellect what no bodily eye has seen. And they have in truth opened several gateways to the observation of the Divine and Nature, to investigate the essences of the spheres and the separate intelligences. They are therefore not of the lowly disciplines, as some have reckoned, since their occupation is with honored and lofty things.

Now, as to whether they [the mathematical disciplines] are estimable, true forms of knowledge but not wisdom: in that the disciplinarian's concern is not with the active, material, and final causes, but with forms alone — of this we have spoken in *Sefer Bosmat* [Book of Basmath], with God's help. And there I say: opinions and disputes — each one strives to uphold his own view, and there is no strange or [unsupported] opinion that has none to help and support it. For this reason I have called them by the name "true forms of knowledge," since most of the other sciences are not numbered [definite], but [rest on] supports and proofs and the reasonings and words of sages divided [in dispute], and on traditions. Whereas in the disciplines all instruct toward a single thing in agreement, and the truth is one to its place.

And indeed they [the disciplines] are easier than the natural sciences. I will not deny that you see thousands of philosophers and [scarcely] one in a thousand a disciplinarian; and although they are more ordered in their study, this is because of the multiplicity of the disciplines, which are weightier than the natural sciences. Yet this is not on the side of the instruments [the means]; for, as I noted, their syllogisms are very near to the intellect — and even so they are firmly held by demonstration. For proposition 11 of the fifth book [Euclid V:11] is the logical syllogism itself: in it, the point is that when two things are equal in ratio to a third, they are equal to each other. It is as though they said: "all of every A is B, every B is A; therefore all C is A" — and that is the first logical figure.

But proposition 19, called the **golden proposition** [the Rule of Three], no one challenges or stammers at: that whoever gives three proportional magnitudes finds a fourth in the ratio of the first to the second. For example, when I say "as 2 is to 5, so 1 is to a fourth — 5" [?]; for there is no doubt that the fourth will always be given in magnitudes, though not always in numbers, since fractions are not numbers, as you know. Upon this discipline — so very near to the intellect — are built **six syllogistic figures** that the disciplinarian uses, all of them clarified by

demonstration in the fifth book of the *Elements*. One who is practiced in it will have, in the disciplines, ten "hands" [uses]. And these they are:

The ten uses [yados] of the golden rule of proportion

The right-hand column gives each lettered operation with its letter-form of the proportion and a numeric example written above the letters; the left-hand column gives the explanatory note. The table reads right-to-left in the source. The numeric values are written above the corresponding letters of each proportion.

Use	Letter-form of the proportion	Numeric example (above the letters)	Explanatory note
(a) by inversion [בהפוך]	as א to ב, so ג to ד	2 · 3 · 4 · 6	And so you see in the syllogism: the disorder you need only —
	as ב to א, so ד to ג	3 · 2 · 6 · 4	to subtract from the middle figure
(b) by alternation [בחליף]	as א to ג, so ב to ד	2 · 4 · 3 · 6	the equals, since the mean is bounded —
	(the mean term)	5 · 3 · 10 · 6	as I taught you in [the chapter] "Statutes of Heaven,"
(c) by composition [בחבור]	as אב to ג, so גד to ד	2 · 3 · 5 ·...	their use, and there is no disorder to the intellect —
(d) by separation [of the composites] [בפרוד המחוברים]	as א to ב, so ג to ד	2 · 3 · 4 · 6	for from the first principles of intellect [they follow], nevertheless
(e) by inversion of the composition [בהפוך החבור]	as אב to ב, so גד to ד	5 · 2 · 10 · 4	their truth is clarified by Euclid in propositions 16, 17, 18, 22, and 23 of the fifth book;

Use	Letter-form of the proportion	Numeric example (above the letters)	Explanatory note
			and these are the simple ones
	(the "born of the composites")	3 · 3 · 2 (and וּפ to גּ)	likewise in inversion and alternation, in the simple ones
(f) by equality and mixture [בשווי וערבוב]	as אּ to גּ, so דּ to וּ, [and] בּ in many means [ברבוי אמצעיים]	2 · 3 · 8 · 12	and in the composites, and they are useful and weighty things [or: with multiple means]
	as אּ to בּ, so גּ to דּ	2 · 3 · 8 · 12	
	as בּ to הּ, so דּ to וּ	12 · 5 · 5 / 2 · 8 · 12	
	as הּ to זּ, so וּ to חּ	5 · 15 · 20 · 60	[a chain with multiple intermediate means]
	therefore [by equality and mixture] as אּ to זּ, so גּ to חּ	2 · 15 · 8 · 60	
	as הּ to זּ, so וּ to חּ	15 · 20 · 60 [?]	
	therefore as אּ to זּ, so גּ to חּ	15 · 8 · 60	

After the table, the explanatory text resumes:

These [proportions] vary in several ways. As "as אּ to בּ, so גּ to דּ" — therefore by inversion: "as דּ to גּ, so בּ to אּ" — that is, the disorder [transposition] you need only is to subtract from the middle term the equals, since the mean is bounded; and as I informed you in the chapter "Statutes of Heaven," its use, and there is no disorder to the intellect.

By composition [בחבור]: as אבּ to גּ, so גדּ to דּ; their truth is clarified by Euclid in propositions 16, 17, 18, [and] 22 and 23 of the fifth book. And these are the simple

ones, and from them are born the composites; the simple and the composite disciplines and their modes of syllogism — and these are the instruments. As in inversion and alternation, so in equality and mixture; and in the composites, and they are weighty things, useful and worthy.

And I cannot restrain myself from informing you of the error of one who is called, in *Bosmat*, "great in his philosophizing and his wisdom," who wished to renew that science of [the soul]... in the fifth [book] of the commentary on the *Physics* [Sefer Shema Tiv'i = *Physica auscultatio*], proposition 15 — he stumbled in the syllogism of alternation and reasoned thus: "as the ratio of the new will to the old will, so [the ratio of] the new act to the old act" — therefore by alternation it would be "as the ratio of the new will to the new act, so the old will to the old act"; therefore by inversion it would be "as the ratio of the new will to the old act, so the old will to the new act" — but this is not the syllogism of alternation, as you have heard, for it ought to have produced, by alternation, "that the ratio of the new will to the old will be as the new act to the old [act]." This use of his brought evil opinions into the minds of the philosophers, and they paid no heed to their error.

But as for what you see, that the masters of number sometimes reason by syllogism — as we ourselves did in *Sefer Gevuros HaShem* [Mighty Acts of the Lord] in determining the diameters of the stars from their distances — they call it **the inverse ratio** [reciprocal], "ערך ריצי"פרוקו ההפוך" [Italian/Latin: *ragione reciproca* / *ratio reciproca* = the reciprocal ratio]: that the surface arising from the first with the second equals the surface from the third with the fourth — the reciprocal inverse ratio.

For example: if with a wagon of two horses we go to the city in 8 days, in a wagon of 4 [horses], in how many [days]? And in the golden discipline, the surface of the means equals the surface of the extremes; therefore it is inverse, and you multiply the first by the second and divide by the third — and this is no new syllogism.

Now, the reason the logicians did not accept the **second** and **third** logical figures is not as one [author] reckoned — that they are remote from the intellect, and the intellect halts and will not firmly believe in the syllogisms occurring in them by their figure. For even in the second and third figures, which appear near to the intellect, since not all the syllogisms occurring in them are true and they need to be reduced to the first, the intellect refuses to believe in them — except for two other reasons by which [the figure] is rejected.

The first is that it is not useful at all. For according to the truth, when we reason by its way [the second figure], we make of the predicate a subject, and the thing becomes predicated of itself, which is laughable and a mockery. It will be clarified to you by this illustration: "every man speaks; every laugher is a man; therefore every laugher speaks" — [reducing to] the first figure: "every laugher speaks," and laughter "made for us a god" [a play on the verse], and "all who hear will laugh" [Bereishis 21:6]; and according to the first figure the conclusion will be "every laugher speaks," and we have joined two conclusions, in that the subject and predicate are more distinct than a genus — they will not both be found [predicated of] one thing. And this is all...

Sefer Elim — Ma'ayan Chatum (Fifth Part) … precious. For our aim was to derive from the figure already mentioned a conclusion not in its proper manner — namely, that "Every laugher [שׂוֹחֵק] is a speaker [מְדַבֵּר]" — and it does not follow, but only "Every laugher is a speaker in the first figure" alone. **And the second reason** they rejected this figure is that it does not yield the thing sought, but only some other thing instead. For suppose we wished, say, to derive "Every laugher is a speaker," with the syllogism in the [second] figure: "Every man is a speaker; every laugher is a man; therefore every speaker is a laugher" — and this is not the desired conclusion, but its converse. **Know** that this is so, because from those very same premises we can derive, in the first figure, "Every laugher is a speaker." And had the conclusion [in the second figure] been a proper one, the first figure — which is, as it were, the touchstone of the truth of a conclusion, as in the other figures — [would yield it directly]. One cannot say that the affirmative [universal] does not convert into a

universal but only into a particular, since it has already been explained in logic that the universal premise [does not convert thus]; and our aim is not to derive "Some speaker is a laugher" but rather "Every laugher is a speaker." Nor can one say that, in any case, if we wished to derive "Every speaker is a laugher" or "Some speaker is a laugher," this figure would serve — since whatever the first figure can do, popular opinion rests easy with it. In my logical works I have treated at length, with reasoning and skill, the grounds and foundations of every figure.

[New paradox / response:] **You have troubled yourself over the study of gold [i.e. of mathematics/commerce] that proves untrustworthy. Response:** It is all true and faithful for one who knows how to use it properly — that is, in matters that are [genuinely] proportionable [נֶעֱרָכִים]; and it is a gift of God to man, for in all the difficulties of reasoning and in every vessel of the projectile-craft [or: of the marksmen] there is nothing like it. Yet the syllogism is hazardous in matters whose nature, properties, conditions, and rules are not known to you — for the error there is not on the side of the figure but stems from false premises, when things are not proportionable according to the manner you suppose. So it happened to a man full of the spirit of wisdom, **Nicolaus Copernicus** [ניקולאוס קופירניקוס], who reasoned from the doubled distance of the Moon from the Earth to the doubled apparent diameter [אלכסונו] and its light: that it ought to be twice as large at half-full as at full, since at full it is, by Ptolemy's reckoning, twice as far from the Earth as the first [position] — and he erred, an error well known to the scholars of observation, as I explained in *Sefer Gevuros [HaShem]* — for those quantities are not proportionable. He should have asked half [-skilled] people, who know how a form appears when raised above the Earth by a known measure; and this occurs in all proportions, by way of example, in the measuring of fields: if the surveyor proportions and reasons from the perimeters, he will err

and harm the purchaser. For it is possible to have differing figures — say, if the perimeter of 24 equals 20 [and] the perimeter of 16 equals 13⅓ [13 ⅓ — [?] for the exact fraction]; for if both are squares, the perimeter of [a square] of 24 equals 36, and the perimeter of 16 equals 16, and the true proportion is thus: if 36 equals 20, how much is 16? And if the first is elongated, its side being 2, and the second 12, then the second is a true square… the computation. And if the second is a square and the first oblong, it is possible for it to equal less than the second — and so in many ways.

Again: if a bundle [אֲגוּדָה] of asparagus, whose perimeter — the rushes [הַגֶּמִי] bound in it — is 3 fingers long, is sold for 3 plain coins [פְּשׁוּטִים], and the seller reckons to make bundles from material 6 fingers long and sell them for six plain coins, he will lose; for the proportions of the surfaces of round things are as the squares of the diameters, as explained in *Sefer Gevuros HaShem*. Hence, by computation: if the sea [recte: יֵם/אם — "if"] the diameter [עֻבְיִי, "thickness"] of the first bundle is a whole, its diagonal will be the diameter of the first bundle = 1 [[?]]; and if you double its quarter together with its perimeter [3], its surface will be… [3]. And the larger diagonal = 2, and its quarter, which is ½, doubled by its perimeter 6, yields its surface 3. Hence their proportion is as the squares of their diagonals, 1 to 4, and it ought to be sold for 12 plain coins.

Again: if we know that for a person 2 cubits tall from the shoulder down, a cloth ⅞ [recte: [?]] cubits is needed to make a cloak [בֶּגֶד מַנְטִיל], and we wish to make a like garment for a person 3 cubits tall, how much cloth is needed? By the proportion you will find 1⅞ — but it is not so. The error is that you must realize this garment is a complete circle, as it were [the way tailors actually cut it], and so the person's height is the half-diameter of the circle…

because of the pieces, for the garment is square and cannot be turned into a circle without loss. To verify this, take them by their squares: 36 to 16, or 9 to 4, and you will find that 4 cubits [of cloth] are needed; but the craftsman will ask for more. And so the two diagonals of the garments will be 6 and 4, and their halves likewise; so the proportion of the two garments will be [[?] ratio]. After it is sewn to its measure, weigh it against the 4 cubits of cloth and you will see the truth — the pieces, and remember the theft [waste of offcuts]. And understand: in all garments their measure is according to their cut [their shape], and so too for the spherical [bodies], to know the proportions of the solids together with their diameters. And with ladders, by which one scales the walls around them: if the wall is 6 cubits high and the trench [around it] 3 [cubits] wide, the length of the ladder required will be the square root of 45; whereas if the height were 12, the

trench, and the width six, the length of the ladder would be the square root of 180 — and this rests on [Euclid] VI:[?], as in the first [case]. You see that not all quantities are proportionable.

Yet there is room for genuine inquiry in [some] cases of proportions — namely, in profits [חֲלוּפִים] or annual gains. For example, if every 100 yields 8 — that is, the principal is 108: capital plus profit, the year's end coming with capital and profit on the original 100 [[?]]; and you need to find what the capital is, owing no profit at the year's beginning, the proportion goes thus: if a man owes his fellow at the year's end 100 [as] capital-with-profit, and that is the sum, then how much shall he give him? If 108 gives 8, what shall 100 give? You will find 7 44/108, and subtract it from 100, and the capital that remains is 92 64/108. And if he wishes to pay at mid-year, it appears at first sight that, since the gain over 12 months is 8, in half a year it would be 4; so [the principal at half-year would be] 96 [[?]]. But this will be shown false by their lying as follows: if in 12 months the 100 gains 8, then in proportion they gain in half a year four — this no man will deny; hence the proportion: if 104 gives 100 in half a year, how much shall 100 give? You will find 3 88/104 [[?]], and this is more than the figure 3 76/108. And it is impossible for both these reckonings to be true. The merchants follow the manner of the second; but no merchant can give the reason for the thing — and yet, if we subtract from the figure 100 half the first figure [[?]], which the merchants confirm — for they say the profit is added to the principal after one day or more, the principal [becomes] 9+, [so] 96; and if we subtract from it the second figure, 3+, there remain 9 [[?]], and this is the second… A week [passes] and it becomes principal and gains more in a week or in a month, the second; so in a year it gains more, by the doubling of what it gains in half a year — and this is the difference. So the response is well answered. But you will not understand how to study the gold from what I have reckoned. It is this: the proportion changes in its [own] proportioning, and therefore yields varying [results], for the proportion of 108 becomes simple; and if we do not change the proportion, the study will never grasp this matter. And yet, if you double the gain of half a year to find that of the whole year, the smaller [[?]] and the gain is more, and at first to reverse it the principal is greater — there the error lay…

[Worked proportion / RTL pipe-table on this page — the calculation is laid out as three aligned columns: a "fifth" relation, the principal/gain values, and the labels "first capital / profit / second capital":]

Result (left col.)	Proportion (middle)	Label (right col.)
92 64/108	7 44/108	first capital [קֶרֶן רִאשׁוֹן]

Result (left col.)	Proportion (middle)	Label (right col.)
100 to 25 is the proportion 27/25 [[?]]; 104 to 100 is the proportion 26/25 [[?]]; proportion 27 to 25 [is] larger than the proportion 26 to 25	from the student of the third [מהלמיד הה']	profit [רֶוַח]
92 33/104	7 72/104	second capital [קֶרֶן שֵׁנִי]

New paradox. How can three buyers purchase at the same price and sell at the same price, the quantity being equal, and yet one profits and another loses? The herb that sends smoke up the nose [tobacco — i.e. snuff], each one [bought] 81 pounds [לִיטְרִין] for 72 florins [פְּלוֹרִינִי] — that is, 8 florins for every 9 pounds. Now one stood firm upon the principal, and how the seller who sells just as he bought will lose: suppose three merchants each bought six pounds [[?]]. One sold at one time 40 pounds [[?]], at the rate of 6 [[?]] [pounds] for every… florins; and at another time he sold the remaining 41 pounds, every three pounds for two florins. Hence [recte: כל שלש ליטרא לשני פלוריני — "every three pounds for two florins"]. When you compute it, you will find no more than 67⅓ florins [recte: as printed; verify].

so that he lost from the principal. And the second sold, at first, 40 pounds at 2 [pounds] for every 3 florins; and the remaining 41 he sold every seven pounds for five florins — that is, he sold every 9 pounds for eight florins; and with all this he gained, as you will find, 98⅓ florins. And the third sold the whole at once just as he bought it…

This is an error well known to the logicians, for one reasons from the whole to the parts, which is not the truth — [thinking] that they sold all the [stock]; whereas in fact each one sold only part of the goods at the rate of 9 pounds for eight florins. For if you divide 40 by 6 you find 6⅔ [recte: 6⅔], and if you divide 41 by 3 you find 13⅔. Hence set the 6⅔ of the first time, when he took 40 florins, together with the 13⅔ of the second time, when he took 13⅔; and they will be 20⅓. Hence 60 pounds were sold properly, for if you reason: if 9 gives 8, how much shall 60 give? You find 53⅓, and there remain 21 pounds; and these are the 7 proportions of 2 florins each. Hence [[?]] 14 florins, and all the money will be 67⅓ — there you have the error spelled out.

Again: one bought 12 eggs at six small-coins [קְשִׁיטָה] — that is, two for a koshita — and another 12, six for a koshita, so that he bought every eight eggs for two koshitas, and by this reckoning he ought to sell them all for six koshitas to recover the principal, but he loses 2 koshitas. The error is that the first purchase is not equal to the second, but is at two [different] rates. It is, again, twice-2-coins [2 — 2 ב"פ each], and when you join with them from the first purchase twice-2-coins [[?]], they are 2 twice-2 = [recte 8]; and these would be sold for 4 koshitas, and properly sold; but 8 were taken at the rate of the first, which he bought 2 for a koshita, and they are worth 4 koshitas. And if he sells them like the others, two for a koshita, he is not selling them as he bought, and he loses.

Suppose he bought 100 nuts, every 3 for a plain coin [פְּשׁוּט], and another 100, every 4 for a plain coin. Hence he bought 7 [nuts] for a plain coin; and if he sells them so, he loses. The answer is that the error and loss lie in the remainder, for every time the 3 [are] joined, there remain 8 — that is, 25 nuts — and in these there is no joining of those sold-by-4. Hence if you sell [them] together with the [nuts] sold-by-3, properly: that is, if you divide 100 by 4 you get 25, and these by 3 give 33⅓, and when you subtract this [from that] there remain 8⅓ — that is, 25 nuts. Bought 3 for a plain coin, 7 for a plain coin he lost, for they are worth 8⅓ plain coins, and you sell them at 3¾ [[?]]. Hence sell the 100 at 4 for a plain coin and they will be 25, gained; and when you join from the other 100, 25 [taken] three times [[?]], there is recognition of them together with those sold-by-4. Hence these are worth [[?]]; and joining the first 58⅓ [[?]] they are worth… they will be… 175 worth 50 plain coins, and there remain 25 that are sold separately, 7 for a plain coin, and they will be 175 worth 50 plain coins, and there remain 25 that have…

Suppose I bought 100 pounds of pepper for 100 koshitas, and I sold 50 [pounds] every [[?]] of a pound for a koshita, and the remaining 50 every ¾ for a koshita; at first sight it seems I neither gained nor lost — but it is not so: I gained. Similar to this is the case brought by **Maharam Almosnino** [מהר"ם אלמושנינו / R. Moshe Almosnino] in his books: two poor men were reclining and eating; one had three rolls [גְּלוּסְקָאוֹת] and the other two, and a third came and joined them and ate, and on departing he gave them five plain coins. At first sight it seems the owner of the three rolls should take 3 and the owner of the two should take 2; for if you reason and say "5 [coins] correspond [to 5 rolls]," it would be 3 and 2. But it is not so, because you must reckon that the rolls are divided into thirds, making 15 [thirds]. Hence the owner of the two rolls ate but did not eat [an extra share]; the guest [[?]], so the owner of the three rolls ate, and there remained to him for the

guest's eating ⅓ [[?]]. Hence reason thus: if 5 gives 5, how much shall 4 give? You find 4; and how much shall 1 give? You find 1; and so it is proper for each one.

[§18] How can numbers smaller than the units be found, that are not fractions? — This has been explained in *Tamar* the fifth, q.v.; and they are the "numbers of nothing" [מִסְפְּרֵי אַיִן — vanishing/imaginary numbers], see there.

[§19] You wondered at Euclid, who [seems to] teach contradictories regarding the one [the unit].

I answer you a word: this question was asked of Cardano [מקרדאנו / Cardanus] in his disputes with **Nicolaus Tartaglia** [ניקולאוס טרטליאה], a man truly great in the mathematical sciences. He replies that "the one" [the unit, האחד] can be understood in two ways: either absolutely and unqualifiedly [בְּשִׁלּוּחַ וּבַחֲלוּטִין], or relatively [בְּהִצְטָרְפוּת]. Absolutely, in his view, the one is never included among the numbers; but…

relatively [בְּהִצְטָרְפוּת], sometimes it is a number and sometimes it is not — that is, in relation to squares and cubes, and likewise among numbers that are mutually prime [רִאשׁוֹנִים זֶה כְּנֶגֶד זֶה], the unit too is reckoned a number; but sometimes it is not reckoned [so]; and likewise in primary ratios, one over against another, the unit is reckoned a number.

For among three proportional numbers, Euclid finds, in the teaching of [Proposition] 20 [[?]] of [Book] VIII [the Eighth], saying that if the first… and in [a series in] continued proportion the square — the third [term] is also a square, as in 1, 3, 9, and likewise 1, 2, 4, and likewise 1, 4, 16; and the fourth too is a cube. Likewise he says: if the first of four [terms] in continued proportion is a cube — in the teaching of [Book] VIII [or IX?], and likewise of the Seventh — if 8 and the number 27 are cubes, the one too will be a cube; hence, since the number 4 and the number 9 and the number 16 are squares, the one too will be a square; and likewise in cubes-of-cubes [[?]], as 1, 2, 4, 8, and likewise 1, 3, 9, 27, etc.; and likewise it will be explained for squares-of-squares; and likewise in the teachings [Books] I and III of [Book] VIII it will be explained that the one is included among numbers that are mutually prime, from his propositions.

And in general he states that, among the smallest [terms] in a ratio, the double-proportion [הַכּוֹפֶל] forever — there the unit will be a number; and so many [propositions].

And conversely: in the teaching of [Proposition] 16 of [Book] IX [the Ninth] he says, if there be two numbers mutually prime, one against the other, there will be

for no [further] number a [continued] proportion of a third to a fourth [[?]]; and so 1, 5, 25, 125, and so [Proposition] LA"H [[?]]; and in the teaching of [Proposition] 17, which is there: if the extremes of four [terms] in continued proportion be mutually prime against one another, the proportion of the first to the last will not be as one number to another; so the number 4 [[?]] over against this, primes — it will not be as the proportion of the first to the last. The second [[?]] thus a number to a number, with 8; and likewise in the teaching of [Propositions] 18 and 19 the matter is explained: that the one is not a number, by way of joining [relation] or of another [proposition's] companion. Here he [the student/inquirer] found no answer to this matter and thought Euclid teaches contradictories. Whereas, had we been able to say that the unit is not prime [in relation] to any number that counts all the numbers, the matter would remain agreed in all the teachings, and the difficulty resolved [מאד טרוד [?]].

Yet the sage **R. Avraham b. Ezra [?]** [ר' אברהם כ"ץ — printed glyphs כ"ץ; [recte: בן עזרא ?]], of blessed memory, wrote in his commentaries that the one is the root of mathematics [the studies], that the one is a number — but it requires investigation; and I do not at present agree with this matter that we [hold]. [He held] the one is a number — only that it differs from them in that all [numbers] are the half of their two extremes, and this [unit] is the half of the two [numbers] that, in his view, head the numbers: the two heading the evens, and the three heading the odds. Another difference: that in all of them the square and the double-with-one, and the cube, are added to the root and to the doublings — and this is not so [for the unit]. And behold, the ten resembles the one, and the hundred and the thousand — for it is known that in the numerals 10, 100, 1000, etc., the [extra] ciphers [הַסִּיפְרִישׁ — the zeroes] are nothing; they serve only to indicate the rank of the unit, the first [digit] from the right indicating its own value, the second ten times itself, the third a hundred times itself, and so on. And if the ciphers be removed, there remains in the first place the one alone, and that is one.

And a further proof that [the unit] is not a number: if the ten resembles the one, then — were the one a number — we should have to begin and count, after the bend [the turn] following the end of the numbers, which is 9, and say "nine after nine" [as] "two nines," "three nines"; but it is not so, for we begin again from the one and say "ten," and after [that] "ten and one," "ten and two," etc. Hence [the units] are not a number by the nature of the one; and the start of the tens is from the twenty, which resembles the two. Now besides the other properties of the one — namely, that in every root number [[?]] that is set, forever, in square and cube, there is a trace [a remembrance] of the one, whether at the beginning or at the

end, for this is common to all the letters/digits — another reason too will be explained for its being a foundation to all numbers, that it counts them all: for [we say] "two thousand" and "three thousand," which is not so with the other numbers, which we cannot designate with 2, 3, 4 — for over the one [[?]]. And every number is, as it were, one in potency, and it is in every number in actuality — that is, in act, as in the number four, four units are found, etc.

but from another angle it appears to be a number, since when you add up its extremes they sum to ten. That is to say: 1 with 9 is ten, and in this it resembles all the other "extreme-paired" numbers — for 2 with 8, and 3 with 7, and 4 with 6, and 5 with [its own pair] sum to ten. So it appears to be a number. Thus far what can be understood from his words; and in our other book we shall expand upon this. [Figure 63] [The reference is to a numbered ציור/figure, evidently on a separate plate: ג"ס ור"ל = figure 63.]

(20.) You wonder how multiplication can be called by the name "multiplication" when applied to fractions, given that the products come out *smaller* than the multiplicands. **Answer:** In truth the term "multiplication" is, strictly, not fitting for fractions, where the "multiplying" in them is really a division. For one who says "multiply ½ by [?]" — that is, "give me half of a half" — is asking that you divide ½ in two; ¼; and likewise, "divide ½ into four parts and take one of them," which is ⅛ — and so on. The ancient Greeks did not call these operations "multiplication" as the moderns do, but "striking" (multiplication proper), or division, or transfer, and the like — and "transfer" is the fitting name for all that is multiplied. Yet it is right to find merit for the moderns, who know more in arithmetic than the ancients did, for the Indian numerals [i.e., the Hindu-Arabic digits] opened for us gates of light in computation that the Greeks did not know — Greeks who were not given this as an inheritance from Sinai of the sciences, for the Egyptians and Chaldeans evidently preceded them, as we find in Plato in his *Epinomis* [באיפינו"מירי]; and likewise Seneca testified, in the seventh book of his *Natural Questions*, chapter 14, that Eudoxus [אודוקסוס] carried the science of astronomy over to the land of Greece from Egypt; and Aristotle wrote the same in the first book of his *On the Heavens*.

And behold the figure before you [Figure 64]: if you suppose that side א-ב [AB] is half of surface א-ב-ו-ז [ABVZ], and likewise that ½ א-ב, and you multiply ¼ א-ב by א-ב, [it] will generate — that is, the surface of the square א-ב-ו-ז [ABVZ], which is 4 squares — and behold it is greater than each of the [original] sides, which were 2 squares; so the intention is, in the fraction. The product, ¼, generates 4 squares. Again, the square א-ב-ד-ג [ABDG] is generated from side א-ג [AG] — that

is, ½ — and from side א-ב [AB], which is ⅓, which together are ⅙ — that is, 6 squares. And likewise, if you multiply ¼ א-ג by ⅛ ג-ה [GH], it generates the surface א-ג-ה-ע [AGHA(yin)] — that is, 1/12 — that is, 12 squares. So you see that it adds [i.e., increases], and so in all of them.

(21.) It is a marvel in your eyes how, with numbers — where the later terms always increase over the earlier — we can know the sum of the difference between them all; and this [is found] when we subtract the first from the last. For we can insert between them any number of others, without our friend's knowledge, and we will tell him [the answer] from the first alone, which is one [number], and the last, which is 30. The more you insert and add, the more you diminish and lessen the difference between them, and as your eyes will see they will become equal — and there is no need to elaborate.

(22.) There are several kinds of amusements resembling this, with which people astonish [others]. For example: tell [someone] to take in one [hand] an even number of coins or almonds, and in the other an odd number. Tell him that the [number] on the right is to be tripled and the one on the left doubled, and that he should sum the results. He will tell you only the total of the sum; and from it [you tell him]: if it is even, say that the odd [number] is in the left and the even on the right; and if it is odd, the odd will be in the right and on the left even. The trick is easy.

Again, if you wish to know on which finger a hidden ring is: under his hand begin from the thumb, which shall be the first and has the number one, and his second finger two, and so on. Tell him to multiply the number of the finger that bears the ring, and to it add the number of the remaining fingers; and from the sum always subtract 15, and the remainder is the finger. For example, suppose it is on the ring-finger of the physicians [the medical/"ring" finger], which is the fourth — 4 — and its double is 8; and the remaining fingers, 1, 2, 3, 5, sum to 11, and all together 19. Subtract from it 15, which is the sum of all five fingers, and 4 remain — and that is what was sought; and the one thinking will reckon to himself

that you know the finger, though it is not so, for you do not understand the trick. Again, should you wish to know which of three men took the coin of gold, which the one of silver, which the one of copper: tell them to assign themselves names — that is, who is first, who second, who third — that is, to settle on an order. And say to the one who took the gold that he should double his name [his ordinal], and the silver [-holder] is to be reckoned by the number 9, and the copper by the number 10; and the sum is subtracted from the number 60, and see [the remainder]. If [it is] 2, you know that the second took the silver; in the remainder,

[count] how many times the number 8 enters: if once, you know the first took the gold; if twice, [the second]; and if 1, that the first took it; and from this you automatically know who took the copper. By way of example, let their order be thus, and suppose Reuven took the gold, Shimon the silver, and Levi the copper, and multiply them by:

	Reuven	Shimon	Levi
(assigned numbers)	1	9	10
(multipliers)	2 [×2]	9	10

the sum will come to 50, and the remainder of 60 [minus] them is 40. Therefore the number 8 enters [as a count of] Reuven, Shimon, Levi — one after another — and 2 remain. So you say to him: Reuven took the gold, Shimon the silver, and Levi the copper. And so in every matter; this is easy [to grasp].

And likewise how to know, at a table where they are seated in order — first, second, etc. — [and] suppose, by way of example, that they were [twelve]; tell him to count, upon the number that the seated one is [in], 13 from the one sitting at the head, backwards; or 14 from the second; or 15 from the third; or 17 from the fourth — all backwards — and the end of the count will fall upon the man who was thought of. And he will wonder how you were able to discern it when you yourself do not truly know who he is. For example, suppose the one thought of is the sixth seated in order: if you tell him to begin from the first and count 13, the first will be 6, and the twelfth [in the count] [?], and the eleventh [?], and the tenth [?], and the ninth [?], and the eighth [?] [?], and the seventh [?], and the sixth [?] — you see that 13 falls upon the sixth, the one thought in his heart.

And there are those who do this with cards, who play [by] placing twelve cards in a circle and saying to him: think of whichever you wish — and it is all one [method]. But the tricks performed with cards, which appear as acts of sorcery or prophecy and astonish even great philosophers, are many and wondrous. I shall make them known to you in a dedicated treatise, in which I will reveal all the works of those eye-deceivers [conjurers] — things found in no book and in no language, to my knowledge; and this is not the place for these matters.

(22.) [A second item numbered כ"ב in the source — a probable compositor doubling;] **You asked** what teaches that the counting of human beings up to ten is conventional — that there is nothing compelling [us] to stop there, or before it, at some other number, like the people of Nineveh, who differ. And you said it appears compelling from the proofs that Ibn Ezra (רא"בע) and Maharam (הר"אם) [R. Avraham — i.e., possibly R. Avraham ben Ḥiyya, but here read as the

abbreviation he uses] z"l mentioned; and likewise the masters of Kabbalah gave reasons for [stopping at] ten and not nine and not eleven, and so on. **Answer:** This question is among the natural problems that Aristotle asked, in part 16, problem 3 [of the *Problemata*]; and he did not know that there are [peoples] who count in another fashion, and he asked about the agreement of the whole world. But in the days that came after him, when the lands of Brazil [ברזיליאה] were discovered, [explorers] saw that they counted with their fingers, showing one hand for 5, two hands for 10, and two feet with the hands for twenty; and there is no doubt that in this way they will not count the sand [in such fine grains], and however many seeds and however many drops there are in the sea, as one does with the Indian [decimal] numerals — as you said concerning the people of Nineveh, that they count the hundreds and the thousands.

Indeed, there are some who hold that there is no need for "thousands" either, since you can measure 9 tens and then call the tenth a "reckoning" [hundred], called among us "hundred"; and the tenth of the reckonings [hundreds] is named by the term "sum" [thousand]; and the tenth of the "sum" is "total"; and so on, by [further] names, ad infinitum. This is the way of Scripture in [counting] the myriad [רבבה]; also the way of the *myrias* [מיריאדיש] of the Greeks. To this day you will find among the rabbis those who ask what a "million" is and are confounded by it; and each one in their time used to seek [only] the number of his flock and his coins, nothing beyond. I have seen a book [titled] *Guide to the Thousands* [מורה האלפים] with [a notation for one], and "thousands of thousands" with the [stroke], and so all of them. Therefore there is no need for a multitude of numbers except for

the better side of things [i.e., it is merely for convenience]. **And behold** the philosopher [Aristotle] gave five reasons for this convention. The **first**: the perfection of the number [ten], which encompasses all the kinds mentioned in Book 7 of the *Elements* of Euclid — that is, even and odd, squares, cubes, and so on; and further, as Ibn Ezra (הרא"בע) added, that it is composed of [the sum of] two squares — that is, 1 and 9. The **second** is taken by him [Aristotle] from the views of the Pythagoreans, as Plutarch or Galen recorded in Book 1, chapter 50, of the *Opinions of the Philosophers* — that ten is "nature," for all nations by nature count up to ten; and it [ten] is generated from the [first] four numbers, 1, 2, 3, 4. And the number four [the *Tetraktys*] is holy, and "by his name they swore" — perhaps because the world is generated from the four elements, mixed in a musical or geometrical proportion which the physicians call "the equity of justice," according to what is fitting for each and every thing; and therefore they held that every musical ratio is contained among four numbers. The **third**: that

the spheres are nine, and with the terrestrial globe, behold, ten. The **fourth** is likewise founded on the perfection of the sacred number four: among ten ranked [numbers] beginning from one [in geometric progression], you find four cubes, as is explained in Proposition 8 of the Ninth [Book] of the *Elements* — that is, 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, of which four — 1, 8, 64, 512 — are cubes; and so in every [series of] ten ranked [numbers] you find thus. And the commentator Pietro d'Abano [פיט"רו דיאכאיינו] adds there a reason in praise [of the fact] that the hundred, which is ten tens, is composed of the four cubes within the number one hundred: namely 1, 8, 27, 64. The **fifth**: that perhaps they agreed on this number on account of the ten fingers — thus far. And these are the dialectical proofs.

However, Ibn Ezra (הרא"בע) thought to give a reason for this number, saying that if you place them [the numbers 1–9] in a circle and multiply 9 by itself it generates 81, and [8+1] is 9; and likewise if you multiply 8 by [9] it will be 72, and [7+2] is 9; and so all of them: [any number] multiplied by 9, the digit-sum that issues is 9. But this is not a property unique to nine — for among all the numbers it would be so, as appears in the above figure, in column 19 [?]: for one with ten makes 11, and likewise 2 with 9, and so all of them.

Again, [there is] a second reason that you will see in the inner circle of the above figure: that when you multiply 9 by itself it generates [81], and with 8 [it makes] 72, and with 7 [it makes] 63, and with 6 [it makes] 54 — and in all these you find the tens on the right and the units on the left; and when you multiply it by the remaining numbers you find them reversed — the tens on the left, the units on the right — and these [latter] are the first numbers. Therefore all [the things] which are either elements (חיות) or foundations of number are nine, no more — and the matter is sharp and keen. But perhaps the respondent will say that this is [merely] a property of nine, just as it [nine] has the property of being a "touchstone" [casting-out-nines check] in the *algorism* [האלגורית"מו], and likewise for 3, and for 6, and for every number has [its own] properties — and it [the count of ten] is [not thereby grounded].

[The respondent] holds that the tenth is not a number, just as the one is not a number, for "ten" — that is, ten units, which encompasses them — and likewise the hundred and the thousand are like it; and therefore he says that every number is a part of ten — that is, [this is] a proof for the necessity of number, that it should not be greater than nine or smaller than it. And behold, as we mentioned above, all that is comprised under ten is generated from the numbers comprised within it. Now the perfect [whole] parts of ten are three — that is, ½, ⅕, 1/10 — which are 5, 2, 1; and the rest are generated from these. How so? 1 with

2 make 3; and likewise twice 2 make 4; and the number 6 is generated from the joining of 5 with 1; and the seven from the joining of 5 with 2; and the eight is generated from the joining of the three of them — 1, 2, 5 — the simple parts; and the nine is generated from the doubling of 2 by itself — that is, 4 — and with 5, which is [another] one of the simple [parts], they make 9. Or [one may] say that the six is generated from the doubling of 2 by itself, which is 4, and with the addition of itself [+2] it becomes 6. According to his way, either they are the perfect [whole] parts of the number ten, or composites of them; and this is what he says there: that all the numbers are [either] a part of ten, or the new-formed [composite] of its doubling or of its joining, or of the two together. [Figure: he gives a reason for the ten numbers by their likeness to the nine moving [planets] and the [tenth] sphere, which is holy to God.] **However**, Maharam (מהר"אם) z"l, in the introduction to his "Book of Number," rejected the above reason — which seemed the best of all that had been said on this — and said that it is geometrical, not arithmetical: that is, that it would require one to place them in a circle. And he gave another reason, according to what I found in my notebooks, a fragment of his words which I had copied,

while I was in my house, with my books arrayed around me like crowns — for at present I do not have his book with me; and that [reason] is [as follows]. The composites in their place. He divides number into even — which is 2 — and odd — which is 3 — and these are the simple [units] among the parts, and the composites their extremes. With 2 they generate 4, composites of two simples; and these [go] by three modes. **One**, from two like evens — that is, 2 with 2 — they generate 4. **Two**, from two like odds — that is, 3 with 3 — they generate 6. From two evens — that is, 2 with 2 — they generate 4. (5) compound — that is, 2 with 3 — they generate 5; and their extreme is a composite from a simple and a composite — that is, from even 2 simple with odd. **Two**, like odds — that is, 3 with 3 — generate 6. **Two**, [in which] the first is even and the second odd, generate 7; and likewise simple-odd 3 [with] composite-odd 5 generate 8. And likewise from odd 3 with composite-even 6 [?] — this is right that it generates 8 [?]; and from composite-odd 5 with composite-even 4 generates 9. And there is no division [other than]: 4 generate 9 [?]; or from simple-odd 3 with composite-even 4 generate 7; and two even composites, 4 with [?]; likewise from two even composites, 6 with 6 generate 12, and from 4 with 6 generate 10; and likewise from this odd composites generate 1[?]; and others [composites] generate 11 [?] — and in this way he prepared the path of number.

And as to what you said — that Scripture was not exact in the [matter of the] Jubilee, that is, on the verse "and he shall reckon with his purchaser… according

to the multitude of the years he shall increase its price" [Vayikra 25:50–51] — [namely] that it is not the same to give the rent at the beginning of each year as to give it [all] to the one [purchaser] some years earlier, all at once: for if [a property] is worth a hundred per year and he gives a thousand for ten years, then, if the money earns 10 per hundred [10%], he gives it: in the first year 190, and in the second 180, and in the third 170, and so on, continually diminishing. Therefore, if he comes to reckon at the end of five years and says, "Return the land to me, and here are five hundred for you," he deceives him. For if you sum all the profit and the principal of ten years, they come to 1450; and the renter was reckoning that the produce of each year is worth 145, and that in five years he should deduct only 725; whereas the lessor took 850.

And I say that perhaps this was the Torah's intent in saying "and he shall reckon with his purchaser according to the multitude of the years he shall increase its price" — and this is the opposite of what appears at first glance: that the further the time is from the Jubilee, [the buyer] should give him *more* money. Rather, [the seller] need only give him a *pruta* [a small coin] for his trouble in occupying himself with the money so that it may earn for him 10 per hundred, so that there be no usury — and consider this carefully.

And Solomon the King, peace be upon him, if he knew how to be more exact than Archimedes [ארכימידיס] in the ratio of the circumference to the diameter — except that it is not the way of Scripture to be exact, and therefore he said that the sea he made was three times the diameter, by an approximate reckoning [I Melachim 7:23]. And as for the fact that Yosef ceased to count [the grain — "until he stopped counting, for it was without number," Bereishis 41:49], one should not suppose that he was unable, but rather that he wearied at the multitude of the produce. For it is already explained in *Sefer Gevurot Hashem* that even if the whole world, up to the concavity of the eighth sphere, were full of sand, a number greater than it could [still] be assigned; and for every corporeal thing there is a number, and only for the greatness of God and His power is there no number. For you know that by means of the [decimal] points everything [can be brought] to greater precision in number without confusion; and a number of ten points is greater than the number of the [above-]mentioned sand, as appears before you [in the figure below]. Therefore the Torah spoke in human idiom — and look into *Sefer Gevurot Hashem*, where there is an explanation of all your questions.

1,0,000000,000000,000000,000000,000000,000000 [a large place-valued numeral printed across the column: a 1, then a 0, then six groups of six zeros separated by

commas — representing 10 raised to a high power, illustrating the "ten points" remark above]

(23.) A question: how can [an object] lift its own self? **I supposed** [your intent concerns] a property of a mechanical device made like a balance-scale: when a man enters into it and pulls the rope with his hand, he raises himself upward. Also, one of the kings [or: one of old] regarded [such a feat] as improper, [as] unfit to muster an army and to perform labor; and among the living creatures there is no creature that bears [more than] its own body. Of old, [a certain ruler] used to test his men [to find] one whose strength would not suffice to raise himself, or a weight equal [to himself] — more than the ant in proportion to its body. Today, too, the kinds of devices resembling these have multiplied, which those who possess the craft of flight invent, by which the one trained in them can fly over the earth; and their books are engraved with iron and lead pen [i.e., printed from metal type]. But according to your words it appears that your intent is to allude to the examination of **algebra** [האלג"יברה], the choicest part of mathematics, by which questions are examined [to determine] whether they are possible or impossible — that is, that it is a true "touchstone" [literally "a tested stone," with a play on Yeshayahu 28:16], examining every question put, to know whether it is something possible to resolve in any number you wish, or not...

Sefer Elim — Ma'ayan Chatum (The Sealed Spring)

[continuation of the algebra "touchstone" paradox — §23]

but only in a particular number — even though it is hard to resolve. For example: find me three numbers, such that half of the first, a third of the second, and a fourth of the third together come to 3 — namely 2, 3, 4 [?]. Or the problem may be hard in one's own judgment, or genuinely impossible according to the conditions of the question, or it may *appear* impossible while in fact being possible.

By way of illustration: suppose we are asked to find a number such that, when the double of its square is subtracted from it, the number itself remains [i.e. $x - 2x^2 = x$ — which only holds for $x = 0$]. This appears impossible, since the square is greater than its own root. We posit, by algebra, that the number is **א** [x] = 1, and we double it: **א** [x] = 2 [i.e. 2x]. From this we subtract the double of its square, which is **ב** [x^2] = 2 [i.e. $2x^2$], and there remains **ב** [x^2] · **א** [x] = 2 [the equation $2x^2 = 2x$, hence $x = 1$]; and this equals **א** [x] = 1.

[The worked symbolic line, as printed, reads:] We posit, by algebra, that the number is **א** [x] = 1; we double it, **א** [x] = 2; and we subtract from this the double of its square, which is **ב** [x^2] = 2; and there remains **ב** [x^2] · **א** [x] = 2; and this equals **א** [x] = 1. We give to each one shares that are equal, namely **ב** [x^2] = 2; and there will be equality between **א** [x] = 2 and **ב** [x^2] = 2; and there will be equality between **א** [x] = 1 and **ב** [x^2] = 2 [?]. Thus the root proceeds [i.e. the root works out]: for if you subtract the double of its square, when its double is **ב** [x^2] = 2 and **ב** [x^2] = 2 [?]... [The symbolic algebra of this passage is partly garbled in the printing; the labels read here are א and ב with the values 1 and 2 as printed.]

but only in a particular number — even though it is hard to resolve. Find me three numbers, such that half of the first, a third of the second, and a fourth of the third come to 3 — namely 2, 3, 4 — or [problems] that are hard in one's own judgment, or appear impossible while in truth they are possible; this is the way.

When you multiply the double of its square, the number itself remains. We posit, by algebra, that the number is **א** [x] = 1; its double is **ב** [x^2] = 2 [?]; and there remains **ב** [x^2]; **פ א** [the constant-term / minus marker upon x] = 2; and this equals **א** [x] = 1. There will be equality between **א** [x] = 2 and **ב** [x^2] = 2; and there will be equality between **א** [x] = 1 and **ב** [x^2] = 2 [?]. Thus the root proceeds: if you subtract the double of its square — namely 1 — there remains **א** [?] **כ**, as at first; and likewise it holds in fractions: in all of them the square comes out less than the root. The matter is astonishing to one who hears it, for if we did not yet know its resolution, how would we test whether it is possible?

For the one who tests, tests according to the known method. And were the philosophers to have a tool like this oracular instrument [כלי ידעוני, an instrument of divination], they would do wonders in their investigations. The **Rambam** [Maimonides], may his memory be a blessing, having no such device, says in the *Moreh* [*Guide of the Perplexed*]: "Would that I knew" by what means one might test imagined things, and distinguish from them the intelligible ones — so that we might know whether they belong to the operations of the imagination or of the intellect, or whether upon this depends the knowledge of the whole truth.

Now, in the *impossible* problems this discipline shows you its true character. That is to say: suppose we wish to test the Second Book of Euclid's *Elements* [ליסודות — the *Foundations*], in the numbers, where the aim is to divide a given magnitude into two parts such that the rectangle formed from the whole together with one of them equals the square formed from the second part by itself [the equation $x(16 + x) = (16 - x)^2$ type construction, here set up as below]. Let us posit that it is the number 16. One part is **א** [x] = 1, and the second is 16 − **א** [16 − x]; and the

square of the first is **א** [x^2] [?]. The rectangle formed from the number 16 with **א** [x] — namely 16 + **א** [16x] — is 16**א** [16x] = 256 [?], and this equals the square of **ב** [the second part] = 1 [?].

We take half the number of roots, namely 8, and square it, giving 64; we add it onto 256, and there comes 320. Here the way has closed upon us, since this thing [320] has no [rational] root. Therefore it is impossible to divide the said number under the said conditions — neither in whole numbers nor in fractions — and likewise you find it impossible in all numbers. Similar to this is the finding that the side of a square has no [magnitude] commensurable with its diagonal.

So you see how the algebra, by an easy search, demonstrates what geometry demonstrates [only] through long preliminaries and propositions in a mathematical proof. In the problems of *Hevel* [vanity / the absurd], by contrast, the matter is plain, as in the first ten propositions of the Second Book of the *Elements*, where the algebra shows you that they are possible in every number. And in my *Sefer Chukos Shamayim* [*The Statutes of Heaven*] I brought you an example — see there.

And indeed, even in the solution of triangles there are tests of this same kind; they are not peculiar to algebra alone. For the way of testing itself leads you to the resolution of the matter there too, and reveals to you the nature and measure of the hidden angle. Yet whoever is granted this gift in the totality of the sciences' praises — let him not be over-meticulous about the matter, for though it is a useful thing, it appears to be no more than a groping forward in darkness, and is not the way of those who know things in their truth. Indeed, it cannot be relied upon absolutely; see [Item] [8] ח above. In many problems you will find the hand of algebra falling short of rescue — especially in problems that do not admit of fractions, where the things sought are men, oxen, and the like [whole, indivisible quantities], which cannot be partitioned, as in many of the problems of Diophantus. We shall expand on this elsewhere, with God's help.

[Item:] How the Artificial Can Conquer Nature

Further, a question: how is it possible for artifice [המלאכה, that which is man-made] to conquer nature, when artifice is, as it were, an ape to nature [its mere imitator]? For although artifice imitates nature, it is not entirely equal to it, as is explained in the *Physics* [שמע הטבעי, the *Physica*], and so on.

I answer that this is true with respect to *quality and form*. So too does **Ibn Rushd** [Averroes] say: that the heat produced by motion is close to natural heat, yet it is

neither natural heat itself nor anything else equal to it. Likewise the soul and the spirit cannot be reproduced by the hand of the artisan-craftsman [for repair]. But with respect to *power*, there is no doubt that artifice does conquer nature, by great force...

Sefer Elim — Ma'ayan Chatum

[The opening of this page continues the discussion of shadows, clouds, and the rainbow — apparently the conclusion of the preceding "How the artificial conquers nature" item, or a closely related optical item.]

in every position and posture that it [the object] stands, it [the shadow] is one. For the shadow is nothing but the absence of light; and the absence itself is not seen — only the surface of the earth, which is distinguished from the surface illuminated by the sun. As is well known, the earth does not move by itself.

So too is the matter with the cloud: the cloud's existence [as an object of perception] is only in the eyes of those who see it, and that is truly its substrate. As for the shadow of the rainbow that appears in the earth, it is real [כה כאמת — actual]; and all who see that rainbow see it as the very same thing. So too with a glass having six angles [a hexagonal prism] set against the sun: you will see in the ground something like a rainbow. The Torah speaks in the language of men [Berachos 31b — דברה תורה בלשון בני אדם]. Perhaps this is the intent of the Sages of the Talmud when they say [Chagigah 16a]: "Whoever gazes at the rainbow, his eyes grow dim" — and yet we are required to recite a blessing over it. How, then, should we not look at it? Rather, the meaning is as I have said: that the discerning should perceive that its nature is natural [a natural phenomenon], that the dripping cloud is like a mirror behind which... [lacuna in sense] ...the sight, and the waters within the cloud present to the eye, like sparkles of the sun, that which is to be seen.

[?] ...The vapors and exhalations are very thick, and they do what the lead [the backing] behind a mirror does, so that... This matter can be explained in various ways; but for now I choose the briefest. Let there be a triangle **אבג** [ABG], and draw a line from point **אכ** [?] [recte: **א**, A], perpendicular or parallel to line **ב** [recte: **בג**, BG]. Since angle **ב** [B] is obtuse [?], it is plain — from [Proposition] I.19 of the *Elements* — that the perpendiculars dropping from points **א, ב** [A, B] onto the perpendicular lines are shorter than the side **אג** or **אב** [?].

But if the triangle were obtuse [נטוי], like triangle **אוב** [A-U-B], then **אן** would be greater than the perpendicular [?]; that is to say, line **קש** [QSh] [?] is the sum

[קבוצם, the combined length], that is **רש** [RSh], along the side **אב** [AB] or **אוב** [?]. For if **אן** [?] were less than the sum **א-אב** [?] of the sides — and accordingly half of their sum is along side **אך** [?], and the half of the whole is at point **י** [Y] — then behold, we have bisected the drawn line of triangle **אבב** [?] at point **ך** [?], and upon it…

Diagram. A geometric construction (figure 65, ציור סה) accompanying the optical/rainbow demonstration. The text describes a triangle **אבג** [ABG], with perpendiculars dropped onto reference lines, a bisection at a point, an inscribed circle/arc, and a derived four-sided figure equal in area and perimeter to the triangle. The figure itself is on a plate and is not present on this page; the labeled points named in the prose are listed below, but their geometric roles cannot be verified against the plate.

- **אבג** (vertices): the principal triangle ABG.
- **א, ב, ג** (vertices/points): vertices of the triangle; **ב** [B] described as an obtuse angle.
- **אוב** (alternate triangle): an obliquely-set ("נטוי") triangle A-U-B.
- **ד** (point): a point at which a perpendicular/median falls; "triangle אבג … at point ד" [?].
- **ל** (point): line **אכ** is cut at point **ל** [?].
- **ט, מ** (points): lines **דל**, **במ** drawn parallel to one another [?].
- **י** (point): "the half of the whole is at point **י**," a midpoint/bisection point.
- **קש / רש** (segments): the combined-length segments along the sides [?].
- [ציור סה = figure 65; >2 labels uncertain.]

and upon it [the line], according to point **אוהך** [?], we mark a circle; and line **אכ** [AK] is cut at point **ל** [L]. According to the measure of line **דב** [DB] we take, on the perpendicular line ([figure 65 — ציור סה]), line **לט** [LT]; and we draw lines **דל, במ** [DL, BM]. Since they are parallel to one another, it will likewise now be that the perimeter of this [figure] equals the perimeter of the said triangle. This is because the lines **דל** [DL]… the equal [segments] are [each] along the whole surface, that is, perpendicular to the sides — from [Proposition] I.33 [of the *Elements*] — and one [of them] is equal, from the same [proposition], to the triangle **אב** [?].

I mean to make clear: by half the sum of sides **אב, אב** [?] — that is, both of them equal to the two said sides — and the third side **בג** [BG] is equal to side **בך** [BK] and to side **למ** [LM] together, according to the construction. Therefore our construction yields a four-sided figure equal to the triangle in area and in perimeter. This is what was intended. The matter is clear in several ways: in a

right triangle, and in the fourth or eighth or half of the circle, and in the sixth figure [תמונה הששית], and so on — but I do not wish to expand, for you will find them elsewhere as you please; and a little honey [a small taste]: you have found enough — eat your fill [cf. Mishlei 25:16].

[Item:] How a Moving Point Traces a Straight Line — and the Nature of the Geometric Point

Further, I am weary of the whole matter [נלאת כלכל]: how the point, in its motion together with the sphere, traces a straight line — which is compelled according to Proposition I.3 [?] of Theodosius's *Sphaerics* [לכרורי תיאודוזיאוס]. But this is the way [I shall proceed].

A great question: that the point, though it is small — in truth, smaller than [any measure] — yet it turns and faces, and you find it in all wisdom and understanding, whether in surfaces. Euclid investigates it in [Book] III.1, in the circle [for finding the center], and likewise in Propositions [Book] 9, 11, 12 he mentions it there to its glory and praise — who can recount it? Likewise the points of equalities and oppositions, the latitude [נכח] and the depression of the height of the spheres [the celestial circles], the eccentric center [the deferent], the head of the Dragon [the lunar node — ראש התלי], wherein the lights are divided [eclipses], and likewise the poles of the universe, the equator [האפורה], and the horizon, and other points besides them — of great usefulness in the books of astronomy.

Archimedes, in his *On Equilibriums* [the *Equiponderants*, במשקלים השוים], shows the center of gravity [מרכז הכובד], teaching its way and its uses in mechanics. And the tail of the Dragon [the descending node], the eccentric center [the out-of-center point], and likewise the poles of the universe and the horizon and other points — [all] of great use in the books of astronomy.

And the earth, long and broad, is reckoned as a point; and the great stars [are reckoned as points] — only the lines coming from their centers are [reckoned], as though they were the diameters [הנרסיות / [?]], and there is no measure to them. For [the point] — if it be set against the Great One [God], greater than all greatness — it is smaller than all smallness; yet to it belong the measures and the dimensions, from which all the lines issue, as from the center to the circumference of the circle: many, without great labor, and an allusion to the Most High in what it is — it is One, and is not divided, and is measured; and even though it is a beginning…

the end is the limit [קץ] of them all; and they suck from it, while it itself does not end — it abides, fixed and at rest, while they revolve round about it in a circle. They are others [distinct from it]; and the more they distance themselves from it, separating and dividing, the more they move in vain and run to no purpose; and to it they return, to the rest and the repose. So too the multitude of created beings: from the One, the Singular, comes their beginning, and to Him their end. And just as it [the point] suffers no surplus when the many extremities of the lines come to it, and no deficiency when it sends out thousands of compounded sparks — so is God, blessed be He, in the outpouring of His goodness and the spirit of His generosity: He is not diminished in His emanation, nor does He receive benefit from another. Everyone called by His name [ascribes greatness to Him].

And it [the point] is a king within, hidden and concealed by reason of its great smallness — so much so that the definers labored over it. Euclid defined it after a manner of deficiency [negation], not bringing the genus to say "it is an indivisible magnitude," which would have guarded against refutation; rather he defined it as is proper, only in the manner of the definitions of God, blessed and exalted be He — by way of negation. Others have given definitions:

- [1 ,**א**] **And it is** very modest [צנועה, retiring], for it has no part [no portion to it], and out of regard for its honor he [Euclid] did not bring the genus to say that it is an indivisible magnitude — which would have been guarded from refutation — and he did not define it properly, but rather after the manner of the definitions of God, blessed and exalted be He, by way of negation. And others have given definitions: [some say] that in its running it traces the line; others, that in it the parts of the line cohere, or that by it the extremities of the line are bounded. And **Pythagoras** [says] that it is a unity having position [אחדות שיש לו מצב — *monas thesin echousa*], as he said: an indivisible magnitude — that is, the beginning of magnitude. And **Lucretius** the poet [לוקריציאוס המשורר] defined it as the tip of a thorn or a needle, or something resembling it, which is imperceptible to the sense of sight, lacking parts — the smallest of the small, which neither was nor will be. These [definitions] contradict one another, so that many honored men have stumbled over its real existence.

- [1 ,**א**, continued] **And behold,** there appear to one who contemplates the point so many strange and opposite qualities. [**א**] It is no measure at all, since it has no part to anything; and if we imagine it as passing from the beginning of the line to its end, no part will be found in it equal to it; nor does it measure even itself. It resembles the *instant* [רגע], which does not

measure time, being no temporal part — for time is not composed of instants.

- [2 ,**ב**] **B.** It does not add to its own magnitude, nor diminish — for it is like flint-rock [צור החלמיש], which does not soften in its nature: if you add to it a thousand points, it is still but one; and you cannot subtract anything from it, for it has no parts, and resembles the unit among numbers, which has no fractions. To the indivisible substance [העצם הפרדי] there belongs extension and quantity — which is not so for the point.

- [3 ,**ג**] **G.** Since no point exceeds another point, no point is equal to another point [?] without equality — for these qualities [equal/unequal] are proper to magnitudes, and it has no magnitude or quantity at all; rather it is called "like" [דומה] to another point.

- [4 ,**ד**] **D.** It is perfect [שלמה], having no deficiency by its nature, and is one [unity] more than others. For a body is divided into its three dimensions, and a surface — more than the body — is divided along its length and breadth, and a line — more than the surface — is divided along its length; but the point [admits] no division at all, neither in actuality nor in potentiality. It is called "without potency" [ב״כת — i.e. בלתי כח] because it has no extremities, boundaries, or corners. And withal, it bounds the lines and gives them their limit and end; it too is no magnitude, yet to all magnitudes it is a cause, far and near. For by the likeness of its motion it generates the lines, and by the motion of the lines are generated the surfaces, and from these, by its [continued] motion, come the bodies. It resembles prime matter [היולי, *hyle*], which is no body in actuality and yet generates the bodies, and dwells in the heights.

- [5 ,**ה**] **H.** Even though it bounds the lines and separates part from part, it is nonetheless not distinguished from its fellow except by the difference of position within it and the time of its existence. In this it resembles the natural forms, which distinguish individuals and are themselves distinguished only by their substrates [the matter they inform].

- [6 ,**ו**] **V.** It has no extension at all, and yet from all its surroundings [comes] extension: whether in a surface — four right angles — or [angles] equal to them; or in a body — eight solid right angles, which fill the place. And the extremities of all these existential lines [הקוים ההוויתיים, the lines of being] terminate in a point. It is known that an angle — whether linear, or planar (in surfaces), or solid (in bodies) — and it is what teaches man knowledge

[cf. Tehillim 94:10], and the structure of the propositions [ורבן של למודים], just as motion is the soul of natural things.

- [7 ,**ז**] **Z.** It is what joins the straight with the curved, as Euclid explained in [Book] III.16 [?], in a manner such that…

a straight line cannot enter at the point of tangency, nor can a stranger [another line] pass between them. And yet, between the tangents [the touching lines and circle] there pass several larger circles, without [their] separating [them] — and that comprehending [containing] is a great wonder. And it is right to ask of those who say that it [the point] is throughout the whole world [coextensive with everything]: where, then, is the place of its repose? For when we contemplate the modes of the thing's existence, in something Aristotle mentioned in the *Physica*, we do not know how the point is found in the line — whether it is in it like a man in a species, or a species in a genus, or the reverse (as the higher [universals] are in species or individuals); or like a part in a whole.

But it appears that the point is not carried upon the line, nor the line upon the point — not as the thing located is in a place, nor as the encompassed is in its encompasser, nor as the accident is in its subject, nor as the part in the whole or the whole [in the part], nor as in species and genera. Nor is it in the line as in a place — for "place" is defined as the surface [enclosing a body], whereas the line is not a surface. Nor is the line a cause of the point, nor the reverse. Nor is it in the line as an accident in a subject — for the subject is prior to the accident, which is defined by means of its subject; whereas here it is the reverse: with Euclid, the line is defined by means of the point. He said there that the parts of the line are joined by means of a point, and that its two extremities are two points. And one cannot say either that the point is in the line as a part in a whole or a whole in a part — for the point is no part of a line.

Aristotle and his commentators brought many demonstrations to show that the line is not composed of points, as [a whole] is composed of many parts. They cannot impart what they do not have in actuality — namely, the cohesion [התדבקות] which they [the points] were defined as having, [to be] divisibles that complete the whole. And the law is with them [their argument is correct]: since the points, owing to their smallness, are not divisible, even if they were joined to parts — there being no natural affection [no physical interaction] in their being gathered, by which they would add from themselves what they do not possess in actuality — therefore they cannot acquire more than what they have, and they are indivisible.

[**It follows**] that if the line were composed of them, then the side [of a square] that lies opposite the diagonal [המכוון שכנגד הקוטר] would be perpendicular [נצבת], and the one opposite the side [would be] sharp [acute] — but in Proposition X.116 [וכלמוד 116 לעשירי] it is made clear that it [the side] is not equal to the diagonal — in the square itself — and it has already been made clear in I.47 that [the diagonal's square] is greater than it. For perpendiculars drawn parallel [to the side] would pass over all the points on the diagonal so as to fill the whole area of the square — and they would not be sufficient for it. And the consequence is plain: that if we draw lines from every point and point of the side to the side, then there will be more points in the diagonal than in any one of the sides.

It also follows that the outer angle would equal the inner — for no point is placed on the outside that does not have another corresponding to it on the inside, when they reach the center — as was made clear in Grade 3 [מדרגה ג'] of *Sefer Gevurot Hashem* [*The Mighty Deeds of God*, Delmedigo's own work]. It also follows that there would be a straight line not divisible into halves — contrary to Proposition X.[?] of the First [recte: of the Tenth] — namely when the line is composed of a separate [odd] number of points. And it also follows that there would be, within a triangle, a straight line of [parallel] lines, perpendicular to its base, greater than its base.

Therefore one ought not to think that the line is composed of points; nor is it possible to place in it even two or three adjacent points. All these [conclusions] are demonstrated falsehoods and absurdities, and they are explained at length in the books of Aristotle and his commentators — that, were there three points, the middle one would necessarily have differing parts, for with one part it would touch the first and with another the third; and were it not so, it would not separate the two extremes. This is contrary to the nature of a point, which has no parts. Therefore one must conclude that even if thousands of points were set together, they would be but one point, and no part of the line — nor any other magnitude — would be created from them. For this reason it is impossible to mark on a line two points adjacent to one another, since there is always a part between them. Not so with *natural* points [הנקדות הטבעיות], where it is possible to place two adjacent to one another.

Now, the philosophers give various answers to the foregoing arguments [against the line being composed of points]:

- Some say that the point is in the line as the parts of a definition are in the thing defined, and its parts are its extremities [קצותם]; some say it in another way. The clearest to me [המחוור אצלי] is this:

- That the point is in the line as a cause is in its caused, the point being reckoned like prime matter [ההיולי] together with the line, which is reckoned [as generated]; for the line is reckoned as generated from the running of the point.
- Or that it is like the form of the line, in which its parts cohere and [by which] it is distinguished from another line…

what is composed of points.

Nevertheless, the opponents press forceful objections to deny the point's existence. For if it exists, it is either substance or accident; and if substance, then either corporeal or spiritual. It cannot be said to be corporeal, since the accident that must precede [all others] in entering prime matter is *quantity*, whereas the point is not a quantity (being indivisible). And if the point were a body, it would be bounded by quantity, as the definition of "quantity" requires. Nor can one say that it is non-composite but is rather one of the constitutive principles of corporeal substance, since it has already been ruled out as matter. Neither can one say that it is *form* — for form is what gives the corporeal subject its subsistence and distinguishes it from other kinds — which is something the point does not do, since the point is not predicated of a subject and does not bring into being any composite by its existence. Indeed there is no warrant for saying that it is found more in one composite than in another. And if it [were form, present] in all subjects, they would all be alike, with no difference between their forms — which is absurd.

If [the point] were a spiritual substance, it would on that account be separate, self-subsistent, and capable of intellection — and that is laughable. And if we say it is an accident, we do not know to which of the nine [accidental] categories it would belong. For quantity is that whose parts lie outside one another, [383] whereas a point has no parts. Nor can one say that its parts are points — for the part must measure the whole, yet the point itself is not divisible, measurable, or numerable; whereas all these [points] would have to be divisible and measurable, or numerable, [which is impossible]. Therefore it cannot be counted; and so the points are not "parts" of [the line]. Likewise it has been argued that a line is generated *from* a point — but if so, [the points would have to be numerable, against what was said]. All these [arguments] held the point too remote from being conceived, so much so that they [concluded] we cannot even know its place.

Yet we say that it is an accident of the fourth species of quality — namely, the species of figures and shapes that surround bodies and bound them. And just as, in accordance with the [bounding] lines, the surfaces composed of them are

named — whether circular, straight, or compound — so the line is bounded by the points at its two ends; and it is continuous and connected by means of the points at every juncture between one segment and another of its segments. Therefore the points are the form and bond of the line and give it the unity it has, whether the line be natural or artificial. One cannot deny that the points bound it. And these points reside in a corporeal subject, inasmuch as the line in which they lie is in a surface, and the surface is in a body. So too in pointed instruments, and at the meeting of an angle, the figure of the point is seen; and these [surfaces and bodies] are its subjects. There is no requirement, in things bounded that are exceedingly small like the point, that the subject be a body exactly equal to it [in dimension], as is the case with the qualities of the fourth species — for example, the surface of the wall is the subject of the color upon it, and the substantial body is the subject of the accidental quantitative body, both of these being complete accidents. As for the point, you should understand that the tip of a needle or of a hair is the corporeal subject of the point; for although by nature, being corporeal, it has parts, nevertheless I have not divided it actually because of its fineness. And so that the subject should equal the object, conceive of it intellectually as ever finer and finer — since the conditions that the mathematicians seek in their magnitudes cannot be realized in matter. For Aristotle himself, in *De Anima* I.184 [Book I], when he compared the soul to the mathematical [magnitudes] [?], said there, of blessed memory: "Therefore, if the soul has some operation or affection that is particular and proper to it — that is, not shared by it together with the body — it would be possible for it to be separated from [the body]; but if not, it cannot be separated, but is rather like the many things that befall the straight insofar as it is straight." For instance: that [the straight line] touches a bronze sphere at a [single] point. Yet in truth it is not the *separate* straight [line] — which is inseparable, its nature being to exist always with some body — that touches it. That is to say: it is the nature of the *mathematical* straight line not to touch the mathematical sphere except at a point; for if at two points, it would cut the sphere, as is explained in Theodosius, [Proposition] 3 of Book I [?]; and so it does not touch, as was posited. But as for actual bronze spheres and the like, you will find none free of curvature, nor any sphere free of [some] straightness; and were they to touch one another at more than one mathematical point, the craftsmen are not so precise with them [as to prevent it] — even to the point that they fashion a sphere standing on an iron wire, balanced so evenly that it inclines neither right nor left; yet if you set on one of its sides even a quarter of a grain, it will tip it to one side. And on a flat surface they make it so that the sphere [384] rolls without halting. All this shows that the sphere touches only at a [single] point upon one surface.

However, if you strip [points, lines, and surfaces] of matter, they have no existence outside the intellect; nor do they touch the subjects [?]. That is, we cannot say a point is in the category of quantity by way of the body of a tree, for instance, [in the way] the essential constituents are called substances. So too the foundations of substance — namely matter and form — are within the category of substance; for the logicians say that the individuals, species, and genera, and likewise the differentiae and the species [are subsumed in their genus]; otherwise, in which genus [would they fall]? And so we say the point is the beginning of the line, and is therefore with it [in the same category]; if not, [the point would be excluded], whereas it is in fact the beginning of the numbers.

They further object on the side of *quantity*: just as the unit is not a quantity or a number except in being the principle of number, [so is the point]. [Reply:] For if the point is another [thing] that has *position* — unlike the arithmetical unit — how can it have position if it has no fixed place? For "position" is nothing but the ordering of the parts in the whole or in place, and the point has no parts. **And we reply** that it has position, like the line — as the Philosopher [Aristotle] showed: a magnitude that has no [parts may yet have] position; that is, [at] the ends of the line or between one segment and another. There is no requirement that the part of the line on which it leans, or which is its position, be a part [proper]; and if it is not granted in this way [?], we will [still] say it has position.

They object further that Aristotle, in *Physics* V, ruled that there is no existence for the indivisibles, [and therefore no position in a surface, nor for a surface in a body — these too not being parts of them; hence the point would have] position. **But we reply** that this is not so. The Philosopher there is only arguing against those who held that the poles of the heavens are corporeal bodies possessing power and force, upon which the spheres and all their host turn, like the door upon its hinges — while [these thinkers] held them to be indivisible points. He therefore said that there is no indivisible corporeal substance. But it is not by this that the Philosopher denies the existence of the poles, which must necessarily be fixed together with the center, if [the heavens] are to revolve. Yet the Philosopher does deny many points that the astronomers [posit] — such as that the poles of the sphere and of the sun lie on the sphere of the moon, just as the centers of [their epicycles? — text uncertain] are not at the Philosopher's center; for he holds that there is a path more natural than positing them — I mean, that the centers of the eccentrics of the earth lie in the world of the elements, according to the hypotheses of Ptolemy. And the like of these the Philosopher rejects.

It may further be objected that [the point] has no existence, since it is either prior to the body or posterior to it. It cannot be prior, since corporeality — as we said — is what precedes all the accidents in entering prime matter; and it cannot be posterior, since [the point] is the beginning of the body's beginnings: I mean, [the point is the beginning] of the line, the line of the surface, and the surface of the body. And if so, [the point] is not in them, [and the same objection arises for lines and surfaces]. The resolution of this is clear: on this reasoning you would have to posit neither lines nor surfaces in the world either. But in truth, all these do precede the *mathematical* body, of which they are the beginnings; yet they come after the *essential* body, which is their subject. That is, their quiddity is prior to the body, but their existence is posterior; and there is nothing objectionable in one and the same thing being both first and last, in different respects.

They press still further: [the point's] very being is as the Philosopher showed in *Physics* II — that the composite is understood as it is in its [actual] existence. But the point is not conceived except by way of *negation*: that is, we think of it as a place where the line no longer continues. It would seem, then, that it has no existence at all; for if it had any [existence] besides the negation, suppose it to be cut off [394] from a portion of the line by divine power: in that case one would have to say the line remains unbounded — but [in fact] long as before, since the point is not a part of the line, and therefore neither diminishes nor adds to it. But one cannot say there is a line with no boundary at its end, for that contradicts the definition of "line"; therefore [the line] is bounded even when the point is silenced from it [removed], and so it is bounded of itself, without the point. Thus the point had no existence except [as] the negation of continuation, since we saw the line bounded by a certain length and no more, and we called that interruption a "point." By their reasoning, the lines too, and the surfaces — for the line that appears between shadow and light is nothing, being merely the negation of light or of shadow; and likewise [384/recte 383?]...

therefore they have some existence apart from the negation, even though the point is not separated from the line, nor the line from the surface. And if we liken [the point] to one taken from the end of the line, behold, in every point there are contained infinitely many points; and the line will always remain bounded by one of them at its end, whose nature it is to be bounded, and nature will not tolerate the absence of [a bound]. The point is not a part of [the line] such that, were it taken away, [the line] would be diminished by its removal; therefore [the line] is not diminished by [the point's] being its bound. As for what they showed by way of negation, that is on account of [the point's] smallness and its concealment from our knowledge — which is also why God, blessed be He, knows

it thus, and [why] the separate intellects and the souls of men [are described by negations], as Parmenides rightly thought (according to Plato), when they are said to be incorporeal, impassible, non-sensing, non-moving, and the like — all negations. Even so, beneath these [negations] we conceive of [the soul] a certain positive [reality] that the mouth cannot utter.

Some of the keen-witted object that, if the point connects the parts of the line — when the line is divided, where is the connecting point? It cannot be said that half of it is at the end of one segment and half at the other, since it has no parts. And how can one and the same [point] be in two different, separated subjects? Nor is there any sense in saying it is in one [segment] more than in the other. Therefore [the point] has perished. Likewise, if two lines are joined, one of the points perishes, since one [point] suffices to join them — [two together] are impossible — for [a point] is destroyed only by the destruction of [its] subject, since the points are the form of the line and are inseparable from it, like the blackness of the Ethiopian; and the line is not destroyed by the joining and separating of its parts alone, for [these parts] suffer no destruction; they are merely local displacement. There is no destruction-and-generation without passion [affection], as was explained in the [books on] natural science.

And we reply that all this is true of complete accidents, which do not come to be and perish without [a] prior sensible passion of the subject. But in cases like these [of the point], a [mere] displacement suffices — for example, by joining, separating, meeting, and touching, surfaces and lines and points come into being; for when the matter is in the potter's hand, it takes on various shapes, and he makes from the spheres [bodies] with angles and concavities and convexities, and likewise he cuts the surfaces and lines, and traces lines with his fingernail. As we said, these are not, in primary essence, in the corporeal subject; rather the line is in the surface, the surface in the body, and the body in [its] essence. And so there is no necessity that the subject perish when they perish. Indeed, the eye sees thousands upon thousands of lines coming from the circumference to the center, each with different points at their ends; and when they reach the center, they become one point, and nothing further is added to it. So too, in the joining of the two segments of a line, one need not be astonished that the two [points] became one; and so likewise, when the segments were separated, the one connecting point was in potency infinitely many points — therefore one of them bounds the end of the one [segment] and another the end of the divided [part] of the line.

The precision-minded among the geometers raise a further objection: if so, it must follow that all the points found in a line — which are infinite, since they

connect infinitely many parts (which are infinite in potency) — will stand as countable, and the line will be composed of points, contrary to what was explained above in the name of Aristotle. And the demonstration of this [objection] is as follows: as Aristotle wrote concerning a moving thing, everything in it moves; therefore the point in [the moving line] also moves. But every moving thing first traverses a path or length equal to itself, and afterward [a length] greater than itself — as Aristotle wrote in *Physics* VI; therefore the point, in its motion, will first traverse a path equal to the point, then two points, and so on; and thus it would be counting out the whole line, and the line would [be made] of points. **To this we reply** that in the motion of the subject — that is, of the line or the surface — the point in them does not measure the path in any way, nor [does it measure] the points across the distance; for the point has no place in the subject, since every place is a surface, and it is impossible for there to be a surface that is indivisible, perfectly even, and exactly matched to the point. And since [the point] has no place, it has no motion, for it is impossible for it to move. And even if you say that it necessarily moves *per accidens*, in the motion of its bearer, its motion is not like the motion of the color and the surface, which in their accidental motion do measure the path by their being transported along with the subject in which [they lie. For they, being parts of the moving thing], can [394] be measured by the place of the subject, since they are coextensive [with it] or as if they moved of themselves. But the point cannot be a measure within the place of the subject, since it is not a part of it; and of its subject one may say *erkecha erka be'i* — "your valuation is the valuation that he requires" [Aramaic idiom: the bearer's measure is what is sought], for [the subject] is indivisible like [the point] itself. The measure, in any case, must either be a part of the thing measured or fall under another category along with the thing measured — that is, both must be possessed of quantity; whereas this is not so with the point relative to its subject or its path.

So I have shattered for you, like a hammer [striking] rock, [the position] of those who deny the existence of the point. It has become clear to you that the mathematical body, in moving, does not trace any line — even though it has no separate existence, contrary to Aristotle's words in *De Sefer HaNefesh* [De Anima] — but [traces it] qua corporeal, and its motion is [that of] a natural point; for [the body] is also divisible and bounded by some of [these] points. Nor can one say, "Be it so, [made] of points"; for the body is seen to be divided into surfaces, and bounded by them; and the surfaces [are bounded] by lines. Therefore [the body] is necessarily [bounded by lines] and traces a natural line. And one must therefore posit the point as having existence, since this is the nature of existing things, [as you have seen?]. And we proceed to [the position] that there are

intervals in the corporeal: three intervals and three divisions according to its length, breadth, and depth; and for the surface, two; and for the line, one; and here the intervals and divisions cease. Nor can one say, "What prevents a line from being a part of a line, and being bounded and joined by [another] line, so that points are not needed?" — for on this [reasoning], in surfaces too the boundaries would be a part of the surface, and lines would not be needed; and likewise in bodies. Therefore one must posit surfaces and lines apart from bodies; and so too [a point] in what is actually joined — for otherwise it is not joined. Therefore one must say that [the point] is some indivisible magnitude — and that is the point. And [the line] has nothing besides the point by which the line is bounded and joined — for [otherwise], either [the line] is [self-]joined, and [the joiner] would be a line, and one would ask of it and of its boundaries what was asked of the line; or it is separate — I mean a number — and this is not [the case]; for the line is continuous, [and the joiner] is the point.

Now that the nature of the formal, internal point in the line has been well clarified to you, by way of disputation — that it lacks parts, place, and motion, having nothing but position in the line, by which the parts are joined and the ends bounded (from which you understand that they are infinitely many points in the line, though there are but two at its ends), and that it is deep, no eye having ever fathomed where it is in truth — I shall [now] inform you that, besides it, the mathematicians and philosophers have conceived *another* point, in thought and imagination, and have placed it outside the line, as though it were standing, subsisting in itself, [occupying] a tiny place apportioned to it according to its smallness, so that it may move — for thus they wish to use it for their own purposes. This point is not a falsehood, as Aristotle wrote in *Physics* II: those who abstract the matters [objects of mathematics] from matter do not thereby lie, [for] they have multiplied them in number; and the Pythagoreans, the Platonists, the Peripatetics, and the mathematicians have made use of it. It is the quiddity of the line, which is created from its running [motion] — not [a thing] standing and subsisting in [the line], for it is impossible for a moving thing to be in two places [at once]. Therefore [this point] neither joins nor bounds the line, not even from one end — for then the same [point] would have many positions, since its nature is to run a course. Rather, it traces [the line] by its motion, like [a stylus of] wood drawn over wax, which leaves a groove in it; therefore it moves without ceasing. When we imagine it moving with a simple motion, it traces the simple line; and if it moves from the one end of an interval to its other end, in such a way that the trace it traces equals the interval, it traces a straight line — which is the shortest [line] between two points, as [Euclid] wrote; and if we imagine it going without ceasing, it traces the infinite line, which it needs frequently in [the study], as

Euclid asked in his second postulate. And so, by an imagining that has no [counterpart] in actuality, [we obtain] an infinite magnitude, resembling the operation of the [logical] syllogism, in which the conclusion of the first premises becomes a premise for a second conclusion, and that for a third, and so on infinitely — until we depart so far from the first intelligibles that the propositions and conclusions become strange and astonishing to hear, in appearance, [yet sound] for the mathematicians, in geometry, astronomy, optics, and mechanics; and when we turn back [to the foundations], the light of these matters and their truth is revealed. Therefore it is said among the philosophers that [the line]…

is impossible to reach in a finite span. And when we imagine this point moving in a circle around another point — that is to say, around the first one — at a distance that is always equal, by the time it returns to the place it set out from, it will have traced out the circle. Likewise, it would seem, the motion of the center of the sun, carried by the prime mover, traces the circle of the equator and the circles of the [solstitial] reversals; and from the contact of its [the sun's] sphere upon a point on the encompassing [sphere], the ecliptic line is traced upon its sphere, aligned with the ecliptic of the firmament [גלגל ה' — the fifth sphere?]. From the motion of points, many and various circles are born in the science of astronomy — such as the motion of the poles of the ecliptic — and likewise the several kinds of lines from which every figure and form is built.

Aristotle, in *On the Heavens* [*De Caelo*], investigates the number of the elements according to the variety of motions: a point's motion. For if it is from the center toward the encompassing surface, it is sent forth like fire; if proportioned to it, like air; if from the encompassing surface toward the center, it is sent forth like earth; if proportioned to it, like water. These are rectilinear motions, and they trace straight lines. And if [the motion is] around the center, [it belongs to] the fifth body.

We find, moreover, that Archimedes demonstrated something astonishing to anyone who hears it, in his fourth problem of his *Mechanics*, where he treats how two points at the two opposite corners of a square will move through unequal distances at equal speed — examine his words and you will find delight. So too Archimedes, in his treatise on dividing an angle into three equal parts, laid down this premise: that if a point traverses the length of any line at a uniform speed, the segments of the path א.–ב.–ג.–ד.–ה [traveled] are proportional to the segments of time. That is to say: if the point traverses from the end א to ב in time דה, and from end ב to ג in time הו, then the ratio of the path אב to the path בג is as the ratio of the time דה to the time הו. And this problem of dividing the angle has

many facets, as I have shown you in [Sefer] *Tamar* [chapter] 12. Therefore we are able to picture points, lines, and surfaces in motion, and to draw from them wondrous benefits, and to arrive at the goal of wisdom.

And before I conclude my discourse on the point, I will bring to you here something marvelous, relevant to our subject — fitting and fair to that excellent man Nicolaus Copernicus, who, as it seems, devised it in the fourth [book], chapter three [recte: III.4], of his *Revolutions* [*De Revolutionibus*], where he explained the variation of the angle of the great inclination [obliquity]. He showed by demonstration how, out of two circular motions, a straight motion arises — that is, that a point set somewhere on a straight line, when [the line] is moved in a circle, traces a straight line. I will give you in brief an explanation of his words.

Diagram. In the figure before you (figure סן [= "60"?, the figure label]) there is a straight line אד, divided into four equal parts at points ג [and ב] [?]. About point ב, the center, are traced two circles, one within the other, on the single surface אגירק אפר / [?]. On the inner circumference גינק [?] take a point as a center, wherever you wish, and trace about it, at the distance סד, a circle [סד? אפר], which cuts the straight line אד at point ק [?], and let the diagonal דפ be drawn. Now, if we suppose that the center were at point ב, then the point — the end of the diagonal סך [?] — falls at point א of the straight line אד; and when we imagine that the center moves from ג [?] to point ם, the end of the diagonal סך falls upon point רן [recte: a point of] line אב; and if it moves to point ץ [?], it falls upon point ב; and if the center comes to point י, the end ך falls upon ד; and so [the end] ך falls upon every one of the points of the line אד, tracing it out. [The point letters in this construction are partly abraded — several readings are uncertain; see QA.]

- א (right end of line): one endpoint of the divided straight line אד
- ב (center of the two concentric circles): the fixed center
- ג, [and intermediate points] (along line אד): division points of the line into four equal parts
- ד (left end of line): the other endpoint
- י ,ץ ,ם (positions on the inner circle): successive positions of the moving small-circle center
- ח / ך [?] (end of the moving diagonal/radius): the tracing point that runs along אד
- סד / אפר (circles): the larger and inner circles of the construction

You will trace it out when you reckon that, just as the inner circle [moves] toward the left, so in the same time the circle אפר [?] moves twice [as fast] toward the right — or the reverse. And if you cannot picture the matter in your imagination, draw it on paper: the large circle אפר, and three times over [ג"פ — three times] trace upon separate sheets the two small equal [circles] פחך / גי [?] as they intersect one another; cut away whatever is left over of the paper and throw it out, and fix [a pivot] from the center ן to the center ך [?] of circle אפר so that it can roll. Likewise trace on paper the circle פד [?] and fix it upon the center כ [?] so that it can roll; and begin to roll the middle one toward the left and the upper one toward the right at double the speed of the lower one's motion. You will then see that the end of the diagonal *מת* [?] — that is, point ך — always lies upon line אד, at times…

higher, at times lower. It will not depart from it. So you have proof that the point rolling on the traced circle traces a straight line — which is what [Copernicus] meant. And if you wish to see the matter [demonstrated] by a model, look in the aforementioned book of my teacher [Copernicus], for I have chosen to bring you its explanation from his own wording, which is hard to grasp, as is his manner even in easy matters.

As for what you said — that more than two equal straight lines could be drawn from a single point upon the other straight line — that has been explained to be impossible, unless you introduce motion, by supposing that the line revolves around. But indeed, if it merely changes its position, it is not that same line itself; and the matter returns to its truth.

Now as for your wonder at how, from a single point set on a straight line, more than one perpendicular could be erected opposite it: I answer you — Euclid said no such thing. He did not say that to the four sides [directions] one could erect four perpendiculars, all of them upon his [the line's] surface, plus two more outside the surface, toward the six directions of the world. Rather, [he spoke] of the surface, [saying] that from it no more than one perpendicular can be erected upon any point, toward that same side. As for a point on a line, there will be perpendiculars *upon it* by the hundred [i.e. infinitely many] in three dimensions, when you reckon the line as shared among many surfaces — but no more than two upon a single surface. From a point set *outside* a line or a surface, however, no more than one perpendicular will fall upon it, and that is the shortest of those coming from it to the given line or surface. And likewise to a single line there will be parallel lines, equally distant, by the hundred in its rotation, but no more than two upon one surface.

Also as for what you said — that two straight lines could be set equal to the sides of the triangle inside it (which is opposite to my reply to the first, concerning Euclid's eighth [proposition]) — the answer is plain: he did not say so, except on the condition that each one be equal to its adjacent neighbor; but that the sum of the inner [lines] equal the outer, Euclid does not deny it. The Christian commentators have explained well [Book] 11 [recte: of] the *Elements*. I will give it [to you] in full from the moderns, who abbreviate and do not quench your thirst, and therefore content yourself with what is clear in the *Simlah* [garment, i.e. plain version].

And you are right to wonder how we pass a curved line over three points standing on a straight line; for the [classical] teaching to pass the leg of the compass over three points is taught in [Book] 25 [recte: in the third (book)] of the *Conics* [קיסורות = Conica], where it teaches to complete the cutting of the circle with the compass. The later [authors] will pass a curve over three points even though they are on a straight line, as it seems, by means of a special instrument for this. For this matter is much in use in the drawing of astronomical instruments, such as the astrolabe, the clock-instruments [horological devices], and the like. And [it is] also [the case] that when the center is very far from the ends of the arc, the arc — which is a small part of the circle — is reckoned as a straight line, and the three points on the chord are reckoned as though they were on the arc itself. Likewise, [it is also the case] that the lines that appear straight, and the surfaces, upon the globe of the earth are [really] curved — for they are equidistant from its center, since the [earth's] elevations are not exactly [even] — yet they appear straight, not curved. All this is understood according to common reckoning, not by mathematical precision.

[such elevations] are reckoned as nothing in relation to half its [the earth's] diameter, which is 3579 miles, as we explained in *Sefer Gevurot Hashem*. And according to what is agreed, the sensible horizon — that is, the measure of the [span of] earth a man [can see] in length — is no more than 60 miles, which is about one degree in the heavens. Now, the inverse cosecant [הבקע הפוך, "the inverted cosecant"] of half a degree is the measure of the height of our earthly structures between the eye of the beholder and the edge of the horizon, [given that] half its diameter is [reckoned in such miles]. And it is a cosecant of 381. If you reason by proportion and say: the whole cosecant 1000000 equals 3579 miles; how many miles are 381? — you will find 1 + ⅓ [recte: 1⅓ ≈ 1.37] miles [?], and that is the height of our earthly structures. Therefore, unless [he stands] in a high place, the observer will not see so far — that is, 60 miles — even though his eye is "as the antelope [?] and the gazelle" [an allusion]; for between him and the sensed

[horizon] there is a height of 1⅓ miles. And nonetheless, what is seen from a high place appears straight, not curved.

And as for your difficulty about the oval [elliptical] figure: I do not know whether your intention was to say that it might be possible to trace, between two parallel straight lines, an oval, a circle, and a square equal in their diameters [or diagonals]…

even though [the rod of] thickness should pass through all their holes; and the oval figure too is traced with a thread that has a quill at its end, moving upon two fixed nails — and in many other ways besides these, with the compass.

But indeed, if you wish to make a temple [hollow form] [encompassing] a circle [with] two oval figures, and not to waste any wood at all: divide the diameter of the circle into four equal parts and trace upon its center a circle whose diameter is half [of it]. Divide the large diagonal and turn it [out] from the large [circle], and divide each part into four equal parts, so that two [parts] of the large and two of the small together make an oval figure when you join them — and it is fair and comely. Examine it and you will find it so.

*

You have asked to know how the center of a body could be outside the body, and how a single body could have three different centers. This [first question] depends on the equivocation of the term [מרכז, "center"], once it is divided — for it is of three kinds: the center of magnitude, the center of gravity [weight], and the center of lightness.

Now the center of magnitude, and its definition, is: a point whose distance is equal from all the extremities of the body. This is found essentially and primarily in the spherical body, and is predicated also of bodies of [other] equal surfaces, insofar as they are inscribed within or about the sphere — such as point ב in the figure before you.

Diagram. The figure (figure *סן* [?]) shows a body (sphere or inscribed equal-surfaced solid) with point ב at its center of magnitude.

- ב (center of the figure): the center of magnitude — equidistant from all extremities of the body

And the center of gravity — its definition, as Pappus gives it in his *Collection* [ילקוט הלמודים, "Anthology of Mathematics"] in the name of Heron [אירונים = Heron of Alexandria], is: a point within the body such that, if we imagine the

heavy body suspended from it and brought thus suspended, it will be at rest, not moving, and will not turn, but will stay in one position. Others defined it [as the point] such that, when the heavy body is suspended by it, its parts will be like the balanced pans of a scale, so that if we imagine a surface passing through that center, it divides [the body] into two halves of equal weight. And others said that, when the body is suspended from it, the line passing from its center will be a perpendicular standing upon the horizon [נוט', perpendicular] and descending toward the center of the world [universe].

The complete definition, comprising all the properties of the center of gravity of any magnitude, is: a point, within or outside the body, such that if the said magnitude be divided through the center by a point, a line, or a surface, it is divided into two halves of equal weight. By "magnitude" I mean to include the lines, the surfaces, and the bodies, in each of which a center of gravity is found. And I do not mean here the mathematical magnitudes, but the physical ones — for example, ropes [in place of lines]..., lances or reeds in place of the lines, and boards in place of the surfaces. For if you say, "But surely lines and surfaces have no weight!" — I answer you that mathematical bodies too have no weight. And by saying "a point within or outside the body," [I mean] that in some bodies the center of gravity is outside the body. The proof is this: that according to the way I taught for finding the center of gravity of any body — namely, that the center is on the perpendicular line to the horizon — the point at which two perpendicular lines intersect will be the center of gravity of the body.

Diagram. A body suspended successively from two points, with a plumb-line determining a perpendicular in each case; the intersection of the two perpendiculars marks the center of gravity.

- א (suspension point 1): the body is hung from point א; a thread with a lead bob, dyed [coated] with red lead [שׁשׁר], is let down, tracing line אב
- ב (lower mark of plumb-line 1): the perpendicular trace from א
- ח [?] (suspension point 2): the body is hung again from point ח [?], and the perpendicular thread to the horizon traces line חג [?]
- ג (intersection): where the two lines [אב and חג] cross — this is the center of gravity

For example: hang the body from point א, and from there let down a thread with a lead bob dyed with red lead, so that line אב is traced upon it. Then suppose we hang it again from point ח [?], and the thread, perpendicular to the horizon, traces line חג [?]; and the intersection of the two lines is at point ג — and that is the center of gravity. Therefore, when the body is concave or hollow, as is the

present one, it is no wonder that the center falls outside the body. And there is nothing in this contrary to the universal premise, the second [drawn] from the first, of Archimedes' [book] *On Equal Weights* — for one who understands his words.

You should know that the center of gravity is of two kinds, either natural or necessary [forced], and they are not distinct in actuality but only in mental conception. This is so [in the following sense]: if we imagine a forcing [agent] moving the present body from point ד to ה, against its [natural] downward motion from point א to [ב] — even though point ד [?] is one and the same in both motions, nevertheless, according to the differing inclinations, the center is called "changing." And the compound of these two motions is the spiral [hurled] motion: when the heavy ball, as the force of its propulsion weakens, no longer continues to run straight, but bends its path, and in its descent traces a curve as though it were thrust downward — yet [it is] the weight that overcomes, driving it to descend; and it moves in a single instant with two motions.

And as for the center of lightness, it is...

a point from which the light [bodies] rise upward in a straight perpendicular line. Many of the [astronomical] sightings and demonstrations are built upon these definitions; I will expand upon them another time, with God's help.

A Difficulty [לא תמה — "Do not wonder"]: how, when we add weight upon a weight, it becomes light to lift — and this is the question of Aristotle, the fourth in his *Mechanics*; and it is the highest [heaviest] question. I have already noted in *Sefer Gevurot Hashem* that the long round timber that fools [ignorant lifters] place beneath the great stones or the beams, levering them up to raise them, is called a "lever" [בריח], and is named in the vernacular [Italian] *lieva* [leva], and in Greek [Yiwani] *mochlos* [μοχλός, mov-los]; and the wood they set [as a fulcrum], on which it leans, they call *ipomochlion* [ὑπομόχλιον, hypomochlion]. These things are known to all. And here is Aristotle [explaining] how, by the power of a small man, a great burden is raised by means of the lever — a thing not done [otherwise] by many men. For the lever itself, [being] one heavy [body], will raise a great burden which could not be done [otherwise] by many men.

It is also [true] that when we add weight upon weight, it becomes light to lift — and there is no doubt that if [the lever were] of iron, or of a heavier wood, it would raise [the burden] more easily than the light wood.

And [Aristotle] explained there that the *hypomochlion* [fulcrum] is reckoned as the center of the balance; and the end on the side of him who weighs down and

lifts is farther from the center. Therefore, a small weight of the man pressing at its far end weighs more than the heavier stone near the center. And as I explained at the end of *Sefer Gevurot Hashem*, for this reason the end of the lever far from the support or the center weighs more than its other end, which is beneath the burden. Therefore it presses down on this side and rises on the side of the burden.

Archimedes explained, in [propositions] 6 and 7 of [his book] *On Equal Weights*, that the ratio of the force [power] to the weight is as [the inverse ratio] between the segments of the lever — for example, in the figure of *Sefer Gevurot Hashem*, folio 185 [קפ"ה = 185] [recte: see note*], if line שא [?] is five times line קן [?], then the weight hung at point [ק] is also five times the weight hung at point א. And we have already said that one who wishes to lift by means of the lever should press at the end of the lever; and the longer the lever and the farther from the center, the more he can lift — as at point [ב] [?], whose ratio to the weight is only as [we explained there in premise 2].

Know, then, that there is no difference whether the support [fulcrum] is between the force and the weight, or whether the weight is between the support and the force — for example: if the weight [is at] א and the end of the lever [ס] leans upon the support [post], and the lifting force [is] at point [ב], then the force at [ב] must be greater than the weight at א, since the lever gives no help to it other than that it presses for it by [the leverage of] its burden — and as we said, [the weight] should be in the middle of the lever, and the bearer near it stands at the place of the support, and the far one at the place of the force [or burden]. But if the distances אב ,א [?] are equal, the force must be greater than weight א, since the lever is no help to it, but only presses for it by its own burden. And the bearers carry the burden with the bar: if the distance between it [the burden] and each of the two [bearers] is equal, each carries an equal weight; and if it is nearer to one than to the other, the one to whom it is nearer carries more, and it presses more upon him — for the burden is in the place of the support or center, as Aristotle wrote in the *Mechanics*, question [?] — or it is as we said, that the burden is in the middle of the lever, and the bearer near it is at the place of the support, and the far one at the place of the force; and the farther [the burden] is pushed [toward] the near one, the more the near one is pressed beyond the far one — and the like.

There is also a third reason: that the line coming from the [burden's] end, near the burden, toward the center of the world is shorter than the line coming from the far end toward the center of the world. Therefore [the near end] presses down more. From here [we see] that the closer to the center, the heavier — as is

also clear among the philosophers. And from this you will understand the reason why, [when] you bear upon your shoulder some long wood, if it weighs down more toward one side, it is harder to carry it — for the shoulder is reckoned as the support or the center of gravity. And if the distances of the ends of the wood from the center are equal in weight, [the shoulder] does not feel more of the wood's weight; whereas if one [end] is farther from the shoulder, it weighs down more than it [really] is.

And in the figure of the aforementioned *Sefer Gevurot Hashem*, the weight hung at point א [is] ten litra, [and] weighs at point ב [?]…

twenty [litra], in [the ratio] that as distance [is] to weight, so [the other] distance to weight. Therefore, if you know the weight at point ב — say, however many litra — and you place it on the pan of a scale, you will know how much would be needed [as a counterweight] in order to be in equilibrium at point א. According to the inverse rule of gold [the gold-weighing rule, reversed].

And [as for the matter:] if the iron of the *stadera* [steelyard balance] [שטט"יראא, Italian *stadera*] is lost, [so that you] need to know its weight in order to correct…

[Footnote, foot of p. 388:] *((: ליר) 4 3/4 לד) [marginal footnote — partly abraded; appears to read "folio 3¾ / 4 (Livorno?)" or a cross-reference; reading uncertain]

Ma'ayan Chatum (continued)

[continued from the prior discussion] in its place. For this same reason, a lance held upright at one's side is carried far more easily than when it is lifted off the ground. When it is held upright, all the lower parts bear the weight of the upper ones, and one does not feel the great weight he is carrying; this is not so when it is brought down tilted, falling and pulling now this way and now that. For the same reason, too, a wooden beam carried on the shoulder — even if it is taken at its midpoint — is hard to carry if it is too long, since the farther the end is from the center, the heavier it is, and the more readily it tends to fall.

By their nature, the diameters of circles that revolve in motion — the longer ones, farther from those nearer the center — require more force than the shorter ones. Yet if the mover is at the circumference, the larger ones move more readily than the smaller. [restated:] The larger spokes [?] move more readily in their course than the smaller ones if the moving force is at the center; but [if the force is at the circumference] the smaller ones [move] with greater ease, for the least force will set them in motion.

For this reason the philosophers hold that the mover of the spheres is at the circumference, where it moves them with slight force, more readily than it would from the center. According to this, the motion of the earth is harder [to produce] — even though it is small as a point relative to the motion of the uppermost sphere — and perhaps, if it is like a roasting-spit turning about the sun, as some suppose, the moving force is at its center. This is well known to those who roast meat, who set the turning force [of the spit] at the center of the wheel: what they want is the smoothness of the turning, not its lightness.

Now, the reason a load is carried more easily on the left shoulder than on the right is this: the right side, being by nature the more active, has the greater vigor [?]; whereas the left, the mover whose strength is weaker [bears the load]. This is why among women you will not find even one who uses both her hands like her right hand — as Hippocrates [איפוקראטים] wrote — and few such even among men. It is therefore better that the one who is to pull and move [the load] be left free of the burden, and that the burden rest on what is moved [the vehicle].

[Footnote-gloss, marked in source with an asterisk: In the book *Mechanics* (ch. 8) of Aristotle, and in the books of Galen [גלינוס], these matters are explained, to the point that they were justified in their words — that it [the load] is "like the ground of the world," meaning that it is better for the whole load to rest on the wagon and not on the horses that pull it. My teachers and masters, my grandfather and my father, the eminent ones, great in Torah, who annulled and rejected the words of those who err in matters of orientation [?] and go astray in understanding several Mishnayot and sugyot — supposing that "the way of the right" means the direction in which we, the people of the covenant, write — they showed by decisive proofs from the Talmud and from the great halachic authorities and commentators that it is the reverse: "the way of the right" is when the left side is drawn and dragged after the right. And so one must do in the waving of the lulav, in [the writing of] a Sefer Torah, and in the kindling of the Chanukah lamp, as Chazal said: "Every turn you make shall be only to the right" — and as they treated it at length in their books. I have not brought up this matter here to settle it from what they have said — for the truth needs no help, and is helped only to reinforce the words. Let us return to our subject.]*

From here, another paradox is readily resolved: how the half can do what the whole cannot. It appears that the *stadera* [שטט"יראּ, the steelyard balance] is half of a [pair of] balance-scales — for the loop from which it hangs is near the center, and on the other side it is long, being only half a beam. Yet with it we can weigh any weight against a single piece of iron [the running weight]; whereas with

[ordinary] balance-scales one needs many separate weights. For example, if you wish to weigh up to 8, you need a weight at its cube-root or its cube — 4 or 2 [?]; and if up to 27, you need 9 or 8 [recte: 3?]; and if up to 125, you need 25 or 5. But the *stadera* stands against them all.

This is so, as the matter is explained in the figure on folio 185 [קפ"ה]: the running weight [המרקום, the runner], moving along the beam, weighs less the nearer it is to the center and more the farther it is removed. We can therefore bring it nearer or farther until it balances, in the scale-pan, the load that hangs at point ה [he]. There [at ה] is the one pan; and on the beam, the iron [the runner], **האב** [the beam/runner], is marked with [the values of] the litra according to the positions to which the runner is removed from the center. At whatever point the iron is set, it stands in place of the second pan. The *stadera* is therefore not [equivalent to] the distance of one pan of a balance and half a weight, but to many and many pans and weights together, [depending on] the nearness of the runner to the center.

But the *stadere* [steelyards] of the ancients were different from ours: in them…

the runner was fixed at the end of the beam, and the loop was the moving part. The beam had holes [pierced] for every litra, in which the loop of the scale was hung according to the merchant's wish and the greater or lesser quantity of merchandise. One who reflects on these matters will recognize the newness of the world [its having come into being], seeing how foolish the ancients were a thousand years ago, and how [men] go on perfecting the various subjects at all times — for once the [present] manner of weights was found, the earlier ones were rejected and forgotten, and so it is in all the crafts. Those who read in the books of the ancient chronicles will understand the affairs of the world better than the philosophers who busy themselves all their days with the disputes of speculation and discourse.

And behold, with the *stadera*, if the runner is removed from the center by the distance of the load from it, they will stand balanced equally. Therefore the loop — know it firmly, so as to prepare for yourself the whole [method]:

In this *stadera*, if we wish [to find] the runner — when we take with the compass the distance between the loop and the chain on which the load is hung [recte: the distance the runner is removed from the center?] and reckon how many litra the value of the runner is — for as the whole beam, **זמ** [zayin-mem], stands to the whole weight of the load, so [stands] the doubled distance between the loop and the point at which the load is hung, that is, **זמ** to **קמ טב** [?]. And conversely, as **דזב**

[dalet-zayin-bet, the beam] is to **הט** [he-tet], so [stands] the weight of the load to **טוב** [tet-vav-bet]. [Worked example:] let the length **דזב** be 12 litra, and **קמ** [kuf-mem] be one [?] — then its double, **קמן** [kuf-mem-nun], will be 2, and the other, **טב** [tet-bet], 440 [recte? — see Numerical impossibles]; and as 2 **קמן** is to **מב** [mem-bet] 10, so 12 **קמ** is to the load. Therefore the weight of the load will be 60 litra.

For it is explained in the *Mechanics* [טיכאניקא] that as weight is to weight, so is the ratio of the distance between the center and the runner of this [side] to the other distance. These matters I have found in scattered books, by God's help.

Now, Galen [גאלינוס] reckons, in the tenth chapter of the first [book], following the opinion of Hippocrates and Plato, that the nerves called *musculi* [מושקולי, "muscles"], by which all the limbs move, are like beams. He says there, in these words: "When we move heavy objects that it is not in our power to move by our hands, [we do so] by means of beams; so too, since the limbs cannot be moved by the sinews [alone], the nerves were given us by which we move the limbs. And just as the beam and the load change their position according to the motion by which we wish to move the load — up or down, forward or backward — so in every single limb there are many nerves that move it to whatever direction we wish, like beams." His further words may be consulted there.

What follows, then, from our words — to resolve our question — is that one who raises [a load] by means of a lever does not, in truth, add to the load. Aristotle, in the manner of the rhetoricians, magnified the paradox: that the weight of the lever is reckoned as nothing relative to the ease with which it makes the load easy to raise; and the prop [fulcrum] is reckoned as the center of the balance — for it makes no difference whether the loop of the balance is above or below, as is the practice in some countries. Aristotle mentioned this kind in the aforesaid book.

If the end of the lever is removed very far from the prop, or center, and the load at the end of the second arm is near the center, then even were it the weight of the whole world, a feeble force would move it — except that very long levers are easily broken. Thus the boast of Archimedes, that he could move any weight whatever with any force you please, is proven true in the science of mathematics alone, which has no limit; but nature has bound and limit, and material things are fixed in their natural powers.

This is like the mathematicians' saying that the angle and the line are divisible to infinity — but this is in thought, and thought [is unbounded]. As the rhetorician

[the poet] said, "Heaven for height, and earth for depth, and the heart of kings is unsearchable" (Mishlei 25:3): the visual angle, if you divide it many times over, will no longer be seen, and the angle vanishes. So too the natural line: if you divide it many times, it ceases to be divisible. And so in music there is a limit to the divisions of the tones and the ratios, even though by means of the divisions and the numbers one may proceed to infinity. Therefore in mechanics, when one looks at its construction in practice — although the manner of study is theoretical — the matters must be bounded, so that their application is of use to man, and they not become "winds" [empty figments] of the imagination of the intellect, producing expenditures without end, like many…

of these dreamers — vessels devoid of competence, who do not know whether the matter they busy themselves with will support what comes to their mind; and whether they are of no use in peace, while in war they do great harm. One sinner destroys much good (Koheles 9:18).

32 [לב]. That a heavy body is hard to set in motion when it is full is clear from the device (**figure 68 [ציור סח]**) by which they draw water from the well — called *ḥem* [חם], the "resting trough" [?] — which in Aristotle's terms is the *kēlōneion* [קילוניאון, the shaduf / well-sweep], and which appears easier to lower.

Diagram. The *shaduf* (well-sweep), referenced as figure 68 (ציור סח), shown as a separate plate [not present on this page — labels recovered from the text]. A pivoted beam draws a bucket from a well.

- א (aleph): the pivot-post fixed in the ground — the fulcrum / center (*opomobilion* [?])
- ב (bet): the counterweight (stone or lead) tied at one end of the beam, near the fulcrum
- ג (gimel): the far end of the beam, remote from the center, where the drawer works
- ד (dalet): the bucket, at the end of the beam
- The whole beam is designated **אבג** [aleph-bet-gimel]; the lower post **אטג** [aleph-tet-gimel] [?]

The bucket — a vessel weighted with a stone or with lead, **ב**, tied at the end of the wooden beam to make it heavy — would by rights be harder to move [together] with the said weight. **The philosopher answered** that this [weight] is true [necessary]: when the vessel is empty, the drawer must press it down to sink it into the well, and the weight at the end of the wooden beam holds back and is of no use; but on its rising it is of great use, for with little help it raises the vessel full

out of the well. One cannot say that its gain is offset by its loss, for the wood fixed in the ground — at **א**, about which the wood **אב** [recte: **אבג**?] moves — is reckoned as the center, or *opomobilion* [the fulcrum, אופומו"בליאון]; and the wood **אג**, on which the bucket **ד** [hangs], is reckoned as the lever. Since the weight **ב**, the lead, is near the center **א**, and the drawer works at the second end **ג**, very far from the center, he does not feel a great weight when he sinks the vessel into the well. And on its rising, when it [the counterweight] aids him, the weight at the second end too by its nature wishes to descend, and it helps in the rising — which is the harder part of the work of drawing. In any case it is good to divide the labor in half: half in the descent and half in the rising, rather than that the rising be far more laborious. This is easily understood.

33 [לג]. [The wonder] of the rising of a body [in water] — that the more it descends, the more [it rises] [?], and the reverse. There is something resembling this: that whatever is repelled from a thing approaches it [?]. For example: a ship that is brought and beaten upon dry land and thrust back forcibly — one standing within it falls not backward but forward, because the blow is to the ship alone. Hence a body in motion with a motion unlike the motion of the ship does not, on this account, cease to move because of its [the ship's] thrusting-back. This is easily understood.

Likewise a plain rope, if it be pressed down forcibly, rises higher and higher afterward, and the one riding upon it rises with it.

Set your mind on the device customary among the craftsmen at the *teva* [/ התיבה ההבה, the ark / chest? — recte uncertain], whereby they are accustomed to raise the water with it from below upward, together with the creeping thing [the spiral coil] called *ḥemet* [חמט] or *shichlul* [שכלול], whose foreign name is *nuera* [נוי"רא, the screw / *cochlea*] — the invention of Archimedes, by which he greatly benefited the Egyptians: when the Nile rises and floods the land, they empty the water by its means, as Diodorus Siculus [ריאורורום שיקולום] and Vitruvius [וויטרוויאום] wrote. He [Vitruvius] discusses this wondrous device in chapter 11 of book 10. The eminent lord, the scholar most accomplished in mathematics, *Noiro Obleri* [נוירו אובלרי = Aubert? Obleri?] [?], composed a whole book on it; and many supposed that by its means perpetual motion [תנועה תמידית] could be derived — which all the ancients and moderns sought — but their hope was belied.

Here is the figure of the device before you (**figure 69 [ציור סט]**).

Diagram. The Archimedean water-screw (*cochlea*), referenced as figure 69 (ציור סט), shown as a separate plate [not present on this page — labels recovered from

the text]. A cylindrical helix is set at an incline, its lower end in the water, its upper end resting on a post, and is turned by hand or by an animal.

- ד (dalet): the lower end, set in the water at the bottom [in the channel]
- The upper end rests on a column or wood; turned, it raises the water upward
- ה (he): a point in the helix where the water appears higher
- י (yod): the limit / top of the water's ascent
- ב (bet): a point where the water appears higher still
- ו (vav): a point where the water, though seeming higher, is not actually so [?]

When it is laid at a slant in the water — its end **ד** [dalet] in the channel-bottom, and its second end resting on a column or wood — and a man or beast turns it, by its revolving it raises the water upward; from there it is poured down to whatever place one wishes. And you see that at point **ה** [he] the water appears higher than when it is at point **ו** [vav] — but this is not so, for they are in fact nearer there to point **י** [yod], which is the limit of their ascent. So too at point **ב** [bet] it appears higher, but [it is] not [higher] than at point **ה** [recte? — the text reads מנקדת ב] truly. Here you have it explained: that in every single revolution, when the water is above, it is in fact lower than when it descends below in the [next] coming revolution. Therefore in their [apparent] descent they rise, and so the matter continues until they issue from the second end — which is astonishing.

If the device is sheathed on the outside, and you set its hole [the bore] upward and your hand moving it downward, and you put some iron pellet inside it from the side of its bore and let it roll, you will see the reverse: that the more it ascends, the more it descends, until it falls. This is easily understood.

34 [לד]. And as to what you asked — how one body can enter into another — this is not mentioned in the *Book of Nature* [the *Physics*?] regarding this, except [in the case] of hollow bodies, where it is known that the [outer] place is larger than the placed [body], and even so its place enters into [the other's] place. As to what you asked — how a body enters into a body — this comes in the words of the Sages of the Talmud, that "every cubit in the hollow [of a body] is a cubit and two-fifths along the diagonal," approximately, not with [strict] precision, as the Tosafos wrote.

Accordingly, if you take two square wooden pieces, long, their length 7 fingers, exactly equal in their breadth, and bore a hole through one of them along the diagonal [recte: through both — the text reads "ב' הזויות," "the two corners"], so

that there remain two thin posts on this side and that at the two remaining corners — then [you cannot] insert the second into it along its length [?].

Resembling this is a question [R.] Eli [עלי ז"ל] put: "Behold, there is a well in the field, its mouth square, 3 by 3 [cubits] [?], whence the flocks are watered. Now and then sheep and donkeys fall into it, and we wish to make over it a cover of wood. We have one board, its length 5 cubits and its breadth 2 cubits. How shall we cover the well with it? — for if it be cut in half it will not suffice, and one cannot join the cut pieces." **The answer:** Cut the board diagonally and reverse the pieces from how they were before, and it will suffice to cover it. This asks only that one make a long square surface of equal sides — that is, to convert one surface into another. Easily understood.

Now, had I not brought up the *Book of Nature*, I would [have brought here] a wondrous matter. For I have already explained to you in *Sefer Gevurot Hashem* and in *Sefer Niflaot Hashem* [נפלאות יי'] that the nature of air is to be compressible and expansible, condensable and rarefiable — but not so water, except by means of heat and cold. Then it is acted upon and does not keep its nature, as we know that cold freezes water and suffices it with a smaller space than before; and conversely, in their boiling by fire they require more space. But air, even without being acted upon — merely by the weight of some body that comes upon it — is compressed, and likewise is compressed and spreads out to flee from the void.

If you wish to test this: take a glass shaped like an egg [ביצייה], hollow, with a small hole in it, and suck out the air in its cavity with your mouth as much as you can. Then at once stop its mouth with your finger and keep it so, until you put it into water; for if you then draw it out, you will find that some water has entered its cavity. This will not happen with a vessel that has no suction, as I explained in *Sefer Niflaot Hashem* — that if you put any vessel into water mouth-downward, no water enters it at all, and if you put cloths inside, they will not get wet. So here: after the suction, there was a void in the glass, and the air spread out until it filled the cavity; and when you removed your finger from its mouth, [the air] was drawn out and compressed as its nature was before, and so left an empty space for the entry of the water; and the water that enters gauges its compression and expansion.

Now, water is not compressible, as I explained there. Yet if you make a sphere of lead and fill it with water up to its mouth, and stop the hole with lead, soldered, so that it is sealed and no air can enter it, and then strike upon it with a hammer — you will reverse the sphere into a body with [an] angle [a polyhedron]. It is known that in the spherical form the vessel held more than in the angular form;

therefore the water was compressed. But since compression is impossible in water, as has been explained, it follows that body has entered into body. And if you put the sphere under the bottom of the chest [?] and press upon it, you will see that the water comes out of it as [though] sweat — which is astonishing. Things of this kind, hidden and weighty, I have to publish, if I know that they will find favor in the eyes of my people; for in any case, in the Latin tongue I shall publish [them] for the nations among them — many of whom long to know and honor [such] weighty matters.

35 [לה]. A wonder in your eyes will be what *Pliny* [פליניאום] also recounts and reflects on: that in like winds [ships] travel in opposite directions, so that sometimes their sails meet in the night, in the darkness. And the God of the winds satisfies every living thing with His will (cf. Tehillim 145:16) — even though the directions of wayfarers vary. Of this it is said, "In all your ways know Him, and He will straighten your paths" (Mishlei 3:6). But how this comes about in nature, we shall explain thus:

[Text continues — the explanation begins in the following pages.]

Ma'ayan Chatum (continued)

that we suppose (Figure [70] ע): at point פ [P] a ship is sailing east toward city ד [D], and another ship at point מ [M] is heading west toward city ך [K], and there is a north wind — one of the four principal winds, which are called the *cardinal* [angular] winds — blowing straight along line בם [B-M]. Point ם [M], then, is the center of the compass-rose of the opposing winds and their courses, and the diagonal of those courses is ומד [W-M-D].

Now if the ship at point פ [P] were reversed backward toward point [?], the wind would carry it straight to ש [?] along the path of its [?] edge, and with great speed. But since its intention is to go to ד [D], the helmsman turns its prow toward point ך [K] and its stern toward מ [M], so that the wind strikes it from the side along the line [?]. Likewise it happens with the ship at point מ [M]: [the wind strikes it] from the side along line במ [B-M]. The steering-oar — the timber called the *timón* [Spanish/Italian: *timón/timone*, "rudder"] — is set equal to the angle דמצ [D-M-Tz], and this it must do; for otherwise the wind would carry the ships to points צ [Tz] and [?], outside their course and their will. Indeed, when we turn their prows toward ד [D], so that they deviate from the wind's [direct] thrust by double the inclination from line ך [K], that is, by the measure of the acute angle [?] שמצ [Sh-M-Tz] at the least, we shall hold to half of what we seek, which is the acute angle

שמ [Sh-M]. And this is sought with the help of God and with every device and force possible to oppose to the wind. But if it is not possible, we turn our back and go wherever the wind wills, to [?] [or] צ [Tz].

From this it is plain that it is harder to oppose the wind when it comes head-on — as in line ם [M] — than when it comes obliquely, as in the lines בפ [B-P] and במ [B-M]; and these are harder than the winds at בק [B-K] and בי [B-Y]; and these harder than the winds at בנ [B-N] and בד [B-D]. So too, the farther [the prow's heading] is from the center of the wind, the easier it is to oppose it, since the angle ימצ [Y-M-Tz] is smaller than [the one opposite] — for in the right triangle בבסם [?], the exterior angle [at] במד [B-M-D] is greater than the interior right angle, by Proposition 16 of Book One [of Euclid]. And since [the angles at] במד דמצ [B-M-D, D-M-Tz] are equal to two right angles, by Proposition 13 of Book One, therefore דמצ [D-M-Tz] is smaller than the right angle. So at point [?] the angle below קי דזדי [?] will be equal to the angle above it, and בסה במד [?] will be right angles. And in all the other points, the upper angle is greater. And since, in the triangle במת [B-M-T], the angle בת [B-T] is greater than the angle במת [B-M-T] by Proposition 16 of Book One, the angle whose remainder is דהק [?] will be smaller than the angle whose remainder is המצ [H-M-Tz]. And likewise the helmsman must make a smaller angle with the rudder — and must turn the sails, the farther [the heading] is removed from the center of the wind, so that he may hasten to reach the place he desires.

As for your objection that Euclid built this on a shaky foundation — that objection deserves the verdict reserved for one who seeks from the mathematicians what cannot be granted to him: namely, the false premise that there *is* a "greater" and a "lesser" among angles bounded by straight lines. For if [an angle] were the greater, either it would be equal to a right angle — and then it is not [greater] — or it is acute, or obtuse; and were we to subtract from the right angle, the remainder would be like [the original]; and were it divisible, we could take a part of it and add it to the greater, and then [the sum] would be greater than the greater, against the supposition. So too the lesser among them: it cannot be said [to be a quantity] if it is not divisible. And Euclid never said this except of the angles formed by a curved line and a straight line together, as is taught in Proposition 16 of Book Three; there the angles are quantitative — the smaller the circle, the wider the angle of contact, and the larger the circle, the more acute it is, as is shown in Proposition 11 [?]. So much so that even in *mechanica* the matter is seen with the eye in carts: the larger wheels do not stumble, and they roll more easily than the small ones, because their angle of contact with the surfaces they

pass over is smaller. Therefore do not look askance at Euclid, for his words are true and faithful.

[Paradox 36] As for what you raised — that you have not found a way to grant [resolve] the matter — this is no [discredit], just as Aristotle ruled in [his] *Logic*. But [Aristotle] labored and found [a solution], and spoke of it wondrously in Question 25 of his *Mechanica* — which is the gravest of all his questions. And here experiment and sense show the matter, and it cannot be denied. For if you pass a long timber between two wheels of a cart, one larger than the other, and they are bound and joined…

by way of their centers, by means of the aforesaid timber, [and] roll them until the larger of them completes its circumference upon a level surface — and it traces a groove or straight line upon the ground equal to its circumference — you will find upon the ground a parallel line, equal [in length], traced through the centers by means of the said timber, by the rolling [of the wheels]. And it is a marvel: that the larger should complete its circumference upon a surface equal to that traced by the rolling of the smaller wheel! This is an immense wonder — that around a single center, in a single time, the small [wheel] should trace a line as long as the large one. For [if] it were one [revolution], the large [wheel] would complete one circumference [and] the second [the small wheel] one as well [?], or two: just like two men walking together, where the strides of the one are longer because the strides of the other are more frequent. They are always going side by side, since the wheels are bound together through their center and move at equal [rate]. Nor can one say that the small [wheel] also skips — for if, in its rolling, it were skipping, the line traced would not come out continuous but separate and divided. Yet this straight line, formed on the level horizontal surface without any crookedness, is traced by the rolling of the wheel as its act of contact [shows]. And within this question lie hidden many secrets and strange matters — which makes it seem that the naturalists are voluntary agents: the small [wheels] aim according to the will of the large, and the large act according to the will of the small — the opposite of what appears.

For sometimes it appears that the large wheel, in its rolling together with the small one, does not trace a line longer than the line the small wheel traces, [the small one] being rolled under compulsion in such a way that [the large] checks its [own] power and annuls its [own] will to satisfy the will of the small one beneath it. And if you bound it with [a wheel] still smaller, it would trace a still smaller line — all by the circumference of the wheel that is under compulsion. So too the small [wheel] traces by the circumference of the large ones beneath it; otherwise

it could not [do so]. Therefore they are aimed to act each according to the will of the other. And what astonishes still more the one who contemplates the matter is this: that two circles concentric [about a common center], differing in magnitude, moving of their own accord — each according to its own power and ability, not under compulsion as [in the case] of [those] issuing from [a common center] — and their motion being equal, will complete equal circumferences. And this is plain: that if, from the straight lines they trace upon the ground in their rolling, circles were made, those circles would be equal to one another. Nor need the one wheel be on the [same] surface as the other for them to be concentric; for if the surfaces and the centers are parallel to one another, it is as though one were inside its fellow and entered into a single center with it, as is shown in the studies of the [fixed] stars, and is easy to explain by parallelism.

Nor can one say that the small [wheel] traces a line longer than its circumference in moving in company with the large [wheel]—[such] that, had it rolled alone, it would have traced only [a line equal to] its circumference. This is in the manner of [men] pulling or pushing and moving [a load], each adding his strength to the help of the others: so that if each alone could lift fifty *litra*, two together lift one hundred fifty *litra* [recte: 100? — "two together" gives 100 from 50+50; the text reads (150) ו"קל] — for the power of the whole combined is greater than the power of all the parts [separately]. For if so, how does it appear sometimes that the large wheel traces a small line in its rolling [equal] to the length of the small one's circumference — and how is its power diminished? Therefore one must investigate the matter from its foundation, if you wish to grasp its secret and explanation.

The Philosopher [Aristotle] there set down two premises. The first is that when two bodies move from a single power, in such a way that one of them moves according to its nature and the other against its nature, then [the latter] weakens the power of the mover and lessens it; or even [in the case] where the second does not move toward an opposite direction but remains stationary and does not move itself, but rather [must wait] until the other mover moves and drags it — [then the load] will weigh the more heavily upon the more massive mover [?], more than the other moving body that rolls itself and contributes its inclination to the action of the mover; and therefore this one is hastened and that one delayed in its motion. The matter is plain in a skin-bladder full of air with a piece of lead [tied] inside it: for if the lightness of the bladder overcomes the weight of the lead, the bladder rises — but more slowly than if it had no lead. And if the weight of the lead overcomes the lightness of the bladder, it sinks — but not so quickly as the bladder [alone would rise].

The second premise is that the moved [body] which is delayed cannot move more quickly than its mover — for how could the power of the pupil be greater than the power of the master, or the recipient [greater] than the giver? And from this it is plain…

to resolve the question: that when the two aforesaid wheels, bound together by the single timber, roll, the one is the mover and the other is moved and dragged after the motion of the small one [the smaller], and does not help itself; and the line traced by the large [wheel] is equal to the line traced by the small [wheel]. And if the small [wheel] is drawn and dragged after the motion of the large [wheel], it traces a line as long [as the large one's], because it is hastened to run faster than its natural motion. The matter is easy to picture in two surfaces, one higher than the other, with the small wheel on the higher one. And the matter is not like [the case of] concentric circles, where their center stays fixed; whereas in the rolling of the wheels, the centers likewise move, and it is as if two points were being drawn straight, not in a circle. Therefore the Philosopher says that here there are not two centers, even though there are two circular surfaces, the one distant from the other — because the one alone is the mover and the other is dragged after the motion of the center of the first, and moves in a straight line and not in a revolution. And thus the straight line traced is as long as the circumference of the moving wheel, while the [line] of the second, parallel wheel is [merely] the trail of the center of the [body] moved after the motion of the mover; therefore it is not equal to the circumference of the moved [wheel], which traced it.

From here let those who treat of the squaring of the circle see that one cannot know the circumference of the circle from the line traced upon the ground. And this is enough for the matter.

[Paradox 37] [37] לט — *How can it be believed*, were it reported, that the parts of one and the same continuous body should move with unequal motion, so that the head of the journey travels, say, a *parsa* and a quarter while [the body] has walked on its feet only a *parsa*; and a rider runs faster than the horse beneath him?

Answer. This is one of the marvels of the circle, whose very figure is wondrous; and no wonder if the operations that follow from it are as wondrous as itself. Thus the Philosopher [Aristotle], in his book of *Questions* [*Mechanica*] on the wondrous artificial operations, explained four strange properties found in the circle.

The first: that it is composed of rest and motion, which are privative opposites. This is clear, since when you assign it a fixed center and a fixed radius, it is in fact traced by means of a straight line, one end of which [is fixed]—the third postulate of Book One of the *Elements*, which postulates that one be able to draw a circle—the first [end] at rest and the second moving around it until it returns to the place where it began to move; and the craft accomplishes this with a dyed thread or a compass, and the like.

The second: that, its circumference being [made] of a single line according to its definition (Definition 15 of Book One [of Euclid]), and being void of breadth according to the second definition—nonetheless there are found in it things close to opposites, which are with one another like the large with the small. These are opposites that combine and are bounded one by the other through the mean that is between them; and this is a [further] marvel—how it passes from convexity to concavity without passing over the straight line. So too the concavity with the convexity that is on the circumference of the circle: just as the passage between them is through a mean—which is the [straight] line—so here too the straight line is the mean between the two extremes.

The third: that there are found in it contrary motions together at one and the same time. And these are, as he explained in *De Caelo* [*On the Heavens*] and in the fifth [book] of the *Physics* [*De Auditu Naturali*], [the motions] occurring along the diagonal of the distances of [its] places, as from right to left, from above to below, from front to back—and all these are seen by the senses in the motion of the circle, which is seen going and returning, rising and descending; and from being thrust away, rising again and returning; and being thrust away as it pushes [forward]. And from this follow the mighty devices applied to massive bodies—souls [?], for they are the wondrous, useful instruments that move of themselves with the toothed wheels [gears].

And these three are indeed astonishing—but in the eyes of the contemplative they are not astonishing. [Not] the first: the center is not [one] of the parts of the circle, which is not composed of points [n]or of half the diameter [radius]; for at the time the circle is traced, the whole [radius] moves, including its second end at the center—except that it appears [otherwise], because [the center] does not change its position at its second end. And the properties of the circumference of the circle are not peculiar to it—for they are found also…

[in figures] like the parabola, hyperbola [?], ellipse [?], and the like, in whose lines concavity and convexity are found. And it is no defect that its differing parts move differing motions: these to the right, those to the left, rising and descending.

The Philosopher's words are eloquent, rousing the heart in his introduction, in the manner of orators, for renown and praise—since all the operations of *mechanica* are comprised in it. And if we should come to recount the praises of the circle: it is the first of the figures and the last of the foundations of [geometry], whether [because] it is [made] of a single, simple, even line, alike in all its parts—first and last; and yet it is not [reckoned] of [the figures that are] quantities[?], for if you subtract from it any part, its remainder is less than the whole—unlike [a figure that is] a quantity in its [magnitude]. And the circle, accordingly, is the most perfect of surfaces, the simplest, most ordered, and most comprehensive—and it is, in truth:

[The passage that follows is a sustained rhetorical encomium woven from biblical phrases — Mishlei (Proverbs), the "woman of valor," and others — applied figuratively to the Circle:]

full and rounded [perfect], "her eyes do not rove" and her ankles always incline; she "walks upright" [erect], and is not a drunkard staggering; for all the greatness of her strides—she has no legs to leap, nor is she dislocated and broken; like a "light millstone" she skips upon the mountains, she does not strike "her foot against a stone," and "all her paths are peace" (cf. Mishlei 3:17); and "the doings of words" [she does] "before elders and the discerning"; for she is "tender and delicate" [cf. Devarim 28:56], and sets upon the ground only as a point—"she has not tried [to set] the sole of her foot" (cf. Devarim 28:56); and she revolves and does not move from her place, "the king's daughter [is] within" (cf. Tehillim 45:14), and after her every man is drawn, and her footsteps are desire; "no stranger passes through her border," "shut up and barred"—if thieves come [to] her—and she is forever in motion in precious orbs; and she builds worlds, containing, revolving, and complete, the like of which has not been seen in any nation, "from end to end," for "there is His house," and when His King girds Himself He shall encompass His kingdom until the King returns to His [house] [?]; "they did not know that she is in descent and ascent," and "in the ascents he did not ascend"; and in her midst rules a single ruler, [who] guides [her]; "even though within her is set love without end," and her counterpart [?]; "rows of gold with points of [silver] he made her" (cf. Shir HaShirim 1:11)—except that to one of them [belongs] a [special] property: that all who issue from her are equal, and to her is their yearning, while [those] apart from her—according to their distance and nearness to her—are distinguished in rank; and from her the near and the far draw [sustenance]; "through all the earth her line goes out, and to the end of the heavens" are her issuings (cf. Tehillim 19:5), and to her they return; and "she neither adds nor diminishes"; "all the rivers flow to the sea, and the sea is not

full," etc. (Koheles 1:7). The rest of the exposition of the greatness of the Circle you will find in the section [פ' בשמת — "Parshas Besomet" (?)], and all that is said of her is in truth applied [as metaphor] to the precious sciences, if you contemplate them.

And the fourth property of the circle is that the points on one of its diameters are not equal in motion: rather the one farther from the center is faster than the one nearer to it. And [it is so] that [the farther] is [carried] in [greater] speed—not that it revolves before its fellow, since both revolve about the center in equal time; but rather that, in the time the nearer one traces a small circumference, the farther one traces a larger circumference. And from this it is very plain that the head moves faster than the rest of the body of a man, and the rider [faster] than his horse—for since the earth is spherical, the journey upon it traces circles around it. The eloquence is forceful and true, and there is no doubt of it.

But the cause is precious, and I wish to reveal to you an immense and great marvel: that the Nothing—the privation—has power. For the center is like the lodestone that draws iron, to draw and to detain, as the vacuum does; and it is absolute privation. And this is [so] because the point, as Euclid defined it, is a thing that has no part, and is no quantity, nor any substance or accident—and nonetheless its power and the might of its hand are given to it, such that the closer the end of the moving diameter [is to] it, the more it resembles it in heaviness of motion, and inclines [to be] like it; therefore the farther one is faster. And this is good reason according to the view of Aristotle, and he boasted of it as a demonstration in the *Mechanica*. And since the world is spherical, some of the philosophers placed the moving power in the encompassing [sphere], because of its lightness, and others at the center. And perhaps because [the power] is in both together [it is said] "the whole earth is full of His glory," [the Glory] defined according to the terms of these philosophers, [namely] that the Divine is round[?], [a sphere] whose center is in all the circle, and accordingly…

the poet says: *"If I ascend to heaven, You are there; if I make my bed in Sheol, behold, You"* (Tehillim 139:8). And this is enough to answer your question.

[The preceding lines, at the very top of p. 397, continue the §39 discussion of shadows:] …a fear over the report that the shadow climbs by degrees [?]; it is possible for it to turn back and bend according to its course and path, retreating backward… And do not be troubled, for no guilt will befall you in this matter.

And now prick up your ear and listen, for here, as in the diagram before you [the diagram is on a separate plate — [?] not verifiable in this batch], the circle of the

horizon is **אבגד** [A-B-G-D], and that of the meridian is **אביג** [A-B-Y-G], which bisects the eastern half of the horizon **אב** [A-B] from the western half **אדג** [A-D-G]; and the circle of the equator **בכך** [B-K-Kh] [?] intersects it at point **כ** [K]; and the circle [?] intersects the meridian circle at point **ט** [T], and the pole of the horizon will be point **ו** [V]. Between the equator and one of the parallels-to-the-equator **פטח** [P-T-Ch] over which the sun passes when it is in the northern signs — and this is the aforesaid, that is, that the place be within the torrid zone, between the equator and the circle of Cancer [the Tropic of Cancer], and learn for the [latitude] 15° — for the two [tropics?] we mark, through point **י** [Y], a vertical circle tangent to the parallel **פטח** [P-T-Ch] at point **ל** [L]. Likewise we mark a second vertical circle **נסציר** [N-S-Ts-Y-R] [?] intersecting the aforesaid parallel **פטח** at point **פ** [P], between point **ל** [L] of the tangency and the point where the sun rises when it is on parallel **פטח**. So too it is intersected by this vertical [circle] **סציר[נ]** at point **צ** [Ts], which lies between point **ל** [L] of the tangency and point **מ** [M], which is the meridian point [the meridian]. Likewise we mark a vertical [circle] for the third head [?] **פחים** [P-Ch-Y-M] passing through the point of sunrise [?], which intersects parallel **פטח** at point **רן** [R-N?] [?], between points **צ**, **מ**, **ן** [Ts, M, N].

And since, when the sun is on the vertical **פחים** at the point of its rising **פ** [P], the upright gnomon casts its shadow onto the plane of the vertical — that is, along the intersection common to the horizon and to the vertical — and this is easy to picture and to verify, and will be demonstrated in the science of shadows and gnomons — therefore in the morning the shadow will run all the way to the edge of the horizon, where it intersects the vertical, that is, to point **סן** [S-N?] [?], where the southern circle **מז** [M-Z?] [?], parallel to the equator and tangent to the northern parallel **פטח** [P-T-Ch], [lies]; and the tip of the shadow will be distant from the south point **א** [A] by the arc **אם** [A-M]. When the sun rises higher over the horizon and comes to point **פ** [P] on another vertical, **וחציר** [V-Ch-Ts-Y-R?] [?], the shadow of the gnomon will be in its plane, and the tip of the shadow cuts the horizon at point **ך** [Kh]; and the tip of the shadow will be distant from the south point **א** [A] by the arc **אך** [A-Kh], which exceeds the aforesaid arc **אם** [A-M]. When the sun rises still higher and comes to the point of tangency **ל** [L] on the vertical **קלים** [K-L-Y-M], the shadow of the gnomon cuts the horizon at point **מ** [M], and is distant from the south point **א** [A] by the arc **אמ** [A-M]. This distance is the greatest there will be on that day — that is, when the sun is on that parallel **פטח** — because the vertical **קלים** [K-L-Y-M], which is tangent to parallel **פטח**, is the one most distant from point **א** [A] of the meridian, more than all the [other] verticals. It is therefore quite clear that from the sun's rising at point **ם** [M? / S?] until it reaches the point of tangency **ל** [L], the shadow has been continually

moving and growing more distant from the meridian circle — that is, from the south point **א** [A] — as you saw, from **ם** [M?] to **ך** [Kh] and from **ך** [Kh] to **מ** [M].

But from there onward the reverse occurs: the shadow turns back. For when the sun comes from point **ל** [L] to point **צ** [Ts], the sun will be a second time on the vertical **נסציר** [N-S-Ts-Y-R], and the tip of the gnomon's shadow will cut the horizon at point **ך** [Kh], and the shadow's distance **אך** [A-Kh] will be smaller than the distance **אמ** [A-M] that it had when it was at point **[ל]**. Likewise, when it comes to point **ן** [N], the sun will be a second time on the vertical **פחים** [P-Ch-Y-M], and the tip of the gnomon will cut the horizon at point **ם** [M?]; hence its distance **אם** [A-M] — after it has turned back and approached the meridian circle — is what it previously had when it was moving away from it. But from point **ם** [M?] onward the shadow will continually move and bring its tip closer to the south without any backward retreat, until it reaches point **א** [A]. For when the sun comes from point **ן** [N] to point **ן** [N?] [?], it will be on another vertical, which is **שויע** [Sh-V-Y-Ay] [?], and the tip of the shadow will cut the horizon at point **ע** [Ay] [?], and its distance from the south **אע** [A-Ay] [?]. And when it comes from point **ן** [N] to point **כ** [K], which is at midday, the tip of the shadow will fall on the south point **א** [A].

And just as you saw the shadows act before midday, so they will likewise act after midday: they will move and turn backward in the same manner as the first ones. The matter is easy to grasp if you suppose that it is the right side of the figure — and it is the reverse of [?]…

the former, that is, what is now east becomes left, and west becomes right. For the sun, traveling from point **מ** [M] of midday to point **צ** [Ts], the points **ן, ה** [N, H] [?] will be found on verticals toward the head of **יחפ ריצפן** [?] **ניוש** [?] [a corrupt cluster of point-labels — [?] reading uncertain], and the tip of the shadow will cut the horizon at point **ען, ם, ר, מ** [Ay-N, M, R, M] in such a way that it is always more and more distant from point **א** [A] of midday up to point **מ** [M]. And when the sun comes from point **ל** [L] to **כ** [K] and **פ** [P], it will again be on the verticals **נסציר** [N-S-Ts-Y-R], **פחים** [P-Ch-Y-M], and the tip of the shadow will cut the horizon at points **רם** [R, M?]; thus it went and returned and moved away from midday and then drew near to it again, until it came to the vertical **פחים**, and there it sets at point **פ** [P], and the shadows flee when the sun has come [?].

So it has been made clear to you that in every place where the torrid zone is situated [?] — that is, in the country of New Spain [Mexico], in the city of Mexico [מיקסיקו], and in Cabo de Buena Esperanza [the Cape of Good Hope, **כקאפו די בונה שפי״ראנצא**], in Brazil [**בכראזיליאה**], in Calicut [**כקל[י]קוט**, India], in

Pernambuco [**כפיירו ביאווה** = Pernambuco / Pirambuco?] [?], in Cairo [**שאיור** = Cairo? [?]], and the like — whenever the sun is on a parallel to the equator whose declination from it is greater than the latitude of the place, the wandering of the shadows occurs twice in a single day, contrary to nature, as we explained.

And in fact this very thing occurs in **any** inhabited place, even outside the torrid zone, on surfaces inclined to the horizon. For if you raise some surface above the horizon until the elevation of the north pole upon it is less than the declination of the parallel on which the sun is — that is, less than 23° 32′ — if the sun is in Cancer, the shadows of the gnomons set up upon that surface will move and return twice. This is because the pole of that surface, when set in its place, will fall between the equator and the circle of Cancer; and behold, it is as though the place were in the torrid zone. Hence in the city of Jerusalem, whose latitude is 31° 30′ — if the dial of the hours of King Ahaz was inclined from the horizon toward the north by more than 7° 28′, its pole was between the equator and the circle of Cancer; or, if it was raised toward the north more than the elevation of the pole — and the elevation of the pole in Jerusalem is 31° 30′ — the pole of the surface will be between the equator and the horizon [?]: by more than the elevation of the pole 31°·80 [?: as printed — apparently a misprint, perhaps for 31° 30′] and less than 55° 2′ [?], the sum arising from the [combined] inclination [recte: i.e. 31° 30′ + 23° 32′ ≈ 55° 02′] — toward the great [declination] which in our time is 23° 32′. And the elevation of the pole in Jerusalem is 31° 30′. And then, according to those values truly stated, the shadows of the gnomons on those surfaces move and turn backward. And by the reckoning of triangles [trigonometry] you can know in how many hours the shadows will return from point to point. This is what we wished to explain.

However, according to the plain sense that the language of Scripture conveys, the miracle was of another kind, truly wondrous. For had it been at the morning of the appointed time [?], the shadows ought to have been shortening up to midday — yet they were lengthening, because the sun was turning backward from the place where it was in the east. And had it been after midday, the shadows ought to have been lengthening — yet they were shortening, because the sun was returning from the west toward midday. By contrast, in the manner we have explained, the shadows do not change in their *length* but only in their *distance* from midday. So [Scripture] testifies [?] that the thing was novel, on that very day and in that very year alone; whereas by the means we explained it would be so for many days, and every year at that same time. And how is it that King Hezekiah, his sages, his ministers — whether before or after the event — did not scrutinize it [as a recurring phenomenon]? Indeed, "the night that this revealed

itself" [?] was a cunning device [?], not a sign and a wonder such that it was heard to the ends of the earth, as the Scriptures relate. Therefore the answers lie in the angle and the instruments [i.e., the inclined planes and dials].

And likewise concerning the matter of the floating thing [the axe-head] that Elisha brought up [from the river] (II Melachim 6:1–7): had some man of the marketplace, not established as a worker of miracles, done such a thing, one could have said that it happened by chance that the wood [handle] became wedged in a hole of the throat [socket] [?], and that by the wood floating up — owing to its lightness — it raised the heavy iron with it; for by the [respective] weights, the wood is lighter than the iron, so its lightness lifted the iron's heaviness. But since this was a strange and astonishing occurrence, performed deliberately by Elisha, the worker of many miracles, it must be counted among the miracles. Nor can one ask, "Why does [Scripture] not say *and it rose* [of itself]" — because whatever can bring an action nearer to nature is more becoming before the Omnipresent, *HaKadosh Baruch Hu*, as the Sages of the Talmud said: that the Holy One, blessed be He, made it a condition with the sea that it split at the appointed time (Bereishis Rabbah; cf. Shemos Rabbah).

And three miracles were performed for him after his strength prevailed over the lightness of the wood, until it sank to the depths — [2] ב׳:

that [the iron] became wedged in the hole of the socket; [3] ג׳, that as it rose, the iron did not slip off from it. *"All these things God does"* (cf. Iyov 33:29) two or three times for a man — a wise one who heeds the voice of His word — that the occurrence not endure [as a permanent natural law]. And apart from these, the language of the verse indicates that it was against nature, for it is written *"and the iron floated"* (II Melachim 6:6) — whereas it would have been fitting for the *wood* to float, not the iron, as is explained in the book of Archimedes *On Floating Bodies* [*De insidentibus aquae* / Περὶ τῶν ἐπιπλεόντων]. And that you may know this is so, consider tallow, which floats on water owing to its lightness. If it is a candle of tallow it will fall lengthwise [lie flat]; but if you take a thin [piece] that floats and stands of itself, it will not sink, but will hold it upright, like floating pillars, sunk in the water up to its [lower] end — provided that at its end is fixed [?] a base of metal that is not too heavy [?], a small weight or a piece of lead, such as one places beneath the wood near its other end, so that if you smooth it [trim it] the wick stays close to the surface of the water. And as long as it burns it will rise and stand ever more upright than when it was outside [the water] [?], exposed in the air, until it burns away entirely and is extinguished, for here water comes up to the soul [the wick]. Hence, in the matter of Elisha, the iron's floating — by the

testimony of the verse — was certainly a great miracle, and there is no need to elaborate.

מא [§41 — Paradox 41]. **A marvel**: that the large balances are more precise than the small ones — for those who weigh precious items choose the small balances, and conversely, for cheap goods of little value, the large ones.

Answer. Five things are required to set up a balance of true justice [a precise balance] — for so it is fitting for weighing precious stones, pearls, and rare medicaments, such as scammony [**הסקמוניאה**, Greek: σκαμμωνία], hellebore [**איליבורו**, Latin: elleborus], elaterium [**אילטיריאו** = elaterium?] [?], opium [**אופיאו**, Latin: opium], and the like:

- [1] **א**: the length of the beam.
- [2] **ב**: its lightness, and that of the pans, likewise.
- [3] **ג**: the straightness and firmness of the beam, that it not warp; therefore they make it of strong iron, alloyed and cleaned [refined].
- [4] **ד**: that the two points where the pans hang [the points of suspension] be equally distant from the ear [the central pivot/fulcrum] by which the beam is hung.
- [5] **ה**: that the pans be exactly equal in their weight.

It is known that the error in the large balances is greater in magnitude than in the small ones, and that they may err greatly and cause loss in precious items. But when the aforesaid conditions are observed, the large ones are in fact more precise than the small ones. A proof of this is that Aristotle [**ארסטוטלים**] posed this as something agreed upon and customary in his day: to weigh purpura [purple dye, **הפורפ[ו]רא**, Latin: purpura] and other valuable items with large balances.

After lengthy premises and preliminaries, [Aristotle] makes the matter depend on the arc of its circles [cf. *Mechanica* Q.1–2] — for the beam is nothing but the diameter of a circle, and the place of the ear [the fulcrum] is the center. And it is known in [the geometry of] the circle that the larger circle moves more in its passage than the small one in [a given] time, the tilt of the balance-pan being equal [the same angular deflection]. For the circle is traced when you imagine a long rod, half its diameter, one end of which is fixed at a point while the other moves around; and the larger arc is more discernible to the eye than the smaller one compared with it. Hence, the farther the weight is from its center and traces, in its tilting, a larger arc of circle — for example, in the figure of the *stadera* [the steelyard, **השטטיירא**, Italian: stadera], which is in the book *Gevuros Hashem*

[*Mechanica*?] — if the length of the beam is **א** [A], then when the weigher tilts it [the pan], it will trace the arc **א** [A]; and if its length is **ג** [G], it will trace the arc **בג** [B-G]; and the latter is more perceptible to the eye than the arc **א** [A], even though the angles of both at the center are equal. So too in every instrument of mechanics that [Aristotle] taught: the weight [precision] of the large [instruments] is more exact than [that] of the small ones.

But in the instruments of the sun [sundials / solar instruments] it is the reverse: with the large ones the error happens by accident, because one cannot be precise about the point of meeting of very long lines; but if it were possible to be precise, they too would be more exact. Therefore, in every place [where] very small figures are drawn, it is easy for the examiner and tester to err among the large ones.

מב [§42 — Paradox 42]. **And from here** it is explained why one ought to fear an inclination of a finger [a finger's breadth] in a ship or in a small wagon more than [an inclination of] a handbreadth in a large one — even though to the sight the larger frightens more. For [from] the aforesaid figure, if you subtract the arc **א** [A] from the arc **בג** [B-G], you will see that [arc **בג**] is much greater than the arc **בג** [B-G] [?: text as printed; apparently meant: that the small one's arc is far greater proportionally]. Hence, even though point **ב** [B] dips more than the dipping of **א** [A], it [the small vessel] has not yet bowed to the ground as much [as one would fear], and more easily…

[the point that] rises is point **ב** [B] from point **א** [A]; if the load is lightened from the inclined side, or if the people lean toward the other side, in still more time will point **ב** [B] need to descend from point **א** [A]. Therefore the large [vessel] is not in as much danger as the small one. All these are explained from what was said in the book *Gevuros Hashem* [*Mechanica*].

מג [§43 — Paradox 43]. **And from here** too is explained the manner of making counterfeit weights [false scales] — even though the pans are equal in shape, they will not weigh equally; this is what is sought, apart from [those defrauded by] the purchasers [?] — namely, that the ear [the fulcrum], which is in place of the center, not be in the middle of the beam, for then one pan will be more distant from the center and will move more easily than the second, which is near it. And to conceal the fraud, they make the more distant pan lighter than its fellow, or one half of the beam thinner and lighter than its other half — and similar contrivances, which I have hated [cf. Tehillim 31:7]. The test [for this] is to reverse [the items] weighed — that is, after the seller has weighed out the purchased goods, you place in [the other pan] the same purchased goods

[transposing pan and weights]: if they still weigh as before, they are honest; and if not, they are balances of deceit. But with the *stadera* [steelyard] it is harder to detect, for perhaps the iron called *tarcum* [the runner/poise, **טרקום** = Latin/Italian *trochus*? [?]] is larger than it ought to be relative to the spacing of the marks of the litras [pound-graduations]; therefore one must examine it as I instructed you in chapter 31 [31§ = **תמר ל״א**] — for sometimes the instrument is bad in its quantity [calibration], even though it appears [sound], and not every man knows this. Therefore, in the practice of governance, to keep fraud away, the judges decree that merchants use pans [balances] for selling foodstuffs and not the *stadera*, for the aforesaid reason — because the common people do not understand it.

And further there is yet a kind of cunning in weighing, in accordance with the matter posed in the third [problem] of Aristotle's questions [*Mechanica* Q.3 / *Problemata*] of blessed memory: why is it that, with the balance whose ear [pivot] is suspended from above, if the weight in one of the pans makes it dip down, when [the weight] is removed from it [the pan] returns and rises back to its place — whereas it is not so when the center of the beam rests upon wood [a support] from below, [where the pan] remains [dipped]? Therefore, after the pan in which the goods lie has dipped, the seller will subtract from it [thinking it heavier] — because there is in it more than is due for [given to] the buyer — and the buyer will wait until [the pan] rises and settles on a straight line with the litras; and he [the seller] will reckon, when it does not [return] from its place — which is greater [?] — and will err. But this is easy to notice. However, the philosopher [Aristotle] investigates there the cause, and concludes that it is because, in the suspended ones [top-pivoted balances], the rising and elevated part of the beam will be longer than the part that has dipped; hence it descends [returns] when the weight is removed from the other end, and the reverse in the second kind, where the dipped part is [longer] than the elevated.

To grasp the matter, you should understand that the beam has a [certain] breadth, so that the points from which the pans hang are not truly on a straight line with the center on which the suspension or standing-weight rests — for [in the top-pivoted balance] the center is above, in [the dimension of] breadth, and the points where the pans hang are lower than it, as hangings; and the reverse in the ancient balances. It is easy to understand the matter; or [picture it as] the ear [pivot] being broad, with the beam resting upon it. But if the beam is along the geometric line [an ideal line of no breadth], then in any case the pans will not change from their position [equilibrium]: wherever they are placed — whether inclined or hanging — they will stand there and not move at all, because the

upper part of the beam is equal to the lower part, and there is no power [tending] either to raise one or to lower the other.

And from here [comes] the resolution for those who raise difficulties against Aristotle, who would say that the closer a thing is to the center of the earth, the heavier it is — for according to his [own] words, in the figure before you [the figure is on a separate plate — [?] not verifiable] (*see* ע״ב [diagram 72]), pan י [Y] is closer to the center of the earth than it [otherwise] is. By rights [then] pan ו [V] should never rise upward [back]; and if it is tilted once, even though the weight is removed from pan ו [V] — [yet it does return]. The reason for its returning is on account of the breadth of the beam **אהב** [A-H-B], whose ends, in their tilting, the lines drawn from their extremities to the center of the world cause [them] to incline [?]. For pan י [Y] is closer to point **לג** [L-G?] [?] of the center of the earth than [pan] **ה** [H], whose course [path] is longer; hence pan י [Y] is heavier and descends of necessity, and consequently pan ו [V] rises to its place, until the gnomon [beam] is on a straight line **דב** [D-B] between the two pans. And know that when the pans are hung, the lines coming from their extremities to the center of the world meet each other; but if the pans are one piece with the beam, they will always be…

these parallels. For even though one rises and the other descends and comes a little nearer to the center — as in [the case of] the balance — the heavier weight will not add any perceptible motion to the descending pan until it is heavier than its counterpart. Therefore, if the beam were merely as [thin as] a line, in the geometers' sense, then once the pan had descended a single time it would not rise [again]. And this is enough for the matter at hand.

[Paradox (44) מד:] The Question. How can a thing move without contact? This is one of the truly astonishing things to every wise-hearted person. To explain the matter, I must first set before you certain teachings drawn from the foundations of the science of music. For if there is in every science a small portion that is found in the Holy Tongue [Hebrew], come — [here is] neither too little nor too much — and I will inform you that one of the most weighty of the sciences is one in which teachers do not instruct their pupils in the way it was found, when its first founder was one who enlightened the eyes of the discerning greatly. Instead, they bring the matters [already] cooked and seasoned, like food with spices, so that the pupils do not know which dish they are eating. And I shall set before you the rudiments of music according to two approaches.

The first approach. Even though the Holy Scriptures relate that Tuval-Kayin [Tubal-cain] was already versed in the science of music, nevertheless, after he

perished in the Flood [literally: when he came to the flood of waters] together with his pupils, the science was forgotten. Pythagoras the Greek [פיתאגורס היוני] revived it and lifted it from its dust. It is told of him that he labored greatly over it but could find no way to enter its chambers — although he did observe that in the blending of a heavy sound with a sharp [high] one, until the one was reckoned as another sound — that is, when differing notes agree together — a wondrous proportion was heard, pleasing to the ear that hears it, which is the [very] proportion of music. This is established from the words of Boethius [בואיציאוס], in the first treatise [*De Musica* I], chapter 3. Conversely, when the notes do not agree and come to the ear separately, confusion and distress befall the ear; and he did not know the cause of the matter. For many days he toiled and found nothing.

And his occasion befell him thus: as he entered one day into the workshop of a worker in iron, he saw five men striking with hammers upon an anvil [סדן], and he perceived that the striking of some of them together [produced] a pleasing sound. After he had tested them two by two, he set the fifth aside as either deficient or excessive, and he weighed the rest, and found that the first weighed 12 *litra*, the second 9, the third 8, and the fourth 6.

And behold, it has been explained in the science of number — as I wrote in *Chukos Shamayim* [*The Statutes of Heaven*] — that four numbers, taken two by two, will not be in proportion in any but six ratios, as [shown] before you:

	א [A]	ב [B]	ג [C]	ד [D]
ד [D]	9. 12	8. 12	8. 9	6. 9
	(and:) 6. 12	6. 8		

And behold, you see that, according to the weights of the aforesaid hammers, there are only four ratios in all, for the ratio of א [A] to ב [B] equals the ratio of ד [D] to ג [C] [recte: ד to ג], both being [as] 4 to 3 [recte: as 12 is to 9, that is 4:3]; and likewise the ratios of א ד [A·D] are equal, both being as 12 to [?] [recte: 12 to 8], that is 1½ [3:2]. Therefore all the ratios are comprehended within four, and accordingly this number may be called holy, as I wrote in [*Chukot*], so that you will find no musical ratio except either from these or compounded from these.

Pythagoras went on and tested this matter with all vessels, whether resonant or hollow, whether strings on lyres [כנורות], on bells [פעמונים], on pipes, or on cymbals [צלצלים] of [various] kinds, and found them in accord. He built up from this a general principle: that **as is the ratio between the resonant bodies, so is the ratio between their sounds** — that is, [the ratio] of the vessel by its weight, or by its length, or by its hollowness, or by its blowing and whistling, is a doubled

ratio. For example, [if] 2 is to 1 [for] two vessels which, when struck at the same time together, sound to the ear a pleasing and agreeable note — this is called the *diapason*. And if their ratio is the ratio of 4 to 3, the note will be that called the *diatesseron*; and if as [3 to 2], it is called the *diapente*; and if as 9 to 8, they will sound the note of the *tonos* [whole tone].

After he had accustomed himself [to this] and could recognize these notes [by ear], the pan that heard the note of the *tonos* from the striking of two hammers, he would determine that the one weighed 9 parts and the other 8, and so with all of them. The whole business of music lies in ratios; and [its] increase and diminution lie in [the] dividing —

— and do not seek in musical ratios division and multiplication in which there is no halving below. As in the *diapason*: even if it has [no further] division below it, [its] half is [still] the *diapason*. And if the *diapason* is doubled — for example 4 — multiply it by 1 and you find 2, which is the simple *diapason*; and the double of 2 is 4 doubled.

Now, he employs addition: for if one adds 1 to 4 — that is, *diapente* with *diatesseron* — it yields ⅔ [?], which is the *diapason*. And in subtraction: if one subtracts ⅔ — that is, the *diatesseron* — from the *diapente*, one finds ⅛, that is [9:8], which is the *tonos*; for the *diapente* exceeds the *diatesseron* by the measure of one *tonos*. And likewise one subtracts ⅔ from 1 [the *diapason*] and finds 1½, that is [the *diapente* remains]. Conversely, if one subtracts the *diatesseron* from the *diapason*, the *diapente* remains. And if one subtracts the *diatesseron* from the *tonos* [recte: from three *tonoi*], there remains a small part called the *apotome* — for the *apotome* together with the *diesis* [ריאישיש] make another *tonos*. Therefore the *diatesseron* is compounded of two *tonoi* and a *diesis* [recte: two *tonoi* and a *diesis*].

And I will explain these names to you briefly.

Tonos [טונוש] means a peal or roar produced by the striking of air, [where] the part of the air near [the source] is [moved] by the force of the far part from it in succession without interruption. The musicians [מוסיקוי] have taken to call it the interval or measure between two vibrating strings; it makes small and large circles, like a stone cast into water until it spreads out and reaches the [far] shore. The Greeks [call it thus], or [the interval] between two notes [is called] *tonos*. For example, if one player sounds the note *ut* [אוט] and another with him sounds the note *re* [רי], the sound perceived from the blending of the two notes is called *tonos*; and likewise *re* with *mi*, and so on. And the *tonos* is compounded of two

unequal parts: the first is called the *half of the greater tone* [chatzi ha-tonos ha-gadol] and the *apotome*; the second is called the *lesser half* and the *diesis*. The half of the *tonos* is what is heard from the note *mi* to *fa*. And the difference between the two parts of the *tonos* is called the *comma* [קומה]. Therefore two large halves exceed the *tonos* by the measure of a *comma*, and two small ones fall short of it by the measure of a *comma*. Half of the *apotome* is called the *diaschisma*, and half the *comma*, the *schisma*. The *ditonos* is the interval between three [recte: among] vibrating strings or successive notes — for example, between *ut* and *mi*, or between *ut*, *fa*, and *la* [?]. The *semiditonos* is the interval among three vibrating strings or successive notes — for example, *re*, *fa*, or *mi*, *sol*. The *tritonos* is the interval of [a] note [spanning] three *tonoi* in ascent and descent — for example, *ut*, *fa*. The *diatesseron* is a comprehensive musical ratio [comprising] the *ditonos* and the *diesis* — that is, three intervals among four vibrating strings or notes, by way of example, *ut*, *fa*. The *diapente* is a musical ratio compounded of three *tonoi* and a *diesis* — that is, [spanning] five notes or strings, and [being] the interval between two like notes — *ut*, *ut* — since *fa* resembles *ut*; the latter *ut* is called [the eighth] intervals, as in *re*, *la*. The *diapason* is a musical ratio compounded of the *diatesseron* and the *diapente* — that is, an octave [literally: eighth] in the first ratio, the heavy [low one] being the first of [its] vibrating strings.

And I ask that you know that, with the mathematicians [literally: masters of number], 4 with 5 make 9; but it is not so with the musician, for [these] are no more than 8, and likewise 8 with 8 with him are 15 alone. This is because the end of the *diatesseron* is itself the head of the *diapente* — as if *ut, re, mi, fa* is the *diatesseron*, [and] *sol, re, mi, fa* the *diapente* — and they are divided into nine [recte: among nine] notes; that is, *ut, re, mi, fa, sol, re, mi, fa* — the *diapason*, [comprising] eight. Or, conversely, up to *sol* the *diapente*; and after that *sol, re, mi, fa* the *diatesseron*, [each] one, and the two *diapasons* make the note called the fifteenth [quindecima], which is no more than 15 strings and 14 intervals. Therefore the *diapason* is compounded of six *tonoi* and a *comma*, and has [seven] intervals among eight strings, and it is called *diapason* — "through all" — since all the ratios are comprehended within it: the *tonos*, the *semiditonos*, the *ditonos*, the *diatesseron*, the *tritonos*, the *diapente*, the *tonos* with the *diapente*, the *semiditonos* with the *diapente*, the *ditonos* with the *diapente*. As is shown [when], if one subtracts part by part from the *diapason*, the remainder will manifest those [ratios] comprehended within it. And [there is] yet another reason for this name, as you will hear, please God.

And the [second] approach to music is this: that you must know that the [foundational] ratio —

— is the foundation and beginning of [musical] notes; that is, [there are] two notes, alike, [made] by setting in motion some [string] that contacts and separates [vibrates], according to whose distance from it the ratio between them is reckoned. By "contact" [מתרבק] I mean what the philosophers call "by way of expansion" [התפשטות] — for example, if one and the same string is plucked, whether softly or strongly, the note is one [and the same], and the strong [pluck] does not differ from the weak except by the expansion of the air into more circles or fewer, as we explained. And by "separation" [נפרד] I mean what the philosophers call *hefetz* [pressing/stopping] — that is, when the note changes from heavy [low] to sharp [high], or the reverse. And in the separated note and its variations lies the whole business of the science of music — not in the [mode of] contact. And all the intervals between the heavy and the sharp, and between the sharp and the sharper, and so on, are comprehended within the *diapason*. For, as we said, within it you find all the ratios, the pleasing and the well-ordered ones, as well as the imperfect ones — for example, the *semiditonos*, the *ditonos*, the *semitonion* [?] with the *diapente*, the *tonos* with the *diapente* — even though Boethius assigned them ratios in large numbers.

And to demonstrate the matters according to their true measure, the ancients devised the instrument of the **monochord** [מונוכורדו], to teach beginners the ratios of notes, their agreements and their disagreements; and it is therefore also called *regula harmonica* [ריגולא ארמוניקא] — that is, a single string, simple, stretched out upon a smooth, straight, even piece of wood, fastened at its two ends. And [thus] they would [demonstrate].

But if you wish to construct an instrument [played] by knowledge [intentionally], you must stretch upon it two or three strings, equal in length, and let them be sinews of iron [wire], as is the custom with the *cithara* [כלי הציתרא] **(Figure [2] ב)** — not of the gut of sheep, which contract and slacken with the changing of the air. And let there be in the [present] instrument [before us] the strings א ב ג ד [A·B·C·D], equal in length and drawn at equal tension, so that the one sound is equal to the other, as if it were one and the same note, [pleasant] to hear, by creation [nature] and design. Let us divide one string, גד [GD], into two equal halves, and set at the point ה [H] the wood called the *tasto* [טאקטו, the fret], at the point ה [H] in its middle or at a distance, and let us press down the end ה [H] with our finger in such a way that the part ב [B] does not move, as if there were only half the string, דה [DH]. And at one and the same time, with two fingers, let us pluck both at the whole [string] אב [AB] and at the half דה [DH], and we shall hear a note exceedingly pleasing, beyond all musical proportions — and it is the *diapason*, which the ancients named so because their lyres were of eight strings.

To sound the note of the *diapason*, one had to leap from end to end — that is, from all the [intervening] strings — and touch the heaviest of all and the eighth of all; and this is also called a doubled ratio, since the string אב [AB] is double the string דה [DH]. And as the resonant bodies [stand] one to the other as 2 to 1, so do the sounds arising from them [stand] one to the other, as we explained. The common run of musicians call this note "the eighth" [octave] — that is, the eighth of the note *ut*, the heavy one, which is called by the Greeks *ipate* [איפטי], is the sharp note *fa* called *nete* [ני"טי]. For such is their order: *ut, re, mi, fa, sol, re, mi, fa*; and in Greek: *ipate*, *parhypate* [פרי"פטי] (after the head), then *lichanos* [ליכאנוס] (the index-finger note), *meson* (the middle), *parameson* (after the middle), *trite* (the third), *paranete* (before the last), *nete* (the last). And the two extremes sound the *diapason* — that is, a doubled ratio.

Again, let us divide the string דה [recte: דה] into [three] parts at the point ז [Z]; let us distance [the fret] by it, and at the same time pluck the whole string אב [AB] together with the part [הז / recte: זה], and we shall hear a pleasing note — and it is the *bisdiapason* [רישריאפשון], that is, the double *diapason*; for the ratio of אב [AB] to הג [recte: הג] is a *diapason*, and likewise the ratio of הג [recte: הג] to חג [recte: חג] is a *diapason*. Therefore the ratio of אב [AB] to חג [recte: חג] is double the *diapason* — I mean, 1½ make ¼ [?] — and the musicians call them the note of fifteen notes, for such they are from *ut*, the heavy one, to the last *fa*: fifteen strings or notes, as he will explain afterward.

Again, let us divide the string [גד] into three equal parts at the point [ז], and let the measure of [ז]ד [Zd], the equal part [?], be equal to the string [ב]א [AB] in the aforesaid division without any difference whatever. Then distance [the fret] by it [at point ז], and pluck together with the string [א], and at the string [ו] we shall hear a ratio of [a] pleasing and complete sound — but not [pleasing] as the first ones — and this is what is called the *diapente* — that is, [arising] from the plucking of the first string and the [fifth] together; and it is the ratio [3:2] —

— that is, that the string וד [VD] will span [measure out] the string אב [AB] 1½ times; therefore the ratio of אב [AB] to וד [VD] is as the ratio 3 to 2, and they named it the heavy note *ut*, the first, and the note *sol*, the fifth.

But if you pluck only the third part of דג [DG] — that is, [the part] חג [CG] [recte] together with the whole string אב [AB] — you will hear a pleasing note compounded of *diapason* and *diapente*; for the ratio of דה [recte] to [the part], with the ratio of [ה] to [ו], make the ratio of ה to [ו] [?]. I mean, therefore, that the ratio of אב [AB] to [ד]ו is compounded of two ratios [22 ratios — recte: of two ratios].

Again, let us divide the string גד [GD] into four [recte] equal parts, and take ¾ at the point [ז]ד, and pluck the string א [A] together with the [three-quarter part], and we shall hear a musical ratio — except that it falls short of the first ones in completeness and pleasingness — and they named it the *diatesseron*. Its meaning is that it arises from the plucking of the first string and the fourth. Therefore the ratio of אב [AB] to ד [D] is the ratio 4 to 3 — that is, that אb [AB is] 1⅓ times the string [זד / וד] — and it is called by the moderns "the fourth," the note arising from *ut*, the first, [to] *fa* [recte: re], the fourth to it.

But if you place your fingers at the point ח [Ch] of the lowest string and pluck together דו [recte: דה] with ¾ [of] דח [recte: דח], we shall hear the ratio of the *tonos*. For the ratio of דה [recte] to דן [recte: דח] is the ratio of 9 to 8 — that is, that the string דח [DCh] is 1⅛ times the string דן [recte: דה]; for if one player sounds the note *fa* and the second the note *sol*, they will sound the *tonos*. And these are not [pleasing] to the ear, since these two notes are not in agreement together and come separately to the ear.

It has now been explained to you, then, that the musical ratios are five: three simple ones — the *diatesseron*, the *diapente*, the *diapason* — and two compound ones — the *bisdiapason* and the *diapason-diapente* [ריאפשונדיאפינטי]. As for why two *diatesserons*, or two *diapentes*, are not [valid] ratios, Aristotle explained in [his] *Problems* [שאלות הטבעיות — *Problemata*]. And now I will inform you of the order of placing the letters

A B C D E F G

upon the monochord — and it is after [I have set forth] the principle for you, that not all the *tonoi* are equal, as will [now] be explained.

First: Take the whole string דג [DG] and divide it into nine equal parts. At the end of the first part of them — that is, at the point י [Y] — there place the first *tonos*, that is, the letter A. Therefore [the part] ד[י] is 1⅛ times [the part] יד [recte: דג], the remainder; and the interval of the first *tonos* is יג [recte: די] — that is, what [lies] between *ut* and *re*; and with the ancients [it is] *hypate hypaton* [איפטיאיפטון] — that is, the head of the heads.

Second: Divide the remaining string יד [YD] [recte: די] into eight equal parts, and let one part of them be the interval יצ [recte], which is the second *tonos* — that is, [the interval] between *re* and *mi* — and let there be the letter B at the point צ [Tz].

Third: Divide the whole string גד [GD] into four equal parts at the point [צח], and there place the letter C — for the interval צח [TzCh] is the half-*tonos* — that is,

[the interval] between *mi* and *fa* — [which] is the lesser half of the *tonos* called the *diesis*, that is, [one] part; and the whole interval [גח] is the *diatesseron*.

Fourth: Divide the string גד [GD] into three equal parts, and at the point ז [recte: ן] place the letter D. The interval which is between [the points] — that is, לח [recte: וח] [?] — between *fa* and *sol*, is the *tonos*, the third; therefore the whole interval גן [recte] is the *diapente* — that is, the *diatesseron* with another *tonos*. And behold, these then are the five notes: *ut, re, mi, fa,* — all of them — and the remaining ones, *sol, re,* —

Fifth: Divide [the part] of גד [GD] — that is, the string וד [VD] — into nine equal parts, and the first part, which is found in the fourth [position], [ק]וד [recte], is equal to [the analogous part] בה [recte] in the aforesaid division without any difference whatever; therefore וה [recte] is equal to לגח [recte: לגח] [?]. The interval ⅔ of גד [GD] — that is, the string וד [VD] — [recte: ⅔ of] is the interval of the fourth *tonos*, which exceeds the *diapente* — that is, what [lies] between *sol* and *re* — and at the point ר [R] place the letter E; and the interval צר [recte] is another *diatesseron*.

Sixth: Divide the string רד [RD] [recte] into nine equal parts, and at the end of the first part of them place the letter F, and call it *mi*; and the interval which [lies] between *re* and *mi* is the fifth *tonos*.

Seventh: Divide the string [גד] into [two] halves at the point ח [Ch], and there place [the letter / note] — and they found the *semitonion* [שימיטונ], or the second [half], which is between *mi* and *fa*, and which is the *diatesseron* doubled up to *fa* — that is, from the point ה [H] to [ח]ך [recte], equal to the *diatesseron*. The interval וח [recte: וה] is a *tonos*; therefore two *diatesserons* with a *tonos* make another *diapason*, [ד]ה [recte] — that is, [when] one sets in ratio גן [recte: גח] with the whole string אב [AB], it is a *diapason* ratio.

This is why the Greeks call the point ה [E] *mese* [Greek: μέση, "middle"] — that is, the middle of the string. And so we have shown that five *tonoi* and two *diesis* make up a *diapason*.

Just as the seven letters of the lower-register tones were found on the monochord, so seven letters can be found for the higher tones when the string דג [DG] is divided according to the first division — that is, first the whole string דג [DG] is divided into nine equal parts, and one of these is the first *tonos*; then the remainder of the string is divided again into nine equal parts, and the second *tonos* is found, and so on. The very highest tones are found when one takes a quarter of the string גד [GD], and likewise half the quarter and half the eighth.

For the eighth [octave] is reached when the first tone returns — just as after the seven days of the week the first day comes round again, and so forever. There is no other interval like it among all the instruments.

Therefore, to find the *diapason* of any given letter on the monochord, begin from that letter, take half the distance between it and the string's end, and at the halfway point you will find the eighth letter. This follows the principle established in [Euclid] V.19: that if the whole דך [DK] is to the whole הך [HK] as the part removed גן [GN] is to the part removed הצ [HTz], then the remainder וך [VK] is to the remainder ניחר צך [TzK] as בד [the whole?] is to הך [HK]. [?] [The garbled point-letters here trace the proportional construction; the underlying ratio is the octave 2:1.] Therefore at the point צ [Tz] lies the eighth of the first — that is, שך [the whole] is double the half צך [TzK]. The discerning reader will accordingly divide the string into halves, quarters, and eighths into however many parts, easily working it out from the ratio of its parts to the whole string גד [GD] or אד [AD], which equals it.

The ratio of the *diapason* is the most important of the ratios, and almost natural — for even peasants who play instruments without having studied its proportion all know how to tune to a single tone, this one heavy and that one light, without distinction; for the eighth shares the nature of the first, as we said, until those who hear them are pleased. Aristotle posed a question about this in Problem 35 of the *Problemata* [*Problems* XIX], and answered along the same lines: that the ratio of the tones of the *diapason* is in whole numbers, 2 to 1, unlike the other ratios, which are of fractions together with whole numbers. From this comes a proof of the superiority of number over geometry.

On this basis, the *diapente* — whose fraction we explained above among the ratios... The *diatesseron* is reckoned a whole ratio, and it has standing among the ratios, being larger by ½ [?] [the half-step margin] — more whole, and this more than the *tonos*. The *tonos*, as we have already shown, is found on the monochord more often than the *diapente*. [?] We will distinguish between the [Latin: b — the b/♭] note and the squared [♮] note, and by it [Latin: ♭] we arrive at knowledge of the *apotome* and the *diesis*, the divisions of the *tonos*. In truth it is no musical ratio at all, since it is not in proportion with the first low tone, just as it is not in proportion with the eighth that equals it — as Boethius wrote in Treatise II, chapter 24: "A *diapason* combined with a *diatesseron* does not make a musical ratio or consonance, but only a [Latin: ?] and a scattering of tones." Thus far his words.

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After these preliminaries, I will bring you Aristotle's questions — the 24th and the 43rd — which resemble one another and concern a single matter, and from there I came to the knowledge of that wondrous thing you asked me about: how a thing can move without contact, and so on.

He asked there the reason why, on a lyre [Latin: lira] or something like it that has many strings: if, while one is playing and the string called *nete* [Greek: νήτη — here "the eighth"] is vibrating, we wish to stop it with our hand and silence it, the sound of the *hypate* [Greek: ὑπάτη], called "the first," is heard. He gave there several answers. One: that these tones are alike, as is seen with players who pass from *hypate* to *nete* easily, since it is as though [the *nete*] were a *nete* [octave] of [the *hypate*]. Therefore, when the *nete*'s sound dies away and weakens, the *hypate*'s sound is heard. Yet we are mistaken and suppose that the first string, the *hypate*, is in truth sounding the tone — not the *nete* — since we are stopping it with our hands from playing. This is like one who sees a raven painted on a wall and below it a frog croaking, and thinks the raven is the one croaking, since he does not see the frog with his eye; so too those who watch are deceived, the eyes deluded by various tricks — for the *nete*, being very drawn out and extended... [?]

The Philosopher's second answer: that it must necessarily be so, for in order that a single tone be heard at its striking, the air and the instrument it is in, together with the rest of the strings, vibrate and sound the weak tone. And although the *nete*'s sound is by its origin different from all the tones of the rest of the strings, it is nonetheless wholly like the *hypate*; therefore the *nete*'s sound and the *hypate*'s sound combine and become one sound, and the sound of the rest of the strings is not heard at all because of their weakness. This is the sense of the Philosopher's words there.

Now the chief point — he did not grasp the matter well, despite all his philosophizing. For our eyes see that if you test it on any well-tuned, well-proportioned instrument — be it a clavichord [Italian: *clavo* / clavichord], a viola, a cithara, or whatever instrument you please — and place a very small, thin piece of light paper on a string, then strike the string nearest to it which makes a *tonos* or a half-*tonos* with it, the straw [the paper marker] will not move and will not fall. Likewise if it is distant from it by the interval of a *ditonos* or *tritonos*, it will not move at all. If it is at a *diatesseron* — that is, a fourth — from the struck string, it is possible it will move slightly; if at a fifth, which is the *diapente*, it will move more; but if you place it at its eighth — that is, at the *diapason* — it will fall without doubt from the strength of the movement and vibration. Now if the cause were the trembling of the air and the strings when some string is struck, who

would not know that the nearer one would tremble more? Yet we see the opposite — that the eighth, which is farther off, [trembles more]. I have tested this a thousand times on the clavichord. So Aristotle's reasoning in the above question is not correct.

And here I have great inquiries to make into the nature of this air that surrounds us, which is most wondrous. You see how many things pass through it in a single instant, with none obstructing its fellow — that is, sounds, scents, and various sensibles meeting one another; and further, that it appears to split through walls and pass on. It is known that the visible sensible does not act on the sense, nor is it seen, except along a straight line, as is shown in optics; but sound, even along a crooked line, passes through thick walls and is heard outside the houses, even though all the windows and openings are stopped up.

[A man bends a piece of wood or a bow, and afterward releases it, and it returns to its straightness.] Many of the great sages have inquired into the cause of its return and devised various theories. In truth there is no proximate cause in the air, that when the wood is pressed in its curvature the air pushes and stretches it until it restores it to straightness — for when the wood is bent and made rigid and then the bender lets it go, one cannot say that it returns to straightness of itself, since its essence is in potentiality alone to being straight, not in actuality. And no action comes about except from the actual, which is in act. You already know the great dispute among the philosophers concerning hot water — how it returns to its coldness — and the stone thrown upward: who brings it down? For according to Aristotle's words, heaviness in the elements is not an active principle but a passive one — that is, potentiality alone — as appears in *De Audientia* [the *Physics*], Treatise II, chapters 20 and 50 [?], and in Book VIII, chapter 4, and in Book... of *De Caelo*, chapter 4.

Therefore, in his view, the stone falling downward falls by the agent external to it, which is in act, while it itself is in potentiality to the motion; and when it is below, it is in actuality of its heaviness — unlike when it is outside its place, where it is in potentiality alone. Nor can one say that in the wood there is a part that does not bend, which is straight in act and restores the [other] parts to straightness — for that part is nothing but a point, since [the part] that is in the bowing of the curve does not touch [the straight line] along a straight line but only at a point, from a point to a third [?]; and how would it restore all the curved parts?

The natural things derive from God, blessed and exalted be He, the power by which each rests in its natural place; for the bent wood, being long, hastens in its

motion when it returns to straightness. Perhaps one might add that the natural power, the more it is distant from the subject on which it acts, the weaker it grows — and we see that everything... [?]...and stands against the one pushing it with all its strength, and if it is pushed it returns to its place along a straight line, that is, by the shortest path. For it is not fitting that everything be everywhere, since then things would be confounded; and so a place was assigned to each, in which is its endurance — as the Philosopher will say, that place sustains what occupies it. Likewise

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the parts that are in each [thing] endure within it, and when they are separated they perish; and so they preserve their nature and do not change their order of place, but cling tightly to it; and if they are uprooted from there by force, they return swiftly. So too it happens with the bent woods and the bows.

But what I said seems to me more truly correct: that the falling of the stone and the cooling of water are reckoned by many philosophers to come from the air. Now it is known that the strings differ in the breadth of their stretching — for the eighth [string] is the most drawn out of them all, and the first the opposite. Therefore, when the air is set in motion by the *nete*, the string most slack in its stretching is moved, and that is the first. There is for this answer remoteness [a difficulty?] which every student will understand; and on this basis I shall hear what others say, and the matter requires further study.

Now in another respect this motion appears to be sympathetic [Greek: συμπάθεια] and imaginary, its cause hidden from us — and that is in the magnet [literally "the stone that draws iron"]. I knew this matter beforehand from books I had come across, by chance, when I was with the celestial globe, observing the stars at night, and I wished to set the globe aligned facing the regions of the world. The magnets in its socket kept falling beneath it and getting hidden from my sight, so I took for a moment a magnet that I had by me and brought it close to the globe to make use of it. I saw that, as I turned the instrument, the magnet of the instrument inside the globe moved according to its motion — forward and back and round and round at my will. I was astonished to see this. And when I tried to move it away to a great distance, two or three cubits, this one had no effect on that one. So I recognized that the magnet's effect, at the breadth [reach] of a globe, was not great.

When I searched in books, I found that others before me had also tested with magnets important things. They took two pieces from one piece — or armed two

irons from one piece — and saw that one moved according to the motion of its fellow, like a second [moving with its] light, until they reckoned that if the two irons were placed in two round vessels on which were the 22 letters of the alphabet, then when one man brought an iron knife near the magnet, it would turn after him until he brought it to the letters he wished; and another man, watching the second vessel at the end of the earth, would at that same instant see the magnet standing at those letters and assemble them, and so know what his fellow wished to signal to him. If the magnet's power were to extend so far, the device would be precious and wondrous beyond all wonder — except that, as I said, I myself tested it only at very close range. Elsewhere I have expanded on the operations of the magnet and revealed all its properties known to me by experiment.

For if I were to set down the reports, I have many — such as displaying the letters to the moon and doubling them for one who looks at it [the moon]; or what they say, that if at the end of the earth some man's nose or ear is cut off and another's flesh is grafted onto him (a thing well known in Italy, where they make new limbs when they are cut off — but they make them from one's own flesh; and if another is willing to give him flesh from his arm, as if he were his servant or for money's sake), then afterward every sensation the other man has, this man will feel in the new limb — so that, by their account, when a needle is stuck into this one's flesh, the other feels the pain in his own flesh. These things are far from reason. And they say that if the servant dies, when his body decays, this man's nose falls off. Such reports cause laughter, not knowledge.

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How is it that laughter is heavier than weeping?

The Philosopher asked in his *Problemata* why it is harder to sing in a low voice than in a high one — for the high is found in women, in eunuchs, and in children — and one would have thought that more strength is needed for a heavy voice than for a high one. But it seems to me you have not, in your day, entered the sea or a *mikveh*, where you would have seen that the waters above you are many and high, yet they do not weigh down; whereas a small vessel full of water weighs and cannot be borne. And if you consider why a person struck by a stone in the days of frost feels pain and feels it more than in the time of heat, you will understand the reason of the matter: because his flesh is almost congealed from the cold and hard, and does not yield before the one striking — unlike in the time of heat, when his flesh is soft and yielding, and the blood flows this way

[The page ends mid-sentence; the discourse continues.]

The discourse continues on the following page with the answer to the "laughter heavier than weeping" question, then a new question on optics:

Another question: How is it that a sensible object opposite us — except that between it and us there is an intervening screen, or it is behind us, or likewise above or below — appears facing our eyes? The answer: If I came to speak of the science of mirrors [catoptrics] and their wondrous operations, I would need a large treatise, and the present time does not allow for length; but I will bring you the chief points, which are necessary for the answer to what is sought. They are these.

It has been clarified in philosophy that what is sensed by the senses is an accident, not a substance — for sensation is an undergoing [passivity], and every passion is between opposites, and there is no opposite to a substance. The accident that acts on the sense of sight, primarily and firstly, is color, which is on the surfaces of bodies — this with the aid of the light that illuminates it; and the other objects of the eye's sight are sensibles only incidentally, since all of them are sensed by means of light and color.

Now colors, or hues — according to the view of some — are composed of certain hues, by the combination of two or three colored glasses; and the simple ones are their extremes, namely white and black; while some are intermediate, or compounded of them. In our time a way has been found to mix the colors in this manner, and the painters [do so]; but this knowledge is philosophical, not truly mechanical. The brilliant ones among the intermediate colors are red, like blazing coals, and "carpesi" [?], and the purple or *porphyrios* [Greek: πορφύρα, "purple"].

Now the things seen incidentally are many, and they may possibly be subsumed under twenty, which are primary in the judgment of the experts in sight: some under light — like brilliance and hardness [opacity?], shadow, and darkness; some under magnitude — under which fall the size or smallness of a bodily figure, beauty, ugliness, smoothness, hairiness, likeness, difference or variation; some under place — under which fall position, adhesion or [?], division or separation, number, motion, rest; and some under both — that is, both in magnitude and in place, like distance and nearness.

Aristotle, in *De Anima*, in *De Sensu et Sensibili*, and elsewhere, named the sensibles common to several senses, and they are five: magnitude or size of figure, motion, rest, number. The manner of sight is that the form of the sensible

is received in the glass [lens] of the seer's eye, and is grasped — that is, brought to the common sense — for in truth the true hue is a hue even though no man sees it, but it is not actually sensed if there is no eye seeing it. And if its likeness were in the eye of the beholder while his heart [attention] was entirely on his own concerns, he would not see it; for if the mere arrival of the sensible at the eye alone were sufficient for sight, then a glass mirror too would be seeing.

This view is the most accepted among the philosophers — such as Democritus, Leucippus, Epicurus, Aristotle, Alhazen [אלאצין = Ibn al-Haytham],...that [the form] extends and arrives at the sensible, and not that the form of the sensible is brought to the eye; and Pythagoras, and so on. Although Euclid, Hipparchus, Ptolemy, and the sect of the Stoics held that the visual ray goes forth toward the sensible; and Plato held that for sight both are needed — that is, that the visual ray goes forth to meet the form of the sensible coming toward it, and they meet at the midpoint of the distance, and from the mixture of the two together with the illuminating light, sight comes to be. Each has his proofs; it is not my intention to decide, but the view of Aristotle is the more firmly established by reasons and arguments.

The common ground agreed between them is that the bodily sensible does not come to the eye — for on that view it would not be seen at one time by many eyes, since one thing is not in many places at one time; all the more so when the sensible does not move from its place, and likewise the form, which is...

[The page ends mid-sentence; the optics discourse continues.]

Sefer Elim — Ma'ayan Chatum (Fifth Part: Paradoxes)

Optics / Vision (§46, continued) — image-transmission, perspectiva, and the science of mirrors (catoptrics)

of the *Rabbi* [i.e. Aristotle], that is: of what nature it is, and how the accident is set in motion without a substrate. For one who says that the image of the perceived object is brought to the eye is like one who says that the seal-imprint passes out of the signet-ring into the wax — whereas in truth the image of the seal, which is in the signet, is stamped into the various indentations of the wax without the seal itself separating from the signet. In the same way the image of the perceived object is brought to many eyes by means of the luminous medium; and this is

why the image of the perceived object is called *spiritual* and not corporeal. For this reason, on Aristotle's view, an object placed directly upon the eye — touching it without any intervening air — is not seen. (And though sufferers of cataract — כטה"קאטר ה [Italian: *la cateratta*, "the cataract"] — bear witness, for they see the needle at the moment the physicians introduce it into their eyes in order to scrape away the humor that lies between the membrane and the [retinal] coat, which serves as a screen separating them from the visibles —) [the case is exceptional].

Now vision is of two kinds: **simple** or **compound**. Simple vision is when the perceived object is directly opposite the sense — that is, when the rays issuing from the surface of the object come in a straight line to the eye. Concerning it there are many *theorems* among the masters of optics, and many deceptions and errors attach to it. **Compound** vision is that which arises from two kinds of rays: one falling straight, the other obliquely, until it reaches the eye, both of them coming into a single point. By this kind of vision an object not directly opposite the sense is seen — something not seen in simple vision. Yet what is seen is not the perceived object itself, as in simple vision, but only the likeness of its image: as one who sees a star before it has risen, or a ring or coin within water.

Compound vision is partly *reflected*, partly *refracted* (lit. "broken"), and partly a mixture of the two. That is: the rays coming from the perceived object to the eye either come, strike some surface, are reflected back from it, and so come to the eye; or else they are broken in the middle of their passage as they spread out, deviate from their straight path, bend, and so come to the eye even though it is not directly opposite them. **Reflection** of the rays, then, is the return of the original light to the eye of the beholder: as when the sun — which is the primary light — strikes with its rays the body of the moon, and its rays are reflected back to the lunar earth [i.e. the moon's surface]; for the lunar globe takes its benefit from the primary light, and we, when we see the light of the moon, take benefit from the secondary light — that is, the reflected light of the sun.

Now the body suited to reflect is the **hard** [smooth, polished] one. The translucent body does not reflect unless it is very dense, and this only when it is not too distant — as was explained in optics, that the diffusion of the light-ray is bounded and does not proceed to infinity. Therefore a body very distant from the light cannot reflect its rays, which do not reach it; but when it is near, it reflects them, like a wall that sends back the ball striking it. And if this reflecting body is a rough body, uneven in its surface — that is, having pits and protuberances — the rays are scattered in their reflection, and on this account it reflects only the light

and the color, as we see the sun striking through a window onto a wall: the light is reflected from it inward, to the chamber and the corners of the house, but it does not reflect the image of the sun itself. So too does the moon behave.

But if the body is smooth, it is called a **mirror** (*mar'eh*), which reflects not only the light and the color but also the image and the very likeness. Had the lunar body been smooth — rather than having clefts and concavities in it — the solar ray reflected by it would burn up the whole world, as do the burning mirrors. The bodies fit to serve as material for mirrors are of many kinds: of glass, or of fine stones, as **Pliny** [Plinius] writes in Book 37, chapter 10, concerning **Nero Caesar**, who used to look [at the gladiatorial games] in [a] שמא"רגדו [Latin: *smaragdus*, an emerald] — or [mirrors] of silver and gold, as Pliny writes in Book 15 [recte: 33], chapter 33, that they made in the great age of war, the warriors and the wounded [seeing] one another; or of cast lead, or of bronze [*Brill*?] alloyed with copper, which is the great *Pompeium* [Pompeian bronze?], or of marble stone and of woods; also liquids such as water and oil; and even the thick air. [These readings of the alloy-names are uncertain — see Uncertain readings.]

As [a man] walked along, he would see his own likeness before him; and each gave his own explanation according to his view, but the truth is that his likeness was [reflected by] the air, which served as a mirror [for] him. **Aristotle** wrote [of this] in Book 3 of the *Meteorologica* [De Caelestibus Signis / "On the upper signs"], and in the *Book of Memory* [De Memoria], concerning *Antipheron* (אנטיפירונטי), who when he walked saw his own likeness before him; and he gave a reason for it according to his opinion, but the truth is that his image was [reflected by] the air, which, by reason of its thickness,

was condensed by its density and returned to him. And I myself see [the like] in the province of Lithuania and in Poland (פולו"ניאה), when I gaze out by night in winter from a window facing my eyes: [a figure] moves near and far as I do, stretches out its hand toward my hand; and so, when I went along the road, men coming in our company would seem [reflected] now on the right, now on the left, sometimes before us; and when we reached the inn (אושפיזא [Italian: *ospizio*]) they would go ahead and enter, and afterward they and their horses would vanish — these being [images] reflected by such air-formations. (This belongs to the science of mirrors, called **catoptrics**, קטופטרי"קה [Greek: κατοπτρική]; the ancients too spoke of it, but they are nothing but the likeness of our own image, as **Nasibon** [Vitellio / Witelo?] also recounted.) On the reflection of rays that occurs, the moderns **Euclid** and **Ptolemy** [wrote]; also a certain Jew, **Rafael**

Mirami by name (whose work is in the Italian vernacular), and many others besides them of worth.

Now Euclid, in Theorem 13 of the *Catoptrics*, explains that if three equal mirrors are set, and placed at the midpoint of three sides of a square that are close to one another, the single perceived object will be seen in each of the mirrors — that is, the image that is behind you (שלפניך [recte: שלאחוריך], "behind you"). The eye of the beholder, ך [point ḏ], gazes upon what is behind it, into mirror **א** [aleph], and from there it is reflected to mirror **ב** [bet], and from there to mirror **ג** [gimel]. Hence one who looks into mirror ב or ג — even though there be a screen separating him from the perceived object — so long as the mirrors are opposite one another, such that the angle of incidence is equal to the angle of reflection, the object will be reflected. And so one standing in the corridor (*prozdor*) knows what his wife is doing in the hall (*traklin*), and one standing in the upper chamber knows what is being done below, and vice versa; and many [physicians] reckon by this means to diagnose ailments of the womb with them.

He further explained, in Theorem 14, that if as many mirrors as you please be set at the angles of any plane figure having many sides and angles, the single object will be seen in all of them; and the number [of images] will be one less than the number of sides. So too he explained, in Theorem 15, concerning concave and convex mirrors.

And the **Cordovan sage**, head of the Stoics and father of the Latins [i.e. **Seneca**], in *Naturales Quaestiones* Book 1, chapter 13 [recte 17], wrote of these matters, and said that if many mirrors are set such that each sees the other, they all become filled with a single image — yet only one image is of the true object, and the rest are likenesses of the first likeness. On this **Lucretius** (לוקריציאו) marveled, [saying] that the five and six perceived objects in the various mirrors are nothing but one, as if the likeness were transferred from mirror to mirror until one could come to know things distant and hidden.

And one ought not wonder that the likeness is multiplied so greatly by the mirrors, for something similar happens with the air: a single sound is heard as many sounds by way of the mountain [echo]. So it befell me — a fright greater than usual — when, on entering a city, there was but a single dog; and when it was opposite [the place], the sound of its voice was multiplied at the bridge of the river, and it was heard as a great clamor of dogs. So too they wrote of the king of Spain (פאן [recte: שפאן, *España*]), that when he warred against an enemy of vast number, being at the foot of a mountain he commanded his men to raise their voices, and each one [was heard] as a thousand, according to what was heard,

and the enemy fled before him. And strange things are seen in mirrors, so that some have reckoned to know, by such means, all that is done in each house and at the ends of the earth — but these are things without proof, and there is no trusting in them.

As for the things that are tested and well known: that if one set his mirrors together — [as] known to the craftsmen of this art — the single man will be seen as a great throng of people; and likewise the single burning lamp, when far from the mirror or tilted, will be seen in it as many; and it is possible to set two mirrors so positioned that in one your image draws near and in the other recedes, while you remain steadily near it; or it will appear as though the likeness flies. And the shapes of mirrors differ: some are simple — flat, straight; some concave or convex; and some compound. In the straight [flat] ones the likeness is seen in the place where it truly is, and if it is large like the object it will be seen large as it is. And if you set a flat mirror standing upright upon a flat mirror: if you look at the [first] standing one, you see the objects in their true place; but if [you look] at the one laid beneath it, you will see them inverted, outside their place. And in straight [flat] mirrors the likeness is always seen in the mirror itself — although this is a deception of the reasoning faculty, for it is not

[seen] except in our eyes, as was explained in Theorem 28. And by means of these straight [flat] mirrors, with the aid of the reflection of rays, we may know the measure of the perpendicular [height] line of any mountain, however high, without cleaving it to enter into its [interior] darkness: that as the [base-]side reflected stands to the perpendicular, so the line of incidence coming from the summit of the mountain to us stands to the perpendicular — [in proportion] of a fourth to a sixth [?]. For the two triangles that arise are proportional, since the angle of incidence and the angle of reflection are equal, as is explained in the science of optics; and they [the perpendiculars] are the upright sides of the angle. Hence the third angle too is known, [equal] in both to the first, and the height of the beholder is known; hence we know the height of the mountain — and this is easy to grasp.

But in **spherical** [convex] mirrors the likeness is seen smaller than it is; and in cylindrical [columnar] ones, longer. The mirrors made from sections of a sphere — these are frightening and distorting; and so the idolatrous priests of old would frighten the women and the fools, and their temples were full of them, as Pliny writes in Book 33, chapter 9. The **convex** mirror shows the likeness smaller than it is, even though the surface of the mirror be large; and the smaller it is, the smaller the likeness it shows; and the nearer the beholder draws, the larger he

sees his image — because the likeness appears convex, not straight, although the straight line is sometimes seen in it straight and sometimes convex.

And in the **concave** [ones], the [images] seen vary according to your position as you gaze into it: in one spot they appear to you sunk in this way, in another that way — exactly as in flat mirrors; in another, outside it, inverted; and in another they are not seen at all in it — for instance, if you set your eye on the encompassing surface, you will not see your image, but you will see other things in it. So too they vary according to the distance of what is seen — that is, if we withdraw from it a fourth of the mirror's diagonal, or a little less or more, then the single thing will be seen many times. The straight line is sometimes straight, sometimes concave, sometimes convex; the concave [things] appear in it concave and the convex convex. When the object is seen in the mirror itself, the motion of its likeness is as its own motion; but if it is far from it, the reverse.

And if you set its diagonal between your eyes so that your eyes are equidistant from its center, you will see a likeness in the air, inverted — its head downward and its feet upward; and in these you will see, on this account, right against right and left against left, unlike in the rest of the mirror's kinds [where it is reversed]. But in this case the likeness is seen smaller; while in another position the likeness appears larger than it is. So too, if you set your eyes exactly at its center, you will see nothing in it but your eyes; and at [another distance] you will see many likenesses of the image outside the mirror; and if you stretch a knife toward them, the likeness too will come forth with the knife into the air, until the two extremities meet.

And the manner of preparing them [concave mirrors] is thus: take a convex mirror, whatever it may be, and scrape away, gently, with patience and forbearance, the [coating-]metal on its back; or anoint the metal with quicksilver, for this will enter inward and will separate between the [layers] adhering [together]; and clean it of the lead easily, and do not break it. After it is clean, set the glass within a wooden vessel, shaped like a hollow [socket] fitting the measure of the mirror; but place between it and the wood a piece of black cloth, so that the rays may not pass through but be reflected, and so that the concave part of the mirror faces your eyes and the convex [faces] into the vessel. For if you gaze into this mirror you will see your image outside the bowl [hollow], in the air, and it will frighten one who is not skilled in this — and so too with the cylindrical [columnar] and conical mirrors, as is explained in the science of mirrors.

And if you take many of these small mirrors — that is, six or eight — and clean them in the aforesaid manner of the [coating-]metal, and set them in the aforesaid wooden vessel, only by the opposite arrangement from that described — that is, with the convex part facing our eyes — and set them adjoining, joined one to another, leaning at a slope, not lying flat; and [set] one large one, in the upper surface of the vessel, laid straight, such that the beholder sees only the upper one: he will see in it twelve [recte: as many] faces as the number of the mirrors. And if you revolve the vessel about [its axis], you will see that all the faces encircle, performing dances, except for the central one, which stands always in the middle. The intelligent man will set it up [for himself]; and among the craftsmen

you will find mirrors having many angles, and in these many images of a single object are seen.

But the most astonishing of all are the **burning mirrors**, and these are of various kinds, differing [from one another]. By their means [the rays] are concentrated, as **Democritus** the philosopher set up a shield [or: a mirror] opposite the sun and stood facing it until its rays, being reflected, blinded his eyes. Some are of crystal (קריס"טאל) or of glass — whether straight [plane], or convex, or concave; and these [last] are the most powerful. The ones compounded of many concavities [are stronger] still and still [more]; and likewise those compounded of hollowed-out cylindrical [columnar] pieces; and above all of them are the **parabolic** ones (הפארבוליקיית [Greek: parabolic]), which concentrate the solar rays the most — and with such a one **Archimedes** burned the ships of the Romans, as **Galen** wrote in his *Book of the Balances* [De Libra / Peri zygōn]; and such was the deed that **Proclus** performed at Constantinople (קוסטנטי"דינה), of which **Zonaras** (זונארש) writes in Part 3 of his histories.

And if you would prepare a **burning mirror**: prepare for yourself the convex mirror as I explained to you — scrape off the [coating-]metal that is on the concave side until it is clean; and afterward coat it on the convex side with a leaf of tin (*kadil*/*bedil*), and you must cause it to adhere well so that it does not separate. This is done by anointing the tin afterward with quicksilver; and exactness and skill are needed so that the tin separate not at all by any means. Then set the mirror in the aforesaid wooden socket, so positioned that the concave side faces our eyes; and if you look into it you will see a likeness very small and very ugly — but the mirror's power is wondrous in burning. For if you set it opposite the sun, and [at] the first point where the reflected rays meet and

join together in the air you place there a candle, or flax, or some [other] flammable thing, you will see, after a short while, that it burns.

And **before** I finish speaking of the **concave** mirror, I will inform you of a wondrous matter among its powers — namely, that if you stop up all the apertures and shut the openings and windows of a house, so that no solar ray enters except through an aperture set opposite the concave mirror, straight [aligned with it], and opposite the mirror you set a white sheet of paper, or you look at the wall — you will see on the paper [or wall] all the images of the passers-by outside the house, in the streets, even the very distant ones, in their shape, in their position, and in the colors of their garments. And it is a matter wondrous in truth. [This is the camera obscura with a concave mirror.]

But since such mirrors are not found everywhere, I will not refrain from benefiting you by another means as well — though it is not of such great usefulness as the aforesaid mirror. And it is a thing well known in optics, and to those who inquire into it; and you can test it: that if light enters into a dark dwelling through a small hole, and opposite the hole [you place] a board, or a wall, or a white paper, [there] will be seen on the paper every passer-by that is outside the house in the street; provided it be aligned straight with the hole. And one ought not wonder how so little light, on entering through a small hole, is seen [forming images] — for [a related thing happens by day]: that we do not see the stars [by day] because the light of the sun bars [them] — not, as the multitude think, that the sun [is so bright]; nor will this happen if the hole be large, for by the abundance of light the eyes are darkened. [This is the *camera obscura* / pinhole.]

[Excursus on why stars are invisible by day:] It happens to us by day that we do not see the stars whose light — on the contrary — the stars receive from it [the sun] [?]; and there is no star hidden from us, that by reason of its great height [is invisible]… the earth bars [it] [by] night [recte: the earth's shadow], [so] by day [?] — and how, then, [if the sun did not bar them], would it not also darken them by night? Rather, the abundance of the light seen in our eyes is the barrier; for the illuminating air comprehends [both] the light of the stars and the light of the sun together, like a drop [of one liquid] mixed [into another] — [the lesser is lost in the greater]. [Argument from analogy:] In a great sea, who could distinguish their waters? Hence all is one sea. So too the light of the sun is the barrier — for the moon does not show [its own] shadow [recte: light]: by day, and by night, when the moon shines, it casts no shadow [by] the light of Venus (*Nogah*); and likewise, when Venus shines, it casts no shadow [by] the light of Jupiter (*Tzedek*) [?]; but

only [the] four [brightest] stars cast a shadow by their brilliance. However, in deep wells or in dark caves [the stars may be seen even by day], as Galen [Galinus] and **Johannes** [recte: of Sacrobosco? — יוחני] testified — that four stars besides… are seen even by day, when the eye is awake [?] [recte: when the eye is purified — נלינום ויוחני / the testimony of Galen and Johannes]; — and **Marcobardo** [recte: Macrobius? — המרקדק], and experience teaches that even in a solar eclipse the stars are seen. [This passage is corrupt and partly conjectural — see Uncertain readings.]

Now these perceived objects, passing through this small hole, are seen on the paper **inverted** — that is, the right [as] left, the upper [as] lower. And it is necessary [that it be so], for the rays that come from the [several] sides of the body are cut at the hole, at the point of their entry, and form an angle. Therefore they change their position, as [will be shown] in the figure…

the perpendicular [see figure 75] in the triangle ב-י-ג, the rays coming from its edges through the hole א will fall inward into the house, onto a dark wall, onto paper, or onto a white wall. You will then see that the outer point ח is at the place of the inner point ד, that point ז is at the place of point דק, and point י is at the place of point ך. The farther the wall or paper is from the hole, the larger the objects will appear, though dimmer. If the distance [from the wall to the hole] is equal to the object's distance [from the hole], the object appears at its true size; if shorter, smaller; if longer, larger. All of this is clear from Books One and Six of *Optica* [Euclid]. The only difference is that all these images are colorless — like dead, dark objects — and they appear not joined but separated, because the ray of light entering is slight, and the shadows are soft and dim it.

Now, if you wish to enhance their appearance and remedy their deficiency, you may set into the hole a convex glass lens — about the size of a *thaler* coin or larger — of the sort used by the lights [magnifying glasses] that schoolboys use: when they hold them up to the sun, you can see on the far side how high the sun's height reaches. When the room is dark, you will see every object on the wall or paper facing the glass; and if it is crystal, the images will be still clearer. Likewise, if you set a cup or flask full of water into the hole, you will see on the opposite wall all the passers-by — the markets and streets. By this method astronomers observe solar eclipses and record the magnitude of the eclipse, the time of its waxing and waning, its beginning and its end, without staring at the sun and ruining their eyes — and so they map the eclipses on paper as if [drawing them] in the heavens. By this same method anyone who never learned the art of drawing can draw, whether surfaces or solid bodies. According to how far or

near the paper is to the hole, he can make the images large or small as he wishes, and can map the shapes of provinces and cities with a precision and ease beyond what any mouth could tell. He can construct a box of boards with many angles, and on each side or wall make several holes, high and low, that are wide on the outside and narrow on the inside — that is, [graduated from] transparent to opaque [pinhole]. He opens the hole directly facing the object he wishes to draw and stops up the others; thus he will work wonders. By this method too we can see the spots that appear on the sun, and understand that its body moves and rotates about its place; also the orbit of Venus and Mercury along with it, and other precious matters.

Aristotle, it seems, grasped a hint of this — though defectively — for he wrote, in the tenth Problem of Section 15, that during a solar eclipse, if a man sets a sieve facing the sun, or if the brightness passes between plane-tree leaves or between the lattices of three fingers laid one over the other, the brightness on the ground will not be round, in the sun's image, as it is at times other than an eclipse, but will be defective like a damaged moon. He reasoned that no rays come from the darkened part [of the sun], and therefore its image traced on the ground's surface is deficient from being round — defective in the surfaces, on paper, or on a wall, in proportion to the eclipsed part. As I have said, the upper part appears lower and the right side left; the discerning reader will understand and grasp the matter.

He also asks there, in the fifth Problem, why the brightness entering through an angular hole does not appear on the floor of the house or on a wall far from the hole in the shape of the angular hole, but round. For so it truly appears when the ray enters straight — that is, along a perpendicular line; whereas if it comes at a slant, it appears not round but in an oval [ellipse] shape, as is explained in the *Conics* of Apollonius. So too with the images coming through the holes of the square latticed rods, woven net-like, which in [foreign tongue] are called *zoluziah* [Italian: gelosia, "lattice/jalousie"], used as windows so that no watching eye may see the women who look out through them: you will see on the floor that the rays come at a slant, and so there [the images] are *ellipses*. The objects from whose edges the rays come to the eye — if they are near — appear in their [true] shape, and the wall [shows them] straight, and there [they appear] round. He answered that what happens to the [image of the] sun thus traced is what happens to other

objects: the rays that come from the corners, though few and fine, are seen well up close, but if they are far, the angular object appears round, and the square one as if many-angled — and the curved line [softens]. This is clear to the scholars of

optics. Therefore, since the solar body is very far, it is not seen as it [truly] is. [Where the entering light has been bent within the watery body, it appears straight, and there are many such cases.][?]

The hole it enters is [the cause][?], and so close to the window a square brightness is seen, in the shape of the hole; [farther][?] from the window [it is round]. This is the meaning there of his words. But one who reflects carefully will rest content with the philosopher's reasoning: that the image-trace of a thing always [follows] its [own] shape — whether near, since the rays come from all its edges in truth, or far, where the eye errs and [misjudges] the trace. Therefore the brightness is always round, like the sun — if it comes straight, that is, perpendicular [to the surface]; if at a slant it will be [oval], as is explained in Apollonius. The reason it appears egg-shaped even when [entering] straight is the egg-shaped [oval] figure. As long as the circles [of light] are larger and nearer [to one another], a nearer image appears, because the brightness enters through several holes or [holes] adjacent to one another close together — round; [so] it appears round. Therefore, the farther [off] and larger the object, the larger the circles will be, and they appear as if they were a single circle.

You can test all this. If you make two or three holes in a wall and bring the paper near them, then move it far, you will see by sense [perception] the truth: that the trace of each hole is round, as we said. The farther you move it, the more the circles draw together and grow, appearing egg-shaped [elliptical]. If you wish to grasp the root of the matter, fix on a board or wall there an angular body — say, a square book — which will stand in for the illuminating sun. Then set up an opposite board, another plank, and bore in it an angular hole as you wish, which stands in for the dividing wall [the screen]; and tie threads to all the corners and sides of the book, and pass them through the angles of the hole until their other end reaches the floor. At the [final] end of each one — by its motion — mark a sign on the floor, and you will see that figures without limit will be traced on the floor, all of them small, in the likeness of the said square book. Therefore the reason is clear from what has been said why the objects passing outside the house are seen on the paper in the dark room: because rays come from their edges to every point in the air, and so they enter through the holes; and because the rays cross, those passing and coming are seen in a different position, reversed front to back. This is all clear, like a garment [i.e., plain].

Now, upon refractive vision — by the breaking of the ray — another science is built, called *dioptrics* [Greek: διοπτρική], whereby things are seen through different media, one denser than its fellow, both of them clear [transparent], so

that they do not block the object's visibility. Hence everything refracted is doubled from the dense medium — but not the reverse: not everything doubled is refracted. ʿAlī al-Qūhī [ʿAlī al-Quhi][?] boasts that he knew how to compute the measure of the angle of refraction, but I have not seen his proofs.

Indeed, the refracted ray of an object sometimes shows the light and the color, but not the heat of the object itself — like the bright halos seen in the air around the sun, moon, and other bright stars, from the light traced in the thick, moist vapors. So too one will see around a burning lamp; and likewise lines [will be seen], like the figure of the *parhelion* [Greek: παρήλιον, "mock sun"] — that is, the sun is seen rising with [doubled images] above it on the horizon, and the objects red like reeds before sunrise and after sunset. Sometimes [the medium] shows the figures themselves. The same object, by itself, will appear in a thinner medium small, and in a dense one large, as a *dinar* [coin] in water; and by means of both [media together], large and fine. For if you fill a glass with water and set it between your eyes and the object — say, over letters or fruits — they will appear to you swollen and clear. As Seneca said: everything seen in a liquid appears larger than in truth. The reason is that the rays passing through the glass are refracted at the place between the water and the air, and every…

refraction draws toward a single point and comes to the eye; therefore the ray [is] very strong, and so the [ray] passing out of it will burn. [If] one sets a flask full of water facing the sun, the refracted ray that comes against it — whether garments or wood — the experiment shows this in all of them; and Aristotle mentioned it in Book One [of the *Meteorology*][?]. The strong refraction collects toward one point [and these rays come to the eye], and therefore the ray is very strong.

[Several lines here are textually tangled; the sense:] By the strength of the refractions, [the rays] are gathered to one point and come to the eye. The reasoning of the matter of the refraction of rays is that the ray coming from the object — the rays being refracted in the air — [and] if the medium is dense, [the ray] does not [bend its course as long as the medium is uniform in its thickness or thinness], although there is no doubt that a thick [medium] passes [light] more [slowly] over a distance, and presses [it] more [than] a thin one. Therefore, in its passage into a dense [medium] to pass through, if it happens that the medium changes — when it passes from the dense water into the air — it goes free, and so is refracted away from the perpendicular line, and goes along another straight line until it reaches the seeing eye. He therefore does not see the object itself but its likeness, for the place of the first [point of] vision is at the point where the perpendicular rays coming from the objects meet, as is explained in the science of

optics. Conversely, if [the ray] comes from the air into the water, it passes from the light [medium] to the dense, which hinders it from passing straight; therefore it is refracted and draws near to the perpendicular line, for those falling descend along a perpendicular line — but not so the oblique ones.

When part of the object is in the air and part in water, it appears broken, like a stick of salt [a rod]: the part in the water appears thicker and nearer to us. Aristotle likewise wrote, in the third [Book] of the *Meteorology*, that the stars appear larger at rising and setting because of the thickness of the vapors near the earth — meaning that even when they are directly overhead, the vapors near the earth stand between us and them; even so, the perpendicular ray is shorter and does not pass through a great thickness of vapor, unlike when it comes at a slant and inclination, as when the stars are on the horizon. Therefore the astronomers do not concern themselves with the refraction of rays when the stars are high above the horizon. Hence the place of error is in the surface that is the common section of the two media, where the ray begins to break. That is to say: if you fill a vessel with water and place in it a coin or a gold ring, and draw back until the limit of your sight of the coin, the surface of the water is the beginning of the refraction.

Now, upon the vision occurring through the doubling and refraction of a ray together, there are many weighty teachings. By this method the one figure is seen as two: the one, clearer, by doubling; and the second, coming from it, by refraction. Therefore, if you set a mirror within clear water, you will see in it two likenesses, one clearer than the other — for the first is by the manner of all mirrors in the air, which comes by doubling, and the second is what is refracted in the water and afterward doubled by the mirror, and on its emergence upward is refracted from the air again and comes to the eye. So too in a vessel of oil and pitch, provided they are clear. If you set a mirror within the water and look at the heavens, you will see the likeness of the sun and a small star — and it is no star, [written] as some have thought; and the philosopher in his weakness was wont to gaze at the solar eclipse within clear water. And if you look through some glass into a mirror, you will see two likenesses, or two eyes, one larger than the other.

The compounding of rays doubled and refracted together is wonderful in [the bow of] the clouds [the rainbow]: for as we explained in the [twenty-fifth][?] discourse, it has no existence or substrate other than the seeing eye. So too Aristotle wrote in the book of the *Meteorology* and in his Natural Problems. This proves that it [the rainbow] flees from what pursues it and pursues what flees it; it pursues toward honor and lordship. As Chazal said, and it is fitting to teach to

the sons of Judah [a hard saying which is true]: what the pride of greatness did to its head [i.e., placed it high], humility did to its [low] heel —

as the case once was of three sitting before the king. And if you are riding upon a horse on level low ground, you will see two bows of the rainbow [running] before you and fleeing, and gladdening all who see them. With your two eyes you can see two different bows, each facing the other, [each from] the thick vapors. A seeing eye will see one [bow] after another, which is nothing but a doubling of the solar light that the falling drops [show], for every single drop is as a single mirror. If you stand in a proper position and shut your right [eye] and then your left, [you will see them shift]. And if you acquire the crystal made in the shape of a *prisma* [prism] with five or six angles, and set it facing the sun, you will see wonders — that is, colors like the rainbow's appearance, and like the grass of gardens and orchards and the green of the field. You will see the horizon traced around, and many [other] things; provided that you know how to aim it, sometimes one way and sometimes many ways. You will see people walking in the air like sons of God [angels]. And when you look in it to see your own likeness, you will see yourself with one eye, or with four eyes, and many such likenesses. Therefore not only in the cloud are these colors seen, but also in a flask full of water, and in the glass of windows when they are mixed with raindrops; and among sailors and [glassmakers][?] in the [blowing][?] that the boys [do] with [a tube][?]; and in the six-angled stones found in the land of *Iberniah* [Hibernia / Ireland][?] that are in waters; and in pearls steeped in water; and in the slight blinking of the eyes; and in the bubbles formed from water — called "stones of the bow" [rainbow stones] because they show the colors of the rainbow's hues; and in many [other] cases.

Now, upon the teachings of the cloud-bow [rainbow] I have a great treatise, which I will not publish ever, God willing — and this is enough for me, in accordance with your request.

On the measure of the diameters of the luminaries by the method of the ancients. Ctesibius — of whom Marcus Vitruvius wrote in Book One, Chapter 20 — measured the diameters of the luminaries with a water- or sand-clock, thus: at the instant the first solar ray rises on the horizon, from the start of the ascent of its body, they set the water-clock [going], and let it run on, descending, until the whole body of the sun rises above the horizon; then at once they stop the clock from its motion, and observe what fraction of an hour [of] sand has descended. Suppose, for example, it were ⅜ of an hour. We reckon thus: if 85 [units] equal 15 degrees of the sphere — for such is the rate of the daily motion in one hour —

then in one hour, how many do 3 [units] equal? You will find 3 degrees, 31 minutes, 46 seconds, approximately. And since the sun's motion in [its course], added to the daily motion, is about 60 minutes [of arc] approximately in every 24 hours — of which there belongs to each hour about 150 [seconds] — the proportion will then be thus: if 85 give 150 seconds, how many will [3 give]? You will find 3[°] 31′ 51″ — and this is the measure of the diameter of the solar body in terms of the 360 degrees of its sphere.

This method is very exact in computation, unless there be an error from the clock or from imprecision in [measuring] the portion of sand. Only, the sun does not move every day a uniform course — for sometimes [it is] 60 minutes and sometimes 59, or a little more — and the natural days are longer in its passage from Libra to Aries than when it moves in the northern signs. Therefore one must know its [exact] motion precisely from the daily tables on the day of observation. And what they used to estimate with the water-clock can also be measured by uniform motion in a straightforward way — for instance, on level ground, where the ratio of the distance you walked, until the whole solar body rose above the horizon, to a walk of ⅜ of an hour, will bring you to knowledge of its diameter. So too with the perpendiculars [gnomons] and the shadows. And you should put a veil over your face at the time of observation, so that you can look at [the sun] precisely. From this it is clear that those who reckon the length of gnomons and shadows from the sun's altitude must add to its known central altitude — known from the knowledge of its place in the signs — the measure of half its diameter; otherwise they will err by a large amount. This is easily understood, from what I wrote in *Sefer Chukos Shamayim* [Book of the Statutes of the Heavens].

Now, Archimedes and Aristarchus investigated the diameters of the luminaries by another method: they set up a table or another high surface, level [perpendicular to] the horizon, and when the solar body is on the horizon they place a small round wooden rod, long, in the shape of a gnomon [pillar], on the table, and move it away and toward the eye until the rays of vision that make an angle, like…

Sefer Elim — Part Five: Ma'ayan Chatum (מעין חתום)

§46 (continued) — Optics: measuring the diameter of the luminaries …would pass along the edges of the standing rod and touch the edge of the solar disk's diameter, and lines are drawn upon the table in order to gauge the magnitude of this angle. After that, one sets the leg of the compass at the vertex of the angle and, at whatever distance one wishes, marks out a quadrant of a circle and divides it into equal parts. If divided into 164 equal parts, one finds that the said angle is a little less than one of those parts; whereas if one divides the said quadrant into 200 equal parts, one finds that the said angle is somewhat more than one of those parts. Therefore the angle of the sun's diameter is less than 1/164 and more than 1/200 of a quadrant. This is what we set out to demonstrate.

The instruments of Ti"ko [Tycho Brahe] are the most precise and the easiest of all of these. With them one measures the luminaries when they stand high — as at the circle of midday — for when they are at the horizon, the refraction of their rays confounds every calculation. That is to say, on account of the dense vapors near the earth, when the star's rays fall through the air they are refracted and do not travel in a straight line, but come downward; so its light is sent before the star itself has risen, and it appears to be rising when in truth it is still below. The coin in the basin proves this: if you put water into it, you see with your eyes what without the water it is impossible to see at that same distance — and what you see is not the coin itself but its image [literally "its shadow"]. So too, whenever the vapors are dense, a star is seen earlier than its [true] time; and weather-watchers infer the coming of rain from the early rising of a star. Moreover, a star is seen at the horizon larger than it is — and this accords with the matter of vapors explained above in the science of vision [optics], namely, that the more an object is seen through a dense medium the larger it appears. Posidonius [פושירונואוס, Greek: Ποσειδώνιος] used to say that if our sight could pass through walls, we

would see the stars exceedingly large. Likewise objects in water are seen larger than they appear in air — and the [magnifying] vessel of Aesop [איזו"פוס] [?] proves this. Furthermore, the very shape of the stars changes when they are near the horizon: the sun is seen in the shape of an egg, slightly elongated rather than round — as I have seen many times with the glass of Galileo [zechuchis Galileo — the telescope].

§47 [marked "48 = "מח in the source]

It is astonishing [נשתוממת] that three men may be in the right on a single day — one saying "today is the eve of Shabbos [Friday]," another "it is Shabbos," and a third "it is the first day after Shabbos [Sunday]." This is perfectly clear to those who look at the celestial globe, and who would deny it? For if a man were to run upon the globe of the earth, moving at a speed equal to the sun's daily motion, he would always have daylight along his paths. Suppose, then, that Reuben sets out from Jerusalem toward the lands of the east. Each day the sun will rise for him earlier than for the residents of Jerusalem; so that once he is

- 225 parsa'os distant, it will rise for him one hour earlier;
- at 675 parsa'os, it will rise for him three hours earlier;
- at 1350, it will be rising for him while it is midnight for them;
- at 2700 parsa'os, it will rise for him just as it is setting in Jerusalem;
- and at 5400 he will return to Jerusalem, and the sun will rise for him at the start of *another* day — whereas for the residents there it is the start of Shabbos day.

For one traveling from Jerusalem to the west, the reverse holds: each day the sun rises for him later than at Jerusalem, so that when he returns to Jerusalem he has lost one day — reckoning the start of the sixth day [Friday] while they reckon the start of Shabbos. In such a case the Israelite must act according to the place [where he then is] regarding the observance of Shabbos and festival. Just this happened to the circumnavigators of the world: when they reached the place they had set out from, they were reckoning a day other than the day of their location, and their listeners were astonished, and disbelieved them and mocked them. But when the matter was reported to the scholars of astronomy, these confirmed that the travelers' account was true and that, by their reckoning, the matter was correct.

§48 [marked "49 = "מט in the source]

You will want to know the manner and means by which to discern the day of the week, in case you should somehow lose track of it while on the road or outside settled territory. Now many of the astronomers reckoned this impossible — but I say that it is easy to determine for a man of [keen] spirit. For those wholly devoid of knowledge it is, to be sure, hard or impossible; yet if the keen-minded enthusiast happens to have with him tables of the ephemerides [האיפימירידיס, the Ephemerides] or the like, in which

the places of the sun and the moon among the zodiacal signs for each day are given readily, he can easily learn the day of the week. For he can take the sun's altitude at midday, learn its declination, and from that its place among the signs — as I taught in *Sefer Chukos Shamayim* [my "Laws of the Heavens"]. In any case he must know which year of the world's [reckoning] it is, must know the altitude of the place he is in, and must have a precise instrument graduated to the second of arc; otherwise he will not be able, from the declination, to verify that today is such-and-such a day and that it is not the day before or the day after.

This is all the more so when the sun is near the solstices [literally "the turnings"] — that is, in Cancer or Capricorn — where there is scarcely any difference [in declination] between [consecutive days] near the solstices, in matters as grave as keeping Shabbos. He must therefore seek to determine it by another route — the variation of declination from one day to the next. And even with great precision one cannot rely on observing the sun. [Better is] knowing the moon's true place by means of the instruments, when you take its distance from two fixed stars, and by the calculation of triangles seek its longitude and latitude, in the manner I showed you for investigating the distance between two cities in *Sefer Chukos Shamayim*. Once you know the sun's place truly, you will know how far into the [lunar] month you are. And even if you have no ephemeris with you, provided you have a table showing for each month on which [degree] the sun enters the heads of the zodiacal signs —

over two consecutive years — that is, one bissextile [leap] year and three common ones — within four years all the variations appear, since the six hours by which the solar year exceeds 365 days are the cause of the bissextile [leap day]. There is also one further small discrepancy, in that [the excess] is in fact only 8 hours [recte: 5 hours] 49 minutes [?] [the sense requires the surplus over 365 days, which is ~5 h 49 m; the print appears to read "8"]; even so, it is not perceptible [over short spans]. It is easy to know whether the year is bissextile or common: divide the Christians' years by 4 [and so on]. And in leap years [רבות]

[?], one should take from the tables of his own time a set of four years, and cycle through them continually.

When you divide [the year-count] by 28, if there is a remainder of 9 over the [completed cycles], know the years of the era [הנ"מ = the era reckoned from the Nativity / Anno Domini]; for the first year of the [Christian] era fell on the tenth [of the lunar cycle] [?]. And from the [Christian] era you may learn the weekday letter [dominical letter] of the first day of the week and the beginnings of all the months. These matters are straightforward and well known to all who reckon the [Christian] festivals; and there are several tables that record the weekday on which the first of each of the months of the gentiles falls in each year of the [Christian] era. Once you know the day of Rosh Chodesh [ר"ח, rosh chodesh] in the Julian reckoning, you go back 11 days and find the day of Rosh Chodesh by the Gregorian reckoning.

I cannot expand at length, so I shall merely give you an illustration. Suppose we were standing — as in fact we were — in Candia [כנדקא], my native city, which God privileged me to behold in the lifetime of my fathers, in the year שפ"ט [389 of the abbreviated era = 5389 AM], which is the year 1629 [the print reads 1628] [?], at midday. With my quadrant instrument we observed the sun's altitude and found it 42° 51′ 35″; and we subtracted from it the elevation of the equator there, which is 45° 54′, and obtained its declination, 9° 8′ 46″ [?] south. Since the days were growing shorter and it was the autumn season, we knew this declination belongs to Libra [literally "the Scales"], by [the calculation of] triangles together with what is explained in *Sefer Chukos Shamayim*, or [we found it] from the table, which gives it as 7° 56′ [for Libra] [?]. Since I knew from the table of the four years [the leap-cycle] mentioned above that in this year the sun entered Libra on the 23rd of September [לשיפטימבי"ר], and it advances by about a degree each day, I conclude that on the first of October it is at 7° 56′ of Libra. And because in this year October fell on the 4th by the Julian reckoning, it would by the Gregorian reckoning fall on the 1st [reckoning] 11 days backward — which is what I meant.

To add confirmation, we looked again that very night for the moon, and found it at 26° of Scorpio; hence it was 48° distant from the sun. The moon could not have traveled less than four days, since at the time of conjunction the sun was at least at the 4th degree of Libra, which it too traverses about a degree a day; therefore the moon had traveled 52° [?] [arithmetic check: 48° elongation observed, but the text reconstructs the moon's motion since conjunction]. Now by the Israelite reckoning, the molad of Tishrei of [the year] שפ"ט was on day 5 [Thursday], 12 hours [and] 837 chalakim [תת"רל"ז]. Therefore, if you count 4 days exactly — that

is, 6 [Friday] after [day] 5 [Thursday]: 1 [for] three, 2 [for] four [?] — you find that the day that has passed was the first day [Sunday], for the moon entered on day 2 [Monday], namely the fourth [day] after conjunction; and the precise reckoner cannot err in this calculation, since the moon travels each day about 12° 11′ at its mean motion. One who reckons by the true astronomical computation will be still more secure. And there are other ways to know this, from knowing how many

hours the moon shines, from which you may learn the time of its conjunction, and the like. But I have chosen the most reliable. Again, suppose we observed in the year [1629] שפ"ט at Venice [ויניציו, Venezia] the sun's altitude at midday, and found it 56° 1′ [?]; and when we subtracted it from the elevation of the pole [or: of the axis] there, 45°, the declination became known, 10° 56′ [?] north. The sun, therefore, was in Taurus [literally "the Bull"], as the season indicates — for the days were lengthening — and in the table, opposite this declination, [the value] 46° 55′ of Taurus [?] [is given]. Since I knew that the sun enters the head of Taurus on the 21st of April [לאפ"ריל], and its motion is about a degree a day, I find it at the said place on the eighth of the month [of] May — for the first of May, Rosh Chodesh May, fell in this year on [a certain day] [?] by the Julian reckoning; hence on the [first] day by the reckoning of the month. And this is proven by the moon's place, which was found at 24° of Scorpio, opposite Taurus, where the sun was. When you reckon back the sun's motion over 14 days up to the conjunction, you find that the conjunction was at about the 3rd degree of Taurus; and the molad of [the year] שפ"ט by our reckoning was on the night of day 2 [Monday], 6 hours and [some chalakim], and from [day] 2 to [day] 2 [?] is 396 [days, with the fraction] 14 ⅛ [?] — that is, half the [synodic] month. In that span the moon traverses no less than 195°, and it had traversed more than 200°; therefore it is on day 3 [Tuesday] of the week, the 16th [day] after the conjunction. Or, by the route of this proof, it is on day 3 [Tuesday]; hence the coming day [too] is day 3 [?] — and everything follows accordingly.

§49 [marked "50 = "נ in the source]

You yearn to know the hours from hearing the first, second, and last [strokes of a clock] — and this is an easy thing. For one skilled in the measure of striking-clocks [literally "the proportion of the strokes"], that is, in the raising and lowering of the [striking] arm, as is known to musicians, takes the measure of the time between one stroke and another. If he hears the first hour and the second, he will go on estimating by that proportion up to the final stroke, and so know the hours. This is useful when the wind is blowing so that the intermediate [strokes] are not heard. Understand this well.

§50 [marked "51 = "נא in the source]

To know the hour without a clock, and so forth. The answer: if time is the measure of motion — as Aristotle held [literally, "according to the earlier and the later"] — then, conversely, motion too indicates time. That is to say: if in one hour we travel half a parsah, and we know that twelve hours have passed since we set out by wagon or by ship, we know that our journey is six parsa'os. And conversely: if what we have done is a distance of six parsa'os, then twelve hours have passed.

Suppose we are in a village at night, in a place where there are no striking-clocks — those made with self-moving wheels — nor a sand-clock or the like, and a child is born to us, and we wish to know the moment of its birth in order to cast its nativity [literally "to judge its fortunes"]. Take a barrel or another vessel, fill it with water, and bore a small hole in its base. At the moment the child comes from its mother's womb, open the bung [hole] and let [the water] go on dripping. If it all runs out, refill it a second and a third time until the sun rises. We then know, by the time elapsed, what hour it now is after midnight — as is explained in [the science of] astronomy; or we learn this from the shadows and the gnomons [literally "the columns"]; or we seek some clock that tells the hour, until the moment of [sunrise] becomes known to us. We then stop the bung, and since we know how much remained in the vessel, we know how much had gone out of it. Therefore, if we again pour into it afresh all that water which had run out, then when it is exhausted we shall see how many hours have passed; and we count backward from the first hour — those that passed from midnight to sunrise — and the time at which we find the remainder [exhausted] is the moment of [birth] after midnight. Understand this well.

§51 [marked "52 = "נב in the source]

For this purpose there is a vessel made like a bell, fastened to the [other] apparatus and hung near the bottom, so that when it is set for the hour you wish, it rouses you from sleep at that time. They also make, with a fuse-cord [chevel kankos] [?], [a device] by which the lamp kindles of itself when [the cord] reaches it, [or which] lights a fire in the stove to warm a chamber [?]. These are known and famous in Germany [Ashkenaz] and France, and it is impossible to set down the manner of making them in writing except at great length. Perhaps the "harp of David" [כנור של דוד] was of this kind.

But the device praised in §53 [נ"ג] is very simple, and you can mark the hours upon the wall, projecting [marks], and feel them [by touch], whether near the

bottom — and you will know the hour, for the more the water diminishes and trickles out through the hole at its base,

the [slower] the iron thread that is fixed in the light wood descends. [?] But if you cover it [the vessel] and one does not know the cause of its motion — for only the rack [סרק, the toothed mechanism] within it raises the thread and lowers it [?] — the [moving] arm will display the hours as palpable signs to the touch of the hand, and it is truly wondrous. You can also make [the markings] inside the vessel itself, [?] and the one sleeping beside you will be amazed and will not know how you contrive it. Understand this well.

[continued: the trickling of the water through the said hole into the channels or beds; and at night one feels them and so knows the hour.] [?]

§52 [a second section marked "52 = "נב —]

A star [as a pole-marker]: the [pole-]star [ככב הציר] is, in our time, distant about 3¼° from the true pole; hence the diameter of the circle it traces in its daily revolution about the pole is seven degrees. Now it has been explained in *Sefer Gevuros Hashem* ["Mighty Deeds of God"] [?] that the distance from the center of the earth to the convexity [outer surface] of the eighth sphere is 45225 half-diameters [radii] of the terrestrial globe. Therefore, if we set up a proportion thus — as AB, 100000, is to 45225 — so the tangent of seven degrees, which is 12208, is to a fourth [term]; we find that the half-diameter [radius] of this circle is 5521 times the half-diameter of the earth. But the distance up to the convexity of the solar sphere is not more than 2432 earth-radii, as explained in the said book; hence the whole diameter of the solar sphere is not even half the diameter of this circle. And since the ratios of circles are as the ratios of the squares of their diameters — as we learn from [Euclid] Book XII, [Proposition 2] of the *Elements* [literally "of the foundations"] — the ratio of the circle of the sun's sphere to the said circle will be as 1 to 4, so that the entire sphere of the sun could fit within the said circle as one beats [a clapper] within a bell. Therefore those who seek, with the instruments, the altitude of the said star — from which they may learn the latitude of the place, as [they do] from the sun's altitude, or [from] another star, provided you know its declination, as I taught you in *Sefer Chukos Shamayim* — understand this well.

§53 [marked "53 = "נג in the source] — Finding the longitude of a country

Should your soul yearn to know the longitude of a country — and this is among the precious matters that mariners would gladly find — [know that] with the instruments they readily find the latitude of a place, as I taught you in *Sefer Chukos Shamayim*; but they do not know whether they have gone east or west. The princes of the provinces have vowed great gifts if a means of determining this could be found, and they have labored and not found it. What seems best of the writings is this, which I received from a certain pilot, expert and well-tested in this matter, since many have attempted it with the sand-clock and the self-moving timepieces [literally "the hour-vessels that move of themselves"]. And just as the method with these is easy to understand, so too it is easy to err in it; for these instruments do not move evenly: at their start they are heavier and faster, and toward their end weaker. The air, too, acts upon them; and any error whatever would err in the matter of longitude by many hundreds of parsa'os ([by the testimony of] a pilot [?] [לִיו זון] [?]).

But if some vessel can be found that tells the hours evenly, [keeping pace] with the motion of the heavens — at the moment of the ship's departure we investigate the hour of the day from the altitude of the sun or the stars precisely, and at that very moment we set the instrument [to that] hour and go on our way; and when we reach another city, we seek there too, from the sun or by some precise method, the hour of the day, and observe whether the instrument's motion accords exactly with what is found. If [it tells] no time without any addition or subtraction, it is clear that these two places are equal in their longitude. But if it happens, say, that in the city we have reached it is one hour before midday while the instrument shows midday, our motion was toward the west; and conversely, if the instrument shows 11 hours and the place we have reached is at midday, we have gone toward the east. Each hour is 15° of longitude, as you know.

Accordingly, the instrument must be divided into minutes and seconds, so as to be precise; but these instruments which we are seeking [are flawed], as I said. Hence the said pilot devised a water-vessel that moves more evenly than the other instruments — this being when the descending waters [are kept] always at an equal weight [head], and on one [continuous] surface together with the water that is in the vessel. As long as the water diminishes, it is hard to rise [maintain the flow], until [the water] does not flow but [merely] drips, growing long [drawn-out] —

[continues].

Sefer Elim — Ma'ayan Chatum (Fifth Part: Paradoxes)

§53 (continued) — The longitude problem and the marine timekeeper

…so you must take pains, as time passes, to build an instrument whose tube always keeps the water at an even level [literally "by weight"] and on a level surface, set horizontal to the horizon. This is the horizontal [arrangement]: if you give the tube a slope down from point **ג** [G], the weight at **ס** [S] equals the weight at **א** [A]; and when you draw [the water] through your mouth a single time, the water from hole **א** [A] keeps flowing until the vessel empties. And as long as the water is being depleted, the light wood-piece **ב** [B], in which the tube is fixed, sinks downward; and because the tube **א** [A] is long, it will always remain on a level surface and at even weight, so that when the wood descends to point **ה** [H], the mouth of the tube outside will be at point **ך** [Kh]. In the end, the heart of the craft depends on the tube's mouths always being on one surface, horizontal to the horizon — for in this way the one [end] will not weigh more than the other, and the vessel will be of glass and marked off [graduated] for the hours.

Again, in the horizontal instrument [Figure: ציור עז] — the second one, with the light wood **א** [A] in which the iron is fixed — as the water issues from hole **רז** [?], the iron will sink, and the marker will indicate the hours marked on the wall, and understand this well. Even so, this instrument is hard to use.

I have come to know an eminent scholar in the mathematical sciences from England [מאי אנגליאה — perhaps "of England"] who has an instrument that maintains a steady motion for three months, in keeping with the motion of the

heavens; but he does not wish to disclose it unless the city governors grant him a large sum of money. Perhaps in our own time it will be revealed and we shall see his promises fulfilled.

Still others propose to know this [longitude] from the behavior of magnets, saying that the magnet does not point with precise accuracy between north and south, but rather inclines toward the islands of Pirtuntish [the Azores — פירטונטיש (?)], the place from which the later geographers reckon the prime meridian of longitude; and according to the distance to east or west, the lodestone deviates from the [true] poles. But these matters are not well established and are not of much use. Some have also proposed to know it by the rising and dipping of this stone toward the [terrestrial] pole [magnetic dip], but in all these the observed experiment does not at all agree with what is supposed.

§54 (גד) — Why a vessel holds more at its bottom than near its top

[54] **גד**. The question: why does the same vessel hold more in its lower part than in its upper, so that one who drinks from it in the cellar [בטרתף — Italian *trattoria*/tavern? (?)] necessarily profits — gets more for less — than one who drinks at the top of a mountain or a tower?

The matter is clear once it has been agreed and confirmed among the philosophers and astronomers that all the elements are spherical and enwrap one another like the layers of an onion. All the proofs that demonstrate the sphericity of the earth demonstrate likewise the sphericity of the water — for those who sail the sea observe that the sun rises and sets earlier or later according to the variation of their course, whether eastward or westward; and this would not occur unless the waters were spherical.

A further proof that they are not flat and level: from the top of the ship's flagstaff [נס] one can see the houses of a city and its towers, which cannot be seen by those standing lower down in the ship. Now it is known that the line of sight coming from the top of the flagstaff to the city stands as the diagonal [hypotenuse], while the line coming from [the deck of] the ship stands as the leg [of the right triangle]. Therefore, since it is shorter than the diagonal, by [Euclid] proposition 19 [book I (?)] of the first [book], it would by rights be that things should appear nearer from the [deck of the] ship than they appear from the top of the staff — as is well established among the masters of optics, that the nearer is seen larger and more distinctly than the farther. Hence this [paradox] follows necessarily from sphericity, for there is no other cause for it.

Indeed, sphericity is shown not only in the whole aggregate, but even in its parts you see this figure [the spherical form] — as in every single drop, which is round to one who observes it closely.

Now the Philosopher [Aristotle], in *On the Heavens* II.31 [De Caelo II], proved by demonstration that the surface of the water is spherical, after first laying down the accepted premise that water flows of its own nature to the lowest [point] and draws near to the center of the world. Let them be in the figure drawn here [Figure: ציור ערן (?)]: the water descends from points **א ו** [A, V], and **א** [A] therefore necessarily flows down to point **י** [Y], which is nearer the center **ך** [Kh] than the points **א ץ** [A, Tz (?)]; and when [the water] accumulates upon point **ו** [V] and rises up to **ד** [D] — which is as far removed by the distance of points **ץ** [Tz] from the center **ך** [Kh] — the waters come to rest there. For otherwise they would never rest, but would keep flowing without repose, contrary to experiment. Therefore the surface of the water is spherical, [arc] **פך קצ** [P-Kh-Q-Tz], which is what we wished to demonstrate. This is according to those who interpret his words [Aristotle's].

But I demonstrate it thus: if there be a cup **פך דצ** [P-Kh-D-Tz] and we pour liquid into it and fill it up to point **פ צ** [P, Tz]...

or up to surface **אי ב** [A-Y-B (?)]; if you say we add more by pouring, the surface **ך** [Kh] rises higher — that is the point above the case at **ד** [D] — and indeed it rises [?]. The case [hole] is **ד** [D], farther off, so the mouth at **ד** [D] [?]; if [the liquid] should fall at point **א** [A] or **ב** [B], the waters from hole **א** [A] would always be flowing until the vessel empties; and so long as the water keeps emptying, the wood-piece **דה** [D-H (?)], and concerning this is what was said [?]. And R. Nachman brought you a proof of this [?] [literally: "and N.Ch. brought you, etc."]. But the architects [ארכימשידים — Archimedeans? (?)] [build] along these lines, generally [?].

[This central stretch of p. 421/422 is partly worn; bracketed letter-tokens are transcribed as printed.]

Here, then, it is explained — as it has been clarified — that the surface of water enwrapping a mountaintop holds less of it [water]; for if you draw two arcs, one inside the other, each about a different center, then to the degree that the smaller circle's arc is shorter, its chord too is shorter than its [the larger circle's chord (?)]; and the line cutting the smaller circle's arc itself cuts off a smaller portion of the smaller arc than [it would of] the larger. Hence the concavity of the lower cup holds more than the concavity of the upper, even though chords **אב גד** [A-B, G-D],

which are the diagonals and breadths of the cups, are equal. Only — in small vessels of slight height the difference is not perceptible, for it has no [appreciable] ratio to the half-diameter of the earth. Even so, the matter is true and necessary by demonstration, to those who understand, and this is what was said.

§55 (נה) — Why falling bodies do not descend by the shortest path

נה ראיה [55, "a proof"]. You have seen how heavy bodies do not descend by the shortest path; and this will be well explained in the following paradox.

There is no doubt that, if their motion were not compelled and natural — as it is — but were [free] descent along the shortest [line], namely toward the center: suppose a rod of even thickness and weight throughout [literally "in all parts of its body"], shaped like the bent curve of a sphere [bent like an arc]. Then in its fall it would mark, at its two ends, two parallel [lines] in the air — for no end has more force than the other to fall and descend faster than it. Therefore the part at its middle descends, and its whole body lies outside the center. Yet the heavy body does not bear itself onto the center of the world, but only [presses] onto the body [beneath it]; therefore the whole heavy body will lie outside the center. And if it strives toward one of the [two] ends to reach the center, it must travel by way of two legs, not by one [the hypotenuse], and so it lengthens the path [?].

The first [point] is only to say that the [body] inclines toward the surface [of the earth], not [exactly toward its center]; for in the following paradox I have shown you, in the figure, that what descends by way of **ך ר** [Kh-R (?)] ascends by way of **ך** [Kh], and they do not ascend and descend by the shortest line, since they incline toward the [terrestrial] center [למרכז], not toward the surface of the earth. For otherwise they would have to ascend and descend perpendicular [נכחיות] to one another, as [in the case] when they were [parallel?] in your figure, [whereas] they were [in fact converging]. So it is clear they do not descend by the shortest line, but along the surface's measure [?], not by the measure of distance from the center. So when the long, even rod falls and its parts [strike] — [it strikes] not [perpendicular] to it [the surface], but obliquely [distant] from it. From this is a proof that the motion of heavy bodies lies outside [the perpendicular] [?] — and there is no need [for] more.

What [holds] the parts [of a body] together in all the force of the heaviness in them, so that they do not separate — that each one [stays] upon its place — [is gravity itself]. And every body — in a stick of wood or a reed — likewise [?]: in bodies, if a heavy body should fall upon the earth, it touches it only at a point,

since a sphere touches a [flat surface] only at a single point, as is explained in [Euclid] book III among the elements of geometry. Therefore the whole world is not disturbed by the fall of [a body] upon the earth. In sum: the center of the heavy body is what descends toward the center of the world, and after it [the body] is drawn…

all the part [parts] of the body, and they are reckoned as one in this — although it is impossible that all the parts should arrive [at the center] together; rather, each part reaches its destined point by motion. As for certain people's view that delay occurs in this [?]: those are the affairs of the would-be philosophers [המתפלספים, dismissively], and there is none [substance] in them. At the far west, when one arrives there [?], the comers [?] [are blown about] on the wings of the wind [Tehillim 18:11 (?)], and there is no fixed place for them here.

But as for those who hold [that falling lines are] parallel — the reverse is in fact true [?]; for this has not been [grasped] in the mathematical sciences. As is explained in the science of optics, which I have learned: [things] appear parallel when they are not — like long ditches [furrows], which appear to converge, and so too their [vanishing] heads, [like] those who go aboard ship lengthwise; in [their] thinking they have not yet [reached] dry land, by reason of [the ditch's] great length — as Pliny [פליניאום — Pliny] reported, in book V, chapter [?]. But in truth the matter will be properly explained by our [reasoning] and by demonstration — namely, that after it has been shown that every heavy body inclines toward the center of the world, if you draw many straight lines from the tops of two towers, or from walls standing upright on the surface of the earth (and so parallel to one another, up to where they meet [?]), and you extend them to infinity, the two will meet and make an angle at the center of the world. Therefore it is impossible that they should be [truly] upright [perpendicular] against [both] the premise and the senses — unless we say that the parallels do meet. And it is true that no [tall] building stands parallel to another along its whole height; for the building is necessarily wider [its parts more distant from one another] at its tops than at its bases — for otherwise the pillars would not stand as pillars and could not stand and endure. Hence a wagon that cannot pass through a narrow lane: if they raise it higher up, it may pass. And so too one boring a pit down to the center of the world must make its mouth wide, and the further it goes the [narrower] it becomes, until its sides meet at the center of the earth; for in any other manner its sides would not stand upright and would not endure.

From this you understand that the fortress [tower] of the generation of the Dispersion [the Tower of Babel], which they intended to raise up to the heavens, should by rights have had a narrow mouth and gone on growing wider, in the shape of a cup; for otherwise it could not stand — as every builder and craftsman knows, [though they] think the opposite. [Some letter-tokens here are abraded; read as printed.]

It is clear from this that those who go up or come down by this means do not descend by the shortest line; for [if they did] they would fall outside the terrestrial sphere. In the present figure [Figure: ציור עד (?)]: if **אב** [A-B] is the surface of the tower and a stone falls from point **א** [A], it does not fall on the line parallel and perpendicular to line **יך** [Y-Kh], by which it would fall outside the earth, but rather falls by a long path along line **פד** [P-D] and is brought to the center, which is what heavy bodies long for — there they come to rest. In low elevations the magnitude is not perceptible, [so it is supposed] that the towers are parallel in every building; but the truth is as I have said. So too with the pillars of [tall] columns: were they extended down they would [meet], and would all converge to the navel [center] of the earth; yet they appear perpendicular and upright. Hence the shadows cast by them are likewise not [truly] perpendicular in truth — only the [optical instruments] deceive [us], even close at hand, with various vessels [lenses]. Since this comes to deceive, [let me reveal] a wonderful thing I have devised, to explain to you with great clarity, God willing — namely, that the rays issuing from point **א** [A] of an illuminated body: the more distant they are from one another, the nearer they become to being parallel to each other, until to the sight of the eye they are reckoned parallel. This I have clarified [in part] in [a figure]: Mechaniki [מיכא״ניקיי — "mechanics" (?)] [Figure: ציור עט]. For if we observe the altitude of two stars — men [observers] **א א** [A, A] with an astrolabe **אשטרו״לביאה** [astrolabe] or [other] instruments — there is no doubt that angle **א** [A] will not appear equal in both; therefore, by [Euclid] proposition 16 of the first [book], the rays of light **א קא** [A, Q-A (?)] will not be parallel. But this will not be verified [as parallel] unless the two observers mentioned above are very near each other; for otherwise [the lines] would not in any way be equal to the angle of the [region] [?], even though the senses cannot perceive the difference at a small interval, until the geometric precision escapes [notice]. But at a slightly greater distance — say a distance of a parasang or more, where the [apparent] horizons seen by the eye change — the difference would be perceptible beyond doubt, and in geometrical learning it is explained thus, that the rays which…

[Geometric demonstration. Figure referenced: ציור עט.] ...[the rays] are equal [in length] and divided by lines **רד בג** [R-D, B-G], [so that] arcs **חח וז בג רד** [Ch-Ch, V-Z, B-G, R-D] are [cut] into equal parts and are parallel — both to the [first] sixth [proportion]. Triangles **שבג שרד** [Sh-B-G, Sh-R-D] are equiangular, and their sides proportional, by [Euclid] fourth [proposition] to the sixth [book VI.4]. They are divided by lines **חח וז בג רד** [Ch-Ch, V-Z, B-G, R-D] into equal parts and are parallel, both to the sixth.

[The proportion-chain, as printed:]

- As **שו** [Sh-V] is to **שח** [Sh-Ch], which is twice **ה** [H] — that is, as **שו** [Sh-V] is to **שף** [Sh-F (?)], which is twice it — so is **וז** [V-Z]; double **חח** [Ch-Ch], and just so **שו** [Sh-V] is to **בג** [B-G] as **1⅓** [literally "1 and a third" — 1⅓], and as **שר** [Sh-R] is **1⅓** times [literally "1 and a third times"] **שב** [Sh-B] [is to] **כד רך** [K-D, R-Kh] as **1⅓**, [so is] **בג** [B-G]. Therefore [Figure: ציור עט]:
- the ratio **וז** [V-Z] to **חח** [Ch-Ch] is greater than the ratio **בג** [B-G] to **וז** [V-Z], and the ratio **בג** [B-G] to **וז** [V-Z] is greater than the ratio **רד** [R-D] to **בג** [B-G].

So the ratio keeps diminishing, the more distant the [ray] roots [points of emanation] become — that is, as lines **רד** [R-D] approach being [equal]: the ratio of **וז** [V-Z] is large to **חה** [Ch-H], while the ratio of **בג** [B-G] to **וז** [V-Z] is greater than the ratio of **רד** [R-D] to **צנ** [Tz-N (?)]. Thus they are more nearly equal — lines **בג וז** [B-G, V-Z] more so [than the others], and these more than lines **וזח** [V-Z-Ch]; and so always, the more distant they grow. Therefore at a great distance — as it is from the center of the earth to its surface, and all the more so up to the sphere of the sun or the stars — they will be very nearly equal, until the joining lines **בר בד** [B-R, B-D] (which are equal) become parallel, by [Euclid] proposition 33 [?] of the first [book]; for the longer the rays **שר שד** [Sh-R, Sh-D] become, the smaller the ratio of lines **רד** [R-D] to **בג** [B-G], and understand this.

[A] More briefly [the matter] will be explained thus. Divide equal portions **רנ דם** [R-N, D-M] of the star's rays into parts so small that they bear no [appreciable] ratio to the distances **שרנ שדם** [Sh-R-N, Sh-D-M], and join them by line **ו** [V]; then **רד** [R-D] will be parallel [to **עם**], both to the sixth, and triangles **שעם שרד** [Sh-Ayin-M, Sh-R-D] equiangular, by proposition 16 to the first [I.16]; their sides proportional, by [the rule] fourth-to-sixth [VI.4] — that is, as **שע** [Sh-Ayin] is to **שר** [Sh-R], so **עס** [Ayin-S] to **רד** [R-D]; and from what follows from proposition 19 to the fifth [V.19], by inversion of the ratios:

- but according to the premise [the working assumption], the difference between **שעץ** [Sh-Ayin-Tz (?)] and **שר** [Sh-R] [≈ **שע** to **ער**], and the difference between **שם** [Sh-M] and **סד** [S-D], are not perceptible at all; just as **שן** [Sh-N] is to **ער** [Ayin-R], so **עס** [Ayin-S] to **סרח** [S-R-Ch], the surplus over [the value] upon **רד** [R-D].

Because of the great length of **שנן שם** [Sh-N-N, Sh-M (?)], the surplus of **עס** [Ayin-S] over **רד** [R-D] is imperceptible, and they are reckoned equal and as parallel to one another; and being joined by lines **ער סד** [Ayin-R, S-D], the lines **ער סד** [Ayin-R, S-D] — the parts of the rays **שע שם** [Sh-Ayin, Sh-M] — appear parallel to one another. This is what we wished to demonstrate.

From this there has spread among the masters of the mathematical sciences the saying that the sun's rays nearest the earth are reckoned parallel — which, as it appears, is not the very truth, just as Pliny and his commentators erred [in supposing] that the parallel rays would be parallel to infinity, and the inclined ones would forever go on diverging and inclining, [as though] distances did not alter what is natural. From what has been said you will understand that, if the whole surface of the earth is spherical, it is impossible for there to be any perfectly even valley or any pavement perfectly flat and level; for just as we showed with [the manner in] which a spherical surface is made, so the soil is made — only that it is not perceptible over a short distance. For so long as two upright posts of equal height are set up and the ray of vision passes through the heads of both, the valley joining them, **כג** [K-G (?)], will be even [level]; yet this cannot be even over [a span of] a quarter-parasang without curving and inclining to roundness. The length of the arrow [perpendicular] indicating the height of the screen separating sight from distance has already been demonstrated in paradox 29. And if you imagine a level, straight pavement longer than a mile's walk, it will not lie on the [true] surface of the earth, but [will be] a building or a sloped figure: one who walks along it from one side rolls and goes down, and from the other side climbs and tires, as one ascending a high, steep mountain, until he can climb no further, drawing near to the heavens and far from the center of the earth. These matters are easy for the discerning and need no [elaborate] figures and lengthy demonstrations.

§56 (נז) — On inclined lines (Maimonides) and curve-drawing instruments

[56] **נז**. On the matter of the inclined lines [the asymptotic lines] mentioned by the Rav, the Moreh [Maimonides, *Guide of the Perplexed*], many have written; and

one of the leading men of my country gathered them into a book of its own, and according to him there are 13 modes [of the demonstration]. I myself have seen, in the Holy Tongue, in manuscript, still more added.

But a straight and short path I have shown to certain of the enlightened by means of the curved line called the conchoid [קונקי״לישׁ — Greek: κογχοειδής, "conchoid"], which is the invention of Nicomedes the Greek [ניכו״מידיס — Nicomedes]. And with my dear friend, the man of [great] learning, Mahara"r [our teacher the Rav] Moshe — may his Rock keep him — you will find an instrument by which this line is traced with great ease. Elsewhere we have described the instruments that draw the hyperbola [חפריבולא — Greek: ὑπερβολή, "hyperbola"] and the parabola [האיפרכולי — Greek: παραβολή, "parabola" (?)] and all these and their like. And there is, moreover, no instrument [but] it can be traced, as the ancients used to…

Ma'ayan Chatum (continued)

§57 [נז] — continued: the conchoid (deer-curve) and its asymptote

we mark the deferent (העגול) first, once the compass has been found (Figure 80). Let קו בת [line BT] be the line, and let קד [KD] fall perpendicularly upon it, the pillar (העמודי) standing at point ה [H]. We set the axis of the conchoid line (הקו הקונקי״לישׁ [the conchoid]) at whatever point we wish on the pillar, only that it must lie beneath line בת, which we shall call the horizontal (אופקי) — for example, let point ך [K] be the lower axis, and the upper axis will lie on the portion of the pillar above the horizontal, for example at point ג [G], at whatever measure you wish, for there is no need for the horizontal to bisect the pillar.

From the lower axis draw out many lines rising and crossing — for example קז, נן, יה, שפ, אע [KZ, NN, YH, ShP, AʿV], and so on. The more lines you draw close to one another, the better. Mark above the horizontal a line cutting off from each of them equal segments of the measure בד [BD], all beginning from the horizontal — for example בף [BF]. The conchoid is then more complete and finer, since it passes over the endpoints of the equal lines just mentioned, as it [passes through] קז, עפג, זה, זסף [KZ, ʿPG, ZH, ZSF]. I call this the **deer-curve** (קו צבייי — a curve resembling the deer-creature [the conchoid; in the vernacular: tishtu"rini (?), Italian for the figure[?]]). After I have drawn this curve I shall teach you its uses in the science of mathematics.

(α) It is impossible for this curved line ever to meet line בת [BT], the horizontal — for of necessity the two bounding points of each of the lines equal to the perpendicular line ה־ג [HG?] lie above the horizontal; and from the universal premise, the eleventh [?] of the Elements, the lines coming from axis ך [K] which intersect the horizontal at בף [BF] make with it the angle that divides the upper portion from the lower of each of them. Therefore they will never meet it, for the angle arises [where] two lines [meet] each other, not from one line falling upon another.

(β) Every point other than the figure-point ג [G] which you mark on the deer-curve will be closer to the horizontal than the perpendicular line ג־ז־ץ [GZTs?] is; and the more the points are larger than the lines of [?] (the seventeenth of the first [book?]), so too the pillar בד [BD] will be greater than each of the pillars [erected] on the horizontal. And since the lines שפ, א [ShP, A?], equal to pillar דב [DB] by the construction, are farther from the axis, they will be closer to the horizontal line ־ב [B-]; as it will become clear when you drop [a perpendicular] from point ם [M] [to] עת [ʿT]. Therefore point ז [Z] is farther from the horizontal than point ם [M] is — and this is the first [proposition]. The second is demonstrated thus: since the exterior angle רשק [RShK] is greater than the apex-angle [of] the sixteenth of the first, the angle פש [PSh], being equal to angle ׳פשה [FShH] of the fifteenth of the first, will also be greater than it [by an amount] עאת [ʿAT]; and since the angles לת [LT] are right (the thirty-second of the first), angle לפש [LPSh] is smaller than angle תעא [TʿA]. Therefore, if you cut off angle הזע [HZʿ] equal to angle פש [PSh], the two triangles עתץ, פלש [ʿTTs, PLSh] will have equal angles (the thirty-second of the first), and the ratio of the fourth term to the sixth will be: as עץ [ʿTs] is to עת [ʿT], so פש [PSh] is to פל [PL]. But אע [AʿV] is greater than עץ [ʿTs], since angle עצא [ʿTsA] is greater than angle [?] (the sixteenth of the first), and angle עאץ [ʿATs] is smaller than it (the seventeenth of the first). Therefore, by the eighth [proposition] of the fifth [book], the ratio of אע [AʿV] to עת [ʿT] is greater than the ratio of עץ [ʿTs] to עת [ʿT], which is greater than the ratio of א [A?] to פל [PL]. Therefore, by the tenth of the fifth, עת [ʿT] is smaller than פל [PL] — and this is what was to be proved: that is, [the ratio of אע to עת is] greater than the ratio of פש [PSh] to פל [PL]; and since אע, שפ [AʿV, ShP] are equal, so likewise the ratio of עא [ʿA] to עת [ʿT] [is greater] — [thus] עת [ʿT] is smaller than פל [PL], and this is what was to be proved.

(γ) Even though the deer-curve never meets the horizontal — since [the curve], even though it comes very near the horizontal, never meets it — nevertheless it necessarily meets every line [drawn from] ג [G]. For let דמ [DM] be a line parallel to the horizontal as well, prolonged to infinity; it cuts the pillar גה [GH] at point

[?]. Let the ratio of הזג [HZG] to גר [GR] be as גח [GCh] to a fourth [term] — for example מ [M] — and let us, taking axis-point ך [K], draw upon it an arc of a circle at distance מ [M]; this will cut line נש [NSh], which is necessary, since line קמ [KM] is greater than line גד [GD], inasmuch as ג [G] is greater than line גד [GD], and they are in ratio. Suppose it cuts it at point ף [F]. Then, in the triangles נף [NF], [by] the proposition that the fourth is to the sixth, as גה [GH] is to גר [GR], so פב [PB] is to פד [PD]. But it was also that, as נה [NH] is to נך [NK], so גח [GCh] is to פו [FV], namely חר [ChR]. Therefore בדז [BDZ] is equal to line בף [BF] (the ninth of the fifth). But it is impossible for them to be equal unless they meet on the deer-curve, for if you mark the deer-curve upon points ב, ם [B, M], line נף [NF] will be cut…

at point ך [K], parallel to the horizontal — only if line נם [NM], or one resembling it, is not at point ף [F]; and this is what was to be proved. There is no need to prolong [the argument], for [even] if we draw a sloping line, all the more will it meet the deer-curve from one of the sides — that is, when the perpendicular stands erect upon line גה [GH], the perpendicular [is] parallel and right to the horizontal בה [BH?]; if not[?]…

[Two lines here are heavily abraded and partly illegible — see QA. The geometric argument continues:]

the lines [?] which we draw [toward] it[?], their endpoints will be farther and farther [apart], cut to infinity; yet even so, the distance will not be of the measure of the perpendicular line גה [GH]. We drop from point [?] perpendiculars onto the horizontal ל [L?] [?], near and equal to one another (the thirty-fourth of the first), like גה [GH], and this is what was to be proved.

(δ) [On] the measure of the deer-curve in dividing an angle: it [divides] right [angles?] [into] equal parts. [?] With this horizontal it [the distance] will never be equal — the distance between the end of the deer-curve and line [?] [is never] equal to the perpendicular line [?]; for the deer-curve [approaches?] the horizontal, [yet never reaches] the limit; and since the end of the deer-curve [does not] … the lines [?] grow farther apart at points [?], which grow distant by [?] points [?]. And this [?] — it [the curve] is longer [?] than [?]; it becomes reversed[?] [?] smaller than that which is [?] easily; and that over which the ancients and the moderns labored to consider, the angle in question [is] one-third the [?].

For in the square (בנצבה [the right-angled/square figure]), all know how to divide [it], since the half-diagonal is one-third of thirty degrees [?] — that is, the sides of

the hexagon; and the breadth (הרחבה) is composed only of a perpendicular (נצבת) and an acute [angle]. Therefore, when you subtract from it the perpendicular, there remains the acute angle [?] which is one-third of the acute angle [?] of the whole [?]; therefore one-third of it is the thing sought, [namely] one-third [?]. Now [as to] dividing the acute angle like the perpendicular: we drop from whatever point we wish on the leg אב [AB] — for instance, let קה [KH?] be a perpendicular אד [AD] upon leg הב [HB], בה [BH?] — we take from point ד [D] toward the right a line double the measure of leg אב [AB], namely line וק [NK?]; and this is the pillar in the conchoid. We reckon line אר [AR] the horizontal, point ב [B] the lower [axis], and point ק [K] the upper axis. We draw the deer-curve as stated above, which is the curved line. We draw out from point א [A] a line אחן [ACh-N?] parallel to line בה [BH], and let it be cut with the deer-curve at point [?]. We draw [a line], and I claim the angle to be one-third of angle אבה [ABH], and that angle אבחז [ABChZ] [or אבח, ז] is one-third of it, [the angle] which is double angle זבה [ZBH] — that is, when you cut angle אבן [ABN?] in half, dividing the whole angle אבה [ABH] into three equal parts. For by the property of the deer-curve stated above, גח [GCh] which lies on the horizontal אד [AD] is equal to the perpendicular line דה [DH]. Therefore גז [GZ?] is double it, [equal to] קאב [KAB?]; and if you bisect it at point טן [TN?], then ה־א [H-A?] is equal to line אט [AT?] or to line מה [MH?]. And taking point מו [MV?] as center, draw a semicircle on diameter דזמ [DZM?]; it will pass over the right angle חזא [ChZA] (the thirty-first of the third); and line וכן [VKN?] [and] angle אמב, אבט [AMB, ABT?] [are] equal to [?] (the fifth of the first). But (the thirty-second of the first) line זא [ZA?] coming to the circumference is equal to lines אב, מח, מג [MG, MCh, AB?]; therefore angles טאה, טחא [TAH, TChA?] are equal, and the exterior angle אטם [ATM?] is equal to the two interior angles זא, טאה [ZA, TAH?]. Therefore it is double angle אחט [AChT?]; and we have proved (the nineteenth of the first) that angle אבן [ABN?] is also double angle חבה [ChBH?] — and this is what was to be proved.

§58 [נח] — Astonishment [רנזה]

[Body heading p. 426: "נח רנזה" — paradox (58) נח, titled רנזה ("astonishment / wonder")]

On the marvelous reasoning [found] among the mathematicians (חכמי הלמודים), whose premises negate, yet their conclusions affirm — and behold, these are the contradictions[?] [?]. I have been calm, having sensed [your] troubles[?] and the hour [?]; yet now I have shown you, in this place[?], great matters such as are found in no book — but with God's help[?], in the simplicity of [?]. As to the

foolishness of the philosophers[?], whose nature is [?]; but now I shall speak briefly.

The cause of astonishment is the contemplation of some matter and the consideration of it with desire and knowledge of its cause. And since the dumb beasts (הבעלי חיים האלמים) contemplate nothing at all, they are never astonished. But the Divine intellect (השכל האלהי), which understands and contemplates [all], knows the causes of every existent thing, for He has created them; therefore He does not wonder at their beauty, nor is He ever astonished. The angels likewise — they are all instruments[?] without inference or astonishment; in a single moment they

[grasp?] their causes. But human beings who contemplate the works of God, so long as they do not know their causes, are bewildered and astonished. From this it is clear that of one who does not know and does not desire to know, it is said: *"A man in honor who lacks understanding is like the beasts that perish"* (Tehillim 49:21) — for astonishment leads to the goal of knowing wisdom; and [it is] the contemplators and the astonished [who attain it].

But those who are placid, settled, and at rest are not complete philosophers, since they do not seek out all the causes. And so are the mathematicians: their whole aim is merely to demonstrate the truths of the theorems through what is marked [in the figures], and they pay no heed to [the causes]. No man investigates except to know whence these astonishing results emerged. The true philosophers, however, are astonished and investigate their causes; and therefore, over these strange theorems, even the great ones of the world have all stumbled.

The one who understands the mathematics is the most complete of the sages, for none of [its results] is hidden from him — he can understand, instruct, and answer every question. I shall bring you a proof of Euclid: the eleventh [proposition] of the ninth [book], or the twelfth according to the ordering of the moderns, in the Elements, who [Euclid] said:

> When there are several numbers in continued proportion (בהתמדות) beginning from unity, and a number other than the first [unit] measures the last of them, then the number coming immediately after unity will also measure it.

Let the numbers in continued proportion be as far as ד [D], and let there be:

ה (H)	ד (D)	ג (G)	ב (B)	א (A)	ו (V)
9	18	36	108	216	846 [recte: 648?]

Let א [A] be the first number, and let it [unity] be the measurer of ד [D], the last. He says that ה [H] will also measure number א [A]. This is proved thus: since ד [D] is the first [?]...

if it does not measure number א [A], let it be first; and since by the proposition (the eleventh of the seventh, ed.) the numbers [א ב ג ד ה] are in continued proportion from unity, [their terms will be] smaller in value down to פג [from ... to G?] (the seventh [book?]), and since ד [D] was posited as measurer of ך [K], then ר״מ [for example] its measurer is so-and-so-many times the number ן [N?]. But it is clear that from doubling א [A] to ו [V] arises ך [K] — this is true in every group of numbers in continued proportion. Therefore, from the second part of proposition 20[?] of the seventh, as א [A] is to ק [K], so ן [N] is to ו [V]; and accordingly from proposition 21 of the seventh, זק [ZK?] measures ך [K], and its measurer is, for example, so-and-so-many times the number מן [MN?]. And since from doubling א [A] by א [A] arises ב [B?], it likewise follows from the aforesaid theorems, in the aforesaid way, that ך [K?] measures אתּ ף [F?]; and suppose its measurer is in the quantity רז [RZ?]. And since from doubling א [A] by itself arises ב [B], it follows from the aforesaid that דק [DK?] measures אתּ א [A]. But it was supposed that it does not measure it; therefore, from the contradiction of [its] not measuring [it], it follows in truth that it does measure [it] — and this is what was to be proved in the aforesaid theorem.

Before I expose the disgrace of the logicians upon us, let me remind you that, just as Euclid wrote in Definition 11 of the seventh [book]: a number is called the "measurer" of some number when, multiplied by some number, it produces it. Take heed of this[?] regarding the words of the aforesaid demonstration, which — I do not know who fathered it, or whether the first expositor[?] [erred], something I cannot believe; or whether some Arab [maintains] that Euclid did not demonstrate it in the aforesaid manner — for without doubt the whole force of the inference lies in those very words. And accordingly, from this [it follows that] proposition 21 of the seventh — וק [VK?] measures ן [N] — and his intent is to prove that if numbers וק [NK?], א [A] are the smallest, and as א [A] is to רק [KK?], so [?] is to ו [V], then ץ [Ts?] necessarily measures ו [V]; and from this he brings [the matter] to the intended conclusion.

But here he erred: Euclid in [Proposition] 21 of the seventh demonstrated only that the smallest numbers in any given ratio measure all the numbers in that

ratio. He did not demonstrate the converse, in negation — that is, that if they are not the smallest, they do not measure some number in that ratio; for the prime numbers (המספרי' הנכחיי') are not the smallest, yet they are in the ratio 2 to 1, called the **ratio "shovro pla"** [שובר"ו פלא — "broken (ratio) wonder"(?); read as printed; an obscure ratio-name (?)]; and as 1 is to ד [D], so א [A] is to ב [B]. And with all this, [the demonstration] shows:

א (A)	ב (B)
2	4

ה (H)	אל (to)	1
ג (G)	ד (D)	2
		1
	6	12

that א [A] measures ב [B], and likewise ב [B] [is] to ד [D] in equal ratio. But [the relation] "equal to equal" does not occur in all the ratios; for in these two, א [A] does not measure ד [D], nor does ב [B] [measure] ד [D], according to the definition of "1 to ו [V?]" the measuring [relation] we set out earlier. Therefore, even though the measurers are the smallest, the result does not follow on account of their being smallest, as from an essential cause — for even

were they not the smallest, [the result] would still follow. Therefore the demonstration is the one brought below, in truth, which shows that they are not [in continued proportion] and that this [class of numbers] is hidden — that prime numbers (ראשוני') [are prime] with respect to one another, and that they are measurers, and so on — without any [further] foundation or supposition; and [this is what] all the complete mathematicians, and many of the commentators on Euclid [hold]. And as for your saying that there are such [errors] as these among the moderns — they are all mistaken in some matter and do not understand the principles of the things and their properties and conditions.

Now, Aristotle, in the first [book] of the *Posterior Analytics*, in explaining that the universal demonstration is more weighty and excellent than the particular, brought an illustration from the marvelous property common to all the plane figures: namely, that the sum of all their exterior angles — that is, those that arise when you prolong each and every side of the figure outside it — is equal to four

right angles. This is proved [by] the thirty-second [proposition] of the first [book] of the Elements; and you must prolong all the sides in one direction, [so] that the figure is shown to have as many angles outside it as the number of its sides. There is no difference whether the figure is a triangle, a quadrilateral, a pentagon, a hexagon, or a thousand-sided figure: for all figures, when [their sides are] prolonged, this matter holds — that the sum of all the exterior angles is no more than four right angles.

א (A)	ב (B)
10	20

ג (G)	ד (D)
6	12

Take heed not to err by prolonging a side in two directions, for then there would be more angles, and the property would not hold over them. (Concerning the marvelous figure called the **Shield of David** [מגן דוד, the hexagram] and its properties, we have spoken in *Sefer Basmat* [ספר בשמת].) Therefore, in a triangle, its three exterior angles are double its three interior angles; in a quadrilateral, the four exterior [angles] are equal to the four interior; the five exterior angles of a pentagon are equal to four right angles; and likewise the six of a hexagon [are equal to] four right angles; and so all of them. Therefore, in all figures, the sum of the exterior angles is equal to [four right angles].

Now if you knew this property of a triangle only — that a triangle of three unequal sides [has this property] — you would have a very particular piece of knowledge. If you knew that this property [belongs] to every species of triangle when [the sides are] prolonged, it would be more universal knowledge, but still deficient. Only when you know that it is universal to **every** figure is the knowledge complete; and He alone knows all the things and their properties in a universal way — and a few universals truly suffice the knower up to this point. But we are deficient in our knowledge, most of which is particular even when it appears universal; for even the knowledge of the aforesaid property, which holds for every triangle, appears to him universal, yet it is particular relative to every [other] figure. (In *Sefer Basmat* I have expanded on this with good taste and judgment.)

The purpose intended by [Aristotle] here is to inform you that one who grasps the aforesaid inference in universal knowledge can demonstrate any [number of?]

particulars in the aforesaid manner. I shall bring you another [illustration], from the [argument of] Archimedes in the first [book] of *On Floating Bodies* [ראשון לצפים]: that in every place where they [waters] are — even in vessels — they preserve the shape of the spheres above, in a place where they are not surrounded by [other] waters, in this manner. And what I noted in *[Sefer] Tamar* [בתמר]: to show you that the waters

of the sea, since by their nature they seek the center, except that the heavier earth among them precedes [them] and seizes [the center], therefore they surround it on every side; and their center is the center of the world, and they are pressed away from it [at] an equal distance. Now, for the clarification, the aforesaid sage [Archimedes] sets out premises there — that is, that the more depressed portion of the waters presses down the portion that is depressed a little less than it, and thrusts it away, this being so when the two portions are close to one another and adjacent, set in some place. And the meaning of "pressing" is: when other waters fall upon it straight down, perpendicular and descending, so that they are pressed by them — so too these [portions], and they press the nearby [portions] and thrust them away. All this is clear; one who understands will not deny the truth of what is said, and will testify that it accords with sense and experience.

After this, [Archimedes] sets out a postulate and explains it, [stating] that it requires the clarification of the aforesaid proposition; and this is its clarification: if some body is cut by some plane in several intersections, and they always pass over one and the same point, and the intersection-line [of the cut] makes a circle whose center is the point at which they intersect, then it follows that —

[Marginal proof-figure premise, restated:] If you cut some body whose plane [is cut by] ב [B?] [and] you make [a] cut, then the mark [or] cut which the cut body makes is spherical, and its center will be that very same point. And when you draw it (depict it), the surface (הכף) will become clear to you — its truth is a circle by [?]; it follows that this body will be a complete sphere at the place of the cut. For from point א [A], at which the body was turned [back] together with the plane, all the lines אב ,אג [AG, AB] coming to the circumference are equal, since, by the supposition, the mark which [the cut] makes is a circle. Therefore, if you mark point א [A] standing in its place, and the plane גב [GB]…

[The proof of the Archimedes postulate continues beyond this opening; no colophon appears on these pages — Ma'ayan Chatum continues.]

Sefer Ma'ayan Chatum (continued)

§58 — "Astonishment" (נח), conclusion of the Archimedes proof

in motion around the body, so that at every point a perpendicular is dropped, and your line will be cut [at] point **א [aleph]**, toward the surface of the body. Therefore all the lines issuing from the center point **א [aleph]** to the surface of the body [?] are radii, and their centers [are at] **א [aleph]** — and this is what was to be demonstrated.

Now it is plainly shown [?] how it is necessary that the surface of waters be spherical: it requires that they be radii, so that [?] the curve [?]…

the surface of the waters is everywhere uniform, on every body; and even so, that body is [?]… as is plainly shown [?] to whoever would believe that the rounded surface of [?]… is radii. For from the aforesaid proposition it follows that this surface… …the cut [portion], the waters seek the spherical compass [perimeter], and it will be made clear so as to be understood [?]… …the surface of the waters together with a surface **ד [dalet]**, which is its center, [in the case] where the surface of the waters does not form a spherical compass — it is required that the radius [?]…

[Several lines here run full-width across the binding gutter; point-letters recovered where legible.] …the radii were uniform [equal] over it, and we posit [?] — and so too [if] the depth [draught] of **ד-ט [dalet-tet]** were greater than… … according as the distance of **ד-מ [dalet-mem]** by a spherical surface **חזמה [the depressed portion]** [?], and it would be according to… …**דן [dalet-nun]** falls short of the line **מקו [from the line] ד-ט [dalet-tet]**, and the mark **ד-ט [dalet-tet]** [makes] a smaller and lesser spherical surface. Now behold, the parts of the water that [are] within a spherical surface **ני [nun-yod]** are at rest; but the waters above them, within the surface **זני [zayin-nun-yod]**…

are larger, and they are heavier; and the part **ד [dalet]** and the depressed portion **ון [vav-nun]** are up to **ד-ם [dalet-mem]** [?], so that the radii are equal — and yet there is no… …their distance from the center **ד [dalet]** is uniform, but the part **טו [tet-vav]** is heavier [?]… For this [reason] the waters are pushed down, even [the part] from **ה-ו [heh-vav]**, and likewise some [parts] remain… …the place that is distant by the center [?] — when the world's center is not uniform with part **א [aleph]**… …if so, they are radii. If you wish [to take it] of a [physical] vessel, then you have here…

we have explained, and it is wondrous, and according to Archimedes [?]… is [?], and this is true [?], that the depressed portion **חתם [chatam — the "sealed"/cut portion]** of the cap [dome] over it presses down… …and the depressed portion **ומטח [vav-mem-tet-cheth]** is from the place where the spherical surface **ד [dalet]** is — that is, the perimeter [compass] of the water; and so too in every place, where the body is in motion around a point. And this is the matter: it is a sphere [round body], and this is what was to be demonstrated.

And so Euclid teaches, in Book XII, to begin with, [for] Theodosius [?], so that the means he employs are not… …we calculated it — for the commentators introduced [it] as the demonstrative middle of my words [?]… …toward a finished construction. And this is the figure of the proposition demonstrated (Figure 82 [ציור פב]) — and the corner [?]… [Figure 82, plate, not legibly present on the page — see Diagrams.]

the body of the **משחף [the cut/intersected figure]** [?], being itself a curved [body] with two centers **אג ובד [aleph-gimel and bet-dalet]** — its center [is] within itself [?]; and the centers **אג [aleph-gimel]** [are] its [own]… …with center **מ [mem]** standing erect, and according as what is drawn from **מי [mem-yod]**, as taught in… …this center of the sphere, and it will be a sphere **דים [dalet-yod-mem]** at point **דז [dalet-zayin]** [?], and just as so it was upright [?]… …its center **מ [mem]**, and according as what is drawn aforesaid, the center of the sphere will be… …for a sphere **א [aleph]** [there are] two centers, and this is false [?]; and the configuration runs… …the two aforesaid circles are one and the same, [together] with the center of the sphere and… …of the largest [circles] in the sphere, as Theodosius teaches first [?], in Book X [?]…

§59 — "But indeed" [a question on day-length] (נט)

§59. But indeed (נט ואולם) — a question: How is it possible that those who dwell [in a place] enjoy more of the solar light…

have they to enjoy more of the solar light? This matter is explained in *Sefer HaKaddur* [Book of the Sphere], chapter 3 [פרק ג'], where the reason is given [?]: that if it pleases [?], upon the light itself — which is called **אזא [?]**… For to those dwelling beneath the artist's [Creator's?] zodiacal belt [?], there will be night… … until the end is reached in the virgin [Betulah/Virgo], and the traveler — this is greater than… …since the day's gathering [day-length] for them is as explained in the *Sefer Iyyun HaSh…* [book of contemplation] [?]… …**ה [heh]**: those who are at the spherical [equator] who bring the solar body — it is at time… …**ו [vav]**: the gathering of days will be **182½** [182 ½], uniform for the gathering of nights…

[Marginal/inline fraction reads 182 with a fraction ½.] …the matter for those dwelling beneath the northern pole, their reckoning is… …who exceed by the days of the south beneath the equator: but… …from the First Light, whether from the Second [Light], even those who dwell beneath… …at the equator [?], which by the corrupt view they add [?], that there be [?]… …upright at the head [zenith], beneath the horizon there will be twilight… …its inclination does not set [sink] for those dwelling beneath the [celestial] axes more [?]… …of the sun, and being [for] the dwellers, like an ox going [in] the morning… …glow [twilight] to them until [their] coming, until **א-א [?]** for the lion [Aryeh/Leo] he goes [?] [for] a time [?]… …**[315] שט"ו** day, and these for those dwelling at the equator, ever-setting permanently… …of the sphere, upright [zenith] for the equator if the sun be at the equinox [?]… …and when it is in Cancer [Sartan], or [when] our candle is distant **[66] ס"ו** degrees…

[66] ס"ו degrees [?]; and these measures, reckoned within a sphere, [are] **24** hours and according to their nature [?]… …toward the east, there it is **24** hours, and according to its own [face?], when one sees… …in five of them which they playfully [?]… …rounded [traced] a complete, large round [circle] in the heavens [?], as one does… …the motions, sometimes swift, sometimes slow [?]… …they were hastening, and afterwards [some go] faster and others slower… …toward an extreme in the motion. **And indeed (ואולם)** — for if they were beneath the moon… …**ד [dalet]** standing erect, and according as what is drawn from **מי [mem-yod]** [?]… …187 [in the margin/inline], and being **24** hours, [?]… …in fact night and day are made equal as… [Numerals on this band: 187 … 24 … 41 … 19 … 1 … 20 … 180 … 365 … 200 … 180. See worked-value list below.]

41 — that is one hour and **49** [49 / מ"ט] minutes [regaim] they travel [?], even though it be… …that is, as if they were all great [?], according as it be the equator [?]… …**1** hour goes up to **20** days, by their closeness, to **180**… …the existent, [from] the twilight and the solar [light], the dwellers travel beneath the equator [?]… … beneath the equator [?], they conduct [themselves] more — and this is what was to be demonstrated. **And indeed (ואולם)** the author of the *Book of the Sphere* came back and said there [?]… …that the sun [at] their place is screened by clouds [?], and there is no power for it to cross [?]… …it is true that the inquiry [investigation] of the rational [intellect] is Torah, according to several of my words, and the experiment [trial]… …for the two who dwell [?], that in them there is a six-month day and a six-month night, as when… …there is no day, nor when [?]… [The polemic continues against those who add days to the reckoning.]

[A heavily figured/numeric passage; values transcribed as printed: **365** (days of the year, doubled in the back-and-forth of the sun), **180**, **200**, **180**.]

being **187** [?] uniform — and behold, they exceed [by] **39** [?]; and behold, they are deficient [?]… [Inline numerals: **187 … 39**.] …if you wish to grasp the pleasing notion of [that] being [?]: whether from the [First] Light… …the axis of the south, they conduct themselves more, in [a place] uniform [equinoctial]… …the sun [at] **שח [308]** [?] and a deficiency of **י"ח [18]** or **כ' [20]**, [for] degrees of ascent within a sphere [?]…

and behold, even though the sun be at the limit of the equinox [?], its inclination [setting] does not… …it indicates an ascent for all the evils [?], at the setting of the sun; and being [?]… …the twilight [glow] of those dwelling beneath the southern axis, and there is no Venus [Nogah] for them [?]… …darkness, by as much as **50** [חמשים] day, and the days of their light [are] **שט"ו [315]** day [?]… …the sun [at] the horizon, at point **צ' [tzadi]**, [for] the ascent of **מ"ש [?]** of the [?]… …for it is there, in a place [where] the round [circle] is upright [at the zenith], at the head; and when it… …and the rest of the points [degrees] [are] according to the perfection of their times [?], up to a fourth of [?]…

[A polemic against **Francesco Patrizi (פרנציסקו פטריציאו)**:] **And indeed (ואולם)** — see what the two [point-]letters **ב' [bet]** [?] did not display [?], the [?]… For if the small enclosures [are] the clouds [?], and yet [the one] who is too small [insignificant] in [?]… …let him not give heed [his heart] to the vanities of **Francesco Patrizi (פרנציסקו פטריציאו)** [Francesco Patrizi, Italian philosopher], who [?]… …the worthless old man [?], for all the known stars of the heavens [are] known and [?]… …Alexandria [אלאכסינא — Alexandria?] [?], **ח' [8]** degrees, and he has no power to make anything new [?] [innovate]…

§60 — "What do you cry out to me" [against Aristotle] (ס)

§60. (ס) What do you cry out to me [Mah titz'ak elai] against Aristotle, who held that the stars… …[are merely] hollows [cavities] [קעירות] in the lunar sphere [galgal], and they are the vapors [smokes] of the earth, the upper air [?], and being oily [fatty] or dry — flaming [?]… …burning, or [reflecting] the light of the stars; and [as for] the testimony of the latter-day [moderns] who testified that… … to the earth — because they [the new stars] are born [?], I do not deny that these were born [?] or… …for they were in motion in the motion of the lunar sphere [galgal], since whoever would deny [?] the daily [motion of the] sphere [?]… …the rational [intellect], or [those] in motion, [acting] against the perverse view [of] the philosopher [?]… …the sphere [galgal] — for there is no compelling force

[necessity] reaching to the dwelling of [?]… …the foundation of fire would not be — there is swift motion in the motion of the sphere [galgal]…

[The argument continues — that a comet/new star cannot belong to the elemental fire-region:] …the first [sphere] in motion — and how does the separated [parted] vapor [matter] [?]… …it parts in a moment, as is the nature of all that is rolled [revolved] in the air, which is not [fixed] [?]… …and we see them standing for several days [?], and more — and they [?]… …are not distant from the earth more than 22 [כ"ב] mil [?], as is explained in [?]… [Inline numeral: **22 = כ"ב**.] …[in] rapid motion [?], for the stars are wondrous, but they appear [?]… …to the eye [vision], but [in fact] close to twelve [12 = י"ב] parsa'ot were there [?]… [Inline numeral: **12 = י"ב**.] …and stars [?] are darkened [eclipsed]. **And further (ועוד)** — that being all in the air, they are all… …and according as it be [?], and these the wandering stars [planets] are in motion, when… …a special motion from the west to the north, and from there to the east, as when… …the investigators [chochrim], their motions and their settings, which in [their] books they recorded… …the moon and the sun in their travel; and even though by my computations [?]…

they were [?]; indeed, the arrangement of their motions, which at the start were hastening… …in order [arrangement], and there were no [parts] reaching to the extreme within the [?]… …it was fitting that it should move in the special [particular] motion more, in the swift [?]… …their motion forever — more than [cut off?] degrees in a day; and the moon, even [?]… …in their travel, it appears that they are above [supralunar] — for the lunar sphere [galgal] is so [?]… …the upper [sphere] is changed by the motions close to the earth. **And there is not (ואין)**… …for the worlds [eternals?] **Seneca the philosopher (סיניקא הפילוסוף)** [Seneca], a wise man, one of the singular ones [?], and likewise… …the latter wise and discerning men [chachamim] — let them affirm that they [the new stars] are above [supralunar] [?]… …he brought a correction [repair], as is fitting for their views, and their reasonings, [recorded] in books that… …the fifth [element] [?], that they [the stars] are existent, and they perish [pass away?] already, as I have shown in proofs [?]…

…and their corrections [identifications?], that they are blooming [appearing] as a row [?], that one flees [vanishes] [?]… …blooming [appearing] in the body [substance] of the stars [?]; and on what day is the eighth sphere [galgal] renewed [?]… …that they were not in the eighth sphere [galgal] — for there could not have been [?]… …that is, the stars of the first eighth sphere [galgal] [?], that they… … **And further** [there were] famous [stars] afterward, in the year **1572** in [?]… [Inline year: **1572**.] …and all the astronomers [tochnim] today hold to the

elevations [appearances] within the sphere [galgal]… …of the philosophers, and among the Christian sages of theology [?]… …**Niccaynos [ניקיאנוס — ?]** and with them many weighty [authorities]; **but indeed (ואולם)**…

it troubled you [stirred your mind] from what was [?] in the *Book of the Powers of HaShem [Sefer Gevuros HaShem]* [?] — for it is alone [?]… …the bowl of the firmament [raki'a] — that those wandering [comets/new stars] were the stars, beneath [?]… …in those years [were] different places, within the sphere [galgal] of the zodiac [Mazalot], which… …drew near to the earth, as is explained there. **And indeed (ואולם)**, these according to [?]…

[Continuation on the 1572 and 1604 novae:] …a flight [vanishing] of pleasing praises [?] arose [?], blooming [appeared], and it is **נצה [bloom/blossom]** [?]; and this is… …the innovation, that three stars [?] which appeared in years became renewed [?]… …there was for them another motion besides the existing [permanent] motions… …the wandering of them in the year **1600** [?], in the eighth [year] [?] of [?], that they remained existing [?]… [Inline year: **1600**.] …**1572** at **Cassiopeia [קאסיאופיאה — Cassiopeia]**, and one [other] in the year **1604** at [?]… [Inline years: **1572 … 1604** — the novae of 1572 (Tycho's Star, in Cassiopeia) and 1604 (Kepler's Star).] …elevations [appearances] within the spheres [galgalim], and already he calculated, as before, [the] *Palatines* [פלטין — Palatinate?] and dwellers [?]… …the Christians **Esbrutziaus [אסברוטשיאוס — ?]** and **Bsuliaus [בסוליאוס — ?]** and **Veranoria'as [ונרינוריאס — ?]** [?] [— garbled Latin author-names; see Persons]. **But indeed (ואולם)** the vision [seeing] was taken from the side of difference of appearance [parallax] [?]…

it is for it alone [?], a [device?] sufficient to depress [bring down] [?] the egg [oval] of the **משטאייב [?]**, within… …beneath the sphere [galgal] — the moon, [these] were seen by those dwelling in the lands [provinces]… …of the zodiac [Mazalot], for the moon itself [shows] a difference of appearance [parallax] [?]… …these, according to what was testified [?], so that it sets toward the east, and [the] west, which were brought [?]…

[The full-width bottom band of page 4 carries the numeral-dense computation of distances. Values transcribed AS PRINTED:]

within Persia [פרנציסקו פורט? — uncertain] they were seen by all in one place [?]… …their appearance, [the new star] going [moving] [?] above [supralunar]; they [were] swiftly [?]… …it appears greater, and the reason for this is null [?] by the shadow [shade] of the earth's body [?]… …or a star, and behold, my words [are] the purest, [for] the stars are sent [?]… …of the air, and the luminaries are

dwelt [seated] [?] [by] them — the vapors… …it appears a [perfect] circle, this — for all the world it crosses, passing over [?]… …**"Pia Tsini'enu" [פיאה צינ"ענו — ?]**, a vulture's [eagle's] bowed neck [?] [?], and what was hard [?]… [garbled phrase, flagged.] …the sparks [scintillations], the twilights [?], that it is impossible to see [?] in a place… …its station [mtzav] is hard [difficult] against it, having changed from its station by [a] difference [?]… …**that the luminary itself [is] in the sphere [galgal] of HaShem** [?] — and already by choice [free will]… …a flight [vanishing], and a purity, as the power of **Tycho [טיכו]** the **Nilliaus [נלילויאוס — ?]** [?] [— probably "Tycho Brahe" + a garbled second author] in their books, and we…

[The terminal computation band — distances of the new star, transcribed exactly:]

and there is not — for if many stars [are] small, close and distant [?]… **And indeed (ואולם)** — to recount the [view] of those who refuse [the supralunar position], here is your eye [?]… …**ופהה [a gap and an opening]** [?] over the center **ם [final mem]**, and the half of [its] Alexandria [diameter — אלכסונה] **חם [chet-mem]**, and [its] perimeter [circumference]… …the star, according to the view of Aristotle, and there will be a horizon [ofek] [of] **עץ [wood/tree — ʿetz, here a point-label]** for the dwellers [?]… …the star, looking [observed] at point **ה [heh]**, only being [seen] in an arc [keshet] **סי [samekh-yod]** [?]… …it does not appear in any way, and we see it — [therefore] when by the measure [?]… …that it [is] beneath them, over the horizon, [a] mark [trace] ascending [appearing] as much as [?]… …it appears that, accordingly, the star did not appear to them, even… … the diameter [span] of the earth **הם [heh-mem]** is, by the view, [an] arc [keshet] **3500** mil, and **יקו הא [?]**… [Inline numeral: **3500**.] …and in a third [?], the angle of the corner [zavit] **סהם [samekh-heh-mem]** [is] from the rib [side, hatzela], and the surface investigated [?]… …to the secret [mystery] of the foundation [yesod], approximately, [the] sum **סם [samekh-mem]** to **ה [heh]** [is] **100000** [?]… [Inline numeral: **100000**.] …**80. 11'** they tremble [?] from ninety [?], and the remaining angle [zavit] **סבא [samekh-bet-aleph]** [is] **9. 49.** [?]… [Inline numerals: **80. 11'** and **9. 49.**] …it [is] the measure [shi'ur] of arc [keshet] **מאי [mem-aleph-yod] 38. 19.** [?], and behold you see a small thing [?]… [Inline numeral: **38. 19.**] …and a half of [your eye's?] glance — and it is not in it [?], and we see all… …in the air, and it is impossible [?] to leave [?] the star, [for] it appears to us [for] a long time [?]…

[Right half of the same terminal band — parallel distance values, transcribed as printed:]

the span [alachsan — diagonal] **52** mil, every [traveling] sum **סם [samekh-mem] 3552**, and in a third [?] standing erect [?]… [Inline numerals: **52 … 3552**.] …angle [zavit] **פסה [peh-samekh-heh]** of [?] **[8] ח'**, turned [?] to a fourth, to the secret [foundation] [?]… …they [are] toward **ם [mem]**, [from] the rib [hatzela] **98536**, and arc [keshet] **80. 11'** [?]… [Inline numerals: **98536 … 80. 11'**.] …and it [is] the measure of arc [keshet] **מא [mem-aleph]**, [the] traveling [hilukh] [of] **כפלה [its double]**, which is arc [keshet] [?]… …so that it does not appear, even [for] one hour and a half [?]… …the night; these stars [in their] travel are not in the air, and not [?]…

among the wandering stars whose passage beneath them [we observe], we must posit ourselves upon a circle as far removed from the earth as the distance of the orbit of the moon. Consider this carefully [ראה א"כן]. Now if there is, with respect to these trustworthy demonstrations, any room to give weight to the ancient Greeks — for according to their account, what appears to the inhabitants at point [ה] would not appear to the inhabitants at point [?] — yet the distance between the two observers is small for one who computes it, and experience itself shows that the [novae] were seen by inhabitants at the two extremities of the world at one and the same time. Therefore believe that Aristotle is a man and not a god: do not rely upon him in all his words without investigation. I have already treated this at length earlier, and I have refuted every error of those who go astray, so there is no need to dwell on it further.

§61 [סא]. Concerning the **mountains of the moon**. There is no doubt that the sense [of sight] testifies that the moon's body is not perfectly round like a smooth, even sphere; for the polished bodies and the glass instruments [telescopes] that have appeared in our day show us crevices and fissures of rock upon it, so much so that one could almost make a *selenography* [שילינוגרפיאה — Greek: σεληνογραφία, "mapping of the moon"], just as men make *geography* [ניאוגרפיאה — i.e. γεωγραφία]. For the understanding of the matter, I copy here ✻ the figure set down on p. 11 of *Sefer Ma'ayan Ganim* [the author's own work].

Diagram. A construction illustrating the heights of the lunar mountains and the depth of a lunar crater, by comparing the moon's diameter to the earth's. The figure was originally printed in the author's *Sefer Ma'ayan Ganim*; here Delmedigo reconstructs its measurements verbally.

- א, ב, ג (circumference): points on a great circle equal to the sphere of the moon, labeled אבג"ש.
- ש (circumference): a point on that circle.

- [Center point]: the center of the lunar circle (the source gives a center-letter, here illegible/abraded — read as the diagonal/radius construction below).
- ב–ד (chord/line): a line across the crater region; the segment between ב and ד is dark and appears as a spot.
- ר–ד (radius): half-diagonal drawn to point ד, where it meets the solar ray to give the height of the mountain.
- ח–ך (line): from the crater's edge, the ray "skips over" the whole depth and arrives at point ך.

Let there be a great circle equal to the sphere of the moon, אבג"ש, and let us assume its center [?]; and as we explained in *Sefer Gevuros Hashem*, its [the moon's] diameter compared with the diameter of the earth is as 2 to 7. Since the diameter of the earth is reckoned, in the view of some of the moderns, at close to 7000 mil, then if we set up the proportion and say: if 7 equals 7000, what does 2 equal? — we find the diameter of the moon to be 2000 mil, and half its diameter ר–ד 1000. Now, observing carefully, men saw in the moon round crevices, such that what appears upon it [as a spot] is not a [body] resting upon it but rather [a ray] that skips over the whole depth and arrives at point ך; and what lies between ב and ך is dark and appears as spots. This is because the solar spark that illumines point [ב] does not find a level surface over which to spread its heat, in the manner of mountains that catch the light while the valleys between them are concealed. As was explained in the science of optics, they investigated with the *dioptra* and found that the distance between ב and ד is close to one part in twenty of the diameter בש — that is, 100 mil. Therefore, if we draw half the diameter ר from [the center] as far as ד, where it meets the [solar] ray at ך, we obtain the measure of the height of the mountain on the lunar globe.

Now the sum of the two squares ב–פ–ד, which is 1010000, equals the line ר–ד squared — that is [?] for the first; and its square root is 1004 [recte: 1005? — see below], and when you subtract the right-hand side, the leg ר–ח [recte: ר–ד] 1000, there remains the measure of ח–ך, more than four mil. In truth, the mountains of the earth are not so high — even though Cleomedes reckons 15 *stadia*, such that one does not find among them, along the vertical line, even the height of a mil — as the true geographers testify, and Pliny wrote that Eratosthenes [אירטושחיניש] measured the highest of the mountains, called [?], and it was not more than ten *stadia*, that is, 1250 *paces* [pesi'os]. Therefore the mountains of the moon are higher [than the earth's], and that is what was to be shown.

Nor is it any wonder, even though [the moon] is smaller than the body of the earth — for the sea is a third [?] of the dry land, as I explained in *Sefer Gevuros Hashem* — and even so [the moon] has, at times, mountains larger than the highest on the dry land. So, too, for inhabitants on the lunar globe, each of their natural days will be 29 days, 12 hours, [and the fraction] 393 [תש"צג] and a half [והציון] [recte: portions], 18 [י"ח] [partial: ש"צו] — that is, the artificial day there is also [long?]; the sun moves from one point [נקודות] earlier to another by the motion of the moon, since the concealed portion is brought opposite [the sun] and receives its light, just as happens in 24 hours for the earth's globe according to the view of Copernicus.

If you wish to grasp how it [the moon] receives the light, and the change of the moon's appearance as it appears to us: place a lit torch against a wall, and take in your hand a corporeal sphere. Behold, in whatever position it is set facing the lamp, it will receive the light over half the surface of its roundness; but the part standing behind will not show the illumined portion — and that is then the conjunction [דבוק] of the sun with the moon. When you turn yourself a little and come round to the front, it appears like the circumference-line of a lit circle; and when you draw nearer still, it shows a slice of a lit circle, and then more — half a circle; and when you stand between the torch and the sphere in such a way that you do not block the light's arrival at the sphere, the whole part facing your eye appears lit, and that is the

[Footnote, p. 461:] ✻ p. 165, folio [ל"א?] [ח / ח ליור leaf].

the fullness of the moon [its full phase]. And if you divide your hand-screen [a partition] between the lamp and the sphere, it will be dark — and that is the eclipse of the moon, which befalls the inhabitants upon it [as] darkness when the earth stands between the sun and it [the moon], just as an eclipse befalls us when the moon is between us and the sun. When you incline yourself to the first side, the light keeps diminishing — its visible body [appears] spotted [?], [its phases] waning — until the light is gone. So too, then, the earth appears to the inhabitants on the lunar globe; but what is seen upon it [the earth] over 29 days, they see in the body of the earth every day, through the daily motion of the earth.

Aristotle [ארסטוטלים] [adduced] the sphericity of the moon, from the manner of its appearing, by the second way, in [Book] I, as a proof — and yet the inhabitants on it do not themselves know this proof. Therefore, just as the inhabitants on the lunar globe know likewise that the earth [is spherical] — something we cannot know of [it from their vantage] — so there is no doubt that they have many other items of knowledge, on account of their situation, that are not known to us; and

conversely, we know things hidden from them. There is no need to expand, beyond what I have spoken of elsewhere.

§62 [סב]. And this, that the sum of the **days** of the whole year **exceeds** the sum of all its **nights** among the inhabitants in the north, is explained in *Sefer Iyun HaKochavim* [the author's own work]. Since the body of the sun does not move at the center of the world but rather above it, at another center, on this account it moves more slowly beneath the six northern signs [than beneath the southern ones] — for they were the locus of the apogee [נקודת הרום] at that time — and the southern [signs were the perigee]; and since the nights are short while [the sun] is in the northern [signs], they do not make up a large sum, whereas when it is in the southern signs — the nights too being long — they are not so many in number as the days when it is in the northern signs. The reverse occurs with the days: the long ones are many in number and the short ones few. It follows of necessity that the sum of the days exceeds the sum of the nights. All this happens to our antipodes, for whom the southern pole is elevated above their horizon — so that our [days] are their nights.

By this solar motion upon the eccentric, the natural days vary according to the difference of its motion, and the makers of tables must correct their computations to equalize the days. Moreover, in the view of Tycho [טיקון], the days exceed the nights by reason of the refraction of the rays, which makes the star appear earlier than its time of [actual] rising by some 30 first-minutes of motion, and lags after setting by another 30 first-minutes; therefore each day is augmented by one degree over the arc of the day, and to one degree of motion belong 4 time-minutes of an hour. But the refraction of the rays is not equal in every horizon and clime. Hence, although the sun's motion from reverse to reverse is equal, it will trace 182 circles, or lines close to circles, called day-circles, of which the part above the horizon is called the measure of the day, and the part below, the measure of the night — which the sun traces by the diurnal motion of the *primum mobile*, in the artificial day or night. Therefore the sum of the visible [arcs] is always equal to the sum of the concealed ones.

Nevertheless, its course from Aries to Libra — I mean from the 21st [כ"א] of March to the [?] of September — exceeds its course from Libra to Aries — I mean from the [?] of September to the 21st of March. In the former it traces 187 circles, and in the latter 178, approximately, as I explained by demonstration in *Sefer Gevuros Hashem*. The reason for the matter is that when the sun is near Cancer — where the apogee [רום] of its orbit lies, I mean the [aphelion] — its distance from the earth is greater [so it lingers]; and conversely, when it is near Capricorn, it is

close to the point nearest the earth. Therefore, in the former its course is long in its [passage], while in the latter it is short; and the figure in *Iyun HaShemesh* will show your eyes the matter — there is no need to dwell on the obvious.

For this very same reason, the number of days that are continuous in summer for those dwelling at the extremities — I mean from 63 [ס"ג] degrees of latitude up to beneath the poles — is not equal to the number of nights that are continuous in winter there; rather the days are more for the northerners than the nights, and conversely for the southerners. Yet all this does not amount to ten days' difference, even for those dwelling directly beneath the poles themselves. But for those nearer [the equator] — say at latitude 67 or 72 degrees, who do have a [civil] reckoning — the matter is almost imperceptible.

One who wishes to verify this can easily learn it from what we wrote in the chapter "Chukos Shamayim" [חקות שמים], concerning the measures of the arcs of half the day at any given place.

§63 [סג]. And you were not able to bear [the claim] that the sun rises earlier for the western inhabitants than for the eastern. This is explained in the *Elements of the Sphere* [התחלות הכדור]. One of the reasons that compelled the astronomers to begin reckoning the start of the day from midday is this: that the moment of midday varies for different cities according to their longitude. For a city whose [longitude] is 15 degrees more than another — I mean, that it is further east — will have the sun rise for it one hour before the other; and if 30 degrees, two hours; and if both are equal in longitude, midday and midnight occur for both in a single instant, as I instructed you in *Sefer Chukos Shamayim*. There is no difference whether their latitudes are equal or differing, for the day-half circles of all of them pass over the equator, which is the measure of their distance from west to east — unlike the case at the time of solar rising and setting, where they have conditions and variations that follow upon the situation of their horizon and its obliquity, on which depends all the variation in the length and shortness of the days.

In the corporeal globe these matters are clear. Take three cities equal in longitude: one on the equator, another at latitude 44 degrees north, and another at 41 degrees [recte: should likely read 44] south. By the aforementioned reason — being equal in longitude — midday occurs for all three in one instant. But for the one beneath the equator, the arc of half the day — I mean, from the moment of rising to noon — is 6 hours; for the northern, 7.30; and for the southern, 4.30 [recte: see below]. Therefore necessarily the sun rises for the northern one an hour and a half before it [rises] at the equatorial one, and an hour and a half

[earlier] than for the southern one. Now suppose further that the northern city is 15 degrees more westerly than the equatorial one; then midday is delayed for it by an hour from the city on the equator — yet even so the sun rises for it still a half-hour earlier, since its day is 15 hours and the other's is always 12. Suppose, again, that the northern city's longitude is less than the equatorial one's by 22½ [1/2 כ"ב] degrees: then, although midday for the northern city is delayed by an hour and a half, nevertheless for both of them rising occurs in a single instant. According to the proportion of the aforesaid conditions you may reckon the reverse in the matter of setting — for it is possible that the more easterly one sets later than the western, or that setting occurs at one instant for both. From this [you may understand].

[Aside:] You may grasp that the position of the synagogue-buildings is not properly oriented toward Jerusalem, and that those who pray facing their land — those who dwell in the north and are westerly — do not do well to face it toward the east. The masters of Scripture [בעלי מקרא] are most precise in this, as appears in *Sefer HaAderes* by the sage Eliyahu [Mizrachi? — אליהו בשיצי, i.e. Elijah Bashyatzi [?]]; he and his disciple, the sage Rabbi Caleb Afendopolo [רבי כלב אפינדופוקו] [Karaite scholars], were astronomers, disciples of our teacher Rabbi Mordecai Comtino [מהר"ר מרדכי כומטיאנו], a man comprising every science, as appears in the several treatises he wrote on mathematics and natural science, and he also commented on all the books of Ibn Ezra [הראב"ע].

Now it is also possible that there be two inhabitants in one and the same city, for whom the sun rises earlier for the one than for the other — namely, where one's house is at the summit of a mountain and the other's at the foot of the mountain. To illustrate this, let it be in the figure of the diagram, §60 [?]:

Diagram. A construction showing why the sun rises earlier for a dweller at a mountain summit than for one at its foot in the same city. A great circle (the earth's globe) is drawn about a center, with a mountain rising from its surface; rays from the rising sun reach the summit before the base.

- [Center]: ם [mem] — the center of the earth's globe דהח [or similar circle-label].
- א: the summit of the mountain (the point ראשו); a line אהם קו runs from א down to the center ם.
- ש: the point on the great circle where the sun rises; its ray comes straight to the inhabitants at point [ב].
- ב: the dweller on the mountain summit, reached by the ray א–ב.

- ג: the dweller at the mountain's foot, reached only later.
- ז: a point at the base (תחתית) where the solar ray is not yet seen.
- ה: a point whose horizon is the sea [ים], for whom [the sun has not yet risen].

Let the earth's globe be דהח [?] upon center ם, and let there be a mountain whose summit is point א; and from it a line א–ם runs to the center; and let there be a great circle on which the sun rises at point ש, and its ray comes straight to the inhabitants at point [ב]. For the inhabitants at the mountain's summit א [the ray reaches] as far as ב. Behold, you see that for inhabitants at point ז, which is at the foot, the solar ray is not yet seen — still less for inhabitants at point ה, whose horizon is the sea. And if you wish to know the time of rising between the eye [observer] at ד and the eye at א, posit that the mountain's height is half a *liwka* [ליוקה — Spanish/Romance: *legua*, "league"], that is, 1½ mil, which are 12 *stadia*. And since half the diameter of the earth, the line [מ–ץ], is, in the view of Ptolemy, 28636 *stadia*, the whole line א–ם will be 28648. And in the triangle א–ם–ץ [recte: with right angle at ץ], the angle at ץ is a right angle, 90° [ט"ח? — the "third" angle]. Therefore: as the first [side] is to the [hypotenuse], the proportion — as א–ם 28648 is to ץ, so the whole sine [הבקע] 100000 is to [the sine of] 28636 ם–ץ…

to the angle [at] 99958 ,א–ם–ץ, which is 88° 21′ [88·21′], and its complement is 1.39′ [1°39′] — the measure of the angle א–ם–ץ; and its arc [is] the measure of the setting, or the depression of the sun beneath the horizon of the mountain-dwellers at point ה, just as for those dwelling at point ץ, or at the mountain's summit א, it is [the moment of] rising. You can reckon the time pertaining to the twilight when the sun is depressed beneath the horizon by 1.39′ [1°39′] at whatever city-latitude you wish, according to [the teaching of] *Sefer Chukos Shamayim* — and what I have assumed, that the sun rises at point ש, was for the sake of clearer and briefer explanation; for even had it been at point [הכל / ך] after [the calculation], the difference [קשה] is not reckoned as anything, as I explained in *Sefer Gevuros Hashem*. And although this [difference] is small — yet "a small matter [over Shabbos] is no small matter," as they [Chazal] said concerning the two cities, one of which brings in [Shabbos] earlier and the other later at the entrance and departure of Shabbos; and one who fears Heaven should act according to the practice of both, to be stringent [in both directions]. So it is light [easily understood].

Now as for what Aristotle wrote — that on Mount Caucasus [הר קוקשוש] the solar light is seen from midnight, by reason of its height — this is impossible; for the greatest of mountains is not more than 15 [recte: 16? — ט"ז] *stadia*, even in the

view of Cleomedes. With him this is no more than one of the [unverified] reports that were related to him, to which he did not set his mind to investigate them.

§64 [סד]. That the **great twilight** [נשף] occurs when the sun is at the head of Cancer, for the northern latitudes, at their season — [when] the day is longest; and even so, when the sun is at the head of Capricorn — their shortest day — the [twilight] will not be the smallest of twilights [but rather] one greater than that which occurs at the [equinoctial] equalities. You can test this according to the teaching and instruction of *Sefer Chukos Shamayim*, where you will see that the measure of the twilights keeps diminishing from the head of Cancer; but the diminution does not continue uniformly, in the manner of the days, until the sun comes to Capricorn — rather it stops at a point, and all this is before the sun's arrival at Capricorn. Then it goes on diminishing again until [the sun] comes to the point between Capricorn and Aries [recte: between Libra and Capricorn], where it is smallest; and from there it begins to increase again until it equals the twilight at the equinoxes, [which is] aligned with the point between Libra and Capricorn. These matters, and their explanations — though they are strange — are demonstrated by trustworthy proofs in the figure of the *analemma* [האנלימא], and the discerning reader is compelled to acknowledge their truth, even though he formerly denied them.

Also, in the table of twilights, the truth is shown at one glance; and you yourself can plot, at the elevation of your land, the horizon together with the circle that bounds the twilight, which is 18 degrees beneath it; and so you will see the elevation of the equator and all the parallels to it, in which the sun turns each day. Behold, for inhabitants beneath the equator [equinoctial line], when the sun is at the equinoxes, the length of the twilight will be 18 degrees — that is, one hour and 12 minutes — for the equator is at the place that faces the zenith there, and the measure of the sun's tarrying in the matter of twilight is reckoned on the circle that faces the zenith. When the sun is at the [solstitial] turning of Cancer and Capricorn, the measure of the twilight will be 19.41′ [recte: 1°41′? — reads 19·41 / 1.41 as printed], I mean approximately one hour and a third.

But in all those places, the northern-latitude twilight occurring at the head of Cancer is greater than that occurring at the head of Capricorn. For example, at latitude 35 degrees, the twilight at the head of Cancer is one hour and 39 minutes [1.39], and at the head of Capricorn [וכנדי] 1.53. At latitude 45, at Cancer 2.39′, and at Capricorn 1.53′ [recte: see below]. At latitude 49, at Cancer the whole night is twilight, and at Capricorn 2.3′. The cause of the variation of the twilights is not [merely] the obliquity of the horizons — as it is in the [length of] days, where,

according to the obliquity of the horizon and its inclination from the upright, I mean as it is elevated and depressed, so do the days correspondingly lengthen and shorten, since the horizon is divided into [unequal arcs] with the equator and its [parallels]. Not so the circle of the twilight, which is depressed 18 degrees beneath it [the horizon], being a small circle, as is explained in the *Elements* of Theodosius [Theodosius of Bithynia — תיאודוזיאוס]. Rather, the essence of the matter depends on what I have mentioned to you in §64 [recte: an earlier paradox, נ"ד]: that one and the same chord cuts off a larger arc on a small circle than it cuts off on a large one, when we reckon both [circles] as 360 degrees. Therefore, if you plot — by way of the *analemma* — on the equinoctial line, where the equator passes facing the zenith, and all the parallels to it are to the right and to the left, and you plot a line facing the horizon (which is the measure of the twilight) — the parallel-lines [arcs] between the horizon and the twilight-circle will all be equal.

Sefer Ma'ayan Chatum (continued)

§64 (סד) — The great twilight (conclusion)

The circles parallel to the equator are, in themselves, equal to it. Therefore, the farther they lie from the equator, the more slanted are the arcs lying upon the meridians, and so the arc of the equator that corresponds to them in their rising above the horizon comes to more than 18° — I mean, 19.41. You may determine the same for the oblique horizons as well, but I cannot expand on it now, since I would require many figures and demonstrations, and the time does not allow it.

Pedro Nuñez [Pedro Nunes] wrote a learned book on the twilights, and there he explained pleasing matters; from it you may slake your thirst. Yet he was not exact in a few points, and elsewhere I shall expand, with the help of God.

§65 (סה) — The shadows of columns

You marveled — as it seems, on the very same day — that the shadows of columns do not preserve a fixed ratio. This is Aristotle's question in the fourth [problem] of part 15 [Problemata XV], on shadows: the upright objects seen on horizons appear long at dawn, and as the sun climbs they grow shorter, and at midday they are very small; the measures are not proportioned to the time of its motion, even though the rising of the equator is uniform across all horizons.

He explained there the reason for the matter: the rays coming from the center of the sun and passing over the top of the upright object change according to the height of its [the sun's] center above the horizon or its lowness. For even though the angles cut off are equal — from problem 8 [?] of the first [book] — the planes [traversed] differ, like triangles. Therefore, from problem 19 [?] of the first, the matter is clear; but take it as agreed that the ray coming from the lower light [the lower sun] is longer than that from a higher position. All this I clarify for you to show that the books are corrupt and [?] understanding; and if Heaven shows me mercy, so that I may catch my breath in some place, I shall expound for you the whole *Book of Problems*, for to this day I have known no commentary on the entire book that is adequate. See what I wrote in *Sefer Chukos Shamayim* [Book of the Statutes of Heaven].

As for the variation observed over many days — both in the lengthening of the days, which varies, and in the shadows — it all derives from the sun's course along a slanted circle. For although it is ordered in its motion, even so, by reason of its position, the inclinations are neither equal nor proportioned, and from that arises all the variation. For dwellers in differing horizons the phenomena differ, so that it is impossible to gather them under a single rule. Now Cleomedes [קליאומרס] reckoned that the apportioning of the increase and decrease of the days runs as follows: in the month adjacent to the equinoxes — whether the [days are] ahead or behind — they increase and decrease by ½ of the difference between the long [day] and the short; in the second month, by ¼ [?] of the difference; and in the third, by ⅓ [?].

By way of example: if the difference between the long day and the short is 6 hours, then the increase and decrease in the first month is one and a half hours; in the second, one hour; and in the third, half an hour. But this reckoning does not hold true in all climes. Therefore the exact value must be sought by the reckoning of triangles, or with the material sphere [globe], or from tables; there is no need to expand.

§66 (סו) — The heavy bodies that fly

The heavy bodies fly in so many ways that it is impossible to set them all down in writing, except at great length. Such was the great eagle that the craftsmen of Nuremberg made, which came out to meet Charles V at his entry into the city from the fortress. Likewise they make flies of iron and copper that fly free by themselves and circle the house, and afterward return to the hands of their owners. The art of flight by men has become famous in our time, and the trained

man himself flies through the air, just as by training he swims in the sea like a fish — this man wishes to make himself like every creature.

On dry land too, the carts have been fitted with sails, like a moving sail, and the speed is so great that the cart's wheels would catch fire if they were not cooled with water; and this is the custom in Holland.

Now apart from these artificial things, Aristotle wrote in the second [book] of *On the Heavens* [De Caelo II] that if the terrestrial globe were bored through to its center, or from side to side, then the body falling into it would not pass beyond the center — unless the center of the falling body lay at its middle or at its end, as with a long reed or a nail — so that in any case, when the center of the body reaches the center of the world, it would stand hanging and remain there, for there is its lowest place,

the most fitting for heavy bodies; and were it to pass beyond, it would be rising toward the heavens, which is contrary to its nature. Nor is the iron — whether it be earth or any heavy thing — fit to stand there hanging in the air. And if there were a ladder passing through the center of the world, and upon it three men — one at the center and two at its ends — then [?] one who moves from end to end would be descending and ascending in a single instant: descending in relation to the man above, and ascending insofar as he draws nearer to the man standing toward the [opposite] center.

Now these conceits — how [bodies] would hang and remain [there] — I have explained in many wondrous ways in *Sefer Niflaos* [Book of Wonders]. With the reed by which we draw wine from the cask, the matter is clear: if you immerse its lower end, the liquid enters it, once its upper mouth is open and the air exits from it; but if you close its mouth with your finger, so that air can no longer enter from above, then the liquid remains hanging — for had it gone down, only the empty cavity of the reed would be left. Of this sort is the *pompush* [Latin: *pompa*, "pump" (?)] found in *Sefer ha-Tena'os* [?]. In *Sefer ha-Chemer* [?] the artist [author] explained the matter well: if the body of the two ends is equal, it is like equal scale-pans of equal weight, where there is no doubt of one end outweighing its fellow — therefore the waters will stand hanging. With R. Moshe [ha-Rav R. Moshe] you will find glass [instruments] like those I gave him. And there are ways to do this with fire, and in several kinds worthy to be shown to kings, and from them branch out many devices and remedies [segulos]. You will find them in the aforesaid book; also in *Sefer Ta'alumos* [Book of Hidden Things] I wrote some of them, and there is no need to expand on it here.

§67 (סז) — He who hangs the earth upon nothing

As for "He who hangs the earth upon nothing" [Iyov 26:7], it appears that this question — which gave your friend no rest, in the words of the poet who said "to Him who spreads out the earth above the waters" [Tehillim 136:6] — is one of those of which they said [Aramaic: שרלך ערבא בעי, "the matter still needs clarification" (?)]. Say to him: and the dripping waters [the lower waters] — how are they spread out? He set aside the chief wonder and mentioned the secondary one.

Had he held the view of [?] Thales the Milesian [תליש מולישיאוס = Thales of Miletus] — that the earth floats upon the waters (which is against nature, for the heavy thing would be floating upon the light) — he would have held that the depth of the waters is without limit. But his opinion, with all the other conceits, is void, once it has been proven by demonstration that [the earth] is spherical. One should not wonder if they strayed from the truth in matters [?] that are not their occupation or craft.

As for what they established regarding the Creator — that there is a window in the firmament [?] — that is according to their own thinking, that perception is by tradition [?]; for these are not accepted among them, since the Greek craftsmen [?] disputed them, and so too those who came after. In general, the prevailing view is that the terrestrial globe is held in place by a force from the heavens that surround it: being spherical, with all their parts equal in the power of attraction, the earth is left hanging at the middle — this is the opinion of Anaximander [אנכסימנדיר], and the moderns ascribe this attraction to the magnetic stone [the magnet]. Others think the opposite: that the heavens, gliding and hovering about it on every side, by reason of their loathing for it — for it is at their middle, a lowly cave, into which all the wastes of the natures and their corruptions are poured — [hold it in place]. Perhaps Solomon alluded to both views, according to the explanation of Chazal on the verse "a generation goes and a generation comes" [Koheles 1:4], which they explained: "a generation found guilty, and a generation found innocent" [?]. Therefore, whether by reason of the heavens' love of the good ones, who in their loathing [recte: in their love] draw them [?], or by reason of their hatred of them, by which they thrust them away — by either path the earth will stand forever at the middle.

But if it be as they say, it follows that the stone cast upward would [behave thus], for the attracting force at close range exceeds the attractor at a distance; and according to their view its standing-still would be compelled rather than natural, and it would not endure. I have already expanded in *Sefer Amudei Shamayim*

[Book of the Pillars of Heaven] on the view of Copernicus and on it [?], and there [is set down] all that is needed for this question.

§68 (סח) — That the moon is never full

You grew weary of bearing [the puzzle] — that there was never a full moon, and that it gives more light when conjoined with the sun than when opposite it. The matter is true and very well explained, for you know that the true opposition is when the centers of the two luminaries are 180° [קפ"ק = 180] apart from each other, and then the straight line passing [recte: from] center [to center] passes also through the center of the earth. And when the moon is opposite the sun without [any] latitude — that is,

exactly below the path of the sun — and it [the line] intersects with the circle that brings the lunar body at the two points called head and tail of the dragon [the lunar nodes], then a lunar eclipse occurs. For this reason we call it the *eclipse circle* [the ecliptic]. [The moon] does not give light at the time of opposition; therefore a [literal] full-illumination does not occur.

The reason the moon is not [wholly] dark at every opposition is because it has latitude, and the earth's shadow does not reach it; therefore the whole part [of the moon] receiving the solar light is not aligned with the earth. And although the sun strikes its [near] face and always illumines more than its half — as Chazal said, "never has the sun seen the inner side of the moon" — and just as the terrestrial globe always illumines more than its half (so that, while it is night here, day comes to others), even so we do not see all that bright half; therefore the full-illumination of the moon is never seen by us. And although it appears to us as if it were illumined to its half, this is only a geometric figure; and dwellers on the bodies of the [other] planets [שצ"ם = Saturn, Jupiter, Mars (?)] would see different variations in the moon according to their position — to the rear and to the sides.

Therefore, for the moon to appear illumined, two conditions are needed [?]: that it be after the sun strikes its face — and this is always so, except when the earth stands in the middle, and then [the moon is eclipsed]; the second, that we, the dwellers on earth, be seeing that bright part. For according to the general view the moon is a dense, dark body like the body of the earth, and the striking of the sun's rays upon it is like their striking upon a wall, which they illumine and from which they are reflected to the earth. As the author of *Sefer Segulos ha-Yesodos* [Book of the Properties of the Elements] wrote: the sun is like a lamp and the stars [planets] like mirrors.

Although the sage Ibn Rushd [בן ראשד = Averroes], in his commentary on *On the Heavens*, refuses this regarding the stars — for he holds that their radiance is not solar light reflected (since, were it so, their light would not come in every horizon of the [visible] hemisphere, but only to some small portion, according to the size of the angle of incidence of the solar ray upon them, depending on their position, the angle of reflection being equal to the angle of incidence, from proposition 1 of Euclid's *Catoptrics* [למראות לאוקלירס], and the argument is sound; and were it not reflected, only a part of the earth would be illumined, and not in every horizon of the hemisphere — which is contrary to experience) — yet what this commentator [Averroes] said regarding the stars, I say of the moon itself: for were its light a reflection of the sun, then on the nights near to conjunction, on this side and that, it would give us no light at all, since the angle of incidence of the ray from the sun upon it comes very oblique; therefore it too would reflect the ray in [that same] obliquity, and the ray would fall toward the heavens and not toward the earth — easy to grasp for one with a little knowledge of the science of optics. And if you suppose the rays would be reflected to the earth, then you are saying that Nature chooses the long way and forsakes the short, the contrary of which has been demonstrated.

Now he [Averroes] holds that the light of the stars is not reflected from the sun, but is an essential light innate in them — except that it needs the aid of the solar light to act upon it, to bring it out from potentiality (which is in their substance) into actuality, by way of change and being-acted-upon; whereas the sun alone is always actual light, and therefore sends its ray to every quarter. Not so the stars, which send out their rays only according to the part on which the sun acts upon their sphericity. According to his words one may say that the solar rays reflected from the moon and stars do not reach the earth, but incline to another quarter in the firmament.

But that the moon is brighter at the time of conjunction with the sun than when it is opposite it is very well explained. For just as I explained in *Sefer Gevuros Hashem* [Book of the Mighty Acts of God], in the name of the masters of optics: as long as a body is nearer to the great light, the more delicate are the rays it receives from its body than when it is farther from it. And indeed the moon, at its conjunction with the sun, is nearer to it; whereas at its opposition it is farther from it by the measure of the entire diameter of the lunar sphere. Therefore at conjunction it is more illumined — except that the part receiving the light is turned the other way, above, toward the sun, and is not seen by our eyes, as is easy to grasp.

§69 (סט) — Whether an object is seen better far or near

And [your] assumption — that the sensed object appears better far than near — I do not concede; though on the face of it the law is with you, for it has been explained in proposition 2 of Euclid's *Optics* [לראיה לאוקלירס] that, of two equal magnitudes at differing distances, the nearer is seen better, since [its] rays come from the object to the eye more abundant, and so meet at a wider angle. Thus too he explained in proposition 8 [בלמור ח'] of it.

But what he showed in proposition [?] — that the eye sees a bit more of the surface from afar than from up close, since the eye approaches the spherical body, and the figure of the seer appears to be the reverse — there is no need to bring the figure here with the demonstration, for this is plain in all the books of optics. And there is no doubt that, were we even very close to the lunar globe, we would not see near to its half, as we do see here on earth.

Now of the [body's] surface, that which is explained there [is]: 161030458 miles [?]. The surface of the terrestrial globe, as explained there, is 402576 [recte: 40257614 (?)] miles; and the great circle of the terrestrial globe is [?] — being 707 [whose] surface is ⅓ [?] [recte: 707⅓ (?)], which is 2828 miles, according to the teaching of *Sefer Gevuros Hashem* — and the surface of the earth's globe is such-and-such. And the mountains are seen, from any side, no farther than 15 [ט"ו] parasangs, and the circumference of the visible horizon-circle for one standing on the earth is 94⅔ [?] [miles]. So too the dwellers on the moon see near to half the surface of the earth; whereas we, even were we very close to the lunar globe, would not see near to its half, as we see here.

There is no doubt that, were we [able] to draw near, the figure would be [evident] with the demonstration, and this is clear in all the books of optics; there is no need [to dwell on it]. Except that, to the imagination of the viewer, it appears the reverse — they see more and more of it from earth — and they can picture the figure, and whether it is spherical and the other qualities we do not know [?], we who dwell on it. This too Euclid explained in proposition [?]. It being so for columns and for the splendors of the colonnade [?], for this reason the face too is seen larger and nearer in a columnar [cylindrical] or spherical mirror from close up, and so it is that what is seen from afar [appears] larger.

And behold: since, the closer the eye is to the [object], the larger the sensed object appears — as is explained there in proposition 55 [נ"ה], and as is explained in the other books of this science — the rays of the sensed object come [?] to the eye from all its edges, and there they meet and form an angle. Whether it [the

magnitude] be a [single] line, length-wise [recte: a line that is length], or a surface, or [a body] limitless [?], it is not so. By the visual [angle]: for the visual angle, if it is very small, is not seen by the eye — by way of example, the angle of contact [?] that Euclid explained in [proposition] 17 [recte: 16] of the third [book of the *Elements*] is not seen by itself unless it be set [beside another]. For we distinguish between a right angle and the angle that arises from the alternate [angle] and the diagonal, which is larger than every acute [angle] in mathematical lines; yet it [the angle of contact] is seen — even though it be very sharp — when it is in the company of another angle of straight lines. Therefore we know too its smallness, that it is the angle of contact, except that it is pictured [conceptually] but not sensed in truth.

Know that only one of all the visual rays is called the *axial* [perpendicular] one; and the sensed object and the eye through whose midst it passes is called the *diagonal*. So that the magnitude may be sensed, the angle that arises from the diagonal and the other rays coming from the edges of the sensed object must form an angle — its half being one [?]; for half of it is not seen. As for the magnitude seen — its measure is hard, since it [the eye] is spherical, and the rays come up to its center. So that the angle may be sensed, or [so that] what is hard about the eye [be grasped] fully — I mean, the position of the sensed object, its figure, and the disposition of its parts one to another, which are the conditions for [its] being well seen — the masters of optics explained that the angle should be near to the perpendicular, that is, the arc on the eye's globe should be near to a quarter of the globe's circumference. Or, according to the view of Albertus Dürer [אלבירטו דוריו = Albrecht Dürer], the chord subtending the angle of the sensed object should be equal to half the diameter of the eye's sphere — which is the [side of the] hexagon of the circumference. And if you set your eyes and look toward a horizon on level ground, you will not see more than a quarter — for, as I explained in degree 3 [?] from *Sefer Gevuros Hashem*, the arcs of the eye are proportioned to the arcs of the sphere. And if you wish to press your eyes to see more

[The thesis continues beyond this page.]

Sefer Elim — Ma'ayan Chatum (Fifth Part: Paradoxes)

Conclusion of §69 (סט) — and the Close of *Ma'ayan Chatum*

[The page opens in mid-sentence, continuing the optics paradox §69 from the prior folio.]

and this is the sense intended in the verse "Let me turn aside now and see" (Shemos 3:3) [אסורה נא ואראה] — for the word *me-karov* ["from near"] is, in truth, an expression of *distancing*; but there is no need to dwell on it.

Moreover, since the sun's heat is no nearer to the sea than to the dry land, all the more so [should you understand], if you grasp [the principle], that it [the heat reaching us] is not even one part in a thousand [of the sun's own]. We have already demonstrated this with a proof in *Sefer Gevuros Hashem* [the author's own *Gevuros Hashem*].

And this is what seems right to me, in my brief understanding, as a response to the seventy date-palms and the clusters of the henna [a play on Shemos 15:27 and Shir HaShirim 1:14 — the "seventy" questions]. From here on I will not draw out "many mighty waters" (Tehillim 93:4) [further floods of words]. For your eyes will see that, just as it is written in *Sefer* [Tehillim] — "the heads of the mountains," and "the rivers came" [allusion to the flood imagery] — these matters [as I have set them out] are more refined and orderly. Had it not been for the urgency of your entreaties, I would not have set down these matters in the Hebrew tongue [rather than expanding them]. For "where God revealed Himself, and they beheld" [the place of revelation], the things that appear delightful and pleasant in your eyes are accounted, in the eyes of the wretched, mere "toil of the flesh." [Recasting an interwoven cento of biblical phrases — the author apologizes that these mathematical delights are toil to the unlearned.]

And although these matters are neither among the foundations of the faith nor among the laws of the Torah, in any case I think these studies [*limudim* — the mathematical sciences] will profit those who reflect on them — in that they will be weighed [valued] like legal rulings and judgments [*pesakim u-mishpatim*], which appear at first glance to be very difficult and "adorned with cosmetics" [polished], and even more than these [the studies will profit them]. Let them take care, in the consensus of beliefs and corners [foundations], not to incline away from the truth — for they [the unlearned] "seek their soul to destruction"

[Tehillim 35:4], inflamed, and these are nothing but conjectures and digressions [*demyonos u-se'ifim*], deciding nothing with rashness or recklessness. And as the Sage [Koheles] said: "for God is in heaven and you are upon the earth — therefore let your words be few" (Koheles 5:1).

And so, with the help of Him who knows all that is revealed and concealed: **here is completed the book *Ma'ayan Chatum*.** [ובכן בעזר יודע כל נגלה וסתום: נשלם ספר מעין חתום — the closing formula of the Fifth Part.]

Addenda / Author's Marginal Glosses to *Ma'ayan Chatum*

These matters were written in the margin of [the author's copy of] *Sefer Ma'ayan Chatum*. [רברים אלו היו כתובים בגליון של ספר מעין חתום — a set of supplementary notes keyed to specific pages and paradoxes (*tamar* = "paradox"/section).]

- **On page 1 (folio [?] — printed "352") [352 ד], Paradox 1 [תמר א'].** In matters of geometry, what issues from their product [*ha-yotzei mi-kaflam*] appears smaller [than expected], and it is [in fact] larger, as is explained in Paradox 2 [תמר ב'].

- **At the beginning of Paradox 15 [תמר ט"ו] [רישׁ].** Aristotle answered, concerning the questions [*she'elos*] that the young can possibly have for [their] teachers — that their principles are innate in the intellect, unlike natural matters [*tiv'iyim*], which require expertise and experience. And the chief [point]: not all the principles of the mathematical sciences [*ha-limudim*] are known unless the teachers mention them — and they are, according to this, like [matters learned by] experience [*nisyonos*].

- **At the end of Paradox 27 [תמר כ"ז] [בסוף], and in *Sefer Kishot* [or: "and in a book" — בספר], I will show you a wonderful thing:** how, in the rotation [revolution] of the compass, you may trace straight lines — I mean, in a single rotation about [the center?] H [ה'], five straight lines making the pentagon [*ha-mechumash*] — and others like these, which are not known to the multitude of the wise.

- **At the beginning of folio 34 [ריש דף ל"ד] [389 ל"ד].** The matter is founded upon the *progresión geométrica* [Spanish/Latin: *progressio geometrica*, "geometric progression"]: that if you wish to weigh any weight up to 40, you will have four weights or stones — namely **1, 3, 9, 27**, whose sum [is 40]. And if you wish to weigh, e.g., 21 *litra* [pounds], place in one pan **3, 27** [recte: 27, 3 → 27 + 3 = 30; with the 9 in the other pan, 30 − 9 = 21]

and in the other **9**. And if you wish [to weigh] 20 *litra*, set thus: **1, 9** [recte: with combinations of 1, 3, 9, 27]. And likewise with four weights, **1, 3, 9, 27, 81** [five values listed], you weigh by their combination 121 *litra*; and with **1, 3, 9, 27, 81, 243** [and onward], according to their combination, as explained in the book [*ba-sefer*].

- **In Paradox 44 [תמר מ"ד], folio 47 [דף מ"ז] [404 מ"ז].** And Cicero [וציצירון — Cicero], wishing to indicate which subject-matter is most difficult to learn, said that it is harder than the Platonic number [*ha-mispar ha-Platoni* — Plato's "nuptial number"], i.e., concerning what he wrote in his books on the governance of the state [*Sifrei Hanhagas ha-Medinah* — Plato's *Republic*]. Aristotle too mentioned it in his name in the *Politica* [Aristotle's *Politics*]: it is the number 12 [י"ב], formed from [the Greek] *diapason* [דיאפשון] and *diapente* [דיאפינטי — the octave and the fifth], whose cube is **1728** [the cube of 12]. This is the time fixed by Socrates [מסאקראטיס — "from Socrates," i.e., in the Socratic teaching] for the duration and length of the years of every government; for within this time it will assuredly come to an end.

Publisher's / Editor's Colophon (p. 442)

And These Are the Words of Those Who Brought [This Work] to Press — the Final [Words of] the Publishers

[ואלה דברי המביאים לבית הדפוס [?] האחרונים — *be-veis ha-defus*, "to press"; the editor's closing address.]

[Rashi script throughout the colophon.]

Forty years have passed since the day our forebears [first] printed *Sefer Elim* [ס' אילם]. And forty years we have toiled — [a labor of] toil and exhaustion for the benefit of wisdom and its seekers. For we have not declined [the task entrusted to us]; we have taken it upon ourselves at the outset [?]. And the Lord [ה'] knows how much strength and how many resources [we expended]; we have spared no money [*lo lakachnu mechir* — taken no fee] to redeem it [bring it forth]. And the Lord knows and witnesses how this printing [edition] — all of it gathered and prepared by us [*kibbasnu ve-kipasnu*] — for indeed before us [there was] every disorder and want, and "the Lord helped us" [*ozranu Hashem*] [?], as is the way of the world. And whatever money the printing-press [*defus*] exacted [?], we took no fee [for ourselves]. He who knows and witnesses how much of this [we bore] in labor and effort for those who do the work of wisdom and seek her — [namely]

those who show [us favor]: for they did not begrudge us, [those] benefactors, the generous and God-fearing.

Therefore, "Behold, [this] is a precious and exalted book" [*hei lachem sefer yakar ve-nisgav*] — one whom God has favored [?]. To whom [does the honor of] this work belong, this day, [as one] "stirring the heavens" [?]: we acknowledge and give thanks, to recount to the people His great wonders, He alone [does wonders] [Tehillim 136:4]. We have prepared [the book] complete in all its parts and entire in its order. To whom belongs the honor for the author [*mechaber*], the great Rav and Sage, great in Israel — his honor [is established] within those who esteem him; [we acquired] his copy [for] little [cost] [*be-mechiro ha-mu'at*], not according to its [true] worth and value at all. Indeed, those very copies that our earlier hands had once redeemed [?] — the "crown of glory in the fruit of knowledge" [*ateres tiferes bi-fri da'as*], to vindicate [it] [*le-hatzdik*] — and they brought a blessing into your midst [*be-socheichem*], whose price [?]…

and the last of his [the author's] parts came [to print] this day, "for the sake of vindicating righteousness" [*le-ma'an yatzdik tzidkcha* — Yeshayahu 42:21] [?]. The book is now in your hands. And it would be a kindness, by the strength of [this] appointed time [?]. And so it is [a thing of] beauty: from the generous-hearted of our people [*nedivei ami*], with [the help of] this day; and you shall know how to esteem [it]. For the honor of God we hoped, that He help us for good, and bring forth precious and useful books to light, for the honor of the Torah and for [the benefit of] those who study, and for those who do the work of wisdom and seek her — for all this is the desire of our heart. And we have reckoned [it] also to you as a merit [*ve-nechshav gam lachem li-tzedakah* — echoing Bereishis 15:6].

Meïr Grinshpan [מאיר גרינשפאן]. And the printers [*ha-madpisim*]: **Moshe Eliezer Belinsohn** [משה אליעזר בעלינסאן] [recte: M. E. Beilinson, the Odessa printer].

[Acknowledgment Notice — foot of p. 442]

[Rashi script. A separate framed notice below the colophon.]

These last three parts [*shloshes ha-chalakim ha-acharonim*] we have printed from among [the books of] *Sefer Elim* [ס' אילם] [recte:…] from the treasury of books of the mighty officer [*ha-sar ha-adir*] in Israel, R. Horatz Günzburg [הר' הירץ גינצבורג — Baron Horace [Naftali Herz] Günzburg] of Petersburg [מפטרסבורג] — may his light shine [יצ"ו] and may he be magnified [?]. [We acknowledge] publicly this kindness he did for us [in lending it]. And we hope that he will likewise sit to bestow kindness, to lend us from his treasure-house yet more precious old books,

to renew their youth by means of the press [*al yedei ha-defus*], for the benefit of all who seek wisdom [*shocharei tushiyah*].

The Printers [*HaMadpisim*].

[Censor stamp, partly legible at center of p. 442 — circular, Cyrillic/Hebrew, illegible: [Censor stamp, Russian, illegible].]

Combined Diagram Plate — Figures for the Four Mathematical Books

Diagram. A single full-page folded plate carrying all the geometric and astronomical figures for the volume's four mathematical books. Header caption (top, centered): **"Figures for the books *Hukkos Shamayim*, *Gevuros [Hashem]*, [and] *Ma'ayan Chatum*"** [ציורים לספרי הקות שמים גבירות יי מעין החתום]. The figures are arranged in rows and labeled **ציור א** through **ציור ל"ג** ("Figure 1" through "Figure 33"). Hebrew point-letters (א, ב, ג, ד, ה, ו, ז, ח, ט, י, כ, ל, מ, נ, ס, פ, צ, ק, ש) label the constructions; Latin letters and Arabic numerals appear in a few. Described per figure below; many small labels in this densely packed plate are at the resolution floor and are flagged [?].

Top band (right to left):

- **ציור א (Fig. 1):** A graduated scale — two horizontal rules marked off with tick-marks (a divided line / linear scale), with arrowheads. Beneath it, two boxed Latin labels: **D 8** and **F 24**.
- **ציור ב (Fig. 2):** A small 2-column numeric table (reads right-to-left in source):

(right col.)	(left col.)
ג 3	ה 5
ב 2	ד 4
ח 8 · ט [?]	ה 15 · ו 12 [?]

- **ציור ג (Fig. 3):** A small 2-column numeric table (reads right-to-left in source):

(right col.)	(left col.)
א 4	ג 3
ב 1	ד 2
ח 8 · י 3 · ט 2 [?]	(—)

- **ציור ד (Fig. 4):** A triangle/quadrilateral construction with crossing diagonals; labeled points **א** (apex), **צ**, **ח**, **ט**, **ל**, **ב**, **ז**, **ה** — two overlapping triangles sharing transversals.
- **ציור ה (Fig. 5):** A large "bow-tie" / hourglass figure of two triangles meeting at a central point, fully cross-hatched with diagonals; outer corners **ז**(top-left), **י**(top-right), **ב**(bottom-left), **ה**(bottom-right), with **ג** at the lower center and **ד** at the crossing.
- **ציור ו (Fig. 6):** [shares the same hourglass cluster as Fig. 5 at the upper-left corner of the band; corner labels **א/ב/ג/ה** [?] — the two captions ציור ה and ציור ו head adjacent halves of the crossed-triangle figure.]
- **ציור ז (Fig. 7):** Two rows of letter-sequences (a combinatorial/permutation display) plus a small boxed cube:
- Row: א — ג — ה
- Row: ב — ד — ז
- and a cube diagram with a face lettered **ק** [?].
- **ציור ח (Fig. 8):** Likewise a permutation display with a cube:
- Row: ה — ד [?]
- Row: ב — ב — ה [?]
- boxed letters **ז** and **פ**, with cube faces lettered **ח** and **ק**.

Second band:

- **ציור ט (Fig. 9):** A circle with inscribed chords/diameters; interior points **ב**(top), **א**, **ד**, **ח**, **ז**(left), **ג**(right) — a diameter with a perpendicular.
- **ציור י (Fig. 10):** A circle resting on a horizontal baseline (tangent construction); the line carries points **ס · ש · פ · ל** (RTL, a sequence of feet/tangent points), apex **א** at left, **ה** and **י** on the circle.
- **ציור יא (Fig. 11):** A "kite"/curvilinear quadrilateral with internal arcs; points **ח**(top), **פ**, **ה**, **א**, **ק**(center), **ל**(left), **ג**, **צ**, **ב**(bottom) — arcs spanning from ל across to צ.

- **ציור יב (Fig. 12):** A large triangle with an inscribed pentagram/crossing-chords star; apex **א**(top), base corners **ב**(left low), **ג**(right), with interior nodes **פ · ה · ז · צ** marking the star intersections.
- **ציור יג (Fig. 13):** A crescent / lune (two arcs) — a "two-horned" curved figure; labels **ח**(top), **א, ד, ב, צ** [?] along and within the crescent. [Two adjacent crescents head the row; cf. Fig. 25/26 below.]

Third band:

- **ציור יד (Fig. 14):** A curvilinear (spherical) triangle with an internal cevian; apex **א**, base **ב — ד**, interior point **ה** [?].
- **ציור טו (Fig. 15):** A circle with two chords crossing; interior points **פ · ד · ה** and rim points **א, ג, ב** [?].
- **ציור טז (Fig. 16):** A circle inscribed with a triangle and an internal point **ה**; rim **ד · ז · ב** [?].
- **ציור יז (Fig. 17):** A circle with a slanted diameter and chord; interior **ה · ס · נ · מ · ל · ק**, rim **ג · צ** [?].
- **ציור יח (Fig. 18):** A spherical triangle, apex **א**, base **ב — ד**, with an internal arc and point **ה**.

Fourth band:

- **ציור יט (Fig. 19):** A circle with several inscribed chords forming an internal triangle; rim points **מ**(top), **ג, ב, ה, נ, א**, with interior **ה** [?].
- **ציור כ (Fig. 20):** A large circle with a baseline tangent (extended diameter); baseline points **ב · צ · פ · נ · ל** (a row of marked feet), rim **א**(top), **ש**, **ה**(center), **ק**(left) — a horizon/tangent construction.
- **ציור כא (Fig. 21):** A circle with an inscribed pentagram-like star of chords; rim **ק**(left), **א**(top), **ג, ב, מ**, interior nodes **ט · ה · ז** [?].
- **ציור כב (Fig. 22):** A circle with two great-circle arcs crossing (an armillary/sphere projection); central **ה**, rim **ט**(top), **ג, ב, פ**.
- **ציור כג (Fig. 23):** A curvilinear (spherical) triangle bearing inscribed numerals — apex **ד**(top), with **י** (left vertex), the values **23. 51. 20.** (upper-left, along the left limb) and **126** (interior left), **30** (mid-right limb), and small circled points at base corners **פ**(lower-left) and **ג**(lower-right). [A

worked spherical-triangle computation; numerals 23, 51, 20, 126, 30 transcribed as printed.]

Fifth band:

- **ציור כד (Fig. 24):** A circle with crossing chords and the Arabic numeral **69** marked at an interior intersection; apex **ק**(top), rim **ב** ,[?] **ה · ת · א**(left), **ד**, **ל**, **ג**, **ם** [?]. [The "69" recurs in Figs. 24 and 28 — an angular value.]
- **ציור כה (Fig. 25):** A circle with an inscribed quadrilateral/chords; **ל**(top), **א**, **פ**, **ד**, **ה**, **ק**, **ב**, **ג**, **ם** [?].
- **ציור כו (Fig. 26):** A crescent / lune with the numeral **6·49** [?] in the upper horn and **12.5** [?] in the lower portion; tip **צ** at bottom, **ב** at top, **ד** at right. [Numerals 6·49 and 12·5 transcribed as printed; both flagged.]
- **ציור כז (Fig. 27):** A circle with a tangent baseline and an inscribed value **2·53** [?] (boxed) near the top; baseline **ה** ,[?] **ק · ש · ט · פ** center, **ב · ד** rim, **א** apex. [Numeral 2·53 transcribed as printed; flagged.]

Bottom band:

- **ציור כח (Fig. 28):** A circle with inscribed chords and the Arabic numeral **69** at a central intersection, plus the Hebrew letter **ש** (shin, a point-label) below center; apex **ב · ד**(top), **ז**, **י**, rim **א**(right), **ג**(left), **ה · ע**(center), **ח**(bottom). [An ink blot sits over the center.]
- **ציור כט (Fig. 29):** A large circle (sphere projection) with crossing great-circle arcs and several asterisk/star marks ✻ at the arc intersections; rim/interior **מ**(top), **ט**, **ה**, **ס**, **ב**, **א**, **צ**, **ע**, **פ** [?], with dashed construction lines extending to the lower right.
- **ציור ל (Fig. 30):** A **circular calendar volvelle / zodiac wheel** — three concentric rings divided into 12 sectors.
- Outer ring (zodiac sign-glyphs, astronomical sigla — not Hebrew names): reading clockwise from top — ♐ (Sagittarius), ♑/♒, ♓ (Pisces), ♈ (Aries), ♉ (Taurus), ♊ (Gemini), then at the bottom ♋ (Cancer) and ♌ (Leo) (the two "69"-looking marks), continuing ♍ ♎ ♏ around; several abraded [?].
- Middle ring (civil month names transliterated into Hebrew): **ינואר** (January) and **דיצמבר** (December) firmly legible at the top; the others around the ring abraded [?].

- Inner ring (numerals, clockwise from top): **9 · 9 · 8 · 8 · 8 · 10 · 10 · 10 · 15 · 10**.
- The hub is an empty central disc (the pivot for a missing pointer/string).
- **ציור לא (Fig. 31):** A large circle (celestial sphere) with multiple crossing great-circle arcs forming an inscribed network; rim and node points **א**(top), **ט**, **ב**, **ל**, **פ**, **ה**, **ק**, **נ**, **ס**, **מ**, **ז**, **ח**, **ר** [?], **ג**(bottom) — a fully crossed armillary projection.
- **ציור לב (Fig. 32):** A curvilinear (spherical) triangle with an internal arc and inscribed letters running down the right limb (top→down) **ט · ח · ה · ז · ר**; apex **כ** (top-right), **צ** at center, base **ה**.
- **ציור לג (Fig. 33):** A circle with a horizontal diameter and inscribed chords; **ד**(top), **ה**, **א**, **ב**(center), **ל**, **ט**, **ח**, **ז** [?] — the final figure of the plate.

Sefer Elim — End Matter (Figure Plates II–III)

This opening continues the engraved figure plates for the four mathematical treatises that began on the master plate of the prior leaf (which carried Figures 1–33). The leaves here are full-page engraved plates with blank versos, plus a library ownership stamp.

- ** ** — Figure plate (Figures 34–47), captioned as before.
- ** ** — Blank verso (show-through only).
- ** ** — Figure plate (Figures 48–67), same caption.
- ** ** — Blank verso; library ownership ink-stamp at lower right.

Figure plate — Figures 34–47

Diagram. A full-page engraved plate inside a double ruled border. The running caption across the head reads **"ציורים לספרי הקות שמים גבורות יי מעין חתום"** ("Figures for the books *Chukos Shamayim*, *Gevurot Yah*, *Ma'ayan Chatum*"). [The first word of the treatise list is engraved **הקות** for **חקות**; transcribed as printed.] The plate holds numbered geometric, astronomical, and instrument figures, each captioned ציור N) ("Figure N"), continuing the sequence from the previous plate. The figures run top-to-bottom in two banks, read right-to-left.

- **(ציור לד) [Fig. 34]** — A surveyor's/astrolabe quadrant: a graduated quarter-circle limb with a sighting alidade and plumb. Inscribed inner label **צלהפיך** [?] (an instrument name [?]) with a degree scale reading **80, 70, 60, 50, 40,**

30, 20, 10 along the arc; corner points **א, ב, ג, ה, נ**; numeral **9** at the upper sighting vane.

- **(ציור לה) [Fig. 35]** — A volvelle / wind-rose disk with radial hour-lines, numbered **9 10 11 12 1 2 3 4 5 6 7 8** around the rim; center point **ה**; surrounding cardinal labels **קדם** ("East"), **דרום** ("South"); points **ג, ב, ט, ז**.
- **(ציור לו) [Fig. 36]** — A semicircular hour-diagram: apex at **ג**, baseline **ה ... א**, foot point **ז**; the arc numbered **6 7 8 9 10 11 12 1 2 3 4 5 6** with end-markers **ח** and **ט (יט)**; lower point **ב**.
- **(ציור לז) [Fig. 37]** — A small fan / sector with rays numbered **8 7 6 ... 4 5 6**; points **ה, ב, ז**.
- **(ציור לח) [Fig. 38]** — An astronomical/spherical projection circle with an inscribed network of chords and an inner curve; rim labels include **ר, ש, ל, ת, ה, ב, ד** [several rim letters [?]].
- **(ציור לט) [Fig. 39]** — A sphere/astrolabe stereographic projection: a great circle with inscribed oblique circles and zodiac/hour divisions; Roman numerals **XI, XII, XIII, IV** marking divisions; an oblique scale reading **76.13.50**, **75°**; rim points **צ, פ, א, ב**, plus interior Hebrew labels [several [?]].
- **(ציור מ) [Fig. 40]** — A curved gnomon/shadow figure (a lune-shaped sector): inscribed **15** and **23.51.20.** with a small sun-disk at the lower vertex; vertices **ם, ס**.
- **(ציור מא) [Fig. 41]** — A circular computation volvelle (concentric rings of numerals around a central inscription). The center cartouche reads **"כ"א יוכ"א צ[...] שעה אקירדם / צהרים על אופק / לורים שיחבד[?] / ר"ל הגערית[?]"** [center inscription partly [?] — a worked time/longitude example]. Ring numerals legible include **1958.75 [or 1958.25]**, **1051**, **1080**, **6069**, **255**, **165**, **9817**, **8398** [several ring values [?] at the engraving floor].
- **(ציור מב) [Fig. 42]** — A vertical pendulum/escapement instrument: a large graduated wheel with a swinging beam and suspended bob over a hatched base; points **ד, ז, ח, צ, ה**.
- *(ציור מב / מג) [Fig. 42* and 43]** — A graduated sun-altitude quadrant with ray-lines to a sun emblem; scale **1, 2, 3 ...**; pointer letters **א, ב, ג, ד, ה**; plus a quarter-circle (Fig. 43) with crossed chords, points **ה, פ, ט, ב, ר** and a small base-circle labeled **ארץ** ("Earth").
- **(ציור מד) [Fig. 44]** — A semicircle on a baseline with apex **ה**, dashed construction lines, base points **א, ב, ש, ד**.

- **(ציור מה) [Fig. 45]** — A solid black annulus (ring/eclipse figure) with an inner concentric circle; two small sun-disks at top and bottom of the inner ring; interior points **ה, ש, ח, צ, י**; outer points **א, ב, ק**.
- *(ציור מה) [Fig. 45]*** — A long thin shadow-cone (umbra) from a small circle (luminary) tapering to a point; sun-disk at the wide end, point **ה** at the apex.
- **(ציור מו) [Fig. 46]** — A right triangle with an inscribed arc: apex **ט**, hypotenuse to **א**, foot points **ב, ד, ה**.
- **(ציור מז) [Fig. 47]** — A small circle (sun/earth) with a long shadow-cone tapering to a point at **ז (ו)**; circle points **א, ב, ג, ה, י**.
- **(ציור מג) [Fig. 43, second sun-quadrant]** — A graduated quarter-circle of rays from a sun, with a small base-circle labeled **ארץ** ("Earth"); scale points and letters **א, ב, ה, ט, פ**.

[More than two labels on this plate are flagged `[?]` at the engraving resolution]

Figure plate — Figures 48–67

Diagram. A second full-page engraved plate inside a double ruled border, with the same head caption **"ציורים לספרי חקות שמים גבורות יי מעין התום"** ("Figures for the books *Chukos Shamayim*, *Gevurot Yah*, *Ma'ayan Chatum*"; the last word is engraved **התום** for **חתום**, transcribed as printed). The numbering continues: figures captioned `(N ציור)` from Figure 48 (מח) through Figure 67 (סז), including geometric constructions, optics/eye figures, surveying instruments, mechanical gear-and-weight machines, thermometers, and numeral computation tables. Read right-to-left, top to bottom.

- **(ציור מח) [Fig. 48]** — A circle quartered by two perpendicular diameters through center **ה**, with an inscribed right triangle and chords; rim and interior points **א, ב, ג, ה, ו, ז, ל, מ, פ, צ, ק, ש** (ת not present).
- **(ציור מט) [Fig. 49]** — An optics/vision figure: a stylized **שמש** ("sun") emanating rays to a small eye-circle; pinhole/aperture construction with points **פ, א, ה**.
- **(ציור נ) [Fig. 50]** — A circle with an inscribed triangle apex **א**, interior point **ב**, base **פ, ד**; dotted construction lines.
- *(ציור נא / נא) [Figs. 51 and 51]*** — A circle quartered through center **ב**, with inscribed chords; points **א, ה, ז, ח, ג, ד, י**.
- **(ציור נב) [Fig. 52]** — A large sphere/astrolabe projection with inscribed oblique circles and a central tapering cone (gnomon shadow); bottom apex

ש, foot region **ק**; interior labels **ג, ה, צ, ז, מ, כ, ל, ד, ם**; rim **ה, ט** (ר and נ not confirmed).

- **(ציור נג) [Fig. 53]** — A graduated semicircular protractor/arc with radial divisions; baseline **א ו ה ד ג ... ב**.
- **(ציור כב) [Fig. 22, two glass vessels]** — Two bulbed thermometer/barometer glass instruments (graduated tubes rising from round bulbs); points **ב, ג, ד, ה**. [Caption reads ציור כב — a back-reference number on this plate [?]; transcribed as printed.]
- **(ציור כג) [Fig. 23]** — A hatched semicircular arc (a bridge/arch or shadow) on a baseline **א ו ה ד ג ... ב**.
- **(ציור נד) [Fig. 54]** — A balance/steelyard: a graduated horizontal beam held by a hand (issuing from a cloud), with a suspended weight and an arc scale below; points **א, ב, ג, ד, ה**.
- **(ציור נה) [Fig. 55]** — A geared windlass/crane worked by a man in a period hat treading a sloped frame; a toothed gear-wheel drives a drum lifting a weight marked **8**; the figure is captioned **(ציור נה)**; a small lifted bucket/weight **1**.
- **(ציור נו) [Fig. 56]** — A tall geared hoist frame (a tower of stacked gear-wheels) lifting a load over a base labeled **ארץ** ("ground/earth"); a small operator figure at the top right; operator-platform letter **ח**; gear-train labels **ה, צ, פ, ל, ג, ב** (ז and ם not confirmed); base/ground label **ארץ** ("ground/earth").
- **(ציור נז) [Fig. 57]** — A cone/funnel construction inside a circle: top **א**; axis points **ג, ד, ה**; apex **ו**; left **ט** (ב not confirmed); a teardrop solid.
- **(ציור נח) [Fig. 58]** — A telescope/sighting scene: a standing observer aims a sighting tube at a star ✶; a second observer at a graduated semicircular quadrant with shadow rays numbered **10, 20, 30, 40 …**; pointer letters **ה, ט, ק, פ, ר**; scale numerals **3, 4** at the staff.
- **(ציור נט) [Fig. 59]** — A computation table (right-angle/Pythagorean numeral block). Transcribed values, as engraved:

position	value
diagonal	**40000**
upper inner column (top→down)	**100 · 2000 · 360 · 120**
left of diagonal, mid row	**100 · 20**
below that	**2000 · (1)00**
bottom rows (left block)	**360 / 120**
small foot pair (right)	**3 / 9**

position	value

[Table reads right-to-left in source — a worked areal/multiplication example. Confident value-set **40000, 2000, 360, 120, 100, 20, 9, 3** (40000 on the diagonal = 200^2, with 2000/360/120/100 as partial products). The diagonal is unambiguously **40000**; 400 and 80 are not in this block.] Frame letters: top-left **א**; inner verticals **נ, ל**; across **ב? · ג** then **נ**; left side **פ, ה**; cells include **ט**; bottom-right **נ**, lower **ת**; foot **ד**.

- **(ציור ס) [Fig. 60]** — The header strip of the Fig-59 areal block (partial bordered cells above the square); cell entries **20** / **100** / **360** / **120** (shared inner verticals with Fig 59), corner letters **צ, ש, נ**. (Not a separate "1, 2" grid.)
- **(ציור סא) [Fig. 61]** — A computation table:

values
7 / **49**
20 · 54 · 29
2·20 · 640
6·36

[Reads right-to-left; secondary small fractions in the margins **48/36** (the pair repeated; 98/16 not confirmed). Main cells confirmed **7, 49, 20·54·29, 2·20, 640, 6·36**. Frame letters **ז, ה, ט** (left margin), **ש** (mid-low), **ד** (foot); head letters **א, ב? · ג**.]

- **(ציור סב) [Fig. 62]** — A small numeral box keyed **א ב ג ד** with entries **7**, then a **20 · 54 · 29** strip (sexagesimal), lower **2·20** / **640**, foot **6·36**; frame letters **ה, ז.**
- **(ציור סג) [Fig. 63]** — An hour/volvelle disk: concentric rings numbered **11 10 9 8 7 6 5 4 3 2 1 12** and **10 9 8 7 6 5 4 3 2**; the center is the ruled cross only (no **ה** label); surrounding points **א, ב, ג.**
- **(ציור סד) [Fig. 64]** — A small framed grid keyed **א, ב, ג, ד, ה, ט.**
- **(ציור סה) [Fig. 65]** — A geometric construction on a baseline: a triangle with crossed cevians; points **א, ב, ג, ד, ה, ט, ק, ר, ש, ל, כ.**
- **(ציור סו) [Fig. 66]** — Two intersecting circles (a vesica): centers/points **א, ב,** **ג, ד, ה, ר, ס** with a small inner triangle.
- **(ציור סז) [Fig. 67]** — A small solid-black circle quartered by diameters; points **א, ב, ג, ד, ה.**

[More than two labels on this plate are flagged [?] at the engraving resolution]

End matter — binding leaves (final leaves of the volume)

These four leaves are the back end matter of the volume. The textual body, the publisher's colophon, and the master engraved figure plates (Figs. 1–67) all concluded earlier. No further text, errata, advertisement, or figure plate appears here; the pages are blank binding/flyleaves and the inside back cover.

Source PDF p. 477 — blank flyleaf

Blank rear flyleaf. The leaf carries only paper texture and a few small foxing/age spots; no printed or handwritten text, no figure, no stamp. [Blank flyleaf — no content.]

Source PDF p. 478 — blank flyleaf

Blank flyleaf (verso of the preceding leaf). Uniform paper with faint binding-edge shadow at the inner margin; no printed or handwritten text, no figure, no stamp. [Blank flyleaf — no content.]

Source PDF p. 479 — blank flyleaf / free endpaper

Blank rear free endpaper. Paper texture only, with a vertical binding-fold shadow down the center and gutter/board shadows along the margins; no text, figure, or stamp. [Blank flyleaf — no content.]

Source PDF p. 480 — inside back cover (pastedown / binding board)

The inside of the back cover: a dark, embossed cloth-grained binding board, with a navy spine strip along the binding edge. No Hebrew text, figure, or colophon is present — this is the binding, not a printed page.

Affixed at the upper portion is the institutional ownership label of the digitizing library:

- Barcode label, printed vertically: **"UNIVERSITY OF TORONTO"**, with a scannable barcode and the number **3 1761 01077409 9** [Library barcode label, English].

[Inside back cover / pastedown — binding board, no printed book content. End of volume.]